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A Belief Network Approach to Modeling of Environmental Change: The Methodology and Prospects for Application

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WP-94-40
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Abstract

In environmental management, assessments and far-reaching decisions must typically be made under very high or extreme uncertainty. The future development of the environment in interaction with societies in transition is very difficult to forecast. This is the case regardless of whether the change is introduced actively at the project or policy level, or passively through accumulated environmental deterioration or climatic change. This study presents a belief network methodology designed specifically for modeling environmental change. Belief networks contain a set of interlinked nodes. Prior probability distributions of nodes are updated with information from the rest of the network, according to transfer information in links. A link can transmit information in two directions. The existing belief network methodology was extended in several ways to meet the multiple requirements of environmental modeling. Most notably, two-layered parallel linking of nodes was allowed: the conventional probabilistic linking, and linking of outcomes of probability distributions using deterministic or logical relations. Moreover, several decision analysis techniques were included. The applicability of the methodology is discussed in reference to the following topics: knowledge acquisition, decision analytic modeling, mechanistic and process modeling, topological and spatial modeling, learning and adaptive modeling, and hybrid use.

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1. Introduction

One of the ways a human being comprehends a context is through naming objects and assuming associations between them. When considering a certain object in this context, he/she sees it simultaneously as a single unit and as a detail in interaction with the rest of the context. Systems involving uncertain information on a set of mutually dependent objects would typically be approached in Bayesian calculus by assigning a prior probability distribution to each object. Thereafter, the strength and character of the dependency between each object pair would be inserted. With the given information, posterior probability distributions can be calculated for each object. This is actually the key idea in belief networks. In belief network terminology, the objects are called nodes, their associations are links, and the context is a network.

Belief networks emerged from the Bayesian tradition in the 1980s (see Pearl 1986, 1988, Shafer and Pearl 1990). The key idea is that any new information introduced in the network can be propagated in all directions in the net, instead of only in a single direction. This is achieved using bi-directional information flow in the links. The nodes are able to merge the information from these systems and update it. Pearl presented a sequence of algorithms starting from a chain, and proceeding through trees and polytrees to networks. The basic problem in network algorithms is to cope with circular references. The algorithms available consist of approximate methods such as simulation (Pearl 1988).

Many of the tasks encountered in modeling and assessing environmental change are those in which (most of) the change has not yet taken place. Because of the high variability in semi-natural systems caused by numerous uncontrolled and controlled factors, the potential for change is often so high that extrapolation from past development gives vague results. The systems are often changed on purpose in a certain direction, and historical records are made partly or totally irrelevant. Numerous challenges face developers of computational methodology for such problems. They include the need to cope with extreme uncertainties, support expert reasoning and judgement (people from several disciplines), handle several types of information, forecast systems that will possibly be subject to structural changes, provide computer implementations that make models easily structurable, and facilitate the expression of and trade-off between various environmental, social, and economic aspects.

The goals of this study were to formulate and discuss a belief network methodology for modeling and forecasting change in environmental systems subject to radical alterations. The methodology is intended to serve the needs of, for instance, environmental impact assessment or climatic change studies. The interdisciplinary and otherwise special character of the subject has led to a need for a number of extensions to the present approaches. These include the incorporation of many decision and risk analysis ideas and the use of deterministic and rule-based links in parallel with probabilistic links. These are important features, since much of the information obtained by environmental forecasts should eventually yield decision support, in one form or another, and causal modeling has a long tradition and extensive use in the field.

Next section presents the extended belief network approach. Prospects and methods for modeling and forecasting environmental change are given and discussed. The potential uses of the belief network methodology are clustered in five groups: belief and knowledge acquisition, use for decision analysis, mechanistic and process modeling, topological and spatial modeling, and learning structures. A hybrid use of all these approaches is also suggested, and, finally, conclusions are drawn.

2. The Belief Network Approach

A belief network consists of nodes, which are connected with links. A bi-directional belief network with n nodes has $n(n\text{-}1)/2$ links in two directions, denoted here as π and λ. That makes the total number of possible links $n(n\text{-}1)$, and yields a substantial number of links even in models with only a few variables. As the number of nodes is doubled, the number of links grows more than four-fold. This underlines the need for models that are as simple as possible.

This section defines and illustrates the basic properties of nodes, links, and networks, with special reference to environmental modeling. The methodology presented here is deeply rooted in the work of Pearl (1986, 1988). It has adopted certain features from the influence diagram methodology of Shachter (1986). A number of extensions have been made.

2.1 Nodes

Each node i in a network contains

- A vector of possible (discrete) outcomes $\mathbf{y}_i$ that can be defined as inputs, or they may depend on the outcome values of other nodes.
- A prior probability distribution $\mathbf{e}_i$, with probabilities $e_1...e_k$ assigned to the k outcomes given.
- A sign indicating the direction of change. It may either be positive (implying, for instance, growth, increase, addition, enlargement) or negative (implying, for instance, decline, decrease, reduction, lessening).
- A posterior probability distribution $\mathbf{Bel}_i$.

The prior probabilities assigned to the outcomes are updated with information linked from other parts of the network, yielding the posterior probability distribution.

Certainty and uncertainty

In general, the nodes are probabilistic (uncertain). They can, however, have one outcome with the value 1 and others with the value 0, and thus be deterministic (certain). If an outcome in a prior distribution has the value 0, then the posterior distribution will also have a 0 value in the respective outcome. Only uncertain information accepts updating from other parts of the model.

