Working Paper

Computable General Equilibrium Models in Economics: A Survey on Theoretical Foundations and Applications

Pekka Sinko

WP-92-26
March 1992

IIASA International Institute for Applied Systems Analysis □ A-2361 Laxenburg □ Austria
Telephone: +43 2236 715210 □ Telex: 079 137 iiasa a □ Telefax: +43 2236 71313

Computable General Equilibrium Models in Economics: A Survey on Theoretical Foundations and Applications

Pekka Sinko

WP-92-26
March 1992

IIASA International Institute for Applied Systems Analysis □ A-2361 Laxenburg □ Austria
Telephone: +43 2236 715210 □ Telex: 079 137 iiasa a □ Telefax: +43 2236 71313

Foreword

This Working Paper gives an overview of the theoretical background analysis and applications of Computable General Equilibrium models. The foundation for the paper was laid during the author's stay at IIASA as a 1991 YSSP-er, when his research was incorporated into the MDA Project.

Computable General Equilibrium Models in Economics: A Survey on Theoretical Foundations and Applications

Pekka Sinko

Contents

0. Introduction

Computable general equilibrium (CGE) models have become an increasingly popular tool in macroeconomic analysis. The model tradition has proved particularly suitable for the study of structural changes in economy and long-run impacts of policy measures. The main fields of application include trade policy, fiscal policy, development strategies and enviromental and energy policies.

The aim of this paper is to give an overview of the modeling tradition, its theoretical foundation, characteristic features and main applications. Because the paper serves as a preliminary report to the Government Institute for Economic Research particular attention is paid to the possibilities CGE-modelling offers to research in public economics.

The structure of the paper is as follows: Chapter one discusses the concept of an equilibrium in economics and introduces two examples of general equilibrium models highlighting their derivation and underlying assumptions. Chapter two examines the basic theoretical properties of general equilibrium models. Chapter three introduces numerical specifications, computable general equilililibrium models and addresses related questions of solving, parametrization and data collection. Chapter four discusses the specific features of CGE-models including dynamic settings and compares CGE-models with so-called econometric macromodels. Chapter five surveys the main applications with emphasis on public economics. The last section of the chapter discusses modeling of government behaviour in CGE-models preparing further researh. A stepwise schedule for the CGE-modelling procedure is given in Appendix 1.

Aknowledgements

This paper was started during my visit to IIASA as a YSSPer in the summer 1991. In addition to IIASA I own a lot to the organizers and participants of the International Summer School and the 5th IIASA Task Force Meeting on Applied General Equilibrium Modelling, both of which took place at IIASA premises in August 1991.

1. Concept of general equilibrium and the models

1.1 Equilibrium in economics; theory and models

An equilibrium may have several meanings in an economic context. The term equilibrium, originating from mechanics, was introduced to economics by Adam Smith. In Smithian sense an equilibrium refers to the state of a real market where opposite market forces - demand and supply - are equal and no adjustment takes place. In this theoretical tradition an equilibrium is assumed to be stable so that deviations from the equilibrium will automatically be corrected by the economic system and a new equilibrium will be found. Thus any single market or whole economy can be seen as a system shifting from one equilibrium to another.

The Smithian view of the world has later led to attempts to formulate mathematical models that try to simulate the working of the underlying economic system. In these models the equilibrium is a built-in property, a simultaneous solution of a group of equations that can be found by calculation. The existence, uniqueness, stability and other equilibrium related attributes are determined by the mathematical formulation of the model.

A common distinction made in economic modelling is that between partial and general equilibria. In partial models the price and the quantity of one good (at a time) are allowed to adjust to the equilibrium in response to changes in exogenous variables. There are no dependencies between different markets. (For example, a change in the equilibrium of commodity markets is assumed to have no effect on the labour market and consumers' income). The lack of interdependencies causes the existence and stability of a "total equilibrium" to be trivial. In principle, partial analysis can be justifiable when the markets considered have only a marginal effect on consumers' income and relative prices of other commodities.

In general equilibrium models a somewhat more realistic approximation of the economic system is reached by allowing for intermarket dependencies that are supposed to characterise real economic systems. These models are in fact mathematical formulations of Smith's world where an "invisible hand" chooses correct price signals to simultaneously clear up the complexity of numerous markets of an economy. One of the attractions of general equilibrium modelling lies in its ability to track responses that differ considerably from those observed via partial analysis; eventually, they may even be of opposite sign.

In multisectoral general equilibrium models the question of the existence of an equilibrium solution becomes non-trivial. Considerable mathematical effort has been put in examination of the properties of these models. Wald (in the 30's) and somewhat later Arrow (in the 50's) proved the existence of a multisectoral equilibrium under some restrictive assumptions. Uniqueness and stability of the equilibrium have thereafter been objects of frequent research. Another field of considerable activities is application of computable general equilibrium models to economic forecasting and policy analysis.

1.2 A two-sector analytical model

To see how a general equilibrium model is constructed, it is useful to consider a simple analytical model. We will derive a model with two goods produced, two factors of production and one representative consumer. The economy is assumed closed so no interaction with the rest of the world takes place. The procedure follows closely that presented by Dinwiddy & Teal (1988).

