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Cost-Effective Sulphur Emission Reduction Under Uncertainty

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Abstract

The problem of reducing SO₂ emissions in Europe is considered. The costs of reduction are assumed to be uncertain and are modeled by a set of possible scenarios. A mean-variance model of the problem is formulated and a specialized computational procedure is developed. The approach is applied to the transboundary air pollution model with real-world data.

Keywords: Environment, Probabilistic programming, Interior point methods.

Cost-Effective Sulphur Emission Reduction Under Uncertainty

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1 Introduction

Reducing the pollution in the environment has become one of the challenges of the present time in industrial countries, and especially in Europe. It is commonly agreed that action should be undertaken to stop the growth of emissions and eventually achieve a substantial reduction of depositions.

One of the issues that attracts the attention of researchers and decision-makers is the emission of sulphur dioxide to the atmosphere, which has a damaging effect on the environment through acid rains. Clearly, this is an international problem because air pollution can move across the borders and damage the environment in other countries. Therefore it is necessary to look for a common European solution for this problem.

There are many ways to approach such problems. One would be to reduce the emissions uniformly over the continent (for example, by 30%). This, however, may prove prohibitively expensive. On the other hand, it might be possible to achieve the same reduction of depositions by a non-uniform reduction of emissions in a more cost-effective way.

To investigate this possibility, the Regional Acidification Information and Simulation (RAINS) model has been developed at the International Institute for Applied Systems Analysis (see [1]). This model simulates the flow of acidifying pollutants (sulphur and nitrogen species) from source regions in Europe to environmental receptors. Comparable models exist for the USA and other regions [9, 11, 12, 20]. The current RAINS model (version 6.1) used in this context covers 38 source regions in Europe: 26 countries, 7 regions in the former USSR, and 5 sea regions (ship emissions). Analysis of deposition is performed for 547 land-based receptor sites with a regular grid size of 150×150 km. The United Nations' Economic Commission for Europe has used the model as its major tool to negotiate on a new second sulphur protocol which was signed in June 1994 (see [5]). Whereas the first protocol stipulates a 30% uniform cut back in emissions, the new protocol requires country specific reductions. In addition to RAINS, two other Europe-wide models were used in the negotiations: CASM (Coordinated Abatement Strategy Model), developed by the Stockholm Environment Institute in York, and ASAM (Abatement Strategies Model), developed by Derwent of the UK Department of the Environment (this is now being run by Imperial College in London). The CASM model has a more detailed assessment of pollution control costs than the RAINS model since it uses data on

individual point sources rather than sectoral data only. The ASAM model uses RAINS cost data. The ASAM model has a more detailed, grid-to-grid, atmospheric transport matrix whereas both RAINS and CASM employ country-to-grid matrices. In contrast to CASM and RAINS, the ASAM model does not use or possess an optimization routine, but instead uses a ranking procedure [8]. The extent of the emission reductions in the new protocol, however, was based on RAINS model runs that searched for the cost minimum solution to meet the targets for the deposition of sulphur at each grid area in Europe. These targets were based on the notion that the difference between the deposition in 1990 and the so-called *critical sulphur loads*, which damage the environment, should be reduced by at least 60%.

The optimization module of RAINS formulates possible strategies to minimize the costs of achieving deposition targets at certain receptors as a deterministic linear optimization problem that can be solved by linear programming techniques [4]. The cost-effective solution requires that the total costs of emission reductions be minimized, subject to the constraint that the desired depositions are met at every receptor.

However, there are many uncertainties in the problem due to inaccurate or missing data, unknown future energy policies for the countries, etc. The purpose of this paper is to formalize the problem of reducing emissions in the presence of uncertainty, to develop a specialized solution procedure and to apply it to the real-world data available so far.

In Section 2, we recall the deterministic formulation of the problem following [5]. In Section 3, we develop a mean-variance model for the problem under uncertainty. The uncertainty is modeled by a number of scenarios of future costs of emission reduction.

Section 4 is devoted to the development of a specialized computational procedure for solving the problem under consideration. The algorithm is a version of a primal-dual logarithmic barrier method.

In section 5, we report computational results obtained for six different scenarios proposed by the modelers. In the last section, we present our conclusions and give propositions for the future work.

2 The deterministic problem

The RAINS model contains a sub-module to assess the potential and the costs for alternative emission abatement technologies. The evaluation is based on internationally reported performances and cost data of control devices [4]. Cost estimates for specific technologies are extrapolated by the model to reflect country-specific conditions such as operating hours, boiler size, and fuel price. In the current version of the model the cost evaluation of the emission reduction techniques is limited to the most relevant measures that have no impact on the underlying pattern of energy use. For the time being, energy conservation and fuel substitution are excluded from the analysis. The following technical options are implemented:

- use of low sulphur fuels and fuel desulphurization;
- desulphurization of flue gases during or after combustion.

For the optimization mode, RAINS creates ‘national cost functions’ for controlling emissions. National circumstances (such as sulphur content and operating hours) result in variations in the costs for applying the same technology in different countries in Europe. Another difference is the structural variations of energy systems, especially in the amount and structure of energy use, which determines the potential for application of individual control options. The national cost functions describe the lowest costs for achieving various emission levels by applying the cost optimal combination of abatement options. These national cost functions depend on future energy use. The RAINS model makes forecasts for energy use and national cost functions for the year 1995, 2000, 2005 and 2010. The costs are expressed in constant prices of 1990 for that specific year. In this paper, we will make use of the (static) cost functions for the year 2010. The cost functions are convex and piece-wise linear, reflecting the fact that for additional reductions, a new, more expensive technology will have to be applied.

Let us formalize the problem. There are K countries (regions) in our model. For each country (region) k we denote by x^k the level of emissions, which will be our decision variable. The cost of reducing emissions to levels $x^k, k = 1, \dots, K$, is expressed by the functions $f_k(x^k)$. They are assumed to be convex and piece-wise linear. Our objective is to minimize the total cost $\sum_{k=1}^K f_k(x^k)$ subject to some environmental constraints and additional policy restrictions.

The environmental constraints are described by a vector $b \in \mathcal{R}^m$ of maximal grid depositions, as illustrated in Figure 1. The dependence between emissions $x = (x^1, \dots, x^K)$ and depositions $y = (y_1, \dots, y_m)$ is assumed to be linear: $y = Tx$, where T is an $m \times K$ matrix.

These source-receptor transfer coefficients, which relate (country) emissions in the diffusion model to deposition at receptor points (for each grid), are based on the acid deposition model developed within the European Monitoring and Evaluation Program (EMEP) [18]. The model includes ten different chemical components in the air, three of which are man-made: SO_x , NO_x and NH_3 . Input data for the model consist of emissions for the three pollutants and meteorological data such as precipitation, wind speed and temperature. Meteorological data are taken from a weather prediction model and direct observations. As far as possible, emission data employed are official data submitted by the different countries. The model calculates transboundary fluxes of oxidized sulphur and nitrogen as well as reduced nitrogen (ammonia and its product ammonium). For this paper, EMEP model results that have been applied, reflect the meteorological average of the years 1985, 1987 to 1990.

The whole optimization problem can be formulated as

$$\begin{aligned} & \min \sum_{k=1}^K f_k(x^k) & (1) \\ \text{subject to} & \quad Tx \leq b, \\ & \quad l \leq x \leq u. \end{aligned}$$

Vectors $l, u \in \mathcal{R}^m$ are policy constraints given by policy-makers.

