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Pathways of Economic Development in an Uncertain Environment: A Finite Scenario Approach to the U.S. Region under Carbon Emission Restrictions

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Abstract

Prediction of future economic behavior is increasingly important for both public and private economic planning. This prediction is, however, increasingly fraught with difficulties because of the uncertainty surrounding the future state of so many key economic parameters. In this paper we consider how stochastic programming may be a valuable tool in the analysis of these kinds of problems. Using the U.S. region of Alan Manne and Richard Richels Global 2100 five region world trade model and a set of eight future state-of-the-world scenarios, we observe how the development paths of several key variables predicted by stochastic programming differ in interesting ways from the paths predicted using deterministic methods. We conclude that the explicit way in which stochastic programming models uncertainty may prove useful to economic analysis efforts and provide additional insight into the nature of economic development in an uncertain environment.

Key words: Economics, environment, stochastic programming

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1. Introduction

Multistage economic models are tremendously important for decision makers, both in government and industry. They are essential when planning how present resources should be allocated because these decisions affect the present state of commerce, as well as the future, in complicated ways. One such model is Alan Manne and Richard Richels' large scale deterministic macroeconomic model, Global 2100 [MaR91]. It is to this model we have chosen to apply stochastic programming techniques.

2. Deterministic Economic Model Description

Manne and Richels developed this model to investigate how different carbon dioxide emissions restriction proposals might impact the world's economy. In particular, what the cost of such proposals would be in terms of reduced GNP, what the level of carbon tax would have to be within each of the world regions included in their model to induce compliance with an emission ceiling, and how differences in this carbon tax rate between regions would make attractive an international market in carbon emission rights trading.

The Global 2100 model divides the world into the five separate regions shown in table 1. Each region is independently described by a deterministic nonlinear optimization model, and is connected to the other regions via the exchange of crude oil and carbon emission credits on the international market. The carbon market allows those regions producing less than their quota of CO_2 to sell the residual to those regions wishing to relax their carbon restriction.

The economy of each region is summarized by the following sectors: Nonelectric and electric energy, capital and labor. Aggregate economic output in each period is a function of the activity in these sectors and is divided into two age classifications. The first is the old output that is the result of inputs to production from previous periods. It, along with these old inputs to production, decrease at a fixed depreciation rate over time. The

Region:
USA
OOECD
USSR
CHINA
ROW

Table 1: World Regions

second is new production which is a concave function (Cobb-Douglas/CES type) of four aggregate inputs to production in the present period as shown in the following equation:

$$
\begin{array}{lll}
YN_t & = & (A * KN_t^{(\rho * KPVS)} * LN_t^{(\rho*(1-KPVS))} \\
 & & +B * EN_t^{(\rho*ELVS)} * NN_t^{(\rho*(1-ELVS))})^{(1/\rho)} \\
\text{where:} & & \\
t & & \text{period t} \\
YN_t & & \text{New output} \\
KN_t & & \text{New capital} \\
LN_t & & \text{Labor force - new vintage} \\
EN_t & & \text{New elec. energy cap.} \\
NN_t & & \text{New nonelec. energy cap.} \\
A & & \text{Const. for cap./labor index (parameter)} \\
B & & \text{Const. for elec./nonelec. energy index (parameter)} \\
KPVS & & \text{Capital value share (parameter)} \\
ELVS & & \text{Electric value share (parameter)} \\
\rho & & \text{Elasticity between K-L \& E-N (parameter).}
\end{array}
\tag{2.1}
$$

These inputs are new labor, new capital stocks, and the new electrical and nonelectrical energy capacity installed during that period. This division of output and input into classes based on age is meant to model the inertia an economy exhibits; its inflexibility to changes in the price signals from inputs to production. Economists refer to this construct as the putty-clay characteristic of the model because existing capital and energy use patterns, like hard baked clay, are fixed, unable to respond to price and availability changes, while new capital and energy use patterns are malleable, like putty, in that they can be modified by the model in response to changes in resource price and availability.

The total aggregate output in each period is allocated amongst the following: Satisfying final consumption in the present, maintaining or expanding the stock of capital, payment for energy utilized during the present period, and net carbon rights purchases.

The mixture of capital and energy inputs to production is influenced by the by-product of energy production, CO_2. Carbon dioxide emissions are required to lie below the sum of that regions quota plus the net of its carbon rights transactions on the world market.

Because Manne and Richels developed the Global 2100 model to explore the interaction between the world's economy, the energy that fuels it, and how the evolution of energy use patterns change in response to environmental pressures, the energy sectors are modeled in much greater detail than the nonenergy sectors.

Technology name	Identification
Existing:	
HYDRO	Hydroelectric, geothermal, and other renewables
GAS-R	Remaining initial gas fired
OIL-R	Remaining initial oil fired
COAL-R	Remaining initial coal fired
NUC-R	Remaining initial nuclear
New:	
GAS-N	Advanced combined cycle, gas fired
COAL-N	New coal fired
ADV-HC	High-cost carbon free
ADV-LC	Low-cost carbon free

Table 2: Electrical Technologies

Technology name	Identification
OIL-MX	Oil imports minus exports
CLDU	Coal-direct uses
OIL-LC	Oil-low cost
GAS-LC	Gas-low cost
OIL-HC	Oil-high cost
GAS-HC	Gas-high cost
RNEW	Renewables
SYNF	Synthetic fuels
NE-BAK	Nonelectric backstop

Table 3: Nonelectrical Technologies

Broadly, they've included both electrical and nonelectrical energy producing technologies. Within each of these categories they've included those technologies which are presently considered conventional and those which are only now being developed and will fill the void in energy capacity left when present technologies are no longer sufficient for whatever reasons. Those that are conventional typically require less capital than new technologies and rely on the availability of easily obtainable natural resources that exist in finite quantities, such as oil, gas or coal. In most cases the pool of any one kind of resource is not homogeneous in its ease of discovery, extraction or refinement. Thus, each resource is divided into grades and the grades are, roughly, used in order of increasing cost incurred in bringing them to market. Those technologies being developed to replace existing conventional sources not only require more capital, but usually rely on an energy supply that is less easily captured. Hence, they are typically more expensive. A listing of both the electrical and nonelectrical energy producing technologies used in the model are shown in tables 2 and 3. Also, the equation representing the total energy cost of all energy technologies being used during a given period in Global 2100 is as follows:

$$
\begin{array}{lll}
1000 * EC_t & = & \sum_{e \in ET} PE_t^e * ECST_t^e \\
 & & \sum_{n \in NT} PN_t^n * NCST_t^n \\
 & & +OGPD * GN_t \\
 & & +(CARP_t + .5 * MXDIF) * CARM_t \\
 & & -(CARP_t - .5 * MXDIF) * CARX_t \\
 & & +.5 * \sum_{e \in EX} ECST_t^e * F(XPE_t^e) \\
 & & +.5 * \sum_{n \in NX} NCST_t^n * G(XPN_t^n) \\
\text{where:} & & \\
EC_t & & \text{Energy costs } (10^{12}\ \$) \\
ET & & \text{Set of elec. tech. (table 2)} \\
NT & & \text{Set of nonelec. tech. (table 3)} \\
EX & & \text{Subset of elec. tech.} \\
NX & & \text{Subset of nonelec. tech.} \\
PE_t^e & & \text{Elec. produced by tech. } e \\
PN_t^n & & \text{Nonelec. produced by tech. } n \\
ECST_t^e & & \text{Elec. cost coef. - mills per KWH (parameter)} \\
NCST_t^n & & \text{Nonelec. cost coef. - \$ per GJ (parameter)} \\
OGPD & & \text{oil-gas price differential - \$ per GJ (parameter)} \\
GN_t & & \text{Gas consumed to meet nonelec. demands} \\
CARP_t & & \text{Int. carbon prices (\$ per ton) (parameter)} \\
MXDIF & & \text{Carbon tax (parameter)} \\
CARM_t & & \text{Carbon imports - } 10^9 \text{ tons} \\
CARX_t & & \text{Carbon exports - } 10^9 \text{ tons} \\
XPE_t^e & & \text{Additional expansion of elec.} \\
XPN_t^n & & \text{Additional expansion of nonelec.} \\
F(.), G(.) & & \text{Quadratic functions.}
\end{array}
\tag{2.2}
$$

For a more detailed explanation of the component parts of this equation, consult [MaR92].

Manne and Richels also account for the role that energy efficiency improvements can play in the economy. They model this with a parameter representing autonomous energy efficiency improvements. These are efficiency improvements made irrespective of energy cost. That is, they are made not as a way to avoid the use of costly energy supplies, but automatically over time as the economy matures. This parameter appears in the model as follows:

$$
\begin{array}{lll}
\sum_{e \in ET} PE_e^t & \geq & E_t * AEEIFAC_t \\
\sum_{n \in NT} PN_n^t & \geq & N_t * AEEIFAC_t \\
\text{where:} & & \\
t & & \text{period t} \\
E_t & & \text{Total electric energy} \\
N_t & & \text{Total nonelectric energy} \\
AEEIFAC_t & & \text{Autonomous energy effic. improvement (parameter)}
\end{array}
\tag{2.3}
$$

The system of equations, summarized above, describes the set of all feasible development paths for each regional economy. The path which is ultimately followed is assumed to be the one which maximizes the utility of consumption within that region. This utility is the discounted sum of logs of consumption in each period. In order to reduce end-of-study effects [Man81] [Gri83], post terminal consumption, a function of the investment during the later periods, is included in the objective function.

The entire Global 2100 model is available from Manne and Richels as a collection of command files and GAMS [BKM92] program and data files. By invoking the name of appropriate command files at the operating systems prompt, the model describing the entire world or separate subregions within it can be run for several different carbon constraint futures. Different scenarios can be explored by changing appropriate data within the GAMS data files. As GAMS is a portable modeling language, versions of Global 2100 can be run on either a PC or Unix_based workstation.

3. Economic Modeling under Uncertainty

Clearly, much uncertainty exists in a model of this type. The model has a very long horizon and, as such, users must assign data values to characteristics of the economy that will only be known with certainty in the distant future. Even in the present, statistical data collection techniques are required to approximate key parameters which strongly influence solutions returned by the model. As deterministic optimization, the technique by which Global 2100 is solved, typically returns an extreme point solution from the set of feasible possibilities, small changes in data values can often dramatically change the values of important decision variables within the model. As a result, solving the deterministic model with only one data set can only approximate how the system will evolve when faced with a multitude of different possible futures. We desire to employ techniques that explicitly take this uncertainty into account when solving the model; preferably techniques that approximate the way economic agents make their decisions when faced with uncertainty about the future, as this is what the model is attempting to predict.

3.1 Scenario Analysis

One common technique for dealing with uncertainty in data is to run the model many times using different data sets representing different possible scenarios. By doing this, one can observe how key decision variables change as a function of the scenario, and probability distributions can be constructed based on the likelihood of the different scenarios. This method is known as *scenario analysis* and has been used successfully by many practitioners of modeling. Along with the derivation of probability distributions, it is useful for determining which uncertain parameters have the largest impact on present decisions, and for deducing how future uncertainties will impact long term trends in the economy. One characteristic of the economy which this method does not attempt to explicitly model is decision making under uncertainty. That is, decisions made in the present which are robust against a variety of future scenarios. The solution returned by each run of a scenario analysis is the string of economic decisions that are optimal for that particular scenario. As such, scenario analysis may return a multitude of different present decisions, each optimal for a different scenario. The real problem being modeled, though, often requires that one present decision be made based on a multitude of different possible futures before it is known with certainty which of those futures will occur. This one present decision cannot be trivially constructed from all the present decisions returned from each run of a scenario analysis. In the next section, we suggest a method for deriving what this optimal present decision should be. We also consider some additional uncertainties that this new method will introduce.