Decisions and objectives

Some nodes may be understood as controllable, decision nodes. One or several nodes can act as a criterion for or constraint on decision making, and constitute an objective function. Uncertainty can be removed from a node, i.e., the probability of one of its outcomes set at 1, in order to simulate a decision or other action. The implications of the action are propagated through the network, and they can be observed at each of the successor nodes. Some of them are usually more critical than the others, involving objectives or constraints, and the adjustment of the actions simulated can be based on observed changes in these nodes.

2.2 Links

A link transfers information from one node to another. In the methodology suggested, the links are in two layers, as described below. When defining the concept of link, Pearl (1988) lists the following four primitives, for which I suggest climate change examples:

- *Likelihood*: Lake eutrophication is more likely to increase than to decrease.
- *Conditioning*: If temperature increases, then lakes will become more eutrophic.
- *Relevance*: Whether the lake will become more eutrophic depends on whether changes occur in policy or climate.
- *Causation*: Higher temperature will intensify lake eutrophication problems.

For more discussion and illustration, see Pearl (1988). I would like to propose another classification based on the information source for the link: (1) deductive; there is prior knowledge, theory, or belief concerning the interdependency of the two nodes, and (2) inductive; there is empirical evidence or data concerning the interdependency of the two nodes.

In order to increase the practical applicability of the belief network approach, a set of definitions and extensions to the mathematical formulation of links is presented. They are:

- Non-informative link.
- Division of links into (a) uncertainty links (denoted here simply as links) and (b) outcome links (denoted here as *o*-links).
- Deterministic *o*-link.

- Logical o-link.
- Asymmetric link matrix.
- Direction-specific link.
- Negative link.
- Information content of a link.
- Link-strength approach.
- Node independence.

These extensions are defined and illustrated below. The link matrix is case-specific and does not generally follow any theoretical probability distribution. However, in specific applications, it may do so.

Non-informative link

For computational convenience, when, for instance, using the fully connected approach by Varis (1992b) in a spreadsheet, it is useful to predefine a network topology that includes all possible connections within the group of applications to be studied. The initial state of links is then non-informative. This implies that all the elements of a link matrix have an equal value. All the new information concerning the probabilistic relations between the nodes is expressed by changing the link matrix element values.

Linking uncertainties and outcomes

An *uncertainty link* is defined as the link matrix $\mathbf{M}_{i|j}$ between two variables i and j. It can have all the properties described below, except deterministic or rule-based presentation. Those two features are the only ones that can be used for outcome links. An *outcome link* presents a relation between outcomes $\mathbf{y}_i$ and $\mathbf{y}_j$ of variables i and j. Because the belief network approach requires that each outcome has a single value, the propagation of outcome values is unidirectional, i.e., a functional relationship exists, $\mathbf{y}_i = \mathrm{f}(\mathbf{y}_j)$. This relation is deterministic, either *algebraic* (numerical) or *logical* (rule-based).

The three-node example in Figure 1 illustrates the idea of two different links in a model: probabilistic links propagated bi-directionally through link matrices $\mathbf{M}_{i|j}$, and outcome links $\mathbf{y}_i = \mathrm{f}(\mathbf{y}_j)$. The computational details for the former are presented in the subsection '*Network Propagation*'. Analogically, logical rules can be used as o-links.

Asymmetric link matrix

If a nonlinear relationship exists between two nodes, there are two ways to take it into account. First, the scales of the outcomes can be made nonlinear, for instance, by using a nonlinear function in the o-link between the variables as shown above. Second, a nonsymmetric link matrix can be used. The example in subsection '*Information Content of a Link*' provides an illustration of both of these features.

Direction specific link and negative link

In Pearl's original concept, the links from one node to another were not direction specific, i.e., $\mathbf{M}_{i|j} = \mathbf{M}_{j|i}$. This is often a reasonable assumption, but not always. Moreover, only link matrices in which the diagonal elements are greater than the off-diagonal elements are conventionally used. This implies that the nodes may only be positively linked to one another. In many cases, however, negative links and non-symmetric links are very useful. If a link represents correlation between two variables, then there is no reason to use non-symmetric links. Negative links are very useful, however. In deductive models or dependencies, non-symmetric links are also highly applicable.

Generally, if the sign (+ or –) of a node is changed, then the signs of the link parameters should also be changed. For instance, if the increase in temperature were to have a positive link to the increase in wheat yield, then the decrease in temperature would have a negative link to the increase in the yield (Figure 2). The same example can also be used to illustrate the idea of a non-symmetric link. Increasing temperature has a positive link to the increase in wheat yield, but the wheat yield has practically no causal influence on temperature. Therefore, there should be no symmetric, causal link in such a case.