To be able to construct the model one needs to define demand and supply schemes for four different markets: markets for the two commodities and the two factors, which are here named labour and capital, according to common practice.

Demand for commodities is determined through the conventional optimization procedure where a consumer maximizes his/her utility function subject to a budget constraint. If the utility function is defined in terms of quantities consumed, the demand functions can be derived in terms of commodity prices and consumer's total income (Equation 1.1). In a closed model without government, all income created in production ends up with the consumers. Thus the consumer's total income consists of capital and labour income and company profits (Eq. 1.2).

Demand for factors of production is derived through a cost minimizing procedure. Given the production function and factor prices, the total costs of production are minimized subject to a given level of output. Demand functions for factors can then be written in terms of quantities produced and factor prices (Eq. 1.3 - 1.4).

The supply of goods is derived through a two-step procedure. Firstly, the total cost function of production is derived by solving the cost minimizing problem described above. The desirable level of production (Eq. 1.5) is then found by maximizing the difference between the total revenue and the total costs. The difference can be presented explicitly as company profit (Eq. 1.6).

The supply of the factors can be derived by including leisure and future consumption in the consumer's utility function and then carrying out the optimization procedure. For simplier models it is common, however, that the supply of factors is assumed to be fixed, i.e. exogenously determined.

Finally, we need to set up the equilibrium conditions for the four markets. This is done by equating demand and supply in different markets simultaneously (Eq. 1.7 -1.9).

In the case of two goods and two factors, we now have a system of 15 simultaneous equations with 15 endogenous and two exogenous variables. Depending on the functional forms of the equations, a unique solution for the system may or may not exist. In the case of non-linear equations the existence of a solution becomes ambiguous and analytically solving the system is very cumbersome. However, a numerical solution (given that it exists) for a certain specification of the model can be found by using iterative computer programs.

$c_i = c_i(p_i, y)$ (1.1)

$y = r*£k_i + w*£l_i + £s_i$ (1.2)

$k_i = k_i(w, r, x_i)$ (1.3)

$l_i = l_i(w, r, x_i)$ (1.4)

$x_i = x_i(w, r, p_i)$ (1.5)

$s_i = x_i p_i - rk_i - wl_i$ (1.6)

$£k_i = k'$ (1.7)

$£l_i = l'$ (1.8)

$c_i = x_i$ (1.9)

Equations for two-sector model. The subindex i stands for the commodity and corresponding sector of production, £ stands for summation over i. Symbols read as follows: c = demand, x = supply, p = price, y = income, s = profits, k = capital, l = labour, w = wage, r =interest rate. k' and l' stand for fixed supply of the respective factors.

1.3 The multidimensional Arrow-Debreu model

The problems related to the existence and finding of a solution to a general equilibrium model become more complex when the number of variables in the model is increased. In a closed economy model similar to the one introduced above, the existence of a solution is especially sensitive to the number of consumers with differing preferences.

The first rigorous mathematical treatment of the existence problem in an n-dimensional model was presented by Arrow and Debreu in the 1950's. Arrow and Debreu showed that under some assumptions, which were mostly in line with neoclassical theory, it is possible to prove the existence of a equilibrium set of prices that simultaneously equate demand and supply in various markets. Mathematically the proof was based on the techniques of convexity and the fixed points theorem.

Before proceeding further it is useful to review the assumptions and equilibrium conditions of the Arrow-Debreu model.

1. Commodities

The number of commodities in the model, L, is finite and is based on the idea that continuous refinement in the categorization of commodities leads to a situation where no utility increasing reallocations can be reached by making the categorization finer. This critical set of commodities is referred to as Arrow-Debreu commodities.

2. Consumers

There are H consumers in the model, each of whom can make consumption plans which lie in a subset of the L-dimensional commodity space, which is closed and bounded from below. Each consumer has well defined, complete, continuous and transitive preferences over all possible consumption plans. Furthermore, it is assumed that consumers are non-satiated and their preferences are convex. Convexity of preferences implies that consumption bundles are infinitely divisible and that mixtures are at least as good as extremes.

3. Endowments

Each consumer in the model is characterized by his/her initial endowments of commodities and ownership shares of the firms.

4. Firms

There are J firms characterized by their initial distribution of owners and a feasible set of production plans with negative components denoting inputs and positive components denoting outputs in the L-dimensional space. For each firm the production possibility set is a closed, convex set containing the origin. The convexity assumption rules out e.g. indivisibilities and increasing returns to scale in production. Furthermore, it is assumed that the aggregate set of production possibilities is compact, which means that the level of production is always bounded. The disposal of commodities is assumed to be free and they are not distinguished according to which company produces them or who consumes them. The economy is assumed to be irreducible, which means that for any two agents a and b the initial endowment of a is positive in some commodity which b could use to make himself better off.

After listing the assumptions, the Arrow-Debreu equilibrium can be defined as an array of prices, quantities consumed and quantities produced that satisfy the following conditions:

(a) For each household the designated consumption bundle maximizes utility among the available bundles.

(b) For each firm the designated bundle of production maximizes profits among all technically possible bundles.

