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A CRITIQUE OF RECENT MODELLING EFFORTS TO DETERMINE THE VALUE OF HUMAN LIFE

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I. Introduction

The purpose of this paper is to critically review the formulation and results of three recent models by Conley (1973), Usher (1973), and Jones-Lee (1974) each of which theoretically specifies what is popularly termed the "value of human life." A result common to each of these models is that this value can be calculated given sufficient information on personal consumption. Because of the importance of this result for the evaluation of public programs, regulations and policies which affect population mortality, the assumptions critical to its formulation will be carefully reviewed.

The problem of evaluating risk to human life is part of a broader problem of the societal acceptance of large-scale technologies. A primary objective of the Joint IAEA/IIASA Research Project is to gain an improved understanding of how societies judge the acceptability of new technologies and how risk concepts can be incorporated into the decision-making process.[1] Decisions which affect population mortality rates

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[1]Risk has been defined (Otway et al., 1975) as a combination of event and consequence probability including the uncertainty of this probability. Generally the consequences are thought of as unfavourable, and the most unfavorable is the probability of human death.

mortality rates are naturally the most difficult since inevitably they require either an explicit or implicit evaluation of human life. Yet, since Schelling (1968) first suggested that such decisions need not be considered solely as moral questions and can be considered as practical questions, the idea of rationally allocating the resources of a society among lifesaving and other social objectives has become generally accepted.

Rational analysis of social projects, where one of the impacts (or attributes of the consequences) can be described as lives potentially lost or saved, is often attempted by government or regulatory agencies. Such decisions were originally confined to those projects characterized by a limited number of impacts (e.g., questions of flood control, highway safety, etc.), but with the development of more sophisticated methods for evaluating societal decisions with multiple impacts or attributes,[2] such analysis is being attempted on much broader social problems. The work of the Joint Project on the evaluation of risk involving human life is thus an important input into the work of the IIASA Energy Project on the comparison of energy options.

1.a Multiattributed Decision Analysis

There are essentially three methods for choosing among options or projects where the consequences are characterized by multiple attributes.[3] The first, sometimes referred to as the judgmental approach, involves a listing of the attributes of each consequence in vector form and allowing the decision

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[2]See, for example, the work of Pratt et al. (1965), and Raiffa (1968).

[3]For a more complete review of these methodologies, see Baecher et al. (1975).

maker to choose, based on his own judgment, what is the "best" option. Alternatively, the analyst can combine the attributes into a scalar measure of desirability by systematically expressing in commensurable units the desirability of each attribute.[4] A convenient unit of measure is money, and cost-benefit analysis, an established procedure for the evaluation of public programs, requires that all the benefits of the program be expressed in monetary units so that they can be compared with the cost.[5] The desirability of the consequences ofthe project can then be expressed as a scalar value simply by summing the money values of each impact or benefit. However, such a procedure is limited to the evaluation of programs for which all of the impacts are preferentially independent, i.e., the desirability of any impact level is independent of the levels of the other impacts (Keeney and Raiffa, forthcoming). A third and more general methodology, sometimes referred to as the utility approach, accounts for possible interaction among the attributes. This methodology requires direct assessment of utility with the use of interview techniques which have been recently developed (Gros, 1974; Keeney, 1973; Keeney and Raiffa, forthcoming) and which are based on a set of axioms of individual behavior.

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[4]If the objective of the decision maker is the well-being of the population, then this measure of desirability should reflect the preferences of the population.

[5]A basic principle of cost-benefit analysis is that the marginal costs of providing lifesaving or safety should be equated with the marginal benefit. The marginal benefit of a program, if the objective of society is the maximization of total welfare or "utility," is measured by its contribution to total utility. However, the policy maker cannot make interpersonal comparisons of utility so it is impossible, in any true sense, for him to maximize total utility. But he can be guided in his decisions by what has been termed (Mishan, 1971) a Pareto improvement, or a policy which makes at least one person better off without making anyone worse off. The logic of the Pareto concept requires that programs which involve some increased or decreased chance of death should be evaluated by reference to what each member of the community is willing to pay or to receive for the estimated change in the risk.

Under certain very specific conditions the monetary approach of evaluating the desirability of program benefits or consequences is comparable to the utility approach. There is a utility basis to these monetary values if they represent the total willingness to pay on the part of the recipients of the benefit, in which case the money values are a measure for changes in utility at the margin. However, ranking projects in terms of the summation of the aggregate willingness to pay for each of the benefits is not always comparable to ranking them with the utility approach. According to Keeney and Raiffa (forthcoming) the willingness-to-pay approach is justifiable only if

> i. the money attribute taken together with any other single attribute is preferentially independent of the others, and
>
> ii. the marginal rate of substitution between money and any other attribute does not functionally depend on the monetary level [pp. 198, 199].

In addition, they point out that the willingness-to-pay approach does not adequately deal with uncertainty of the impact occurrance.

1.b <u>Defining the Value of Human Life</u>

Recognizing that only under certain conditions can willingness-to-pay be considered an appropriate measure for the preferability of an attribute or impact, we nevertheless will define for purposes of this paper the "value of human life" as the willingness to pay at the margin for changes in survival probability. This definition is consistent with the terminology of the Conley, Usher, and Jones-Lee models which has been adopted for the application of cost-benefit principles to programs which affect human mortality.

Defining the value of human life as a probabilistic term represents a break in the traditional method of valuing life for purposes of evaluating public policies. The most common method has been to estimate the expected number of lives saved (or lost) and to value this benefit (or disbenefit) by aggregating the expected discounted earnings of each respective individual. This "human capital approach"[6] has been criticized by economists (e.g., Schelling, 1968) on the grounds that it ignores the individual's own desire to live and concentrates solely on society's ex post loss in GNP. Such a measure is not appropriate from the standpoint of economic welfare criteria which suggests instead that an appropriate measure should take into account each individual's ex ante desire to avoid the impending probability of death. Thus, programs affecting population mortality should be evaluated according to the change in the probability of survival for each relevant person since ex ante the impacts are known only probabilistically (it is almost never the case that a public program deals with identifiable individuals who will live or die with perfect certainty).[7] This desire on the part of the individual to

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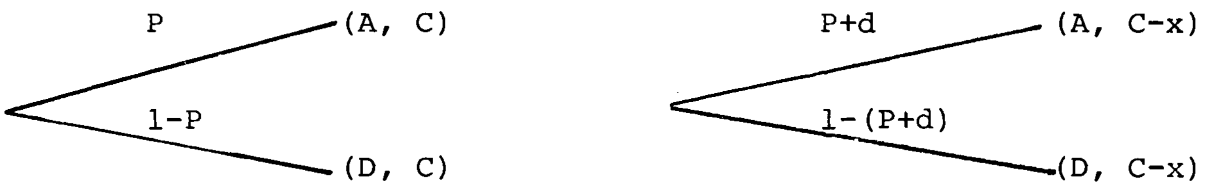
[6]Originally, the "human capital" measure was calculated for purposes of estimating optimal life insurance (e.g., Woods and Metzger, 1927; Dublin and Lotka, 1940); later, it served as an approximate measure for the ex post societal losses from accidents, diseases, etc. (e.g., Fisher, 1909; Reynolds, 1956; Weisbrod, 1965); and finally, it has been adopted as an ex ante measure of benefits from lifesaving programs (e.g., Fromm, 1965; Lave and Seskin, 1970; Otway et al., 1971; White House Office of Science and Technology, 1972; National Highway Traffic Safety Administration, 1972). For this latter use, other suggested methods have been derived from jury awards (Thedie and Abraham, 1961), from insurance decisions (Fromm, 1965), and from implicit values from past political decisions on lifesaving (Morlat, 1970; Starr, 1969). For a survey of these methodologies, see Linnerooth, 1975.

[7]In practice, expenditures on lifesaving take two forms, the prevention of statistical deaths where the identity of the victims is not known (i.e., highway safety) and the rescue of identifiable persons in peril (i.e., kidney transplants). A theoretical treatment of these separate problems can be found in Linnerooth et al. (1975).

increase his survival probability is conveniently valued in terms of his willingness to pay for this increase.