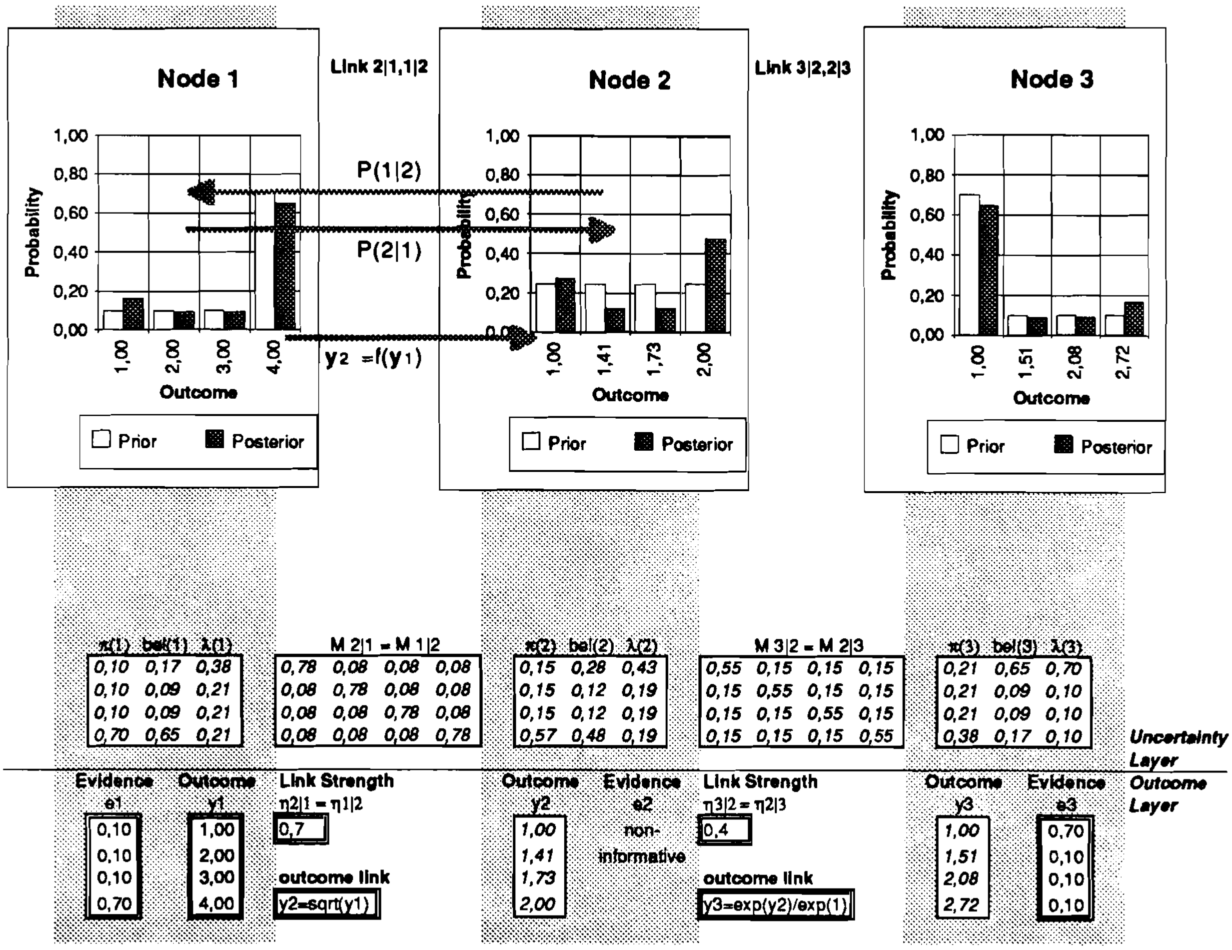

Figure 1. An example of a three node belief network (chain) with deterministic outcome links. Computed values are set in italics, and inputs are in cells surrounded by double lines. The uncertainty layer with links in two directions is above, and the outcome layer with links in one direction is below.

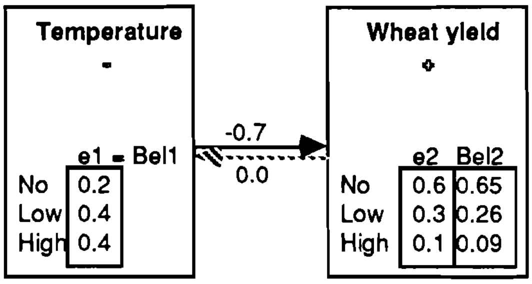

Figure 2. An example of a negative link (from Temperature to Wheat yield) and of a direction-specific link. The links are presented using link-strength parameters defined in the text.

Information content of a link

When we are dealing with uncertain link information, it is useful to define the degree of uncertainty of a specific link numerically. A scale, say from 0 to 1, can be fixed easily at both ends. A non-informative link matrix from node i to j, in which all elements have an equal value, evidently has the lowest information content. Hence, information content $\eta_{j|i} = 0$. An identity matrix **I** as a link matrix contains the maximum amount of information, $\eta_{j|i} = 1$. The intermediate values can be obtained in many ways. One possible index is:

$$\eta_{j|i} = \sqrt{\frac{1}{k-1}\sum_{s}\left(m_s - \frac{1}{k}\right)^2} \qquad (1)$$

where k is the number of rows in **M**, and m_s is the sth element of **M**. It may also be worthwhile to consider the information contents of distributions assigned to single outcome pairs between two nodes.