Every function $f_k(x^k)$ is defined on an interval $[x_{j(k)}^k, x_0^k]$ which can be divided into subintervals $[x_j^k, x_{j-1}^k], j = 1, \dots, J(k)$, such that $f_k(x^k)$ is linear in each of them (note

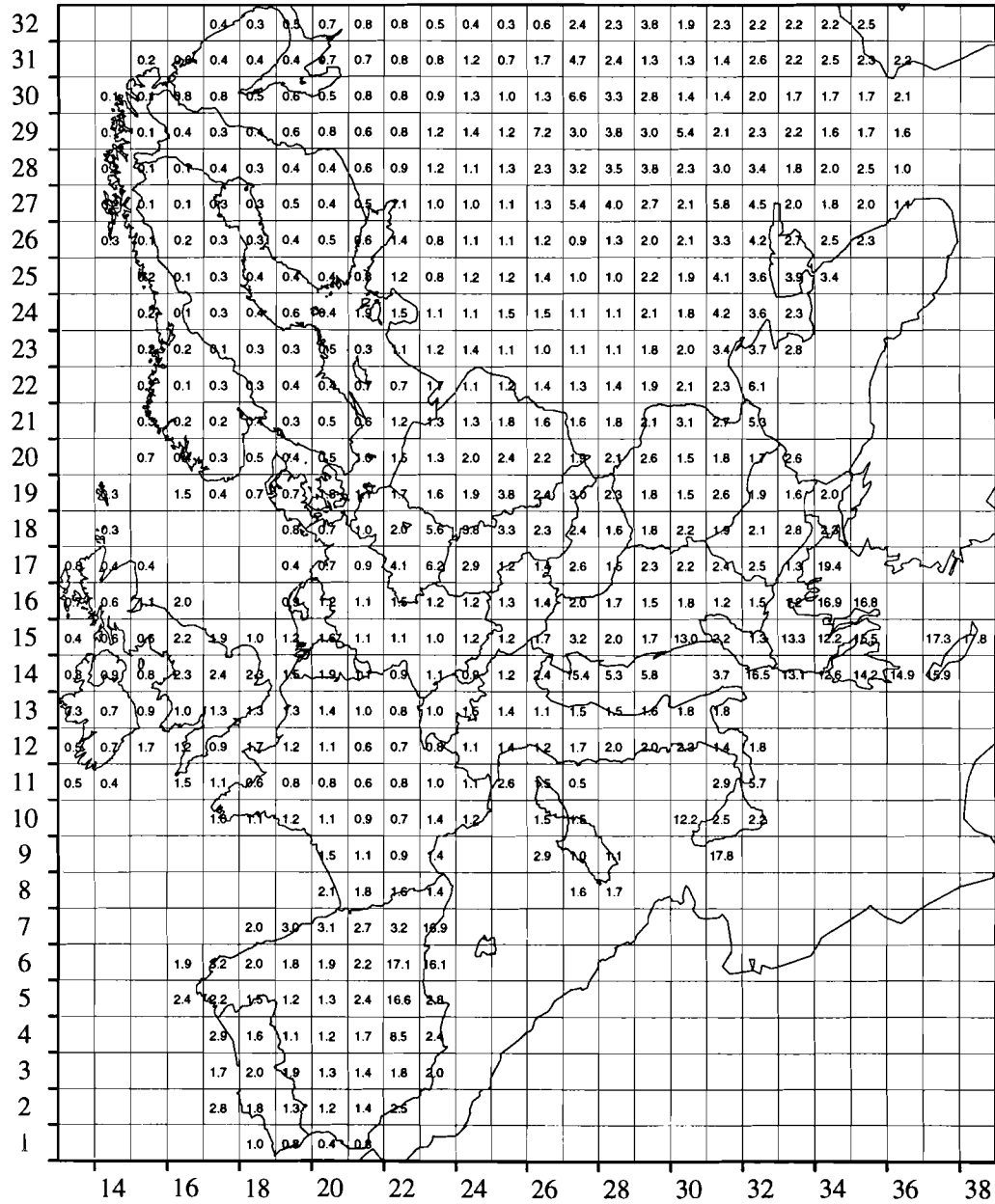


Figure 1: Target deposition levels [$\frac{\text{g of sulphur}}{\text{m}^2 \text{ year}}$].

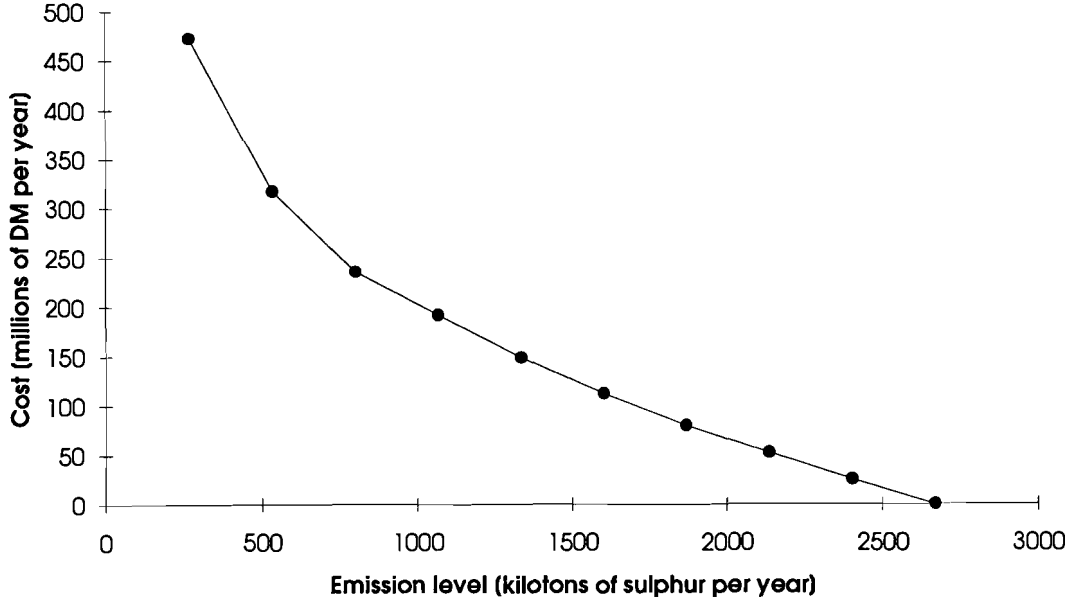


Figure 2: The cost curve for Russia; the annual cost of reducing emissions as the function of the emission rate.

that the break points $x_j^k, j = 0, \dots, J(k)$ are numbered from the largest to the smallest one). The unit cost of emission reduction in the j -th interval will be denoted by $c_{j,k}$. Formally

$$c_{j,k} = \frac{f_k(x_j^k) - f_k(x_{j-1}^k)}{x_{j-1}^k - x_j^k}.$$

By the convexity of f_k ,

$$c_{1,k} \leq c_{2,k} \leq \dots \leq c_{J(k),k}. \quad (2)$$

This is illustrated in Figure 2.

