Working Paper

Evolutionary Theorizing on Economic Growth

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Preface

The research project on *Systems Analysis of Technological and Economic Dynamics* at IIASA is concerned with modeling technological and organisational change; the broader economic developments that are associated with technological change, both as cause and effect; the processes by which economic agents – first of all, business firms – acquire and develop the capabilities to generate, imitate and adopt technological and organisational innovations; and the aggregate dynamics – at the levels of single industries and whole economies – engendered by the interactions among agents which are heterogeneous in their innovative abilities, behavioural rules and expectations. The central purpose is to develop stronger theory and better modeling techniques. However, the basic philosophy is that such theoretical and modeling work is most fruitful when attention is paid to the known empirical details of the phenomena the work aims to address: therefore, a considerable effort is put into a better understanding of the 'stylized facts' concerning corporate organisation routines and strategy; industrial evolution and the 'demography' of firms; patterns of macroeconomic growth and trade.

From a modeling perspective, over the last decade considerable progress has been made on various techniques of dynamic modeling. Some of this work has employed ordinary differential and difference equations, and some of it stochastic equations. A number of efforts have taken advantage of the growing power of simulation techniques. Others have employed more traditional mathematics. As a result of this theoretical work, the toolkit for modeling technological and economic dynamics is significantly richer than it was a decade ago.

During the same period, there have been major advances in the empirical understanding. There are now many more detailed technological histories available. Much more is known about the similarities and differences of technical advance in different fields and industries and there is some understanding of the key variables that lie behind those differences. A number of studies have provided rich information about how industry structure co-evolves with technology. In addition to empirical work at the technology or sector level, the last decade has also seen a great deal of empirical research on productivity growth and measured technical advance at the level of whole economies. A considerable body of empirical research now exists on the facts that seem associated with different rates of productivity growth across the range of nations, with the dynamics of convergence and divergence in the levels and rates of growth of income, with the diverse national institutional arrangements in which technological change is embedded.

As a result of this recent empirical work, the questions that successful theory and useful modeling techniques ought to address now are much more clearly defined. The theoretical work has often been undertaken in appreciation of certain stylized facts that needed to be explained. The list of these 'facts' is indeed very long, ranging from the microeconomic evidence concerning for example dynamic increasing returns in learning activities or the persistence of particular sets of problem-solving routines within business firms; the industry-level evidence on entry, exit and size-distributions – approximately log-normal – all the way to the evidence regarding the time-series properties of major economic aggregates. However, the connection between the theoretical work and the empirical phenomena has so far not been very close. The philosophy of this project is that the chances of developing powerful new theory and useful new analytical techniques can be greatly enhanced by performing the work in an environment where scholars who understand the empirical phenomena provide questions and challenges for the theorists and their work.

In particular, the project is meant to pursue an 'evolutionary' interpretation of technological and economic dynamics modeling, first, the processes by which individual agents and organisations learn, search, adapt; second, the economic analogues of 'natural selection' by which interactive environments – often markets – winnow out a population whose members have different attributes and behavioural traits; and, third, the collective emergence of statistical patterns, regularities and higher-level structures as the aggregate outcomes of the two former processes.

Together with a group of researchers located permanently at IIASA, the project coordinates multiple research efforts undertaken in several institutions around the world, organises workshops and provides a venue of scientific discussion among scholars working on evolutionary modeling, computer simulation and non-linear dynamical systems.

The research focuses upon the following three major areas:

1. Learning Processes and Organisational Competence.
2. Technological and Industrial Dynamics
3. Innovation, Competition and Macrodynamics

Evolutionary Theorizing on Economic Growth

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INTRODUCTION

While an evolutionary perspective has been urged upon economists since at least Marshall 1890 (see Hodgson 1993 for a recent reiteration), what has been lacking until recently, at least for a large portion of the economics profession, has been a body of formal theory and quantitative analysis on an explicitly evolutionary basis. This has changed since the work of Nelson and Winter in the 1960s and 1970s (summarized in Nelson and Winter 1982), which operationalized and extended many of the concepts going back to Schumpeter 1919, Schumpeter 1947, Alchian 1951, Downie 1955, Steindl 1952, and others. Since then a number of authors have been enlarging on this foundation and systematically extending the evolutionary economics paradigm in a number of directions. A survey of some of these can be found in Nelson 1995.

In this chapter we intend to deal with the basics of a formal evolutionary approach to technical change, economic dynamics and growth. In so doing we will leave out for the most part the burgeoning new areas of application of evolutionary ideas to game theory, learning dynamics and bounded rationality, organization theory, financial markets, industrial organization, and the interface of economics, law and culture, most of which are dealt with elsewhere in this volume. Instead we will concentrate on a restricted class of interrelated models of growth and dynamics to see whether a viable alternative paradigm to the mainstream, neoclassical approach, as well as a new class of insights, are emerging.

There are essentially two reasons for believing that an evolutionary approach is applicable to economics. One is based on analogy and an appeal to the type of explanation common in biology: that forms of competition, innovation, variation and selection have analogues in the two subjects and thus that similar reasoning can profitably be applied in the nonbiological domain. Here most authors stress that the analogy should not be taken too seriously, so that it is useless to search for whatever corresponds exactly to genes, sexual reproduction, crossover or mutation in the economic sphere. Moreover, discredited forms of evolution such as Lamarckianism, the inheritance of acquired characteristics, may be perfectly conceivable in the socioeconomic realm.

The second takes a more universalist perspective. It argues that, just as biological evolution has passed through distinct stages (prokaryotic and eukaryotic life, asexual and sexual reproduction, as well as a prebiotic stage), so modern industrial society is just a distinct stage of this single process, subject to the same underlying laws if constrained by specific features of its current realization. Thus economic evolution would be an intrinsic component of a larger evolutionary process, and not merely something accidentally amenable to certain forms of reasoning by analogy.

What reasons might we have to believe this? Lotka (1924) proposed the concept of "energy transformers" to capture the common thermodynamic features of all life forms. This is quite similar to what later was termed dissipative systems (Nicolis and Prigogine 1977), i.e., thermodynamically open systems, far from equilibrium, which maintain a high state of internal organization by importing free energy from their environment, consuming it for purposes of self-repair and self-reproduction, and exporting the resulting waste as high entropy back to the environment. Thus the apparent paradox of life, already pointed out by Henry Adams (1919), of complex structure emergence in the face of the Second Law of Thermodynamics (that in thermodynamically closed systems entropy, i.e., disorder, must increase) is transcended.[1] Life (or at least carbon-based life as we know it until the industrial revolution) can be seen as a sea of such "converters" living off the waterfall of free energy flowing between the sun and the low-value infrared radiation reflected by the earth into deep space.[2]

From this perspective human civilization is distinguished from earlier forms of biological evolution by the fact that the information carriers of the selforganizing structures, rather than being encoded in a form like DNA internal to the organism, now have attained an *exosomatic*[3] (Lotka 1945) form. Information is encoded both in an intangible sphere existing between human minds known as culture, and a more tangible sphere consisting of writing and other forms of representation, and cultural and industrial artifacts. But the fact remains that, within the constraints imposed by the various physical substrates of information storage and transmission, evolution still must proceed along the basic Darwinian lines of (random) variation and selection. The complication associated with modern socioeconomic evolution is that we now have to deal with a mosaic of simultaneous biological (DNA), culturally tacit (existing in the human pyschomotoric systems of individuals and groups) and culturally codifiable (existing in exosomatic artifacts) information transmission and variation mechanisms, the latter category being increasing machine based.

