Working Paper

An Evolutionary Model of Long Term Cyclical Variations of Catching Up and Falling Behind

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WP-95-09
February 1995

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Preface

The research project on *Systems Analysis of Technological and Economic Dynamics* at IIASA is concerned with modeling technological and organisational change; the broader economic developments that are associated with technological change, both as cause and effect; the processes by which economic agents – first of all, business firms – acquire and develop the capabilities to generate, imitate and adopt technological and organisational innovations; and the aggregate dynamics – at the levels of single industries and whole economies – engendered by the interactions among agents which are heterogeneous in their innovative abilities, behavioural rules and expectations. The central purpose is to develop stronger theory and better modeling techniques. However, the basic philosophy is that such theoretical and modeling work is most fruitful when attention is paid to the known empirical details of the phenomena the work aims to address: therefore, a considerable effort is put into a better understanding of the 'stylized facts' concerning corporate organisation routines and strategy; industrial evolution and the 'demography' of firms; patterns of macroeconomic growth and trade.

From a modeling perspective, over the last decade considerable progress has been made on various techniques of dynamic modeling. Some of this work has employed ordinary differential and difference equations, and some of it stochastic equations. A number of efforts have taken advantage of the growing power of simulation techniques. Others have employed more traditional mathematics. As a result of this theoretical work, the toolkit for modeling technological and economic dynamics is significantly richer than it was a decade ago.

During the same period, there have been major advances in the empirical understanding. There are now many more detailed technological histories available. Much more is known about the similarities and differences of technical advance in different fields and industries and there is some understanding of the key variables that lie behind those differences. A number of studies have provided rich information about how industry structure co-evolves with technology. In addition to empirical work at the technology or sector level, the last decade has also seen a great deal of empirical research on productivity growth and measured technical advance at the level of whole economies. A considerable body of empirical research now exists on the facts that seem associated with different rates of productivity growth across the range of nations, with the dynamics of convergence and divergence in the levels and rates of growth of income, with the diverse national institutional arrangements in which technological change is embedded.

As a result of this recent empirical work, the questions that successful theory and useful modeling techniques ought to address now are much more clearly defined. The theoretical work has often been undertaken in appreciation of certain stylized facts that needed to be explained. The list of these 'facts' is indeed very long, ranging from the microeconomic evidence concerning for example dynamic increasing returns in learning activities or the persistence of particular sets of problem-solving routines within business firms; the industry-level evidence on entry, exit and size-distributions – approximately log-normal – all the way to the evidence regarding the time-series properties of major economic aggregates. However, the connection between the theoretical work and the empirical phenomena has so far not been very close. The philosophy of this project is that the chances of developing powerful new theory and useful new analytical techniques can be greatly enhanced by performing the work in an environment where scholars who understand the empirical phenomena provide questions and challenges for the theorists and their work.

In particular, the project is meant to pursue an 'evolutionary' interpretation of technological and economic dynamics modeling, first, the processes by which individual agents and organisations learn, search, adapt; second, the economic analogues of 'natural selection' by which interactive environments – often markets – winnow out a population whose members have different attributes and behavioural traits; and, third, the collective emergence of statistical patterns, regularities and higher-level structures as the aggregate outcomes of the two former processes.

Together with a group of researchers located permanently at IIASA, the project coordinates multiple research efforts undertaken in several institutions around the world, organises workshops and provides a venue of scientific discussion among scholars working on evolutionary modeling, computer simulation and non-linear dynamical systems.

The research focuses upon the following three major areas:

1. Learning Processes and Organisational Competence.
2. Technological and Industrial Dynamics
3. Innovation, Competition and Macrodynamics

1. Introduction

One of the most popular themes in the applied literature on growth during the last decades has been the issue of convergence in GDP per capita. Numerous contributions have stressed the (post World War II) tendency for GDP per capita to converge[1]. In its crudest form, this convergence reveals itself by means of significant negative correlations between initial GDP per capita levels and their subsequent growth rates. What seems to stand out in the debate so far is the particularly strong tendency for convergence during the 1950-1975 period in the OECD area. It is also widely known that countries outside the OECD-area have been much less successful in catching up to the world GDP per capita frontier[2]. Most developing (African) countries have seen the gap between themselves and the developed world increase significantly during the postwar period. Newly Industrializing Countries such as The Republic of Korea, Taiwan, Singapore and Hong Kong are known exceptions to the general rule that countries which are too far behind will not be able to catch up to the frontier.