It is convenient to rewrite (1) as a linear problem by introducing (for each k) new variables $d_k(j), j = 1, \dots, J(k)$ in such a way that

$$0 \leq d_k(j) \leq \bar{d}_k(j),$$

where $\bar{d}_k(j) = x_{j-1}^k - x_j^k$, and

$$d_k(j) = \begin{cases} \bar{d}_k(j) & \text{if } x^k \leq x_j^k, \\ x_{j-1}^k - x^k & \text{if } x^k \in (x_j^k, x_{j-1}^k), \\ 0 & \text{if } x^k \geq x_{j-1}^k. \end{cases} \quad (3)$$

Then we can express emissions as

$$x^k = x_0^k - \sum_{j=1}^{J(k)} d_k(j). \quad (4)$$

We can interpret variables $d_k(j)$ as successive reductions of the emission x^k starting from the maximum level x_0^k and moving down through the break points of the function $f_k(x^k)$. Under (3), the total cost can be expressed as a linear function

$$\sum_{k=1}^K f_k(x^k) = \sum_{k=1}^K \sum_{j=1}^{J(k)} c_{j,k} d_k(j).$$

Owing to that, problem (1) can be reformulated as a linear programming problem

$$\begin{aligned} & \min \sum_{k=1}^K \sum_{j=1}^{J(k)} c_{j,k} d_k(j), & (5) \\ \text{subject to} & & \\ & Tx \leq b, & \\ & l \leq x \leq u, & \\ & x^k = x_0^k - \sum_{j=1}^{J(k)} d_k(j), \quad k = 1, \dots, K, & \\ & 0 \leq d_k(j) \leq \bar{d}_k(j), \quad j = 1, \dots, J(k), \quad k = 1, \dots, K. & \end{aligned}$$

It is interesting to observe that we need not include condition (3) explicitly to problem statement (5). Condition (2) with strict inequalities immediately implies that the solution of (5) must satisfy (3).

We can solve (5) by standard linear programming techniques.

3 The mean-variance model

Unfortunately, the costs of controlling the emission are not deterministic quantities. There are many possible scenarios of energy production, consumption, fuel characteristics and installed emission control measures. Therefore our problem is a decision problem with uncertainty.

There are many ways to formalize such decision problems. We can, for example, use the worst-case approach and require the decision to be the best for the worst possible conditions. This usually leads to very conservative and expensive solutions.

An approach that found many successful applications is to model uncertain quantities by random variables. Then we can use various concepts of the theory of probability to express our objectives and constraints. This leads to *stochastic programming models*.

In our case, only the costs are uncertain; they can be modeled by assuming that the unit costs $c_{j,k}$ are random (but still satisfy (2)). To be even more specific, we shall restrict our considerations to the case of finitely many *scenarios* $s = 1, \dots, S$. Each scenario s has some probability $p(s)$, such that $\sum_{s=1}^S p(s) = 1$, and is characterized by a collection of unit costs in subintervals

$$c_{1,k,s} \leq c_{2,k,s} \leq \dots \leq c_{J(k),k,s}, \quad k = 1, \dots, K.$$

Nevertheless, we still have many possibilities of expressing our objective. The simplest solution would be to minimize the expected cost

$$E = \sum_{s=1}^S p(s) \sum_{k=1}^K \sum_{j=1}^{J(k)} c_{j,k,s} d_k(j). \quad (6)$$

This is equivalent to solving the problem with one average scenario having unit costs

$$\bar{c}_{j,k} = \sum_{s=1}^S p(s) c_{j,k,s}.$$

A significant drawback associated with the expected value approach is that it essentially ignores uncertainty of the cost.

Another possibility would be to define a nonlinear utility function and to optimize its expected value. However, it is not clear how such a function should be defined in our case.

Therefore, we decide to use the mean-variance approach to our decision problem. With such an approach, the quality is measured by two outcomes: the mean value (6) and the (weighted) variance

$$V = \sum_{s=1}^S p(s) \sum_{k=1}^K w_k^2 \left[\sum_{j=1}^{J(k)} c_{j,k,s} d_k(j) - e_k \right]^2, \quad (7)$$

where

$$e_k = \sum_{s=1}^S p(s) \sum_{j=1}^{J(k)} c_{j,k,s} d_k(j) \quad (8)$$

denotes the expected value of the cost for the k -th country (region) and w_k 's are some weighting coefficients.

The variance will be used to measure the risk associated with a decision. The weighting coefficients w_k can be used to bring the variance components associated with different countries (regions) to some common measure. In particular, we could make w_k inversely proportional to the GDP of the k -th country (region), which would measure the risk relative to the economic strength of the region rather than in absolute terms.

Both outcomes are used to form a composite objective

$$G = E + \alpha V, \quad (9)$$

where $\alpha > 0$ is a user-defined parameter. The main idea of the mean-variance model is to replace the objective of (5) by the composite objective (9). The constraints remain unchanged.

By varying α one can generate a family of solutions with different trade-offs between the expected cost and the variance of the cost. This is so-called *efficient frontier* in a multiobjective interpretation of our problem, with objectives E and V .

Clearly, the mean-variance approach is only one of many possibilities to incorporate the risk component into the objective of the decision model. However, its simplicity and

clarity made it a successful tool to approach uncertainties of costs/profits, especially in financial planning problems.

We shall adapt the mean-variance approach to our case. We denote by

$$q_{k,s} = \sum_{j=1}^{J(k)} c_{j,k,s} d_k(j) \quad (10)$$

the cost of reduction at source k under scenario s . Then we can rewrite (6) and (7) as

$$E = \sum_{s=1}^S p(s) \sum_{k=1}^K q_{k,s},$$

$$V = \sum_{s=1}^S p(s) \sum_{k=1}^K [w_k(q_{k,s} - e_k)]^2.$$

The mean-variance problem can be now formulated as follows

$$\min \left[\sum_{s=1}^S p(s) \sum_{k=1}^K q_{k,s} + \alpha \sum_{s=1}^S p(s) \sum_{k=1}^K [w_k(q_{k,s} - e_k)]^2 \right] \quad (11)$$

subject to

$$Tx \leq b,$$

$$l \leq x \leq u,$$

$$x^k = x_0^k - \sum_{j=1}^{J(k)} d_k(j), \quad k = 1, \dots, K,$$

$$0 \leq d_k(j) \leq \bar{d}_k(j), \quad k = 1, \dots, K, \quad j = 1, \dots, J(k),$$

$$q_{k,s} = \sum_{j=1}^{J(k)} c_{j,k,s} d_k(j), \quad k = 1, \dots, K, \quad s = 1, \dots, S.$$

Let us observe that the definition (8) of e_k need not be repeated in the problem formulation, because the quadratic term in (11) is minimized by expected values:

$$e_k = \sum_{s=1}^S p(s) q_{k,s}, \quad k = 1, \dots, K.$$

To allow application of efficient computational techniques for solving quadratic problems, we shall transform (11) to a problem with non-negative variables and with a convex separable quadratic part of the objective (i.e. a weighted sum of squares of decision variables). First, we split $q_{k,s} - e_k$ into the positive and the negative parts, defining new variables $q_{k,s}^+$ and $q_{k,s}^-$ by

$$q_{k,s}^+ - q_{k,s}^- = q_{k,s} - e_k, \quad q_{k,s}^+ \geq 0, \quad q_{k,s}^- \geq 0. \quad (12)$$