The task of an evolutionary theory of economic growth, then, might be to formulate a population dynamics of this multilevel evolutionary process, taking account both of the human components and of the increasingly sophisticated forms of artifactual energy and information transformers collectively referred to by economists in a rather undifferentiated manner as capital.[4] But even if we agree that this more fundamental perspective on economics as an integral part of the evolutionary process has a certain validity, the "genetic code" of the various non-DNA based levels still remains to be discovered. Even in biology, in fact, where a firm understanding of the molecular basis of genetics has emerged since the 1950s, many extreme simplifications of a phenomenological sort still have to be made in formal models of population genetics and evolution.[5] Thus from a practical point of view it may not make much difference whether we apply evolutionary thinking to economics as an

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[1] The observation that open systems (in particular, organisms) can seemingly circumvent the second law of thermodynamics by exporting entropy to the environment (or equivalently, importing "negentropy" or free energy, i.e., energy of a higher "quality" than the ambient heat, which can be converted to mechanical work) goes back at least to Bertalanffy (1932) and Schrödinger (1945).

[2] "Summarizing we may say that selforganization is necessarily connected with the possibility to export entropy to the external world. In other words, selforganizing systems need an input of high-valued energy and at the same time an output of low-valued energy. In the interior of selforganizing systems a depot of high-valued energy of another form is observed. The evolution processes on our planet are mainly pumped by the "photon mill" with the three levels sun-earth-background radiation (let us mention however that the geological processes are pumped by the temperature gradients between the centre of the earth and the surface). On the cosmic scale the general strategy of evolution is the formations of islands of order on a sea of disorder represented by the background radiation." (Feistel and Ebeling 1989, p. 91)

[3] There is of course another level of *endosomatic* information processing based on the neuronal system of animals, which Edelman (1987) hypothesizes to function according to neuronal group selection. This allows organisms to learn from experience during their lifetimes, i.e., is a type of acquired characteristic with clear survival value. However, until the advent of language and culture, which permit *intergenerational* transmission, the neuronal system in itself cannot serve as a basis for long-term evolution but must still rely on the DNA substrate to generate further development.

[4] This is the theme of Boulding (1978) and Boulding (1981), without the author proceeding very far down the road of formal modeling, however.

[5] Thus one often assumes asexual rather than sexual reproduction to simplify the mathematics.

exercise in restrained analogizing or regard the economics of human societies as a specific stage in a universal evolutionary process, until such time as canonical descriptions of the "genetic deep structure" of socioindustrial processes can be agreed upon.[6] For the time being we will have to make do with more or less plausible and heroic assumptions about the entities and variation and transmission mechanisms implicated in economic evolution, and judge them on the basis of a limited range of micro and macroeconomic "stylized facts."

BEHAVIORAL FOUNDATIONS AND FORMAL EVOLUTIONARY MODELING IN THE ECONOMICS OF GROWTH AND SCHUMPETERIAN COMPETITION: SELECTION

Formalization of evolutionary thinking in biology began with Fisher (1930), who introduced what are now called *replicator equations*[7] to capture Darwin's notion of the survival of the fittest. If we consider a population to be composed of n distinct competing "species" with associated, possibly frequency-dependent fitnesses $f_i(x)$, where x is the vector of relative frequencies of the species $(x_1, x_2, \ldots, x_n)$, then their evolution might be described by the following equations:

$$\dot{x}_i = x_i(f_i(x) - \bar{f}(x)),\ i = 1,\ n,\ \text{with } \bar{f}(x) = \sum_{i=1}^{n} x_i f_i(x).$$

The intuition is simple: species with above-average fitness will expand in relative importance, those with below-average fitness will contract, while the average fitness $\bar{f}(x)$ in turn changes with the relative population weights. If the fitness functions f_i are simple constants, then it can be shown that the species with the highest fitness will displace all the others and that average fitness will increase monotonically until uniformity is achieved according to

$$\frac{d\bar{f}}{dt} = var(f) \geq 0,$$

where var(f) is the frequency-weighted variance of population fitness. Thus average fitness is dynamically maximized by the evolutionary process (mathematically, it is referred to as a Lyapunov function). This is known as Fisher's Fundamental Theorem of Natural Selection, but it should be noted that it is only valid for *constant* fitness functions. In the event of frequency-dependent selection, where fitness depends on population shares, including a species' own share, and increasing and decreasing "returns" may intermingle, multiple equilibria are possible and no quantity is *a priori* necessarily being maximized (see Ebeling and Feistel 1982 for an extensive discussion of maximal principles). The replicator equation only describes the relative share dynamics and thus takes place on the unit simplex S^n (where $\sum_{i=1}^{n} x_i = 1$), an n-1 dimensional space. To derive the absolute populations it is necessary to

[6] One difference, however, is the central importance placed upon energetic and environmental constraints associated with the latter perspective. These, for better or worse, will not play any explicit role in the following discussion.

[7] See Sigmund (1986) and Hofbauer and Sigmund (1988, pp.145-6) for a discussion of their basic form and various applications.

introduce an additional equation for the total population level. An alternative description due to Lotka and Volterra is based on growth equations for the population levels y_i (with the frequently used log-linear version on the right hand side):

$$\dot{y}_i = g_i(y) = r_i y_i + \sum_{j=1}^{n} a_{ij} y_i y_j.$$

A theorem due to Hofbauer asserts that Lotka-Volterra and replicator systems are equivalent (see Hofbauer and Sigmund 1988, p. 135).

Most evolutionary economics models to a considerable extent consist of giving the functions f_i or g_i economic meaning in terms of market competition or differential profit rate driven selection mechanisms. The former usually defines a variable representing *product competitiveness*, which may be a combination of price, quality, deliver delay, advertising and other variables (for examples see Silverberg, Dosi and Orsenigo 1988 or Kwasnicki and Kwasnicka 1992). The latter assumes that product quality and price are homogeneous between producers (or subject to fast equilibriating dynamics compared to the evolutionary processes of interest) but unit costs of production differ, so that firms realize differential profit rates. If their growth rates are related to profits, as seems reasonable, then their market shares or production levels (corresponding to x_i and y_i in the biological models) can be described by replicator or Lotka-Volterra equations, respectively.

All of the models we will discuss in this chapter focus primarily on technical change as the central driving element of the evolutionary processes with which they are concerned. They differ considerably, however, in their representations of technology and how it interfaces with firm strategies and the market. A major distinguishing characteristic is whether technology is *capital embodied* or *disembodied*, i.e., whether changes in technological performance are primarily (though not necessarily exclusively) related to investment in new equipment or not. In the former case technical change is highly constrained by investment in physical capital (as well as possible complementary factors); in the latter case it is not and can be almost costless. Yet even on the assumption of embodied technical change, there can be important differences in formal treatments. The classical approach to embodied technical change uses the *vintage* concept going back to Salter (1960), Solow (1960) and Kaldor and Mirrlees (1962), as in essence do national statistical offices with the perpetual inventory approach to the measurement of the capital stock. One assumes that at any given time there is a single best-practice technology in which investment is made. The capital stock consists then of the vintages of past investment going back in time until the scrapping margin, i.e., that oldest vintage on the verge of being discarded due to technological obsolescence and/or wear and tear. This defines a technological lifetime of capital equipment.[8] The aggregate capital stock is a sum or integral (in the discrete and continuous time cases, respectively) over the vintages during this lifetime, and average technical coefficients (labour productivity, capital/output ratios) are the corresponding vintage-weighted sum or integrals. Vintage capital stock may be easy to compute from data, but they have two disadvantages which detract from their realism and tractability. First is the assumption of a single best-practice technology, which rules out multiple competing technologies at the investment frontier, a topic dear to the hearts of most evolutionary economist and students of innovation diffusion. This can be

[8] Except in the case in which capital is assumed to decay exponentially according to some presumed depreciation rate, in which case its lifetime is infinite, although older vintages rapidly become insignificant.

overcome to some extent by assuming multiple, parallel vintage structures of distinct technologies, as in Silverberg, Dosi and Orsenigo (1988). The second is that, although particularly discrete-time vintage capital stocks can be easily calculated from data, when they are embedded in a dynamic framework with endogenous scrapping they can lead to awkward mathematical complications. Delay difference or differential equations and even age-structured population dynamics become involved whose mathematical properties, except under extremely simple assumptions, are still poorly understood compared to systems of ordinary difference or differential equations.