Willingness-to-pay can be illustrated (Acton, 1973) by a very simple model of an individual's choice between consequences characterized by the attributes (alive, A, or dead, D) and lifetime consumption, C. (Instead of lifetime consumption, we could use alternatively wealth, W, or lifetime income, Y. But, for our purposes, we will assume that the three are equivalent.) Consider an individual who in a given period has a probability P of surviving and a probability (1-P) of dying. He can purchase an increase d in his survivial probability by deducting an amount x from his lifetime consumption. The problem is to determine the individual's willingness to pay or that amount (x) which makes him indifferent between the following two lotteries:

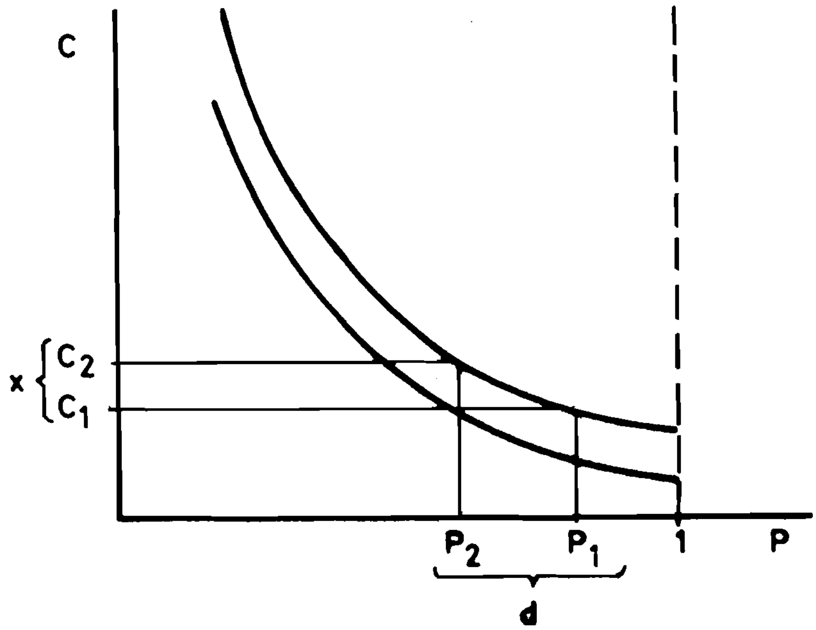


Lottery 1 Lottery 2

(I-B-1)

It would be expected that willingness to pay varies as a function of the probability of death. It will be shown in the next section that under certain circumstances as this probability approaches one, willingness to pay approaches the infinite; yet, individuals willingly accept small nonsurvival probabilities for finite compensation. Figure I-B-1 illustrates one possible indifference mapping where each curve represents a trade-off between lifetime consumption and survival probability.

Figure I-B-1



Since each point on the indifference curve represents equal satisfaction or utility, the slope of the curve represents the individual's willingness to pay to affect small changes in his survival probability. For example, referring to Fig. I-B-1, if the individual has initial consumption Co and survival probability Po, he is willing to pay an amount x to increase his probability of survival by d. It is this trade-off x for d, or the slope of the indifference function, which has been defined (Hirshleifer et al., 1974) as the value of life appropriate for estimating the benefits of lifesaving programs.[8]

[8]This definition, however, ignores the possibility of interdependent utilities or the willingness to pay on the part of other individuals in the society to extend the life expectancy of one of its members. It has been argued (Mishan, 1971) that the familial interest as well as society's general interest in reducing mortality rates is an important and probably inmeasurable factor. Yet, there is some economic justification for this exclusion if we assume that societal or familial interest are internalized in the individual's demand for his own life (an assumption which is implicit in most of welfare economics). In any case, the individual's demand for his life is most likely the dominant factor, and the slope of this indifference function will for our purposes define the value of life.

Empirically, it is very difficult to estimate willingness to pay for changes in survival probability. There are three possible methods. The first is to rely on market data and estimate, for example, the demand or necessary compensation to induce individuals to accept hazardous jobs (Thaler and Rosen, 1973). A second approach, originally suggested by Schelling (1968) and recently attempted by Acton (1973), is to rely on indirect questionnaire procedures. Unfortunately, neither approach has yielded definitive results primarily because of the host of extraneous variables which make it difficult to analyze market decisions and also because individuals find it difficult to give meaningful responses to questionnaires. A third and more indirect approach to determine an individual's preferences for increasing survival probability is to postulate specific behavioral axioms and objectives of the individual and by so doing theoretically reduce the problem to that of determining certain functional relationships. This is the approach of the three recent articles by Conley, Usher, and Jones-Lee. Each postulates a lifetime objective function dependent on lifetime consumption, and by assuming that the individual is an expected utility maximizer determines his rational behavior toward accepting decreased survival probability. A result common to each of these models is that this behavior can be calculated with sufficient information on income utility. There is no need to obtain data on the types of decisions where the individual trades off income for life expectancy. Hirshleifer et al. (1974) further point out that this value is empirically correlated with the human capital approach of evaluating loss of life by calculating and discounting to the present the individual's expected future earnings.

The purpose of this paper is to critically review the formulation and results of these three models. Because of the similarity of this approach with the utility approach which has on one occasion been applied to this particular problem by Raiffa (1969), the two approaches will be compared. This paper will begin with a brief review of each of the three models including the Hirshleifer-Bergstrom-Rappaport interpretation. It will be established that the results are dependent upon somewhat weak assumptions regarding the individual's lifetime objective function. A more plausible specification of this objective function will result in a more complicated relationship between lifetime income and willingness to pay for increased expected longevity. A technique for determining this relationship has been developed in the aforementioned work by Raiffa which will be summarized in the last section.

II. The Models of Conley, Usher, and Jones-Lee

2.a A Brief Review

The problem is to structure an individual's preferences for "lifesaving." Since death is inevitable, we can think of lifesaving as a probabilistic term for increase of life expectancy, and the demand for lifesaving as willingness to pay for these increases. Consider an individual who commences the current period with full information on his expected lifetime. We can denote as p_t his probability of surviving any given period, i.e., his age-specific mortality rate, and P_t as his probability of being alive in any given period. If p_t occurs on the first day of the period, then $P_t = S_t p_t$ where S_t is the probability of surviving until year t or $S_t = p_{t-1} \cdot p_{t-2}, \ldots, p_o$. It follows that the demand for an increase in life expectancy can be expressed in terms of the demand for an increase in any age-specific mortality rate p_t. This demand will likely depend on, among other things, the individual's current age or expected lifetime, his expected income, and the number of his dependents, as well as on the nature and timing of the probabilistic death.

The models of Conley, Usher, and Jones-Lee attempt to model this willingness to pay on the part of the individual for an increase in his own survival probability with the important assumption that the individual behaves as an expected utility maximizer.[9] If a utility is assigned to each year of a person's life, then he behaves in such a way as to maximize his expected utility, or his total utility weighted by his probability of survival. In this way expected years of life enter the individual's lifetime objective function indirectly as a weighting factor. The person is assumed to make a quantity-quality trade-off by purchasing increases in his survival probability with his income until he has maximized his expected utility.

The analytics of the Conley, Usher, and Jones-Lee models are straightforward and can best be reviewed by considering the problem in its simplest form. For this purpose we will assume that the individual is a lone bachelor; i.e., he has no family, friends, etc., who have an interest in his survival, thus eliminating the complication of interdependent utilities as well as any motive on the part of the individual to bequest his wealth. We will further assume that the individual regards his

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[9]Different sets of axioms which imply that the individual maximizes expected utility are presented in von Neumann and Morgenstern (1947); Savage (1954); and Luce and Raiffa (1957).

lifetime as fixed if he survives the initial period; i.e., he either does or does not live through the period. Whether or not he survives is determined on the first day when he faces some probability of dying denoted $1 - P_o(P_o = p_o)$. Because the resolution is instantaneous, he suffers no anxiety. In addition, we will assume that the individual has a certain fixed sum of money which he will distribute throughout his lifetime. There are no opportunities for saving or investing (or for bequesting), so this sum of money can be regarded as lifetime consumption denoted C. (We will use upper case letters to refer to lifetime variables and lower case letters to refer to variables relevant to shorter periods of time, typically one year.)

To be consistent with the Conley, Usher, and Jones-Lee, models, we will postulate an objective function in which the individual maximizes his expected lifetime utility expressed as a function of lifetime consumption. Since the individual begins the initial period with a certain fixed amount of money to be considered lifetime consumption, this objective function can be written

$$E(U) = P_o U(C) \qquad \text{(II-A-1)}$$

where E(U) represents expected lifetime utility, P_o is probability of surviving the current period, and U(C) is lifetime utility of lifetime consumption. The individual's tradeoff of lifetime consumption for survival probability can then be expressed

$$\frac{\partial C}{\partial P_o} = \frac{U(C)}{P_o U'(C)} \qquad \text{(II-A-2)}$$

where U'(C) represents the marginal lifetime utility of lifetime consumption. Referring to Fig. I-B-1, we can see that equation II-A-2 specifies the slope of the indifference function and is thus the value of life. Although this model is clearly too simple, it does illustrate the intuitive. As the probability of survival approaches zero, willingness to pay to increase this probability approaches infinity.[10]

2.a.1 Jones-Lee's Model

The models of Conley, Usher, and Jones-Lee offer only slight variations to the above formulation. Jones-Lee[11] introduces the possibility of a bequest motive in which case the individual, who again enters the current period with a certain amount of money or wealth, faces two contingencies: either he survives the period, in which case he and his dependents will enjoy his wealth, or he does not survive the period, in which case his dependents will enjoy whatever

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[10]As Bergstrom (in Hirshleifer et al., 1974) has pointed out, this formulation resolves the paradox that an individual, although he likely places an infinite value on his own life, willingly accepts small probabilities of death for finite compensation. However, an infinite willingness-to-pay on the part of persons who face immediate and certain death has direct ramifications on the evluation of rescue programs. It leads to the second paradoxical result that society should (following the logic of willingness-to-pay) allocate far greater resources to rescue than to prevention or to programs which save persons facing certain death (i.e., persons in need of a kidney transplant) to those facing probabilistic death (i.e., automobile drivers). This leads us to question the slope of the indifference function as an appropriate definition of the value of life. This is the topic of a forthcoming paper by this author.