3.2 A Stochastic Programming Model

We can picture the decision process modeled by stochastic programming as the scenario tree frequently used in decision analysis. Travel along a particular path in the tree corresponds to the realization, over time, of one scenario, and nodal points along that path correspond to points in time when decisions must be made. Each path in the tree typically starts from an initial node; a decision that must be made prior to any specific knowledge of the future. From this initial node, paths branch off from one another as the scenarios represented by those paths, over time, become distinguishable. The process continues until one has reached the limits of the horizon and the single limb has repeatedly branched into the forest of different possible scenarios. The relationship between the original independent scenarios and the decision tree that embodies the information structure is illustrated by the example in figure 1. At the top of the figure is pictured the

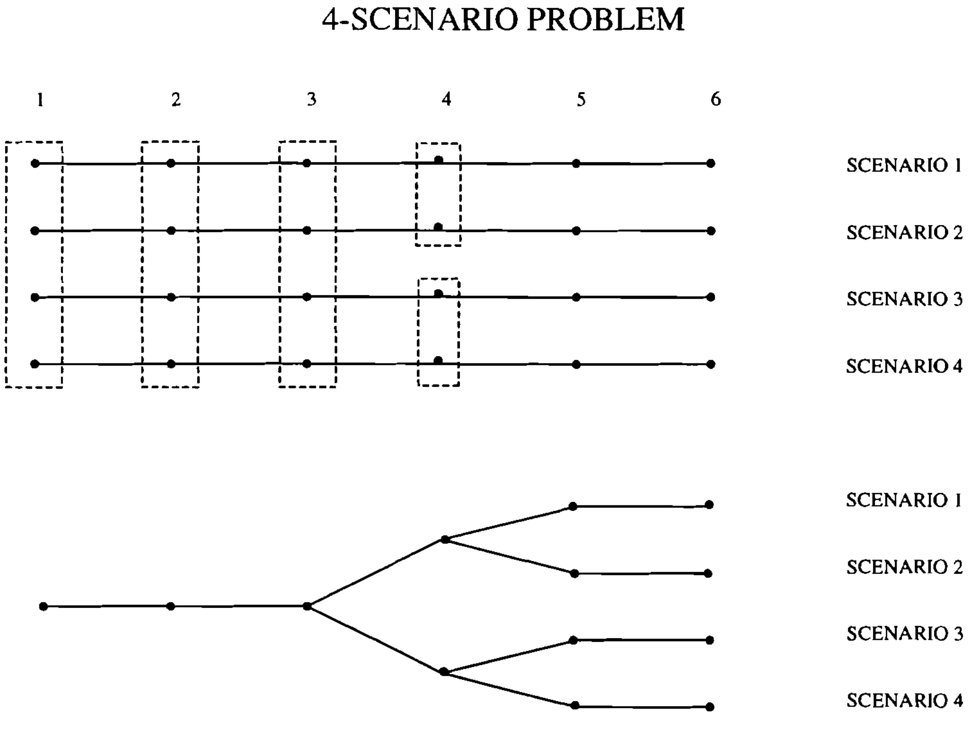

Figure 1: Scenario Tree

four time lines designating the stream of decisions made during each period based on each scenario. During periods one, two, and three, it is not yet known in which scenario one will be. Thus, the dashed boxes surrounding and connecting the nodal decision points of the four different scenarios during these periods require that decisions be the same. This type of constraint enforces nonanticipativity on decision variables [RoW91]. The set of points surrounded by each dashed box is mapped to a single point in the decision tree.

During period four, the decision maker learns that he is in one of two groups of scenarios (the group containing scenarios 1 and 2 or the group containing scenarios 3 and 4). The two dashed nonanticipativity boxes surrounding the decision nodes at this point indicate that the decision maker can make this distinction. The mapping results in a branch in the decision tree. Finally, in period five, all information is revealed and the decision tree branches again into the four possible scenarios. To see how the constraints of the new stochastic program are constructed from the original deterministic model, equation 3.1 shows how the constraints in equation 2.3 become stochastic for a simple two period case with two scenarios.

$$\begin{array}{ll}
\sum_{e\in ET} PE_e^t(s) & \geq E_t(s) * AEEIFAC_t(s) \ \forall\, t = 1,2 \ \forall\, s = 1,2 \\
\sum_{n\in NT} PN_n^t(s) & \geq N_t(s) * AEEIFAC_t(s) \ \forall\, t = 1,2 \ \forall\, s = 1,2 \\
\left.\begin{array}{lcl}
PE_e^1(1) & = & PE_e^1(2) \\
PN_n^1(1) & = & PN_n^1(2) \\
E_1(1) & = & E_1(2) \\
N_1(1) & = & N_1(2)
\end{array}\right\} & \text{Nonanticipativity constraints}
\end{array} \tag{3.1}$$

A decision tree is a good method for modeling the way in which decisions are made under uncertainty. In order to weigh one scenario against another, though, each scenario's likelihood must still be considered, and this is where the complexity of this method introduces additional uncertainty. The original deterministic utility function must be replaced by an expected utility function for the stochastic program. This expected utility function is the convex combination of independent scenario objectives, weighted according to their subjective probabilities. These probabilities must, ideally, be the aggregate weighting of the beliefs of all decision makers within the economy regarding the likelihood of all of the scenarios. Clearly, summarizing all decision makers beliefs about the likelihood of each scenario with a single probability is difficult, if not impossible. Thus, the probabilities that are used are only an estimate of this ideal aggregate weighting and, hence, are uncertain. This additional uncertainty is in some sense inevitable as the stochastic program attempts to model more of the problems complexity. One reason for proceeding with stochastic programming in spite of this seeming difficulty is that even without a completely accurate description of the likelihood of all the scenarios, the stochastic programming solution will still be of an entirely different quality than the deterministically derived solutions. It will suggest strategies not obtainable solely through determinisitc means and, as such, will provide additional insights into the impact of uncertainty [BiR93]. It is our belief that this type of modeling, along with scenario analysis, will increase our understanding of the underlying decision process, and that this increased understanding will improve our decision making in the present as we better describe the interactions of decision makers, environment and technology within the economy.

4. Construction and Solution of Stochastic Model

4.1 Stochastic Elements of Model

The solution of a stochastic programming version of Global 2100 provides a first order approximation of the world economy's evolution in the face of uncertainty. To build such a model, we have developed an assortment of scenarios based on the suggestions of energy/economic experts working on the ECS (Environmentally Compatible Energy Strategies) project at IIASA. Based on extensive knowledge they have accumulated from their work with this and many other models, they have found the following parameters to be both uncertain and to profoundly impact the solution returned by the model:

- Cost estimates of energy technology (esp. Advanced Coal and Gas)
- The nonelectric base price of energy for each region
- The extent to which non-cost justified energy efficiency improvements are and will continue to be made

The first uncertainty is important because coal and gas tend to be substitutes for one another in the production of electrical power. Uncertainty in future cost streams may lead decision makers, contemplating the construction of power facilities with long planning periods and lifetimes, to develop capacity in both technologies. This diversified portfolio of electrical production technologies provides a hedge against the uncertainty surrounding which technology will ultimately dominate the other in terms of price. The second uncertainty is important because the nonelectric base price of energy, a figure that must be estimated each year from the world's energy markets, characterizes how important nonelectric energy is to economic output. It reflects the functional relationship that exists between factors of production and aggregate output each year. Its estimation helps energy producers predict how much of what kind of capacity should be developed to meet consumer and producer demand. Finally, the extent of efficiency improvement has an impact on the rate at which capacity should be developed for all types of energy supply technology. These sources of uncertainty impact the constraints of the model in equations 2.1, 2.2 and 2.3.

4.2 Augmented Lagrangian Decomposition Method

In order to solve the stochastic model we have constructed, we use an implementation of the Augmented Lagrangian Decomposition Algorithm. What follows is a brief description of the general algorithm. For a more complete explanation, please consult [MuR92], [MuR91] and [Rus92].

As already informally discussed in an earlier section, a stochastic programming problem whose uncertainty is fully described by a finite number of scenarios can be conceived of as a finite number of independent deterministic models, each of whose structure and data set correspond to a different scenario. These submodels are connected to one another by a constraint requiring that decisions made in scenario problems whose scenarios are identical up until a certain time stage be identical until that time stage. This requirement is known as a nonanticipativity constraint. This is formalized in the following fashion.

Suppose that in the multistage stochastic programming problem x_i represents the sequence of decisions that are made at each time step $t = 1, 2, \ldots, T$

$$x_i = (x_i(1), x_i(2), \ldots, x_i(T)),$$

within each scenario $i = 1, 2, \ldots, L$. Each scenarios system dynamics are independently and completely described by some nonempty closed convex subset X_i of R^{n_i}. As well, the objective driving the scenario problem to a self-consistent and optimal solution is described by some concave function $f_i(x_i) : R^{n_i} \to R, i = 1, 2, \ldots, L$. The nonanticipativity constraint which binds the separate scenario decision vectors together can be described by a specialized system of sparse linear equations [MuR92]:

$$\sum_{i=1}^{L} A_i x_i = 0.$$

Each row of this matrix sum contains a relation of the form

$$x_i(t) - x_j(t) = 0, i = 1, \ldots, L, t = 1, \ldots, T-1,$$

where j is a scenario indistinguishable from scenario i in period t as in (3.1). All this, together with the fact that each scenario has an associated probability p_i, gives us the following problem to solve:

$$\begin{aligned} \max f(x) &= \sum_{i=1}^{L} p_i f_i(x_i) \\ \text{such that} & \\ \sum_{i=1}^{L} A_i x_i &= 0 \\ x_i \in X_i, \quad & i = 1, 2, \ldots, L \\ X &= X_1 \times X_2 \times \cdots \times X_L \end{aligned} \tag{4.1}$$

Several successful techniques have been developed for solving problems of similar structure using the ordinary lagrangian:

$$L(x, \pi) = \sum_{i=1}^{L} f_i(x_i) + \pi^T \sum_{i=1}^{L} A_i x_i$$

The fundamental results of duality theory for convex programming [Roc73], [RoW91] show that a solution to

$$\min_{\pi \in R^m} g(\pi) \tag{4.2}$$

where

$$g(\pi) = \sup_{x \in X} L(x, \pi) = \tag{4.3}$$

$$\sum_{i=1}^{L} \sup_{x_i \in X_i} (f_i(x_i) + \pi^T A_i x_i) \tag{4.4}$$

is a solution to the original problem. This method has merit because the relaxation of the nonanticipativity constraints leads to a natural separability of the scenario problems. This allows them to be solved independently of one another, making possible the use of parallel computing techniques. Unfortunately, the solution of (4.2) typically requires that an outer linearization of $g(\pi)$ (master problem) be constructed from solutions to (4.4).

The number of cutting planes in this outer linearization that are necessary to adequately approximate $g(\pi)$ grows unmanageable as the number of scenarios in the problem becomes large. Hence, the growing master problem creates a bottle neck that reduces the benefits intrinsic in the separability of the subproblems.

In order to get around the aforementioned problem, we consider the augmented lagrangian

$$\Lambda(x,\pi) = \sum_{i=1}^{L} f_i(x_i) + \pi^T \sum_{i=1}^{L} A_i x_i - \frac{1}{2}\rho \left\| \sum_{i=1}^{L} A_i x_i \right\|^2 \tag{4.5}$$

with penalty parameter $\rho > 0$. Analogously to the ordinary lagrangian, one can solve the problem

$$\min_{\pi \in R^m} (\gamma(\pi) = \sup_{x \in X} \Lambda(x,\pi)) \tag{4.6}$$

to obtain a solution to (4.1). In this case, though, the solution procedure requires no master problem and, hence, no attending bottle neck. The solution is obtained via the "Method of Multipliers" algorithm which follows:

Step 1. Given π^k, determine x^k, a solution to

$$\max_{x \in X} \Lambda(x,\pi^k). \tag{4.7}$$

Step 2. If $\sum_{i=1}^{L} A_i x_i^k = 0$, stop. Otherwise

$$\pi^{k+1} = \pi^k + \rho \sum_{i=1}^{L} A_i x_i^k, \tag{4.8}$$

iterate k and goto Step 1.