(c) For each commodity the total amount consumed does not exceed the sum of the initial endowment and the net quantity produced.

(d) For commodities for which the total consumption is strictly less than the sum of the total quantity initially available and the total quantity produced, the price is zero.

2. Properties of a general equilibrium

2.1 Existence and uniqueness of a general equilibrium

As was noted in the previous section, it is not always clear whether a general equilibrium system of equations has a unique solution or a solution at all. A numerical approach to this problem would be to specify a computable version of the model and use iterative methods to find a possible solution. Another, more general, approach is to study the existence analytically.

A common formulation for the analytical approach has been to define the minimum conditions under which the existence of a solution can be proved. Generally, these conditions set some limits to the specification of the consumers' tastes and production technology of the models. In addition to existence, interest is paid to the uniqueness of the possible solution.

The earliest approach to the existence problem - used e.g. by Walras - was to count the numbers of equations and unknowns. It was stated that a unique solution exists if the numbers are equal. This method, however, only works when there are no inconsistencies or functional dependencies among the equations of the system. Furthermore, even if a unique solution exists, it is not guaranteed to be meaningful in the sense that quantities purchased would be non-negative.

A more advanced approach to the problem is to make use of so-called excess demand functions, which are obtained by subtracting the supply of a commodity or factor from the demand for the same commodity or factor.

$$E_i(p) = D_i(p) - S_i(p) \qquad (2.1)$$

The equilibrium for an n-commodity economy is then defined as a set of non-negative prices p* such that all excess demands are non-positive and if the excess demand for a certain commodity or factor is negative, then its price has to be zero.

$$\begin{aligned} &p^* \geq 0 \\ &E_i(p^*) \leq 0 \text{ , for all } i \\ &\text{if } E_i(p^*) < 0 \text{ then } p_i^* = 0 \end{aligned} \qquad (2.2)$$

The existence of a solution to such a system can be proved, if the excess demand functions satisfy the following conditions: First, the excess demand functions have to be single valued and continuous. Second, they have to be bounded from below:

$$E(p) \geq b \text{ , for all } p \qquad (2.3)$$

where b is a column vector with finite components and E(p) a column vector with single excess demand functions as components. Boundedness means that the supply of any commodity or factor is always limited. Third, the excess demand functions have to be homogenous of degree zero in all prices:

$$E(ap) = E(p) \text{ , for all } a > 0 \qquad (2.4)$$

i.e. only relative prices matter in the model. Finally, the excess demand functions have to satisfy Walras' law, which states that the market value of demand equals the market value of supply at all non-negative prices:

$$pE(p) = 0 \ , \ \text{for all } p \geq 0 \qquad (2.5)$$

Under these assumptions the existence of an equilibrium solution can be proved by Brouwer's fixed point theorem. According to the homogeneity assumption, prices can be normalized to sum to unity. Thus the price vector lies in an n-dimensional unit simplex, S:

$$S = \{ p: p_1+..+p_n = 1 \ , \ p \geq 0 \} \qquad (2.6)$$

If we now define a transformation of prices from p to p' as follows:

$$\begin{aligned} p_i' &= p_i \text{ if } E_i(p) = 0 \\ p_i' &= p_i + d \text{ if } E_i(p) > 0 \\ p_i' &= \max \{ 0, p_i - d \} \text{ if } E_i(p) < 0 \qquad (2.7) \end{aligned}$$

the new prices p' will also belong to S. The continuity and boundedness assumptions ensure that this transformation is a continuous mapping from a non-empty compact convex set into itself. By the Brouwer theorem, there exists a fixed point p# for which p' = p, i.e. the transformation leaves all prices p_i unaltered. This in turn implicates that all excess demands must be equal to zero and thus p# must be the set of equilibrium prices p*.

For the existing solution to be unique, an extra assumption is required. The aggregate demand functions have to satisfy the weak axiom of revealed preference, which basically states that aggregate excess demand functions behave as if they were functions for a single individual. This assumption can be justified if income effects are not too large in the aggregate. An alternative condition for uniqueness is that all commodities are gross substitutes, which means that an increase in the price of any good brings about an increase in the excess demand for any other good (holding other prices constant) (see e.g. Varian, 1984).

2.2 Stability of the equilibrium

Once the existence of an equilibrium has been proved, there is still the problem of actually attaining it. To be able to answer this question we need to define the way prices adjust over time. The resulting system is called stable if the time path of prices converges to an equilibrium.

If it is assumed that prices adjust proportionally to the excess demands, the following dynamic path emerges:

$$dp_i/dt = k_i E_i(p) \qquad (2.8)$$

where k_i can be interpreted as the speed of adjustment, which may differ between commodities. Examples can be given that show that this dynamic system is not necessarily stable even if the equilibrium exists.

Three alternative sufficient conditions for stability of the system have nevertheless been stated. The system is stable if (1) all the commodities are gross subtitutes or (2) the market satisfies the weak axiom of revealed preference or (3) the Jacobian of the excess demand functions has a dominant diagonal, all elements of which are negative. The third condition states that stability holds when the excess demand for a commodity is much more affected by a change in its own price than by any other price changes.