[11]Jones-Lee develops theoretically Mishan's (1971) concept of compensating variations for changes in the probability of an individual's own fatal accident. Compensating variations can be interpreted as either the amount of money necessary to fully compensate the individual for accepting some probability of death or the amount he, the individual is willing to pay to reduce some initial probability of death.

portion of his wealth is bequethable.[12] The objective function can be written

$$E(U) = P_O U_A(W) + (1 - P_O) U_D(W) \qquad \text{(II-A-3)}$$

where U_A is the utility of wealth conditional on the state of the world "alive" and $U_D(W)$ is the utility of wealth conditional on the state of the world "dead." The maximization of E(U) is straightforward and results in the condition which we have defined as the value of life, or

$$\frac{\partial W}{\partial P} = \frac{U_A(W) - U_D(W)}{P_O U_A'(W) + (1 - P_O) U_D'(W)} \qquad \text{(II-A-4)}$$

where $U_A'(W)$ and $U_D'(W)$ are the marginal lifetime utility and the marginal bequest utility with respect to wealth, respectively. This condition compares with equation II-A-2, except as might be anticipated, a man's desire to leave a certain amount of his wealth to his dependents decreases his willingness to pay for his own safety.

2.a.2 Conley's Model

The models thus far have merely described the shape of the indifference function--"the tradeoff between immediate consumption and immediate hazard that would leave the individual in an equally preferred position, if the infinitesimal changes in these variables (and only these variables) were imposed on him" (Rappaport, in Hirshleifer et al., 1974, p. 8). In contrast, Conley develops a model whereby consumption and survival probability enter as behavioral variables. Describing

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[12] We now use the term wealth instead of consumption since we can no longer assume that the individual consumes all his monetary assets in his lifetime.

his model as a "full information, competitive, zero transaction cost, no externalities (except for time of death) model of an individual's choices," he expresses the objective function

$$E(U) = \sum_{t} \lambda(t) u(\bullet) p(\bullet) \qquad \text{(II-A-5)}$$

whereby expected lifetime utility E(U) is separable into discounted single period utilities; $u(\bullet)$ is a single period utility function; $p(\bullet)$ is the probability of being alive; and $\lambda(t)$ is the utility discount function. Since our purpose is to establish the relationship between consumption and $p(\bullet)$, we will note only that both $u(\bullet)$ and $p(\bullet)$ are functions of all lifetime activities, including consumption. Thus, the single period utility function can be written

$$u(\bullet) = u(x^{1t}, x^{2t}, \ldots, x^{nt}) = u(X^{t}) \qquad \text{(II-A-6)}$$

for n activities denoted x in period t, and where X^t is the vector of activities. An activity is called consumption if the individual must expend cash outlays, production, if it results in cash income, and neutral if neither. When there is no activity, i.e., $x^{it} = 0$ for all i's, then the individual is no longer alive. The utility of the state "death" can be arbitrarily set at zero, or

$$u(0,0,\ldots,0) = 0 \quad , \qquad \text{(II-A-7)}$$

and therefore we can say that the utility is determinable up to a multiple constant. Also, in Conley's model $p(\bullet)$ is a function of X^t (along with several other important variables such as age, outside influences, etc.), and the objective function can be rewritten

$$E(U) = \sum_t \lambda(t)u(X^t)p(X^t) \quad . \qquad \text{(II-A-8)}$$

The individual maximizes his objective function or expected utility subject to two constraints. The first, a monetary constraint, requires that expected lifetime consumption equals expected lifetime income plus wealth, or

$$\sum_t \frac{1}{1+r} \sum_t \left\{ \$^{it}x^{it}p(x^{it}, \ldots) \right\} - W = 0 \qquad \text{(II-A-9)}$$

where r is the market rate of discount; $\$^{it}$ is the monetary value associated with each unit of x^{it} such that $\$^{it} > 0$ when x^{it} is a consumption activity and $\$^{it} < 0$ when x^{it} is productive; and W is wealth. The second constraint, the time constraint, takes the form

$$\sum_{i=1}^{n} m^{it}x^{it} - T = 0 \qquad \text{(II-A-10)}$$

where $m^{it} > 0$ represents the marginal time required (if any) of activity i; and T is the total time available in a period.

To put it in words, the equilibrium conditions for maximization of II-A-8 subject to II-A-9 and II-A-10 are such that the marginal utility of activity i must equal the sum of the expected loss of lifetime utility, the marginal utility of the monetary cost and expected insurance loss (Conley includes a term for insurance coverage), and the marginal utility of time used. From this equilibrium condition Conley derives the value of human life, L, "appropriate for life-saving investments" as discounted expected lifetime consumption divided by the elasticity of discounted lifetime utility with respect to lifetime consumption

$$L = \frac{U}{\partial U/\partial C} = \frac{C}{\dfrac{\partial U}{\partial C}\,\dfrac{C}{U}} \qquad \text{(II-A-11)}$$

which is consistent with our previous formulation given values of P close to unity.

The relationship is verified by Conley by considering a state of technology such that all safety expenditures can be separated from all consumption expenditures. The individual maximizes

$$E(U) = P(S)U(C) - \lambda(S + C - Y) \qquad \text{(II-A-12)}$$

where S is expenditure on safety; P(S) is the probability of surviving the current period; and C and Y are lifetime consumption and lifetime income, respectively. Maximizing expected utility $\{P(S)U(C)\}$ subject to the budget constraint $(Y = C + S)$ yields the following first-order condition:

$$-\frac{\partial S}{\partial P} = \frac{\partial C}{\partial P} = \frac{U(C)}{P(S)U'(C)} \qquad \text{(II-A-13)}$$

which is exactly equivalent to our previously derived value of life.

Conley's verification equation II-A-12 differs from his original objective function (equation II-A-8) in two important ways. First, lifetime utility, expressed originally in terms of all lifetime activities, becomes a function of only consumption activities. Secondly, the verification model is, again, a one period analysis which avoids thus complications of discounting future utility.

2.a.3 Usher's Model

Usher[13] also formulates a multiperiod model, but avoids the controversial problem of discounting utility by expressing lifetime utility as a function of all past consumption. Total expected lifetime utility becomes

$$E(U) = \sum_{t=0}^{n} R_t U_t(C) \qquad \text{(II-A-14)}$$

where R_t is the probability of living exactly t years; n is the maximum length of life; and $U_t(C)$ is a function of consumption in each year in which the individual is alive, or

$$U_t(C) = U_t(c_0, c_1, \ldots, c_{t-1}) \quad . \qquad \text{(II-A-15)}$$

Since

$$R_t = (1 - p_t) S_t \qquad \text{(II-A-16)}$$

where $(1 - p_t)$ is the mortality rate in year t and S_t is the probability of surviving until year t, then expected lifetime

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[13] Usher is concerned with the problem from the point of view of developing social indicators which reflect an expanded concept of national welfare from the usual income or GNP measures. The inclusion of one such welfare change, decreased population mortality rates, requires some method of valuing or pricing these changes. For this purpose Usher develops a model of life cycle planning and inquires as to what role uncertain mortality plays. That is, what is worth to the individual in terms of foregone consumption to decrease his risk of dying in any given (present or future) period(s)?

becomes[14]

$$E(U) = \sum_{t=0}^{\infty} (1 - p_t) S_t U_t(c_0, c_1, \ldots, c_{t-1}) \quad . \tag{II-A-17}$$

If we assume that lifetime utility over t years is an additive function of utility in each year,[15] i.e., that the weights are equal to one, or

$$U_t(c_0, c_1, \ldots, c_{t-1}) = u_0(c_0) + u_1(c_1) + \ldots + u_{t-1}(c_{t-1}) \tag{II-A-18}$$

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[14]Prof. Martin McGuire (Dept. of Economics, University of Maryland) has pointed out to me that there is a contradiction in Usher's formulation. Since the maximum length of life n is fixed, then the sum of the age-specific mortality rates $(1 - p_t)$ must be equal to unity, i.e., $\sum_t (1 - p_t) = 1$. Therefore, if $\Delta(p_t - 1) > 0$, there must be a compensating $\sum_{j=0}^{n} \Delta(p_t - 1)_i < 0$ $(j \neq i)$. This suggests that Usher's formulation ignores a necessary constraint.

