Working Paper

Generalized Urn Schemes and Technological Dynamics

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Foreword

Adaptive (path dependent) processes of growth modeled by urn schemes are important for several fields of applications: biology, physics, chemistry, economics.

In this paper several macroeconomic models of technological dynamics are studied by the means of adaptive processes of growth. One of the models tackles the case when there is a separation within the pool of adopters which can be interpreted as the outcome of adaptive learning on the features of the new technologies by imperfectly informed agents. Others deal with dependence of final market shares of two technologies on the pricing policies of the firms which produce them. The stochasticity of the processes is caused by some mixed strategies used by the adopters or/and imperfectness of the information which they posses.

To study these conceptual problems some modifications of the basic results concerning the generalized urn scheme are given.

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1 Introduction

The competition among new technologies is clearly a fundamental aspect of the process of growth and transformation of contemporary economies. So it is the competition among different firms which embody different technologies, different expectations and, possibly, show different market behavior. In turn, it is increasingly acknowledged that technological innovations are likely to involve some forms of dynamic increasing returns, i.e., some positive feedbacks along their diffusion trajectories (cf. Dosi *et al.* (1988) and Anderson *et al.* (1988); for an interpretation of the empirical evidence, Dosi(1988)). To study the underlying competitive process a wide variety of mathematical approaches has been suggested within and outside economic analysis, e.g., ordinary differential equations (as in Polterovich and Henkin (1988)), in particular with trajectories on the unit simplex, i.e., of the population type (as in Silverberg *et al.* (1988)), and generalized urn schemes which generate stochastic dynamic systems with discrete time and trajectories from the unit simplex (cf. Arthur (1988) and Arthur *et al.* (1983) and (1987c)). In the following we shall present some extensions of the later approach able to handle positive feedbacks that are only "local" – in the sense that they occur only under particular states on the diffusion trajectory –, or the co-existence of both positive and negative feedback mechanisms in the competitive process (the simplest variants of the models presented here can be formally reduced to the ones considered in earlier works (Arthur (1988), Arthur *et al.* (1983) and (1987c)). We shall apply these generalized urn schemes to two topics concerning competing technologies.

In the first topic, we study the dependence of limit market shares from some mixed strategies used by risk-averse adopters of the technologies. Conceptually, these mixed strategies, which generate some separation within the pool of adopters, can be interpreted as the outcome of adaptive learning on the features of the new technologies by imperfectly informed agents. For example, one may assume that the later decide by observing the choice of previous adopters and "trusting" them, to different degrees. Such a behavior is also easily interpretable in terms of "bounded rationality", and/or some interdependence in the returns to individual adopters, depending on the relative frequences of the chosen technologies. Economic examples of such interdependences are particular clear with respect to innovation and innovation diffusion (cf. Arthur (1988), David (1985) and (1991), Dosi (1988), Hanson (1985)) whereby dynamic increasing returns and various sorts of externalities are generally observed. However, the modeling techniques suggested here could be in principle applied, with the proper modifications, to other economic domains involving interdependence of expectations (such as those discussed in e.g., (Frydman (1982) and Frydman and Phelps (1983))) and speculation.

The second topics we consider concerns the dependence of final market shares of two technologies on the pricing policies of the firms which produce them. In the following, we suppose that each of the firms until they reach a certain market share (measured by the proportion of units of the technology they produce among all units of all technologies which have been sold up to that time). Above that share, prices are increased. An economic interpretation of such a behavioral hypothesis is that firms - as often found in the business literature - follow strategies aimed at market pre-emption and at learning economies until they reach a dominant market position which they can exploit thereafter. Whether such pricing policies can be derived from strategies of intertemporal profit maximization under imperfect information or not is by no means essential to the model. In principle, it is meant to analyze the share dynamics of different technologies with endogenous prices (no mater whether the later microfounded on intertemporally optimizing agents or not). At each time, prices of each technology can be different but adopters may not instantly switch from one to another due to e.g., imperfect information, network externalities, etc. Indeed, the stochasticity of the process is caused by some mixed strategies used by the adopters in the case of approximately equal prices on competing technologies.

Well beyond the two specifications of the model which we are going to present here, one of our aims is to illustrate the general applicability to economic and technological dynamics of generalized urn schemes which generate discrete time stochastic dynamic systems with multiple equilibria. Stable of them turn out to be attainable (i.e., they realize with positive probabilities). They represent those limit proportions of competing technologies which are feasible. In the following we shall use known results (Arthur *et al.* (1987a), (1987b), (1988), Hill *et al.* (1980))

concerning generalized urn schemes also some further developments.

In section 2 we shall present the basic theorems on generalized urn schemes. Section 3 studies technological competition with imperfect information and endogenous preferences for the two technologies. In section 4 we analyze the case with endogenous prices.

2 The Theory of the Generalized Urn Scheme

Think of an urn of infinite capacity with black and white balls. Starting with $n_w \geq 1$ white balls and $n_b \geq 1$ black balls into the urn a ball is added into the urn at time instants $t = 1, 2 \ldots$. It will be white with probability $f(X_t)$ and black with probability $1 - f(X_t)$. Here $f(\cdot)$ is a function (it is called sometimes (Hill *et al.* (1980) ***urn function***), which maps $R(0,1)$ ($R(0,1)$ stands for the set of rational numbers from $(0,1)$) in $[0, 1]$. By X_t we designate the proportion of white balls into the urn at time t. Then the dynamics of X_t is given by the relation

$$X_{t+1} = X_t + (t + n_w + n_b)^{-1}[\xi_t(X_t) - X_t], t \geq 1, X_1 = n_w(n_w + n_b)^{-1} .$$

Here $\xi_t(x)$, $t \geq 1$, are independent on t random variables such, that

$$\xi_t(x) = \begin{cases} 1 & \text{with probability} \quad f(x) , \\ 0 & \text{with probability} \quad 1 - f(x) . \end{cases}$$

Designate $\xi_t(x) - \mathbf{E}\xi_t(x) = \xi_t(x) - f(x)$ by $\zeta_t(x)$, then we have

$$X_{t+1} = X_t + (t + n_w + n_b)^{-1}\{[f(X_t) - X_t] + \zeta_t(X_t)\}, t \geq 1, X_1 = n_w(n_w + n_b)^{-1} . \tag{1}$$

Due to $\mathbf{E}\zeta(x) = 0$ the system (1) shifts on average at time $t \geq 1$ and fixed $X_t = x$ on the value $(t + n_w + n_b)^{-1}[f(x) - x]$. Consequently limit points of the sequence $\{X_t\}$ have to belong to the set B of zeros of the function $f(x) - x$ (for $x \in [0,1]$). The following statement confirms this hypotheses.

Theorem 1 *(Arthur et al. (1987b)) The sequence $\{X_t\}$ converges a.s. to the set B.*

Because we do not require here continuity of the function $f(\cdot)$, then the set B has to be defined properly. Put

$$B = \{x \in [0,1] : \quad [\underline{a}(x), \bar{a}(x)] \ni 0\} ,$$

where $\underline{a}(x) = \inf \underline{\lim}_{k\to\infty}[f(x_k) - x_k]$ and $\bar{a}(x) = \sup \overline{\lim}_{k\to\infty}[f(x_k) - x_k]$. Here "inf" and "sup" are taken over all sequences of rational numbers $\{x_k\}$ converging to x.