Another approach to the stability problem is to specify alternative adjustment processes for prices instead of equation (2.8) and examine their stability conditions that may be less restrictive. For example, adjustment processes that allow transactions to take place at non-equilibrium prices have been analyzed in the literature on "non-tatonnement" processes (e.g. Hahn and Negishi, 1962).

2.3 Optimality of the equilibrium

Optimality in a general equilibrium context normally refers to Pareto optimality. An allocation is said to be optimal if it is feasible and the utility of one consumer cannot be increased without making someone else worse off. There are two theorems linking Pareto optimality and the Walrasian general equilibrium:

(1) Any Walrasian equilibrium is Pareto optimal.

(2) Any Pareto optimal allocation is a Walrasian equilibrium corresponding to some distribution of initial endowments among individuals.

So if a general equilibrium exists and can be found, we know that the corresponding allocation is Pareto optimal (1). On the other hand, we can reach any Pareto optimal allocation through proper distribution of endowments, provided that the conditions for existence and stability of the equilibrium are satisfied (2).

An important exemption from Pareto efficiency is a case where externalities occur. Externalities cause both theorems to lose their validity (see e.g. Kreps, 1990).

2.4 Comparative statics

Parallel to partial equilibrium analysis, in the general equilibrium context one may want to examine the impact of exogenous changes on equilibrium prices and quantities. Unfortunately, the answer to this question is more problematic in multidimensional general equilibrium models and only one general theoretical result exists (Morishima, 1960). If one's model satisfies certain conditions, comparative statics can be applied locally in some environment of a given equilibrium.

In computable specifications that are introduced in the next section, the calculus-related problems of comparative statics are overcome by using numerical methods.

3. Computable specifications (CGE-models)

3.1 Why computable models ?

Besides the theoretical interest paid to general equilibrium models, a strong tradition of numerical applications exists. These numerical models are referred to as computable (CGE) or applied general equilibrium models (AGE). The fast development of computers together with some improvements in suitable solution algorithms started the wide application of CGE-models in the late 70's.

The basic reason for the use of numerical models is the mathematical complexity of solving even the simplest general equilibrium models analytically. Numerical solutions can be found much easier with the aid of computers. A particular attraction of computable models is that the assumption of only infinitesimal changes necessary for analytical comparative statics can be relaxed. In computable models, equilibrium solutions for alternative exogenous parameters are calculated and the results are then compared.

3.2 Solving a non-linear system of equations

From a mathematical point of view a CGE-model is equivalent to a set of simultaneous, typically non-linear, equations. In terms of excess demand functions, the system of equations can be written as follows:

$$E_i(p) = 0 \qquad \text{for all } i=1,\dots,m \qquad (3.1)$$

where E_i is the excess demand in market i, p the equilibrium price vector and m the number of markets in the model. The functional form of E_i depends on the formulation of demand and supply functions in different markets. Usually E_i's are non-linear which complicates the solution procedure.

As it was noted in the last section, it can be shown that under some restrictions a unique solution exists for such a system. The mathematical methods of finding such a solution are based on iterative procedures, where the outcome is calculated for some initial guesses for prices and guesses are then revised according to some rule. This goes on until the outcome satisfies condition (3.1) within some given tolerance.

The first algorithm designed to solve CGE-models was developed by Scarf in the late 60's. Scarf's algorithm utilizes the fixed point theorem which was earlier used to prove the existence of the solution. The algorithm is guaranteed to converge, but not very effective from a computational point of view.

Later on some more effective solution methods have been developed. Some of them stress the speed of convergence over the absolute guarantee for finding a solution. Solution algorithms are discussed among others by Scarf and Shoven, 1984.

3.3 Data and parametrization

In order to apply a CGE-model to an empirical case, we need a data set corresponding to the structure of the model. The availability of suitable data is often poor and some adjustments become necessary. In addition to the variables of the model, data is needed for parameter estimates. (Related to this section, a stepwise procedure for constructing a CGE-model is drafted in appendix 1.)

For a static CGE-model a consistent data set over one time period (e.g. a year) is needed. This set includes data of different aspects of the model like production activities, consumption, foreign trade, taxes and government expenditure. Usually, the data set has to be collected from different sources which may not be consistent in terms of time period and classification. Even the aggregates of the same time period may differ from source to source. In such situations one has to rely on the source considered most reliable and fit the rest of the data to be compatible.

In general, the level of aggregation and classification of the data sources differ from those of the model. Therefore some final aggregation and manipulation may be necessary before the data set can be used as an input for the model.

Parametrization of the model is usally done by fitting the model to the base year data to define the parameter values. This procedure is called "calibration" and the parameters thus determined are said to be "shift and share" parameters. Calibration makes parametrization totaly deterministic, which is one of the most criticized features of CGE-models.

Many models incorporate functional forms (like CES-functions) whose parameters cannot be determined solely by fitting the base year data. In this case, some exogenous parameter estimates are required as well. The values of these exogenous parameters, which are most often elasticities, are usually chosen among available estimates in the literature. Some efforts to estimate a complete CGE-model econometrically have been made, but the large number of parameters involved makes this approach problematic (see e.g. Jorgenson, 1982).