[15]Originally Meyer (1969) and later Keeney (1974) showed that an additive utility function under certain conditions implies that the individual is "temporally risk neutral." In particular Meyer assumes that the decision maker's liking for future consumption streams is independent of past consumption levels and that the decision maker's liking for consumption in the near future is independent of his consumption levels in the distant future, whatever those levels may be. Meyer then proves that a proper utility function for lifetime consumption streams must be either the sum or product of utility functions for consumption in each year. There are, in fact, two product forms, which we call the negative product form and positive product form. What differentiates the two product forms and the sum form is a property discovered by Meyer, called temporal risk aversion. "Suppose the decision maker holds a lottery determining the next ten years' consumption, which will be resolved independently for each of the next ten years. Furthermore, suppose the decision maker is indifferent between receiving consumption $X for certain in each year and the lottery. Now suppose that instead of independent lotteries for

then Usher's equation II-A-17 can be rewritten

$$E(U) = \sum_{t=0}^{n} S_t p_t u_t(c_t) \qquad \text{(II-A-19)}$$

where $S_t = p_{t-1} \bullet p_{t-2}, \ldots, p_0$. From equation II-A-19 we can specify the individual's trade-off between consumption and survival probability for any period of his life.[16] Considering only the initial period to conform to our simple model, we can write this trade-off

$$\frac{\partial c_0}{\partial p_0} = \frac{u_0(c_0) + \sum_{t=1}^{\infty} S_t p_t u_t(c_t)}{p_0 u'_0(c_0)} \quad . \qquad \text{(II-A-20)}$$

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15 cont. each of the ten years the lottery will only be resolved once and this single resolution determines the decision maker's consumption level for all ten years. If the decision maker will accept less than \$X for certain in each year in place of this single ten-year lottery then he is <u>temporally risk averse</u> and his utility must be of the negative product form (providing, of course, that he follows our other behavioral assumptions). If he will still take \$X for certain in each year, then he is <u>temporally risk neutral</u> and his utility must be of the sum form. Finally, if his certainty equivalent is now more than \$X per year, then he is <u>temporally risk seeking</u>" [Richard, 1972, pp. 1-3,4].

[16] From equation II-A-14, Usher derives the trade-off between initial period consumption and the increase in survival probability for any year t as

$$\frac{\partial c_0}{\partial p_t} = \frac{\sum_{j=t}^{\infty} \frac{P_j}{P_t}(u_j - u_t)}{\sum_{j=1}^{\infty} P_j(\partial u_j/\partial c_0)} \quad .$$

The numerator in the above equation expresses expected lifetime utility given that the individual survives the hazard of the initial period, compared to the numerator of equation II-A-2 which expresses known lifetime utility given the individual survives the hazard of the initial period. Again the value of life is expressed as a function of lifetime utility of lifetime consumption, in this case lifetime expected utility.

2.b The Results

An important result of the Conley, Usher, and Jones-Lee models is the implied relationship between the value a person places on his life and his personal income or lifetime consumption. Conley states at the outset that he is interested in determining the "relationship between one's income and one's value of human life." According to his model this value is equal to a man's expected lifetime (discounted) consumption divided by his lifetime consumption elasticity of lifetime utility. In principle, the Usher and Jones-Lee models agree with this result.

The significance of this result is that given an assumption on the form of an individual's consumption utility, the value of life can be calculated from data on personal consumption. As an illustration, Usher calculates this value from Canadian time-series data on net national product. Assuming a utility of consumption function common to economic analysis, or

$$U_t(C) = \sum_{i=0}^{t-1} \frac{Ci^b}{(1 + r)^i} , \qquad \text{(II-B-1)}$$

Usher is able to estimate values for $U_t(C)$ (in the aggregate) by postulating a range of values for b and r, where 1/b represents a measure for the degree of diminishing marginal utility of consumption and r is a subjective parameter representing the degree of utility time-preference. By substituting this measure of $U_t(C)$ into his derived value of life (equation II-A-20) and with data on age-specific mortality rates, Usher is able to estimate the value of life (in aggregate). For example, choosing r = 3 per cent and b = 50 per cent, the average value of life (in 1961) calculated by Usher was \$91,000. (The values ranged from \$1,331,000 (r = 1%; b = 5%) to \$33,000 (r = 5%; b = 100%).)

2.c The Hirshleifer-Bergstrom-Rappaport (H, B, and R) Interpretation

It is clear form the specifications of the Conley, Usher, and Jones-Lee objective functions that given no bequest motive, the utility of death is implicitly zero. H, B, and R go one step further and assumes that the utility of zero consumption is also zero U(0) = 0. "We now assume, as a special case, that death corresponds to an income of C = 0 - the person, when alive, considers death as the equivalent of an income of zero" (p. 26). He emphasizes the "special case" recognizing that most people are not indifferent between dying with zero income and dying with a positive income--in most cases there is a bequest motive.

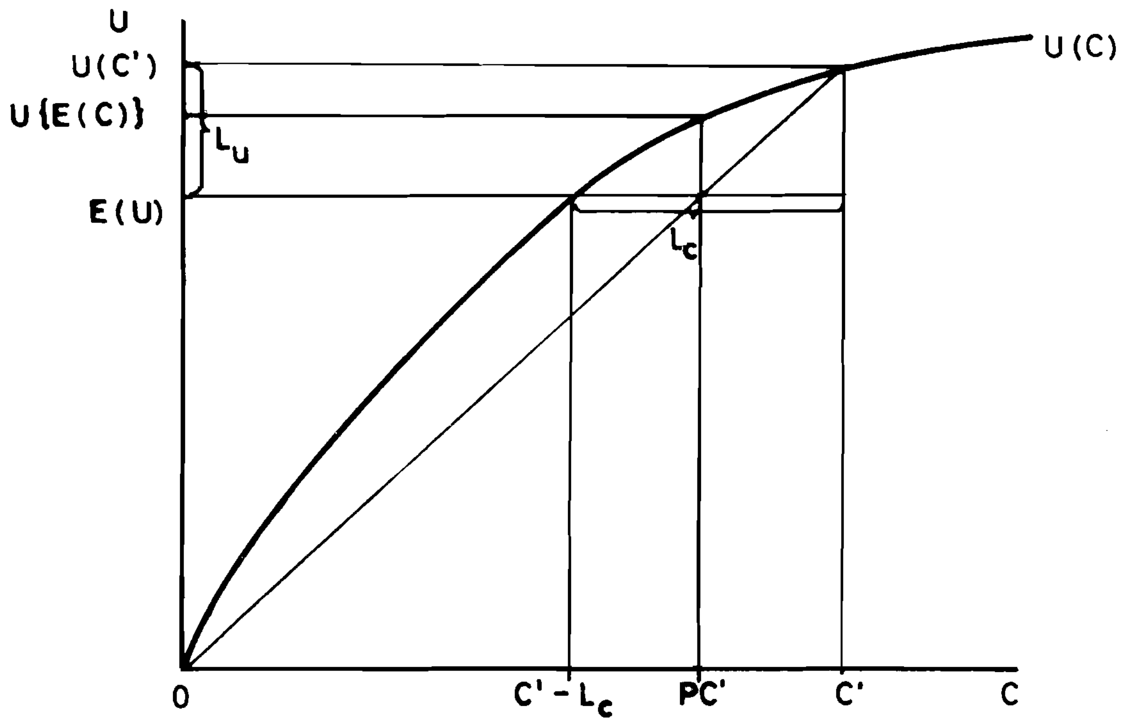
Given this assumption, H, B, and R analyze the individual's response to a risky project yielding a P chance of income C' and a Q = 1 - P chance of zero income. Since the utility of zero income is zero, the expected utility E(U) of this prospect can be expressed

$$E(U) = PU(C') + QU(0) = PU(C') \quad . \qquad \text{(II-C-1)}$$

Postulating a cardinal utility of lifetime consumption, H, B, and R's interpretation of this prospect is illustrated in Figure II-C-1.

The utility function displays diminishing marginal utility, $U'(C) < 0$, and passes through the origin.

Figure II-C-1



If the individual has a lifetime consumption of C', then the expected loss in utility from the hazardous prospect can be seen as L_u, representing the difference between U(C') and E(U). E(U) is simply P times U(C'). The monetary equivalent (the amount of money the individual must be compensated to restore him to his original utility position of U(C')), noted L_c, is derived from the slope of the function and can be approximated for very small values of Q as

$$L_c = \frac{L_u}{dU/dC} \quad . \qquad \text{(II-C-2)}$$

Since $L_u = QU(C')$, equation II-C-2 can be rewritten

$$L_c = Q \frac{U(C')}{dU/dC} \quad . \tag{II-C-3}$$

Because q is the probability of zero consumption which represents in turn the probability of death, the other factor of the above equation, $\frac{U(C')}{dU/dC}$, represents, according to H, B, and R, the value of life. Since this derivation is valid only for very low Q's or $P \simeq 1$, this value is consistent with the value of life, $\frac{U(C)}{P(dU/dC)}$, derived by Conley, Usher, and Jones-Lee.

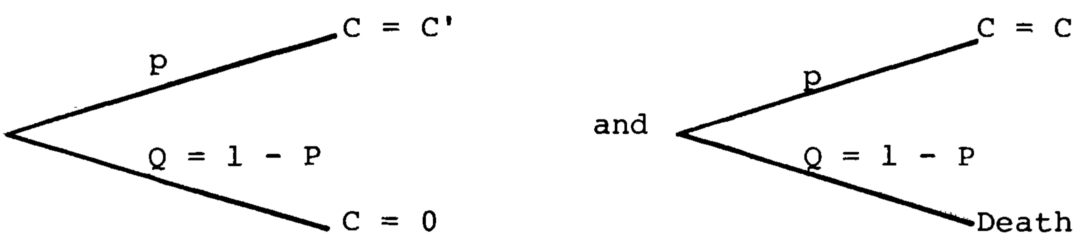
III. A Critique

From the last section we can conclude that the value of human life as formulated by Conley, Usher, and Jones-Lee is formally similar and empirically correlated with the human capital measure. In fact, if Usher's special form of consumption utility is hypothesized, the value of life differs from lifetime earnings only to the extent of diminishing marginal utility of lifetime consumption.