The particularly strong convergence over the 1950-1975 period seems to have led to a consensus on the hypothesis that extraordinary catching-up forces were at work during that period. Abramovitz (1994) gives an overview of the most likely candidates for such factors. This interpretation of the postwar growth pattern seems to emerge from a broader view that (economic) history is dominated by specific, period related phenomena, which are not to be generalized into more general tendencies characterizing longer periods, such as the period after the industrial revolution as a whole. Maddison's (1991) interpretation of the post 1850 growth pattern as a succession of specific 'phases of growth' is an example of such an approach.

While we are not unsympathetic towards the argument that 'history matters', or that specific historical circumstances are important for explaining observed growth patterns in the world, we nevertheless wish to argue that catching up and falling behind patterns can be interpreted in a more general setting than the one advocated by the 'historical school'. There are at least two interpretations of history that would support such as view. One is the idea of 'stages of growth', as proposed for example by Rostow (1960) and Gomulka (1971). In this view, a country's growth pattern is predicted to go through different stages, which are linked in a specific order by certain factors that are believed to underlie the process of economic growth. Gomulka (1971) interpreted catching up as one specific stage in such a process, which had to be preceded by a stage of building up of (imitative) capacity.

A second approach to history as an interconnected sequence of periods is the literature on long waves in economic development. Pioneered by Van Gelderen and Kondratiev, the idea of long waves was given an explicit theoretical basis in the theory of technological innovation by Schumpeter (1939). The long wave hypothesis predicts that the history of economic development is a sequence of upswings and downswings. The actual existence of long waves is a controversial topic, mainly because of numerous technical difficulties with regard to statistically testing the hypothesis of long-term variations in the rhythm of economic growth, unemployment, prices, or other economic variables. The recent application of filter techniques by Metz (1992) seems to provide the state-of-the-art with regard to the

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[1] See for example Cornwall (1977), Abramovitz (1979), Baumol (1986), Fagerberg (1988), Fagerberg, Verspagen and von Tunzelmann (1994).

[2] See for example DeLong (1988) and Verspagen (1991).

measurement of long waves. With regard to Schumpeter's idea that each new long wave is fuelled by a sequence of major technological breakthroughs, Mensch (1979), Kleinknecht (1987), Freeman Clark and Soete (1982), and Silverberg and Lehnert (1993) have discussed the empirical evidence for such an interpretation.

In an attempt to integrate the theories of long waves and catching up, Lundvall (1989) takes a somewhat intermediate position on the question of specific vs. general factors explaining historical growth patterns. From a Schumpeterian point of view, he argues that profit rate differentials are a general cause underlying the cyclical development of capitalism, and that profit rate differentials between countries after World War II were one factor causing a major upswing in the world economy. Thus, although Lundvall recognizes a general phenomenon underlying long-run development over different historical periods, he nevertheless points to a number of very specific factors underlying economic development during the postwar period.

A similar argument, but applied to shorter 'Kuznets cycles', is advocated by Solomou (1987). He finds evidence for approximately 20-year period cycles in international variables such as the terms of trade and international investment flows. His interpretation of this finding is that international capital movements react to international differences in profitability, which displays a wave-like pattern. When it comes to the postwar catch-up boom, however, Solomou (1987: 161-3) strongly supports the 'historical' interpretation pointed to above.

In this paper, we wish to put forward an interpretation of the long-run variations in catch-up patterns along the lines of these more 'general' interpretations of economic history. To this end, we apply a model which is an extension of the model in Silverberg and Lehnert (1993). In the latter paper, it was shown that randomly distributed innovations in an evolutionary context can produce patterns of technical change (or economic growth) in which long-term fluctuations are prominent. In the next section, we will discuss the specific interpretation of this result, which readily extends to the model in this paper, in a long wave context. As a result of treatment of R&D as an endogenous engine for economic growth, the present model also supports a 'stages of growth' interpretation of economic growth[3]. A more detailed overview of the structure and results of the present model, although in a single country case, is given in Silverberg and Verspagen (1994a,b).