An important step in the CGE-modeling procedure is the so-called sensitivity analysis, where the sensitivity of the results to changes in exogenous parameters is examined by running simulations with different values of exogenous parameters. The model is considered to be acceptable if small changes in parameters do not cause drastic changes in the results.

4. Characteristics and limitations of CGE-models

4.1 General features of CGEs

A characteristic feature for CGE-models is their sound microeconomic foundation in the neoclassical optimization behaviour of the agents. The model integrates the behaviour of single agents in a systematic way. Because of the complexity of the models, the results are often unpredictable. However, no theoretical surprises can occur as long as the model is correctly specified.

As it was noted in section 2, the existence of a solution requires the demand and supply functions of a CGE-model to be homogenous of degree zero in prices. The homogeneity implies that for each solution point there is a corresponding array of multiple solution points where all the prices are multiplied by some constant. This fact generates the need for a numeraire i.e. fixing one price (normally equal to unity) and expressing other prices in terms of the numeraire. Instead of a single commodity, a bundle of goods can be chosen as a numeraire.

Also, the excess demand functions of a CGE-model have to satisfy Walras' law (2.5). This gives the models the following property: if all agents satisfy their budget constraints and all but one of the markets are in equilibrium, then also the last one has to be in equilibrium. (In a mathematical sense, one of the equations thus becomes redundant, but after fixing one of the prices we still have an equal number of equations and unknowns.)

The requirement of theoretical consistency sets limits for the plausible functional forms used in the model. Also, the functions should be analytically tractable, i.e. demand and supply responses should be relatively easy to evaluate for any price vector. A commonly used form in the demand side is the Cobb-Douglas utility function (see section 1.2) which, however, has some serious restrictions concerning elasticities. Somewhat more flexible forms employed are CES- and LES-functions. In the supply side CES-functions are also common. A hierarchical CES is appropriate when more than two factors are involved.

A serious weakness of CGE-models is their lack of empirical validation, which is related to the problems in parametrization mentioned above. The calibration procedure actually presumes that the base year data set represents a general equilibrium which, of course, is rather unlikely in reality. The assumption of general equilibrium makes it impossible to check the validity of the model against time series data. It also emphasizes the importance of selecting the base year. Nevertheless, it may be reasonable to say that "CGE-models do not pretend to forecast reality but rather to indicate long-term tendencies around which the economy will fluctuate" (Borges,1986).

Because CGE-models are orientated to long-term simulations, they do not usually pay attention to adjustment processes. The traditional static CGE-models move from one equilibrium to another without specifying the transition path. This defect has made CGE-models less attractive for applications where the adjustment period is of particular interest. On the other hand, there is no inherent reason not to include some adjustment cost in CGE-models (Borges,1986).

The fundamental idea behind CGE-models is the assumption of simultaneous equilibria, which enables the use of numerous endogenous variables. This is why no real disequilibrium can occur in any market of the model. However, it is possible to fix the price for some particular market and let the quantity adjust instead. Applications of this approach are, for example, models with minimum wage and endogenous unemployment or models with an endogenous current account deficit and fixed exchange rate.

The progress in solution algorithms and computers have made it possible to build highly disaggregated CGE-models. Disaggregation enables the study of structural changes in the economy which often are larger than overall changes. This makes CGE-models a powerful tool in analysing things like distributional effects of tax policy changes.

A typical feature of CGE-models is the explicit presentation of utility functions that provide a measure for welfare. By calculating so-called Hicksian or equivalent variations one can translate utility changes into money metric changes that can then be compared to GDP or some other relevant quantities (Shoven and Whalley, 1984).

4.2 Dynamic specifications

Some specific fields of applications have given rise to the need for dynamics in CGE-models. Dynamic modeling becomes necessary once we want to know how adjustment to final equilibrium takes place. This is particulary interesting if there is reason to believe that the adjustment process is not symmetric in time but includes some short-term effects that are different from corresponding long-term effects. Also, a dynamic setting is appropriate if the problem at hand incorporates as a central element a stock variable that is changing over time.

Applications of dynamic CGE-models include natural resource management, investment promotion policies, foreign debt accumulation, life-cycle behaviour of consumers, intergenerational equity, government deficits and many others.

A simple definition for a dynamic model is that it incorporates at least one equation that links variables at different points in time. An example of such an equation could be one that defines next year's capital stock as a sum of this year's stock and net investment. A more complex specification may include intertemporal optimization. In this case one also needs to define how expectations of the future are formed.

Technically, a dynamic specification corresponds to increasing the number of variables and equations in the model. Thus the dimensions of the model tend to double as one time period is added. This makes dynamic models less pleasant to handle and may be one reason why dynamic CGE-models are still quite rare.

There are basically two alternative ways to proceed in solving a dynamic CGE-model. A straightforward method is to regard the same variable at different instants in time as a set of independent variables and make no difference between intratemporal and intertemporal equations. Then the whole model is solved simultaneously. An alternative way is to solve the model recursively, period by period, and fit the temporal equilibria together to satisfy intertemporal equations. The latter approach is appropriate if the number of intertemporal equations is small (see e.g. Codsi et al., 1991).