It is easy to see that if all of the connected component of B are singleton, then the convergence to B implies convergence of the sequence $\{X_t\}$. As it was shown in (Arthur *et al.* (1987b) and Hill *et al.* (1980)), if the set

$G = \{x \in B : \forall \epsilon > 0 \quad \exists y_\epsilon^-, y_\epsilon^+ \in R(0,1)$, which belong to $(x-\epsilon, x)$ or $(x, x+\epsilon)$, such, that $f(y_\epsilon^-) < y_\epsilon^-$ and $f(y_\epsilon^+) > y_\epsilon^+\}$

is nowhere dense, then the sequence $\{X_t\}$ turns out to be a.s. convergent even in the case, when B can contain intervals.

An isolated point $\theta \in B$ we call *stable* if there exist $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such, that for $\epsilon_1 \leq |x-\theta| \leq \epsilon_2 (x \in R(0,1))$ the following inequality holds

$$[f(x) - x](x-\theta) < \delta(\epsilon_1, \epsilon_2) < 0\ .$$

We shall say that an isolated point $\theta \in B$ is ***unstable*** if there exists $\epsilon > 0$, such that

$$[f(x) - x](x-\theta) > 0\ ,$$

for $x \in R(0,1) \cap [(\theta-\epsilon, \theta) \cup (\theta, \theta+\epsilon)]$.

Theorem 2 *Let $\theta \in B$ be a stable point, $\theta \in (0,1)$. Also there exist $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such, that the following inequalities hold*

$$f(x) > 0 \quad \text{for} \quad x \in R(0,1) \cap (\theta - \epsilon_1, \theta)\ ,$$

$$f(x) < 1 \quad \text{for} \quad x \in R(0,1) \cap (\theta, \theta + \epsilon_2)\ .$$

Then $P\{\lim_{t\to\infty} X_t = \theta\} > 0$ for every $X_1 \in (\theta - \epsilon_1, \theta + \epsilon_2)$.

Proof of the theorem is essentially similar to ones given in the papers (Arthur *et al.* (1988) and Hill *et al.* (1980)). We shall note only, that the requirements on $f(\cdot)$ allow to shift to the left for $X_1 > \theta$ and to the right for $X_1 < \theta$ inside $(\theta - \epsilon_1, \theta + \epsilon_2)$ with positive probability through a corresponding finite number of steps.

Theorem 3 *Suppose that $\theta \in B$ is an unstable point, $\theta \in (0,1)$ and one of the following conditions holds true:*

1. *into a neighborhood of θ the function $f(\cdot)$ is continuous;*
2. *there exists $\epsilon > 0$ such, that for $x \in (\theta - \epsilon, \theta + \epsilon) \cap R(0,1)$ it will be*

$$[f(x) - x](x-\theta) \geq \lambda |x-\theta|^{1+\mu}, f(x)[1-f(x)] \geq \underline{c} > 0, \underline{c} > \bar{c}(2\lambda+1)^{-1}\ ,$$

   where $\mu \in [0,1]$, $\lambda > 0$ and $\bar{c} = \sup_{x \in (\theta-\epsilon, \theta+\epsilon) \cap R(0,1)} f(x)[1-f(x)]$;

3. *into a neighborhood of θ for $x \leq \theta (x \geq \theta)$ one of the conditions 1) or 2) holds true and for $x > \theta$ $(x < \theta)$ it will be $f(x) = 1 (f(x) = 0)$.*

Then $P\{\lim_{t\to\infty} X_t = \theta\} = 0$ *for every* X_1.

Remark 1 *The condition 3) differs from conditions 1) and 2) because it permits that* $f(x)[1-f(x)]$ *equals zero in a neighborhood of* θ. *(Indeed, condition 2) postulates positiveness of the value. In the case of condition 1) from continuity of* $f(\cdot)$ *at* θ *we have* $f(\theta)[1-f(\theta)] = \theta(1-\theta) > 0$ *and, consequently, the positiveness of* $f(x)[1-f(x)]$ *take place in a neighborhood of* θ.*)*

Proof The case with continuous $f(\cdot)$ was studied in (Hill *et al.* (1980)). Under condition 2) we can apply results from (Arthur *et al.* (1988)). Let condition 3) hold and $f(x) = 1$ for $x > \theta$. We argue in the following way.

Note, that

$$P\{\lim_{t\to\infty} X_t = \theta\} = P\{\lim_{t\to\infty} X_t = \theta, X_s \leq \theta, s \geq 1\} + \tag{2}$$

$$P\{\lim_{t\to\infty} X_t = \theta, X_s > \theta \quad \text{for some} \quad s \geq 1\} .$$

It is clear, that the second term here equals zero. (Because the process X_t, $t \geq 1$, cannot move to the left from a point lying to the right of θ.) If $f(\cdot)$ is continuous to the left of θ we put

$$\tilde{f}(x) = \begin{cases} f(x) & for \quad x \leq \theta , \\ \min[1, x + k(x-\theta)] & for \quad x > \theta , \end{cases}$$

where $k > 1$. Otherwise

$$\tilde{f}(x) = \begin{cases} f(x) & for \quad x \leq \theta , \\ (1+\theta)/2 & for \quad x > \theta . \end{cases}$$

Using results of the papers (Arthur *et al.* (1988) and Hill *et al.* (1980)) we have for the process $Y_t, t \geq 1$, corresponding to $\tilde{f}(\cdot)$,

$$P\{\lim_{t\to\infty} Y_t = \theta\} = 0$$

for every Y_1. Hence for $Y_1 = X_1$ we obtain

$$P\{\lim_{t\to\infty} X_t = \theta, X_s \leq \theta, s \geq 1\} = P\{\lim_{t\to\infty} Y_t = \theta, Y_s \leq \theta, s \geq 1\} \leq P\{\lim_{t\to\infty} Y_t = \theta\} = 0 .$$

Consequently the first term in (2) equals zero too. The case, when $f(x) = 0$ for $x < \theta$ can be studied similarly.

The theorem is proved.

Conditions of convergence with positive probability to points 0 and 1 are given by the next theorem.

Theorem 4 *(Arthur et al. (1983)).If* $f(n_w(n_w + n_b + t)^{-1}) < 1$ *for* $t \geq 0$ *and* $\sum_{t\geq 0} f(n_w(n_w + n_b + t)^{-1}) < \infty$, *then* $P\{\lim_{t\to\infty} X_t = 0\} > 0$. *Also if* $f((n_w + t)(n_w + n_b + t)^{-1}) > 0$ *for* $t \geq 0$ *and* $\sum_{t\geq 0}[1 - f((n_w + t)(n_w + n_b + t)^{-1})] < \infty$, *then* $P\{\lim_{t\to\infty} X_t = 1\} > 0$.