Thus, although in a stylized way, our model stresses both aspects of 'analyzing economic history in a general setting' discussed above. In the present paper we generalize these interpretations from the single country context to the case of multiple economies in interaction with one other in a world economy. In doing so, we wish to focus on the long wave interpretation of the model's outcomes, and investigate to what extent these long-wave properties extend to the multiple country context when there is technological catching up between countries.

On the basis of the model we will argue that although specific historical circumstances (not likely to be covered by any model) might play a large role in explaining the historical peculiarities of international growth, there might nevertheless be a complex interplay between a limited number of general factors that allow for seemingly unique features such as the strong convergence era during the postwar period. The model we use to analyze this 'complex interplay' is admittedly simple, with many abstractions necessary to keep it tractable. Perhaps the most limiting assumption made is that interactions between countries are restricted to technological and behavioral imitation only, and that purely economic interactions such as

[3] See Silverberg and Verspagen (1994b) for a detailed account of this aspect of the model.

trade, foreign direct investment and exchange rate movements are not taken into account.

The rest of this paper is organized as follows. In section 2, the empirical stylized facts on long-term trends in convergence are discussed. This section also introduces the specific interpretation of long waves that comes out of the paper by Silverberg and Lehnert (1993), and that seems to be reasonably well supported by the data on long-term catching up trends. Section 3 sketches the structure of the model. Section 4 presents the analysis of the model, which is done by simulation techniques. The last section summarizes the results and draws some conclusions.

2. Catching up and falling behind: the long-term stylized facts

There is no need to recapitulate the empirical facts on catching up and falling behind in great detail here, as they are by now well-known and extensively documented in the references cited above. Therefore, we will limit the discussion to particular aspects of the empirical evidence that are also addressed by our model. Figure 1 presents the log of the ratio of GDP per capita in the United States to that of six European countries for the period 1870-1989. The data are taken from Maddison (1991), and are probably the longest time series available for this large a sample of countries[4].

Several conclusions emerge from the figure. First, it is clear that taken over the whole period, there is no strong trend in any of the series. Second, there are wide fluctuations in the ratios for each country, although there is no clear cycle of any fixed length. Third, the postwar period indeed seems to be a rather exceptional one, although it is also clear that a large part of the actual catchup after 1950 was only possible because the war itself created such a huge gap in the first place. At the end of the period, the gaps, as measured by this specific indicator, were only somewhat lower than those existing at the end of the 1930s.

To what extent does Figure 1 support an interpretation of the history of catching up over the last century as a process subject to some general tendencies, as opposed to a more specific interpretation of the postwar period? One way of investigating this question is to use the data in Figure 1 for a spectral analysis, and check the results in order to see if any regularities arise in the power spectrum of the series[5].

Figure 2 is a double log plot of the country-ratio spectral densities against frequency (in units year^{-1}). The figure demonstrates an extraordinary regularity across countries. All power spectra are downward sloping and approximately linear over the range 0.02-0.2, corresponding to cycles of period 5 to 50 years. Such a linear downward sloping power spectrum has been termed $1/f$ noise (or more properly $1/f^{\alpha}$ noise, where $-\alpha$ is the slope and f is frequency) to indicate the power law relationship between spectral density and frequency[6]. Below a frequency of 0.02 the spectrum levels off abruptly.

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[4] GDP is measured in 1985 US$ using purchasing power parity conversions. Although Maddison (1991) provides data for nine more countries, we focus on the subset of only six (Belgium, Germany, France, United Kingdom, Italy and Sweden) for which the time series go back to 1870 in order to obtain the longest possible dataset.

[5] Spectral analysis is a technique which decomposes a time series into individual harmonic components and estimates the contribution of each to the overall motion (the spectral density or power). We employed the SPSS for Windows 6.0 Package with five Hamming windows in all of the following spectral plots.

[6] For an elementary introduction see Takayasu (1990).