To be able to calculate the base case equilibrium of a dynamic CGE-model one needs to have a data set over the whole range of time periods covered by the model. This requirement causes troubles if the model covers time periods in the future, as often is the case. The way to handle this problem is to make guesses about the future path of the exogenous variables of the model.

There are different scenarios one can make about the future behaviour of the exogenous variables. The basic scenario is a steady state where exogenous variables are assumed to be constant over time and the starting point data is adjusted so that the equilibrium will be the same as long as the exogenous variables do not change. Then our base case equilibrium would consist of a string of temporal equilibria that all look alike. Also more complicated scenarios may be used: the exogenous variables may be assumed constant, but some endogenous state variables, like capital stock, are changing over time or alternatively, the exogenous variables are allowed to vary.

4.3 Comparison of CGE- and macroeconometric models

Applied models used to analyze macroeconomic policies can be roughly divided into the two categories of macroeconometric models (ME) and computable general equilibrium models (CGE).

Most macroeconometric models are based on the Keynesian view of the workings of an economy. That is, the prices are assumed to be sticky and quantity adjustment takes place. This leads to drastic changes in productivities with multiple effects on the rest of the model via prices. Moreover, ME-models do not satisfy Walras' law, which means they allow for disequilibria in the current account as well as in the public budget and labour market. As noted in the preceding sections, CGE-models are supply-driven and satisfy Walras' law. These fundamental differences cause the results of the two types of models to be unequal and even opposites in policy simulations.

An attempt to analyse the differences between the two model traditions is given by Capros et al. (1990). The authors construct two small-scale representative models, one from each tradition, and run policy simulations with a common database for both models. The authors admit that their models are compromises, but insist that the models are able to capture the central characteristics of the traditions they represent.

In their policy simulations Capros et al. get very different and in many cases opposite results from the two models of the same economy. They conclude that differences are largely due to different mechanisms for supply and demand adjustment. In most cases the CGE-model can be interpreted to describe the long-term equilibrium whereas the ME-model captures the short-term adjustment process.

An example of the simulations by Capros et al. is the case of a one per cent increase in foreign demand of a small open economy. The impacts of this change in the ME-model are clearly positive due to a rise in exports and increased productivity. This in turn launches a deflationary and growth cycle. In the CGE-model supply constraints bind the economy and increased foreign demand leads to an inflationary cycle. The "investment equals saving" - closure rule chosen for the CGE-model causes increased exports to prevent investment, which has negative effects on growth.

Despite the contradictory results, Capros et al. conclude that both types of models could be succesfully used in a combined manner for policy analysis. When an ME-model is used as a reference, a CGE-model can provide insight into the long-run equilibrium and give recommendations for additional macroeconomic measures. A CGE-model should be used as a reference in cases where normative power is needed. Then, an ME-model can be used to analyze short-term disequilibria pressures that should be taken into account.

An important difference between the two types of models arises from the way they are estimated. The equations of ME-models are usually econometrically estimated from time series data. Empirical validity of single equations and the model can thus be tested. There are no limits for feasible functional forms unlike in CGE-models (see section 4.2).

5. Applications of CGE-models

5.1 Main fields of application

Traditional applications of CGE-models are dealing with tax policy evaluation and international trade. The tradition goes back to the earlier analytical general equilibrium models of Johnson and Harberger in the 50's and 60's.

The emphasis of CGE-studies has been on examining the efficiency and distributional impacts of certain policy proposals, e.g. implementation of a new tax regime or abolition of import tariffs.

In their pioneering work Shoven and Whalley (1972) study capital tax coincidence and integration of capital income taxation with personal income taxation in a static CGE-model. Later on, dynamic models have become increasingly popular in tax policy evaluation. A survey of tax models is given in Shoven and Whalley (1984) and of dynamic specifications in Pereira and Shoven (1988). CGE tax models are discussed in more detail in section 5.2.

CGE trade models are divided into single and multicountry models. The former tradition examines how changes in foreign trade and payments affect an individual economy. The latter is interested in global issues such as effects of free trade areas on member economies.

One of the standard results of CGE trade models is that the gains from trade liberalization are relatively small in terms of GDP growth. The terms-of-trade effects can, on the contrary, be dramatic. Harris (1984) has shown that allowing for economies of scale and imperfect competition may increase the welfare gains from trade liberalization. A survey of trade models is given in Shoven and Whalley (1984).

A third class of applications are so-called development policy models that were originally linked to World Bank advice projects in the Third World countries. These models are somewhat less theory oriented and are designed to support preparation of government policy decisions. In this branch of study the social accounting matrix (SAM) was introduced as a basis for modelling procedures. A survey of development policy models is given in Dervis, De Melo and Robinson (1982).

Another important field of application is energy policy which by its nature calls for a general equilibrium framework. In energy policy models the emphasis is on the assumed substitutability between energy and labour and complementarity between energy and capital. Pioneering work in this field has been done by Dale Jorgenson, whose writings have also contributed to the use of more sophisticated functional forms and econometric estimation of the parameters in the CGE-context. For a short survey and bibliography, see Borges, 1986.