The following statement gives conditions, which ensure convergence of $\{X_t\}$ with positive probability to nondegenerate intervals in every point of which $f(x) = x$. It can be proved by arguments, which are similar to ones given in (Arthur *et al.* (1988)).

Theorem 5 *Let* $(a, b) \subseteq B$, $a < b$, *and* $f(x) = x \;\; \forall\, x \in (a, b) \cap R(0, 1)$. *Also suppose, that there exist* $\epsilon_1 > 0$ *and* $\epsilon_2 > 0$ *such, that* $f(x) > 0$ *for* $x \in (a - \epsilon_1, a) \cap R(0, 1)$ *and* $f(x) < 1$ *for* $x \in (b, b + \epsilon) \cap R(0, 1)$. *Then* $P\{\lim_{t\to\infty} \rho(X_t, (a, b)) = 0\} > 0$ *for every* $X_1 \in (a - \epsilon_1, b + \epsilon_2)$. *(Here* $\rho(y, Y)$ *is the Euclidean distance in* R^1 *from the point* y *to the set* Y*.) Moreover, if* X_t *a.s. converges to* X_0*, then* $P\{X_0 \in (a, b)\} > 0$ *for every* $X_1 \in (a - \epsilon_1, b + \epsilon_2)$.

In conclusion we shall note, that for $f(\cdot)$, which depends on time, i.e., at time instant t balls are added with probabilities $f_t(X_t)$ and $1 - f_t(X_t)$, in such a way that $\sum_{t\geq 1} t^{-1}\alpha_t < \infty$, Theorems 1–5 are valid too. Here $\alpha_t = \sup_{x\in[0,1]\cap R(0,1)} |f_t(x) - f(x)|$.

Now we are ready to formulate the main conceptual results of the paper.

3 Sharing a Market of Risk Averse Adopters with Two New Competing Technologies

On the grounds of the foregoing apparatus let us examine the dynamics of two competing technologies. Consider an adoption of a unit of the **A** technology as an addition of a white ball into an urn and an adoption of a unit of the **B** technology as an adoption of a black ball. The problem can be easily put into the framework of the generalized urn scheme. Let us generalize the model introduced by W.B. Arthur in (Arthur *et al.* (1983)).

Suppose that the two technologies, **A** and **B**, are identical in terms of some utility measure for the adopters. However, the latter are only imperfectly informed about them so that they make their choices by asking an odd number $m \geq 1$ of adopters who are already using the technologies. An alternative hypothesis to the same effect is that there are positive (or negative) externalities in adoption which change the returns to the user along the diffusion process, but adopters in order to estimate them can only sample a fixed number of users. In both cases, we assume that any new adopter will choose with probability α the technology used by the majority of the sample m and with probability $1 - \alpha$ the technology of the minority of them.

For $\alpha = 0$ or $\alpha = 1$ the model coincides with ones considered in (Arthur *et al.* (1983)). The probability to choose **A** as a function of $x_{\mathbf{A}}$, has the following form:

$$p_{\mathbf{A}}(x_{\mathbf{A}}) = \alpha x_{\mathbf{A}} + (1 - \alpha)(1 - x_{\mathbf{A}}) = (1 - \alpha) + (2\alpha - 1)x_{\mathbf{A}} \quad \text{for} \quad m = 1$$

and

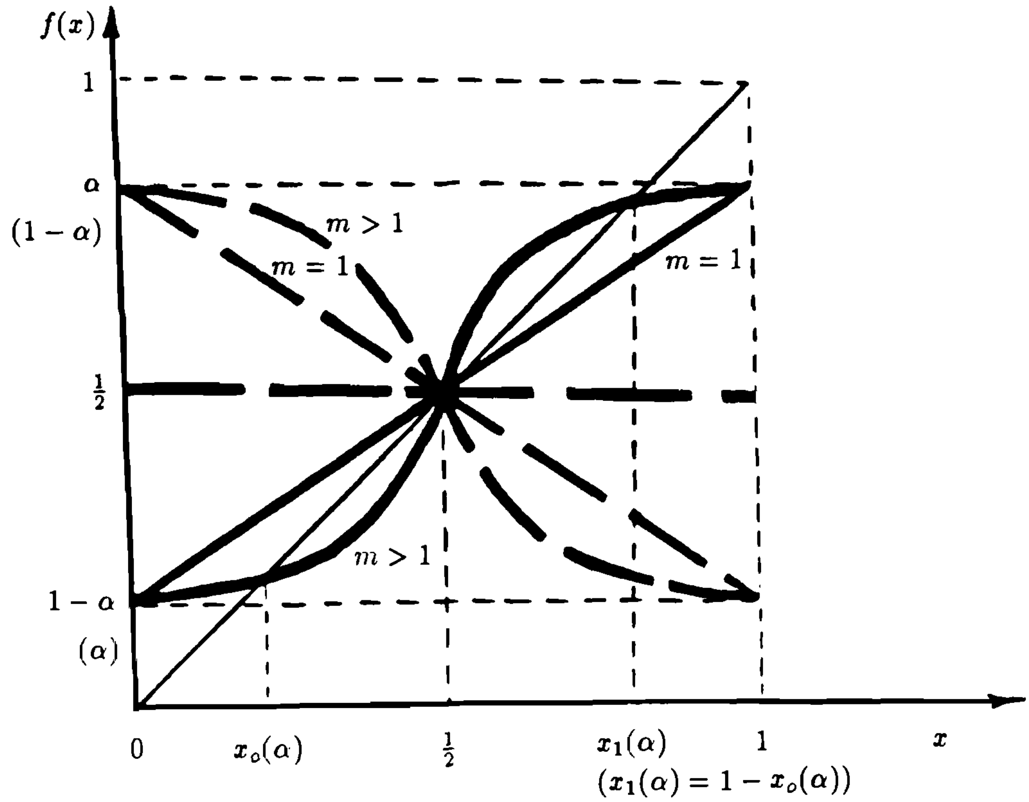

Figure 1

$$
\begin{aligned}
p_{\mathbf{A}}(x_{\mathbf{A}}) &= \alpha p(m, x_{\mathbf{A}}) + (1-\alpha)[1 - p(m, x_{\mathbf{A}})] + o(1) = 1 - \alpha + \\
&\quad (2\alpha - 1)p(m, x_{\mathbf{A}}) + o(1) \quad \text{for} \quad m > 1 ,
\end{aligned}
$$

where $p(m,x) = \sum_{i=(m+1)/2}^{m} C_m^i x^i (1-x)^{m-i}$($C_m^i$ is the number of combinations form m to i), also $o(1)$ goes zero as $O(n^{-1})$ uniformly on $x \in [0,1]$ for $n \to \infty$ (here n designates the number of consumers who have adopted one of the technologies). The function $f(x) = 1-\alpha+(2\alpha-1)p(m,x)$ for $m \geq 1$ is given graphically in Figure 1. Continuous lines correspond here to the case, where $\alpha > 1/2$, broken lines – to the case, where $\alpha < 1/2$ and the horizontal line – to the case, where $\alpha = 1/2$. Using results from section 2 we come to the following conclusions.