Silverberg and Lehnert (1993) have interpreted the $1/f$-noise processes they found in the time series generated by their model (which is essentially a simpler version of our model) as a form of long waves dynamics. Although many long wave theorists have explicitly pointed to the possibility of long waves being combinations of cycles of different periodicities[7], this interpretation is still rather unusual in the literature. And in contrast to the results from the Real Business Cycle and some of the endogenous growth literature discussed below, it is distinct from a random walk.

Figure 2 suggests that this specific interpretation of long waves, or rather, low frequency periodic motion, is not only relevant for time series of individual countries (as in Silverberg and Lehnert, 1993), but also applies to variables describing relative growth patterns. In order to validate the results in the figure against a different measure of relative growth in our sample of countries, we also performed a spectral analysis of the coefficient of variation of GDP per capita. This variable, which is common in the debate about convergence[8], is defined as the ratio of the standard deviation to the mean of per capita GDP in the sample of countries[9].

One advantage of this measure is that it does not rely upon the definition of the frontier as one particular country. The choice of the USA as the frontier implies that the data for GDP per capita in this country is used for each of the series in Figure 1, so that one might argue that peculiarities in the US growth patterns caused the specific outcome of $1/f$ noise, and its interpretation as long waves, in Figure 2.

Figure 3 presents the power spectrum for the coefficient of variation. It is clear that for this alternative definition of convergence, the presence of $1/f$ noise also stands out. This leads us to the conclusion that this feature of the time series is a rather strong one, allowing the finding to be interpreted as a 'stylized fact' of convergence, to be added to the long list of such facts already identified in the literature. We must note, however, that we do not have sufficient data to test generalizations of this phenomenon to other sets of countries[10].

3. An evolutionary model

What are the prospects of a theory trying to explain catching up and falling behind as a long wave phenomenon? Standard growth theory tends to explain convergence of per capita incomes by initial differences in the capital-labour ratio which, combined with the standard assumption of decreasing marginal returns to capital, leads to the conclusion that backward countries (with little capital relative to labour) have high growth opportunities. The immediate implication is global convergence, a conclusion that has been criticized from many points of

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[7] For example, Schumpeter (1939) extensively discussed the complex time series resulting from superimposing a number of cycles of different period (usually referred to as quasi-periodic behaviour in the dynamical systems literature).

[8] Dollar and Wolff (1993), for example, make frequent use of this measure.

[9] It is also rather common to define the coefficient of variation as the inverse of the measure used here. Our definition corresponds to the measure applied in Figure 1, where a downward trend indicates convergence.

[10] We did a spectral analysis on the data for 6 Latin American countries in Hofman (1993). Although the time series were 30 years shorter than the ones used above, the power spectrum for the coefficient of variation in this sample showed a clear negative slope over its range of relevant frequencies. These results are not documented out of space considerations, but are available from the authors on request.

view.[11] This critique, and the mostly linear structure of the standard growth models, makes mainstream growth theory an unlikely candidate for explaining the complex *1/f* noise pattern found above.

A better candidate might be found in those new growth theories that try to combine growth and cycles, as for example in the 'neo-Schumpeterian' growth literature à la Aghion and Howitt (1992) and Cheng and Dinopoulos (1992, 1994). The models in these papers generate cyclical growth paths usually of a random walk nature. Moreover, these models rely upon intertemporal equilibrium, perfectly rational behaviour and rational (technological) expectations that do not seem convincing in the context of such complex and uncertain phenomena as technological change.[12] We therefore choose to extend an earlier model of ours, which shares some concepts with the 'neo-Schumpeterian' endogenous growth literature (such as the stochastic representation of innovations), but tries to remedy some of the objections related to the above-mentioned assumptions.

While our model dispenses with such traditional neoclassical concepts and opts instead for a disequilibrium, Schumpeterian approach based on bounded rationality, it nevertheless makes a number of drastic simplifications. For example, it neglects the role of 'social capabilities' in catching up[13]. It assumes that technology spillovers and behavioral imitation are the only forms of interaction between countries, at the expense of such economic factors as international trade, capital and labour mobility and exchange rate movements. We therefore see this model as the 'Model T' of a range of evolutionary models dealing with economic growth and endogenous technological change in an international context. It should be understood as an experiment to explore the minimal set of assumptions necessary to generate complex dynamic phenomena such as the *1/f* noise found above.