The addition of the quadratic regularizing term in (4.5) makes $\Lambda(x,\pi)$ inseparable in x_i. To restore this separability and make the solution of (4.7) decomposable, consider the set $i = 1, 2, \ldots, L$ of functions $\Lambda_i : R^{n_i} \times R^n \times R^m \to R$,

$$\Lambda_i(x_i, \tilde{x}, \pi) = f_i(x_i) + \pi^T A_i x_i - \frac{1}{2}\rho \left\| A_i x_i + \sum_{j \neq i} A_j \tilde{x}_j \right\|^2. \tag{4.9}$$

Given a π^k, we can use this set of functions to obtain the minimizing x of (4.7). This is done via a Jacobi type algorithm. An initial $\tilde{x}^s$, $s = 0$, is chosen. Each $\Lambda_i(x_i, \tilde{x}^s, \pi)$ is maximized in x_i, independent of all other $\Lambda_j(x_j, \tilde{x}^s, \pi)$ maximizations, $j \neq i$. The resulting vector of optimal x_i^* from the subproblems, $x^* = (x_i^*, i = 1, 2, \ldots, L)$, is used to construct a new $\tilde{x}^{s+1} = \tilde{x}^s + \tau(x^* - \tilde{x}^s)$. This process repeats until the vector x^* is identical to $\tilde{x}^s$. [Rus92] establishes that the structure of the nonanticipativity matrix allows us to set the step size $\tau = \frac{1}{2}$. This makes the Jacobi algorithm convergent at a linear rate. In addition, because the $\Lambda_i(x_i, \tilde{x}, \pi)$ functions are separable, the algorithm can be implemented in parallel. It is this algorithm that I have implemented to solve our stochastic model.

4.3 Implementation of Stochastic Model under GAMS

We have developed two different computer codes, using the Augmented Lagrangian Decomposition method, for solving the stochastic economic model.

Our first code runs entirely within the confines of GAMS, using the **LOOP**, assignment and **SOLVE** facilities provided therein [1]. As such, it is accessible to any modeler having equipment no more sophisticated than a PC running GAMS and a suitable mathematical programming solver like MINOS [MuS87]. To protect against the possibility of a system crash, we ensure that models that terminate prematurely can be restarted from an advanced point by maintaining an updated copy of the multiplier values in a file at all times. Finally, all results from the model are output in a form readable by popular spreadsheets like Excel. In this way, results can be quickly and easily analyzed.

Our (see figure 2) second code is a C program that runs in parallel on a network of

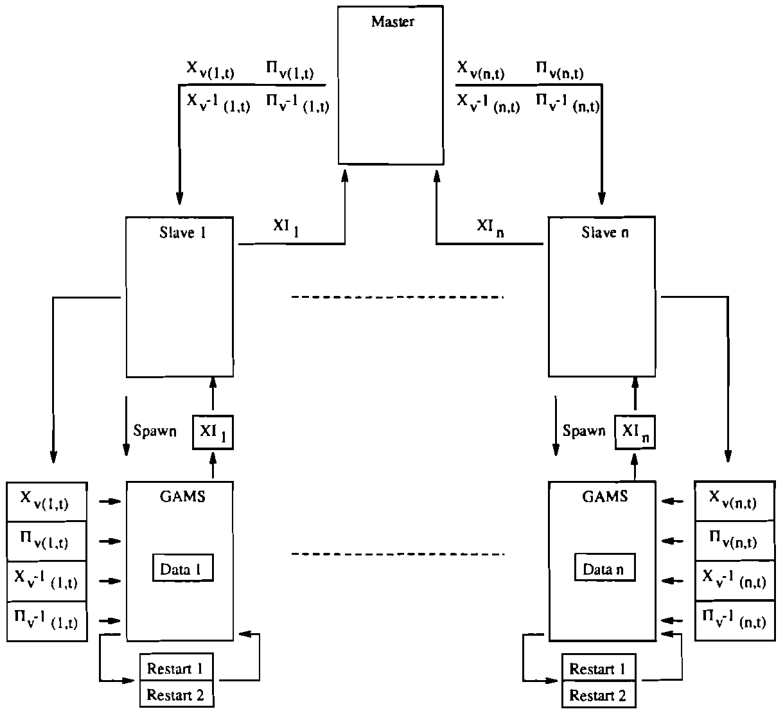

Figure 2: Parallel GAMS stochastic programming solver

UNIX workstations using the software library of routines, PVM. Like the first program, the model is solved using GAMS and an available mathematical programming solver. The linkage between GAMS and the parallel program is affected via the system level UNIX routines, fork(), exec(), and wait(), that enable the C program to spawn the GAMS program as a temporary child process and wait for it to finish. The spawned GAMS process reads input files, suitably modified by the C program, and produces results in an output file which is subsequently read by the C program. The input files that are repeatedly read by the GAMS process during the algorithm play the role of restart files in the event of a system crash or node failure within the virtual parallel machine constructed by PVM.

Regarding our design criteria for the second implementation, our chief aim was to create a flexible stochastic program modeling system. By keeping all the specifics of the particular model within the confines of separate deterministic dynamic GAMS modules,

[1]A portion of the GAMS code, showing the main loops of the algorithm, is shown in the appendix

Constraints	398
Nonlinear Constraints	25
Variables	610
Nonlinear Variables	85
Multipliers	408

Table 4: Scenario Model Characteristics

modifying the system to work with a completely new model requires only a modest amount of time. In this way, practitioners who are working with deterministic GAMS models can very quickly supplement their investigation of deterministic model behavior by performing an explicit stochastic analysis.

In addition to the modeling flexibility afforded to the practitioner by our systems use of the GAMS modeling language, our use of PVM allows the modeler to flexibly take advantage of what ever access he has to a network of workstations. Depending on the size of network available, he can use any number of machines he wishes to boost the algorithms performance. Of course we don't pretend that our system will perform anywhere near that level attained by systems that have been tailored to their particular problem structure. We feel, though, that the ease of model development more than makes up for this deficiency. In the next section, we present results we obtained while running our stochastic energy model.

4.4 Algorithm Performance

The stochastic energy model we solved had eight separate scenarios. The characteristics of each of these individual scenario models is summarized in table 4.

Using a stopping criteria for the outer loop of $\epsilon = 10^{-4}$, a stopping criteria of half of this for the Jacobi steps in the inner loop, a cold start [2], and three SUN 4 workstations running the operating system SOLARIS and coordinated via PVM, we were able to consistently obtain solutions to the problem in 18-20 hours. In a representative run, the algorithm performed 59 outer loops and 705 inner loops, each inner loop requiring the solution of the eight separate GAMS scenario models. The inner loops were mainly concentrated at the beginning of the algorithm as indicated in figure 3 because of the cold start. This suggests one obvious means of speeding up the algorithm: Using problem structure and intuition to locate a good initial solution. The maximum relative error in nonanticipativity over the course of the algorithm is displayed in figure 4. The algorithm initially converges linearly as the changing multipliers on the relaxed nonanticipativity constraints move the solution along linear constraints. The progress is later impeded by the presence of the nonlinear constraints and variables whose properties slow the search for the appropriate penalties. Finally, figure 5 plots the number of nonanticipativity constraints violated at each multiplier step. Again, very fast progress at the start is followed by a slower linear rate of convergence until the end.

[2] multipliers and Jacobi updates all set to zero

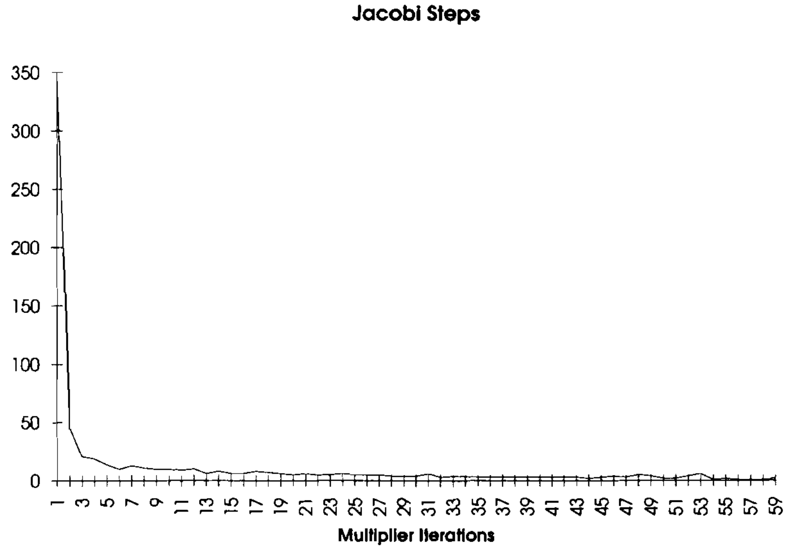

Figure 3: Number of Jacobi steps in each outer loop

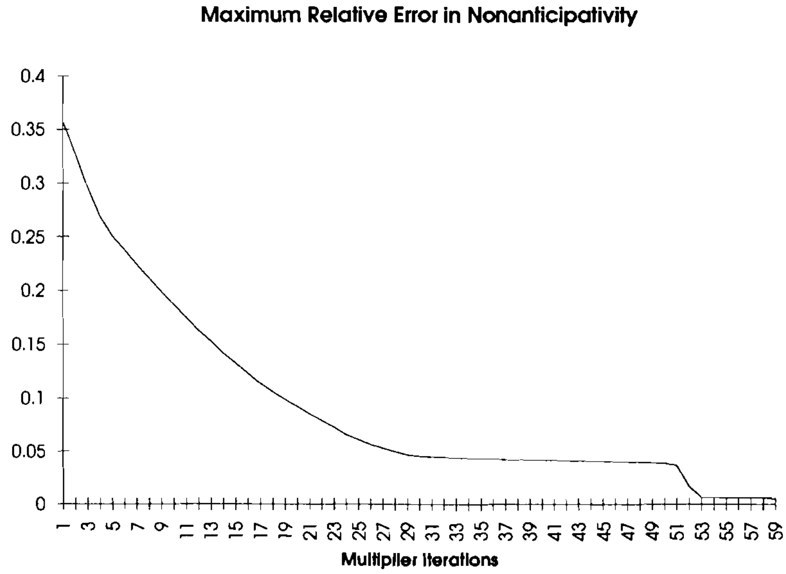

Figure 4: Max. relative error

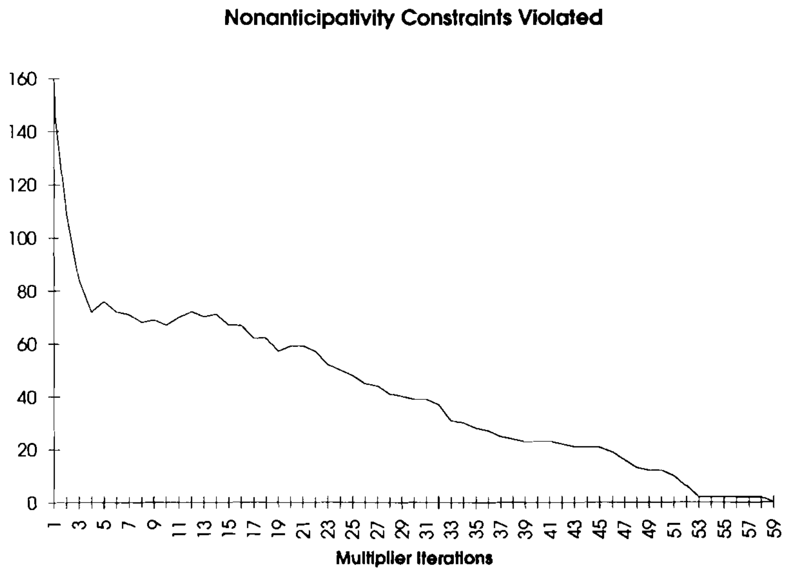

Figure 5: Number of nonantic. violated constraints

As is clear from the numbers displayed, a quick solution time is not one of our systems virtues. Its performance suffers on a number of fronts. Most significant is the requirement that GAMS be restarted from binary files each time an individual scenario problem is to be solved. A modification here would lead to an appreciable improvement in overall performance.

