# Working Paper

## Interactive Specification of DSS User Preferences in Terms of Fuzzy Sets

*Janusz Granat and Andrzej P. Wierzbicki*

WP-94-29
May 1994

IIASA
International Institute for Applied Systems Analysis □ A-2361 Laxenburg □ Austria
Telephone: +43 2236 715210 □ Telex: 079 137 iiasa a □ Telefax: +43 2236 71313

# Interactive Specification of DSS User Preferences in Terms of Fuzzy Sets

*Janusz Granat and Andrzej P. Wierzbicki*

WP-94-29
May 1994



IIASA International Institute for Applied Systems Analysis ◻ A-2361 Laxenburg ◻ Austria
Telephone: +43 2236 715210 ◻ Telex: 079 137 iiasa a ◻ Telefax: +43 2236 71313

## Foreword

This paper illustrates one of the results of a long-term cooperation of the Methodology of Decision Analysis (MDA) Project with several research institutes in Poland working on the theory, methodology and software tools for decision support systems (DSS). One of the principal problems of decision support is the way of specifying the preferences of the user of a DSS (sometimes called the decision- maker, although there are various types of DSS users). In a multiple- citeria decision situation, the preferences of the user are often vague and are modified during a decision process. The authors of this paper propose a novel way of an interactive specification and modification of such preferences in terms of fuzzy sets, related to the reference point methodology developed earlier in the MDA Project and supported both by theoretical analysis and by a interactive computer graphic implementation. A software module for such interaction, developed due to the cooperation with the Institute of Automatic Control of Warsaw University of Technology (where both authors are located) is available as public domain software in the MDA Project.

# Abstract

The specification of reference levels in the aspiration-led methodology of multiobjective decision support is usually imprecise. Such imprecision can be included in the mathematical formulation of a decision problem by using membership functions and fuzzy set theory. This approach can also be treated as a further development of fuzzy interactive methods. A method of constructing order-consistent scalarizing function based on membership functions is presented as well as a related, extended fuzzy interactive method.

# Contents

# Interactive Specification of DSS User Preferences in Terms of Fuzzy Sets

*Janusz Granat*and Andrzej P. Wierzbicki**

## 1 Introduction

A contemporary decision support system (DSS) is a collection of tools (data, models, algorithms, interfaces) which enable the decision maker - or any user of this system - to actively participate in the process of decision elaboration; a good DSS should help the user to learn and should enhance his/her intuition about a given decision problem.

In any decision process, the preferences of the decision maker are initially vague and are gradually better specified as the decision problem is analyzed. One of accepted ways of supporting such gradual specification is an interactive multiple criteria decision process. In such process, the user is supported in the formulation of a multiple criteria decision problem and further in the control of the process of generating and evaluating various options of efficient decisions ( called also vector-, or Pareto-, or multiobjectively-optimal. The interaction of the user with the DSS is an essential element of such a process; during the interaction, the preferences of the user are either explicitly or implicitly specified. In order to organize such an interactive procedure, it is necessary to choose what kind of information should or could be presented to the user, how this information should be presented, how to represent the preferences of the user in the system and how the user should influence the decision process and guide the interaction.

An explicit specification of user preferences takes usually the form of identifying his/hers utility or value function; the initial assumptions concerning the form of such a function are essential in such a case. An implicit specification of user preferences might allow him/her to control the decision process better and be more adequate to his/hers needs. One possible form of such an implicit specification is the reference or aspiration level approach - see e.g. [11]. In this approach, the user is asked to specify a reference point - called also an aspiration level point - in the criteria space. The DSS responds then with an efficient decision such that its outcomes are either uniformly close to or uniformly better than the reference point. This response is based on a maximization of a scalarizing function, typically of a piece-wise linear form, which can also be interpreted as an adjustable proxy value function, controlled by the specification of the reference point.

The specification of a reference point is usually adjusted by the user, because his/her initial perception of preferences and aspirations is typically imprecise. This imprecision

*Institute of Automatic Control, Warsaw University of Technology, 00-665 Warsaw, Poland.

can be included in the mathematical formulation of the decision problem by using fuzzy sets, as suggested originally by Zadeh [13]. We propose in this paper some further developments of fuzzy interactive methods as proposed by Seo and Sakawa [7], [10]. Their original fuzzy interactive methods assumed that the membership functions related to attainment levels for separate objectives or criteria values are specified a priori and the interaction is controlled by specifying reference values for given membership functions.

In this paper, another approach is presented which allows the user to change - during the interaction - the form of the membership functions for separate objectives. Thus, the membership functions are not only elicited at the beginning of interaction, but can be also changed and substantially modified at further stages; their appropriate convolution represents the changing preferences of the user due to the learning implicit in the decision process. Such a convolution of membership functions can be used as an order-consistent scalarizing function.

A method of constructing such order-consistent scalarizing functions based on membership functions is presented; since the membership functions describe the satisfaction of the user with the attainment of separate objectives, the aggregate function can be interpreted as a proxy value function of the user. Moreover, graphic tools of interaction in terms of membership functions are presented, related issues of representing the results to the user are addressed and some ways of such representation are proposed. Some numerical examples and an example of using the approach in cooperative group decision-making are also presented.

# 2 The construction of order-consistent scalarizing functions based on membership functions specified by the user

## 2.1 Basic concepts of decision making in a fuzzy environment

The concepts and methods of decision making in fuzzy environment have long research tradition. After recalling basic concepts of fuzzy decision making, we shall proceed to their multi-objective versions and to some reasons for seeking further extensions of such concepts and methods.

Bellman and Zadeh [2] defined three basic concepts for decision making in a fuzzy environment: fuzzy goals, fuzzy constraints and fuzzy decisions. A set of alternatives $X$ is considered. A *fuzzy goal* is defined as a fuzzy set $G$ in $X$, whose membership function is given by $\mu_G : X \mapsto [0, 1]$; a *fuzzy constraint* is defined as a fuzzy set $C$ in $X$, whose membership function is given by $\mu_C : X \mapsto [0, 1]$; then *a fuzzy decision* is defined as a fuzzy set D, which is the intersection of $G$ and $C$:

$$D = G \cap C$$

The set D can be characterized by a corresponding convolution of the component membership functions, e.g. by the typical minimal convolution operator, which results in the membership function:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)), \quad x \in X$$

The maximizing decision is then defined as follows:

$$\underset{x\in X}{\text{maximize}}\, \mu_D(x) = \underset{x\in X}{\text{maximize}} \quad \min(\mu_G(x), \mu_C(x)) \tag{2.1}$$

The minimum operator, introduced by Bellman and Zadeh [2], is only one example of a fuzzy convolution operator. Other examples can be found e.g. in [7].