The macroeconomic heart of the model is a coupled investment and wage formation mechanism. Let hats above variables denote proportional growth rates, w be the (real) wage rate, v the employment rate (persons employed as a fraction of the labour force), and m and n parameters (both positive). Then the wage rate in each country[14] is determined by the following differential equation:

$$\hat{w} = -m + nv. \tag{1}$$

It is assumed that there is a fixed number q of firms in each of the u countries, while each of these firms has a variable number p_q of different types of capital goods that it utilizes to produce a homogeneous product. New capital arises from the accumulation of profits, a process described by the following equation:

[11] Romer (1986) and Mankiw, Romer and Weil (1990) are examples from within the neoclassical tradition of endogenous growth theory, while Verspagen (1991) discusses the issue in an evolutionary framework and Amable (1993) in a post-Keynesian one.

[12] This critique, which is shared by most scholars in the evolutionary tradition (e.g., Dosi, 1988), has been discussed in more detail in earlier papers, such as Silverberg and Verspagen (1994a,b).

[13] See Abramovitz (1979, 1994) for a historical and institutional analysis of this phenomenon, and Verspagen (1991) for a more model-oriented approach.

[14] In order not to complicate the equations too much, we suppress subscripts indicating countries. Wherever we do not explicitly define variables as firm-, technology- or world-level, all variables are defined for countries.

$$\hat{k}_{ij} = (1-\gamma_i)r_{ij} + \alpha(r_{ij}-r_i) - \sigma. \tag{2}$$

The capital stock is denoted by k, r stands for the profit rate, and σ is the exogenous rate of *physical* depreciation of capital (technological obsolescence is an endogenous component of the model itself). The subscript i $(1..q)$ denotes the firm, and j $(1..p_q)$ the type of capital (the absence of any these indices indicates an aggregation over this particular dimension). Eq. (2) assumes that the principal source for type ij-capital accumulation is profits generated by ij-capital. This is modelled by the first term on the rhs of (2), i.e., $(1-\gamma_i)r_{ij}$. A firm-specific portion of profits (denoted by γ_i) is used for the development of knowledge (R&D). R&D is restricted to cases where there are positive firm profits (i.e., when $r_i<0$, γ_i is set to zero).

However, profits may also be redistributed in such a way that more profitable types of capital accumulate even faster, less profitable even slower, than would otherwise be the case. The mechanism used to model this was first proposed by Soete and Turner (1984), and is represented by the second term on the rhs of eq. (2). By changing the value of α, redistribution of profits takes place faster (larger α) or slower (smaller α).

It is assumed that each type of capital is characterized by fixed technical coefficients, c and a (for capital coefficient and labour productivity, respectively). The capital coefficient is assumed to be fixed throughout the economy (and time), while labour productivity is assumed to change under the influence of technical progress. The profit rate of ij-capital is then given by $(1-w/a_{ij})/c$.

The principal variable used to describe firm dynamics is the share of the labour force employed on each capital stock. Production is assumed to be always equal to production capacity (the influence of effective demand is absent), so that the amount of labour employed by each capital stock is equal to $k_{ij}/(a_{ij}c)$. Dividing this by the labour force (assumed to grow at a fixed rate β) gives the share of labour employed, v_{ij} (called the employment share hereafter). The expression for the growth rate of this variable is

$$\hat{v}_{ij} = \hat{k}_{ij} - \beta = (1-\gamma_i)r_{ij} + \alpha(r_{ij}-r_i) - (\beta+\sigma). \tag{3}$$

R&D also has an employment effect. We assume that the ratio between R&D expenditures and R&D labour input is equal to a fraction δ of the economy-wide labour productivity. The employment rate v_q resulting from production is then found by summing v_{ij} over i and j. Under these assumptions, it can then be shown that the overall employment rate v is equal to $(1+\delta\gamma(1-w/a))v_q$.