As an example, assume that the algal biomass in a lake is predicted on the basis of the orthophosphate (PO4) concentration of the water. The outcomes are:

$$\mathbf{y}_{\text{algae}} = \mathbf{y}_{\text{PO}_4} = \begin{bmatrix} 1 \\ 10 \\ 100 \end{bmatrix} \qquad [\mu\text{g/l}] \qquad (2)$$

and the link matrix from PO4 to algae is:

$$\mathbf{M}_{\text{algae}|\text{PO}_4} = \begin{bmatrix} .3 & .1 & .1 \\ .3 & .5 & .2 \\ .4 & .4 & .7 \end{bmatrix} \qquad (3)$$

With prior probabilities for the nodes:

$$\mathbf{e}_{\text{PO}_4} = \begin{bmatrix} 0.1 \\ 0.7 \\ 0.2 \end{bmatrix} \quad \text{and} \quad \mathbf{e}_{\text{algae}} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \qquad (4)$$

one gets the posterior distribution for algae

$$\mathbf{Bel}_{\text{algae}} = \begin{bmatrix} .12 \\ .42 \\ .46 \end{bmatrix} \tag{5}$$

Returning to the link matrix $\mathbf{M}_{\text{algae|PO4}}$, we can calculate its information content using Equation 1, $\eta_{\text{algae|PO4}} = 0.387$. Moreover, an information index can be calculated for each of the outcome pairs separately, yielding values $[0.1 \;\; 0.361 \;\; 0.557]^T$. This implies that evidence of 1 mg/l of orthophosphate is less informative than 10 mg/l, which in turn is less informative than 100 mg/l, as regards prediction of algal biomass.

Link strength approach

The information content of the link can also be used in the reverse manner, but only for square link matrices, i.e., in cases where the number of outcomes in the linked nodes is equal. It is often practical to present the strength of each link using a single parameter, instead of inserting values for each link matrix component separately. The number of links grows rapidly as the number of nodes increases. There are numerous ways of doing this. The following very practical method is presented as an example.

The link-strength parameter is denoted as $\mu_{j|i}$, $i \neq j$. $\mu_{j|i} \in [-1, 1]$. A symmetric, k x k link matrix $\mathbf{M}_{j|i}$ is constructed as a function of $\mu_{j|i}$. μ is now used as an input. The diagonal elements of $\mathbf{M}$ are obtained by

$$m_{r|r} = 1/k + (1 - 1/k)\mu \quad , \quad \mu \geq 0 \tag{6a}$$

$$m_{r|r} = 1/k + (1/k)\mu \quad , \quad \mu < 0 \tag{6b}$$

and the off-diagonal elements by

$$m_{q|r} = (1 - m_{r|r})/(k - 1) \qquad q \neq r \tag{6c}$$

For instance, the link-strength parameter value 1 implies an identity matrix, the value 0 implies a non-informative link matrix, and the value 0.7 implies the following matrix, which is a 3 x 3 matrix for demonstration purposes:

$$\mathbf{M} = \begin{bmatrix} .8 & .1 & .1 \\ .1 & .8 & .1 \\ .1 & .1 & .8 \end{bmatrix} \tag{7}$$

Node independence

The nodes in a network may have different grades of independence. Some nodes may be totally independent of other network variables, and some may be highly dependent on other variables. For instance, weather conditions may be independent variables in a wheat crop model, while crop size may be highly dependent on weather conditions.

Using the link-strength parameter idea, a simple index for the dependence d_j of a node j on other nodes in a network is the sum of the absolute values of all links ending up in the node:

$$d_j = \Sigma\, |\mu_{j/i}| \qquad , \qquad i \neq j \qquad (8)$$

Another index could be the mean link strength from all informative links to the node:

$$d_j = 1/\kappa\ \Sigma\, |\mu_{j/i}| \qquad , \qquad i \neq j \qquad (9)$$

where κ is the number of informative links to the node. Node independence d'_j can be defined as

$$d'_j = 1 - d_j \qquad (10)$$

2.3 Network Propagation

An algorithm for propagating uncertain information in a network is presented below. It is based on Pearl's (1988) polytree algorithm. Two independent polytree messages are computed, and the updated belief is obtained as the convolution product of these messages and the prior belief. The nodes are linked with link matrices that can be direction-specific. Positive and negative dependencies between variables are allowed. Computationally, all nodes are interlinked, and a non-informative link (link parameter $\mu = 0$) implies no connection. The polytree approximation does not update messages in cases where the propagation direction is changed.

Inputs

Let us consider a fully connected network with n nodes that can be arbitrarily linked to one another. Since a Bayesian network has to be directed, an ordering from the first to the last node must be defined. The inputs to the system are:

- A set of possible (discrete) outcomes $\mathbf{y}_i$ for each node i.
- The prior belief on each node, expressed in terms of probabilities $e_1 \ldots e_k$ assigned to k outcomes given, summing up to unity. These constitute a k dimensional vector $\mathbf{e}_i$ for the node i, also known as an evidence vector. If no prior belief exists, a non-informative prior distribution – a unit vector, for instance – is used.
- The information for each non-informative link. There is an indefinite number of ways to construct the link matrix $\mathbf{M}$. The algorithm requires the use of a square link matrix.

Top-down propagation

The next question is how to make use of the links and their strengths to calculate posterior belief distribution vectors $\mathbf{Bel}_j$ for the nodes. Since the network is directed, two information propagation directions can be distinguished in the network: top-down and bottom-up. The calculation is performed symmetrically, but directions up and down are used for verbal convenience.

