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AN EXPERIMENTAL MULTI-OBJECTIVE GAME "HUMBLE SHALL BE REWARDED" - RULES, ANALYSIS AND PRELIMINARY CONCLUSIONS

Andrzej P. Wierzbicki[1]

Summary.

The papers presents an experimental multi-objective game with a simple mechanics and rather complicated payoff structure. The game has been devised as a tool of study how people learn - not only to play a game, but to recognize new strategic perspectives, how easily they change a pre-conceived problem formulation. The game can be also applied for testing creativity and learning capabilities of students at graduate level as well as of investigating the impact of professional and cultural backgrounds on adaptive decision making. The game illustrates the fact that real-life game-like situations are often multi-objective, while some objectives and their hierarchy or preference structure might be not apparent from the beginning. Effects of multiplicity of equilibria and the possibility of conflict escalation in such cases are also illustrated by the game. The paper presents the rules of the game, an analysis of its various aspects and some preliminary conclusions.

1. The rules of the game.

The game can be played by one or more pairs of participants. It can be played on a table, or in a computerized version[2]. For the "table" version, the organizer provides for each pair: 10 plastic tokens (chips) in two different colors (say, 5 red and 5 blue); 200 yen (cents, any other small coins) in 10, 5 and 1 yen coins - 100 yen as an initial capital for each player; a box marked "Fund for next games", a written set of rules and a form on which the results of each round of the game are noted. If a player does not use up the funds received, he/she might keep them together with additional rewards obtained in the game; this contributes to one of the objectives in the game.

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[1] Institute of Automatic Control, Warsaw University of Technology, Nowowiejska 15/19, 00-665 Warsaw, Poland; in summer 1992 in the International Institute for Applied Systems Analysis (IIASA), A-2361 Laxenburg, Austria. The game described in this paper was originally developed during a stay of the author in Kyoto Institute of Economic Research, Kyoto University, in 1989.

[2] A computer-based version of the game was recently implemented by Marek Makowski in collaboration with John Rohrbaugh, in the Project on Methodology of Decision Analysis at IIASA.

The game lasts 50 rounds or is stopped earlier if any of the players has used up his initial capital. In each round, both players take their tokens in hands and each selects from 0 to 5 tokens, in such a way that the other cannot see it. Then they put their closed hands with tokens on the table and open simultaneously.

The player which has put more tokens on the table wins the round and gains winning points:

* 2 if he/she won by 2 or more token difference;

* 1 if he/she won by 1 toke difference;

* 0 if he/she lost or there was a draw (equal number of tokens).

Winning points decide who wins the entire game. The second objective is money. Both players start with "a vista" capital of 100 yen and:

* If a player wins a round, the other must give him 10 yen.

* If there is a draw, then each player puts 5 yen plus additionally 2 yen for each token played into the box "Fund for next games".

There are also additional rewards paid by the organizer after the game is stopped:

* For each token "saved" - not played in a round - 1 yen of additional rewards is counted (if, say, player A displayed 4 tokens and player B displayed 1 token, 1 yen for A and 4 yens for B are noted).

* The winner of so-called "lucky chance" round gets additional reward 10 yen and the loser even 20 yen (to recompense his/hers emotional losses). "Lucky chance" rounds are defined as such that there was a difference of 1 token and the winner played an odd number of tokens.

If the game is stopped because a player has used up his/hers initial capital, the due payments in the last round are subtracted from the additional rewards.

There are also some restricting rules. Players are not allowed to communicate when devising a strategy nor to comment on strategies before and during the game; they might think about cooperative strategies, but they must implement them through empathy or by suggesting them through the way they are playing, not through direct communication. If they start to communicate or if they are "too lucky", that is, if a "lucky chance" round happens for more than 12 times in total or in more than 4 consecutive rounds of the game - no matter whether this happens unintentionally or through empathy with the opposite player - the game is interrupted, declared as invalid, and players must give up the money received.

The winner of the entire game is the player who had more winning points; the accumulated financial rewards are a secondary objective. There is a kept record of best results (both winning points and financial results) in the history of playing the game.

The game might be played either without preparation or after some time given for studying the rules and devising a strategy, in a single run or repetitively. To summarize the rules, the results of a single round of the game are represented by two tables of bi-matrix payoffs to players A and B given at the end of this text, where (i, j) denote the numbers of tokens played by respective players, (x, y) denote winning points, (w, z) denote net financial gains per round including all costs and rewards paid after the game is stopped. But the players are warned that, beside the two apparent objectives, there are also hidden objectives in the game.

Table 1 - payoffs in winning points for a single round of the game

i \ j	0	1	2	3	4	5
	(x,y) - winning points for moves (i,j):					
0	(0,0)	(0,1)	(0,2)	(0,2)	(0,2)	(0,2)
1	(0,1)	(0,0)	(0,1)	(0,2)	(0,2)	(0,2)
2	(2,0)	(1,0)	(0,0)	(0,1)	(0,2)	(0,2)
3	(2,0)	(2,0)	(1,0)	(0,0)	(0,1)	(0,2)
4	(2,0)	(2,0)	(2,0)	(1,0)	(0,0)	(0,1)
5	(2,0)	(2,0)	(2,0)	(2,0)	(1,0)	(0,0)

Table 2 - payoffs in net financial gains in yen per round

i \ j	0	1	2	3	4	5
	(w,z) - net financial gains for moves (i,j):					
0	(0,0)	(15,24)	(-5,13)	(-5,12)	(-5,11)	(-5,10)
1	(24,15)	(-3,-3)	(-6,13)	(-6,12)	(-6,11)	(-6,10)
2	(13,-5)	(13,-6)	(-6,-6)	(13,22)	(-7,11)	(-7,10)
3	(12,-5)	(12,-6)	(22,13)	(-9,-9)	(-8,11)	(-8,10)
4	(11,-5)	(11,-6)	(11,-7)	(11,-8)	(-12,-12)	(11,20)
5	(10,-5)	(10,-6)	(10,-7)	(10,-8)	(20,11)	(-15,-15)

A form for recording the results of the game is given in Appendix.

2. An analysis of the game.

While the mechanics of the game is rather simple, the rules of financial rewards are more complicated, with several purposes. The game should present challenge to participants; students are often most motivated by intellectual challenge. The financial rewards for playing the game are relatively small: they cannot dominate the winning points. Thus, each player in the game might have two objectives: to accumulate more winning points and to get higher financial reward. Various intuitive hierarchies or tradeoffs between these objectives might result from different cultural backgrounds. The complicated rules allow also to observe differences between the game played without preparation and after some time for analysis as well as in the capability to learn during the game. In fact, there are several layers of other objectives or perspectives of looking at the game implied by the rules, with corresponding distinct strategies of playing the game. The experimental runs of the game observed until now suggest that it is not easy and requires some creativity to change such a perspective and devise a more effective strategy. We shall present here an analysis of the game starting with the most natural perspectives.