As already stressed though, our systems main strength is the ability it gives practitioners to quickly explore the impact stochastic programming has on deterministically formulated GAMS models. In the next section we consider this impact on the results returned by our energy model.

5. Policy Analysis

We have applied stochastic programming methodology to the USA region of the Global 2100 model, leaving the other regions in the model, and the resultant trade issues (oil and carbon credits), for a future study. We have solved two versions of the stochastic programming model. The first is with the CO_2 constraints suggested by Manne and Richels [MaR92] and the second is with no CO_2 constraint. This will help us to establish whether there is any relationship between the expected value of perfect information (the difference between the expected return with perfect information and under uncertainty) and the restrictiveness of the carbon emissions constraint.

For both cases we included eight scenarios to form a three-stage stochastic programming problem. As mentioned before, the sources of uncertainty we included are the future costs associated with coal and gas sources of both nonelectric and electric energy, the rate of autonomous energy efficiency improvement expected, and the structural importance of nonelectric energy in the production function over time. This last uncertainty is summarized by the reference nonelectric price of energy ($/GJ) which is used, along with the assumption that the price of an input is equal to its marginal productivity, to derive all

Scenario	AEEI	PNREF	Cost(Coal)	Cost(Gas)
1	0.5%	1.0	low	high
2	0.5%	1.0	high	low
3	0.5%	4.0	low	high
4	0.5%	4.0	high	low
5	2.0%	1.0	low	high
6	2.0%	1.0	high	low
7	2.0%	4.0	low	high
8	2.0%	4.0	high	low

Table 5: Scenarios of the Stochastic Model

the parameters that control the specific shape of the production function.

After consulting with the energy experts at IIASA about the timing and level of uncertainty, we decided that in period 2020 the rate of efficiency improvement will either increase or remain constant at its base case value. Also in this period we assume that the reference price of nonelectric energy will either increase or decrease from the level assumed in the base case. This translates into four branches of the tree. We assume that each of these branches will occur with equal probability. After the resolution of uncertainty regarding efficiency improvements and the structural form of the production function, we assume the ultimate costs of gas and coal technology are revealed in 2030. We consider only two possibilities. Coal becomes 20% less expensive and gas 25% more expensive than the base case or vice versa. These two cases, also occurring with equal probability, correspond to an additional branch at the end of each of the first four branches, leading to a total of eight scenarios. We make the assumption that each of these branches also occurs with equal likelihood. Because we've assumed for this initial study that these uncertain events are independent, we can conclude that each of the eight scenarios is equally probable. The scenarios are summarized in table 5.

5.1 Carbon Limits

As the world economy evolves over time, decision makers attempt to develop investment, consumption, and production strategies that are optimal in the face of the uncertainty they perceive regarding the future. These strategies are optimal in the sense that they hedge against the many different possible states that important economic parameters may be in over the horizon. The objective that they frequently maximize to produce these strategies is the expected value of the figure of merit, whether it be profit or utility. Stochastic programming is a tremendously valuable tool for modeling this very phenomena. We use this in the U.S. submodel of the macroeconomic model with which we are working and show that it produces results that differ markedly from those returned by scenario analysis. We stress that our results are only indicative of the additional information that a stochastic programming analysis can offer to the policy debate. They are not themselves policy suggestions.

Energy Capacity Development:

One of the principal problems that modelers have when using deterministic models to derive hedging strategies is the sensitivity of these models solutions to the scenario being considered. Scenarios that are identical in the early part of the horizon and differ only later on may still give rise to initial period decisions that are quite different. This makes it difficult to derive initial policy decisions that are robust for a wide variety of scenarios. The energy modelers at IIASA, having encountered this so-called "bang bang" property of deterministic models in some of their work, have devised various methods for dealing with it. We now investigate how stochastic programming may also be able to help in this endeavor.

The USA submodel that we worked with exhibited this "bang bang" behavior. For example, we observed that the system, under scenario 4, a high cost coal/low cost gas outcome, invested almost exclusively in its capacity of new gas technology while investing nothing in its advanced coal technology (Figure 6). Conversely, under scenario 1, a high cost gas/low cost coal outcome, the coal technology was developed while the gas technology languished (Figure 7). This phenomena arises in part because of the putty clay nature of the Manne and Richels model. A technology that is in use cannot suddenly be turned off. Its capacity declines gradually as it becomes obsolete. As a result, a scenario which might have preferred to use a technology for only a short period of time does not use it at all if the cost of maintaining the technology during the period of decline is too great. Clearly, this "bang bang" behavior of the model was not helpful for determining a here and now strategy of capacity expansion. The stochastic program provides this "here-and-now" solution by balancing the needs for investing in low cost technology with the uncertainty regarding what that technology will be (Figures 8, 9).

Similarly, we observed that under scenarios for which nonelectric energy was assumed to be highly productive (scenario 4), the nonelectric backstop technology was introduced early in the horizon, while in scenarios for which nonelectric energy was less important to the economy (Scenario 1), it is not introduced until later. In both of these scenarios, the stochastic programming solution suggests more rapid initial development (Figure 10). Later, when it is learned just how critical nonelectric energy is to economic growth, the nonelectric back stop technology can be scaled back or expanded. Interestingly enough, for the case of synthetic fuels development the solutions for scenarios one and four differ very little. This would seem to suggest that the hedging strategy would also be similar. As can be seen from Figure 11, this is not the case.

Increased investment occurs in both of these alternative nonelectric energy technologies because of the uncertainty regarding the future price of all grades of gas for nonelectric energy use. The uncertainty is just large enough to make it beneficial to invest, at least to some extent, in synthetic fuels and the nonelectric backstop because their price structure is known with certainty. The extreme expense of both the advanced electrical technologies means that even with the uncertain price of coal and gas electrical generating technologies, they are still not pursued until later in the horizon. In this latter case, the price of certainty is higher than the price of uncertainty.

Energy Mix in the Economy:

These differences in the rates of development of various technologies lead to a sizable difference in the mix of electrical and nonelectrical energy within the economy as seen in figures 12 and 13. In particular, the optimal hedging strategy emphasizes electrical energy over nonelectrical energy because of the uncertainty regarding the structural importance of nonelectrical energy to economic output in the future.

Imports of Oil:

In the model, oil is imported into the USA region from the world market at a price that changes over time based on estimates made by energy/economy experts. As we consider only the USA region, the region is supplied with as much of this resource as it desires at the given price. There is no attempt to balance supply and demand as we consider only the one region. Our scenario analysis suggested that oil imports would remain constant until the turn of the century, at which point it would likely increase. The path from this point on was dependent upon which scenario would ultimately occur (figure 14). In contrast to this, the stochastic hedging strategy suggests that the level of foreign oil use decreases from 1990 levels until the turn of the century. At this point, oil use either increases or continues to decrease depending on future uncertain events. In all cases, though, it remains below that predicted by scenario analysis for the next half century. This difference between the predicted level of foreign oil use returned by scenario analysis and the actual hedging strategy is most probably the result of the accelerated development of alternative energy forms, the decreased reliance on nonelectric fuel types that occur in the economy as a result of uncertainty regarding the ultimate importance of nonelectric fuels to economic output, and the greater application of natural gas to nonelectric applications as seen in figure 15.

Natural Resource Utilization:

Another area in which our scenario analysis began to exhibit the "bang bang" property of deterministic programming was in modeling the utilization of domestically produced low and high cost oil and gas. Figures 16, 17, 18, and 19 show the time path of the stock of these resources as returned by our scenario analysis solution and the optimal hedging solution. Scenario analysis indicated that the path of resource utilization is not impacted by the scenarios in question. The low cost resources are used first and as they are depleted, the higher cost resources are developed. In contrast to this, the hedging strategy dictates that higher cost resources start to be used earlier along with the lower cost resources in order to partially conserve the lower cost resources for the future, and offset losses due to decreased imports of foreign oil. In the future, the lower cost resources can then be used more quickly in order to offset the increased cost of alternative energy forms. The hedging solution is qualitatively quite different from the scenario analysis solution.

It is quite clear from the examples listed above that the hedging strategy, derived using stochastic programming, can be fundamentally different from the strategies observed using scenario analysis. For this reason it is profitable to supplement traditional scenario analysis of large scale economic models that are known to have uncertain data with this technique. Of course, the benefits of analyzing a model with stochastic programming vary with the structure of the model. That is, depending on the structure of the model, the uncertainty inherent in the data may or may not significantly impact present decisions.

In the next section we consider the results of a stochastic programming analysis of the United States with the constraint on carbon dioxide emissions removed. One principal question will be whether the absence of these constraints reduces the impact that uncertainty over the future has on present decisions and, thus, reduces the need to perform a stochastic programming analysis.

5.2 No Carbon Limits

We consider this model in order to determine how the U.S. economy will develop in the absence of constraints on CO_2 emissions. It can be considered as a sort of base case against which to measure the losses to economic output of varying emissions constraint regimes. We make all the same assumptions originally made by Manne and Richels in their deterministic model regarding the development of oil trade, and the path that nuclear power and other existing conventional technologies follow. We also assume the same future as described by the previous decision tree.

Energy Capacity Development:

As with the previous case, the hedging strategy capacity expansion plans differ from those of the individual scenario strategies, but with some interesting differences.

Unlike the previous case, the predicted path of electrical power production using advanced coal and gas generation techniques differs almost not at all between the two solution methodologies. In fact, in the case of new gas production, the evolution is the same across all scenarios (figures 22 23). New gas production peaks shortly after the turn of the century and then declines as stocks dwindle. Electricity production from new coal technology grows steadily until the early part of the next century without the concern for carbon emissions. After 2010, coal use declines given an expensive coal future and continues to increase otherwise (figure 33).

The generic advanced low cost electricity generating technology becomes the chief competitor to coal use. In those scenarios for which coal is expensive (i.e. scenarios 1,3,5,7), the alternative dominates (figure 34). The robust strategy returned by the stochastic programming solution indicates early development of this technology, identical to the results returned by those scenarios for which coal is expensive. If it becomes known with certainty in 2020 that coal will be inexpensive, the robust strategy removes the existing capacity as do the strategies returned by those scenarios for which coal is cheap which only develop a small amount of the alternative technology in 2010 (figure 24). Apparently it is better to over invest in the alternative than be caught short of electrical generating capacity and have to make up the difference with prohibitively expensive new coal technology.

The hedging strategy methodology suggests more rapid development of the renewable nonelectric energy technology than any of the separate scenario strategies (figure 30). Along with this, it suggests a rate of synthetic fuels production that lies closest to that suggested by scenario four (i.e. expensive coal) (figure 31). This makes sense in light of the uncertainty regarding the future pricing of both coal and gas which are both so critical to nonelectric energy production.

Energy Mix in the Economy:

The robust strategy predicts that the development of both overall electrical capacity and nonelectrical capacity is very similar to the rate of development predicted by the scenario solutions (figures 20 and 21). Though this is similar, the individual technologies producing the two forms of energy differ. In particular, the robust strategy predicts that greater quantities of nonelectrical energy will be produced via the burning of natural gas

Scenario	Scenario	Hedging
1	59793.3	59772.8
2	59755.9	59734.9
3	59556.9	59537.9
4	59487.5	59467.7
5	59949.2	59928.7
6	59920.8	59899.9
7	59819.5	59800.3
8	59766.3	59746.4
Expected Value	59756.2	59736.1

Table 6: Utility in Each Scenario and Expected Value - No CO_2 Limits

during the early part of the next century (figure 25). This, together with greater reliance on renewables, leads to an appreciably lower level of carbon emissions during this same period (figure 35) than that predicted by scenario analysis.