Closely related to energy policy models are environmental models which deal with the impacts of antipollution policies. The results are usually reported with respect to GDP growth and sectoral allocation of resources in the economy, so the welfare measure does not incorporate environmental quality. A recent application in this field is the GREEN multicountry model developed by the OECD (Burniaux et al., 1991).

Besides models that can be classified in one of these categories, there is a number of so-called multipurpose models that may be used to analyse several types of problems. However, these "whole economy" models often have to be adjusted for a particular problem by using only a part of the model. The question has been raised whether CGE-models should be kept as small as possible and issue-specific or go for larger models. One answer to the question is to start with an existing problem and build a model that incorporates all the necessary aspects for this particular case. One of the most famous multipurpose model is the ORANI model of the Australian economy (see e.g. Adams, Dixon and Parmenter, 1991).

5.2 CGE-models and public economics

There are several aspects that make CGE-modelling interesting from the point of view of public economics and policy planning. Most importantly, CGE-models provide a tool to evaluate the effects of policy changes on resource allocation and income distribution which are not well covered by macroeconometric models.

A typical field of application in the public sector is tax policy evaluation, where CGE-models allow incorporation of several taxes simultaneously in the analysis. Distortions caused by taxes in other markets will thus be taken into account and the results may considerably differ from those of partial analysis. Examples of cases where CGE-modelling has been used successfully are integration of personal and corporate taxation, introduction of V.A.T. and many other issues where the interesting question is who bears the tax burden and what will be the effect on sectoral allocation of resources after the change.

The main difficulty in modeling tax systems lies in the interdependency of taxes, demands and supplies. Tax revenues are determined by the levels of final and factor demand which in turn depend on taxes. A way to deal with this problem is to introduce an equilibrium condition for taxes which usually requires tax revenue to be equal with transfer payments in the model. In other words, the government budget is balanced.

Another difficulty arises when one tries to "translate" the diverse tax systems of real economies into the language of the model. It is not always straightforward how a corporate tax, for example , should be modelled. Also, it may be difficult to select the model-equivalent values for tax rates. One way to proceed is to calculate the effective average rate from the data by dividing the tax revenue by the tax base. Theoretically, marginal rates would be more appropriate, but the calculation of marginal rates may get rather cumbersome.

The earlier CGE-tax models were static in nature and did not include transactions over time. This, of course, was a serious defect for the analysis of tax policies that obviously affect savings and investment decisions and are likely to involve adjustment costs. In response to this fact a body of dynamic tax models were developed in the mid 80's (see section 4.2).

In addition to tax models CGE-models have been applied to many other fields that are of interest to economic policy planning. Environmental, agricultural and trade policy questions typically involve large scale government intervention, whose impacts spill over many sectors in the economy. This intervention may take the form of emission rationing, import licensing, subsidies etc.

5.3 Modeling the public sector in CGE-models

A further aspect in CGE-models that is of interest from the point of view of public economics is the modelling of government behaviour. Traditionally, government plays a rather passive role in the CGE tax models. The behaviour of government is restricted to satisfy the annual budget constraint, that is the government income transfers are set to equal the taxes collected.

Also, government may produce goods and thus hire factors. The income or profits that government receives from its production activities is incorporated in the budget constraint. The volume of government production is often defined as a fixed share of total production. The technology that government uses may be assumed to be similar or less effective than the one used by the private sector.

The level of government expenditure is usually adapted to satisfy the budget constraint, i.e. to equal the tax revenues plus profits from public production (which may be negative). The distribution of government expenditure across different categories (consumption, investment, transfer payments) is then given exogenously (see e.g. Jorgenson and Kun-Young Yun, 1986). Alternatively, the level of government expenditure could be exogenous and the budget deficit would adjust accordingly. This would, of course, be just another way to express the annual budget constraint of the government.

A more realistic analysis of government deficits and public debt requires a dynamic model. In a dynamic setting "genuine" annual deficits and surpluses may occur. The time path of government expenditure is in most models exogenously determined, for example bound to the population growth rate (Auerbach & Kotlikoff, 1987). Government then faces an intertemporal budget constraint: today's deficits have to be paid by increased taxes in the future. Auerbach and Kotlikoff conclude that public deficits cause major crowding out effects where private investment is displaced by deficit financing.

In addition to these constraint-based approaches a few attempts to model government as an optimizing agent have been made. Given the tax structure, government can be modelled as to maximizing a social welfare function subject to an intertemporal budget constraint. Another improvement suggested, but not widely implemented, is the inclusion of public expenditure in the consumers' utility function. These refinements in the modelling of government behaviour might change the welfare effects of policy changes such as public deficit growth caused by tax cuts.

6. References

Adams, P.D., Dixon, P.D. and Parmenter B.R. (1991): Prospects for the Australian Economy to 2001-02: ORANI-F projections...; Paper presented to the 5th IIASA Task Force Meeting on Applied General Equilibrium Modeling, Laxenburg, August 27-29.

Arrow, K.J. and Debreu G. (1954): Existence of an Equilibrium for a Competitive Economy; Econometrica 22, 265-290.