The expected value and the variance now have the forms

$$E = \sum_{s=1}^S p(s) \sum_{k=1}^K (q_{k,s}^+ - q_{k,s}^- + e_k),$$

$$V = \sum_{s=1}^S p(s) \sum_{k=1}^K [w_k(q_{k,s}^+ - q_{k,s}^-)]^2.$$

Unfortunately, the quadratic part of the objective is not separable now. However, it can be proved (cf. [6]) that the solution of our problem does not change if we replace (V) by a separable function:

$$V' = \sum_{s=1}^S p(s) \sum_{k=1}^K w_k^2 [(q_{k,s}^+)^2 + (q_{k,s}^-)^2],$$

because at the solution at most one variable from each pair $q_{k,l}^+, q_{k,l}^-$ will be different from 0. Finally, we obtain the following problem:

$$\min \left[\sum_{s=1}^S p(s) \sum_{k=1}^K (q_{k,s}^+ - q_{k,s}^- + e_k) + \alpha \sum_{s=1}^S p(s) \sum_{k=1}^K w_k^2 [(q_{k,s}^+)^2 + (q_{k,s}^-)^2] \right] \quad (13)$$

subject to

$$\begin{aligned} Tx &\leq b, \\ l &\leq x \leq u, \\ x^k &= x_0^k - \sum_{j=1}^{J(k)} d_k(j), \quad k = 1, \dots, K, \\ q_{k,s}^+ - q_{k,s}^- + e_k &= \sum_{j=1}^{J(k)} c_{j,k,s} d_k(j), \quad k = 1, \dots, K, \quad s = 1, \dots, S, \\ 0 &\leq d_k(j) \leq \bar{d}_k(j), \quad k = 1, \dots, K, \quad j = 1, \dots, J(k), \\ e_k, q_{k,s}^+, q_{k,s}^- &\geq 0. \end{aligned}$$

It is worth noting that the definitions of the variance and of the expected value are implicit in this formulation. Indeed, assuming that the variables $d_k(j)$ are fixed, minimization with respect to $e_k, q_{k,s}^+$ and $q_{k,s}^-$ yields

$$\sum_{s=1}^S p(s) q_{k,s}^+ = \sum_{s=1}^S p(s) q_{k,s}^-, \quad k = 1, \dots, K, \quad (14)$$

and $q_{k,s}^+ q_{k,s}^- = 0$ for all k and s . Therefore, we need not incorporate equation (8) into problem's formulation. We shall call (13) the *model with two-sided risk penalty*.

Clearly, (13) is not the only possible way to incorporate the risk term into the objective. For example, instead of quadratic costs for $q_{k,s}^+$ and $q_{k,s}^-$ one can use linear costs as in [7]. We decided to use quadratic penalties, because individual magnitudes of deviations from the expectation, not their sum alone, matter in our case. The disadvantage of the linear penalty is that it does not distinguish between two solutions having equal sums of deviations, but non-equal distributions of them.

Alternatively, one might penalize only positive deviations $q_{k,s}^+$ in (13). Then, we have to explicitly incorporate a constraint defining expectations, such as (8) or (14) into the problem:

$$\min \left[\sum_{s=1}^S p(s) \sum_{k=1}^K (q_{k,s}^+ - q_{k,s}^- + e_k) + 2\alpha \sum_{s=1}^S p(s) \sum_{k=1}^K w_k^2 (q_{k,s}^+)^2 \right] \quad (15)$$

subject to

$$\begin{aligned} Tx &\leq b, \\ l &\leq x \leq u, \\ x^k &= x_0^k - \sum_{j=1}^{J(k)} d_k(j), \quad k = 1, \dots, K, \\ q_{k,s}^+ - q_{k,s}^- + e_k &= \sum_{j=1}^{J(k)} c_{j,k,s} d_k(j), \quad k = 1, \dots, K, \quad s = 1, \dots, S, \\ 0 &\leq d_k(j) \leq \bar{d}_k(j), \quad k = 1, \dots, K, \quad j = 1, \dots, J(k), \\ \sum_{s=1}^S p(s) q_{k,s}^+ &= \sum_{s=1}^S p(s) q_{k,s}^-, \quad k = 1, \dots, K, \\ e_k, q_{k,s}^+, q_{k,s}^- &\geq 0. \end{aligned}$$

It is a matter of elementary calculation to show that (14) guarantees that e_k is the expected cost. We shall call (15) the *model with one-sided risk penalty*. The coefficient two before the penalty term has been added to make both models (13) and (15) equivalent in the case of symmetric distributions of the costs around their expectations.

It is also possible to consider (formally) a model without condition (14), but then the interpretation of the linear and the quadratic term of the objective becomes rather obscure.

4 Solution method

The problem (13), as mentioned before, is a quadratic programming (QP) problem. Let us rewrite it in the standard form. First, we define some constants and variables. The box constraints

$$l \leq x \leq u$$

for the problem (1) will be shifted to obtain lower bounds equal to zero. We define new variables $x' = x - l$, for which we have

$$0 \leq x' \leq u - l.$$

Next, we introduce to the constraints $Tx \leq b$ a vector of slacks $t \in \mathcal{R}^m$ to get

$$Tx' + t = b',$$

with $b' = b - Tl$.

For simplifying formulas, we define the constant $SJ = \sum_{k=1}^K J(k)$. A vector d is constructed from variables $d_k(j)$ defined in (3),

$$d = (d_1(1), \dots, d_1(J(1)), \dots, d_K(1), \dots, d_K(J(K))) \in \mathcal{R}^{SJ}.$$

Next, vectors q^+ and q^- are constructed from variables $q_{k,s}^+$, $q_{k,s}^-$ defined in (12),

$$q^+ = (q_{1,1}^+, \dots, q_{K,1}^+, \dots, q_{1,S}^+, \dots, q_{K,S}^+) \in \mathcal{R}^{K*S},$$

$$q^- = (q_{1,1}^-, \dots, q_{K,1}^-, \dots, q_{1,S}^-, \dots, q_{K,S}^-) \in \mathcal{R}^{K*S}.$$

The vector of expected costs is defined as

$$\bar{e} = (e_1, \dots, e_K) \in \mathcal{R}^K.$$

The entire vector of unknowns ξ is built from subvectors previously used, i.e.

$$\xi = (x', t, d, q^+, q^-, \bar{e}).$$

Its dimension will be denoted by n , where $n = 2 * K + m + SJ + 2 * K * S$. The entire vector of the linear part of the objective is defined as follows:

$$g = (\mathbf{0}_K, \mathbf{0}_m, \mathbf{0}_{SJ}, p(1)\mathbf{1}_K, \dots, p(S)\mathbf{1}_K, -p_1\mathbf{1}_K, \dots, -p_S\mathbf{1}_K, \mathbf{1}_K) \in \mathcal{R}^n.$$

We use $\mathbf{0}$ to denote the vector of zeros with the subscript denoting its dimension. Similarly, we denote by $\mathbf{1}$ the vector of ones.