2.1. Equilibria in pure strategies.

We can start the analysis by computing possible equilibria of a single-round game, treated single-objectively; pure strategies of players in such a game will be called moves or actions, combined actions of both players - multi-actions, combined payoffs or outcomes - multi-payoffs or multi-outcomes. Treated multi-objectively, the game illustrates some relations between Nash equilibria of partial single-objective games and so-called Pareto-Nash solutions of the multi-objective game (see e.g. Bergstresser and Yu, 1977, or Wierzbicki 1992).

In Table 3, the simple parentheses ( ) below a multi-payoff (x,y) indicate a Nash noncooperative equilibrium in pure strategies for the single-objective game in winning points (there are 4 symmetric pairs of them and a symmetric one, together 9, but none of them is strict), [ ] indicate that this multi-payoff belongs to a multi-objective Pareto-optimal response set either of player A to the given move of player B or vice versa, finally { } indicate that this point is a Pareto-Nash noncooperative solution in pure strategies to the multi-objective game; these are such decisions and outcomes that are Pareto-optimal (efficient) for each player, given the decisions of other players. There are 3 symmetric pairs or 6 of them. The same convention applies to Table 4 below; there are 3 pairs of Nash noncooperative equilibria in pure strategies in the single-objective game in financial rewards.

Table 3 - pure strategy equilibria in winning points							
H.R.1	j	0	1	2	3	4	5
i		(x,y) - winning points:					
0		(0,0)	(0,1) {[ ]}	(0,2) [ ]	(0,2)	(0,2)	(0,2) ( )
1		(1,0) {[ ]}	(0,0)	(0,1) [ ]	(0,2) [ ]	(0,2)	(0,2) ( )
2		(2,0) [ ]	(1,0) [ ]	(0,0)	(0,1) {[ ]}	(0,2) [ ]	(0,2) ( )
3		(2,0)	(2,0) [ ]	(1,0) {[ ]}	(0,0)	(0,1)	(0,2) ( )
4		(2,0)	(2,0)	(2,0) [ ]	(1,0) [ ]	(0,0)	(0,1) {[( )]}
5		(2,0) ( )	(2,0) ( )	(2,0) ( )	(2,0) [( )]	(1,0) {[( )]}	(0,0) ( )

Table 4 - pure strategy equilibria in financial gains							
H.R.2	j	0	1	2	3	4	5
i		(w,z) - financial gains:					
0		(0,0)	(15,24) {[( )]}	(-5,13) [ ]	(-5,12)	(-5,11)	(-5,10)
1		(24,15) {[( )]}	(-3,-3)	(-6,13) [ ]	(-6,12) [ ]	(-6,11)	(-6,10)
2		(13,-5) [ ]	(13,-6) [ ]	(-6,-6)	(13,22) {[( )]}	(-7,11) [ ]	(-7,10)
3		(12,-5)	(12,-6) [ ]	(22,13) {[( )]}	(-9,-9)	(-8,11) [ ]	(-8,10) [ ]
4		(11,-5)	(11,-6)	(11,-7) [ ]	(11,-8) [ ]	(-12,-12)	(11,20) {[( )]}
5		(10,-5)	(10,-6)	(10,-7)	(10,-8) [ ]	(20,11) {[( )]}	(-15,-15)

When determining noncooperative Pareto-Nash solutions, we can start with defining the sets $\hat{I}(j)$ or $\hat{J}(i)$ of Pareto-optimal responses of one player to a given move of another one; in these sets, no single objective of a player can be improved without deteriorating other objectives. There are various ways of determining such sets or even directly the sets of Pareto-Nash solutions - see e.g. Wierzbicki (1986, 1992). For this simple example, however, we can employ a graphical way represented in Fig. 1.

Because the game is fully symmetric, we can assume $\hat{I}(i) = \hat{J}(i)$ where $i = j = 0, 1, \ldots, 5$. From Fig. 1 it follows that $\hat{I}(0) = \hat{J}(0) = \{1,2\}$; $\hat{I}(1) = \hat{J}(1) = \{0,2,3\}$; $\hat{I}(2) = \hat{J}(2) = \{3,4\}$; $\hat{I}(3) = \hat{J}(3) = \{2,4,5\}$; $\hat{I}(4) = \hat{J}(4) = \{5\}$; $\hat{I}(5) = \hat{J}(5) = \{4\}$.

Since $1 \in \hat{I}(0)$ and $0 \in \hat{J}(1)$ as well as $0 \in \hat{I}(1)$ and $1 \in \hat{J}(0)$, the multi-decisions (1,0) and (0,1) are a pair of Pareto-Nash noncooperative solutions to the multi-objective game; an analogous argument applies to the pairs of multi-decisions (3,2) and (2,3) as well as (5,4) and (4,5). It can be checked that no other multi-decisions (in pure strategies) satisfy the definition of Pareto-Nash noncooperative solutions. For example, take the multi-decision (3,5) that is a noncooperative Nash equilibrium in the single-objective game H.R.1. For player B, $5 \in \hat{J}(3)$; however, for player A, $3 \notin \hat{I}(5) = \{4\}$. But why does it happen?

We can see the reason after examining the multi-payoff tables and Fig.1. The Nash equilibria of the game H.R.1 are not strict. If player B chooses $j = 5$, player A is actually indifferent in his response while playing the game H.R.1; all his responses yield the same payoff $x = 0$. But if he has also another objective, this can and does exclude same of his responses, in particular the response $i = 3$. In the part of Fig. 1 corresponding to $j = 5$, the outcome of the response $i = 3$ is Pareto-dominated by the outcome of the response $i = 4$. This is because we used the concept of strict Pareto domination in the definition of Pareto-Nash noncooperative solutions; if we used the concept of weak Pareto domination instead, we could observe that the outcomes of all responses of player A to $j = 5$ are in fact weakly Pareto-nondominated and all noncooperative Nash equilibria of underlying single-objective games would be also (weak) Pareto-Nash noncooperative solutions to the multi-objective game.