An alternative implicitly exploited in the models in Metcalfe (1988), Iwai (1984a,b), Henkin and Polterovich (1991), Silverberg and Lehnert (1993, 1994) and Silverberg and Verspagen (1994a,b, 1995a,b), might be termed a *quasi-vintage* framework. Capital "vintages" are labelled by their type instead of their date of acquisition, so that the service age no longer plays any role, only the technical characteristics (although decay by type independently of age is still possible). Thus several qualitatively distinct technologies can diffuse simultaneously into and out of the capital stock. Furthermore, only ordinary differential (or difference) equations are needed to handle the quasi-vintage structure, a considerable mathematical simplification. This gain in realism and tractability is compensated for by an inability to track the vintages by chronological age, however. But quasi-vintages lend themselves more naturally to the kind of multiple replacement dynamics investigated by Marchetti and Nakicenovic (1979), Nakicenovic (1987), and Grübler (1990). And one view on evolution holds that its essence resides exactly in the sequence of such replacements (Montroli 1978), whether related to technologies, behavioral patterns, or social structures.

The disembodied side of technical change (disembodied at least in the sense that it is not representable by tangible equipment) is still even more of a black box than the embodied side. It can reside in (tacit) human skills or organizational and societal capabilities, but little is known of a very fundamental nature about how it is accumulated, stored, and refreshed. *Learning by doing* (Arrow 1962) is a standard phenomenological approach finding expression in power laws for the relationship between productivity and cumulative investment or production. Recently, it has also become central to much of the neoclassical endogenous growth literature. The effects of *technological spillovers* between competitors have also received considerable attention. One possible way of combining learning by doing and spillovers in a dynamic framework is Silverberg, Dosi and Orsenigo (1988), but nothing along these lines has been attempted in an evolutionary growth model, to our knowledge. The net effect of both of these phenomena is usually one form or another of increasing returns, such as increasing returns to adoption or agglomeration, network externalities, etc. (see Arthur 1988, 1994). Within the replicator framework this means that the fitness functions $f_i(x)$ truly depend on the frequencies $\mathbf{x}$, resulting in multiple equilibria, threshold phenomena, lockin, etc.[9]

[9] The increasing returns phenomenon was studied by Arthur, Ermoliev and Kaniovski using the Polya urn stochastic tool, which assumes an indefinitely increasing population to establish asymptotic results. The alternative case of a fixed population size with stochastic effects can be studied using Master equation methods (see Feistel and Ebeling 1989 and Bruckner, Ebeling, Jiménez Montaño and Scharnhorst 1994, and especially Jiménez Montaño and Ebeling 1980 for a stochastic formulation of the Nelson and Winter model). We will only make limited use of stochastic tools in the following, so that the deterministic replicator equation will serve our purposes.

BEHAVIORAL FOUNDATIONS AND FORMAL EVOLUTIONARY MODELING IN THE ECONOMICS OF GROWTH AND SCHUMPETERIAN COMPETITION: INNOVATION AND LEARNING

Evolution would soon come to an end were it not for the continual creation of new variety on which selection (as well as drift) can act. This is especially crucial for growth models, where the ongoing nature of the technical change process is at the fore, although other aspects may well converge to stable stationary patterns. Thus considerable attention has to be devoted to how innovation is realized by firms, individually and collectively. In principle most scholars agree that innovation should be modelled stochastically, to reflect the uncertainty in the link between effort and outcome. The details on how this is done may very considerably, however. The classical formulation is due to Nelson and Winter, described in more detail later in this chapter. Nelson and Winter lump technologies and behavioral rules/strategies together under the concept of *routines*. Since technical change is disembodied in their model, this equivalence is perhaps admissible, since a change in technique for a firm's entire capital stock requires only the expenditure necessary to undertake innovative or imitative search, not investment or training per se. While there is technological learning at the economy-wide level, firms themselves are completely unintelligent, since they operate according to given search and investment rules that cannot be modified as a result of experience. Instead, the firm is subject to selection as a consequence of the technologies it has stumbled upon. A somewhat peculiar aspect is the very literal application of Simon's notion of satisficing to mean that firms only undertake innovative search if their performance is unsatisfactory.[10]

An interesting elaboration of search activity and entry in the original Nelson and Winter model is presented in Winter (1984),[11] where firms are broken down into two types: primarily innovative or imitative. Further, the notion of technological *regime* is introduced (going back to the early or later Schumpeter) depending on whether the source of technical progress is external to the firm (e.g., from publicly available scientific knowledge bases) or from firms' own accumulated technological capabilities. These regimes are referred to as the *entrepreneurial* and the *routinized* and are exogenously imposed by means of specific parameter settings. Although firms can be of two types, neither type is capable of learning. Instead, the market is shown to select between the two depending on the technological regime. Entry of new firms also assumes a greater importance than the mere supporting role to which it is relegated in most evolutionary models, being stimulated in the entrepreneurial regime.

While learning based on selection/mutation dynamics has begun to play a major role in the evolutionary games literature (e.g., Kandori, Mailath and Rob 1993, Young 1993), very little has found entrance into evolutionary models of a general economic orientation. A first stab at changing this state of affairs for the theory of growth was undertaken by Silverberg and Verspagen (1994a,b, 1995a,b), drawing on the evolution strategy literature (Schwefel 1995). Here mutations are local around the current strategy, and the probability of imitation is an increasing function of dissatisfaction with current performance and the size of the

[10] This should be contrasted with the Silverberg and Verspagen models, where firms undertake behavioral *imitation* with increasing probability the more unsatisfactory their performance is.

[11] The discussion of the model is couched in terms of *industry* dynamics, not economy-wide growth, although there is nothing in the basic assumptions to preclude analysis of the latter.

imitated firm. In contrast to the Nelson and Winter tradition, strategies and technologies are treated separately. The learning algorithm applies only to the firms' R&D expenditure strategies; their technological performance then follows in a somewhat complex manner from these decisions and market feedbacks. In this way it is possible to implement simple boundedly rational decision rules gleaned from actual business practice, such as targeted R&D/total investment or R&D/sales ratios, or a combination of the two.

Genetic algorithms and classifier systems have also been gaining favour in recent years as mechanisms for operationalizing learning with artificial agents.[12] Although these appeal even more directly to a discrete genetic mechanism of inheritance à la biological DNA than social scientists may feel comfortable with, they may also be employed agnostically simply as algorithmic tools to allow learning to happen, if not as models of how learning actually happens. The goal of an *artificial economics* modeling philosophy as espoused by Lane (1993) is to put together a basic web of economic interactions between artificial agents endowed with a *tabula rasa* knowledge of their environment, but fairly sophisticated abilities to learn, and see what sorts of markets, institutions and technologies develop, with the modeller prejudicing the developmental possibilities as little as possible. Something along these lines has already been implemented to a certain extent in the 'sugarscape' model of Axtell and Epstein (1995), paralleling the artificial worlds movement in the biology domain (cf. Langton 1989 and Langton, Taylor, Farmer and Rasmussen 1992). While this direction of research has generated much excitement, it has not avoided the fate of many overhyped scientific trends in the form of a sceptical backlash (see Horgan 1995). Be that as it may, in the following we will limit ourselves to those models rooted in the economics tradition that promise to address issues of long-standing empirical interest.