For $m = 1$ the function $f(x) - x$ has for every $\alpha < 1$ the only root $x = 1/2$. Consequently Theorem 1 shows that X_t (the proportion of the **A** technology on the market at time t) converges with probability 1 as $t \to \infty$ to 1/2. Consequently in this case the market is shared in the limit by **A** and **B** in the proportion 1:1. For $m > 1$ and $\alpha \leq 1/2$ completely the same arguments show that in the limit market is shared by **A** and **B** in the proportion 1:1. For $m > 1$ and $\alpha > 1/2$ the function $f(x) - x$ has three zeros: $x_o(\alpha)$, 1/2 and $x_1(\alpha) = 1 - x_o(\alpha)$. Consequently Theorem 1 gives convergence X_t with probability 1 as $t \to \infty$ to the set $B = \{x_o(\alpha), 1/2, x_1(\alpha)\}$. As far as both $x_o(\alpha)$ and $x_1(\alpha)$ turn out to be stable, Theorem 2 assures that X_t converges with positive probability to either $x_o(\alpha)$ or $x_1(\alpha)$ (from every initial approximation). Finally the observation that 1/2 is an unstable point and Theorem 3 (the case when condition 1) takes place) show that X_t converges to 1/2 with zero probability (from every initial approximation). Consequently for $m > 1$ and $\alpha > 1/2$ in the limit the market can be shared by **A** and **B** in proportions (each with positive probability) $x_o(\alpha) : [1 - x_o(\alpha)]$ and $[1 - x_o(\alpha)] : x_o(\alpha)$.

The arguments presented here demonstrate how results of section 2 can be used to describe limit states for a given win function. Further we just mention that corresponding results follow from theorems of section 2.

Consequently the main new elements of this model comparatively with one, considered in (Arthur *et al.* (1983)), are the following:

a) if $m = 1$, then for every $\alpha < 1$ there is only one feasible limit market sharing in contrast with the case $\alpha = 1$, where firstly, feasible limit market shares coincide with the whole closed interval [0, 1] and secondly, these limit market shares belong to each of subintervals $(a, b) \in [0, 1]$ with positive probability;

b) if $m > 1$, then for every $\alpha < 1$, unlike the case $\alpha = 1$, there is no monopoly market shares (because $x_0(\alpha) > 0$ and consequently $x_1(\alpha) < 1$) in spite of the fact, that $x_0(\alpha) \to 0$ and, consequently, $x_1(\alpha) \to 1$ as $\alpha \to 1$.

From an economic point of view, the result shows, it is an "imperfect" process of information-acquisition (or endogenous preference formation) which curbs the tendency toward technological monopoly and allow an equilibrium co-existence of variety. (Note also that this variety may simply be based on equilibrium distributions of diverse expectations on otherwise identical technologies, in terms of utility derived from them.)

4 A Model of Competition under Implicit Preferences of Consumers in the Case of Approximately Equal Prices

Let us now introduce a price dynamics for the two technologies. Suppose, that two firms compete for a market of infinite capacity. Designate the firms and their products (technologies) by letters **A** and **B**. Also suppose that they use the following strategy: until a certain level of market share, defined by the proportion of the product of this firm among all products which have been sold until the current time (usually greater than 1/2) they reduce the price. Above that level they increase it. Let us consider the simplest (linear) case of this policy. It is graphically represented in Figure 2. Here $Pr_{\mathbf{A}}(x_{\mathbf{A}})$ designates the dependence of the price of the technology **A** as

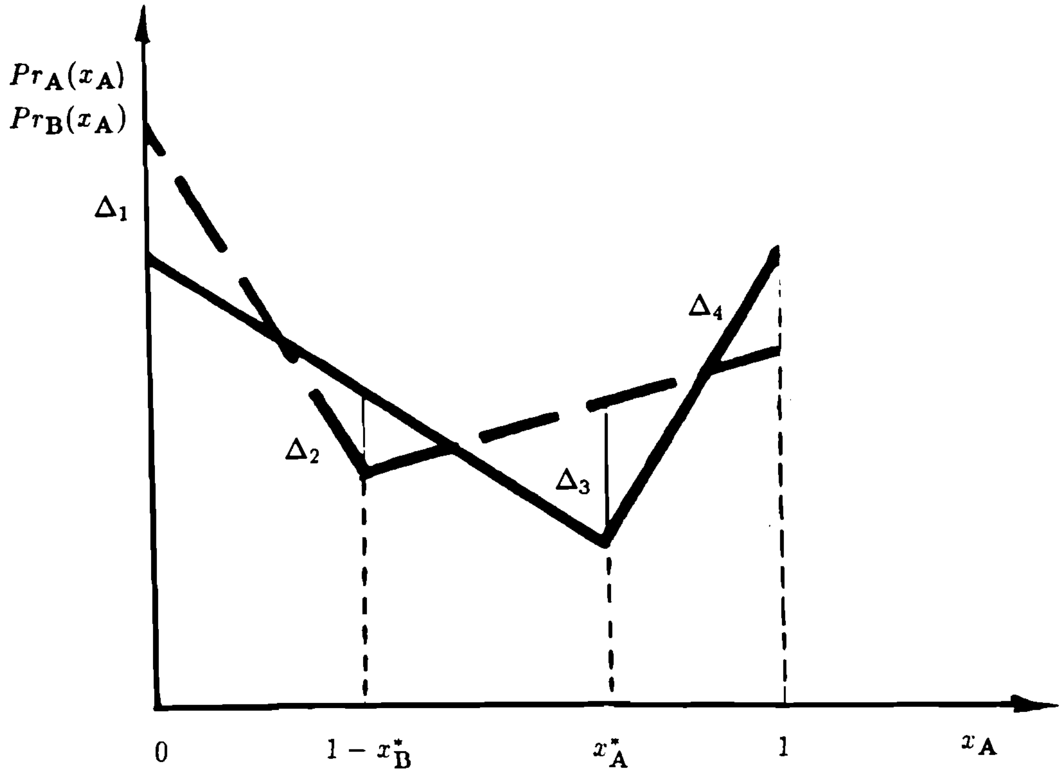

Figure 2

a function of its proportion $x_{\mathbf{A}}$ among adopters, who are using one of the technologies. Also $Pr_{\mathbf{B}}(x_{\mathbf{A}})$ designates the dependence of the price of the technology **B** as a function of $x_{\mathbf{A}}$. (Note, that the proportions of the technologies **A** and **B** are related by: $x_{\mathbf{A}} + x_{\mathbf{B}} = 1$.) By $x^*_{\mathbf{A}}$ and $x^*_{\mathbf{B}}$ we designate the levels of market shares which switch from falling- to rising-price rules. Hence the dependence of the price of the **A** (**B**) technology on its proportion on the market $x_{\mathbf{A}}(x_{\mathbf{B}})$ is given by four parameters: $Pr_{\mathbf{A}}(0)$, $x^*_{\mathbf{A}}$, $Pr_{\mathbf{A}}(x^*_{\mathbf{A}})$, $Pr_{\mathbf{A}}(1)(Pr_{\mathbf{B}}(1)$, $x^*_{\mathbf{B}}$, $Pr_{\mathbf{B}}(1-x^*_{\mathbf{B}})$, $Pr_{\mathbf{B}}(0))$. Note, that we account also for the circumstances, when $Pr_{\mathbf{A}}(1) \leq Pr_{\mathbf{A}}(x^*_{\mathbf{A}})$ $(Pr_{\mathbf{B}}(0) \leq Pr_{\mathbf{B}}(1-x^*_{\mathbf{B}}))$ such as, when $x^*_{\mathbf{A}} = 1(x^*_{\mathbf{B}} = 1)$: in this case, firm **A**(**B**) still reduces the price on its product as its proportion on the market increases.