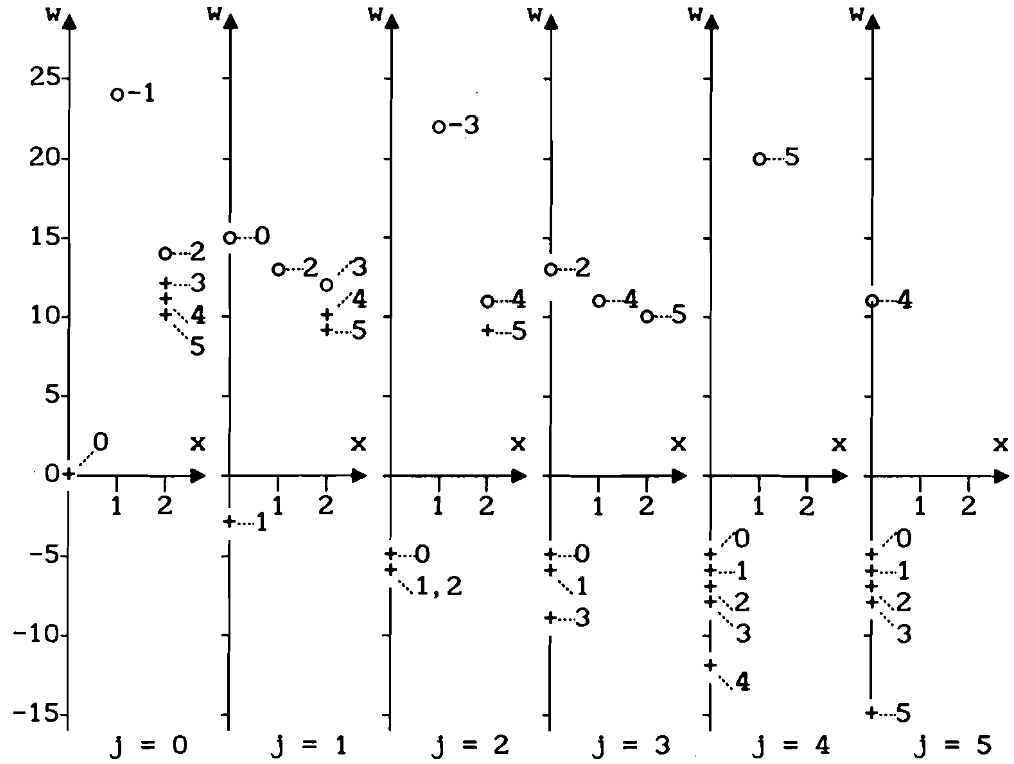

Fig. 1. Multi-objective outcomes of various responses i of player A (denoted by numbers at consecutive points indicated by + while Pareto-optimal responses are indicated by o) to assumed moves j of player B;

However, as this example illustrates, accepting weakly Pareto-nondominated outcomes would mean sacrificing common sense in order to obtain neat theoretical features. Any reasonable player A would reject weakly Pareto-nondominated outcomes of his responses to j = 5, as illustrated in Fig. 2, and keep to to the strictly Pareto-nondominated one, would choose i = 4. Thus, we should rather conclude that it is the weakness of the concept of (non-strict or weak) noncooperative Nash equilibrium that is responsible for this phenomenon, which would be eliminated when using only strict Nash equilibria (see also Young, 1991, for some further-reaching advantages of strict Nash equilibria).

On the other hand, weak noncooperative Nash equilibria can sometimes also contribute to Pareto-Nash noncooperative solutions. Consider the noncooperative Nash equilibrium (5,4) for the game H.R.1. It is strict for player A, since i = 5 is the unique best response to j = 4 in this single-objective game. However, it is weak for player B, since any response j to i = 5 yields the same payoff y = 0; on the other hand, the second objective z in the multi-objective game contributes to choosing j = 4 as the unique Pareto-optimal response to i = 5.

To conclude this part of analysis of the multi-objective game in pure strategies, all parts of Fig. 1 are overlapped and a graph of all possible outcomes of the game is obtained, see Fig. 2.

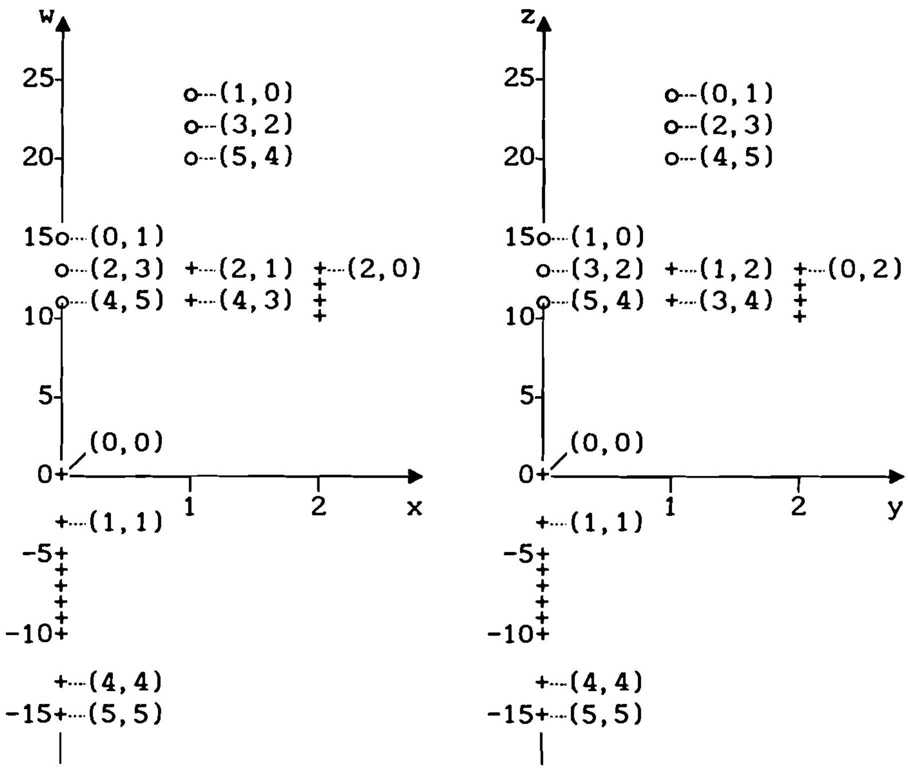

Fig. 2. Multi-outcomes of all possible multi-actions of players A and B in the multi-objective game "Humble shall be rewarded" (only more characteristic multi-actions are denoted by (i,j); multi-outcomes are indicated by + while Pareto-Nash noncooperative solution outcomes are indicated by o).

Imagining a Cartesian product of these graphs is difficult, but we can check also by other means - e.g. preparing bar charts for four objectives of two players as outcomes of more characteristic multi-actions - that cooperatively Pareto-optimal outcomes (in the space of four objectives) correspond only to the multi-actions (1,0), (0,1), (2,0) and (0,2). Of these four, only (1,0) and (0,1) are also Pareto-Nash noncooperative solutions - which indicates that achieving more than 1 winning point in a round of such game played noncooperatively is rather difficult. No symmetric solution in pure strategies is cooperatively Pareto-optimal. The pairs (3,2) and (2,3) as well as (5,4) and (4,5) are also not cooperatively Pareto-optimal for the entire game, although they are noncooperative Pareto-Nash solutions to this multi-objective game. This is a frequent phenomenon in more complicated games; but some mixtures of these pairs are rather probable strategies when this game is actually played, because of conflict escalation phenomena that can easily develop in this game, see later comments.

Some students preparing separately for this game and arriving at this point of analysis might interpret the title of the game in the sense that they should stay humble while not trying to accumulate too many winning points and should concentrate instead on best financial results. Since such results are obtained by playing 0 and 1 tokens and a strategy of playing them alternatively is actually prohibited (by the rule excluding too many "lucky chance" rounds), they might take it as a hint to develop a mixed strategy of playing 0 and 1 with some probability.

