TEMPORAL HIERARCHY OF THE

PRODUCTION MANAGEMENT SYSTEM

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December 1975

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Abstract

The problem considered here is that of managing and controlling the industrial system to achieve efficient performance. Emphasis is placed on structuring the decision making and control functions, taking into account the fact that the production process runs continuously with no starting or end point, and undergoes major changes in product specifications, quality requirements, equipment characteristics, resource availability, and the like. Since all these changes are time functions, the time factor plays a very important role in decision making and control and in functional structuring.

It is shown that the functional structure is a multilevel hierarchical mode with horizontal and vertical decomposition planes. The vertical planes represent temporal decomposition, reflecting the subordination of decision making and control for each time duration or time horizon. The horizontal planes form layers related to each time horizon and consist of the set of decomposed subproblems to be solved in coordinated mode.

The philosophy of this functional hierarchical structure is discussed and some motivation for time horizon estimation is given.

Introduction

Performance of the production system depends on a variety of factors, including product specifications and the technology used for product manufacture, the nature of available resources and environmental constraints, allocation of resources, scheduling of operating sequences, etc. We distinguish two phases of system evolution with respect to information processing and decision making functions.

a) Design phase: Here decisions are made concerning production process performance for the time horizon considered. This phase is called production planning and scheduling, and relates to the preparation of the production process by means of a model reflecting plant capability and boundary conditions imposed by links

with the environment. The functions of this phase are estimation of the requirements for fulfilling the given assignment of goods to be manufactured during the time interval considered. For a given plant capability, the required actions for assignment fulfillment can be considered as the control actions distributed over the time interval (or horizon) in order to obtain the optimal trajectory of production process performance for satisfying the given objectives.

b) Operating phase: Here the control actions defined in the design phase are implemented. Disturbances not predicted by the design phase which influence production process performance cause deviations from the estimated optimal trajectory; to reduce this influence additional control actions are generated.

Model Creation

Models of a real process can reflect only the "main" variables that greatly influence process performance; but other variables, not considered by the model, cause variation of the model parameters. These parameters usually are estimated by statistical methods during investigation of the process. Since the production process is influenced by the environment, whose behaviour is of a random nature, all the main variables are random functions of time $x(t)$; $y(t)$; $z(t)\ldots$, and the production process performance simulated by the model is also a random function:

$$x(t) = \{x(t); \; y(t); \; z(t)\ldots\} \quad . \tag{1}$$

It is obvious that the more variables are included in the model, the less will be the deviation of the simulated process from the real one. But increasing the number of variables is impractical because of the great increase in model complexity.

The model used in practice thus has a limited number of variables, and the relation among them is of deterministic nature. The deviation of the simulated process from the real process is considered as the influence of the "disturbances" affecting the real process. These disturbances are random time functions of different frequency spectra.