The quadratic part of the objective is non-zero only for terms including q^+ and q^- . Formally, we define the quadratic matrix Q of dimension n in the following way:

$$Q_{i,j} = \begin{cases} 2\alpha * p(s)w_k^2 & \text{if } i = j \text{ and the } i\text{-th component of } \xi \text{ is a } q^+ \text{ or } q^- \\ & \text{variable and } s \text{ and } k \text{ denote the adequate number} \\ & \text{of scenario and country;} \\ 0 & \text{otherwise.} \end{cases}$$

Let us now define the constraint matrix of the problem. We can combine equations (10) and (12) and write dependencies between q^+ , q^- , \bar{e} and d in a matrix form, i.e.

$$q^+ - q^- = Cd - J\bar{e},$$

where C is a matrix of dimension $(K * S) \times SJ$ defined as follows:

$$C = \begin{pmatrix} c_{1,1,1} \dots c_{J(1),1,1} & & & \\ & \ddots & & \\ & & c_{1,K,1} \dots c_{J(K),K,1} & \\ & & \vdots & \\ c_{1,1,S} \dots c_{J(1),1,S} & & & \\ & & \ddots & \\ & & & c_{1,K,S} \dots c_{J(K),K,S} \end{pmatrix}.$$

Equations (4) can be written in a matrix form

$$x' + Ed = x'_0,$$

with $x'_0 = x_0 - l$ and

$$E = \begin{pmatrix} \overbrace{1 \dots 1}^{J(1)} & & 0 \\ & \ddots & \\ 0 & & \overbrace{1 \dots 1}^{J(K)} \end{pmatrix}.$$

Now we can write the full constraint matrix A ,

$$A = \begin{pmatrix} T & I_m & 0 & 0 & 0 & 0 \\ I_K & 0 & E & 0 & 0 & 0 \\ 0 & 0 & C & -I_{K*S} & I_{K*S} & J \end{pmatrix}.$$

The submatrix T is the constraint matrix of the problem (1). The symbol I denotes the identity matrix of dimension determined by the subscript. The matrix J is defined as

$$J = \begin{bmatrix} I_K \\ \vdots \\ I_K \end{bmatrix},$$

where the submatrix I_K is repeated S times. When problem (15) is solved, A is additionally augmented with the block row

$$\begin{bmatrix} 0 & 0 & 0 & P & -P & 0 \end{bmatrix}$$

with

$$P = \begin{bmatrix} p(1)I_K & p(2)I_K & \dots & p(S)I_K \end{bmatrix}.$$

It is worth stressing that in both cases, A is a very sparse matrix, which matters a lot for the solution procedure.

The right hand side vector r is defined as follows

$$r = (b', x'_0, \mathbf{0}_{S*K}) \in \mathcal{R}^{m+K+S*K}.$$

Again, when (15) is solved, b is augmented with zeros, corresponding to the new rows of A . The vector ρ of upper bounds (all lower bounds are equal to zero) is given by

$$\rho_i = \begin{cases} u_i - l_i & \text{if the } i\text{-th component of } \xi \text{ is an } x' \text{ variable,} \\ \bar{d}_k(j) & \text{if the } i\text{-th component of } \xi \text{ is a } d \text{ variable,} \\ +\infty & \text{otherwise,} \end{cases}$$

where, given i , the values of j and k are found as follows: let

$$i_1 = i - (K + m),$$

then k is such that

$$\sum_{\kappa=0}^{k-1} J(\kappa) < i_1 \leq \sum_{\kappa=0}^k J(\kappa)$$

and

$$j = i_1 - \sum_{\kappa=0}^{k-1} J(\kappa).$$

We have not yet defined the number $J(0)$, it will be equal to zero.

Finally, we can formulate the complete QP problem in the standard form:

$$\begin{aligned} & \min \left[g^T \xi + \frac{1}{2} \xi^T Q \xi \right], \\ \text{subject to} & \quad A \xi = r, \\ & \quad \xi + \sigma = \rho, \\ & \quad \xi, \sigma \geq 0. \end{aligned} \tag{16}$$

The dual of (16) is given by

$$\begin{aligned} & \max \left[r^T \lambda - \rho^T w - \frac{1}{2} \xi^T Q \xi \right], \\ \text{subject to} & \quad A^T \lambda + z - w - Q \xi = g, \\ & \quad z, w \geq 0, \end{aligned} \tag{17}$$

where $\lambda \in \mathcal{R}^M$ and $\xi, z, w \in \mathcal{R}^n$; by M we denote the number of constraints equal to $2 * K + m + SJ + 2 * S * K$.

For solving (16) and (17) we shall use the logarithmic barrier method (see, e.g., [13, 15]). In such a method, the objective is augmented by adding to it a logarithmic barrier term (with some coefficient $\mu > 0$), which yields the problem

$$\begin{aligned} & \min \left[g^T \xi + \frac{1}{2} \xi^T Q \xi - \mu \sum_{j=1}^n (\ln \xi_j + \ln \sigma_j) \right], \\ \text{subject to} & \quad A \xi = r, \\ & \quad \xi + \sigma = \rho, \\ & \quad \xi, \sigma > 0 \end{aligned} \tag{18}$$

and an analogue of (17):

$$\begin{aligned} & \max \left[r^T \lambda - \rho^T w - \frac{1}{2} \xi^T Q \xi + \mu \sum_{j=1}^n (\ln z_j + \ln w_j) \right], \\ \text{subject to} & \quad A^T \lambda + z - w - Q \xi = g, \\ & \quad z, w > 0. \end{aligned} \tag{19}$$

The first order optimality conditions for (18) and (19) have the form

$$\begin{aligned}
A\xi &= r, \\
\xi + \sigma &= \rho, \\
A^T\lambda + z - w - Q\xi &= g, \\
\Xi z &= \mu\mathbf{1}, \\
\Sigma w &= \mu\mathbf{1}, \\
\xi, \sigma, z, w &> 0,
\end{aligned} \tag{20}$$

where Ξ, Σ, Z and W are diagonal matrices with the diagonal elements ξ_j, σ_j, z_j and w_j , respectively, and $\mathbf{1} \in \mathcal{R}^n$ is the vector of ones.

The system (20) has a solution dependent on the parameter μ . When $\mu \downarrow 0$, this trajectory (so called *central path*) approaches the solution of the primal-dual pair (16)-(17). To approximate the central path, we shall use a quadratic analogue of Mehrotra's higher order method [14]. This method computes a Taylor approximation of the optimal trajectory that starts at a given point and leads to the optimum of (16) and (17).

Mehrotra's method belongs to the class of continuation methods (cf. [16]) for solving systems of nonlinear equations. In these methods, a family of parametrized problems is considered. One element of the family is our original problem (e.g. for the parameter equal to 0), while other problems have a perturbed right hand side vector. For at least one value of the parameter we know the solution of the problem (e.g. for the parameter equal to 1). Moving iteratively from the problem with known solution to the original one (i.e. changing the right hand side vector), we can find a better approximation of the solution.