Eqs. (1) and (3) together create a selection mechanism in our artificial economy. Eq. (3) describes how more profitable (i.e., with above-average labour productivity) technologies tend to increase their employment share, whereas more backward (below-average) technologies tend to vanish. The real Phillips curve eq. (1) ensures that real wages tend to track labour productivity in the long run. In a situation in which new technologies are continually being introduced, this implies that all technologies, after an initial phase of market penetration and diffusion, will eventually vanish from the production system.

The introduction of new technologies is modeled by a process of endogenous R&D. Firms devote resources (R&D) to the systematic search for new production possibilities (i.e., new types of capital). The outcome of this search process is assumed to be stochastic. Each time an innovation occurs, the firm creates a new type of capital. The labour productivity of this type of capital is given by the following process:

$$a_{i,t}^{*} = (1+\tau)a_{i,t-1}^{*}, \tag{4}$$

where τ is the fixed proportional increase in labour productivity between innovations and $a_{i,t}^{*}$ is the firm-specific best practice labour productivity. The new type of capital is seeded with a small employment share (say 0.0001). In order to keep the total employment rate constant, this seed value is (proportionally) removed from the other types of capital of the innovating firm. The number of technologies employed by any given firm may vary in time.

As real wages rise over time, every technology will generate negative profits at some stage (because of its fixed labour productivity). It is assumed that these losses are financed by an equivalent decrease of the capital stock. In other words, losses imply that capital will be scrapped, and scrapped capital can be "consumed" to cover the losses. Note that for individual capital stocks, the point at which scrapping occurs lies *prior* to the point where profits are negative, due to the α-related diffusion term in eq. (2). When a technology's employment share falls below a specified (very small) value E, it is scrapped completely.

A firm's R&D activities (measured by a variable h_i, to be defined below) determine the firm's probability of making an innovation, according to a Poisson process with arrival rate ρ_i. The simplest relation is simply linear:

$$\rho_i = A h_i + \rho_{\min}, \tag{5}$$

where ρ_{min} is the (small) autonomous probability of making a fortuitous innovation without doing formal R&D, and A is the innovation function slope.[15]

The firm-specific R&D level h_i is defined to be the ratio of a moving average of firm R&D investment to its total physical capital stock. A ratio is used to normalize for firm size, since otherwise such a strong positive feedback between R&D and firm growth exists that monopoly becomes inevitable. While *a priori* it is by no means clear why the size of individual R&D effort should not directly relate to innovative success, a pure scale effect must be ruled out by the continuing existence of competition and the ability of small countries to remain or even advance in the technology race. The exponential moving average RD_i on R&D for a lag of L (or a depreciation rate of $1/L$) is given by the following differential equation:

$$\frac{d}{dt}RD_i = (\gamma r_i k_i - RD_i)/L. \tag{6}$$

Hence the firm-specific R&D level is

$$h_i = RD_i/k_i. \tag{7}$$

An innovation can be defined in a narrow or a wide sense. In the wide sense, the adoption of any technology not yet employed by a firm (or a country) is an innovation to that unit. In the narrow sense, only technologies that have never been employed before anywhere

[15] In Silverberg and Verspagen (1994a), experiments were carried out using a logistic form for this function instead of the linear one used here, and in which other firms' R&D enter the innovation probability function (i.e., there are R&D spillovers).

are considered innovations at their time of introduction. If firms innovate according to the above Poisson arrival rates in the narrow sense, however, a very considerable intertemporal externality is created, because firms' innovations always build on each other. Thus there can be no duplication of effort and, as long as firms maintain a minimal level of R&D, no cumulative falling behind.

On the other hand, once an innovation has been introduced somewhere, either in the domestic economy or elsewhere in the world, it should be progressively easier for other firms to imitate or duplicate it; it should not be necessary to reinvent the wheel. We capture this by introducing a catching-up effect. Let the labour productivity of the (world-wide) best practice technology be a^*, and the best practice technology of firm i a^*_i. Then firm i's input in the innovation probability function (h_i) will be augmented by a measure of its distance from the (world-wide) best practice frontier:

$$h_i' = h_i(1 + \kappa \ln(a^*/a_i^*)). \tag{8}$$

Thus, adopting an old innovation is facilitated for backward firms (or countries as a whole), but they are still required to invest in their technological capacity to reap these catchup benefits. Here, however, R&D efforts should be interpreted in the larger sense of technological training and licensing, reverse engineering, or even industrial espionage (all costly activities, if not as costly as doing state-of-the-art R&D).