In order to show the influence of the disturbance frequency spectra on process evaluation, depending on the time considered, let us investigate machine tool performance. Over a short period of time, this performance can be considered as quasi-stationary; over a longer interval, we must regard it as non-stationary, due to the influence of tool wear (Figure 1). We can again consider it as quasi-stationary, due to periodic readjustment of the tool, if the time of process observation is greatly increased. Evaluating performance for a year or even longer, the process will show itself to be non-stationary, again due to wear of the machine tool itself.
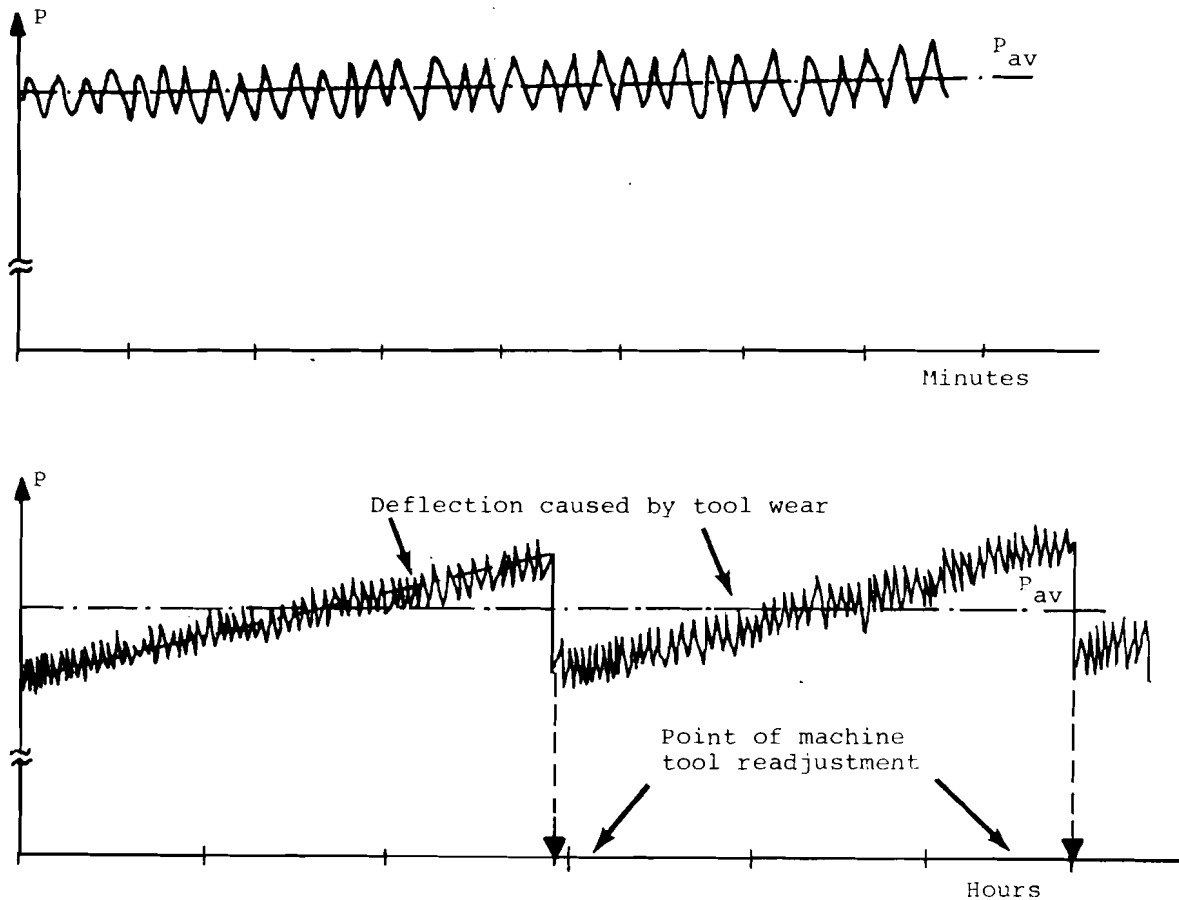
Figure 1. Process evaluation for different time intervals.
(P = evaluation index, P_{av} = averaged value of P).

Knowledge of the time behaviour of the disturbances helps in creating models for process performance evaluation for different time horizons. These models permit an estimate of prior control actions to be taken, in a sequence corresponding to the different time intervals, in order to obtain optimal process performance. The production process, having no starting or end state, can be considered a stationary process over a long time horizon; this means that many variables being averaged during this time interval have zero "expectation" and need not be considered as model variables.

This fact can be interpreted in the following way: disturbances, being periodic functions of high frequency in relation to the long time horizon, need not be included in the model.

Thus, for a long time horizon the model may have a small number of variables without loss of the required precision in process performance evaluation. But for shorter time horizons the frequency of the disturbances may be relatively low and their influence, averaged over the shorter interval, cannot be considered as equal to zero.

In some cases the time behaviour of the disturbances can be defined by considering physical phenomena (e.g. tool wear), but in more general cases, the process relations are very obscure and statistical methods should be used.

Let us presume that we have the simplified model of the process; the question is for which time horizon (T_m) this model is sufficient. A computational technique, similar to that used in statistics for confidence interval T estimation, can be applied to the definition of the time horizon:

$$e_r = \frac{1}{T} \int_0^T \{x(t) - x_m(t)\} \, dt \quad , \tag{2}$$

where $x_m(t) = f\{x_1(t), x_2(t), \ldots x_n(t)\}$ is the process performance evaluated by the model; $x(t) = g\{x(t), y(t), z(t), \ldots\}$ is the real process performance evaluation; T is the time period considered by the model; $y(t), z(t), \ldots$ are variables not considered by the model and influencing the process performance as disturbances. With this technique, computation of the integral is performed through the time of process observation until the value of e_r equals the estimated value $\Delta \approx 0$. The current time, when the computation is stopped, is the value T_m that we are searching for.