These concepts have found applications also in multiobjective programming. A *fuzzy multiobjective optimization problem* can be defined by specifying several fuzzy goals [10]:

$$\begin{gathered} \text{fuzzy\_minimize}\, f_i(x) \quad \text{or} \\ \text{fuzzy\_maximize}\, f_i(x) \quad \text{or} \\ \text{fuzzy\_equal}\, f_i(x) \\ s.t. \quad x \in X \end{gathered} \tag{2.2}$$

and by the following multiobjective optimization problem:

$$\underset{x\in X}{\text{maximize}}(\mu_{f_1}(x), \mu_{f_2}(x), \ldots, \mu_{f_m}(x))$$

Similarly to (2.1) a *fuzzy multiobjective decision problem* can be defined by:

$$\underset{x\in X}{\text{maximize}}\, \tilde{\mu}_D(\mu_f(x)) \tag{2.3}$$

where

$$\mu_f(x) = (\mu_{f_1}(x), \mu_{f_2}(x), \ldots, \mu_{f_m}(x))$$

When the minimum convolution operator is chosen to define $\tilde{\mu}_D$, then (2.3) is equivalent to

$$\underset{x\in X}{\text{maximize}} \min_{1\leq i\leq m} (\mu_{f_1}(x), \mu_{f_2}(x), \ldots, \mu_{f_m}(x))) \tag{2.4}$$

Under additional monotonicity conditions that are discussed later in more detail, a solution of a fuzzy multiobjective decision problem is efficient or Pareto-optimal. However, when the goal *fuzzy equal* is included, the concept of Pareto optimal solutions cannot be directly applied, therefore the following concept of M-Pareto optimal solutions has been introduced [10].

**Definition 2.1** *A decision $x^* \in X$ is said to be a (local) M-Pareto optimal solution to (2.2), if and only if there does not exist another $x \in X \cap N(x^*, \delta)$ such that $\mu_{f_i}(x) \geq \mu_{f_i}(x^*), i = 1, \ldots, m$, with strict inequality holding for at last one $i$ ( $N(x^*, \delta)$ denotes the set $\{x \in R^n : \|x - x^*\| < \delta\}$).*

Several interactive methods have been proposed for fuzzy multiobjective programming. A short survey can be found in the book [10] by Seo and Sakawa. The augmented minimax method in fuzzy programming, proposed for fuzzy problems by Sakawa and Yano [8], is in a sense similar to the aspiration-led methodology of multiobjective decision support, presented in the next subsection. In this method the DSS user determines the membership functions for each of the objectives and then specifies reference levels for membership functions. Thereafter, the following augmented maximin problem is solved:

$$\underset{x \in X}{\text{maximize}}\Big(\min_{1 \leq i \leq p}(\mu_{f_i}(x) - \bar{\mu}_{f_i}) + \rho \sum_{i=1}^{p}(\mu_{f_i}(x) - \bar{\mu}_{f_i}\Big) \tag{2.5}$$

Further concepts of an intelligent DSS have been introduced by Seo and Nishizaki [9]. Such a system gives possibilities of a deeper insight into the problem being solved, especially in fuzzy environment.

However, there are reasons to seek further extensions and improvements of fuzzy interactive methods. One of arising problem is that the aggregated membership functions $\tilde{\mu}_D(\mu_f(x))$, e.g. the functions maximized in (2.4) or (2.5), might have a constant value on subsets of $X$, if all $\mu_{f_i} = 0$ or all $\mu_{f_i} = 1$. Therefore, the solution of (2.4) or (2.5) can be nonunique and not efficient (not Pareto-optimal), see Figure 1, where the dashed area shows the set of optimal solutions of the problem (2.4). Similar problems appear in other fuzzy optimization formulations; in order to achieve unique solutions, the membership function corresponding to the solution should belong to the range $0 < \mu < 1$. In a further subsection, we propose an extension that eliminates such problems.

Another extension is related to the specification of the membership function by the user at the beginning of the interactive process. When practical problems are solved, the DSS user might not know exactly how to specify this function, and should have a possibility to change it during the process of interaction.

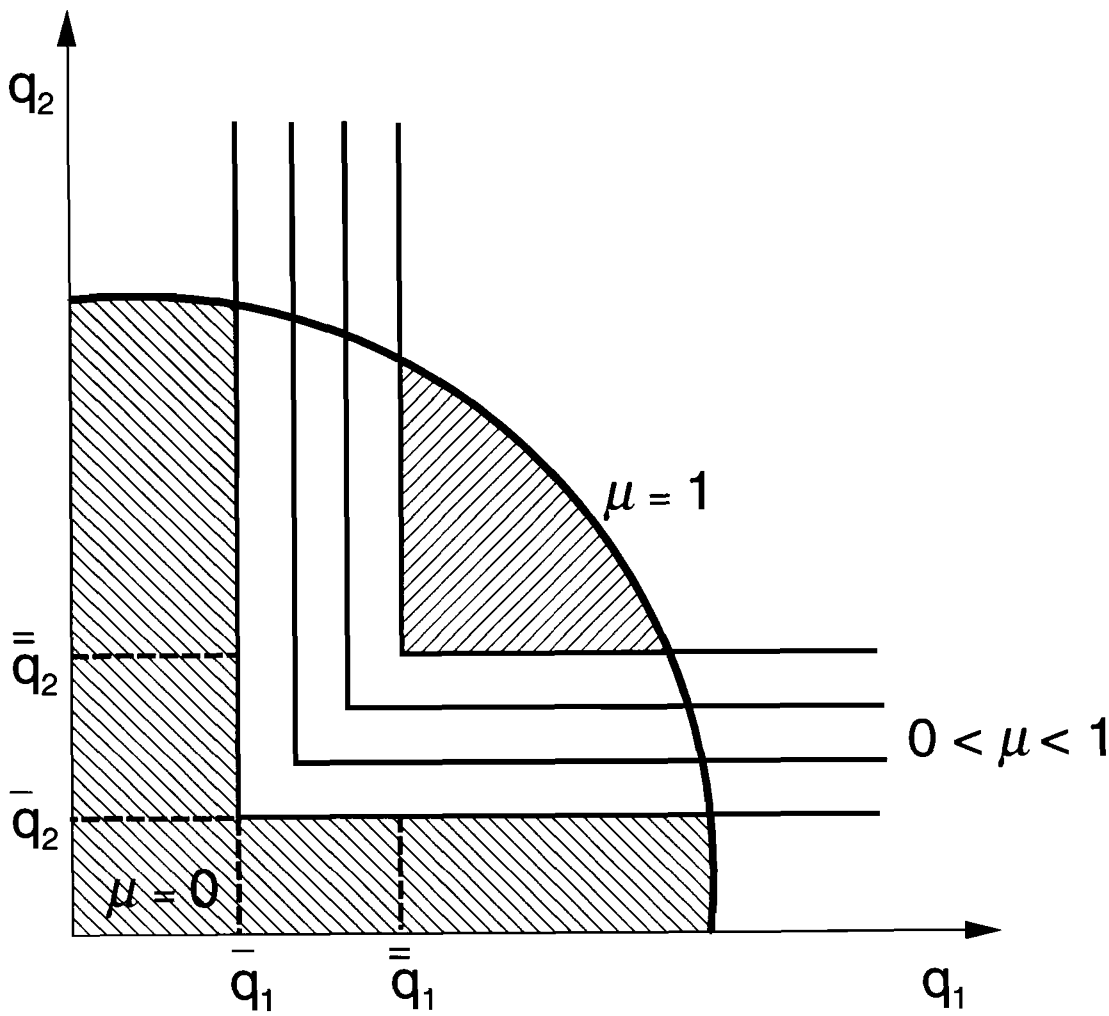


Figure 1: Level sets of the aggregated membership function, where $q_1 = f_1(x), q_2 = f_2(x)$ are fuzzy maximized goals, simplest piece-wise liner membership functions $\mu_{f_1}(x), \mu_{f_2}(x)$ and the minimum convolution operator were assumed.