This rather surprising relationship between what an individual is willing to pay to reduce some probability of instantaneous death and the individual's lifetime consumption is clearly dependent on the assumed objective functions of the three models. The only variable entering the individual's lifetime utility is assumed to be lifetime consumption (plus bequests in the Usher model). Lifetime consumption, according to Conley, is defined as total lifetime activities which incur positive monetary outlays. It appears then that this univariate treatment of lifetime utility focuses only on the more materialistic side of life. Yet, the univariate assumption is central to the arguments of Conley, Usher, and Jones-Lee, and for this reason it is important to give it careful consideration. In particular, we will analyze the H, B, and R justification of this assumption.

3.a The Hirshleifer-Bergstrom-Rappaport Justification of Univariate Lifetime Utility

Consider once again H, B, and R's risky prospect where the individual faces a $Q = 1 - P$ chance of losing his income C', expressed $(C', 0; P, Q)$. We can recall that such a prospect, according to Hirshleifer's assumption that zero consumption means certain death, is equivalent to a risky prospect offering a $Q = 1 - P$ chance of death and otherwise life at income C', expressed $(C', \text{Death}; P, Q)$. In other words, given this assumption, the individual must be indifferent between the following two lotteries



(III-A-1)

where the first lottery represents a P chance of $C = C'$ and a Q chance of $C = 0$, and the second lottery represents a P chance of $C = C'$ and a Q chance of immediate death. This indifference follows, of course, from the fact that since $C = 0$ means death, the two lotteries are exactly equivalent.

However, the problem arises when H, B, and R then conclude that the utility (which can only be interpreted as lifetime utility which we have denoted U) of the risky prospect to an expected utility maximizer is

$$U = U(C', 0; P, Q) = PU(C') \quad . \qquad \text{(III-A-2)}$$

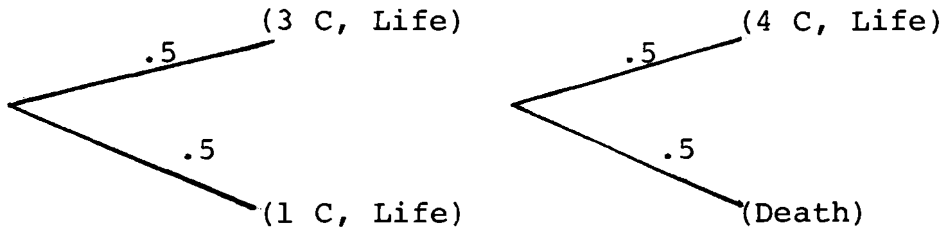
The reasoning seems to be as follows: since U (Death) = 0 and U(C = 0) = 0, then U (Death) = U(C = 0) which in turn implies that U = U(C). The last implication does not necessarily follow from the former, from H, B, and R's assumption (plausible under his special conditions) that U (Death) = U((C = 0).[17] Yet, in order to express the lifetime utility of the risky prospect as PU(C') in the above equation, it is necessary to assume that U = U(C).

To appreciate further the fallacy of this reasoning, we will extend it to the case where the options are not zero consumption and death, but positive consumption and death. The two conditions necessary for H, B, and R's interpretation of the value of life are 1) that the individual maximizes expected utility, and 2) that U (lifetime) = U (lifetime consumption) or U = U(C). With these two assumptions it can easily be shown that, given constant marginal utility of consumption,[18] the individual is also indifferent between the following two lotteries:

---

[17] Another problem, not discussed by Hirshleifer-Rappaport, is that the assumption of zero income being equivalent to death holds only in the aggregate. Rappaport writes that "...income = 0 means that (the individual) would be starving, thirsting, and exposed to the elements; i.e., he would be dead soon anyway" (p. 15). But, in reality, most people do not view a total loss of their income or wealth, i.e., bankrupcy, as certain death. This is then essentially an empirical matter-- will persons choose U = (C = 0) over U (Death)?

[18] Professor Hirshleifer has pointed out to me that this indifference is less obvious with diminishing marginal utility of consumption. To make the point, it would be necessary to show that the necessary compensation (for indifference) is greater than the difference occurring only to diminishing marginal utility. However, with sufficiently small C, diminishing marginal utility is negligible.



(III-A-3)

where C is some level of consumption greater than subsistance. Setting again U (Death) = 0, the expected utility of the first lottery, $(0.5 \bullet (3) \bullet U(C) + 0.5 \bullet U(C))$ is equal to the expected utility of the second lotter, $0.5 \bullet (4) \bullet U(C)$. However, we would not expect the average person to be indifferent between two such lotteries. It can be concluded that the univariate utility function is quite unappealing.

3.b An Alternative Interpretation of Lifetime Utility

We now turn to reconsider the concept of lifetime utility which we have denoted U. We will begin by taking a second look at Conley's model and reinterpreting the explanation which both H, B, and R and Conley, give to the results.

We can recall from equation II-A-8 that Conley's objective function is

$$E(U) = \sum_{t} \lambda(t)u(X^t)p(X^t) \qquad \text{(III-B-1)}$$

where X^t is a vector of activities in period t. Conley calls an activity consumption if the individual must expend cash outlays, production, if it results in cash income, and neutral, if neither. From equation III-B-1, it is clear that discounted lifetime utility, U, is

$$U = \sum_t \lambda(t)u(X^t) \quad . \tag{III-B-2}$$

If we denote the set of all activities which can be considered consumption as c^t, the set of activities which are neutral or nonconsumptive as n^t, and the set of all activities which are productive as b^t, then (assuming c^t, n^t, and b^t are separable) equation III-B-2 can be rewritten

$$U = \sum_t \lambda(t)u(c^t, n^t, b^t) \quad , \tag{III-B-3}$$

and lifetime consumption C is simply $\sum_t c^t$. The term U of Conley's derivation of the value of human life (see equation II-A-9)

$$L = \frac{U}{\partial U/\partial C} = \frac{C}{\frac{\partial U}{\partial C}\ \frac{C}{U}} \tag{III-B-4}$$

can only be interpreted as total lifetime utility which is a function of both lifetime consumption activities C, non-consumption activities N, and production activities B.

It is curious, then, that Conley interprets U as a function of C alone (see equation II-A-12), deriving thus the following value of life

$$\frac{\partial C}{\partial P} = \frac{U(C)}{P(S)U'(C)} \tag{III-B-5}$$

which is determinable given sufficient information on the individual's utility of consumption or income. It seems that a more reasonable interpretation, given the original intent of

Conley's model, would be to retain a relationship between the utility of living through a period and all activities of the period: consumptive, productive, and neutral. Keeping our notation where C denotes lifetime consumption activities, N lifetime nonconsumption activities, and B lifetime productive activities, we can rewrite Conley's value of human life as

$$\frac{\partial C}{\partial P} = \frac{U(C,N,B)}{P(S)U'(C,N,B)} \qquad \text{(III-B-6)}$$

where again U'(C,N,B) denotes the marginal lifetime utility of lifetime consumption. For P(S) very close to one the above equation can be expressed

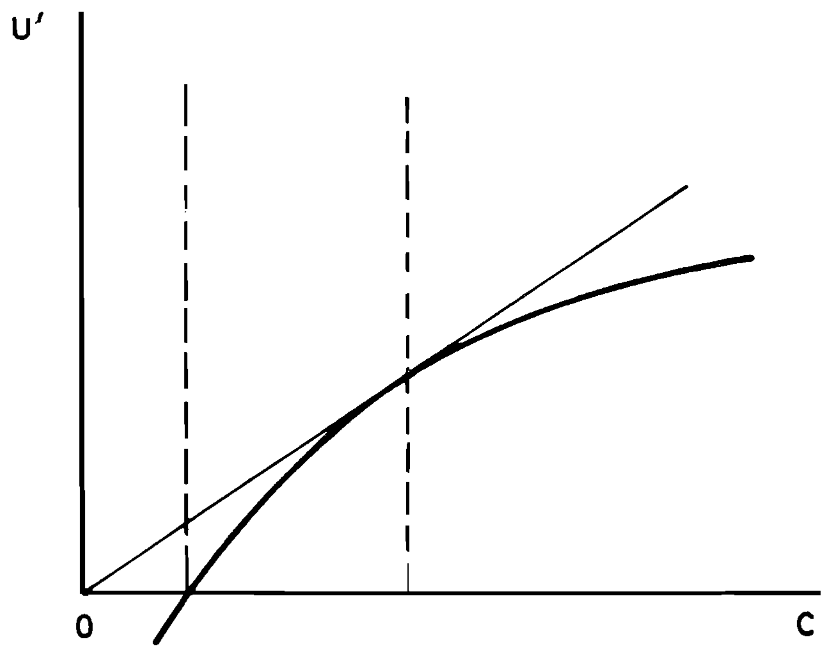
$$\frac{U(C,N,B)}{U'(C,N,B)} = \frac{C}{\frac{\partial U}{\partial C}\ \frac{C}{U}} = \frac{C}{\alpha} \qquad \text{(III-B-7)}$$

where again α represents the elasticity of lifetime utility with respect to lifetime consumption, but in this case U is a function of all activities.