AN OVERVIEW OF EVOLUTIONARY GROWTH MODELS

In this section, we will discuss the similarities and differences between several growth models that have been developed over the last decades, and which were based upon the evolutionary principles that we have outlined so far.[13] The first model that will be discussed is the one presented in Nelson and Winter (1982). This model can be seen as the first evolutionary growth model, and, as will be shown in the rest of the discussion, can be regarded as the pioneering effort in the field. The Nelson and Winter model is a model with an explicit microeconomic foundation, which consists of modelling the behaviour of firms in their search for more advanced techniques. Basically because of the complexity arising from the simultaneous existence of multiple firms with different search behaviour and, hence, different technological levels, the Nelson and Winter model is analyzed by means of computer simulations.

One class of more recent growth models in the evolutionary tradition follows the Nelson and Winter perspective of adopting a microeconomic foundation. Consequently, these models also resort to computer simulations for analysis. In this group of models, the main

[12] See Booker, Goldberg and Holland (1989), and Goldberg (1989) for basic theory and methodology and Holland and Miller (1991), Kwasnicki and Kwasnicka (1992) and Lane (1993) for some economic applications.

[13] The papers that we discuss by no means form an exhaustive list of 'evolutionary growth theories'. However, limiting ourselves explicitly to papers in which mathematical models with a clear 'population perspective' are the core of the analysis, we hope that the present list covers at least the most prominent contributions.

contributions are to extend the original Nelson and Winter setup by introducing more realistic representations of technology, to extend the analysis to a multi-country framework, or to extend evolutionary principles to the issues of behavioral strategies, instead of just technological change.

A second broad group of evolutionary growth models does not take the explicit microeconomic perspective proposed by Nelson and Winter, at least not in the sense of modelling the individual firm. Consequently, the similarities to the original Nelson and Winter model are less pronounced in this group of papers. The main reason for not taking into account the microeconomic foundations explicitly, seems to be the desire to keep the models analytically tractable, or to keep the complexity of a simulation model within bounds, so that extensions to, for example, a multi-country context, or more systematic analysis of the closed economy case, becomes easier.

Because this second group of papers does not have clear roots in any specific approach, these contributions are necessarily more heterogeneous than those of the first group. Still, it is possible to find two broad approaches here. The borderline between the two subgroups is the distinction between analytical solutions and computer simulations.

The guidelines for our discussion of these different approaches within the field of evolutionary growth theory will be four different points. The first three of these points correspond to the three basic principles of the evolutionary process that we have discussed: heterogeneity of the population (usually firms, or alternatively countries, or techniques), the mechanism for generating novelty in the population (mutation, usually in the form of technical innovations), and, finally, selection (related to the economic environment in which the population operates). The last point we will discuss is the economic interpretation, or outcomes of the models.

The Nelson and Winter model

We start our discussion with a brief summary of the model presented in Nelson and Winter (1982, Part IV), which can be regarded as the pioneering effort in the field of evolutionary growth models.[14] This model (the NWM for short) will be used as a benchmark case in the rest of this paper.

In the NWM, heterogeneity is defined in terms of firms. Firms use production techniques which are characterized by fixed labour and capital coefficients (a_L and a_K, respectively). Output is homogeneous, so that we have a pure model of process innovation.[15] Thus, firms produce using a Leontief production function, which does not allow for substitution between labour and capital. Over time, technical change may be biased (i.e., changes in a_L and a_K are not proportional), so that a phenomenon that resembles substitution between labour and capital may result (this is a key result in the outcomes of the model, so that we will come back to this below).

The generation of novelty occurs as a result of search activities by firms. Search is undertaken in a (given and finite) pool of existing techniques (i.e., combinations of a_K and a_L). At any point in time, some of the techniques available in the pool are known, while others remain to be found in the future. Search activities are determined by satisficing

[14] The discussion in Nelson and Winter (1982) largely focuses around an earlier article by Nelson *et al.* (1976).

[15] Gerybadze (1982) has extended the NWM to the case of product innovation.

behaviour, i.e., firms only engage in search if their rate of return falls below an arbitrarily set value of 16%. The mutation or search process may take two different forms: local search or imitation. In the first case, firms search for new, yet undiscovered techniques. Each undiscovered technique has a probability of being discovered which linearly declines with a suitably defined technological distance from the current technology (hence the term *local* search). By varying the skewness of this distance function either labour or capital bias can be introduced into the search process. In the second search process, imitation, a firm searches for techniques currently employed by other firms but not yet used in its own production process. Thus, this aspect of the search process does not generate novelty in the strict, aggregate sense. Rather, it produces novelty at the microeconomic level. The probability of success in imitation is proportional to the share in output of each technique.

Given that a firm engages in search (i.e., that its rate of return is smaller than 16%), it can only engage in one type of search. Which type of search is being undertaken is a random event, with a fixed probability for each type. If the search process is successful, i.e., if the firm finds a new technique, it adopts this new technique only if the expected rate of return is higher than its present rate of return. Expectations are subject to error with regard to the true values of the capital and labour coefficients.

An additional source of novelty in the economy is entry by firms which were not engaged in production previously. This is conceptualized by "empty" firms, with a capital stock equal to zero, but which are active in the search process. If such an "empty" firm discovers a production technique which promises a rate of return over 16%, there is a 25% probability that it actually enters the market. If entry occurs, a value for its capital stock is drawn randomly.

The selection process is thus largely driven by the rate of return on techniques. This rate of return depends on the (real) wage rate, which is a function of exogenous labour supply and endogenous labour demand. The latter is a function of output, which, in its turn, depends on the capital stocks and the techniques currently employed. Net investment in capital is equal to firm profits (minus a fixed fraction that it must pay as dividends) minus depreciation (at a fixed rate). Insufficient profits lead to negative investment, i.e., firms which make losses see their capital stock shrink. Thus, selection takes place simultaneously on firms and production techniques, where one may think as firms as the phenotype, and techniques as the genotype.

Like most models we will discuss here, the NWM has to be simulated on a computer to obtain an impression of its implications. The model, which is calibrated for the case of the Solow (1957) data on total factor productivity for the United States in the first half of the century, yields an aggregate time path for the variables capital, labour input, output (GDP), and wages (or labour share in output). The analysis in Nelson and Winter (1982) is confined to 16 runs, in which four main parameters (the localness of innovation, the emphasis on imitation search, dividends, and the labour saving bias of local search) were varied between a high and a low state.

Nelson and Winter primarily address the question whether these time series correspond in a broad qualitative sense to the ones actually observed by Solow. Given the affirmative answer to this question, they argue at length that "it is not reasonable to dismiss an evolutionary theory on the grounds that it fails to provide a coherent explanation of ... macro phenomena" (p. 226). More specifically, it is argued that although both the neoclassical explanation of economic growth offered by Solow (as well as later work in this tradition) and the NWM seem to explain the same empirical trends, the underlying causal mechanisms between the two perspectives differ greatly:

> the neoclassical interpretation of long-run productivity change ... is based upon a clean distinction between 'moving along' an existing production function and shifting to a new one. In the evolutionary theory ... there was no production function. ... We argue ... that the sharp 'growth accounting' split made within the neoclassical paradigm is bothersome empirically and conceptually. (Nelson and Winter 1982, p. 227).