It is natural to suppose, that in the case when quality of the technologies is approximately the same and potential consumers know about it, the technology which is cheaper, has more chances to be sold, i.e., the **A** technology is bought if $Pr_{\mathbf{A}}(x_{\mathbf{A}}) - Pr_{\mathbf{B}}(x_{\mathbf{B}}) < 0$. But if the prices differ slightly or consumers have some specific preferences (which can be characterized only statistically or on average), that lead sometimes to adoption of the more expensive technology, then the situation we can mathematically formalize in the following way (see also Hanson (1985)). The **A** technology is bought if $Pr_{\mathbf{A}}(x_{\mathbf{A}}) - Pr_{\mathbf{B}}(x_{\mathbf{A}}) + \xi < 0$, where ξ is a random variable. Then the probability $f(x_{\mathbf{A}})$ to choose the **A** technology, as a function of $x_{\mathbf{A}}$, equals to $P\{\xi <$

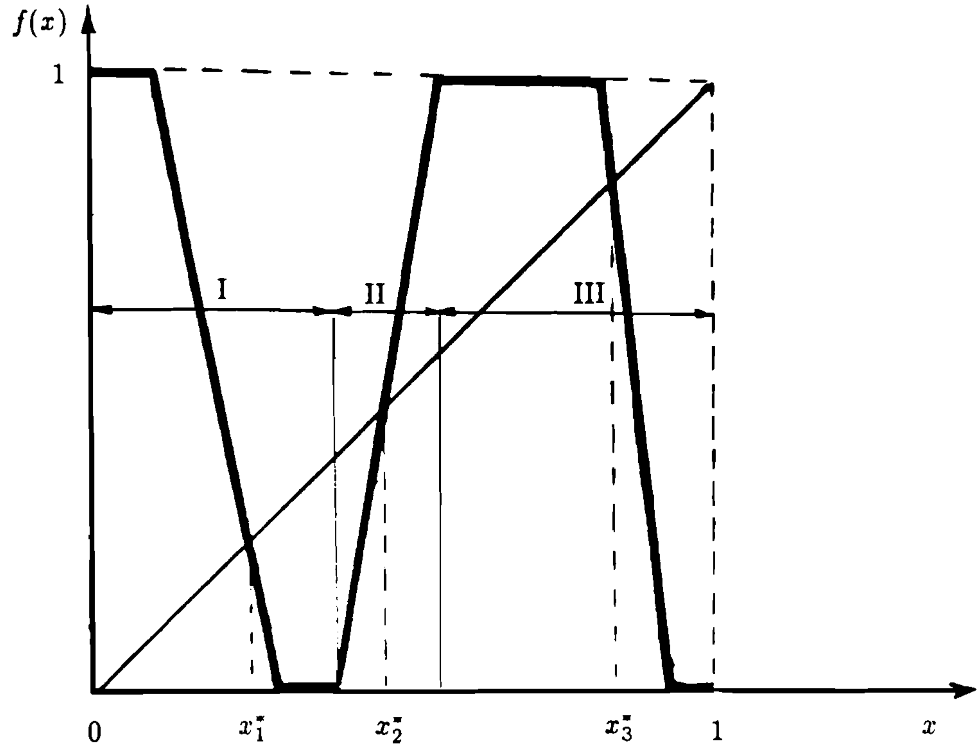

Figure 3

$Pr_{\mathbf{B}}(x_{\mathbf{A}}) - Pr_{\mathbf{A}}(x_{\mathbf{A}})\}$. To avoid unnecessary sophistications of the model, we shall suppose, the ξ possesses density with respect ot the Lebesgue measure in R^1 (otherwise the event "$Pr_{\mathbf{B}}(x_{\mathbf{A}}) - Pr_{\mathbf{A}}(x_{\mathbf{A}}) + \xi = 0$" can have positive probability). Also, it is natural to suppose, that ξ has a bounded support. It means, that $P\{\xi \in [-\alpha, \alpha]\} = 1$ for some $\alpha > 0$. That is, adopters have a "threshold" decision rule: above a certain price differential they choose deterministically; between they follow randomized strategies. To simplify our considerations, suppose, that ξ has the uniform distribution on $[-\alpha, \alpha]$. Since random factors appear, when prices on **A** and **B** are approximately equal, then the following inequality $\alpha < \min_{i=1,2,3,4} \Delta_i$ holds. The probability to choose **A** as a function of $x_{\mathbf{A}}$ in this case has the form

$$f(x_{\mathbf{A}}) = \begin{cases} 1 & \text{for} \quad Pr_{\mathbf{B}}(x_{\mathbf{A}}) - Pr_{\mathbf{A}}(x_{\mathbf{A}}) \geq \alpha \,, \\ 0 & \text{for} \quad Pr_{\mathbf{B}}(x_{\mathbf{A}}) - Pr_{\mathbf{A}}(x_{\mathbf{A}}) \leq -\alpha \,, \\ [Pr_{\mathbf{B}}(x_{\mathbf{A}}) - Pr_{\mathbf{A}}(x_{\mathbf{A}}) + \alpha]/2\alpha & \text{for} \quad -\alpha < Pr_{\mathbf{B}}(x_{\mathbf{A}}) - Pr_{\mathbf{A}}(x_{\mathbf{A}}) < \alpha \,. \end{cases} \tag{3}$$

This is graphically represented in Figure 3. Hence, the model embodies a positive feedback mechanism of diffusion: prices fall with increasing market shares possibly due to learning economies, dynamic increasing returns, etc. and/or, on the behavioral side by market-penetration strategies. But the mechanism is bounded: above a certain market share, the price starts to rise, possibly due to the monopolistic behaviors by the producer(s) and/or the progressive exhaustion of technological opportunities to lower production costs. Finally, market adjustments as a function of differential prices are "imperfect": within boundaries, differently priced technologies both face positive demand. What can one say on the limit shares of such a bounded-increasing-returns process of diffusion? Using results of section 2 we have the following:

1. convergence to x_1^* with probability 1 takes place from the domain I;

2. from the domain II there is convergence with positive probability to both x_1^* and x_3^*;

3. convergence to x_3^* with probability 1 takes place from the domain III.

In the terms of competing technologies these results can be conceptually treated in the following way. If the initial proportion of adopters of technology **A** belongs to the domain I(III), then the technologies **A** and **B** share the market in the proportion $x_1^* : (1 - x_1^*)(x_3^* : (1 - x_3^*))$. Also if the initial proportion of adopters of technology **A** belongs to the domain II, then the **A** and **B** share the market in the proportion $x_1^* : (1 - x_1^*)$ or $x_3^* : (1 - x_3^*)$ (furthermore in each of the cases with positive probability).