## 2.2 Basic concepts of an aspiration-led methodology of multi-objective decision support systems

The methodology of aspiration-led decision support systems, developed by one of the authors and many other researchers - see [6], relates to a multi-objective programming problem defined as:

$$\underset{x \in X}{\text{V\_max}}\ q = f(x), \quad f(x) = (f_1(x), f_2(x), \ldots, f_p(x))^T \tag{2.6}$$

where

$$X = \{x \in R^n : g(x) = (g_1(x), g_2(x), \ldots, g_m(x))^T \leq 0\}$$

V_max - denotes multi-objective optimization in the Pareto sense,
$q = f(x)$ - is the vector of objectives,

$X$ - is the set of admissible decisions
$x$ - is the vector of decision variables

Suppose $q_{i,min} \leq q_i \leq q_{i,max}$ for all $i = 1, \ldots, p$. An order-consistent achievement scalarizing function can be defined as:

$$s(q, \overline{q}, \overline{\overline{q}}) = (\min_{1 \leq i \leq p} z_i + \frac{\rho}{p} \sum_{i=1}^{p} z_i)/(1+\rho) \tag{2.7}$$

with

$$z_i = \begin{cases} \gamma(q_i - \bar{q}_i)/(\overline{q}_i - q_{i,min}) & \text{if } q_{i,min} \leq q_i \leq \overline{q}_i \\ (q_i - \overline{q}_i)/(\overline{\overline{q}}_i - \overline{q}_i) & \text{if } \overline{q}_i < q_i < \overline{\overline{q}}_i \\ \beta(q_i - \overline{\overline{q}}_i)/(q_{i,max} - \overline{\overline{q}}_i) + 1 & \text{if } \overline{\overline{q}}_i \leq q_i \leq q_{i,max} \end{cases} \tag{2.8}$$

where

$\beta > 0, \ \gamma > 0$
$\overline{q}_i$ - a reservation level, $q_{i,min} < \overline{q}_i < q_{i,max}$
$\overline{\overline{q}}_i$ - an aspiration level, $q_{i,min} < \overline{q}_i < \overline{\overline{q}}_i < q_{i,max}$
$\rho$ - a weighting coefficient, $0 < \rho \leq p$
$p$ - the number of objectives

By solving the problem:

$$\max_{x \in X} s(f(x), \bar{q}, \bar{\bar{q}}) \tag{2.9}$$

Pareto-optimal or efficient solutions can be obtained; more precisely, the solutions of the above problem are properly efficient with a priori bound $1+p/\rho$ on trade-off coefficients for the rescaled objectives $z_i$, see Wierzbicki [12]. During the process of interaction, the user specifies aspiration and reservation levels and the DSS responds with a solution which outcomes are either uniformly close to or uniformly better than the specified aspiration or reservation levels. After several steps of interaction, the user either has learned enough about possible decisions and their outcomes to choose the best solution, or he/she can continue the interactive process.

## 2.3 Order-consistent scalarizing function

In this subsection, we propose the background for further developments of fuzzy interactive methods, including experiences and theory developed in fuzzy interactive methods as well as in aspiration-led methodology.

Consider a continuous vector valued objective function $f : X_0 \mapsto R^p$, where $X_0 \subset R^n$ is the compact set of admissible decisions. Hence the set of attainable outcomes $Q_0 = f(X_0)$ is compact. The partial ordering in the objective space is implied by a positive cone D, given by

$$D = \{q \in R^p : q_i \leq 0, i = 1, \ldots; q_i \geq 0, i = p' + 1, \ldots, p''; q_i = 0 i = p'' + 1, \ldots, p\}$$

The outcomes $1,\ldots,p'$ are minimized, the next $p'+1,\ldots,p''$ are maximized and the last $p''+1,\ldots,p$ are kept at some given levels.

**Definition 2.2** *The decision $\hat{x} \in X$ such that $\hat{q} = f(\hat{x})$ is called $D_\epsilon$-efficient if belongs to the set:*

$$\hat{Q}_0^\epsilon = \{\hat{q} \in Q_0 : \hat{Q}_0^\epsilon \cap (\hat{q} + \operatorname{int} D_\epsilon) = \emptyset\}$$
$$\operatorname{int} D_\epsilon = \{q \in R^p : dist(q, D) < \epsilon\|q\|\}$$

*where $\epsilon$ is a given small number.*

It can be shown that $D_\epsilon$ - efficient solutions are properly efficient with a prior bound $1 + 1/\epsilon$ on trade-off coefficients, see Wierzbicki [12]. The definition (2.2) allows to consider as well "fuzzy equal" goals, thus including the concept of M-Pareto optimality.

The specification of the reference levels in the aspiration-led methodology is usually imprecise. Such imprecision of the specification of aspirations by DSS users can be included into the scalarizing function by following means:

- Instead of stating aspiration and reservation levels the user can specify fuzzy numbers which represent those levels, and then the fuzzy linear or nonlinear mathematical programing problem can be solved:

$$\text{maximize}\, s(x, \tilde{\bar{q}}, \tilde{\bar{\bar{q}}})$$
$$x \in X$$

  where
  $\tilde{\bar{q}}$ - is the reservation level treated as a fuzzy number
  $\tilde{\bar{\bar{q}}}$ - is the aspiration level treated as a fuzzy number

- An order-consistent scalarizing function can be found that depends on the membership functions of satisfactory values of objectives or fuzzy goals. This approach would combine fuzzy interactive methods and aspiration-led methodology.

We shall consider the second possibility, looking for a scalarizing function of the form:

$$s(q, \mu(q))) \tag{2.10}$$

where $\mu$ is the vector of membership functions of satisfactory values of objectives or fuzzy goals. We assume the following classes of membership functions for fuzzy maximized goals, proposed e.g. by Seo and Sakawa [10], [7].

Denote, as before
$q_i = f_i(x)$,
$\bar{q}_i$ - is a (least acceptable) reservation level of $q_i$,
$\bar{\bar{q}}_i$ - is an (desirable) aspiration level of $q_i$,
$M$ - is a class of functions defined below:

- **linear**

$$\mu_{q_i}(x) = [q_i - \bar{q}_i]/[\bar{\bar{q}}_i - \bar{q}_i] \quad \text{for} \quad \bar{q}_i \le q_i \le \bar{\bar{q}}_i \tag{2.11}$$

- **piece-wise linear**

$$\begin{aligned} \mu_{q_i}(x) = \mu_{i,j-1} + (\mu_{ij} - \mu_{i,j-1}) * (q_i - \bar{q}_{i,j-1})/(\bar{q}_{i,j} - \bar{q}_{i,j-1}) \\ \text{for} \quad \bar{q}_{i,j-1} \le q_i \le \bar{q}_{ij}, \quad j = 1, \ldots, N_i \end{aligned} \tag{2.12}$$

where

$$\mu_{i,0} = 0 < \mu_{i1} \ldots < \mu_{i,j-1} < \ldots \mu_{i,N_i} = 1$$

$$\bar{q}_{i,0} = \bar{q}_i < \bar{q}_{i1} \ldots < \bar{q}_{i,j-1} < \bar{q}_{i,j} < \ldots \bar{q}_{i,N_i} = \bar{\bar{q}}_i$$

- **exponential**

$$\mu_{q_i} = 1 - \exp[-\alpha_i(q_i - \bar{q}_i)/(\bar{\bar{q}}_i - \bar{q}_i)] \tag{2.13}$$

with $\alpha_i = -\ln(1 - \bar{\bar{\mu}}_i) > 0$, where $\bar{\bar{\mu}}_i$ s an assumed value of the membership function (a quantile) for $q_i = \bar{\bar{q}}_i$

- **hyperbolic**

$$\mu_{q_i} = 0.5(tanh(\alpha_i(q_i - \beta_i)) + 1) \tag{2.14}$$

with $\alpha_i = (\bar{\bar{a}}_i - \bar{a}_i)/(\bar{\bar{q}}_i - \bar{q}_i)$; $\beta_i = (\bar{\bar{a}}_i\bar{q}_i - \bar{a}_i)\bar{\bar{q}}_i/(\bar{\bar{a}}_i - \bar{a}_i)$; $\bar{a}_i = artanh(2\bar{\mu}_i - 1)$; $\bar{\bar{a}}_i = artanh(2\bar{\bar{\mu}}_i - 1)$, where again $\bar{\bar{\mu}}_i$ and $\bar{\mu}_i$ are assumed membership function values (quantiles) for $\bar{\bar{q}}_i$ and $\bar{q}_i$.