Looking below the surface of the broad qualitative resemblance between the simulation and the actual empirical data, Nelson and Winter arrive at some interesting conclusions with regard to the effects of variations in their four parameters. They find that decreasing the localness of search leads to higher values of technical change, a higher capital-labour ratio and lower market concentration. Search biased towards imitation of other firms (rather than local search for new techniques) leads to a higher capital-labour ratio, and lower concentration. Higher capital costs (dividends) lead to lower technical change and a lower capital-labour ratio. Finally, labour saving technical change leads to a higher capital-labour ratio. All of these effects (which were established by regressions on the simulation results) have some plausible explanation from the point of view of the evolutionary theory provided by Nelson and Winter.

Thus, the NWM seems to provide two sorts of outcomes. First, there is the 'mimimalistic' point of view that an evolutionary model may explain the macro facts about economic growth on the basis of a 'plausible' microeconomic theory (i.e., a theory which can account for the observed heterogeneity between firms at the micro level).[16] While this a useful result, there are at least two reasons why one should not be satisfied with it as the sole basis for further development of evolutionary growth models. First, a more 'positive approach' to scientific development would require an evolutionary theory to provide fresh results of its own and not only benchmark itself against neoclassical results, even if the latter have dominated economic discourse until now. Second, the empirical validation of the NWM is highly specific to a single dataset, i.e., the one used by Solow. After the events of the 1970s (such as the productivity slowdown, or productivity paradox), the stylized facts about economic growth that were predominant in the period when Nelson and Winter formulated their model are no longer uncontested.

The second type of result of the NWM, i.e., the relations between the four main parameters in the model and the macroeconomic predictions, can be seen as a first attempt at a more 'positive' approach. But perhaps more important than these results, which only play a minor role in the exposition, is the paradigmatic function of the model as such. As we will see below, the NWM has set the stage for a number of more elaborate evolutionary models capable of analyzing economic growth as an evolutionary process, using much more refined assumptions and model setups, and arriving at conclusions that go beyond broad similarity to the 'stylized facts' developed in the 1950s. Some of the ways in which these newer models refine the NWM concern the endogenization of the mutation and imitation process, the extension of the model to one in which firms in different countries interact, or in which there

[16] See also Nelson (1995) for an extensive argument along this line. The fact that growth accounting with an aggregate production function can lead to a deceptively high goodness of fit even with a microeconomy of heterogeneous firms inconsistent with aggregation or even absurd underlying production functions has been pointed out repeatedly in the literature. See Houthakker (1956), Phelps-Brown (1957), McCombie (1987), Shaikh (1974, 1980, 1990), Simon and Levy (1963), and Simon (1979).

are input-output relations between firms. The common roots of most of these models in the NWM is evident, however.

Evolutionary 'macro models'

Perhaps the most important aspect of the NWM is its explicit microeconomic foundation. As was argued above, this seems to be the basis for the most important conclusion regarding the outcomes. Among the evolutionary growth models inspired by the NWM, there are models with an explicit microfoundation, but also models formulated only at the macroeconomic level. The discussion here will start by outlining the latter category. The microeconomically founded models will be discussed in the following subsection. The models considered in this subsection are Conlisk (1989) (CON), Metcalfe (1988) (MET), Verspagen (1993) (VER) and Silverberg and Lehnert (1993, 1994) (SL).

The first two of these models can be solved analytically, whereas the last two follow Nelson and Winter in using computer simulations for analysis. The analytically solvable models necessarily have to make extensive simplifications relative to the rich picture of the NWM that has become somewhat of a standard for the second group of evolutionary growth models discussed below. In the case of the Conlisk model, these simplifications go so far that it is arguable whether or not the model is still a truly evolutionary one. As will be shown below, however, some of the most important assumptions and results of evolutionary theory remain in the Conlisk model, so that we have no hesitation to discuss it here alongside other models. The abstractions necessary to yield analytical solutions should not be regarded in a dogmatic way leading to the exclusion of these models from the evolutionary category. It is in the interests of the discipline to explore the boundaries of what is analytically possible while at the same time exploring more complex models by means of simulation techniques.

The assumptions on the role of heterogeneity in the CON, MET and SL models are quite similar. In all three models, production techniques are the most basic entities. These techniques differ with respect to their technological levels, for which labour productivity is the sole indicator. This is the main source of the heterogeneity on which selection operates. In the VER model, heterogeneity occurs between sectors within countries, i.e., the sector is the smallest unit of analysis. Sectors differ with regard to the product they produce, which might have different income elasticities in different countries, and also with regard to labour productivity, as an indicator for technology.

The way in which novelty is generated varies the most in the models in this group. The simplest approach is found in the MET model, where novelty is assumed to be absent. To keep the model tractable, the analysis is confined to the selection process operating upon a given set of techniques. Only a little more advanced is the assumption in VER, where technical progress is purely deterministic, and specified in the form of a 'Kaldor-Verdoorn' type of process, which stresses learning-by-doing and dynamic scale economies. Basically, a higher output growth rate leads to faster productivity growth, although the 'returns' to output growth in this process are diminishing.

More squarely in the evolutionary tradition are the novelty generating processes in CON and SL. In these models, a stochastic mechanism is at work in which new techniques are generated from a random distribution. In CON, this is a normal distribution of labour productivity increments with a positive mean, whereas in the SL model, innovations arrive according to a time-homogeneous or inhomogeneous Poisson distribution. In the latter case, whenever an innovation occurs, the new production technique is assigned a labour productivity equal to (1+a) times the prevailing best practice technique, where a is an endogenously fixed constant.

Selection is crucial in all models in this group. In this case, the simplest representation is provided by the CON model. Here, there is a ranking of techniques according to their productivities.[17] At any point in time, the search process is based upon the first n techniques in this ranking: the mean of the distribution from which new techniques are drawn is a weighted mean of these first n techniques. This means that "[s]ince new plant technology will build on the innovative plants from the past rather than on the average plants of the past, productivity will grow. In the absence of randomness, all plants would be alike; hence there would be no innovative plants to induce growth. Thus, randomness is essential." (Conlisk 1989, p. 794).

In the VER model, the selection mechanism is represented by a replicator equation in which sectors from different countries compete with each other on the basis of production costs (profits are assumed to be zero). Production costs are a function of the technological level of the sector, the wage rate, and the exchange rate. Wages depend upon productivity growth and the unemployment rate, and exchange rates adjust slowly to achieve purchasing power parity between nations in the long run. There is no explicit economic basis for the replicator equations other than a short reference to the idea that consumers (in the absence of quality differences between producers) prefer those products with the lowest price, and that adjustment to these long-run preferences is slow. At the aggregate level, selection in the VER model is a function of sectoral shares in total consumption, which evolve according to different real-income elasticities in different countries.

Finally, the selection mechanisms in the SL and MET models are quite similar. In these models, the replicator mechanisms result from explicit economic theorizing. In both models, profits are the driving force for selection. Confronted with economy-wide wage and output price levels, techniques with different levels of labour productivity will yield different profit rates. The assumption is that profits are reinvested in the same technique[18], so that the share in productive capacity of techniques with above-average productivity increases.

In the SL model, real wages are a function of the unemployment rate, and effective demand does not play a role (production is always equal to productive capacity). This leads to a model which is essentially a multi-technique version of Goodwin (1967). This model, in turn, is the standard economic example of a predator-prey model, and yields the same outcome as the original Lotka-Volterra model. In the MET model, nominal wages are given, while the price of output is found by confronting demand and supply. The demand curve is given exogenously, whereas the supply curve is found by aggregating over the different production techniques, which are assumed to supply all their output at the cost level determined by the wage rate and labour productivity. The price of output is found at the intersection of the demand and supply schedules. All techniques with cost levels higher than

[17] Conlisk's techniques can be ranked in two different ways. The first is by means of their actual productivities at any point in time. Because labour productivity partly depends on capital depreciation in this model, the labour productivity of a technique varies over its lifetime. Techniques can also be ranked on the basis of their productivities at the time of invention. This is the relevant way of ranking in the rest of the discussion here.