Increasing the number of variables reflected by the model, we may find the new time horizon $T_{m1} < T_m$ which satisfies the condition:

$$e_r = \frac{1}{T_{m1}} \int_0^{T_{m1}} \{x(t) - x_{m1}(t)\} \, dt = \Delta \approx 0 \quad , \tag{3}$$

where x_{m1} is the new process model with the increased number of variables, thus reflecting the process more precisely.

The time behaviour of different variables can be established by means of correlation analysis. Thus by calculating the correlation function of the influence of a given variable on the process performance measured during the experiment, we may find the time (τ), which corresponds to the attenuation of this function.

Therefore, this technique can be used for creation of the simplified model for the increased time horizon. The model with a longer time horizon is created not only by excluding the variables that do not influence process performance, but also by aggregating the remaining variables.

Let us consider reheating furnace control. The model which is used for metal heating optimization takes into account the variations of furnace temperature during the heating cycle. But the model used for scheduling furnace operations does not include furnace temperature as an explicit variable, since the temperature variations averaged over several heating cycles should have effectively zero expectation. For this latter model, one of the variables will be heating cycle time, which is a function of the heating condition; the fluctuation of the heating cycle time is caused by variations of the mass and thermal properties of the metal charged in the furnace, which, averaged over a long period of time, may be considered as having zero expectation. Thus, for a period of a month or more, the heating cycle time may be considered as a standard with respect to monthly planning of furnace operation.

Hierarchical Structure of Models

The models are used to define future performance of the process as close as possible to the optimal. The optimal process performance in terms of control theory is represented by the optimal trajectory: the track of the process state in multidimensional space.

Since the model for the longer time horizon is less detailed and its parameters are more averaged than those for shorter time horizons, the optimal trajectory found by the former is also of averaged mode. Since the model corresponds to the shorter time horizon, the optimal trajectory found by it has a shorter duration and represents a more detailed segment of the trajectory found for the larger mode.

Since the models for shorter time horizons and with a larger number of variables are more complicated, there may be some difficulty in the estimation of optimal trajectory segments and control actions. To overcome this difficulty the decomposition technique can be used: for a shorter time horizon, instead of one multivariable model, the set of decomposed models can be used, each having fewer variables.

Thus, as can be seen from the above, the structure of models used for control of the continuously running production process is of pyramidal form (Figure 2). On top of this pyramid is located a model of more averaged type, having fewer variables and less detail, and by means of this model an averaged optimal trajectory for a long time interval is found.

The next lower layer of the structure has the set of decomposed, more detailed submodels, by means of which a segment of