When propagating messages downwards in a network, all messages coming to a node, say j, from an another node, say i, are denoted by $\mathbf{p}_{j|i}$ and messages leaving node i are denoted by π_i. For any node j, preconditioned by any node i ($i < j$):

$$\mathbf{p}_{j|i} = \mathbf{M}_{j|i}\,\pi_i \tag{11}$$

The vectors $\mathbf{p}_{j|i}$ and π_i consist of the following elements:

$$\pi_i = \begin{bmatrix} \pi_i^1 \\ \pi_i^2 \\ \vdots \\ \pi_i^k \end{bmatrix} \quad \text{and} \quad \mathbf{p}_{j|i} = \begin{bmatrix} \mathrm{p}_{j|i}^1 \\ \mathrm{p}_{j|i}^2 \\ \vdots \\ \mathrm{p}_{j|i}^k \end{bmatrix} \tag{12}$$

For elements r, the π_i^r message is the convolution product of the message $\pi_i^r|_{1..i-1}$ and the prior belief e_i^r.

$$\pi_i^r = \pi_{i|1..i}^r = \alpha\, e_i^r\, \pi_{i|1..i-1}^r \tag{13}$$

where α is a scaling constant, scaling the sum of the k vector elements of π_i to unity. The incoming message $\pi_{i|1..i-1}$ is the convolution of all the messages, $\mathbf{p}_{i|1}$ to $\mathbf{p}_{i|i-1}$, from the node's $i-1$ predecessors:

$$\pi_{i|1..i-1}^r = \prod_{k=1}^{i-1} p_{i|k}^r \tag{14}$$

Starting from the first node, the $\mathbf{p}_{1|0} = \mathbf{1}$ and $\pi_1 = \mathbf{e}_1$, $\mathbf{p}_{2|0,1} = \mathbf{M}_{2|1}\pi_1$, and so on.

Bottom-up propagation

Bottom-up propagation is quite similar to top-down propagation. Only the direction is reverse. All messages coming to node i from node j are denoted by $\mathbf{l}_{i|j}$ and messages leaving the node j are denoted by λ_j. For any node i, preconditioned by any node j, with $i < j$.

$$\mathbf{l}_{i|j} = \mathbf{M}_{i|j}\,\lambda_j \tag{15}$$

The λ_j message is the convolution of the message $\lambda_{j|j+1..n}$ and the prior belief $\mathbf{e}_j$.

$$\lambda_j^r = \lambda_{j|j..n}^r = \beta\, e_j^r\, \lambda_{j|j+1..n}^r \tag{16}$$

where β is a scaling constant. The incoming message $\lambda_{j|j+1..n}$ is a convolution of all the messages, $\mathbf{l}_{j|j+1}$ to $\mathbf{l}_{j|n}$, from the node's $n - j$ successors:

$$\overset{r}{\lambda}_{j|j+1..n} = \prod_{k=j+1}^{n} \overset{r}{l}_{k|j} \tag{17}$$

Posterior beliefs

For each node j, the posterior belief distributions $\mathbf{Bel}_j$ can now be calculated on the basis of the prior distribution $\mathbf{e}_j$, updating it with the information from the sub-network above and below the node, i.e., vectors $\pi_{j|1..j-1}$ and $\lambda_{j|j+1..n}$, respectively:

$$\overset{r}{Bel}_j = \gamma \overset{r}{\pi}_{j|1..j-1} \; \overset{r}{e}_j \; \overset{r}{\lambda}_{j|j+1..n} \tag{18}$$

where γ is a scaling constant.

Round-the-corner message

The most problematic simplification in the algorithm presented above is that the model contains two independent message polytrees, π and λ. These messages are only connected when the posterior beliefs (**Bel**) are calculated (see Equation 18). The **Bel**s should also influence the messages, though, as in Pearl's original formulation. However, this simplification greatly reduces computational time, makes computation insensitive to the enlargement of the network and the number of outcomes, and facilitates, for instance, interactive, on-line use in a spreadsheet, which has proved to be an important feature in practical environmental management work. With an algorithm requiring iteration or simulation, the time required for calculation would expand.

These "round-the-corner" propagation problems of both π and λ messages are illustrated in Figure 3. Evidence $\mathbf{e}_1$ will be propagated to all nodes through the π mechanism. However, the link from node 4 to node 2 does not take this message into account. $\mathbf{e}_3$ has no influence on $\mathbf{Bel}_2$, and $\mathbf{e}_2$ has no influence on $\mathbf{Bel}_3$, although those nodes are indirectly linked in two directions. This problem can be eliminated by making also links $\mathbf{M}_{1|4}$ and $\mathbf{M}_{2|3}$, and vice versa, informative. But this may then make the semantic interpretation of the network more difficult. If this is considered a major drawback in a specific application, it would then be better to adopt an another, more computation-intensive approach.

3. Prospects for Modeling Environmental Change

Belief networks provide a variety of ways of modeling environmental change. On the basis of my own experience, I see potential for modeling in the following five areas, which have no sharp borders: (1) belief and knowledge acquisition, (2) use for decision analysis, (3) analytical, mechanistic and process modeling, (4) spatial and temporal correlations, and (5) learning and adaptive modeling.

In the following, these areas are discussed, and illustrated with tentative examples. In practice, a belief network can include properties from each of these categories.