Auerbach, A. and Kotlikoff, L. (1987): Dynamic Fiscal Policy; Cambridge, Cambridge Univ. Press.

Ballard, C.L., Shoven, J.B. and Whalley J. (1982): The Welfare Costs of Distortions in the U.S. Tax System: A General Equilibrium Approach; NBER Working Paper No. 1043, December.

Balasko, Y. (1988): Foundations of the Theory of General Equilibrium, Boston, Academic Press.

Barro, R.J. (1990): Government Spending in a Simple Model of Endogenous Growth; Journal of Political Economy, Part 2, 98(5), 103-125.

Borges, A.M. (1986): Applied General Equilibrium Models; OECD Economic Studies No.7, Autumn.

Burniaux J.M., Martin, J.P., Nicoletti, G. and Oliveira-Martins, J. (1991): GREEN - A Multi-Region Dynamic General Equilibrium Model for Quantifying the Costs of Curbing CO2-emissions; OECD Department of Economics and Statistics Working Papers, No. 89.

Capros, P., Karadeloglou, P. and Mentzas, G. (1990): An Empirical Assessment of Macroeconometric and CGE Approaches in Policy Modeling; Journal of Policy Modeling, 12(3), 557-585.

Codsi, G., Pearson, K.R. and Wilcoxen, P.J. (1991): General Purpose Software for Intertemporal Modeling; Paper presented to the 5th IIASA Task Force Meeting on Applied General Equilibrium Modeling, Laxenburg, August 27-29.

Dervis, K., de Melo, J. and Robinson, S. (1982): A General Equilibrium Models for Development Policy; New York, Cambridge Univ. Press.

Devarajan, S., Lewis, J.D. and Robinson S. (1991): From Stylized to Applied Models: Building Multisector CGE Models for Policy Analysis; Paper presented to the 5th IIASA Task Force Meeting on Applied General Equilibrium Modeling, Laxenburg, August 27-29.

Dinwiddy, C.L. and Teal, F.J. (1988): The Two-Sector General Equilibrium Model: A New Approach; Oxford, Philip Allan.

Hahn, F. and Negishi, T. (1962): A Theorem on Non-tatonnement Stability; Econometrica 30, 463-469.

Harris, R. (1984): Applied General Equilibrium Analysis of Small Open Economies with Scale Economies and Imperfect Competition; American Economic Review, Vol. 74, 1016-1032.

Jorgenson D.W. (1982): An Econometric Approach to General Equilibrium Analysis; Hazewinkel and Rinnooy Kan (eds.): Current developments in the interface...Dordrecht, D.Reidel,1982.

Jorgenson, D.W. and Kun-Young Yun (1986): The Efficiency of Capital Aloocation; Scandinavian Journal of Economics, Vol. 88, 85-107.

Kreps, D.M. (1990): A Course in Microeconomic Theory; Hertfordshire, Harvester Wheatsheaf.

Manne, A. (ed.) (1985): Economic Equilibrium: Model Formulation and Solution; Mathematical Programming Study 23, Amsterdam, North-Holland.

Mas Colell, A. (1985): The Theory of General Equilibrium: A Differentiable Approach; Cambridge, Cambridge Univ. Press.

Morishima, M. (1960): On the Three Hicksian Laws of Comparative Statics; Review of Economic Studies 27, 195-201.

Pereira, M. (1988): DAGEM: A Dynamic Applied General Equilibrium Model for Tax Policy Evaluation; Univ. of California San Diego Working Papers 8-17.

Pereira, M. and Shoven, J.B. (1988): Survey of Dynamic Computational General Equilibrium Models for Tax Policy Evaluation; Journal of Policy Modeling, 10(3), 401-436.

Piggot, J. and Whalley, J. (eds.) (1985): New Developments in Applied General Equilibrium Analysis; New York, Cambridge Univ. Press.

Scarf, H.E. (1981): The Computation of Equilibrium Prices; Arrow and Intrilligator (eds.) Handbook of Mathematical Economics, Vol. 2, 1007-1061.

Scarf, H.E. and Shoven J.B. (eds.) (1984): Applied General Equilibrium Analysis, New York, Cambridge Univ. Press.

Shoven, J.B. and Whalley J. (1972): A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S.; Journal of Public Economics 1.

Shoven, J.B. and Whalley J. (1984): Applied General Equilibrium Models of Taxation and International Trade: An Introduction...; Journal of Economic Literature, Vol.XXII, 1007-1051.

Varian, H.R. (1984): Microeconomic Analysis; 2nd ed., New York, Norton.

APPENDIX 1

The CGE-modelling procedure

1. Specify model structure and functional forms.
2. Define exogenous parameter values.
3. Create mutually consistent base year data set.
4. Define endogenous "shift and share" parameters through calibration: the model should reproduce the base year data set.
5. Fix parameter values to "base year equilibrium."
6. Specify in the model the problem you want to examine i.e. change the value of some exogenous variable(s).
7. Compute new equilibrium values for endogenous variables.
8. Compare new equilibrium with base year equilibrium and draw conclusions.
9. Do experiments with different exogenous parameter values in step 2 to determine the sensitivity of the results.