2.2. Equilibria in mixed strategies.

Consider thus a game H.R.2 restricted to decisions 0 and 1 only. There are two interesting mixed strategies.

The first is obtained by computing Nash noncooperative mixed equilibrium strategy. Let $p_A(0)$ denote the probability of playing i = 0 by player A and let $p_A(1) = 1 - p_A(0)$, $p_A(2) = p_A(3) = p_A(4) = p_A(5) = 0$ because we consider the restricted game only; let the same conventions apply to $p_B(i)$. Computing the mixed equilibrium strategy is a simple exercise (as in the "battle of sexes" or "game of chicken" cases): it can be obtained that $p^*_A(0) = p^*_B(0) = 3/7 = 0.43$ corresponds to a noncooperative Nash equilibrium for the restricted game with the expected payoff 60/7 = 8.5 yen per round and the expected number of winning points 0.243 per round (for a game of 50 rounds correspondingly 425 yen above the initial 100 yen and 12.1 winning points).

The second is a mixed cooperative strategy: we assume that both players will apply the same probability $p_A(0) = p_B(0) = p$ and use it jointly to maximize their equal expected payoff $39p(1-p)-3(1-p)^2$. The optimal cooperative value is $\hat{p} = 15/28 = 0.54 > p^*_A(0)$ and the expected payoff is also larger, 507/56 = 9.1 yen per round; the expected number of winning points is also slightly larger, 0.249 points per round.

However, the chances that the number of "lucky chance" rounds will exceed 12 are too high for both these strategies. Moreover, these strategies are highly unstable. Consider the noncooperative Nash equilibrium in mixed strategies. Any small deviation from such an equilibrium would lead to optimal responses of the other player that converge in a repetitive game to pure strategies - see Fig. 3; moreover, mixed strategies are here in fact dominated by noncooperative Nash equilibria in pure strategies (both the winner and the loser receive more payoffs when adopting pure strategies).

This is one of the reasons for conflict escalation. Although both players might have assumed when starting the game that a restricted mixed strategy of playing 0 or 1 token would be the best one, but one of them - say, player A - might be tempted to increase the probability of playing 1 token, thus winning more than his opponent. But player B is not really constrained to play only 0 or 1 token; he can respond by introducing a non-zero probability of playing 2 or 3 tokens. Then player A replies in kind - and they end playing 4 or 5 tokens, which they did not intend originally.

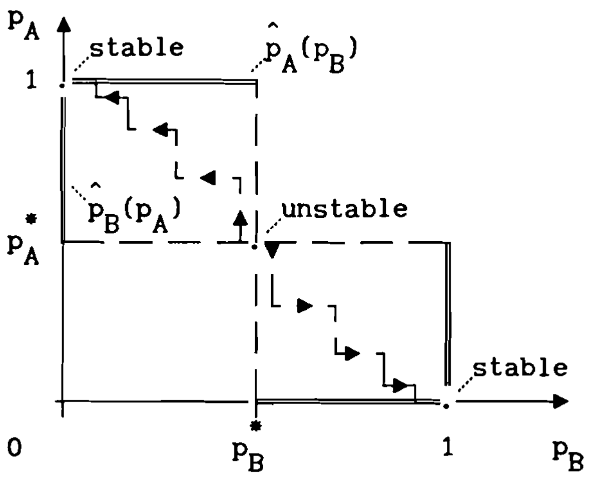

Fig. 3. The optimal response functions in mixed strategies starting with small deviations from p_A^* and p_B^*.

It is easy to prove that the restricted strategy of playing 0 or 1 tokens cannot be a noncooperative Nash equilibrium for the full game H.R.2.

Suppose player A decreases $p_A(0)$ and $p_A(1)$ to $(1-q)p^*_A(0)$ and $(1-q)(1-p^*_A(0))$, respectively, while including $p_A(2) = q > 0$. When doing this, he increases his expected payoff by $(13-8.5)q = 4.5q$ yen per round. If player B responds in kind, their expected payoffs decrease again, below the original 8.5 yen per round, but by repeating such analysis for $p_A(i)$, $i = 3, 4, 5$, we can show that any mixed strategy that does not include playing 5 tokens with probability greater than 0 cannot be a noncooperative Nash equilibrium for the full game H.R.2. On the other hand, when applying an algorithm for computing mixed Nash equilibria - say, one based on repetitive linear programming - one obtains positive probabilities $p^*_A(4)$ and $p^*_A(5)$, but probabilities for all other moves must be zero in the mixed strategy noncooperative equilibrium.

In fact, $p^*_A(4) = p^*_B(4) = 13/29 = 0.45$ and $p^*_A(5) = p^*_B(5) = 16/29 = 0.55$ are a noncooperative Nash equilibrium in mixed strategies for the full game H.R.2, with the corresponding expected payoff 0.69 yen per round and the expected number of winning points 0.247 per round. It is easy to check that the expected payoff for any of the players would decrease if he unilaterally included a non-zero probability of any other move. But again, this Nash equilibrium is highly unstable and dominated by the noncooperative Nash equilibria in pure strategies (5,4) and (4,5). On the other hand, applying a coordinated strategy of alternatively playing (5,4) and (4,5) is actually prohibited - by the rule about too many "lucky chance" rounds; in fact, even the Nash equilibrium in mixed strategies for one-round game is not an equilibrium of the dynamic game of 50 rounds, because the expected number of "lucky chance" rounds is then $50(p^*_A(4)p^*_B(5)+p^*_A(5)p^*_B(4)) = 24.7$ and essentially exceeds the admissible number 12. Moreover, neither of the players would like to be the underdog in a perfectly symmetric game; thus, the possibility that both players would try to out-face each other by playing repeatedly (5,5) is quite high.

When trying to determine a noncooperative mixed Nash equilibrium for the single-objective game H.R.1 we obtain even worse result: $p^*_A(5) = p^*_B(5) = 1$, the only symmetric noncooperative Nash equilibrium is the pure strategy (5,5).

But these are only noncooperative Nash equilibria in mixed strategies for single-objective games; there is also a continuum of mixed Pareto-Nash noncooperative solutions for the multi-objective game. In pure strategies, the outcomes of each possible response of player A to assumed moves of player B formed a discrete, essentially non-convex set - cf. Fig. 1, 2. When considering mixed strategies, these outcome sets are convexified, each outcome in their convex cover becomes attainable, because probabilities of playing various moves can be interpreted as coefficients in convex combinations of discrete outcomes.

We can thus apply for each player separately a standard method of multi-objective optimization on convex polytopes : every Pareto-optimal decision and its outcome can be obtained in such a case as the result of maximizing an aggregated objective - the sum of both maximized objectives with some positive weighting coefficients[3]. We could have applied this method also for pure strategies, but at a risk of missing some Pareto-optimal responses and thus possibly some Pareto-Nash noncooperative solutions: a maximum of weighted sum of maximized objectives with positive coefficients is always Pareto-optimal, but not all Pareto-optimal solutions can be obtained this way in a non-convex case. Consider the response sets to $j = 1$ and $j = 3$ in Fig. 1, where by applying this method we would miss the Pareto-optimality of the responses $i = 2$ or $i = 4$ (although - fortunately - these responses do not contribute to the set of Pareto-Nash noncooperative solutions).