Decision pattern

Adaptation pattern

Time horizon T_1

Time horizon T_2

Time horizon T_3

Time horizon T_4

M_1

M_2

M_3

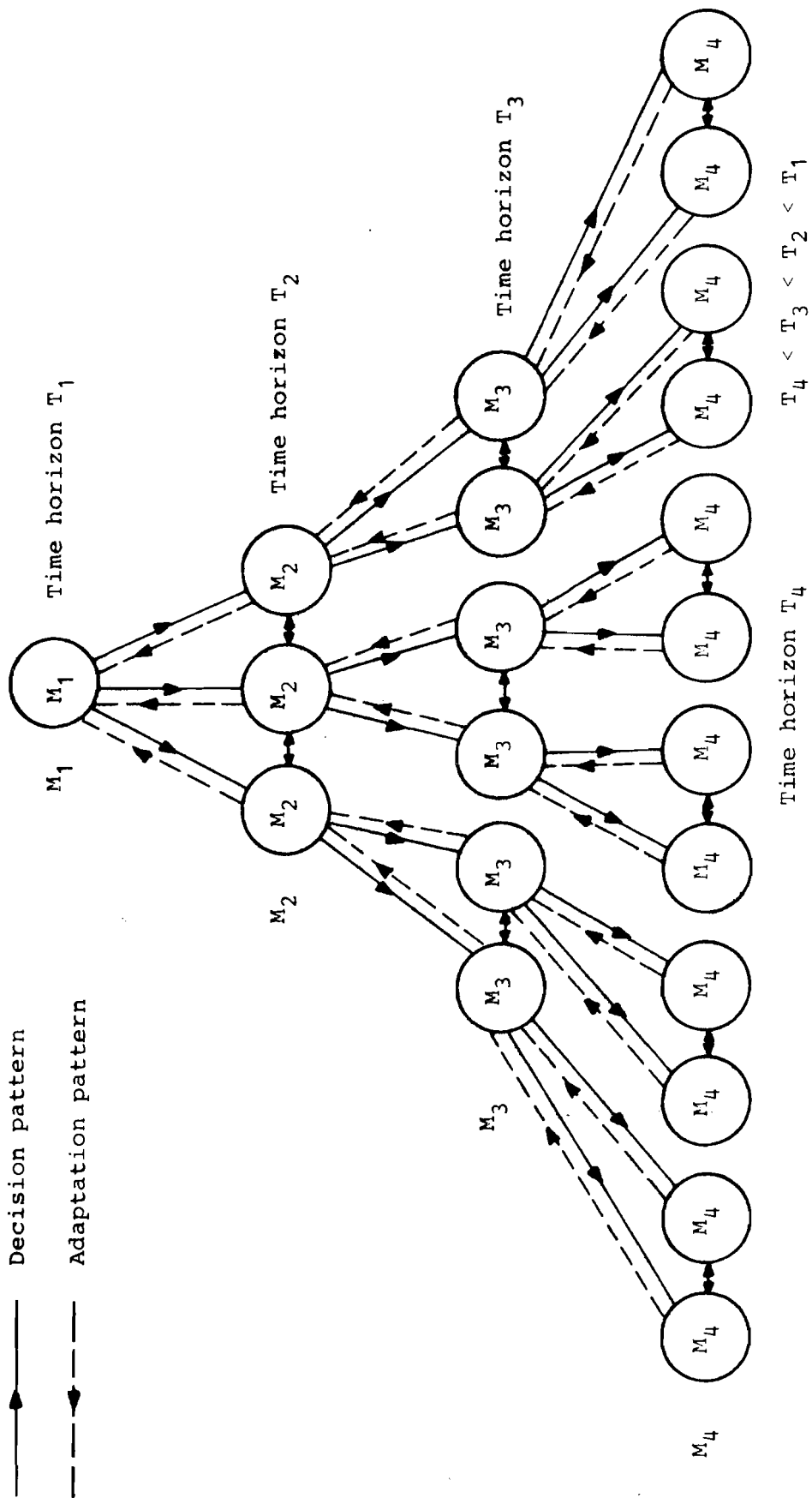M_4

$T_4 < T_3 < T_2 < T_1$

Figure 2. Pyramidal structure of the hierarchical model system.

a previously found trajectory is defined more precisely for a duration corresponding to the time horizon considered. The following lower layers of the structure have models of still more detailed mode, such that more submodels are located in this layer.

The base of the pyramid is composed of a set of models for real-time horizons, and thus reflects current process conditions with the highest possible accuracy (depending on the number of process variables available for measurement).

Each of the layers with submodels is of two levels, since the control actions created by separate submodels must be coordinated.

The pyramidal model structure described is of a temporal multilayer hierarchy mode, since each layer includes models of different time horizons and the lower layers are subordinated to the upper. This subordination means that the set of control actions generated by the upper layer can be considered as the assignment to be fulfilled by the lower layer.

Decision Making and Control in the Multilayer Type System

To design the decision making and control systems for a production process, the conceptual framework should be created. In describing this framework let us consider a production process to be controlled as a plant which can be defined in deterministic form as:

$$y = g(m,z,s,w) \quad , \tag{4}$$

where y, m, z, s, w denote vectors of output variables, controlled inputs, disturbances, state variables and external inputs as the objectives of the process performance.

During the design phase the real process does not yet exist and therefore disturbances are equal to zero ($z = 0$). The control is found by maximizing the function

$$p = f(m,y,w) \quad . \tag{5}$$

The result of maximization implies a relationship of the form

$$m_d = m(y^*,w^*) \quad , \tag{6}$$

where $m_d < M_d = \{m/w = g(m,y,s), \ h(m,y,w) > 0\} \quad .$

In other words, controls to be applied to the real process after it starts are defined by the model reflecting plant input-output relation $g(m,y,s)$, the constraints $h(m,y,w)$ and the external input vector w^*, which is the plant assignment.