More generally: diffusion with endogenous prices and bounded dynamic increasing returns yields under the assumption of the model to market-sharing rather than monopoly. Still, limit market shares are path-dependent: they are determined by history of the diffusion process. The model, however, allows a qualitative analysis – by no means restricted to the price dynamics assumed here – of the ensuing limit proportions dependent on the relative frequencies of adopters of the different technologies.

Let us now turn to the effects of different degrees of "market stickiness", as approximated by α, on diffusion dynamics.

If $\alpha = \Delta_i$ for some i, then the corresponding horizontal part of the graph of $f(\cdot)$ converts into a "sharp", where $f(\cdot)$ attains 0 or 1. If the distribution of ξ is not uniform, then sloping (straight) line segments of the graph of $f(\cdot)$ convert into curve linear ones. And, finally, if $\alpha > \max_{i=1,2,3,4} \Delta_i$ (in particular, when ξ has an unbounded support, as the normal distribution), then all of horizontal segments transform into "sharps" of corresponding highness (from (0, 1)). In this case we can have the graph given in Figure 4. Here (as it follows from the results of papers (Arthur *et al.* (1987a) and (1988), Hill *et al.* (1980)) convergence with positive probability (from every initial approximation) takes place to both x_1^* and x_3^*. Consequently regardless of the relation between the initial numbers of adopters of the technologies the **A** and **B** share the market in the proportion $x_1^* : (1 - x_1^*)$ or $x_3^* : (1 - x_3^*)$. Which one depends on chance.

Implicit preferences of adopters (or, which is basically the same, preferences with imperfect information and 'market-stickiness') can be formalized in a slightly different way. Suppose that

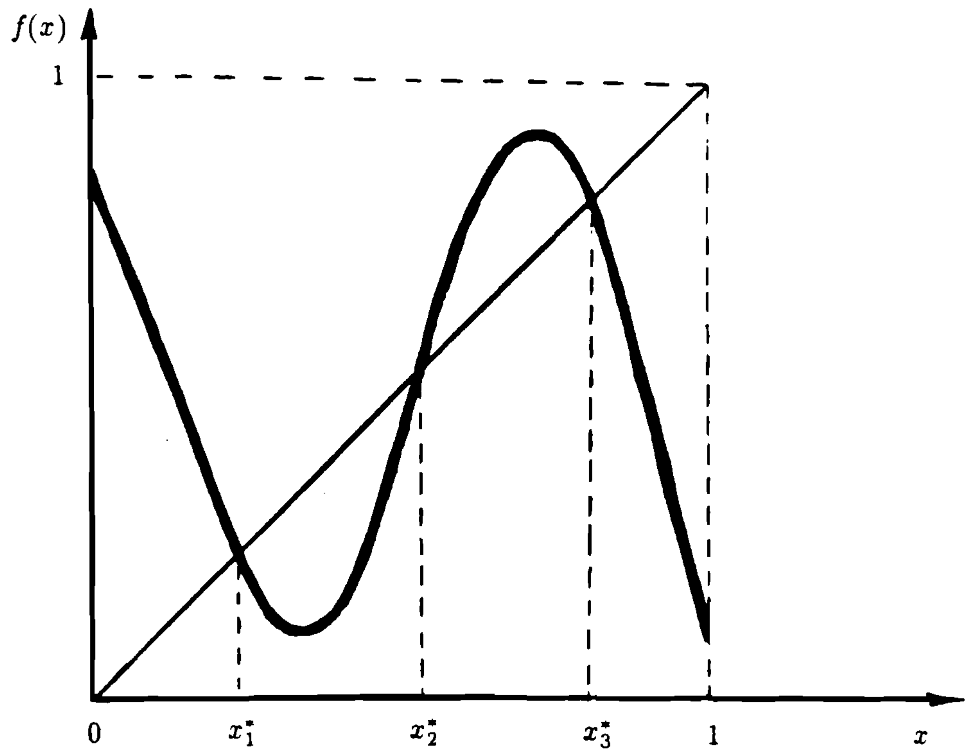

Figure 4

if the difference of the prices is not less than $\alpha > 0$, then the cheaper technology is bought. If the difference is less than the value, then consumers to choose a technology use some stochastic experiment (i.e., a mixed strategy). Here $\alpha < \min_{i=1,2,3,4} \Delta_i$. Consider the following examples:

1. **A** is chosen with probability $p \in (0,1)$ and **B** - with probability $1-p$ (usually, when there are a priori no preferences, $p = 1/2$);

2. would-be adopters sample an odd number $m > 1$ of adopters who have one of the technologies and choose the technology, which is used by the majority (minority) of them.

Then the probability $f(x_{\mathbf{A}})$ to choose **A** as a function of $x_{\mathbf{A}}$, is given graphically in Figures 5 and 6 (in the last case we neglect the term, which goes zero (see section 3). Note that in Figure 6 in order to designate those parts of the graph where $f(\cdot)$ does not attain 0 or 1, we use continuous (broken) line for the cases, when the choice follows the majority (minority) of the sample.

Using results in section 2 we see, that in the first case: (i) from the domain I convergence to x_1^* with probability 1 takes place; (ii) there is convergence with positive probability to x_1^*, p and x_4^* from the domain II; (iii) convergence to x_4^* with probability 1 takes place from the domain III. Hence, the limit market-shares properties are similar to those considered earlier in this section.

However, consider now the case of endogenous prices, as above, with endogenous preferences