This is the only way in which we allow for technology spillovers between countries. As already stressed above, this representation of spillovers is a very simple one[16]. We leave a more realistic model of catching up to a future paper, because we believe that, even given this simple context, some interesting conclusions are already suggested by the model.

We have also experimented with innovations in the narrow sense, but the results on strategic selection are rather ambiguous. This is not surprising, since the import of the intertemporal externality is indeed quite large. We consider the Ansatz in eq. 8 therefore to be a justifiable first formulation, since technology adoption decisions are never passive, but rather require technological efforts of the adopting firm. However, it does place too much of the burden of catching up onto R&D.

In the artificial economy modelled here, entry of a new firm occurs only as a result of exit of an incumbent firm. Exit occurs whenever a firm's employment share (excluding its R&D employment) falls below a fixed level E. While exit of incumbent firms is completely endogenous, entry only occurs in case of exit, so that the total number of firms is constant. Naturally, this feature of the model is not very realistic, as in reality entry may be independent of exit and the total population of firms may vary. However, it is not the aim of this model to describe the phenomena of entry and exit as such. Instead, the main function of entry and exit is to maintain potential variety in the population of firms while providing for firm elimination.

Whenever entry occurs, the entrant is assigned a single technology with an amount of capital corresponding to an employment share of $2E$ (the remaining employment is

[16] We have experimented with one additional formulation, only a bit more realistic than the one offered here. In this case, a firm's R&D potential was augmented by both a catch-up term relative to the world best-practice frontier, and a similar term for the domestic economy. By setting the values of the two parameters associated to this term, a situation could be created in which it is easier to catch up to the domestic frontier and harder to catch up globally. The results of this implementation are not different from the ones presented below, which adopt the simpler, global representation of catching up.

proportionally removed from other firms so that total employment remains constant). The labour productivity of this technology is drawn uniformly from the range $[(1-b)A,(1+b)A]$, where A is the unweighted mean value of labour productivity of all the firms in the economy, and b is a parameter. The values for h and γ are (uniformly) drawn from the range existing in the economy at the time of entry.

The final step in the construction of the model deals with the way in which firms change their R&D strategy parameters γ. Learning enters the picture in the form of two "genetic" operators: mutation and imitation. These are summarized below:

$$\begin{aligned} &Prob\ \Pi:\ \gamma_{it} = Min(1, Max(\gamma_{it-1} + \varepsilon, 0)),\quad \varepsilon \sim N(0, s_i), \\ &Prob\ \Pi_i^c:\ \gamma_{it} = \gamma_{jt-1},\ j(\neq i) \in [1..uq]_i, \\ &Prob\ 1 - \Pi_i - \Pi_i^c:\ \gamma_{it} = \gamma_{it-1}. \end{aligned} \tag{9}$$

With probability Π each decision period, which is set exogenously and equal for all firms, a firm will draw from a normal distribution and alter its strategy within the admissible range [0,1] (mutation). With variable probability Π^c_i the firm simply imitates the strategy of another firm. The imitation probability is partly endogenous to reflect satisficing behaviour. Only firms with unsatisfactory rates of profit with respect to economy leaders will choose or be forced (for example by their stockholders or by hostile takeovers) to adopt the strategy of a competitor:

$$\Pi_i^c = my\left(1 - \frac{y_i - y_{\min}}{y_{\max} - y_{\min}}\right) \tag{10}$$

y_i is the firm's rate of expansion of physical capital (defined as $min(r_i, (1-\gamma_i)r_i)$), $y_{\max}$ and y_{min} are the maximum and minimum values of y in the country, and μ is the (exogenously determined) maximum imitation probability. Thus, the more profitable a firm is relative to the other firms in its country, the less likely it will change its strategy by imitating another firm. The most profitable firm in the country has an imitation probability equal to zero; the least profitable the maximum probability μ.