Thus, let us consider objectives $xw_\lambda = \lambda x + (1-\lambda)w$, $yz_\mu = \mu y + (1-\mu)z$, with $\lambda \in (0;1)$ and $\mu \in (0;1)$. For any such λ and μ, there is a corresponding single-objective game H.R.(λ,μ) and its noncooperative Nash equilibria are Pareto-optimal for each player assuming given strategy of the opposite player, hence they are Pareto-Nash noncooperative solutions in mixed strategies to the original multi-objective game. Let us analyze some representative solutions.

For $\lambda \in (0;0.6)$ and $\mu \in (0;0.6)$ nothing changes in the structure of equilibria, only the probabilities of moves in the Nash noncooperative mixed equilibrium - originally of the game H.R.2 if $\lambda = 0$ and $\mu = 0$ - change continuously with λ and μ in the game H.R.(λ,μ):

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[3] At least theoretically. When applying a standard linear programming method to compute the maximum of the aggregated objective, we most likely obtain only the Pareto-optimal outcomes that are situated on a vertex of the polytope, not such that are situated on its verges or facets. There exist, however, ways to deal with this difficulty - and even to deal with a possible non-convexity of outcome sets, such as for pure strategies - see e.g. Wierzbicki (1986, 1992).

$$p_A^*(4) = \frac{26(1-\mu)}{\mu + 58(1-\mu)}; \qquad p_A^*(5) = \frac{\mu + 32(1-\mu)}{\mu + 58(1-\mu)};$$

$$p_B^*(4) = \frac{26(1-\lambda)}{\lambda + 58(1-\lambda)}; \qquad p_B^*(5) = \frac{\lambda + 32(1-\lambda)}{\lambda + 58(1-\lambda)}$$

where $p_A^*(4)$ and $p_B^*(4)$ decrease and $p_A^*(5)$ and $p_B^*(5)$ increase while λ, μ increase. It is also easy to check that the corresponding expected financial gains and expected numbers of winning points decrease while λ, μ increase. Note that the mixed strategy of one player depends on knowing the weighting coefficients which the other player attaches to his two objectives. Since such an assumption is not realistic when actually playing the game, it is improbable that such a mixed strategy could be implemented, a conflict escalation would rather develop instead. Anyway, all these strategies are unstable and dominated by equilibria (5,4) and (4,5) in pure strategies.

At $\lambda = \mu = 0.6$ the structure of equilibria in pure strategies of the aggregated game changes. Consider the multi-payoff table for the game H.R.(λ,μ) with $\lambda = \mu = 0.6$ below. The equilibria (5,4) and (4,5) remain strict, while the equilibria (1,0), (0,1), (3,2), (2,3) became weak at this particular value of λ and μ (compare the underlined entries in Table 5 below); they disappear after any further increase of λ, μ. The equilibria (5,4) and (4,5) remain strict and dominate the unstable equilibrium in mixed strategies defined as above. This equilibrium is the only noncooperative equilibrium in mixed strategies for all $\lambda \in (0;1)$, $\mu \in (0;1)$. Note in the tables below that no player would profit by unilaterally including a nonzero probability of any other move than 4 or 5, if the other player applies the mixed strategy defined above. In the following tables, we use some convenient scaling coefficients (10 or 5) for the aggregated multi-payoff (xw_λ, yz_μ), in order to facilitate comparisons. As before, noncooperative Nash equilibria in pure strategies are denoted by ( ); they coincide in this case with Pareto-Nash noncooperative solutions denoted by { }.

If there is an asymmetry in weighting coefficients, such as $\lambda = 0.7$ and $\mu = 0.4$ illustrated by the multi-payoff Table 6, only some equilibria in pure strategies disappear - in this case (0,1) and (2,3).

Table 5 - equilibria for the linarly aggregated game, $\lambda = \mu = 0.6$

HR(λ,μ) j	0	1	2	3	4	5
i	($10xw_\lambda$, $10yz_\mu$) - aggregated multi-payoff at $\lambda = \mu = 0.6$					
0	(0,0)	(60,102) { ( ) }	(-20,64)	(-20,60)	(-20,56)	(-20,52)
1	(102,60) { ( ) }	(-12,-12)	(-24,58)	(-24,60)	(-24,56)	(-24,52)
2	(64,-20)	(58,-24)	(-24,-24)	(52,94) { ( ) }	(-28,56)	(-28,52)
3	(60,-20)	(60,-24)	(94,52) { ( ) }	(-36,-36)	(-32,50)	(-32,52)
4	(56,-20)	(56,-24)	(56,-28)	(50,-32)	(-48,-48)	(44,86) { ( ) }
5	(52,-20)	(52,-24)	(52,-28)	(52,-32	(86,44) { ( ) }	(-60,-60)

Noncooperative Nash equilibrium in mixed strategies:

$p_A(4) = p_B(4) = 0.437$, $p_A(5) = p_B(5) = 0.563$

Expected winning points: E x = E y = 0.246 points/round

Expected financial gains: E w = E z = 0.58 yen/round

Table 6 - equilibria for the linearly aggregated game, λ=0.7, μ=0.4

HR(λ,μ) j	0	1	2	3	4	5
i	($10xw_\lambda$, $5yz_\mu$) - aggregated multi-payoff at $\lambda = 0.7$, $\mu = 0.4$					
0	(0,0)	(45,72)	(-15,43)	(-15,40)	(-15,37)	(-15,34)
1	(79,45) { ( ) }	(-9,-9)	(-18,41)	(-18,40)	(-18,37)	(-18,34)
2	(53,-15)	(46,-18)	(-18,-18)	(39,68)	(-21,37)	(-21,34)
3	(50,-15)	(50,-18)	(73,39) { ( ) }	(-27,-27)	(-24,35)	(-24,34)
4	(47,-15)	(47,-18)	(47,-21)	(40,-24)	(-36,-36)	(33,62) { ( ) }
5	(44,-15)	(44,-18)	(44,-21)	(44,-24)	(67,33) { ( ) }	(-45,-45)

Noncooperative Nash equilibrium in mixed strategies:

$p_A(4) = 0.443$; $p_B(4) = 0.431$; $p_A(5) = 0.557$; $p_B(5) = 0.569$

Expected winning points: E x = 0.240, E y = 0.252 points/round

Expected financial gains: E w = 0.53, E z = 0.64 yen/round

As before, the weighting coefficient of player A determines the structure of equilibria preferred by player B also in Table 6. None new equilibria in pure strategies appear for all $\lambda \in (0;1)$ and $\mu \in (0;1)$: if they would appear, this would mean additional Pareto-Nash noncooperative solutions - which is impossible, because we determined such solutions earlier by directly applying their definition.