We now want to ask what difference the above derivation of the value of human life makes to the results of the Conley, Usher, and Jones-Lee models? In particular, in what direction does it affect the relationship, illustrated by H, B, and R, between the willingness-to-pay method of valuing life (where $\frac{\partial C}{\partial P}$ is willingness to pay) and the human capital method (discounted lifetime income)?

H, B, and R's illustration of Conley's value of life is based on Conley's postulated relationship between consumption and lifetime utility. According to Conley, Figure III-B-1 presents the general case, in which a strictly concave lifetime utility function, with $U' > 0$, will have three regions. In the first $U' < 0$ and thus $\alpha < 0$. In the second, $U' > 0$ but $\alpha > 1$, and therefore $L < C$. In the third, which may be called the general case, $0 < \alpha < 1$, and $L > C$; that is, for a value of expected lifetime consumption above some critical value where $\alpha = 1$, the value of human life exceeds expected lifetime consumption.

Figure III-B-1

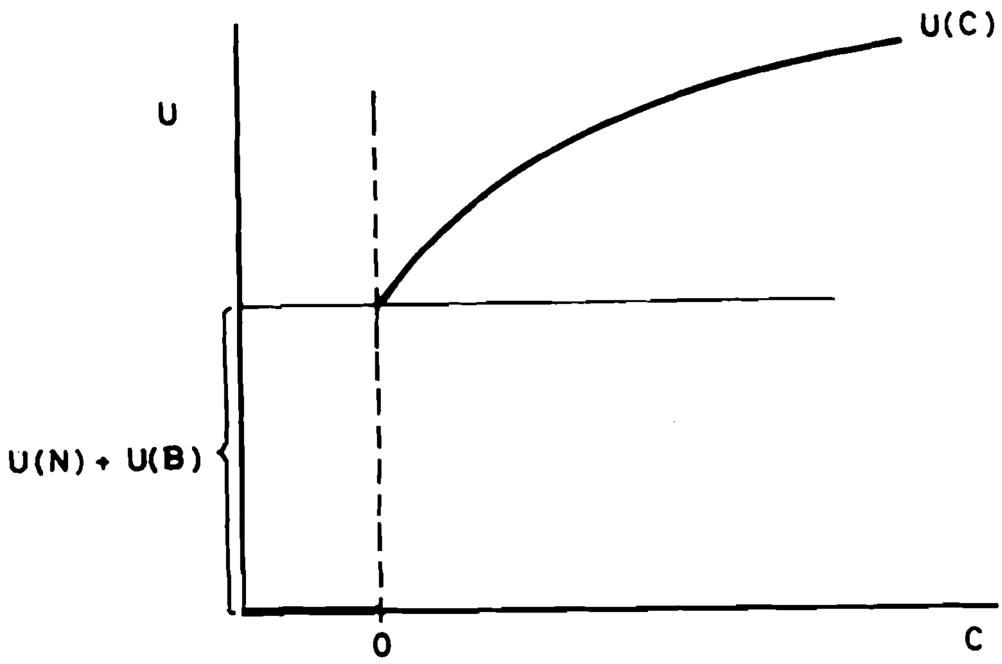


We have shown Conley's lifetime utility as a function of nonconsumption and production activities as well as consumption. If for the sake of simplification we imagine that the utility of consumption activities, the utility of nonconsumption activities, and the utility of production activities are separable and additive, then we can redraw Conley's graph with an additive transformation of the curve. As such H, B, and R's assumption that zero consumption is a state of zero utility or death represents simply a discontinuity in the curve. The relationship therefore between lifetime utility and lifetime consumption (assuming that $C > 0$ is subsistence) can be expressed:

$$\begin{aligned} U &= U(C) + U(N) + U(B) \qquad && C > 0 \\ U &= 0 && C \leq 0 \end{aligned} \qquad \text{(III-B-8)}$$

This relationship is illustrated in Figure III-B-2.

Figure III-B-2

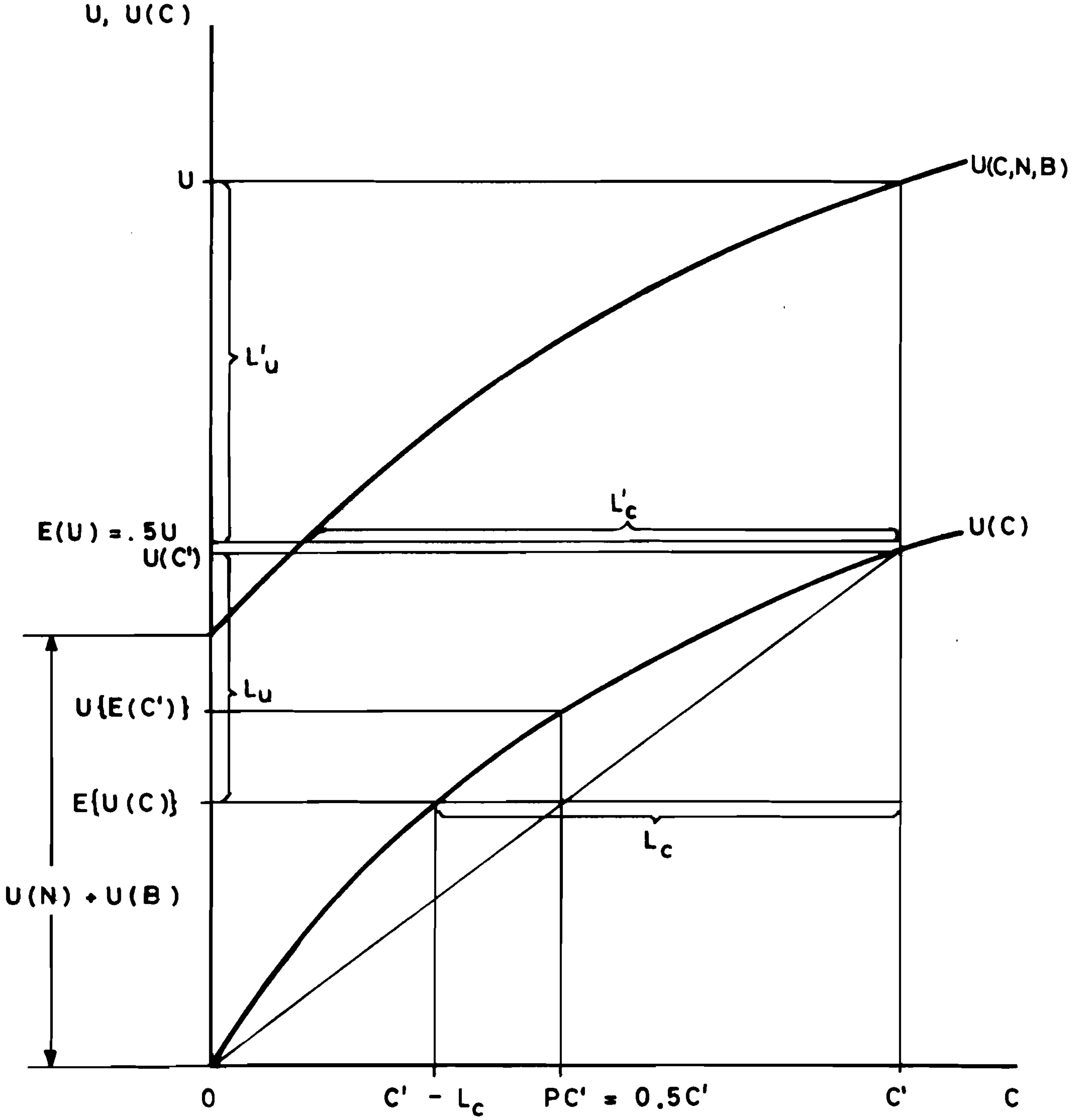


With this interpretation the elasticity of lifetime utility with respect to lifetime consumption α depends on the values of U(N) and U(B) as well as U(C). As U(N) increases, α decreases--thus confirming the intuitive notion that the higher the value an individual places on his nonmaterial (nonconsumption) life, the higher he values his life.

The relationship between lifetime utility and lifetime activities (C and N) pictured in Figure III-B-2 does not, of course, represent reality. We would expect a much more complicated relationship between lifetime utility and both consumption and nonconsumption activities. This relationship does, however, show the apparent fallacy in the reasoning of Conley, Usher, and Jones-Lee when they postulate lifetime utility as a univariate function of lifetime consumption. We will recall that we can accept their hypothesis only if we are willing to accept that the individual is indifferent between lotteries III-A-3.