Insofar as the models are different for different time horizons, let $g_1(\cdot)$ describe the model of the highest layer of the hierarchical structure and m_{1d} be the control or decision making function found by the model to which external inputs w^* have been applied:

$$m_{1d} = m(w^*,y^*) \qquad m_{1d} < M_{1d} \quad ,$$

where

$$m_d < M_{1d} = \{m/w = g_1(\cdot), h_1(\cdot) > 0\} \quad . \tag{7}$$

In general w^* can be a vector function of time (for instance, assignment for different manufacturing of goods with different delivery time); so also m_{1d}, which represents decision making or control actions distributed along the given time horizon.

In accordance with the subordination of the layers, vector m_{1d} can be considered as the external assignment w^* for the next-lower-layer model.

As for the first layer, we may find for the second-layer model

$$m_{2d} = m(w_1{}^* = m_{1d}) = w_2{}^*$$

$$m_{2d} = \{u/w = g_2(\cdot), h_2(\cdot)\} > 0 \quad , \tag{8}$$

and for the i-th layer

$$m_{id} = m\{m_{(i-1)d}\} \quad ,$$

where

$$m_{id} < M_{id} = \{m/w = g_i(\cdot), h_i(\cdot)\} > 0 \quad .$$

As has been mentioned above, the complexity of the shorter time horizon models, in spite of the shorter time considered, can make the problem of control estimation formidable, requiring the use of decomposition techniques based on a multilevel approach. In the multilevel hierarchy the subsystem problems are solved at the first step. But these solutions have no meaning unless the model interaction constraints are simultaneously satisfied. This is the coordination problem that is solved at the second step by the iterative procedure. There are a variety of coordination schemes that have been proposed: price adjustment coordination, primal coordination, penalty function, etc.

Figure 3 illustrates the top part of the pyramidal structure involving the top layer which we denote as layer No. 1, and two lower levels, Nos. 2 and 3, having time horizons T_1, T_2 and T_3 respectively. Let us presume that the top model is a simple one and that, to solve the control problem by means of this model, the decomposition technique need not be used. Being of simplified nature this model operates in multidimensional space, which has more dimensions than the vector w of external inputs (the assignment for the whole plant for the time horizon T_1). The difference of dimensions results in some of the components of the vector w not being reflected by the decision vector m_{1d}, which represents the aggregated assignment for layer No. 2, whose models are much more detailed and thus may be of higher dimension. In order that the assignment w_2 of higher dimension conform with the decision vector m_{id}, this vector is decomposed into the set of sub-vectors $m_{1d_2}^1, m_{2d_3}^2, m_{3d}^3$, which form the more detailed assignments

$$w_2^1, \quad w_2^2, \quad w_2^3$$

for the second layer. The number of sub-vectors and their components are defined by the scope and structure of the second-layer models. The decision made by the first layer is of such a mode that components of the sub-vectors m_{1d}, m_{2d}, m_{3d} are coordinated, i.e. the assignments for layer No. 2 take account of the capabilities of the plant's divisions for a time duration $T_2 < T_1$.