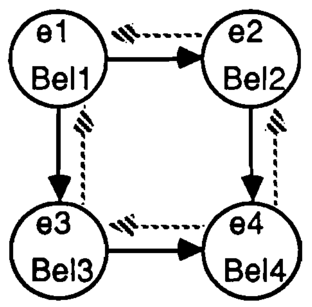

Figure 3. An example of the round-the-corner propagation problem in a four-node network. π messages are shown in solid arcs and λ messages with dashed arcs.

3.1 Belief and Knowledge Acquisition

Under this approach, a model is constructed by an expert who names the most essential quantities (nodes) in the system, given a specific purpose, and assigns the most important links between the nodes. The interface can be an interactive checklist with a relatively low number of linguistic outcomes for the nodes. The nodes can be renamed, and the associations can be easily modified.

This approach may be very useful when we are dealing with very uncertain problems, such as assessing the impacts of climatic change on water quality in a watershed. More conventional checklist approaches are fairly common in environmental impact assessment studies (cf., Biswas and Geping 1987, World Bank 1991). The suggested checklist approach could be of benefit in such applications. It is related to certain associative decision analysis approaches, such as the Analytic Hierarchy Process by Saaty (1980). Preliminary experience has shown that there can be reasonable deviations between prior and posterior distributions within a model. This indicates expert's inconsistency. Iterative use can help the expert to formulate a consistent network of beliefs.

Using a belief network in this way is evidently appropriate as such. In addition, it can be used as the first stage in a modeling procedure, yielding a conceptual network that can later be refined using a higher number of outcomes, especially numerical ones, and *o*-links, decision analysis tools, etc.

3.2 Use for Decision Analysis

Decision analysis attempts to structure and quantify – typically uncertain and subjective – information, to find the most important influences within a system, to detect critical components of total risk and uncertainty, to identify proper variables and policies to be controlled within the system, and to provide scenarios and sensitivity studies to support the above targets.

Varis (1992a) and Varis, Kuikka and Taskinen (1993) list a set of properties that are particularly useful in the use of probabilistic, environmental models, including belief networks, for decision analysis. Those include utility theory, risk attitude analysis, and the value of information and control. In addition, the comments on decisions and objectives in the '*Nodes*' subsection are relevant here.

A special case of a belief network is a model in which (1) all links have only one direction and (2) no cycles are included. Important computational extensions can be made in such networks, including the use of dynamic programming. Influence diagram methodology (Shachter 1986) is based on such networks. Influence diagrams are a very efficient decision analysis approach, tested in various environmental applications (Varis, Kettunen and Sirviö 1990, Varis and Kuikka 1990, Kuikka and Varis 1992, Varis, Kløve and Kettunen 1993). The reader is referred to these case studies for examples of the use of the influence diagram approach.

3.3 Mechanistic and Process Modeling

There is a reasonable limitation on the use of the belief network approach in mechanistic and process modeling because the equations have to be in an analytically solved form. Therefore, the belief network approach is not a suitable approach for most of the complex, dynamic models used in, for instance, climatic change impact assessment studies. However, the belief network approach can be utilized in many practically applicable models, such as many pollution models, growth models, age-structured models, population models, chemical equilibrium and process models, financial models, and econometric models. The strength of the corresponding probability link can be understood as the belief in the appropriateness of the equation for the o-link (cf., Varis, Kuikka and Kettunen 1993). This provides far-reaching potential for describing the structural uncertainty of models.

Take an example from river pollution models, to illustrate how the well-known Streeter and Phelps (1925) model can be presented as a belief network. The amount of biologically degradable pollution in a river (measured as biological oxygen demand BOD), and its impact on the dissolved oxygen concentration in the water are modelled using equations

$$b(t) = b(0)\exp(-k_1 t) \qquad (19a)$$

$$c(\tau) = c_s - (c_s - c(0)\exp(-k_2 \tau) + k_1 b(0)(\exp(-k_1 \tau) - \exp(-k_2 \tau))/(k_1 - k_2) \qquad (19b)$$

where b is BOD, c is the oxygen concentration in the water, t is the time from the discharge of the effluent in the river, and c_S is the saturation concentration of oxygen in water. k_1 and k_2 are parameters. In the resulting network, the Equations (19a, b) are used as the o-links to the nodes $b(t)$ and $c(t)$. The uncertainties in the model can be updated either on the basis of time, parameters and initial values to state variables, or vice versa.

In addition, the uncertain outcome of complex models – such as climate change scenarios computed off-line – can easily be used as prior distributions in belief networks. A belief network can be a meta-model, containing the outputs from several complex models. Analogically, a solved trajectory of a complex model can serve as the basis of an o-link.

Dynamic models can be constructed in many ways, for instance, by cloning a model made for one time step, to create a structure in which one layer represents one time step. Thereafter, time-dependent processes can be linked between those layers, and the value of the time step information node can be changed.

When calculating the node independence parameter, one should be aware that different nodes have very different interpretations. For instance, from a parameter to a state equation, the information content of the link is often close to 1. In a state equation with several prior parameters, the node independence approach is not suitable as such. One can, for instance, omit the information from parameters, and thus obtain a more informative index for the independence of a node containing a state equation.

3.4 Topological and Spatial Modeling

A belief network can also be used to present a topological structure in nature. There are many ways of using the belief network approach with spatial and time-dependent modeling, such as correlation and autocorrelation models and geostatistical models.