The membership function for a fuzzy goal can be interpreted as a function which specifies the preferences of the user as well as a function which implies an ordering in the decision or objective space. A contour of such a function for two objectives has been is shown in Figure 1. A common disadvantage of the linear and piece-wise linear functions (as well as the exponential one if $q_i \le \bar{q}_i$ ) is that they are equal to 0 or 1 below $\bar{q}_i$ or above $\bar{\bar{q}}_i$.

In order to distinguish alternatives such that $f_i(x) < \bar{q}_i$ or $f_i(x) > \bar{\bar{q}}_i$, we have to relax the traditional interpretation of a membership function as a (multi-valued) logical expression and to admit as well values $\mu_{q_i}(x) < 0$ for $f_i(x) < \bar{q}_i$ or $\mu_{q_i}(x) > 1$ for $f_i(x) > \bar{\bar{q}}_i$. This can be done for the linear function (2.11) or the piece-wise linear one (2.12) in the following way, analogous to (2.8):

$$\eta_i = \begin{cases} \gamma(q_i - \bar{q}_i)/(\bar{q}_i - q_{i,min}) - 1 & \text{if } q_{i,min} \le q_i \le \bar{q}_i \\ \mu_{q_i} & \text{if } \bar{q}_i < \bar{q}_i < \bar{\bar{q}}_i \\ \beta(q_i - \bar{\bar{q}}_i)/(q_{i,max} - \bar{\bar{q}}_i) + 1 & \text{if } \bar{\bar{q}}_i \le q_i \le q_{i,max} \end{cases}$$

where $\eta_i$ is an **extended-valued membership function**. Similar extensions can be made for other forms of membership functions. An order-consistent scalarizing function has then the usual form:

$$s(q, \mu_q) = (\min_{1 \le i \le p} \eta_i + \frac{\rho}{p} \sum_{i=1}^{p} \eta_i)/(1 + \rho)$$

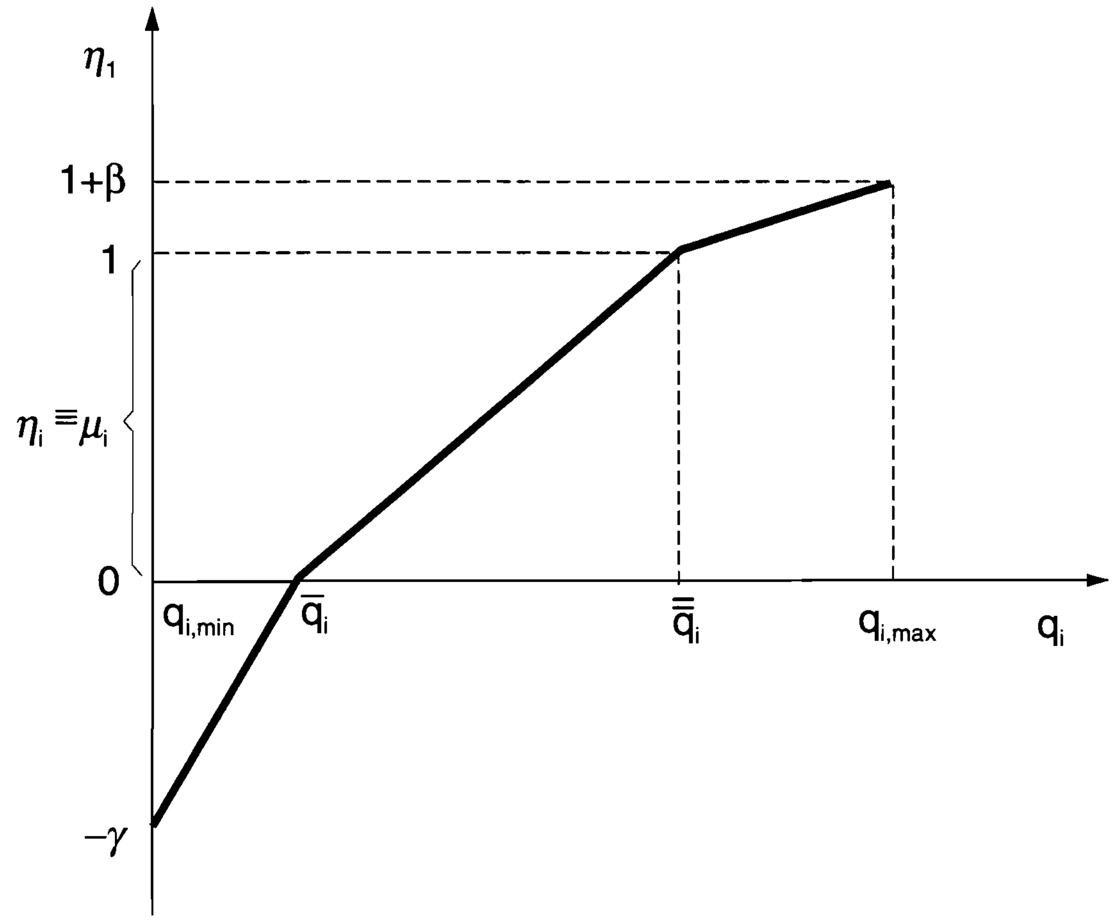


Figure 2: An extended-valued membership function

and can be as well interpreted as an extended-valued convolution of membership functions, since its values are also between 0 and 1 if $\bar{q}_i \leq q_i \leq \bar{\bar{q}}_i$ for all $i = 1, \ldots, p$.

Between $\bar{q}$ and $\bar{\bar{q}}$ the values of the function $\eta$ coincide with the membership function and can be used to represent a (multivalued) logical expression as well as to order alternatives, but for $q < \bar{q}$ or for $q > \bar{\bar{q}}$ we use the values of $\eta$ only to order alternatives. If we choose the linear membership function, then the convolution $s$ is the same as (2.7).