It remains to compare graphically this derivation with that of H, B, and R's. Figure III-B-3 illustrates both H, B, and R's interpretation of Conley's value of life and our derivation or reinterpretation of this value. For purposes of comparison we assume, as did H, B, and R, that zero consumption represents zero utility or death. U(C), in Figure III-B-3, represents the univariate utility which is assumed by Conley, Usher, and Jones-Lee, and U represents the multivariate lifetime utility function which we have assumed to be an additive function of consumption, neutral, and production activities. U and U(C) are assumed parallel, the vertical distance between the two curves being U(N) and U(B). U becomes discontinuous at C = 0, at which point U = 0.

Figure III-B-3



The lower part of Figure III-B-3 is recognizable as H, B, and R's representation of Conley's value of life (see Figure II-C-1). It can be easily seen from the upper part of Figure III-B-3 that given our alternative interpretation of U, the H, B, and R derivation underestimates the value of life. Again, L_u' represents the loss in expected utility, in this case U - .5 U, and L'_C represents the necessary compensation to restore the individual to his original level of utility. Depending on the value of U(N) and U(B), $L < L'_C$, and therefore the Conley, Usher, and Jones-Lee derivations underestimate the true value of life.

We can conclude that the value of life can in no way be correlated with expected lifetime consumption unless the investigator has, in addition, information on the utility of other variables making up lifetime utility. H, B, and R appear to recognize this limitation. Commenting on Usher's univariate specification of lifetime utility, they write: "This function gives no weight to the fact of death except as it means loss of potential consumption income.... No one has yet developed a utility function that captures the value of living per se" [p. 35]. Yet, it seems that to ignore this "value of living per se" is to ignore the essence of the problem of deriving a value of life.

IV. An Alternative Formulation

We have shown in the previous section that the value of life measure derived by Conley, Usher, and Jones-Lee is valid only if one is willing to accept that consumption is the only argument entering lifetime utility. If, on the other hand, one accepts Conley's original formulation of lifetime utility as a function of all activities--consumptive, productive, and neutral--then it follows that the value of life is a much more complicated relationship of both lifetime consumption and non-consumption activities. We have postulated lifetime utility as an additive function of U(C) and U(NC), but to be realistic we would need, instead, to specify the many variables affecting lifetime utility and the interrelation of these variables with lifetime consumption--a process which is extremely complex. In addition, we would need to include an "anxiety" variable expressing the fears and anxieties associated with dying. "Dying" cannot be considered simply as the state of death.

Economists do not, however, usually find it necessary to investigate all the variables (psychological and other) which enter into a utility function in order to determine demand. It is usually sufficient to postulate that the good provides utility to the consumer and then to analyze the relationship between the demand for the good and its price. If we apply this same reasoning to the demand for lifesaving, the value of life becomes essentially an empirical matter. The problem is to specify this demand for longevity or survival probability. Estimates could be made with appropriate data on market prices for personal safety measures. There has been one such attempt (Thaler and Rosen, 1973) in this direction, but the results are inconclusive primarily because of the host of extraneous variables which make it difficult to analyze market data. In light of these difficulties it has been suggested (Schelling, 1968) that a second approach might be to use questionnaire data. This method has also been questionned (Fromm, 1965) because individuals find it difficult to give meaningful responses to questions involving small probabilities. However, there has been at least one attempt (Acton, 1973) to use questionnaire data, and recently there has been much work (Raiffa, 1969; Keeney, 1974; Meyer, 1969) on developing questionnaire procedures for specifying preferences.

The approach by Raiffa, which we will briefly present in this section, is part of more general procedure of ranking of preferences for multiattributed consequences appropriate for making decisions under conditions of uncertain consequences. If expected utility can be considered an appropriate guide for decision making, then probabilistic outcomes ranked in terms of von Neumann-Morgenstern utility are sufficient for assessing decision problems. In this context Raiffa has been concerned with the specification of multiattributed utility or value functions and, in one case, the specification of preferences over the dual attributes, lifetime consumption, and survival probability. In this section we will discuss this line of research after a brief review of utility or preference theory.

4.a A Review of Preference Theory

Preference theory, or utility theory, concerns itself with the quantification of an individual's judgment of the probability, or value of various "goods."[19] A utility function associates a numerical index with each of several possible goods reflecting their preferential ranking. There are two types of utility functions. The first, which is called an ordinal utility or value function, provides an ordering relation between well defined alternatives, e.g., different goods or commodity bundles, and therefore its meaning remains unchanged under order-preserving transformations. The second type of utility function, which was originally developed by von Neumann and Morgenstern and which we will refer to as

---

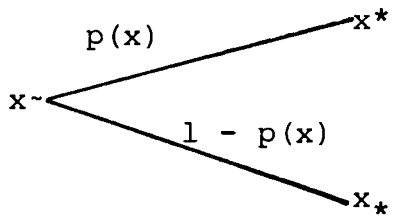
[19] "Goods" in this sense, refers to the set of all possible alternatives--things, services, situations, outcomes, etc., with which we are concerned.

N-M utility, provides an ordering relation between outcomes which are uncertain, that is, outcomes whose characteristics are not definite, but rather distributed according to a probability distribution.

A simple example (Oksman, 1974) illustrates the difference between ordinal and N-M utilities:

> Let ℓ_1 and ℓ_2 be two outcomes corresponding to the characteristic x assuming the values $x = x_1$ and $x = x_2$ respectively. If u(x) is an ordinal utility function for the characteristic x, then ℓ_1 is preferred to ℓ_2 if $u(x_1) > u(x_2)$. Any positive monotone transformation of $u(\bullet)$ preserves this preference structure. However, if ℓ_1 and ℓ_2 are "lotteries" for the characteristic x with probability mass function $p_1(x)$ and $p_2(x)$ respectively, and our decision maker is "rational," then ℓ_1 is preferred to ℓ_2 and only if
>
> $$E_{p_1}(x)u(\tilde{x}) > E_{p_2}(x)u(\tilde{x}) \qquad \text{(IV-A-1)}$$
>
> where the symbol $\tilde{}$ indicates that x is a random variable and $E(\bullet)$ refers to expectation with respect to the appropriate probability distribution.
>
> When expectations are taken, equivalent ordinal utility functions do not necessarily lead to an identical preference structure for certain outcomes. On the other hand, any positive linear transformation of u(x) leads to the identical preference ordering under uncertainty as u(x) itself. Thus, if u(x) is the N-M utility function of a particular individual, then $v(x) = a + bu(x)$, $b > 0$, is also a valid N-M utility function for this individual, and the two functions are said to be <u>decisionally equivalent</u>. Any function which is not a positive linear transformation of u(x) is not an equivalent N-M utility function [pp. 7,8].

One possible (although controversial) way of determining the preferences of an individual in order to specify his N-M utility function is to ask questions about indifference probabilities for certain types of lotteries. For example, if x^* and x_* are designed as the best and worst possible values of the outcome respectively, then supposedly the decision maker can find a probability, $p(x)$, for an intermediary value of x such that he is indifferent between "x for sure" and a chance $p(x)$ at x^* with a completmentary chance $\{1 - p(x)\}$ at x_*. This indifference value of x is called the certainty equivalent of the lottery. Diagrammatically we show this as



(IV-A-2)

By asking assessment questions for several values of x it is possible to establish a function (x) ranging from $p(x_*) = 0$ to $p(x^*) = 1$. This function is a proper N-M utility and any positive linear transformation $u(x) = a + bp(x)$ is a decisionally equivalent proper N-M utility.

In many instances the possible outcomes of a decision problem are described by several characteristics in which case we have a <u>joint utility function</u> which can be denoted $U(x_1, x_2, \ldots, x_n)$. The function $U(x_1, \ldots, x_k, \ldots, x_n)$ when viewed as a function of $(x_1, \ldots, x_k)$ with $(x_{k+1}, \ldots, x_n)$ held fixed is also a proper utility function for ranking lotteries on $(x_1, \ldots, x_k)$ given value of $(x_{k+1}, \ldots, x_n)$. This is called a <u>conditional utility function</u>. If the decision maker can resolve lotteries on some subset of attributes, say $(x_1, \ldots, x_k)$, irrespective of the value of $(x_{k+1}, \ldots, x_n)$, then we can think of his <u>marginal utility</u>[20] for $(x_1,\ldots,x_k)$. Marginal utility functions only exist if preferences between the different characteristics are independent in such a way as to permit appropriate decompositions of the joint utility.[21]

4.b <u>Raiffa's Derivation of U(C,P)</u>

Given this background, Raiffa considers the problem of assessing an N-M utility function over the dual attributes, lifetime consumption C and survival probability P. The problem, according to Raiffa, is greatly simplified if we can assume that this utility is an additive function of the dual attributes, C and P, or

$$U(C,P) = \lambda_1 U_1(C) + \lambda_2 U_2(P) \qquad \text{(IV-B-1)}$$

where λ_1 and λ_2 are weighting factors. It has been proven (Fishburn, 1966) that a necessary and sufficient condition for additivity is that the individual's preferences depend only on the marginal probability distribution of each attribute. We can illustrate this assumption with a simple example. Suppose that the individual is faced with the two lotteries shown below:

$$\ell_1 = \begin{cases} \xrightarrow{.5} (C,P) \\ \xrightarrow{.5} (C_*,P_*) \end{cases} \qquad \ell_2 = \begin{cases} \xrightarrow{.5} (C,P_*) \\ \xrightarrow{.5} (C_*,P) \end{cases} \qquad \text{(IV-B-2)}$$

---

[20] This definition of marginal utility is different from the economists' concept which has been previously referred to in this paper, namely $\frac{\partial U}{\partial x_i}$.