Let $\xi^0, \sigma^0, z^0, w^0 > 0$ and λ^0 be the current estimate of the solution of (16) and (17). Then

$$\begin{aligned}
\pi_r &= A\xi^0 - r, \\
\pi_\rho &= \xi^0 + \sigma^0 - \rho, \\
\text{and } \pi_g &= A^T\lambda^0 + z^0 - w^0 - Q\xi^0 - g,
\end{aligned}$$

are the resulting residuals in the primal and dual constraints. Next, we consider the parametric system of equations

$$\begin{aligned}
A\xi(\gamma) &= r + \phi_1(\gamma)\pi_r, \\
\xi(\gamma) + \sigma(\gamma) &= \rho + \phi_1(\gamma)\pi_\rho, \\
A^T\lambda(\gamma) + z(\gamma) - w(\gamma) - Q\xi(\gamma) &= g + \phi_1(\gamma)\pi_g, \\
\Xi(\gamma)z(\gamma) &= \phi_1(\gamma)\Xi^0z^0 + \phi_2(\gamma)\mu\mathbf{1}, \\
\Sigma(\gamma)w(\gamma) &= \phi_1(\gamma)\Sigma^0w^0 + \phi_2(\gamma)\mu\mathbf{1}, \\
\xi(\gamma), \sigma(\gamma), z(\gamma), w(\gamma) &> 0,
\end{aligned} \tag{21}$$

where ϕ_1 and ϕ_2 are non-negative continuously differentiable functions determined on the interval $[0, 1]$ such that $\phi_1(0) = 0, \phi_1(1) = 1, \phi_2(0) = 1, \phi_2(1) = 0$, and $\phi_1(\gamma) \in (0, 1)$ for

$\gamma \in (0, 1)$. The system (21) for $\gamma = 1$ differs from the system (20) by the right hand side. For $\gamma = 0$ both system are identical.

Let $\Gamma(\gamma) = (\xi(\gamma), \sigma(\gamma), \lambda(\gamma), z(\gamma), w(\gamma))$ be the solution of (21) for given parameter γ . We let $\Gamma(1) = (\xi(1), \sigma(1), \lambda(1), z(1), w(1)) = (\xi^0, \sigma^0, \lambda^0, z^0, w^0)$, so (21) is satisfied for $\gamma = 1$. Thus $\Gamma(0)$ represents the solution of (21) for $\gamma = 0$, which is equal to the solution of (20), and, consequently, an approximate solution of (16) and (17). The key point of Mehrotra's approach is to use local higher order information available at point $\Gamma(1)$ to construct a direction towards $\Gamma(0)$. In our implementation $\phi_1(\gamma) = \gamma$ and $\phi_2(\gamma) = (1-\gamma)^2$, which refers to Mehrotra's Algorithm II.

Since $\Gamma(\gamma)$ is a solution of (21) for a given γ , the appropriate higher order terms of Taylor polynomial approximation of correction $(\Delta\xi, \Delta\sigma, \Delta\lambda, \Delta z, \Delta w)$ to the current estimate $(\xi, \sigma, \lambda, z, w)$ result from the recursive differentiation of (21). The i -th order term of the correction vector can be obtained from

$$\begin{bmatrix} A & 0 & 0 & 0 & 0 \\ -Q & A^T & 0 & I & -I \\ I & 0 & I & 0 & 0 \\ Z^0 & 0 & 0 & 0 & 0 \\ 0 & 0 & W^0 & 0 & \Sigma^0 \end{bmatrix} \begin{bmatrix} \Delta\xi^{(i)} \\ \Delta\lambda^{(i)} \\ \Delta\sigma^{(i)} \\ \Delta z^{(i)} \\ \Delta w^{(i)} \end{bmatrix} = \begin{bmatrix} \eta_1^{(i)} \\ \eta_2^{(i)} \\ \eta_3^{(i)} \\ \eta_4^{(i)} \\ \eta_5^{(i)} \end{bmatrix}, \quad (22)$$

where

$$\begin{aligned} \Delta\xi^{(i)} &= \frac{\xi^{(i)}}{i!}, \\ \Delta\sigma^{(i)} &= \frac{\sigma^{(i)}}{i!}, \\ \Delta\lambda^{(i)} &= \frac{\lambda^{(i)}}{i!}, \\ \Delta z^{(i)} &= \frac{z^{(i)}}{i!}, \\ \Delta w^{(i)} &= \frac{w^{(i)}}{i!} \end{aligned}$$

and

$$\begin{aligned} \eta_1^{(i)} &= \frac{1}{i!} \phi_1^{(i)}(1) \pi_r, \\ \eta_2^{(i)} &= \frac{1}{i!} \phi_1^{(i)}(1) \pi_g, \\ \eta_3^{(i)} &= \frac{1}{i!} \phi_1^{(i)}(1) \pi_\rho, \\ \eta_4^{(i)} &= \frac{1}{i!} [\phi_1^{(i)}(1)^0 z^0 + \phi_2^{(i)}(1) \mu \mathbf{1}] - \sum_{l=1}^{i-1} \Delta \Xi^{(l)} \Delta z^{(i-l)}, \\ \eta_5^{(i)} &= \frac{1}{i!} [\phi_1^{(i)}(1) \Sigma^0 w^0 + \phi_2^{(i)}(1) \mu \mathbf{1}] - \sum_{l=1}^{i-1} \Delta \Sigma^{(l)} \Delta w^{(i-l)}. \end{aligned} \quad (23)$$

Let us observe that for every i the matrix involved in all these linear systems is the same. Owing to that, the factorization of (22) need be computed only once. For the linear case (cf. [2], [3]) we can compute the search directions in the primal and dual spaces as

$$\begin{aligned}
d_\xi &= - \sum_{i=1}^{l_p} (-\gamma_p)^i \Delta \xi^{(i)}(1), \\
d_\sigma &= - \sum_{i=1}^{l_p} (-\gamma_p)^i \Delta \sigma^{(i)}(1), \\
d_\lambda &= - \sum_{i=1}^{l_d} (-\gamma_d)^i \Delta \lambda^{(i)}(1), \\
d_z &= - \sum_{i=1}^{l_d} (-\gamma_d)^i \Delta z^{(i)}(1), \\
d_w &= - \sum_{i=1}^{l_d} (-\gamma_d)^i \Delta w^{(i)}(1),
\end{aligned} \tag{24}$$

where l_p and l_d are orders of Taylor polynomials in the primal and dual spaces, respectively. The parameters γ_p and γ_d in (24) are the largest numbers in $[0, 1]$ for which

$$\begin{aligned}
\xi - d_\xi &\geq 0, \\
\sigma - d_\sigma &\geq 0, \\
z - d_z &\geq 0, \\
w - d_w &\geq 0.
\end{aligned}$$

Unfortunately, for quadratic problems the use of (24) can cause the loss of feasibility in the dual equality constraints. If we have a feasible solution $(\xi, \sigma, \lambda, z, w)$ then the new point $(\xi - d_\xi, \sigma - d_\sigma, \lambda - d_\lambda, z - d_z, w - d_w)$ need not be dual feasible as in the linear case. In a quadratic problem, the primal variable ξ appears also in the dual constraints, and then

$$A^T(\lambda - d_\lambda) + (z - d_z) - (w - d_w) - Q(\xi - d_\xi) = c + Qd_\xi.$$

To overcome this disadvantage we can follow [19] and use $\gamma_p = \gamma_d$ and $l_p = l_d$ for primal and dual space. But it implies that $\gamma = \min(\gamma_p, \gamma_d)$ and $l = \min(l_p, l_d)$, which slows down the method.

In our computations we decided to use different γ and l for the primal and the dual space, similarly to the linear case. Our experiments show that with formulas (24) the algorithm works much faster.