[18] In fact, in the SL model, a certain fraction of profits is redistributed towards the more efficient techniques. Hence, more advanced techniques attract a more than proportional share of total profits. This is, however, not essential to the working of the selection process, although it tends to speed up selection.

the current output price are assumed to be scrapped from the market. New techniques enter at the lower end of the supply schedule, and thus achieve high profit rates.

Despite the similarities in model setup in this broad group of 'aggregate' evolutionary models, there is not much similarity between the outcomes of the different models. Under the assumption that technology advances are indeed random (see above), Conlisk shows that the growth rate of the aggregate CON economy is a function of three variables: the standard error of the productivity distribution of new plants (which can be interpreted as the average innovation size), the savings rate (which is defined somewhat unconventionally), and the speed of diffusion of new knowledge. Moreover, by changing some of the assumptions about the specification of technical change, the CON model emulates three standard specifications of technical change found in growth models in the neoclassical tradition. In this case, the first and third factor no longer have an impact on growth (they are specific to the 'evolutionary' technical change specification of the model). However, the impact of the savings rate can be compared between the various model setups. Conlisk finds that using purely exogenous technical change (as in the Solow model), or learning by doing specifications as in the model by Arrow (1962) or Romer (1986), the savings rate does not have an impact upon (long-run) economic growth. This result, which is in fact also well known from standard neoclassical growth theory, marks an important difference between these models and his more evolutionarily inspired specification.

The other analytical model discussed in this section, the MET model, does not aim at deriving such specific results. Instead, the aim seems to be to provide an exposition of the workings of a possible selection mechanism on the growth pattern of an open economy. Due to the many simplifying assumptions that are necessary to arrive at an analytical solution (such as the constancy of countries' shares in world demand, and fixed nominal wages and exchange rates), it is not easy to link the results to actual empirical trends. Nevertheless, the model clearly shows how a country's share in world demand and its technological level shape the interaction between the trade balance and the growth rate of the economy. The model is thus clearly one in which growth depends on openness and competitiveness of the economy. The long-run outcome of these forces is that the share in world production of the technologically most advanced country tends to one, although production in the more backward country may still be positive. Moreover, applying comparative statics, the model predicts the effects of events such as currency devaluations or protective tariffs.

The VER model can be seen as an attempt to analyze the same issues as in the MET model, but here the emphasis is more on the long-run dynamics of technical change, wages and the exchange rate, rather than on the adjustment process. Verspagen uses simulations to analyze the effects of differences in technological competence between countries, or differences in demand patterns between countries. Because the model is multi-sectoral, endogenous specialization patterns arise, and countries' technical performances depend upon their specialization. These differences in technological competitiveness in turn have an effect upon unemployment and the wage rate, which again feeds back upon competitiveness. In essence, this model highlights the interaction between specialization and growth, and the outcomes show that in a world in which there are differences between technological potentials of sectors and countries, growth rate differentials between countries may be persistent, although not exactly predictable (due to the nonlinear nature of the model).

The SL model predicts a complex pattern for the rate of technical change in which long-run fluctuations of a $1/f^{\alpha}$-noise character dominate, although the stochastic input is simple white (Poissonian) noise. The time series for technical change and growth generated by their simulations are analyzed by means of spectral analysis, in order to decompose them into harmonic oscillations of various frequencies. The result is a downward sloping linear curve in a plot of the log of spectral density *vs.* the log of the frequency of the oscillations, known as $1/f^{\alpha}$-noise, and is interpreted by Silverberg and Lehnert to be a form of long or Kondratiev waves which are neither strictly periodic nor a random walk. In fact, they show that these series have characteristics of deterministic chaos, allowing more precise short-term prediction than a random series would warrant. They term this finding 'evolutionary chaos'. Moreover, technological replacement shows the same robust pattern of successive logistic diffusion into and out of the economy as has been repeatedly revealed in the empirical literature.

Summarizing, perhaps the most important common factor in these models is the role of technological differences between sectors, technologies or countries. These differences are continually modified by a selection process which, no matter how specified, is the driving force for economic growth in all four approaches. It is clear that although these models share a number of general evolutionary principles in their approach to the issue of economic growth, there is no standard set of assumptions, nor does a common set of results emerge.

Evolutionary 'micro models': in the footsteps of the NWM

We continue our discussion of recent evolutionary models of economic growth by considering a number of models resembling the original NWM in the sense that they are rooted in an explicit microeconomic theory of firm behaviour. Once again, the discussion will be organized around four themes: heterogeneity, the generation of novelty (mutation), selection, and the economic outcomes of the analysis. We will discuss models by Chiaromonte and Dosi (1993) (CD), Dosi *et al.* (1994) (DEA) and Silverberg and Verspagen (1994a,b) (SV).

All three models follow the NWM in assuming that technological differences are the prime source of heterogeneity between firms. They also follow the NWM in adopting process innovation as the sole form of technological progress, and thus use the labour and capital coefficients to characterize technology. SV adopt the formalism for dealing with capital-embodied technical change (which we termed the *quasi-vintage* structure above) from SL and assume that each firm may apply a number of production technologies at any point in time. In the CD and DEA models, a firm is characterized by a single labour coefficient. DEA explicitly take an open economy perspective with firms operating in different sectors and different countries (characterized primarily by different labour markets and exchange rates). The firms are located in a home country, and when they serve a market in a different country, the flow of goods is counted as exports.

All three models potentially allow for a second source of heterogeneity in the form of behavioral differences between firms. In SV, these behavioral differences are the R&D strategies, whereas in DEA and CD, the firm strategies may also extend to decisions on price setting (markups), although no systematic study of the effects of heterogeneity or of selection on these strategies is undertaken. In CD, the pricing strategy is based upon demand expectations, which may also vary between firms. The firms in these models are thus characterized by their technological capabilities (in the form of input coefficients), and by economic strategies, which determines how much resources they invest in the search for new technologies, or how they price their products.

In the NWM, local search and imitation were the two means by which firms could generate novelty. This is where the newer models discussed here start expanding on the original NWM approach. In CD, the search process takes place in a complicated two-dimensional space. One dimension in this space corresponds to 'typologies', or 'technological paradigms', and is formally defined as the labour coefficient of producing a unit of productive capacity of a certain type. Within each of these typologies, the labour coefficient for producing a homogeneous consumption good by means of the unit of productive capacity defines the other dimension in the two dimensional space. In CD, firms either produce 'machines' (each of which is characterized by a set of coordinates in the two dimensional plane), or they produce consumption goods (i.e., they use machines as inputs). The evolution of the plane itself, as well as the specific trajectories realized by individual firms in the plane, is a complex stochastic process depending on a number of assumptions with regard to the cumulativeness of technology as well as the realized history of the model.

In DEA, the search space is more similar to the one in the NWM, with the probability of an innovation depending on R&D employment, and the productivity improvement in the event of an innovation also being a random event. In CD, the innovation process differs between the two sectors in the economy. In the first sector, which produces capital goods, the success of innovation is determined by a similar stochastic procedure to DEA, i.e., success depends on the number of R&D workers. When successful, the new capital good's productivity is drawn randomly. In the consumption goods sector, firms possess a skill level for each available capital good type. This skill level evolves by a learning process, which has both public and private features (i.e., a firm using a certain type of capital good improves its own as well as the publicly available skill of working with this machine). Firms are not able to predict their skill level precisely but rather under or overestimate this level by some systematic value. Actual labour productivity is a function of the capital good's characteristics and the firm's skill level. Firms in the consumption goods sector maximize a function involving labour productivity, prices, and the order backlog, and thereby choose which capital good they want to use.