Imports of Oil and Domestic Natural Resource Utilization:

Because of the accelerated development of synthetic and renewable fuels, and a utilization curve for domestic gas and oil resources which is similar to the constrained emission case (figures 26, 27, 28, 29), the predicted level of oil imports from the present until the early part of the next century is lower for the robust strategy (figure 32) than any of the individual scenarios looked at during the scenario analysis.

All the examples listed above demonstrate that the presence of uncertainty has a profound effect on the predicted hedging strategy even for this unconstrained case. In other words, the hedging strategy cannot be trivially reconstructed by looking at each of the scenarios independently.

5.3 Expected Value of Perfect Information

It is often interesting to determine from a model an upper bound on the value of having a perfect state of knowledge regarding the future. This upper bound is referred to as the expected value of perfect information and is the absolute difference of two quantities: The expected objective value corresponding to the optimal hedging strategy and the expected value of the optimal objective values returned from the scenario analysis. This value is, of course, greater than zero as one can always develop superior planning strategy when it is known with certainty what the future holds.

We first consider the case of no carbon limits. Table 6 contains the utility values of scenarios from both the scenario analysis and the stochastic programming problem. As predicted by theory, the scenario analysis utility values are higher for each scenario because they correspond to decisions made under perfect information. This leads to a higher expected utility and to an expected value of perfect information equal to 20.1 utils, a value only 0.03 % of the optimal return obtained when the hedging strategy is used. It is clear that in using the hedging strategy, one is doing almost as well as is possible. This, coupled with previous discussion regarding the differences between the economic

Scenario	Scenario	Hedging
1	59663.4	59607.4
2	59657.6	59600.2
3	59194.1	59133.1
4	59199.6	59132.1
5	59849.2	59791.5
6	59837.6	59778.5
7	59559.4	59495.1
8	59564.8	59494.3
Expected Value	59565.7	59504.0

Table 7: Utility in Each Scenario and Expected Value - CO_2 Limits

path of evolution predicted by the robust strategy and that predicted by the scenario strategies indicate that there is much value in using stochastic programming to model decision making under uncertainty.

Table 7 shows the corresponding values for the case in which CO_2 emissions are limited.

In this case, the expected value of perfect information is now 61.7 utils which is 0.1 % of the expected return when one follows the hedging strategy. This value is three times greater than the previous case and is approaching the magnitude of difference expected between the unconstrained and constrained CO_2 cases (59756.2 - 59565.7 = 190.5 (0.3 %)). It is clear that the increased pressure on the economy caused by emissions limits accentuates the impact of uncertainty. The economy has less room in which to maneuver and, hence, is less able to effectively hedge against all the possible scenarios.

6. Conclusion

Prediction of future economic behavior is fraught with difficulties given the monumental uncertainty regarding the future state of many key economic parameters. In this paper we have used stochastic programming to model how uncertainty regarding this might impact decisions made by economic agents. In so doing we have seen that the predicted path of economic development can change in rather dramatic ways, depending on the problem, from those paths predicted by scenario analysis. For this reason, we feel that stochastic programming is a valuable supplement to scenario analysis in the analysis of economic systems over time and under uncertainty.

Also in this paper, as a way to ease the development cost for modelers of all disciplines who are interested in exploring what impact the explicit consideration of uncertainty might have on their deterministic problems, we have developed a modest parallel stochastic programming solver, based on the Augmented Lagrangian Decomposition algorithm, that incorporates the commercially available GAMS modeling system. The system, though still in the development stage, allows for a relatively quick stochastic analysis of dynamically formulated deterministic GAMS models. Experience thus far indicates that, while solution times are much slower than those obtained for solvers designed for particular problem structures, its generality makes it valuable as a research tool in the analysis of uncertainty on a variety of problems.

7. Acknowledgements

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A Stochastic Programming GAMS Code

```
* Outside loop controlling the changing pi values
LOOP(ITEROUT$(continueou EQ 1),

* Inside loop controlling the changing x values
LOOP(ITERIN$(continuein EQ 1),

* Solving each scenario problem
LOOP(SCEN,

$INCLUDE initparam

$INCLUDE logicreg

* This file contains two solves: One for the Carbon Limit with No trade
* One for the Carbon Limit with Trade
* One of these should be commented out for the duration of a run
*$INCLUDE logicp12
* This file contains the instructions for running the unconstrained
* business-as-usual case.
$INCLUDE logicp3

* update xi values for each scenario
$INCLUDE logicsetxi
);
*end of SCEN loop

* If xi is not sufficiently close to x, continue iterating in the inner loop
LOOP(SCEN,
$INCLUDE checkxstep
);
*end of SCEN loop

* Update values of x based on values of xi
LOOP(SCEN,
$INCLUDE logicsetx
);
*end of SCEN loop

);
*end of ITERIN loop

* If the nonanticipativity constraint is not sufficiently approximated, continue iterating
in the outer loop
```

```
LOOP(SCEN,
$INCLUDE checkpstep
);
*end of SCEN loop

* Update values of pi based on x values
LOOP(SCEN,
$INCLUDE logicsetp
);
*end of SCEN loop

);
*end of ITEROUT loop
```