After computing the search directions we define step factors f_p and f_d as in [14]; new approximations to the optimal point are given by

$$\begin{aligned}
\xi &:= \xi - f_p d_\xi, \\
\sigma &:= \sigma - f_p d_\sigma, \\
\lambda &:= \lambda - f_d d_\lambda,
\end{aligned}$$

$$\begin{aligned} z &:= z - f_d d_z, \\ w &:= w - f_d d_w. \end{aligned}$$

In this way, we generate a new ‘starting point’ for which μ is reduced and the whole procedure is repeated.

Let us briefly discuss how the whole idea can be implemented. Elimination of $\Delta\sigma^{(j)}$, $\Delta z^{(j)}$ and $\Delta w^{(j)}$ reduces (22) to

$$H \cdot \begin{bmatrix} \Delta\xi^{(i)} \\ \Delta\lambda^{(i)} \end{bmatrix} = \begin{bmatrix} -\Theta^{-1} & A^T \\ A & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\xi^{(i)} \\ \Delta\lambda^{(i)} \end{bmatrix} = \begin{bmatrix} h^{(i)} \\ \eta_1^{(i)} \end{bmatrix}, \quad (25)$$

with

$$h^{(i)} = \eta_2^{(i)} - (\Xi^0)^{-1}\eta_4^{(i)} + (\Sigma^0)^{-1}(\eta_5^{(i)} - W^0\eta_3^{(i)})$$

and

$$\Theta = (Q + (\Xi^0)^{-1}Z^0 + (\Sigma^0)^{-1}W^0)^{-1}, \quad (26)$$

where $\eta_1^{(i)}$, $\eta_2^{(i)}$, $\eta_3^{(i)}$, $\eta_4^{(i)}$, $\eta_5^{(i)}$ are defined by (23). Further, we reduce (25) to the equation

$$(A\Theta A^T)\Delta\lambda^{(i)} = A\Theta h^i, \quad (27)$$

and we compute sparse Cholesky factorization of the positive definite matrix $A\Theta A^T$.

It is now clear why separability of the quadratic part was so important. With a diagonal Q all terms in (26) are diagonal and computation of Θ is very easy. Furthermore, the sparsity pattern for $A\Theta A^T$ is the same for every Θ and we can use the same techniques of symbolic factorization as in the linear case [10]. Hence, the method used for solving (27) is the same as in the linear case. We can also use the same techniques for finding starting points.

5 Numerical results

Before the Second Sulphur Protocol was signed in June 1994 in Oslo, the latest model calculations of the RAINS model, that were used as input to the negotiations, were performed in June 1993. These calculations minimized the total European costs subject to two conditions:

- the deposition at each grid area had to be lower or equal to the depositions that would result if the difference between the deposition in 1990 and the critical sulphur deposition values for sensitive ecosystems were reduced by 60% at each area (see Figure 2);
- countries would at least carry out reductions that they were currently planning to undertake anyway.

Of the 38 regions in the model, two cannot control emissions (the Mediterranean Sea and the Black Sea). Hence, 36 countries remained, in which the emissions could be controlled ($K=36$). For each country, 6 different scenarios of cost curves for the year 2010 were

available. These different cost curves were developed at the International Institute for Applied Systems Analysis and the University of Karlsruhe (see [17]). The cost functions were based on the following scenarios:

1. Reference scenario. This is based on fuel prices and energy demand projections of the "Energy 2000" and "Energy 2010" studies by the Commission of the European Communities.
2. Alternative fuel prices, assuming a doubling of oil and gas prices and a 50% increase in coal prices.
3. CO₂ emission reduction scenario, which ensures a severe, country specific limitation in these emissions by the year 2010.
4. Lower demand for final energy by ca. 20% (country specific).
5. Higher demand for final energy by ca. 20%.
6. Country specific scenario accounting, for example, for constant nuclear power contribution in Finland, a 25% decline in nuclear in France and increases in Italy and the Netherlands. Moreover, a faster penetration of natural gas in the United Kingdom and renewable energy in Spain was assumed together with an increase in domestic coal use in Hungary and ignoring current domestic coal contracts in Germany.

The environmental constraints imposed at the reception areas (as in Figure 2) were at first filtered to eliminate those that could not be active at the solution. Those were the constraints that were satisfied by the present emissions and the constraints which were dominated by other constraints (had identical transfer coefficients, but a larger right hand side). In this way, the number of environmental constraints has been reduced to 169 reception areas ($m = 169$). As a whole, the problem (13) had 36 x' variables, 169 t variables, 236 d variables, 216 both q^+ and q^- variables, and 36 \bar{e} variables; the total number of variables was 909. There are 169 constraints connected with the vector b' , 36 with vector x'_0 and 216 others; together we had 421 equality constraints.

The solution of the model with two-sided risk penalty (13) for various values of the parameter α is presented in Tables 1 and 2. We used rather large values of α to account for the difference of orders of magnitude between the unscaled expected cost and the normalized variance. The solution of the model with one-sided risk penalty (15) for various values of the parameter α is presented in Tables 3 and 4 (clearly, the results for $\alpha = 0$ are for both models the same).

We can see from the results that both models lead to similar qualitative changes in the solution when more stress is put on the quadratic risk term (α increases), although numerical values of the solutions are slightly different. Generally, the 'safe' solutions require smaller reductions from the countries with the relatively high ratio of emissions to the GDP, thus reducing the relative risk associated with the cost uncertainty. Still, one has to stress that the currently available scenarios (cost curves) have substantial similarities in their qualitative behaviour and differ mainly in the scale of costs rather than in the shape of the function.

	$\alpha = 0$	$\alpha = 10^4$	$\alpha = 10^5$	$\alpha = 10^6$
Albania	0.63704E+02	0.63608E+02	0.63587E+02	0.63585E+02
Austria	0.38888E+02	0.38888E+02	0.38888E+02	0.38888E+02
Belgium	0.57276E+02	0.43902E+02	0.41583E+02	0.41085E+02
Bulgaria	0.25981E+03	0.25981E+03	0.25981E+03	0.25981E+03
Czech and Slovak Rep.	0.38133E+03	0.41695E+03	0.41957E+03	0.41995E+03
Denmark	0.24088E+02	0.24088E+02	0.24088E+02	0.24088E+02
Finland	0.58153E+02	0.58153E+02	0.58153E+02	0.58153E+02
France	0.42800E+03	0.36986E+03	0.30413E+03	0.30177E+03
Germany-W.	0.25963E+03	0.25963E+03	0.25963E+03	0.25963E+03
Germany-E.	0.17000E+03	0.17000E+03	0.17000E+03	0.17000E+03
Greece	0.23545E+03	0.16482E+03	0.15123E+03	0.14895E+03
Hungary	0.48045E+03	0.46555E+03	0.46445E+03	0.46433E+03
Ireland	0.27268E+02	0.47992E+02	0.47757E+02	0.47958E+02
Italy	0.52973E+03	0.52339E+03	0.52570E+03	0.52576E+03
Luxembourg	0.34023E+01	0.48174E+01	0.49430E+01	0.49483E+01
Netherlands	0.53130E+02	0.53130E+02	0.53130E+02	0.53130E+02
Norway	0.15823E+02	0.15823E+02	0.15823E+02	0.15823E+02
Poland	0.41634E+03	0.57392E+03	0.60654E+03	0.60951E+03
Portugal	0.92994E+02	0.80659E+02	0.67832E+02	0.66636E+02
Romania	0.54639E+03	0.52621E+03	0.52510E+03	0.52501E+03
Spain	0.74977E+03	0.75060E+03	0.75156E+03	0.75163E+03
Sweden	0.39099E+02	0.39099E+02	0.39099E+02	0.39099E+02
Switzerland	0.23892E+02	0.21237E+02	0.21237E+02	0.21237E+02
Turkey	0.83348E+03	0.83348E+03	0.83348E+03	0.83032E+03
United Kingdom	0.73334E+03	0.73310E+03	0.74634E+03	0.74678E+03
f. Yugoslavia	0.27231E+03	0.42090E+03	0.42714E+03	0.42763E+03
Baltic Sea	0.14652E+02	0.14652E+02	0.14652E+02	0.14652E+02
North Sea	0.34652E+02	0.34652E+02	0.34652E+02	0.34652E+02
Atlantic Ocean	0.63320E+02	0.63320E+02	0.63320E+02	0.63320E+02
Baltic States	0.14082E+03	0.14082E+03	0.14082E+03	0.14082E+03
Byelorussia	0.13500E+03	0.10800E+03	0.11028E+03	0.11132E+03
Ukraine	0.84778E+03	0.75583E+03	0.75583E+03	0.75583E+03
Moldavia	0.11563E+03	0.11563E+03	0.11563E+03	0.11563E+03
Kola-Karelia	0.12664E+03	0.12664E+03	0.12664E+03	0.12664E+03
S. Petersburg	0.15334E+03	0.97485E+02	0.10236E+03	0.10269E+03
Russia	0.18642E+04	0.12812E+04	0.91461E+03	0.87483E+03