Note the following interesting asymmetry of the game. Player B attached more preference to financial gain than to winning points; in order to devise his mixed strategy he had to know and take into account the preferences of player A, who attached more preference to winning points than to financial gain. Therefore, player B had to apply a greater probability $p_B(5)$; however, this resulted in player B getting both more financial gain and more winning points, since it has placed his mixed strategy closer to the dominating, locally stable equilibrium (4,5) in pure strategies. This confirms that all noncooperative mixed equilibria in this game are unstable and an escalation of conflict is in this sense inevitable.

2.3. The impact of stopping rules.

However, all above analysis and conclusions apply only to an incomplete model of the game that we have considered so far. We have already indicated that the game is actually a dynamic one - we must include in the analysis the stopping rules and some indicators of the current state of the game, see e.g. Basar and Olsder (1982). The stopping rules contain in fact powerful incentives to devise cooperative strategies in this game; therefore, the above, incomplete analysis is quite inadequate.

Note that we counted in the above analysis net financial gains only, while all additional rewards are paid to the players first after stopping the game. The game, however, stops before 50 rounds if one of the players has used up his "a vista" disposable capital starting with 100 yen. This capital is used up not only after he loses 10 rounds without winning, but even faster if there are some draws . Note that this capital suffices not quite for 7 "out-facing" rounds (5,5). Moreover, the game might stop before 50 rounds if there are 5 consecutive "lucky chance" rounds or more than 12 such rounds in total, in which case all winning points and financial gains are annulled. The probability of such an event in a 50 round game is very high, if the players apply the mixed strategy of playing 4 or 5 tokens.

However, each round of playing the game might bring additional rewards; thus, the game is different from repeated games with a fixed finite number of rounds and more similar to repeated games with infinite number of rounds. Both players are strongly motivated to "survive" entire 50 rounds of the game. Moreover, each of them is motivated also to take care that the other player "survives". As an example, consider a scenario in which player A applies a mixed strategy of playing $i \in \{0,1,\ldots,5\}$ with equal probability $p_A(i) = 1/6$, because he/she wants to avoid draws and decrease the probability of too many consecutive "lucky chances"; but player B wants to exploit this and plays $j = 5$ all the time. The expected results of such strategies are $E\ x = 0$, $E\ y = 1.5$ points/round, $E\ w = -6$, $E\ z = 7.5$ yen/round. However, these are only net results, including the rewards paid after the game is stopped; the a vista capital of player A will diminish by 10 yen each round he is playing less than 5 tokens and 15 yen if he is playing 5 tokens, 10.83 yen/round on average. Therefore, the game will stop after about 10 rounds and player B will gain only about 15 winning points and about 75 yen above the initial 100 yen, much less than he could achieve if he would prolong the game to 50 rounds by letting player A win from time to time.

Thus, an adequate model of the game is dynamic, accounting for the current state of disposable capital of each player at every round. A full analysis of such a dynamic game is rather complicated and will not be presented here. However, essential features of best strategies in such dynamic game are clear: for almost 50 rounds, each player should treat the other one as a partner, not an adversary - and not out of compassion, but in his own best interest - watch carefully not only his own disposable capital but also that of his partner, let the partner win if that capital is dangerously low and thus help him to "survive" the entire game, think about best strategies of avoiding draws that diminish joint disposable capital of both players. Only a few rounds before 50 the partnership might be "dissolved" and a noncooperative game can start, when the players believe that the disposable capital of each of them at this point is sufficient to last through a noncooperative play until the round 50.

These essential features of strategies for the dynamic game can be approximated by introducing a third, hierarchically dominant objective: "avoid draws", that is, minimize payments to the box "Funds for next games" (in order to maximize joint disposable capital of both players), and by insisting on symmetric strategies for almost 50 rounds of the game. The game in "avoiding draws" can be represented by following Table 7, where not net but immediate costs of draws are accounted for. For this game, it is possible to apply one of the standard methods of computing noncooperative Nash mixed equilibrium strategy with results represented also in Table 7. However, this game should be rather played cooperatively, as a game with implicit communication and a correlated strategy (see e.g. Myerson, 1991) by assuming that both players would jointly use the probabilities of each move to minimize the cost of draws (by a standard single-objective optimization technique); the corresponding results are also presented in Table 7.

Table 7 - immediate multi-payoffs for the game "avoid draws"

H.R.3 j	0	1	2	3	4	5
i	(ξ,ϑ) - immediate multi-payoff by avoiding draws					
0	(-5,-5)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
1	(0,0)	(-7,-7)	(0,0)	(0,0)	(0,0)	(0,0)
2	(0,0)	(0,0)	(-9,-9)	(0,0)	(0,0)	(0,0)
3	(0,0)	(0,0)	(0,0)	(-11,-11)	(0,0)	(0,0)
4	(0,0)	(0,0)	(0,0)	(0,0)	(-13,-13)	(0,0)
5	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(-15,-15)

Noncooperative Nash equilibrium in mixed strategies:

$p_A(0) = p_B(0) = 0.318$; $p_A(1) = p_B(1) = 0.176$; $p_A(2) = p_B(2) = 0.151$; $p_A(3) = p_B(3) = 0.132$; $p_A(4) = p_B(4) = 0.117$; $p_A(5) = p_B(5) = 0.106$

Expected immediate costs of draws: $E\,\xi = E\,\vartheta = -\,1.465$ yen/round

Expected winning points: $E\,x = E\,y = 0.67$ points/round

Expected net financial gains: $E\,w = E\,z = 4.31$ yen/round

Cooperative mixed strategy:

$p_A(0) = p_B(0) = 0.290$; $p_A(1) = p_B(1) = 0.208$; $p_A(2) = p_B(2) = 0.161$; $p_A(3) = p_B(3) = 0.132$; $p_A(4) = p_B(4) = 0.112$; ${}_Ap\ (5) = {}_Bp\ (5) = 0.097$

Expected immediate costs of draws: $E\,\xi = E\,\vartheta = -\,1.452$ yen/round

Expected winning points: $E\,x = E\,y = 0.67$ points/round

Expected net financial gains: $E\,w = E\,z = 5.06$ yen/round

We see that correlated mixed strategy gives indeed different results than the noncooperative mixed equilibrium: although the expected cost of draws is only slightly lower and the expected number of winning points practically the same, the expected financial gain is considerably higher, each player could expect 253 yen of net financial gains (above the original 100) as well as about 33 winning points after 50 rounds.

The probability of a "lucky chance" round is 0.185 and the expected number of them in a 50 round game is 9.24, hence the players can rather safely[4] apply this strategy. When following this strategy, however, little room remains for noncooperative play at the final rounds: after 47 rounds, the expected level of disposable capital is only 31.8 yen, hence only 3 or 2 last rounds can be played in a noncooperative way.