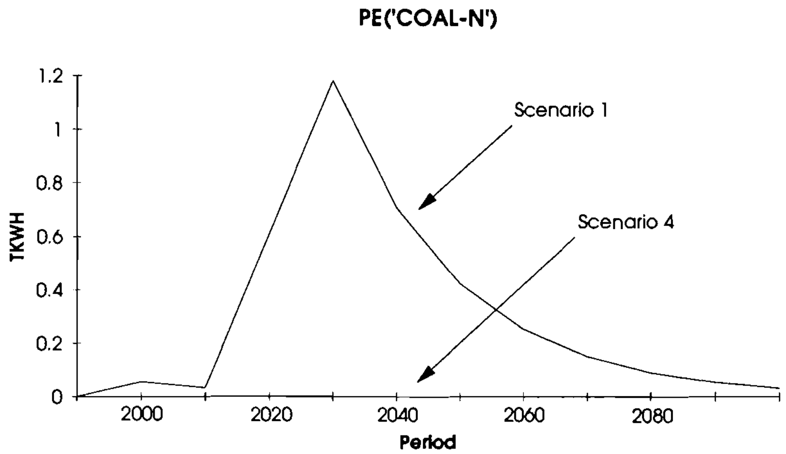


Figure 6: New Coal Tech. Capacity

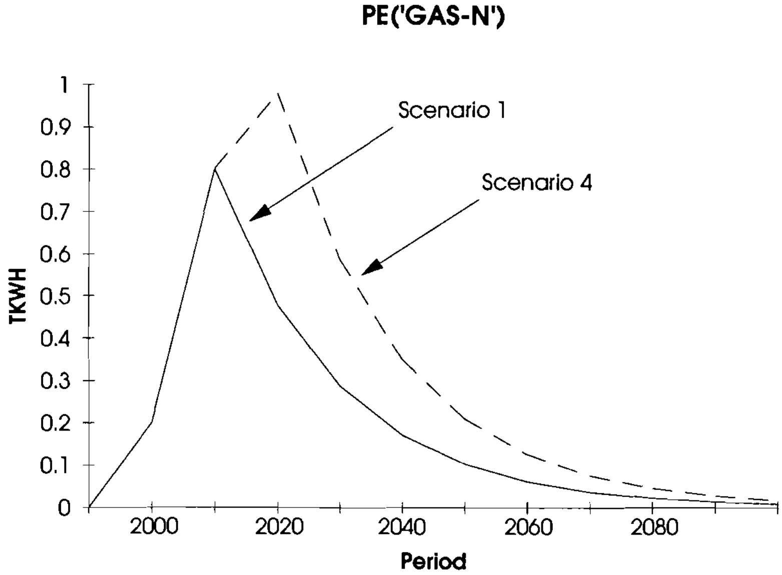


Figure 7: New Gas Tech. Capacity

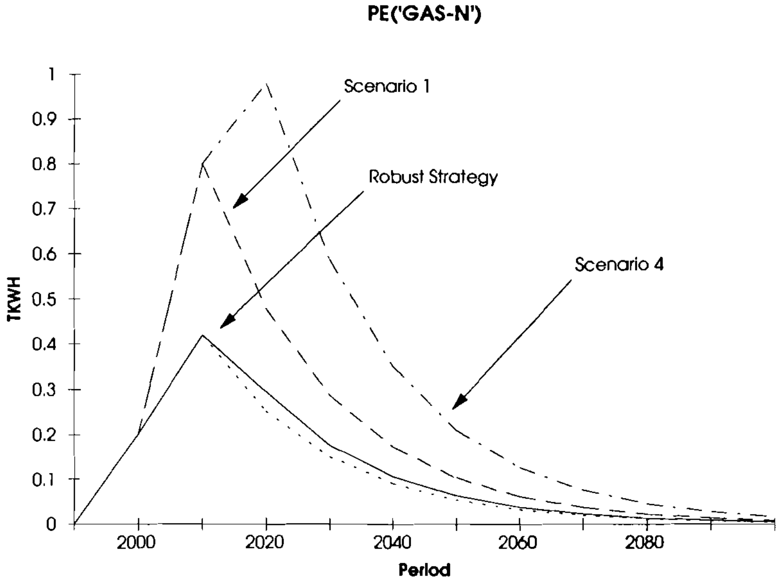


Figure 8: New Gas Tech. Capacity

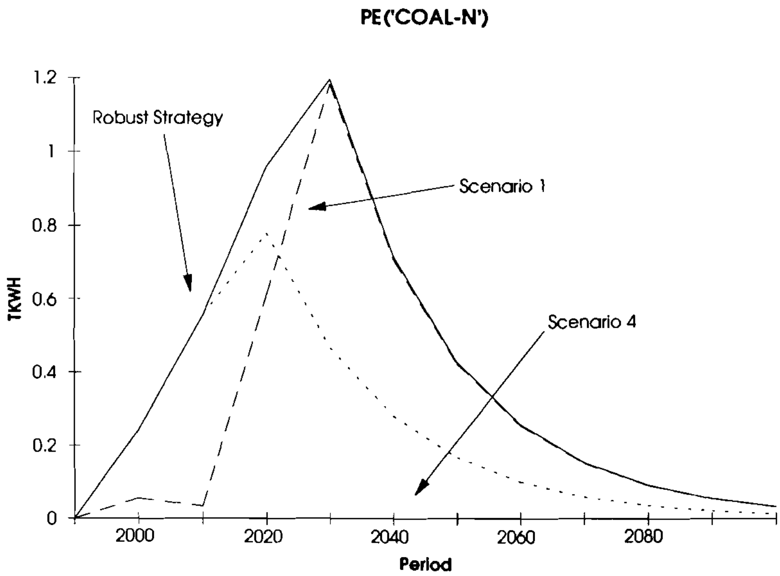


Figure 9: New Coal Tech. Capacity

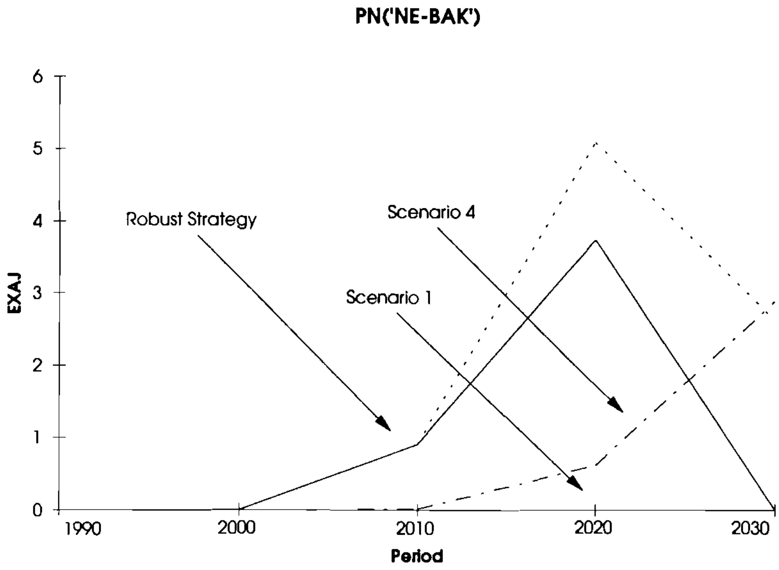


Figure 10: Nonelectric Backstop Capacity

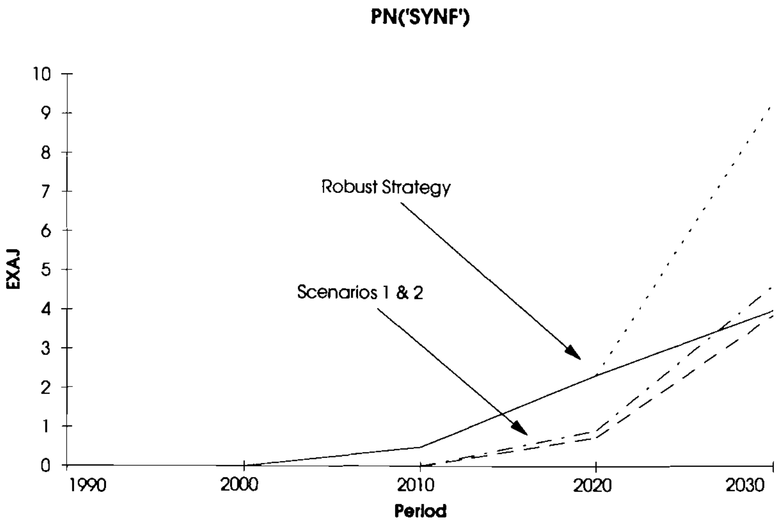
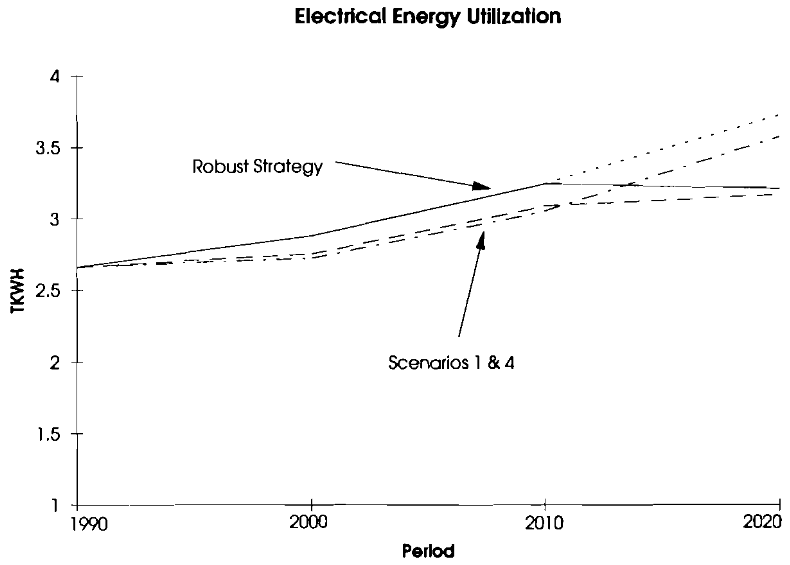


Figure 11: Synthetic Fuels Capacity



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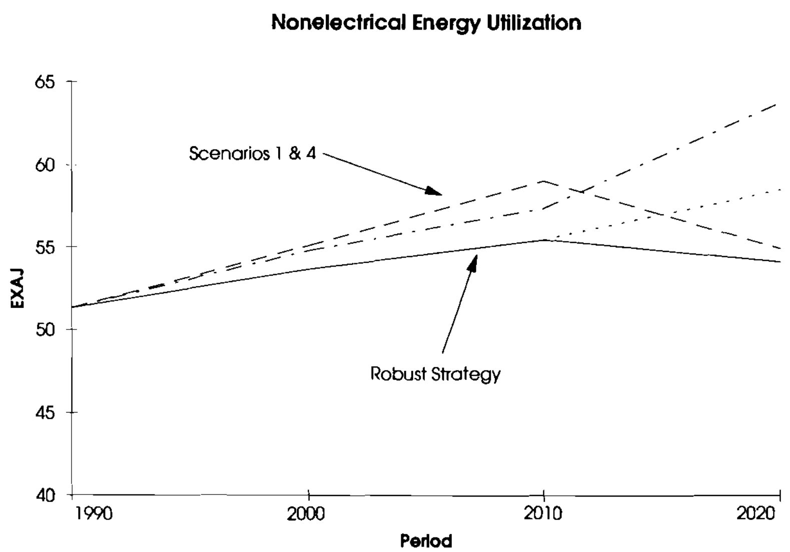

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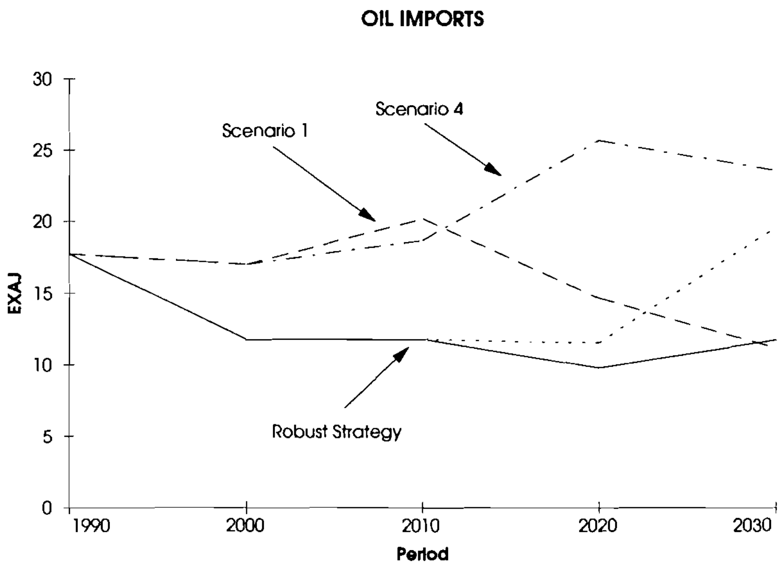

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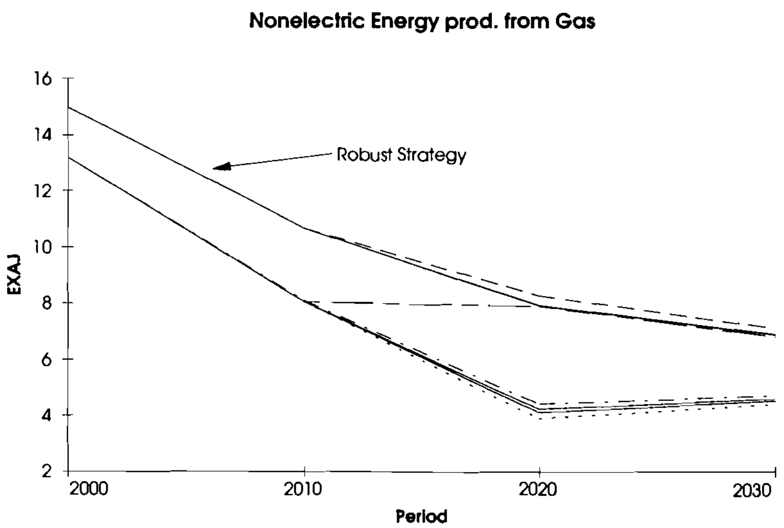

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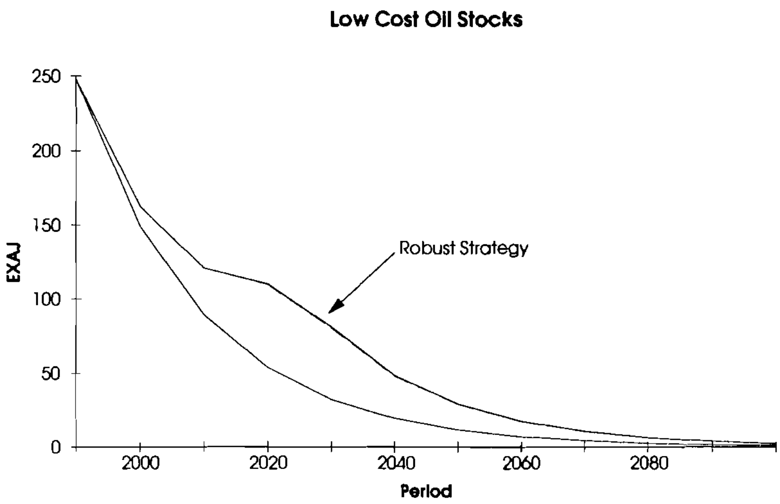

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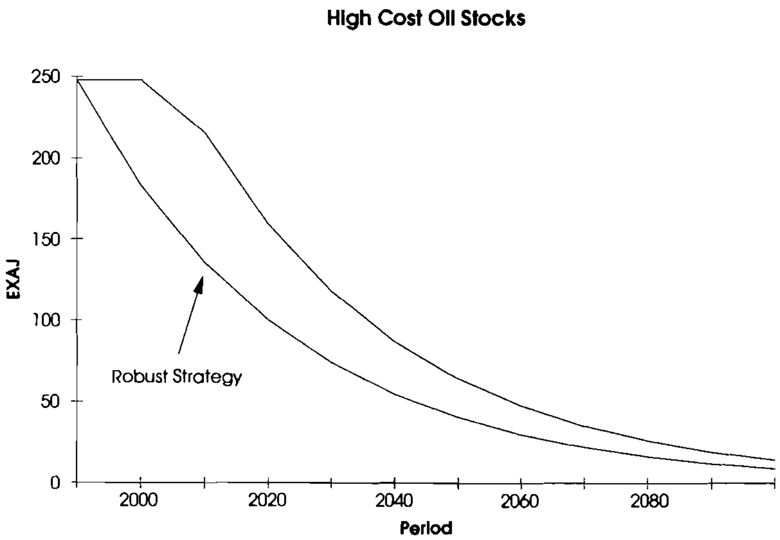

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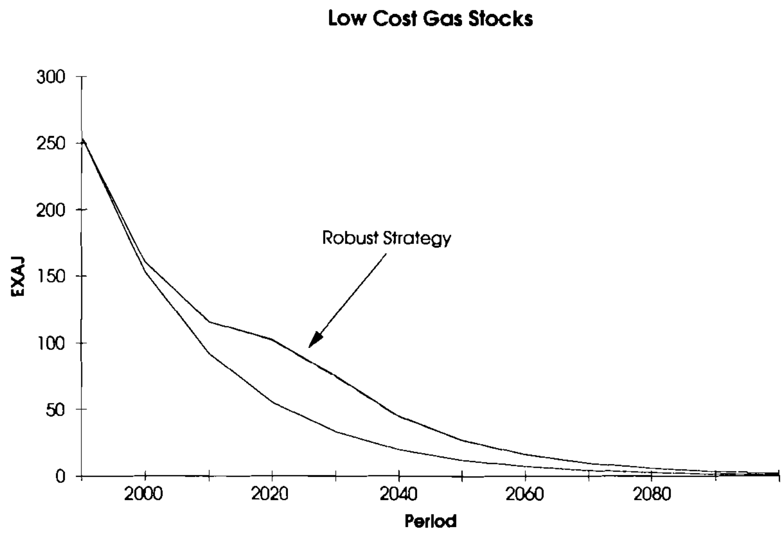