A particularly attractive idea is to use a belief network as an on-line model of the state of the environment in a certain region. Each new observation could be propagated through the network, and it would update our knowledge about the system. Risk analysis approaches can also be easily connected to such a model. As an example, consider a sequence of water quality gauging stations along a river (Figure 4), operating on an on-line principle. The uncertainty system is propagated bi-directionally through the entire network in real time. The outcome system may possess a physical model structure. Additional nodes in the network could be, for instance, meteorological information sources. Analogical uses can be found for off-line problems.

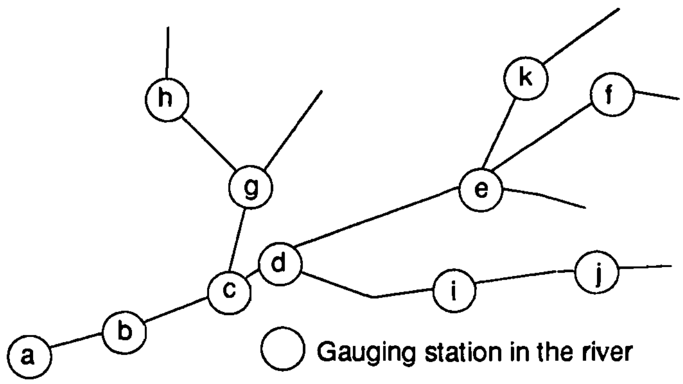

Figure 4. An example of a belief network corresponding to the topology of a river and its tributaries.

3.5 Learning and Adaptive Modelling

Belief networks can also include inductive components that perform estimation, learning, or structural adaptation. Many statistical estimation procedures can evidently be included in a belief network.

Consider as an example the inclusion of a neural network, which is an adaptive and learning model structure, in a belief network. In principle, it would be possible to construct an interface between such networks. The input layer of a neural network would receive information from the belief network, and the output layer would produce an input to the same or another belief network (Figure 5). One would need a set of information to teach the network to produce forecasts. All of the uncertainty information can be propagated through the interface, for instance, by coding each outcome of each interface node as a separate input layer node to a neural net and using the corresponding system in the output layer.

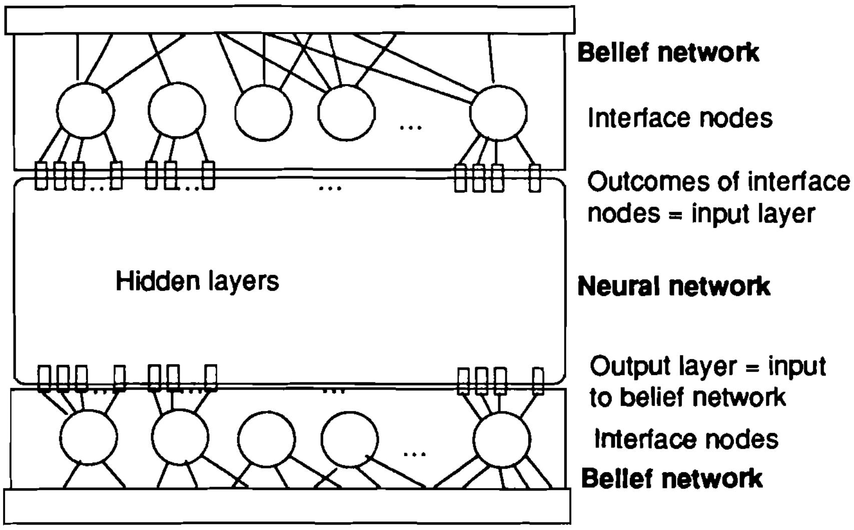

Figure 5. An interface structure between belief networks and neural networks.

3.6 Hybrid Use

Many of the technical details of the five approach categories discussed above partly overlap (Figure 6). In applications, it would evidently be worthwhile to be able to use the most appropriate features of these five areas to produce proper models. There is no reason why a belief network model could not incorporate nodes, links, or sub-networks that make use of all these approaches simultaneously.

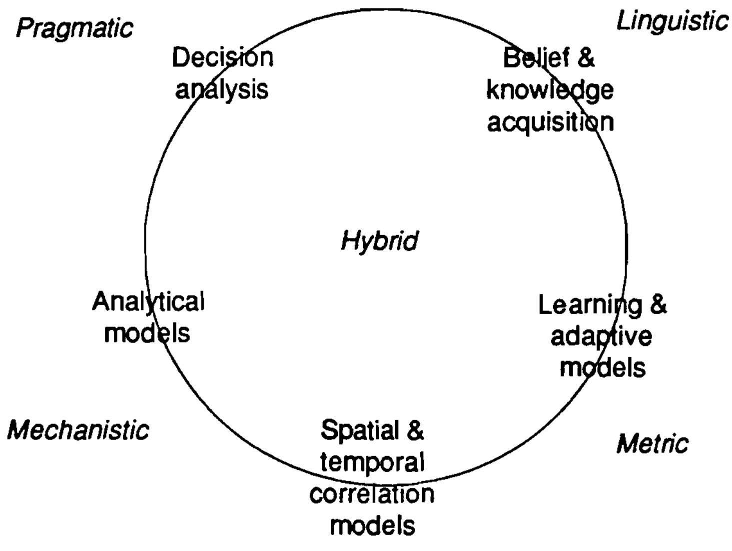

Figure 6. The belief network approach facilitates the combined use of several methodological and paradigmatic (in italics, cf. Beck 1991) facets that are often seen as being far from one another.