If the game is played cooperatively all 50 rounds, one could in fact sacrifice some disposable capital at the end of the game[5] and consider an aggregated payoff for all three objectives with some positive weighting coefficients. But such an aggregated game should not be considered in the noncooperative sense - it is easy to check that it would lead again to results very similar to the noncooperative strategies for the game H.R.(λ,μ) and to conflict escalation. Thus, both players should understand this danger and the necessity to play cooperatively in their best long-term interests in the dynamic game. Moreover, it can be checked that reasonable results (corresponding to non-zero probabilities of all moves, necessary when avoiding draws by a mixed uncoordinated strategy) can be obtained in such an aggregated game only if the weighting coefficients at the objectives ξ and ϑ are rather close to 1 - which practically means the hierarchical dominance of the value of avoiding draws.

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[4] Still, there is a probability that the game will be stopped before 50 rounds or even declared invalid if the players will be considered "too lucky" either because there will be more than 4 consecutive "lucky chance" rounds or more than 12 of them in total; see the note below.

[5] But we cannot go down with the disposable capital to zero. We are speaking here about expected net financial gains, winning points and disposable capital at the end of the game only in an approximate sense, because an unfortunate sequence of events could exhaust this capital and thus stop the game earlier. Precisely speaking, the expected net financial gains and winning points should be computed while using a nonlinear averaging scheme accounting for the probability of such events - such as in a Markov chain with a non-zero probability of stopping. When using only approximate, linear averaging, we must provide for a sufficient reserve in the disposable capital until the last round of the game.

2.4. Cooperative, alternating and coordinated strategies.

The analysis of the game is yet incomplete - because until now, as it is usually done in game theory, we excluded the possibility of such communication between the players that would result in a coordinated strategy of alternating, reciprocated moves. Although explicit communication is prohibited in the game, the sequence of moves of each player in consecutive rounds is an admissible communication medium. When using this medium, concentrating on avoiding draws and on "survival" of both players, an intelligent and perceptible player can achieve much better results than presented above. His/hers first goal is then to teach the partner - the opposite player - to avoid draws completely. This can be done by developing adaptively, through the example of one's own strategy and through empathy, an admissible coordinated alternating strategy. The strategies of alternatively playing (0,1) and (1,0) or (2,3) and (3,2) or (4,5) and (5,4) are actually excluded by the rule prohibiting too many "lucky chance" rounds. But the concentration on these strategies results from a noncooperative treatment of the game, with dominant objectives of net financial gains and winning points. If we concentrate on avoiding draws instead, there are obviously several coordinated strategies not excluded by the rules of the game. Two of such strategies are particularly attractive.

The first one is playing alternatively (0,5) and (5,0). It avoids draws completely and gives the maximum symmetric gains in winning points, although it results in rather low financial gains - but still 2.5 yen/round, better than any noncooperative strategy that does not recognize the dominant objective of avoiding draws. Moreover, this strategy has two other important advantages. It is easy to teach the partner: when player A starts the game with playing alternatively 0 and 5 tokens, he gives to player B a clear enough signal to follow his example and develop a partnership and coordinated strategy through empathy. A difficulty can arise if player B wants also to implement the same strategy and starts with the same move as player A. However, intelligent players would resolve such difficulty by playing chance moves in next round, say, with equal probability of each move. If a next draw happens, they repeat the mixed strategy; if one of them wins, it is his turn to play 0 in the next round and let the other player win.

The second advantage of this strategy is its robustness: even if player B had not analyzed the game deeply enough and thinks about playing it in a noncooperative way, he cannot do much against such consequent strategy. Player B will be forced to relent if he can predict that in the next round player A will play 5 tokens - he will not be motivated to "outface" player A by also playing 5, because he knows that he will anyway win half of the rounds. If player B is tempted by a higher financial gain and responds 4 tokens to a predictable 5 tokens of player A, thereby diminishing also the winning points of player A in such a round, player A should respond in kind: after 4 rounds of playing alternatively (5,4) and (4,5), it will be player B who is forced (by the rule about too many "lucky chances") to change strategy and to give more winning points to player A. Later, player B can try to repeat the sequence of alternative (5,4) and (4,5), but not more than 3 times or 12 rounds in total - which would result in each player gaining about 280 yen instead of only 125 yen of financial rewards above the original 100 yen but in decreasing the number of accumulated winning points from 50 to about 44 after 50 rounds of the game.

Once player B has learned to play alternatively 0 and 5 tokens following the example of player A, the latter can start to introduce the second one, more profitable but also admissible coordinated strategy of playing alternatively (0,2) and (2,0). This strategy results also in the maximum symmetric number of winning points but increases the net financial gain to 4 yen/round. If the players applied alternatively (0,5) and (5,0) before, player A can give a signal and example to his partner by playing 2 instead of 5 tokens at his turn. Player B might again be tempted, by various moves. He could play 3 or more tokens when he predicts player A to play only 2 tokens; player A should respond in kind at the next round (applying a kind of "tit for tat" strategy). But player B should have learned until then that conflict escalation is not in his own long-term interest.

If a coordinated strategy of playing alternatively (0,2) and (2,0) is developed this way, the game can proceed to the round 40 with little or no loss of disposable capital. A carefully counted number of symmetric "lucky chance" rounds can be included, say, by playing 3 instead of 0 when the partner is supposed to play 2 and expecting him to reciprocate in the next round. After a long phase of alternating, cooperative play, the partnership might be "dissolved" somewhere between round 41 and 44: both players can start a noncooperative phase, trying to get better results than the opponent. However, an intention of breaking a record in accumulated winning points requires rather that the partnership should be continued to the round 47 or 48.

We come here to another, deeper layer of traps set by the rules of the game: in order to break a record in winning points, a player must let his partner win at alternative rounds. Does this imply that his/hers strategy must be fully **predictable ?** A predictable strategy is open to exploitation by the other player. If this exploitation is obvious, this would lead to a retaliation and conflict escalation - which a good player must avoid. However, this exploitation might be also subtle - a player might let his partner win alternatively in an almost every second round, but introduce small variations that might give him a some advantage. For example, since the alternating, cooperative play might be varied by introducing "lucky chance" rounds in various ways, the way of initiating and ending such intermissions might be aimed, say, at gaining some more winning points or additional rewards than the partner. Various attempts at such expert strategies were observed when the game was played by more advanced players.

Finally, playing the game noncooperatively but subtly for a last few rounds is also possible. For example, a player might pursue the alternating cooperative strategy almost until the end, reserve some number of "lucky chance" rounds until the round 47, then initiate such rounds but modify them in the last round to get 2 winning points instead of 1. A specific strategy of playing last rounds must depend on the full state of the game - a vista capital and other accumulated results of both players, as well as on the sequence of last moves - and is thus rather difficult to prepare. It remains open, whether such noncooperative but subtle strategy aimed at breaking "the absolute record" - gaining more than 50 winning points - might be developed for the last two or three rounds of the game.