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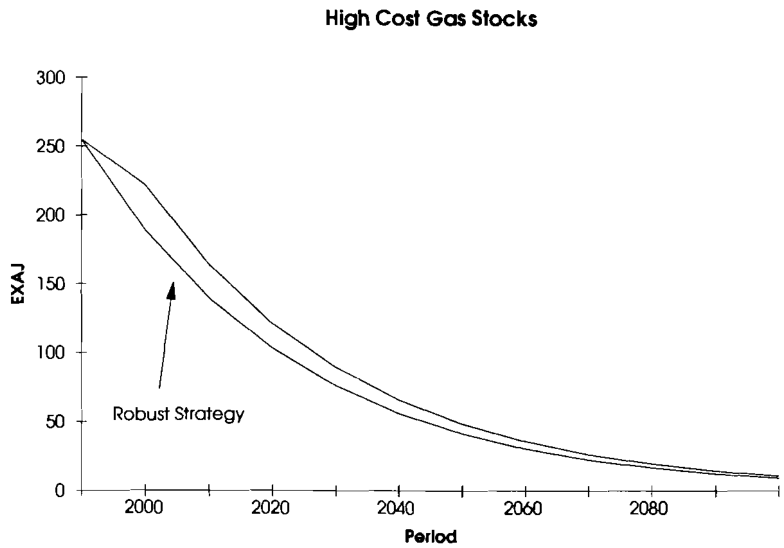

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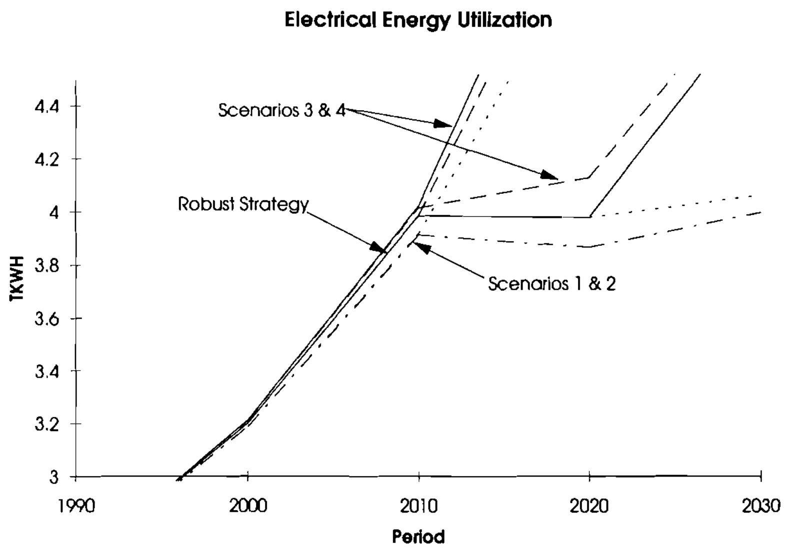

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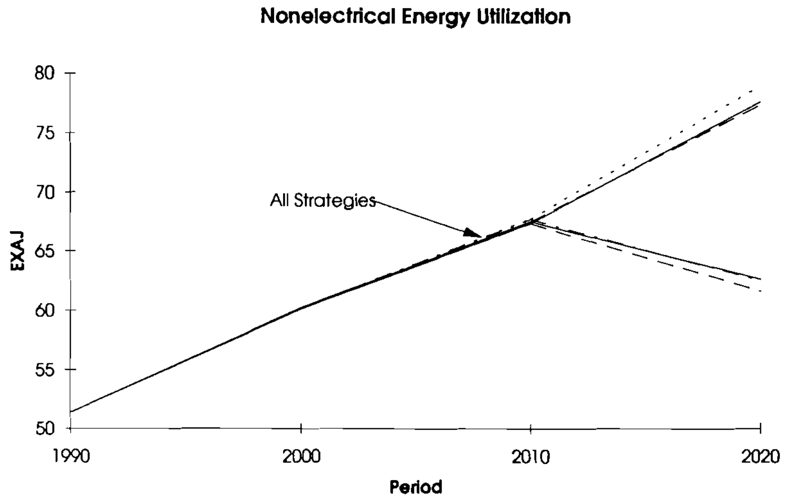

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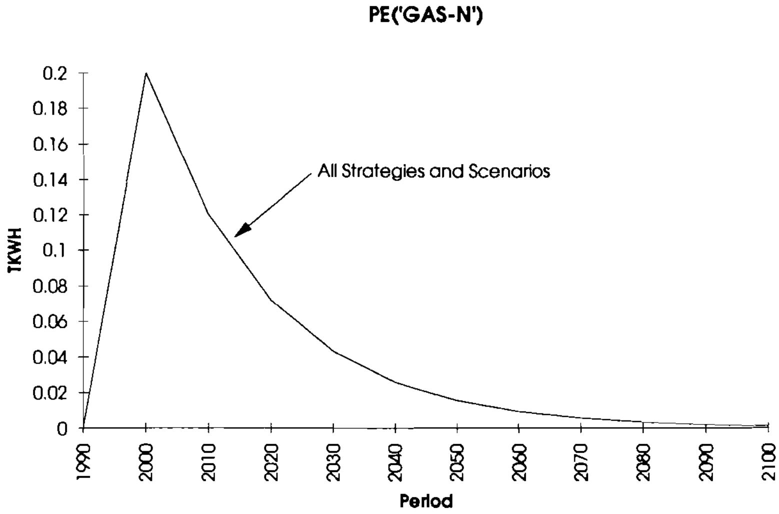


Figure 22: New Gas Tech. Capacity

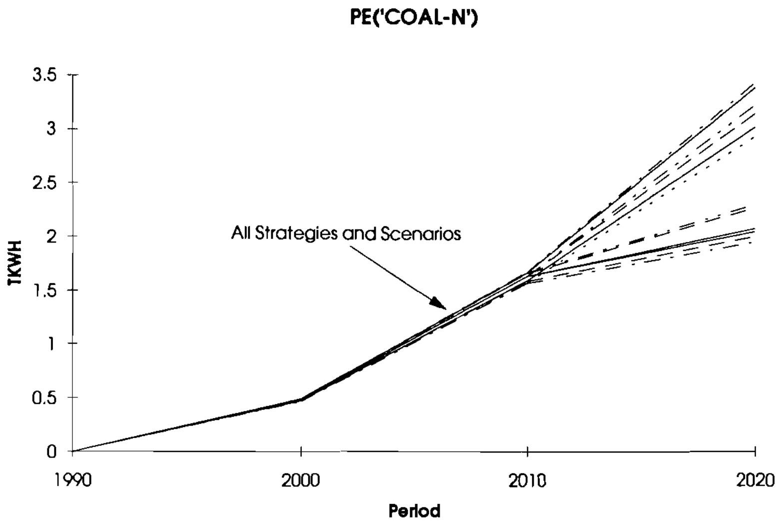


Figure 23: New Coal Tech. Capacity

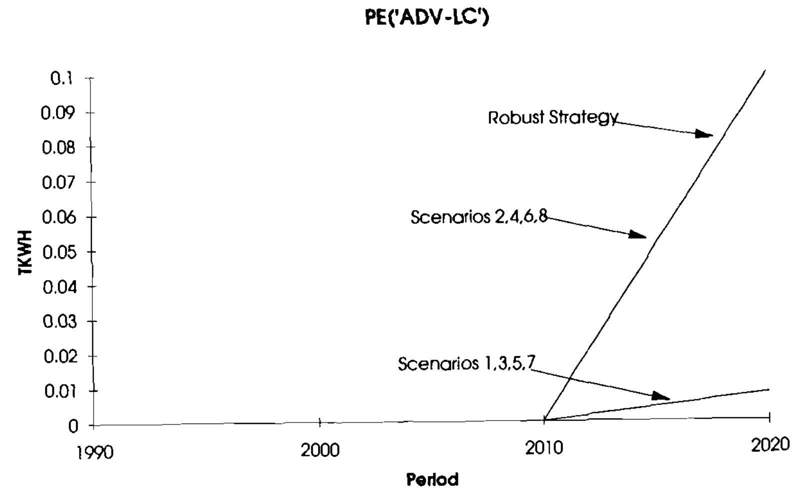
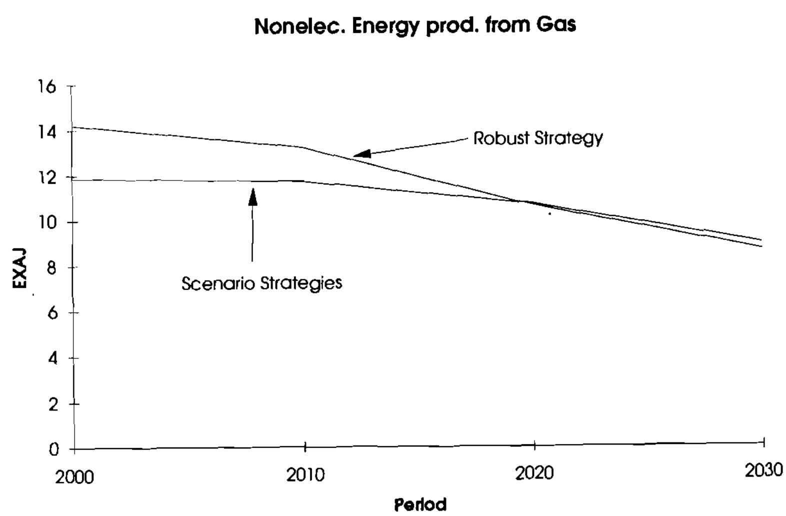


Figure 24: Advanced Low Cost Elec. Energy Capacity



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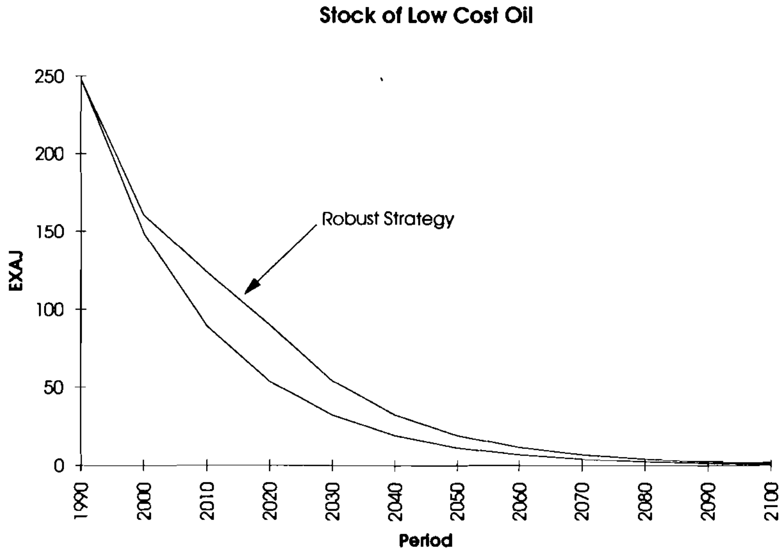

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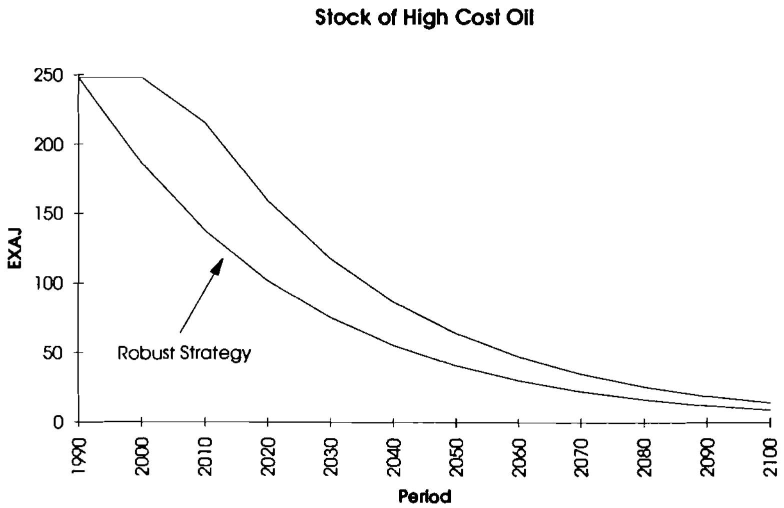