In SV firms may also invest in R&D, and the probability of innovation depends on their R&D effort. When an innovation is made, it is introduced as in SL. Firms that are behind the economy-wide best practice frontier have a higher probability of making an innovation (i.e., adopting the next technology, which brings them closer to the frontier but does not advance the latter itself) than would be the case if they were currently on the frontier. This reflects the diffusion of technological knowledge between firms, i.e., technological spillovers. However, they still assume that this form of technological catchup requires R&D investment of the backward firms and is thus not costless. The main difference between SV on the one hand and DEA and CD on the other hand is that the former allows for the evolution of the R&D strategies themselves, in other words, behavioral learning. In CD and DEA, a firm's R&D and price strategies remain fixed for its entire lifetime. In SV, there are actually selection, mutation and imitation processes with regard to these strategies, so that evolution takes place at two levels.[19] It is assumed that firms have a (small) probability of changing their R&D strategies every period (mutation of strategies). If this occurs, the firm adds a random increment to its present strategy, where the increment is drawn from a normal distribution with mean zero. Thus, mutation of strategies is a local process, with a low probability for the firm to make large jumps in parameter space. There is also a variable probability that a firm imitates the R&D strategy of another firm. This probability decreases with a measure of firm success, the firm's growth rate, so that laggard firms are more likely to imitate than successful ones (to reflect satisficing behaviour). Which

[19] In Silverberg and Verspagen (1995a), firms are characterized by a combination of two different R&D strategies: one targeting the R&D to total investment ratio, and one targeting the R&D to sales ratio.

firm is imitated is also a random process, with the probability of being imitated equal to market share. In DEA, the probability of innovation depends on the number of past and present R&D workers. A successful innovation increases firm-wide productivity by a random step.

Selection takes place according to a replicator process in all three models. In SV, the process is essentially the same as in SL (discussed above), which means there is a Phillips curve determining the real wage rate, and firms expand their productive capacity at a rate equal to their overall (averaged over technologies) profit rate. Thus, there is a predator-prey process in which more efficient technologies tend to extend their market share, and thus firms applying these technologies will grow faster. Exit of firms occurs whenever their market share falls below a threshold, and a new firm with random characteristics enters the place of the old firm.

In CD and DEA, the selection process is represented by a replicator equation which is not specifically founded in any theory, as in the VER model discussed in the previous section. Prices and exchange rates (in DEA) are the variables determining competitiveness in these models. Thus, technological competences (labour productivity), aggregate characteristics of the economy such as wages, as well as other behavioral variables (pricing rules) enter directly into competitiveness. In CD, competitiveness of a firm also depends on the backlog of orders (i.e., unfulfilled demand in the previous period). The market shares following from the replicator equation are translated into actual production levels by considering the size of the aggregate market, which is endogenous to the model. The total size of the market is the minimum of aggregate demand and supply.[20] Aggregate demand is found from the total wage bill (the consumer goods sector in DEA and CD), or total firm demand for machines (the capital good sector in CD).

SV provide a relatively systematic, although by no means complete, search of the parameter space of their model. They arrive at three different types of results. First, they find that for sufficiently high values of technological opportunity (which links the R&D to capital ratio of the firm to the probability of innovative success), firms tend to converge after a considerable adjustment period to a common R&D strategy.[21] In this long-run evolutionary equilibrium of the system, R&D strategies converge to well-defined values (around which the system fluctuates randomly) quite comparable to values observed in high-tech industries in advanced countries. The growth rate of the economy in this state is characterized by the same $1/f^{\alpha}$ noise pattern found in SL. Second, SV find that after initializing the economy with zero R&D strategies, convergence to the equilibrium strategy takes the economy through different growth phases. These phases are characterized by different R&D levels, growth rates, and market concentration patterns in the following sequence. The economy starts out in a low R&D, low technical progress, and near monopoly regime (with the monopoly firm being replaced by a different firm at more or less regular intervals). After passing through a state of intermediate values for all variables, the long-run R&D equilibrium is characterized by low concentration (a nearly even size distribution of firms) and a high rate of technical change. Finally, SV find that by varying such parameters as technological opportunity, R&D spillovers and mutation and imitation rates of the R&D strategies, there are systematic variations in the level of technical progress and market concentration consistent with economic intuition.

[20] Neither CD or DEA discuss very extensively what happens when supply falls short of demand.

[21] This should be compared with the analogous results from neoclassical endogenous growth theory such as Aghion and Howitt (1992).

The discussion in DEA and CD is less systematic, and does not arrive at clear-cut relations between the parameters values and the outcomes of the model. In fact, CD do not provide results for more than one particular run, and DEA provide very little information about alternative runs. Neither of the two papers provides systematic summary statistics for multiple runs, whether for different parameter sets or identical parameter sets with different random seeds.[22] Keeping the 'preliminary' nature of the results in mind, the following seems to be the main outcome of the CD model. In the Nelson and Winter tradition, they put much emphasis on the interpretation of their results as empirically plausible, yet rooted in a more sophisticated microeconomic foundation (compared to mainstream theory):

> ... one can only say that the generated series of income and average productivity seem 'plausible' (...): we conjecture that the aggregate dynamics might show econometric properties similar to those empirically observed. As with 'real business cycle models', one cannot distinguish between transitory (cyclical) and permanent (trend) components in the generated time series. However, unlike the former models, innovations do not take the form of exogenous stochastic shocks but, rather, are generated endogenously by agents themselves. (p. 56)

Thus, it seems as though the evolutionary model has evolved (from NWM to CD) as has the 'adversary mainstream' model (from the Solow model to the real business cycles and endogenous growth models), but nothing else has changed.

While DEA put some emphasis on this property of their outcomes, their main interest relates to growth rate differentials between countries. They find that for the 55 countries in the particular runs for which results are presented, there is a significant trend for GDP per capita levels to diverge. This is tested by using a linear functional form that relates the growth rates of GDP per capita to the initial level of this variable. A significantly positive slope is found. Applying a 'post-selection bias' and testing only for those countries which at the end of the period turn out to be developed, they obtain a negative coefficient (pointing to convergence), which is, however, not statistically significant.[23] Given the available empirical evidence for long time periods and large cross-country datasets, it is not clear whether this property of the simulated data is in close correspondence with reality. Most authors in the field of empirical 'convergence' have found significant convergence for a group of relatively advanced economies in the 1950-1973 period. Divergence seems to prevail in a larger sample of countries (including, e.g., the African countries). It is also clear that convergence in the relatively rich group of countries was much weaker, if present at all, in earlier periods.[24] Thus, it seems as if the DEA results are (at least partly) compatible with a particular period in time (pre-WW II), but not necessarily so with the strong post-war convergence period observed in the OECD.

[22] DEA state that "the results that we shall present appear to be robust to rather wide parameter variations" (p. 235), without presenting statistics to support this statement, or specifying how "wide" the variations actually were. CD, discussing one particular outcome, state that it "holds across most of the simulations that we tried" (p. 58).

[23] This experiment seems to be derived from DeLong's (1988) critique of Baumol (1986) who, as did many other authors before and after him, estimated the convergence equation tested by DEA.

[24] See, e.g., Verspagen (1995) for further characterizations of the convergence debate.