An order-consistent scalarizing function is defined generally by two properties:

- **the sufficiency property**
  For each $\mu_q$ from (suitably extended) class $M$

$$\operatorname*{Argmax}_{q \in Q^0} s(q, \mu_q) \subset \hat{Q}_0^\epsilon \tag{2.15}$$

- **the necessity property**
  For each $\hat{q} \in \hat{Q}_0^\epsilon$, there exist such an extended membership function $\hat{\mu}_q$ from (extended) class $M$

$$\hat{q} \in \operatorname*{Argmax}_{q \in Q_0} s(q, \hat{\mu}_q) \tag{2.16}$$

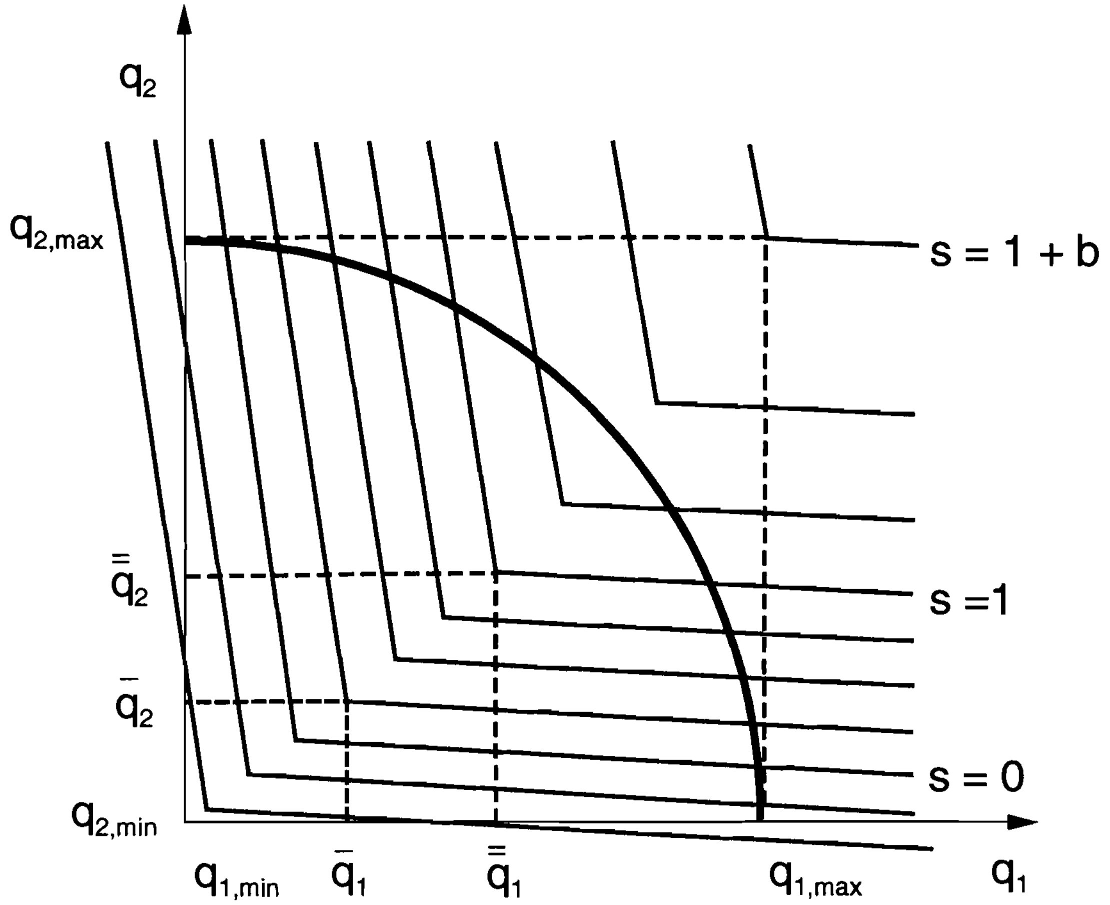


Figure 3: Level sets of an extended-valued membership function

To obtain these general properties, two basic concepts are needed: *monotonicity* and *separation of sets.* The following theorem shows the importance of monotonicity:

**Theorem 1 (Wierzbicki [11],[12])** *Let a function $r : Q_0 \mapsto R^1$ be strongly monotone, that is, let $q' > q''$ (equivalent to $q' \in q'' + \tilde{D}$, $\tilde{D} = D \setminus \{0\}$, imply $r(q') > r(q'')$. Then each maximal point of this function is efficient. Let this function be strictly monotone, that is, let $q' \gg q''$ (equivalent to $q' \in q'' + intD$) imply $r(q') > r(q'')$. Then each maximal point of this function is weakly efficient. Let this function be $\epsilon$-strongly monotone, that is, let $q' \in q'' + \operatorname{int} D_\epsilon$ imply $r(q') > r(q'')$. Then each maximal point of this function is properly efficient with bound $\epsilon$.*

**Definition 2.3** *A function $r : R^p \mapsto R^1$ strongly separates two disjoint sets $Q_1$ and $Q_2$ in $R^p$, if there is such $\beta \in R^1$, that $r(q) \geq \beta$ for all $q \in Q_1$ and $r(q) < \beta$ for all $q \in Q_2$.*

The strong separation property by a function with conical level sets is used in the following definition of a **F-order-consistent achievement function**:

**Definition 2.4** *F-order-consistent achievement function is defined generally as such continuous function $s(q, \mu_q)$, which is $\epsilon$-strictly monotone as a function of $q \in Q_0$ for any $\mu_q$*

*from the class $M$ and moreover posses the following property of order approximation:*

$$\bar{q} + D_{\bar{\epsilon}} \subset \{q \in R^p : s(q, \mu_q) \geq 0\} \subset \bar{q} + D_\epsilon$$

*for all $\mu_q$ from the class $M$, with $\epsilon > \bar{\epsilon} \geq 0$.*

We need yet the definition of **F-properly efficient decisions with bound** $\epsilon$:

**Definition 2.5** *An F-properly efficient decision with bound $\epsilon$ is such an efficient decision that the trade-off coefficients between the values of component membership functions $\mu_{q_i}$ and $\mu_{q_j}$, for any $i \neq j \leq p$, are bounded by $1 + 1/\epsilon$.*

Similarly as in [11], [12], the following theorems can be proved:

**Theorem 2** *Let $s(q, \mu_q)$ be an F-order-consistent achievement function. Then for any $\mu_q$ from the class $M$ each point that maximizes $s(q, \mu_q)$ over $q \in Q_0$ is efficient.*

**Theorem 3** *If $\hat{q}$ is F-properly efficient with bound $\epsilon$, then there is such a $\mu_{\bar{q}}$ from the class $M$ that the maximum of $s(q, \mu_{\bar{q}})$ with $\bar{q} = \hat{q}$ over $q \in Q_0$ is attained at $\hat{q}$ and is equal to $0$.*

It is easy to check that the achievement function is F-order-consistent; however, it is also nondifferentiable. In the (piece-wise) linear case, its maximization can be rewritten as a linear programming problem. Otherwise, the NOA1 solver (developed by Kiwiel and Stachurski [4]) for nondifferentiable optimization can be used. We can also approximate this function by a smooth function and then use a standard nonlinear solver.

Until now, we dealt primarily with objectives to be fuzzy maximized. However, all results either directly relate also (as indicated by the definition of $D_\epsilon$ efficiency) or can be generalized to the cases of objectives which are fuzzy minimized and fuzzy equal.