[21] For a good discussion of these concepts, see Oksman (1974).

where $C_* < C$ and $P_* < P$. If there is any complementarity between C and P, we would not expect that the individual be indifferent between the two lotteries because they offer different prizes. However, if his utility function is additive, and the marginal utility functions for C and P are $U_c(C)$ and $U_p(P)$ respectively, then

$$U(\ell_1) = .5 \left\{U_c(C) + U_p(P)\right\} + .5 \left\{U_c(C_*) + U_p(P_*)\right\}$$

$$U(\ell_1) = .5 \left\{U_c(C) + U_p(P_*)\right\} + .5 \left\{U_c(C_*) + U_p(P)\right\} \tag{IV-B-3}$$

and, therefore

$$U(\ell_1) = U(\ell_2) \quad . \tag{IV-B-4}$$

If we substitute various values of C and P into lotteries IV-B-2, it becomes apparent that most individuals would be indifferent between ℓ_1 and ℓ_2 only for very high values of P. Thus, to assume additivity, we must constrain our analysis to very small nonsurvival probabilities. (It can be recalled Section II-A -- that Conley also analyzed the low probability case with the justification that most relevant decisions fall within this range.) For a range of P's over which additivity does hold and by the assumed existence of a continuous utility function U(C,P) we can express IV-B-4 as

$$.5\ U(C,P) + .5\ U(C_*,P_*) = .5\ U(C,P_*) + .5\ U(C_*,P) \quad . \tag{IV-B-5}$$

If we let $P_* = 0$ and arbitrarily set $U(C_*,P_*) = 0$, then by substitution we get

$$U(C,P) = \lambda_1 U_c(C) + \lambda_2 U_p(P) \tag{IV-B-6}$$

where the λ's represent weighting factors and $U_c(C)$ and $U_p(P)$ represent, again, the marginal utility function, or $U(C,P_*)$ and $U(C_*,P)$, respectively.

The problem now becomes one of specifying the marginal utility functions, $U_c(C)$ and $U_p(P)$. It is easily shown (Raiffa, 1969, p. 88) that, given no anxiety, $U_p(P)$ must be linear with respect to P. This follows from the nature of the units on P.[22] If we normalize this utility, letting $U_p(P_*) = 0$ and $U_p(P = 1) = 1$, then

$$U_p(P) = P \ . \qquad \text{(IV-B-7)}$$

Substituting IV-B-7 into IV-B-6

$$U(C,P) = \lambda_1 U_c(C) + \lambda_2 P \ . \qquad \text{(IV-B-8)}$$

Since an N-M utility function is only meaningful up to a linear transformation, IV-B-8 can be expressed in the form

$$U(C,P) = U_c(C) + \beta P \qquad \text{(IV-B-7)}$$

where $\beta > 0$. The critical parameter β can, according to Raiffa, be estimated given two points on the individual's indifference function. If

$$U(C_1,P_1) = U(C_2,P_2) \ , \qquad \text{(IV-B-8)}$$

---

[22] To illustrate with an example, if the survival probability P depends on whether a given event turns out H or T with equal probabilities P_1 and P_2, then the probability of survival from the gamble must be $(P_1 + P_2)/2$.

or

$$U_c(C_1) + \beta P_1 = U_c(C_2) + \beta P_2 \quad , \qquad \text{(IV-B-9)}$$

then

$$\beta = \frac{U_c(C_1) + U_c(C_2)}{P_1 + P_2} \quad . \qquad \text{(IV-V-10)}$$

Raiffa concludes that in order to assess U(C,P), where we are willing to assume additivity and where there is no anxiety, we merely have to assess a utility function for consumption (with some constant probability of survival) and find two (C,P) pairs over which the individual is indifferent. The derivative of this function, then, represents according to our definition the value of life.

4.c Comparing Raiffa's Derivation with that of Conley, Usher, and Jones-Lee

Because of the similarities between the Raiffa assumptions and those of Conley, Usher, and Jones-Lee, it is interesting to compare the results. In general, the models address the case of the expected utility maximizer who has no family, dependents, etc., and who faces a beginning of the period probability of death. What distinguishes the Raiffa analysis form that of Conley, Usher, and Jones-Lee is that Raiffa specifies U(C,P) and thus $\frac{\partial C}{\partial P}$ by asking the type of questions which directly specify this trade-off (assuming) additivity, he need only find two points). On the other hand, Conley, Usher, and Jones-Lee qualify this trade-off indirectly by specifying the demand for survival probability as the demand for "life." In other words, in the Conley, Usher, and Jones-Lee models the probability of survival P has no direct utility U(P) as in the Raiffa case; preferences over P can be assessed only as an expected value, PU (life), which is simplified to PU(C), and thus for the expected utility maximizer one needs to assess U(C). The Raiffa analysis circumvents this problem of specifying a utility of life. By taking the derivative of IV-B-7, we can formulate Raiffa's value of life as

$$\frac{\partial C}{\partial P} = \frac{\beta}{U'_C(c)} \quad . \qquad \text{(IV-C-1)}$$

We can estimate β, in turn, by isolating two (C,P) points over which the individual is indifferent. Indirectly, the individual by specifying these preferences, assesses his utility of living. This is in contrast to the Conley, Usher, and Jones-Lee analysis where we need only assess the utility of consumption.

V. Conclusion

The purpose of this paper has been to critically review the results of the Conley, Usher, and Jones-Lee models as they relate to the theoretical specification of the value of human life. This value has been defined as the willingness of an individual to pay for changes in his survival probability and it is graphically represented as the slope of the individual's indifference curve between survival probability and income or consumption. It has been shown that, in principle, the formulations of the three models are equivalent, and that the value of life can be expressed as the individual's lifetime utility of lifetime consumption divided by his initial period survival probability times the marginal utility of his lifetime utility with respect to his lifetime consumption, or

$\frac{\partial C}{\partial P_0} = \frac{U(C)}{P_0 U'(C)}$. Hirshleifer, Bergstrom, and Rappaport have pointed out that this value is formally similar to the traditional human capital measure; in fact, if it can be assumed that $U(C) = C^{\beta}$, then the human capital measure understates the value of life by the degree of risk aversion or $1/\beta$. The importance of this formulation is that it is relatively simple to estimate. One need only determine empirically a lifetime utility function.

This surprising result is precluded by the form of the assumed objective function; in the case of all three models this objective function or lifetime utility is an univariate function of consumption. If one factors is an adjustment to this utility to account for the value of living independent of the value of being alive to consume, then the value of life turns out to be understated in the Conley, Usher, and Jones-Lee formulations. The seriousness of this understatement depends on the extent to which the utility of life is greater than the utility of lifetime consumption.

Rejecting the Conley, Usher, and Jones-Lee hypothesis that the value of life is correlated solely with lifetime income, the problem becomes empirical. It is necessary to obtain data on the individual's trade-off between survival probability and income. One possible approach, although controversial, is to specify these choices with the use of questionnaire data. Raiffa, in the context of specifying preferences for decision analysis, develops an approach which is applicable to this problem. He shows that for very low nonsurvival probabilities it is necessary to isolate only two points of the individual's preference function and obtain information on the individual's utility of consumption. From this data, the entire preference function over the relevant range of survival probabilities can be specified and thus the value of life can be calculated over this range.

However, one must be reminded that this solution is dependent on several somewhat restrictive assumptions (no dependents, immediate probability of death, etc.) and, in practice, cannot be generalized. It is also important to note that one cannot assume that the preference function for survival probability is identical for different types of accidents or causes of death. Usually death is accompanied by periods of "pain and suffering," and the individual's perception of death is influenced, as well, by a multitude of psychological factors. Pahner (1975) has suggested that these psychological factors are a possible explanation for what appears, in light of the scientific estimates of the risk, to be an irrational reaction on the part of the public to the acceptance of nuclear power plants. An important feature of preference functions, however, is that they do allow for the psychological factors. To quote Mishan (SCOPE, 1975): "If, for example, a person costs the risk of death in an auto accident as, say $100,000 and costs in the same risk of death through a nuclear power plant accident as, say $1,000,000, the economist accepts it" (p. 5).

The major problem with the willingness-to-pay or preference approach is the difficulty of empirical estimating a problem relevant to any decision technique which requires that the disbenefit of risks affecting human mortality be quantified. The modelling approach to this problem, for reasons discussed in this paper, is limited. However, there are presently some promising efforts on the specification of these preferences with the use of questionnaires. The solution to this problem will be an important input into the work of the IIASA Energy Project on the comparison of energy options.

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