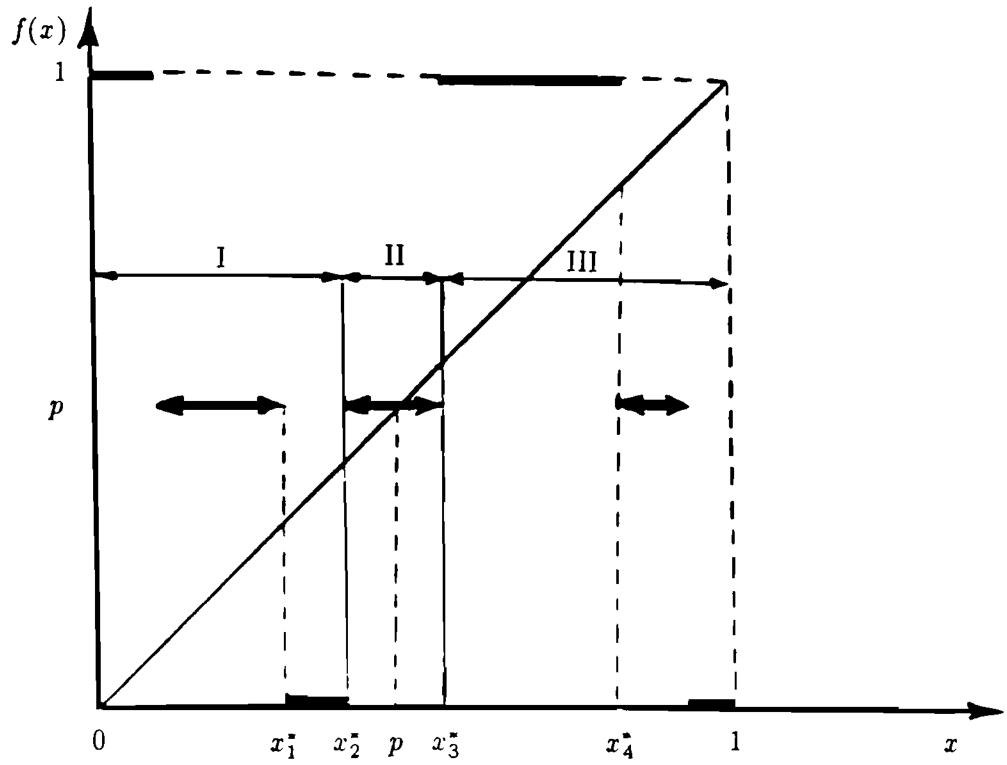

Figure 5

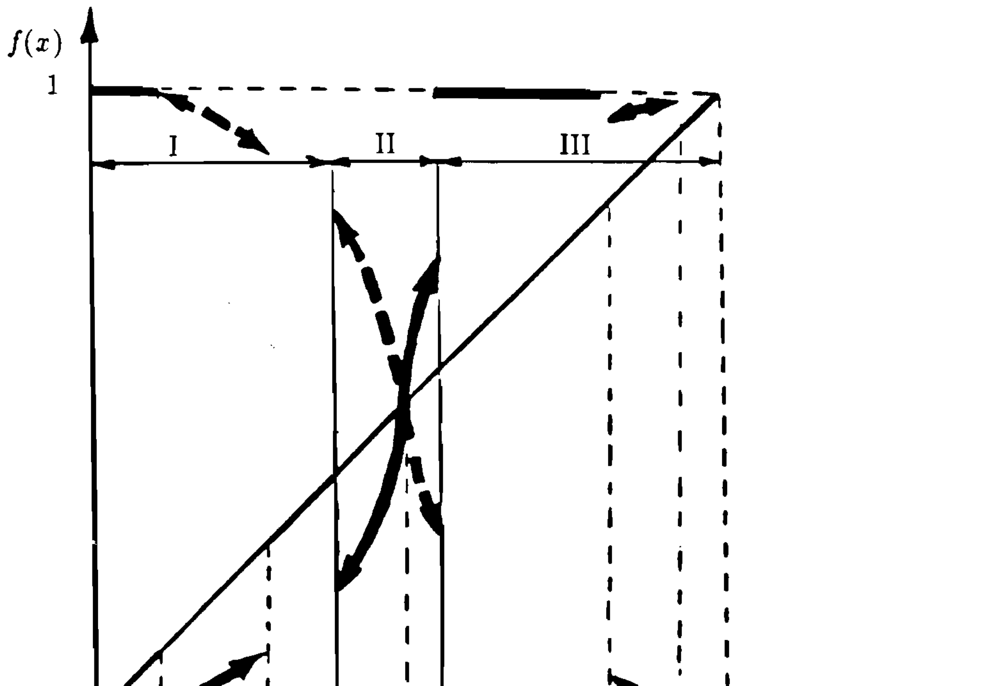

Figure 6

for either technology, whereby choices corresponds to the option of the sampled majority. We have the following limit shares: 1) from the domain I convergence to x_1^* with probability 1; 2) convergence with positive probability to both x_1^* and x_2^* from the domain II; 3) from the domain III convergence with probability 1 to x_2^*.

In all of the considered cases monopoly (i.e., the situation when one of the technologies conquers the market) is impossible. To obtain monopoly one must change the relation between prices on **A** and **B**. As it might be intuitive if systematically $Pr_{\mathbf{A}}(0) > Pr_{\mathbf{B}}(0)$, then we can have monopoly of the **B** technology. Moreover, if $Pr_{\mathbf{A}}(1) < Pr_{\mathbf{B}}(1)$, then we can have monopoly of the **A** technology.

More generally, though, the model highlights the crucial importance of specific price dynamics in the determination of limit market shares. Some (more "evolutionary" inclined) economists might interpret the result as an analytical collaboration of the confecture that out-of-equilibrium "boundedly rational" behavioral norms do affect system-level asymptotic states. Alternatively, one may argue that all this simply emphasizes the dependence of limit market shares upon expectations and intertemporal discount procedures of supposedly perfectly rational but imperfectly informed producers of each technology. Irrespectively of the precise microeconomic assumption, again, the model allow qualitative analyses of the relationships between endogenous price changes, adoption frequencies and limit market shares.

As an illustration, suppose for example that the switch point between price-decreasing and price-increasing strategies occurs at less than 1/2 market shares. In this case, we have the picture of Figure 7. Moreover, if price-dependent choice involve a random error uniformly distributed on $[-\alpha, \alpha]$ (with $\alpha < \min_{i=1,2,3,4} \Delta_i$), then the probability to choose **A** as a function of $x_{\mathbf{A}}$ is illustrated in Figure 8. Using results from section 2, we have: 1) if the initial proportion of adopters of the technology **A** belongs to the domain I (V), then **B**(**A**) conquers the market; 2) if the proportion belongs to the domain III, then **A** and **B** share the market in the proportion $x_2^* : (1 - x_2^*)$; 3) if the initial proportion of adopters of the technology **A** belongs to the domain II (IV), the **B**(**A**) conquers the market in the proportion $x_2^* : (1 - x_2^*)$.

Consequently, under the given hypotheses concerning the behavior of adopters, massive introduction of one of the technologies (domains I and V), which is essentially cheaper in the domain, leads to its monopoly. Under less massive introduction (domains II and IV) we can have monopoly or alternately, **A** and **B** share the market in the proportion $x_2^* : (1 - x_2^*)$. In the case of comparable initial numbers of adopters of **A** and **B** (domain III) these technologies share the market in the proportion $x_2^* : (1 - x_2^*)$.

Note also that the formal apparatus developed here can be used to study all cases, whereby prices depend on the current concentration of one of the technologies on the market in an