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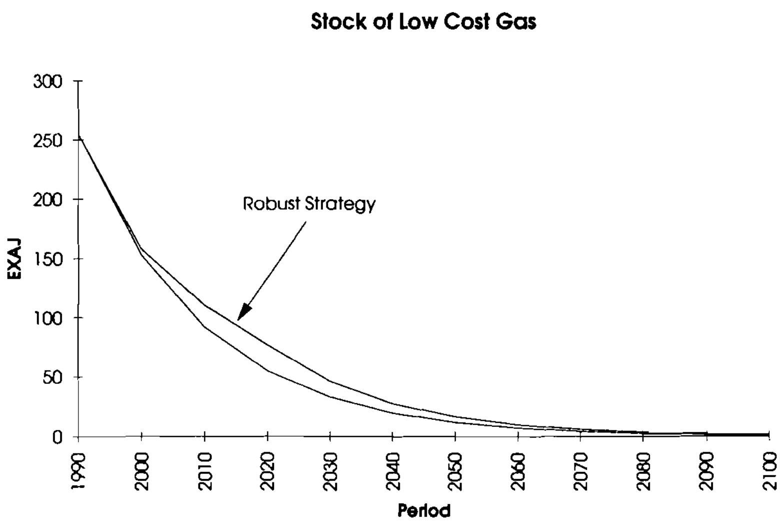

Figure 28:

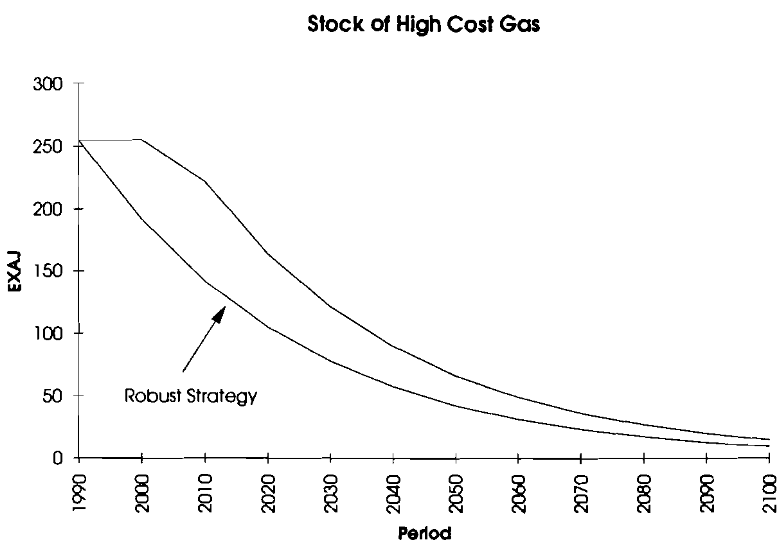

Figure 29:

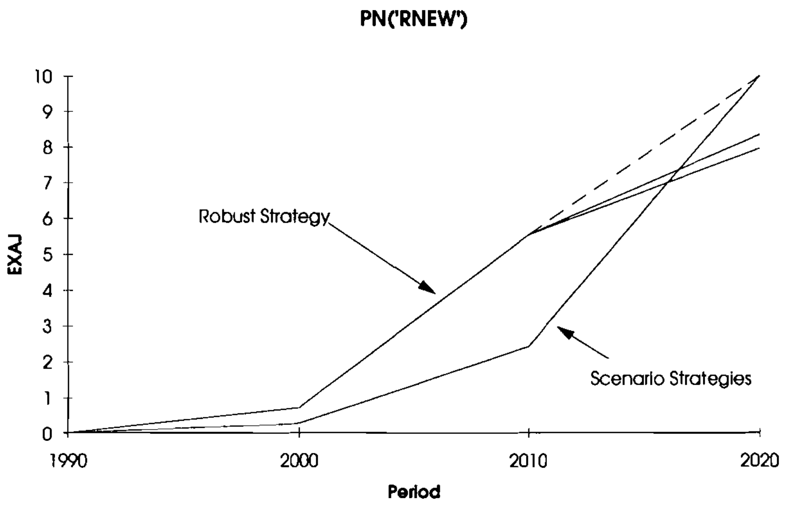

Figure 30: Renewable Fuels Capacity

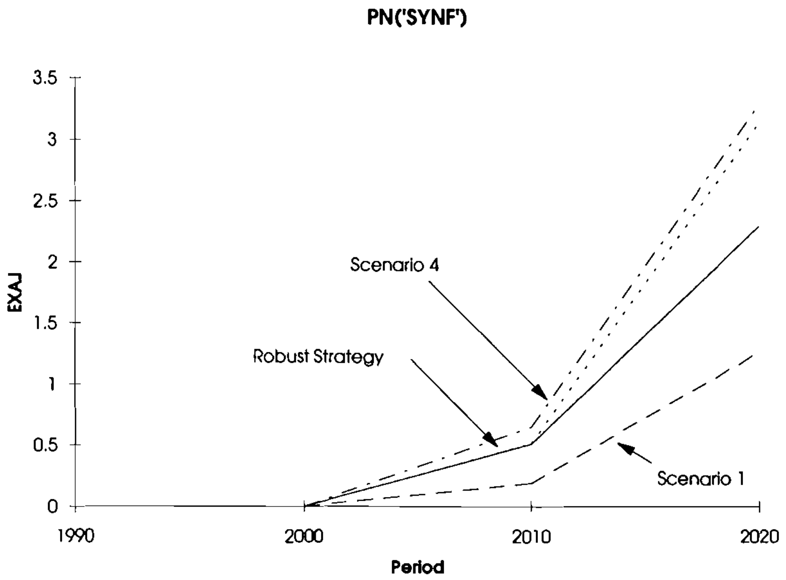

Figure 31: Synthetic Fuels Capacity

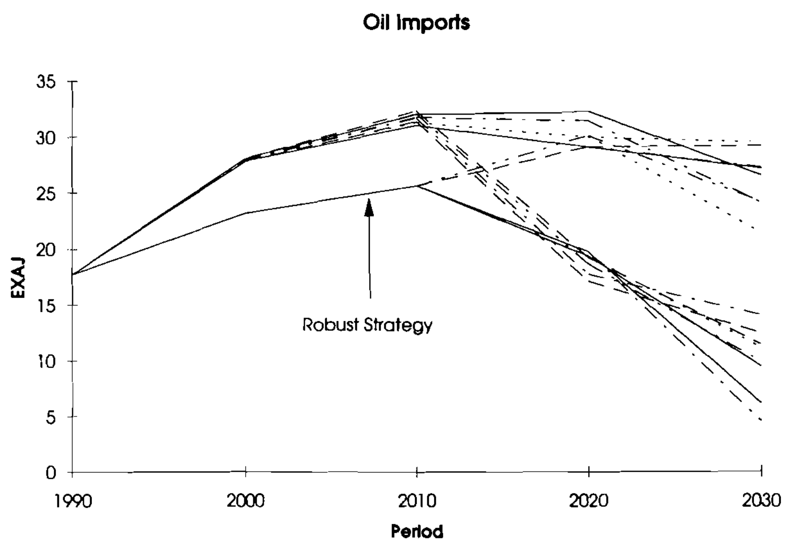

Figure 32:

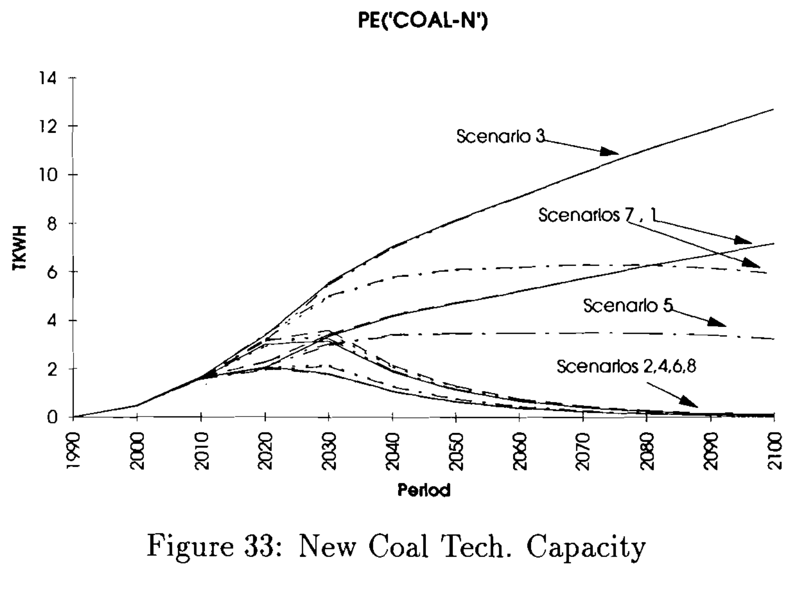

Figure 33: New Coal Tech. Capacity

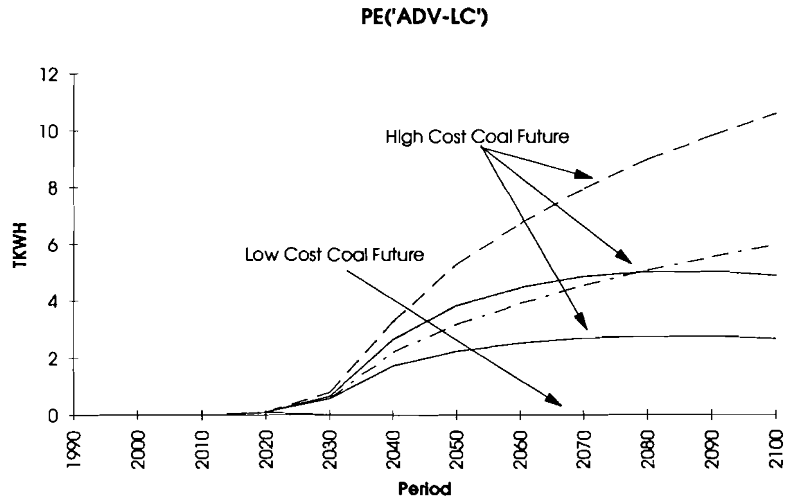

Figure 34: Advanced Low Cost Elec. Energy Capacity

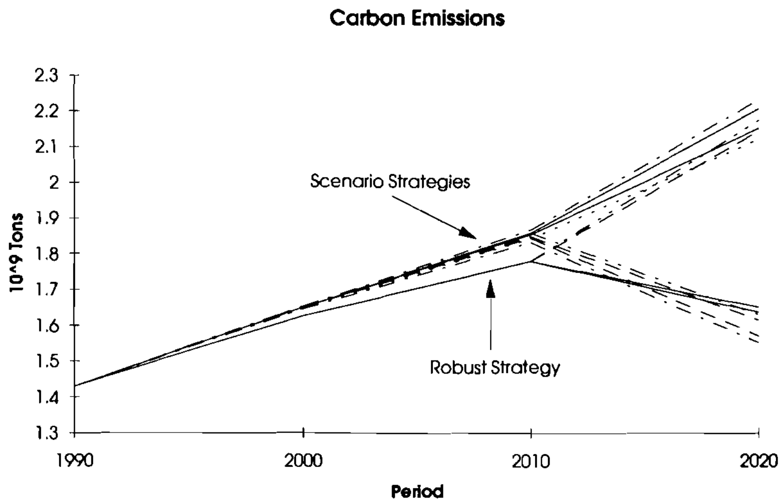

Figure 35: Carbon emissions over time