Table 1: Emissions of SO₂ (in kilotons of sulphur) in the model with two-sided risk penalty.

	$\alpha = 0$	$\alpha = 10^4$	$\alpha = 10^5$	$\alpha = 10^6$	$\alpha \rightarrow \infty$
Expected Cost	0.3870E+04	0.4334E+04	0.4596E+04	0.4628E+04	0.4637E+04
Risk Penalty	5.4678	5.2158	5.2044	5.2033	5.2033

Table 2: The values of the objectives in the model with two-sided risk penalty.

	$\alpha = 10^4$	$\alpha = 10^5$	$\alpha = 10^6$
Albania	0.63613E+02	0.63586E+02	0.63584E+02
Austria	0.38888E+02	0.38888E+02	0.38888E+02
Belgium	0.55015E+02	0.52763E+02	0.53136E+02
Bulgaria	0.25981E+03	0.25981E+03	0.25981E+03
Czech and Slovak Rep.	0.41388E+03	0.41631E+03	0.41664E+03
Denmark	0.24088E+02	0.24088E+02	0.24088E+02
Finland	0.58153E+02	0.58153E+02	0.58153E+02
France	0.36738E+03	0.30617E+03	0.28540E+03
Germany-W.	0.25963E+03	0.25963E+03	0.25963E+03
Germany-E.	0.17000E+03	0.17000E+03	0.17000E+03
Greece	0.19737E+03	0.18341E+03	0.18270E+03
Hungary	0.46646E+03	0.46518E+03	0.46505E+03
Ireland	0.49276E+02	0.49442E+02	0.49333E+02
Italy	0.52369E+03	0.52564E+03	0.52642E+03
Luxembourg	0.48664E+01	0.49847E+01	0.50085E+01
Netherlands	0.53130E+02	0.53130E+02	0.53130E+02
Norway	0.15823E+02	0.15823E+02	0.15823E+02
Poland	0.56378E+03	0.59825E+03	0.60285E+03
Portugal	0.86265E+02	0.75936E+02	0.74852E+02
Romania	0.52809E+03	0.52641E+03	0.52619E+03
Spain	0.75029E+03	0.75109E+03	0.75123E+03
Sweden	0.39099E+02	0.39099E+02	0.39099E+02
Switzerland	0.21237E+02	0.21237E+02	0.21237E+02
Turkey	0.83348E+03	0.83348E+03	0.83172E+03
United Kingdom	0.72874E+03	0.74070E+03	0.74464E+03
f. Yugoslavia	0.41172E+03	0.42047E+03	0.42192E+03
Baltic Sea	0.14652E+02	0.14652E+02	0.14652E+02
North Sea	0.34652E+02	0.34652E+02	0.34652E+02
Atlantic Ocean	0.63320E+02	0.63320E+02	0.63320E+02
Baltic States	0.14082E+03	0.14082E+03	0.14082E+03
Byelorussia	0.10800E+03	0.11027E+03	0.11374E+03
Ukraine	0.75583E+03	0.75583E+03	0.75583E+03
Moldavia	0.11563E+03	0.11563E+03	0.11563E+03
Kola Karelia	0.12664E+03	0.12664E+03	0.12664E+03
S. Petersburg	0.98666E+02	0.10380E+03	0.10458E+03
Russia	0.13355E+04	0.94089E+03	0.7357E+03

Table 3: Emissions of SO₂ (in kilotons of sulphur) in the model with one-sided risk penalty.

	$\alpha = 10^4$	$\alpha = 10^5$	$\alpha = 10^6$
Expected Cost	0.4574E+04	0.4574E+04	0.48978+04
Risk Penalty	3.7802	3.7802	3.8265

Table 4: The values of the objectives in the model with one-sided risk penalty.

	$\alpha = 0$	$\alpha = 10^2$	$\alpha = 10^4$	$\alpha = 10^5$	$\alpha = 10^6$	$\alpha = 10^9$
Iterations	20	18	21	24	30	32
Time (s)	35.09	32.66	36.58	41.66	48.37	51.53

Table 5: Performance of the interior point method

It is also worth noting that the numerical method suggested in the paper proved to be quite efficient for this class of problems. In Table 5, we summarize its performance for different values of the parameter α in the model with two-sided risk penalty. The method never failed, although the required precision was high (10^{-6}). Both the number of iterations and the execution time are rather low. All computations were done on a SUN Sparc 2 workstation.

6 Conclusions

The methodology used in this paper can be used not only for this specific transboundary air pollution problem, but also for a wide class of problems with uncertainty in the costs. By using a mean-variance model, we can properly treat a non-deterministic problem with small data collections, when more sophisticated stochastic methods are not useful. The mean-variance model has a nice interpretation in terms of risk. Furthermore, this method leads to quadratic programming problems that are well described in the literature.

The numerical method used here, the quadratic version of an interior point method of [2], proved to be efficient and robust. In this method the most recent computational techniques are implemented, such as symbolic Cholesky factorization of sparse matrices, splitting dense columns, minimum degree ordering and many others. The application problem described in the paper motivated the development of the general quadratic programming solver for sparse and large scale problems.

Unfortunately, not all scenarios available so far are complete and their set does not seem to be variable enough. Having a richer collection of data, we might obtain more interesting results with greater role played by risk. Still, it is also possible that very tight deposition constraints do not leave much room for stochastic optimization in this case.

In the current model only costs are uncertain. However, depositions depend on many uncertain factors, especially on weather. In a more sophisticated approach, one might incorporate uncertainty into the constraints, which requires further research and cooperation of experts in both environmental modeling and optimization.

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