All these coordinated strategies require implicit communication and present various challenges of correctly interpreting the signals given by the partner, of resolving unlucky chance draws, of not being tempted by short-term opportunities that might lead to conflict escalation. After organizing and studying a number of experimental runs of the game, the author found it to be a very good predictor of adaptability and the ability to learn and communicate of players.

3. Preliminary conclusions.

The intended meaning of the title "Humble shall be rewarded" of this game is that players should deliberately give up short-term attempts to achieve high immediate winnings and certainly should not try to get better than the other player by brute force; they should recognize that it is in their best long-term interests to form a partnership or even to achieve an emphatic understanding with the partner.

The game teaches several lessons - that any realistic decision situation should be rather treated as a dynamic process than a static, well defined game; that in such a dynamic process one can learn to recognize hidden but often more important objectives than those assumed as obvious at the beginning; that the definition of an opponent in a game can also change into that of a partner. When constructing this game, the author of the paper wanted to prepare yet another experimental illustration of the spirit underlying Rapoport "tit for tat" strategy: the belief that an informed, non-naive altruism is often best long-term strategy for ones own interest[6].

The game "Humble shall be rewarded" was preliminary tested by experimental runs in various professional and cultural environments: by game theorists, by their graduate students, by international graduate students - with results that were sometimes predictable but often counter-intuitive. Some players - even specialists in game theory, no matter how long time was given for a study and analysis of the rules of the game - have difficulties with the development of the coordinated, alternating cooperative strategy. Only one pair of graduate students (an Australian and a Polishmen, without game-theoretical background) developed such a strategy when playing the game for a second time; the maximum number of winning points recorded was thus raised to 40. Another similar pair (an American and a Polishmen, also without game-theoretical background) has played the game brilliantly for the first time, very quickly learning, but having some difficulties in establishing an effective communication; the record of their play is given in Appendix.

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[6] Such an interpretation of the philosophy of "tit for tat" strategy was stressed by Rapoport in several discussions; see also his book (1989).

These preliminary tests suggest that the game might have future practical applications: it could be used as a test of the ability to learn, adapt and communicate, important when hiring personnel for certain positions or when selecting graduate students. But the total number of experimental runs of the game - about 15 - is yet too small to allow for definite conclusions. A computerized version of the game was recently developed for the purpose of studying the properties of the game more closely, in an experimental design involving various conditions of information availability.

This experimental game has also some other values, related to the intellectual challenge of its analysis or to its possible entertainment value. However, the author believes that main values of this game are educational - it illustrates several important points:

- Many real-life game-like situations are in fact multi-objective, because it is not realistic to assume that the players would correctly estimate the preferences and the value or utility functions of their opponents;

- A multi-objective game has usually a large number - often a continuum - of Pareto-Nash solutions - and it is difficult to exclude some of them on purely theoretical grounds. This multiplicity of solutions can lead to conflict escalation processes;

- It is an inalienable right of players (or, more generally, of decision makers in a more complicated game-like situation) to learn and change their minds about the importance and hierarchy of values and objectives. Such changes of perspective are often necessary to avoid conflict escalation and require some creativity;

- Multi-objective games can be analyzed theoretically (see also Blackwell 1956, Contini 1966, Yu 1973, Zeleny 1976, Bergstresser and Yu 1977, Wierzbicki 1992), though their analysis might be quite complicated;

- An essential reason of conflict escalation processes - be it in multi-objective games or in real life - is not the viciousness of opponents but their inability to communicate and learn. Thus, further theoretical studies should include adaptive coordinated strategies based upon various media of communication between the players; ideas such as expanding habitual domains by Yu (1990) or evolving conventions of strategies by Young (1991) might be helpful in such studies.

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Appendix: a record of playing the game "Humble shall be rewarded"

Player A: NNNN (an American)

graduate student, theoretical physics and engineering

Player B: MMMM (a Polishmen)

graduate student, computer science and decision support

(this is an example; for further use delete the entries)

In the table below, enter the number of tokens played in the columns i and j. In the column "Winner" enter A, B or D if there is a draw. If the winner won by 1 token difference, mark the column "By 1 token". If, additionally, the winner played an odd number of tokens, mark the column "Lucky chance". The columns "Winning points" and "Addl. (additional) rewards" are filled out according to the rules (0, 1 or 2 winning points, 1 yen of additional reward for each token not played, 10 yen to the winner and 20 yen to the loser of a lucky chance round).

Round	i(A)	j(B)	Winner	By 1 token	Lucky chance	Winning points		Addl. rewards	
						A	B	A	B
1	1	1	D			0	0	4	4
2	1	4	B			0	2	4	1
3	3	1	A			2	0	2	4
4	2	2	D			0	0	3	3
5	1	0	A	V	VV	1	0	14	25
6	0	3	B			0	2	5	2
7	5	2	A			2	0	0	3
8	2	5	B			0	2	3	0
9	1	0	A	V	VV	1	0	14	25
10	1	1	D			0	0	4	4
11	0	1	B	V	VV	0	1	25	14
12	1	0	A	V	VV	1	0	14	25
13	0	2	B			0	2	5	3
14	1	0	A	V	VV	1	0	14	25
15	0	1	B	V	VV	0	1	25	14
16	2	2	D			0	0	3	3
17	2	3	B	V	VV	0	1	23	12
18	1	0	A	V	VV	1	0	14	25
19	4	4	D			0	0	1	1
20	4	1	A			2	0	1	4
Sub-total						11	11	178	187

Round	i(A)	j(B)	Winner	By 1 token	Lucky chance	Winning points A	Winning points B	Addl. rewards A	Addl. rewards B
Sub-total						11	11	178	187
21	3	5	B			0	2	2	0
22	2	5	B			0	2	3	0
23	4	0	A			2	0	1	5
24	2	3	B	V	VV	0	1	23	12
25	3	4	B	V		0	1	2	1
26	0	0	D			0	0	5	5
27	0	2	B			0	2	5	3
28	0	0	D			0	0	5	5
29	2	2	D			0	0	3	3
30	5	1	A			2	0	0	4
31	2	2	D			0	0	3	3
32	5	1	A			2	0	0	4
33	0	4	B			0	2	5	1
34	0	3	B			0	2	5	2
35	5	0	A			2	0	0	5
36	4	0	A			2	0	1	5
37	2	1	A	V		1	0	3	4
38	0	3	B			0	2	5	2
39	0	0	D			0	0	5	5
40	0	0	D			0	0	5	5
41	0	1	B	V	VV	0	1	25	14
42	5	0	A			2	0	0	5
43	5	2	A			2	0	0	3
44	0	5	B			0	2	5	0
45	4	0	A			2	0	1	5
46	0	2	B			0	2	5	3
47	2	0	A			2	0	3	5
48	0	2	B			0	2	5	3
49	0	0	D			0	0	5	5
50	1	1	D			0	0	4	4
Total						30	32	312	308
Remaining capital								-5	15
Total financial payoff								307	323