The problem solution for a time duration $T < T_2$ is coordi- by the controller CR_2, considering all the constraints related to the time interval $T < T_2$. This coordination is performed through the model's interconnection variables q_2^1, q_2^2 and q_2^3. Decisions made by means of each of the models $(g_2^1, h_2^1), (g_2^2, h_2^2),$
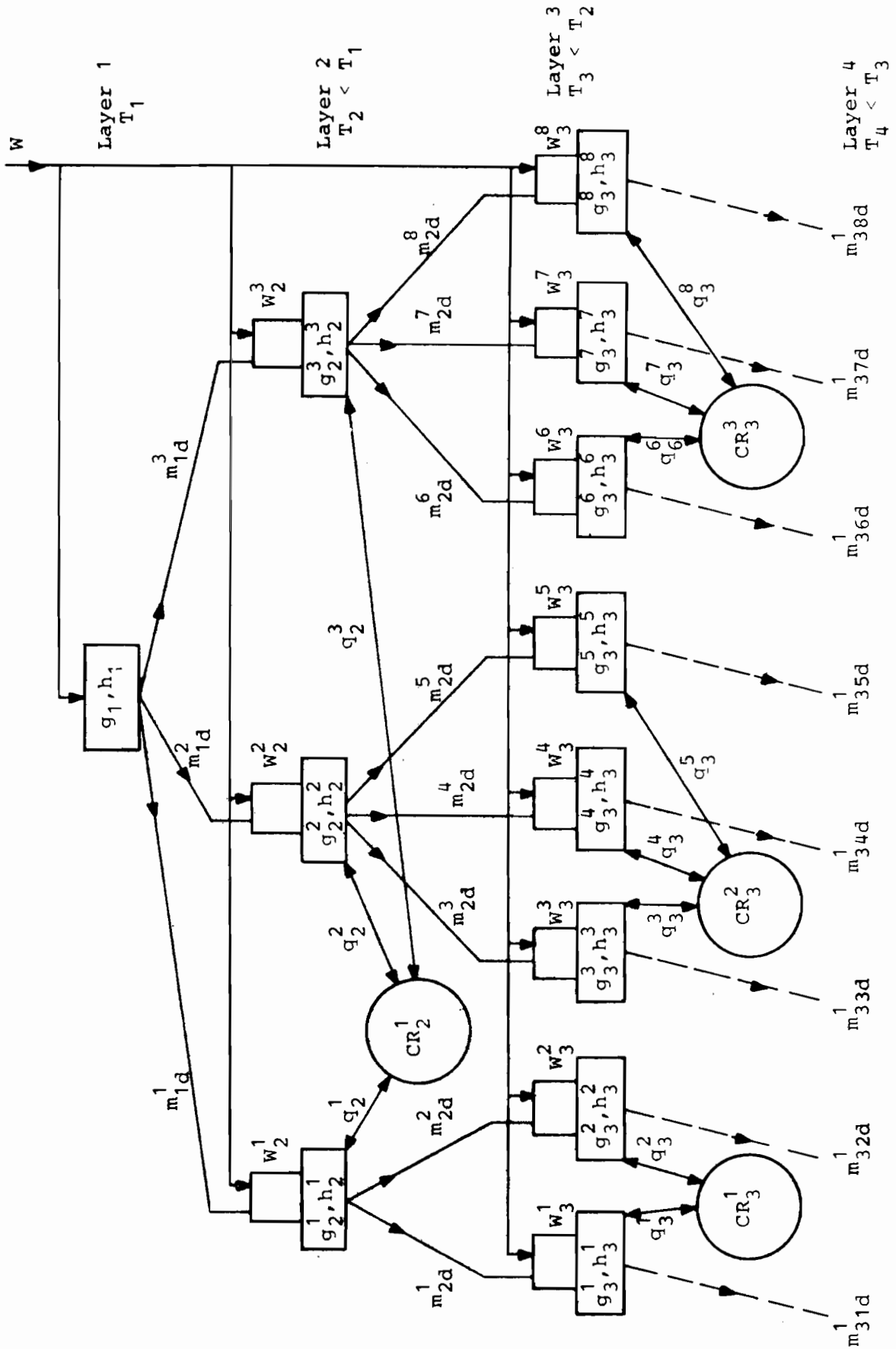
Figure 3. Upper layers of the model structure.

(g_2^3, h_2^3) form the assignment for the third layer by the same mode as for the second layer. Thus, the task of each layer is to form the assignment for the next-lower layer coordinated for a time duration equal to the layer time horizon. Inside the layer coordination is performed in the ordinary way by the controller CR.

All the described functions are carried out during the designing phase--that is, before the defined control and decisions are implemented in the real process. If, during the operating phase, the disturbances do not influence the plant, the process will run as was predicted in the design phase. But in reality disturbances are always present, and the process always deviates from the estimated trajectory.

As shown by equation (4), to compensate for the influence of disturbances, feedback and feed-forward control is used. Since feed-forward control is performed by means of models, its quality greatly depends on the degree to which the model is adequate to the reality. By means of model adaptation this adequacy can be increased and thus reduce the influence of the low-frequency disturbances on process performance.

Usually, for the production process, the longer the time horizon considered, the more uncertain are the external inputs; in many cases this fact makes it useless to define detailed control actions and decisions for the layer time horizon. In practice they are usually estimated for the whole time horizon only on the top layer. For the next-lower layer, the solution is defined only for a first part of the entire interval, equal to the time horizon of this layer. Only after this time interval has passed is the next detailed part of the control estimated. The same methods of time horizon sliding are generally used for all the layers.

Conclusions

The concepts of temporal hierarchy described can be applied not only to production process control systems, but also to all decision making systems that consider continuously running processes.

The following benefits accrue from the temporal multilayer hierarchy:

a) Simplification of the models of the higher layers, by means of variable aggregation and reduction of their number;

b) Reduction of the effect of uncertainty, since the lower layers that have shorter time horizons can be easily adapted.

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