A fisheries management example

Varis, Kuikka and Kettunen (1993) have used a combination of a deterministic, age-structured fish population model and a set of regression models in a belief network framework to support group work and decision making by an international committee that issues annual Baltic salmon quota recommendations. The stocking of reared salmon has enhanced the region's salmon fisheries, and wild stocks are under severe threat of extinction. The goal of the stock assessment procedure is to produce information that is of value in formulating international policy to safeguard the existence of wild salmon stocks. The economic rationale for compiling empirical data is far too low to enable empirical stock forecasts. Furthermore, the Baltic Sea as a system with ecological, social, economic, and political facets, is undergoing practically unpredictable changes and transitions.

In the case of salmon stock assessment, the information and experience available allow the use of empirical, regression-type models for certain relations between sub-stock data, growth parameters, water quality data, etc. The Virtual Population Analysis (VPA) equations (Beverton and Holt 1957, Gulland 1983) have also been found very useful, although they are not identifiable from data and the parameter values are assessed by experts. Experts play a crucial role in the production of age-structured stock forecasts from this – rather diverse – information. Some experts prefer to use selected empirical models, while others prefer the VPA. Clearly, any contemporary assessment technique suffers from severe limitations, and all possible, relevant information and models should be taken into consideration.

A belief network environment was produced (Varis, Kuikka and Kettunen 1993) that allows inclusion of empirical models and the VPA in one frame. The uncertain and diverse information can be merged in expert workshops. The interactive system allows the detection of disagreements in information, the weighting of different models, the tuning of the VPA, calculation of forecasts, and definition of the fisheries quota decision. This has been done using a belief network in which the above-mentioned models have been embedded (Figure 7).

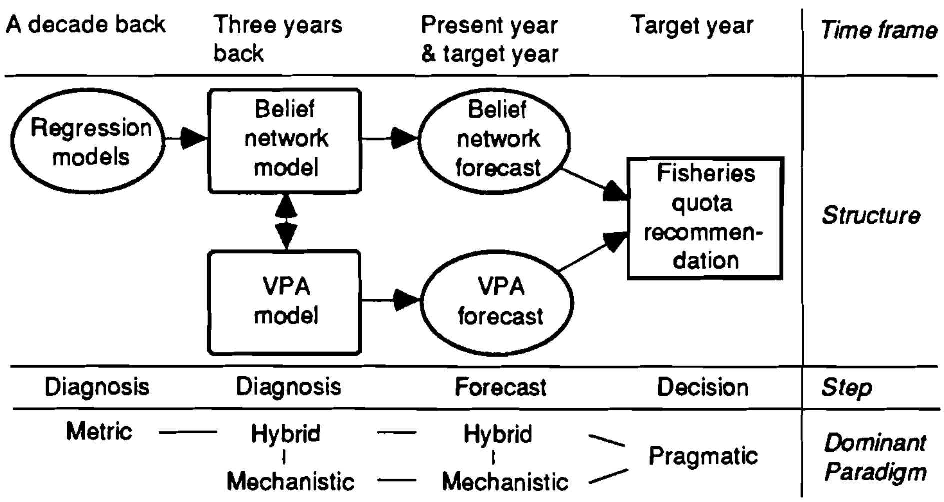

Figure 7. Schematic diagram of the structure of the assessment procedure for Baltic salmon (Varis, Kuikka and Kettunen 1993). The more angular the module, the more important the expert judgement component.

4. Conclusions

Uncertainty and subjectivity are important features of environmental forecasting, particularly in cases where the environment may be subject to radical change. Such forecasts are often made to produce information that is useful to decision-makers. In addition, such forecasts usually require a proper diagnosis of the problem. Subjective expert knowledge and value judgements are often among the major sources of information (cf. Henderson-Sellers 1990). Such information should be handled formally more often than at present. When using computer models, it is essential that the entire inference and decision support process be considered as a whole, and not, for instance, only as parameter or state uncertainty, as is often the case. The Bayesian approach to management of uncertainty provides various possibilities. It deserves more attention in research and in practice in the forecasting of environmental change.

Evidently, environmental forecasts usually serve decision-making situations in which risk-neutral behavior is rare. In contrast, risk-averse behavior appears to be rather typical of these situations (see Laurmann 1991), i.e., the management objectives include reducting of the level of uncertainty involved. Risk-prone cases also exist, but they are less frequent. Therefore, a form of probabilistic modeling facilitating a risk attitude analysis is needed, as its exclusion leads to the assumption of risk neutral behavior.

Belief networks, as presented here, appear to have many properties that help to cope with the above problems. The three most crucial are:

- Advanced handling of uncertainties (propagation & presentation, objectives, and structure).
- Ability to include modeling techniques from many methodological families that are usually considered far from one another (e.g., metric, mechanical, linguistic, and pragmatic).
- Support for the acquisition of expert knowledge and structural construction of a model.

A challenging extension to the approach presented would be to use continuous distributions instead of discrete ones. Such a methodology already exists for influence diagrams (Shachter and Kenley 1989). Though our examples came from environmental management, the methodology presented here is readily applicable to many other fields as well.

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