# 3 Interaction by changing membership functions and the interpretation of solutions in terms of fuzzy sets

An achievement scalarizing function that characterizes the set of F-properly efficient solutions was constructed in the previous section. However, such a function is an internal feature of a DSS. The user of a DSS is more interested in interactive tools which can help him in knowledge acquisition during a decision process and in reaching a proper decision. The scenarios of interaction with the user depend on how many membership functions have been changed as well as on the kind of changes of a membership function. Four types of changing membership functions can be considered:

- moving $\bar{q}$

- moving $\bar{\bar{q}}$
- moving $\bar{q}$ and $\bar{\bar{q}}$
- changing the character (or other characteristic points) of the membership function

Three graphs of membership functions for given objectives are presented on the left side of the Figure 4. Points $P_1', P_2', P_3'$ denote the solutions for specified membership functions. If e.g the user changes a membership function, as it is shown on the right side of Figure 4, the solution will change. The thick lines show the sensitivity of the solution to the changes of the membership function for the objective $q_1$. If the value of function $\eta_i < 0$ or $\eta_i > 1$ then the solution point is projected on the line $\eta_i = 0$ or $\eta_i = 1$, as it is shown on the graph for objective $q_2$.

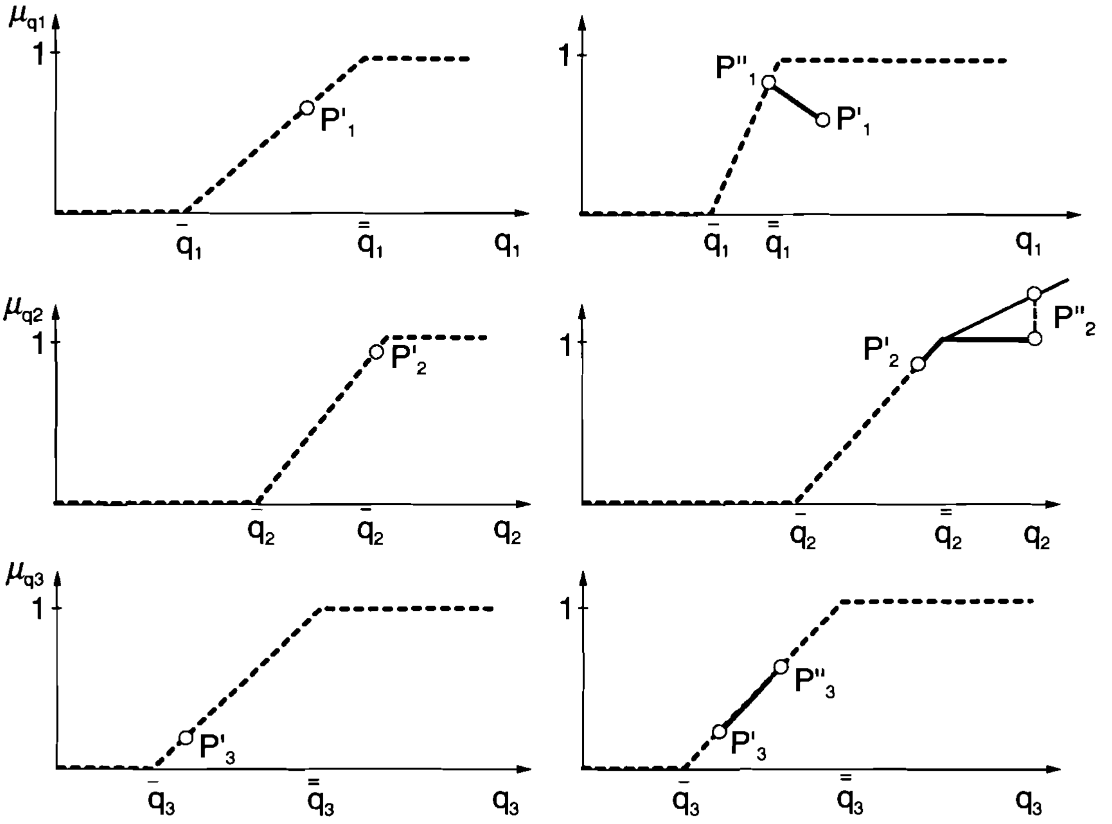


Figure 4: The prototype of an interaction screen

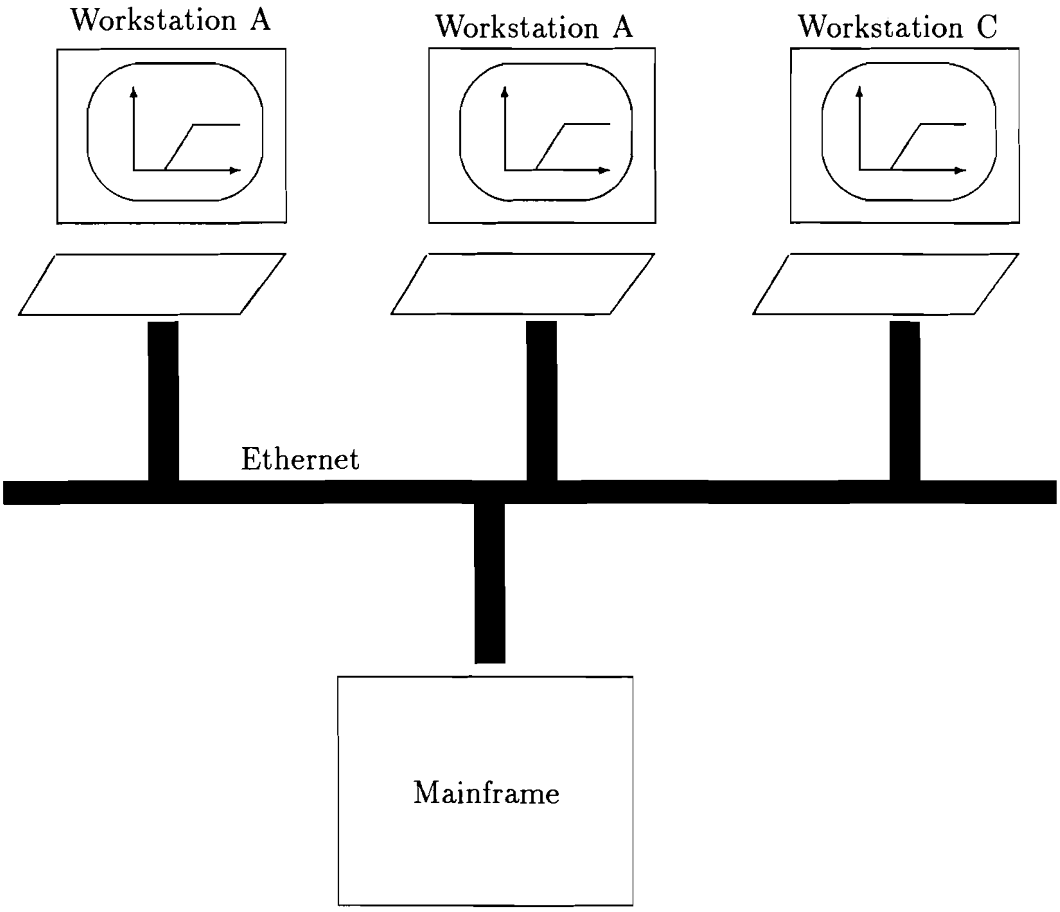


Figure 5: A hardware configuration for group decision making

# 4 Extensions for the case of group decision making

The approach presented above can be also extended for group decision making. It can be assumed that each decision maker in the group is responsible for some decision goals - which can be of the types fuzzy min, fuzzy max and fuzzy equal. Then each decision maker can specify and subsequently modify his membership function for each of these goals. These membership functions can be used to build an aggregated function:

$$\mu_{f_i} = \Phi(\mu_{ij}(x))$$

where:

$i$ -is the index of the objective function
$j$- is the index of the decision maker.