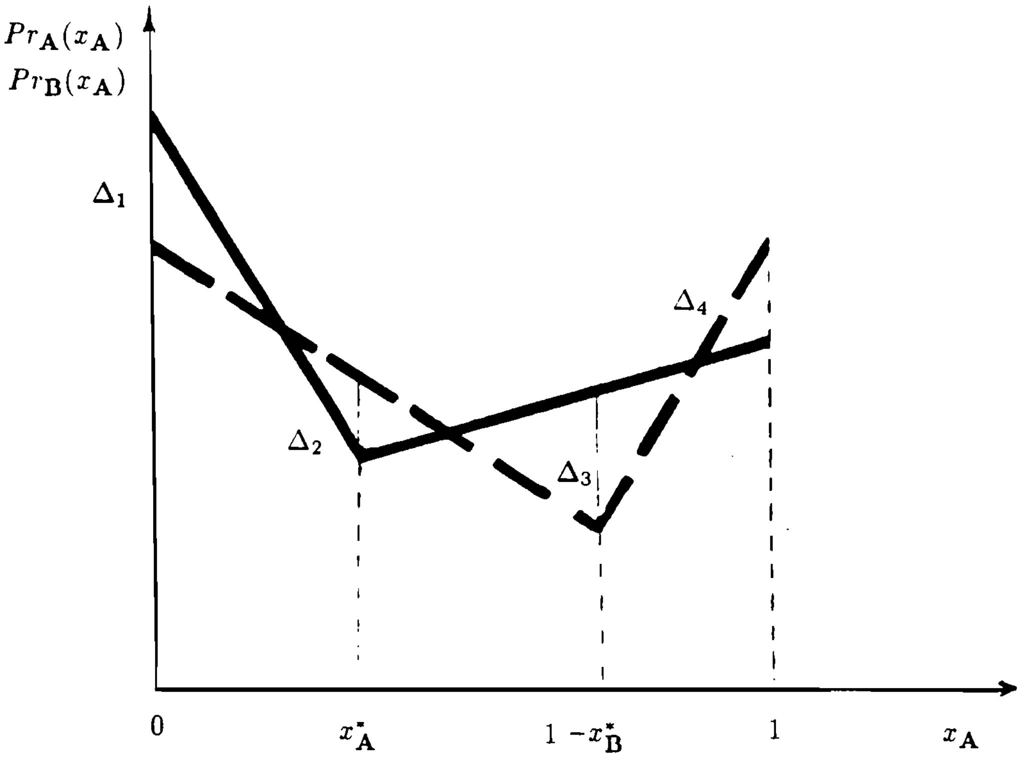

Figure 7

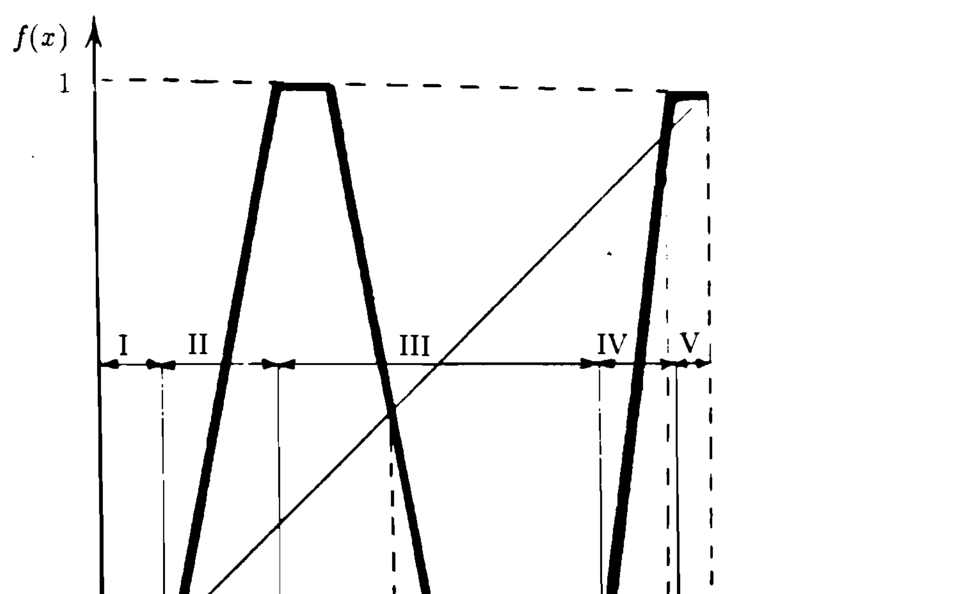

Figure 8

arbitrary way (while here for the sake of simplicity we restricted the discussion to cases where these functions are piecewise linear).

5 Conclusions

Innovation and technology diffusion generally involve competition among different technologies, and, most often, endogenous changes in the costs/prices of technologies themselves and in adopters' choices. In the economic domain (as well as in other disciplines) the formal representation of such processes involves some dynamics of competing "populations" (i.e., technologies, firms, or even behavioral traits and "models" of expectation formation). A growing literature on such dynamics has begun studying the properties of those (generally non-linear) processes that innovation and diffusion entails. As by now robustly established, multiple equilibria are normally to be expected and "history matters", also in the sense that out-of-equilibrium fluctuations may bear system-level consequences on notional asymptotic outcomes. Developing on previous results showing – under dynamic increasing returns – the likely "lock-in" of diffusion trajectories onto particular technologies, we have presented a formal modeling apparatus aimed at handling the interaction between diffusion patterns, on the one hand, and endogenous preferences formation and/or endogenous price formation, on the other. As examples, we presented two stochastic models of shares dynamics on a market of infinite capacity by two competing new technologies. In the first of them, we assumed that the adoption dynamics is essentially driven by endogenous changes in the choices of risk-averse, imperfectly informed adopters (or, in a formally equivalent analogy, by some positive or negative externality imperfectly estimated by would-be users of alternative technologies). In the second example, we considered an endogenous price dynamics of two alternative technologies, driven by e.g., changes in their costs of production and/or by the intertemporal behaviors of their producers.

In both cases, the diffusion process is allowed to embody some stochasticity, due to e.g., "imperfect" learning from other people's choices, marginal and formally undetectable differences in users' preferences, or some inertia in adjusting between differently prices but identical-return technologies.

The formal apparatus presented here, based on a few refinements on generalized urn schemes, allows quite general analytical accounts of the relationships between some system-parameters (e.g., proxies for information "imperfection" by adopters; dynamic increasing returns and monopolistic exploitation of new technologies by their producers) and limit market shares. While path-dependency (i.e., "history matters") applies throughout, the foregoing analytical techniques appear to be able, at the very least, to discriminate those which turn out to be feasible limit equilibria (i.e., those which are attainable with positive probabilities) and, also, to "map" them into relative frequences of adopters.

As the foregoing modeling illustrations show, "market imperfections" and "informational imperfections" often tend to foster technological variety, i.e., the equilibrium co-existence of different technologies and firms. Moreover, stochasticity in the choice process may well bifurcate limit market-shares outcomes. Finally, it is shown, corporate pricing strategies-possibly based on rationally-bounded procedures, imperfect informational and systematically "wrong" expectation-formation mechanisms – are generally bound to influence long-term outcomes. Under all these circumstances, the foregoing modeling techniques allow, at the very least, a "qualitative" analytical assessment of diffusion/competition processes by no means restricted to those circumstances whereby microeconomic expectations, on average, represent unbiased estimations of the future.

If all this analytical representation is empirically adequate, there seem to no *a priori* reasons to restrict it to technological dynamics. In fact, under suitable modifications, it may apply as well to interdependent expectations, decisions and returns on e.g., industrial or financial markets. Ultimately, what we have tried to implement is a relatively general analytical apparatus able to handle at least some qualitative properties of dynamic stochastic processes characterized by both positive, and, possibly negative, feedbacks of a functional form as "badly-behaved" as possible. Indeed, we believe, quite a few of the processes of economic change fall into this category, related to technological change but also to interdependent (possibly "disequilibrium") changes in e.g., industrial structures, but also financial or product-market expectations and behaviors.

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