Evolution, history and contingency as the driving forces of economic growth: an attempt at a synthesis

Having outlined the assumptions and results of a number of contributions to 'evolutionary growth theory', it is time to ask whether this discipline has added to our understanding of the phenomenon of economic growth. We have already seen that the results of many evolutionary growth models are not very specific in the sense that they do not provide insight into exactly which factors play which role in the growth process. Compared to other approaches in growth theory, such as the neoclassical model with its highly practical 'toolbox' of growth accounting, it might seem at first glance as though not much could be learned from their evolutionary alternatives.

As was already stressed by Nelson and Winter (1982), it is indeed one aim of evolutionary models to demonstrate that the sense of precision offered by the mainstream models is to some extent illusory. The causal relationships between the main variables in these models, Nelson and Winter argue, are not so clear once one adopts a microeconomic framework in which heterogeneous firms, disequilibrium, and bounded rationality are the key ingredients. The implications of this point of view may certainly be far reaching. The importance of this argument resides perhaps not so much in the critical attitude towards mainstream theory as in the argument that models of economic growth are just not able to make precise predictions on the basis of exact causal relations. This idea is quite well illustrated by a quotation from Nelson (1995, p. 85-6):

> "There is no question that, in taking aboard this complexity, one often ends up with a theory in which precise predictions are impossible or highly dependent on particular contingencies, as is the case if the theory implies multiple or rapidly shifting equilibria, or if under the theory the system is likely to be far away from any equilibrium, except under very special circumstances. Thus an evolutionary theory may not only be more complex than an equilibrium theory. It may be less decisive in its predictions and explanations. To such a complaint, the advocate of an evolutionary theory might reply that the apparent power of the simpler theory in fact is an illusion .... Such a framework would help us see and understand better the complexity of the economic reality .... But it will not make the complexity go away."

Nelson thus seems to argue that we must simply accept as a fact of life the inability to predict and precisely explain that characterizes many of the evolutionary growth models outlined above. Although we sympathize with this line of reasoning in general, we wish to argue that there are indeed ways in which evolutionary growth theory can take up Nelson's gauntlet of 'complexity' in a more positive way. We suggest going back to an old discussion in evolutionary biology, which focuses on the interaction of 'chance and necessity'.

This debate, which was stimulated by Monod (1970), inquires into the consequences of adopting a view of evolution in which random events, such as genetic mutations, or random changes in the selection environment (such as the by now famous meteorite which supposedly led to the extinction of dinosaurs on earth) have an impact on the general characteristics of 'life as we know it'. In the words of Gould, the question is whether the biological diversity on earth would be different if 'the tape were played twice'.[25] As far as chance and contingency are concerned, the answer to this question would be a firm yes: if evolution completely depended on random events, a literally infinite number of natural histories would be possible, and there is no reason why any of them would turn up more often than others in imaginary experiments.

Applying the analogy to economics and the history of technology, the question is if the tape were played twice, would textile innovations and mechanical power be the technological stimulus for an industrial revolution, and, if so, would England again be the place of origin of such a revolution? Taking this reasoning a step farther, would a Great Depression always occur, and would the equivalent of the USA always surge to economic and technological dominance, inducing a period of sustained catch-up and convergence in part of the world after the second World War?

The economic historian's explanation for such events rests on specific historical circumstances not obviously connected to a more general causal mechanism extending across time periods. For example, Maddison (1991) points to specific institutional and policy factors that led to a succession of growth phases in the modern world since 1820. Although Maddison does not discuss the causal mechanisms underlying these factors at great length, it is obvious that there is a considerable degree of contingency associated with these factors, making them hard to explain from an economic point of view.

However, the biological discussion also highlights the role of more systematic factors in the evolutionary process, suggesting the hypothesis that some 'histories' are more likely than others. Taking this argument even further, Fontana and Buss (1994), on the basis of simulation experiments, have argued that there are certain characteristics of biological life that seem to be generic and robust to different randomizations of the model. They argue that "these features ... might be expected to reappear if the tape were played twice" (p. 757) . Hence the dual relationship between 'chance' and 'necessity', which leads to a world view in which there is considerable uncertainty with regard to exact outcomes and causal mechanisms, but in which there is also some limit to the randomness of history. Thus the basic SV model and its derivatives point to a definite value of R&D, and distinct preferences for particular strategic routines over others, as an emergent outcome of this process of chance and necessity. In fact, the stochastic component of learning models can actually reduce the number of possible outcomes as compared to the equivalent deterministic one (Foster and Young 1990, Young 1993).

The evolutionary growth models we have discussed almost all rely upon stochastic technical change as the driving force of economic growth. In many of the models, one outcome of this stochastic process after a selection process has acted upon it is that a wide range of 'economic histories' are possible, some of which seem to be compatible with the 'stylized facts' of actual empirical observations. While these results are often used to argue the 'minimalist' position that an evolutionary theory can explain the phenomena explained by mainstream theory but with a more realistic (Nelson 1995, p. 67) microeconomic foundation, we wish to argue that this approach should be extended along the lines suggested by the debate on 'chance and necessity' in biology.

Viewed this way, evolutionary growth models would have to be more precise on the possible range of outcomes they predict, by outlining the general features of the histories

[25] See Fontana and Buss (1994) for a discussion of Gould's question in the context of an abstract evolutionary model of self-organization.

generated in the simulation experiments. For example, in a model of international growth rate differentials as suggested by DEA, the main question would be under what circumstances a fairly 'narrow' bandwidth of outcomes would exist, for example in the sense of a small range of values for the coefficient of variation of per capita GDP in the different countries. Such an approach would admittedly not help us much in understanding specific events in economic history. It would not give us an answer to the question why the industrial revolution took place in England, or why the productivity slowdown occurred in the mid-1970s. However, given the inability of evolutionary theory to identify clear-cut causal mechanisms explaining these facts, it would certainly provide a powerful tool of analysis, which would take the field a step ahead of the currently available results.

In an extension of the SV model to the international economy we have taken a first step in trying to establish results along these lines (Silverberg and Verspagen 1995a). There we show that one only needs a fairly simple set of assumptions to robustly generate artificial time series of international economic and technological leadership similar to those observed empirically. We argued that this exercise, although still of a preliminary nature, shows that historical events such as the postwar catch-up boom can be seen as broadly compatible with an evolutionary model of international growth rate differentials. What is robust is not any particular sequence of events but the $1/f^{\alpha}$-noise pattern of the time series that will always generate such patterns if one waits long enough. In other words, we argue that, despite the impact of 'random' events such as US leadership over much of the 20th century, we would expect that similar patterns would have arisen had we been able to 'play the tape twice.' In order to stimulate other contributors to the evolutionary debate to take a similar perspective in the future, further work on methodological issues, such as the status of simulation experiments relative to analytical results, or the statistical evaluation of results generated by computer simulations, is obviously required.

Finally, there remains the issue of the relationship between neoclassical theory and evolutionary growth models. We have already seen that evolutionary theorists, whether old or new, tend to benchmark their results against those of neoclassical growth theorists. Following the logic of the above debate on 'chance and necessity', we would argue that the usefulness of comparing the two perspectives is not very high. The possible directions for evolutionary theory we have emphasized imply that the results of evolutionary simulation models would be of a different class than those derived from conventional models. Just as Newtonian mechanics remained useful after the development of the theory of relativity, the sort of evolutionary results we have in mind would definitely have to say something about the circumstances in which neoclassical predictions are useful, but they would also paint a broader picture in which the role of historical contingencies in the process of economic growth on the one hand, and specifically evolutionary invariant features on the other, would be highlighted.

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