The specific way of aggregations might use minimum, maximum or linear convolutions of membership functions; its choice might express various assumptions about the nature of cooperation within the group. The interactive decision process can also be organized variously. If we assume team-like behavior of the group with full information sharing, each decision-maker in the group could have access to the displays of membership functions specified by each other decision-maker, together with the corresponding decision outcomes and their changes.

Advanced computer technology makes it possible today to build distributed decision support systems. For developing graphical user interfaces in such systems, X Window system is especially interesting. This system allows to run a X client on one computer and display graphic information on another computer. Additionally, the UNIX mechanisms makes it possible to organize the inter-process communication over the network (Transport Layer Interface or Berkeley Sockets). An example of hardware configuration is shown in Figure 5. The tasks in fuzzy group decision making can be divided in the following way: there is one X client for each of the decision makers. This X client is responsible for interaction with help of membership functions and for the presentation of the results to the user. Another process, which can be run on the mainframe, can perform all optimization calculations.

# 5 A numerical example

A simple example with real data is presented in this section. The data and main idea of this example come from the RAINS model which was developed at IIASA. We consider here three countries: Poland, Germany and (treated jointly because of the availability of historical data) Czecho-Slovakia. Germany is represented by two parts (one including the region of old FRG an the other the region of old GDR, because of differences in the levels of emissions $SO_2$). Cost functions of emission reduction and formulae for calculating deposition of $SO_2$ are given for each of these countries. The cost functions assumed are approximations of the National Cost Curves published by Amann and Kornai [1]. The transfer coefficients are based on the work of Eliassen and Saltbones [3]. It should be pointed out that this example and data are rather rough approximations of real situation and are used only to illustrate the applicability of the proposed method. The problem is to minimize cost and minimize deposition in each of countries considered. The multicriteria optimization problem is defined as follows:

```
Formulae:
========
    cost_cze    := 0.0006 * SQR ( er_cze ) + 0.9      * er_cze

    cost_ger    := 0.0001 * SQR ( er_e_ger ) + 0.9    *  er_e_ger  +
                   0.0009 * SQR ( er_w_ger ) + 1.5796 * er_w_ger

    cost_pol    := 0.0007 * SQR ( er_pol ) + 0.0001   * er_pol

    dep_cze     := 0.386  * ( i_em_cze   - er_cze   ) +
                   0.1199 * ( i_em_e_ger - er_e_ger ) +
                   0.1088 * ( i_em_w_ger - er_w_ger ) +
                   0.076 *  ( i_em_pol   - er_pol   ) + 649.6

    dep_e_ger   := 0.0448 * ( i_em_cze   - er_cze   ) +
                   0.298  * ( i_em_e_ger - er_e_ger ) +
                   0.0562 * ( i_em_w_ger - er_w_ger ) +
                   0.0184 * ( i_em_pol   - er_pol   ) + 151.8

    dep_w_ger   := 0.03884* ( i_em_cze   - er_cze   ) +
                   0.0707 * ( i_em_e_ger - er_e_ger ) +
                   0.371  * ( i_em_w_ger - er_w_ger ) +
                   0.0144 * ( i_em_pol   - er_pol   ) + 531.2

    dep_pol     := 0.1088 * ( i_em_cze   - er_cze   ) +
                   0.1277 * ( i_em_e_ger - er_e_ger ) +
                   0.0529 * ( i_em_w_ger - er_w_ger ) +
                   0.425  * (  i_em_pol  - er_pol   ) + 532.1
Variables:
=========
    Name        Units      UpperB        Value         LowerB
    ----        -----      ------        -----         ------
    er_cze      tysTSO2   2.350E+0003   1.266E+0003     0.0
    er_e_ger    tysTSO2   4.900E+0003   3.379E+0003     0.0
    er_w_ger    tysTSO2   2.850E+0003   1.462E+0003     0.0
    er_pol      tysTSO2   4.100E+0003   2.515E+0003     0.0

Parameters:
==========
    Name        Units      UpperB        Value         LowerB
    ----        -----      ------        -----         ------
    i_em_cze    tysTSO2                 2.350E+0003
    i_em_e_ger  tysTSO2                 4.900E+0003
    i_em_w_ger  tysTSO2                 2.850E+0003
    i_em_pol    tysTSO2                 4.100E+0003
```

```
Outcomes:
========
    Name        Units       UpperB        Value         LowerB
    ----        -----       ------        -----         ------
    cost_cze    mln_DM     1.000E+0006   2.101E+0003     0.0
    cost_ger    mln_DM     1.000E+0006   8.415E+0003     0.0
    cost_pol    mln_DM     1.000E+0006   4.428E+0003     0.0
    dep_cze     tysTSO2    1.000E+0006   1.522E+0003     0.0
    dep_e_ger   tysTSO2    1.000E+0006   7.609E+0002     0.0
    dep_w_ger   tysTSO2    1.000E+0006   1.219E+0003     0.0
    dep_pol     tysTSO2    1.000E+0006   1.591E+0003     0.0

Goals:
=====
                          minimize cost_cze
                          minimize cost_e_ger
                          minimize cost_w_ger
                          minimize cost_pol
                          minimize dep_cze
                          minimize dep_e_ger
                          minimize dep_w_ger
                          minimize dep_pol
```

where:

cost_x - cost of emission reduction in region x
dep_x - deposition in region x
i_em_x - initial emission in country x
er_x - emission reduction in country x
cze - Czecho-Slovakia
e_ger - east part of the Germany
w_ger - west part of the Germany
pol - Poland

The calculation has been performed by DIDAS-N system [5], to which fuzzy interactive possibilities were added. The first interaction screen - Figure 6 - presents cost functions and a current solution. Then it is assumed that the DSS user moves the reservation level $\bar{q}$ value for the cost in the Czecho-Slovakia. The results are presented in Figure 7.

# 6 Conclusions

Experiments with the proposed, extended fuzzy interactive method show that it can be useful in solving practical problems. In further research, a nondifferentiable solver will be used for optimization computations; moreover, the distributed version for group decision making will be further developed.

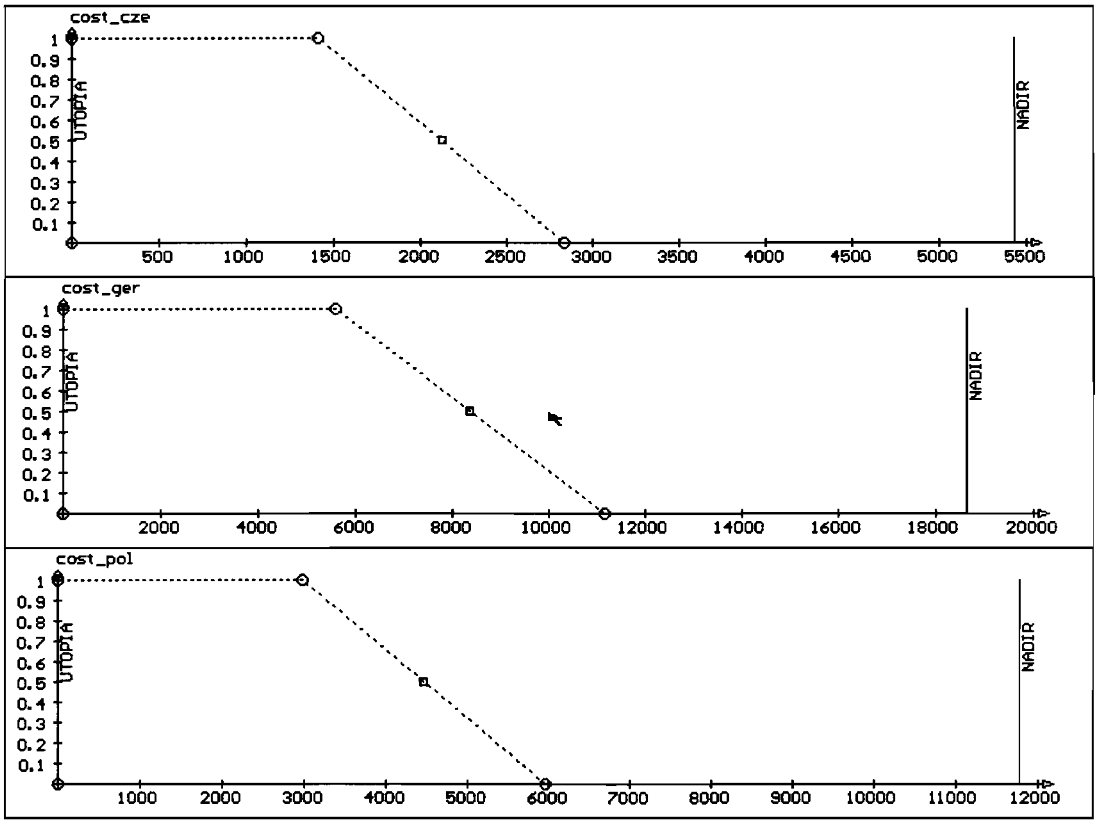


Figure 6: First interaction screen

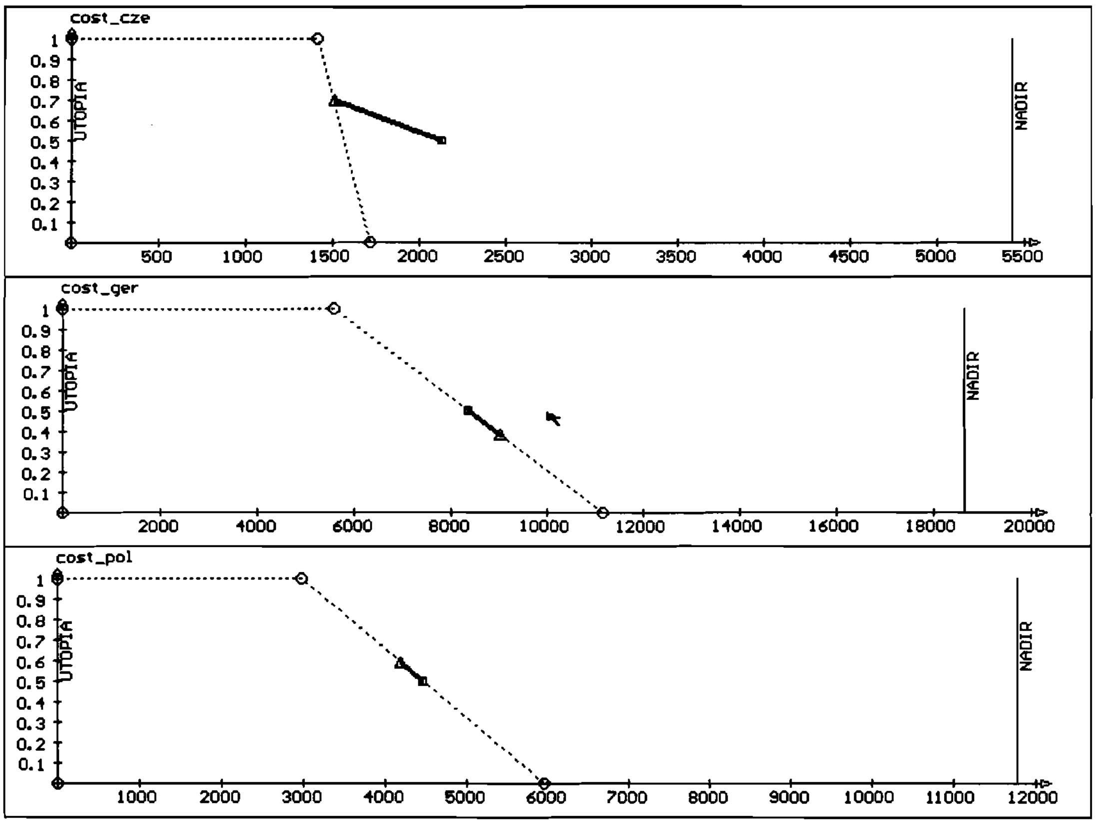


Figure 7: Second interaction screen

# References

[1] Amann M., G. Kornai. (1987) Cost Functions for Controlling SO2 Emissions in Europe". IIASA WP-87-065.

[2] Bellman R. E., L. A. Zadeh. (1970) Decision making in a fuzzy environment. *Management Science*, vol. 17, no. 4, pp. 141-164.

[3] Eliassen A., J. Saltbones. (1983) Modeling of Long-Range Transport of Sulphur over Europe: A Two-year Model run and some Model Experiments. *Atmospheric Environment* vol 17. No. 8.

[4] Kiwiel, K. and A. Stachurski (1988) NOA1: A FORTRAN Package of Nondifferentiable Optimization Algorithms, Methodological and User's Guide, WP-88-116, IIASA, Laxenburg, Austria.

[5] Kreglewski, T., Granat J., Wierzbicki A.P. (1991) IAC - DIDAS - N: A Dynamic Interactive Decision Analysis and Support System for Multicriteria Analysis of Nonlinear Models , CP-91-010, IIASA, Laxenburg, Austria.

[6] Lewandowski, A., Wierzbicki A.P. (Eds.) (1990) Aspiration Based Decision Support Systems, Springer. Springer Verlag, Berlin.

[7] Sakawa M. (1993) Fuzzy Sets and Interactive Multiobjective optimization. Plenum Press. New York/London.

[8] Sakawa, M., Yano, H. (1985) An interactive fuzzy satisficing method using augmented minimax problems and its application to environmental systems, *IEEE Transactions on Systems, Man and Cybernetics*, SMC-15, 720-729.

[9] Seo F., I. Nishizaki. (1993) A Configuration of Intelligent Decision Support Systems for Strategic Use: Concepts and Demonstrations for Group Decision Making. In Wessels, J. and Wierzbicki A.P. eds.: User-Oriented Methodology and Techniques of Decision Analysis and Support. Springer Verlag, Berlin.

[10] Seo F., M. Sakawa. (1988) Multiple Criteria Decision Analysis in Regional planning. Reidel Publishing Company. Dordrecht.

[11] Wierzbicki, A. P. (1986) On the Completeness and Constructiveness of Parametric Characterizations to Vector Optimization Problems. *OR Spectrum*, Vol. 8, No. 2, pp. 73-87.

[12] Wierzbicki, A.P. (1990) Multiple Criteria Solutions in Noncooperative Game Theory. Part III: Theoretical Foundations. Discussion Paper No. 288. Kyoto Institute of Economic Research.

[13] Zadeh, L. A. (1965) Fuzzy sets. *Information and Control* vol. 8, pp. 338-353.