Once a firm has decided to imitate, it randomly selects a firm to imitate. In principle, the probability of each firm being imitated is proportional to its market share (i.e., larger firms are more likely to be drawn as examples of successful behaviour). The possibility of imitating foreign firms is regulated by a parameter F, which lies between 0 and 1. In the draw for a 'partner' for imitation, the market share of each foreign (relative to the firm that is imitating) firm is multiplied by F. Thus, if F=1, foreign firms are as likely 'partners' for imitation as domestic firms (*ceteris paribus* their market shares), whereas if F=0, imitation of foreign firms is ruled out. The parameter F introduces a second behavioral interaction between countries in addition to technological catching up, namely the possibility of cross-country behavioral and organizational learning. The mechanism specified here is admittedly very primitive, since it does not consider behavioral rigidities or the costs associated with such learning. As we shall see in the following section, its effects on global behaviour are not particularly realistic, but it does show the way toward future extensions of the model.

4. Simulation analysis

We use simulation techniques to explore the outcomes of the model specified above. The set of differential equations is solved using a fixed-step, fourth-order Runge-Kutta algorithm. The innovation decisions are executed during each computational step (using Poisson arrival rates scaled by the computation time step), and when an innovation is made, the corresponding changes in initial conditions, number of equations, and coefficients are made for the next step. Mutation and imitation of strategies are only performed at fixed intervals of one "year", which may be many times the step employed in the Runge-Kutta algorithm. In the following simulations the number of countries is set to five and the number of firms to ten in each country.[17]

In previous work outlining the outcomes of a single-country version of the model, two main features were identified. First, it turns out that under quite general parameter settings, the model leads to a long-term evolutionary steady state in which firms converge to similar R&D strategies. The country-wide *ex post* value of the ratio of R&D to investment converges to a stable value, which is only subject to small random variations due to continual 'mutant firms', which are not successful in breaking away from the evolutionary stable state. Only when the innovation slope parameter A is set to a relatively low value (indicating low technological opportunities) does this evolutionary steady state break down.

Second, it was found that the model, when initialized with no R&D being undertaken by any firm, generates stages of development. The market structure (degree of concentration) plays a major role in these stages. Three stages can be differentiated. In the first stage, the market is strongly concentrated, and technical change is low. The intermediate stage implies a market that is slowly getting more competitive, and technical change goes up slightly. The final stage, i.e., the evolutionary steady state described above, is characterized by rapid technical change and a competitive market. These three 'regimes' are separated by sudden 'jumps' between the broad levels of the concentration and R&D variables.

At the individual country level, these outcomes extend to the multicountry case of this paper. We choose not to go into the question of stages of development here, however, but rather concentrate on the implications of the model in the evolutionary stable state, and investigate the properties of convergence indicators under these conditions. In order to do so, the model is initialized in a random state, where parameters are fixed to the values described in the appendix, and firm R&D strategies are drawn from a uniform distribution between 0 and 1. All simulation experiments extend over a period of 5000 'years'. In virtually all cases, the model converges to its evolutionary steady state within the first 1000 periods, so that we concentrate on presenting and analyzing the outcomes of the last 4000 years.

The first figure describing the outcomes of the model, Figure 4, summarizes the outcomes for the aggregate R&D to investment ratios at the country level, as well as the evolution of the coefficient of variation of labour productivity between the five countries. The diagrams in Figure 4 can be interpreted as follows. On the horizontal axis, the value of the parameter F is depicted, and on the vertical axis, there are bins for the observed values of economy-wide *ex-post* R&D to investment ratios, and the coefficient of variation. The shades between black and white in the diagram indicate peaks (black) or valleys (white) in the (3D) histograms arising from the simulation experiments. Thus, a black area in the diagram

[17] The results readily extend to other values for the number of countries or firms. Even in the 2-country case, the main results are preserved.

indicates that a high frequency for the corresponding value on the vertical axis in the experiments with the value of F found on the horizontal axis. The figures are based upon the last 1000 of 5000 'year' runs for five pooled runs with the same parameter values, but different random seeds. In the case of the R&D diagram, there are five observations (countries) per year.

The narrow black band in the upper diagram of Figure 4 indicates the evolutionary stable state of R&D behaviour of firms. All firms in all countries are converging to the same R&D level. Only for zero values of the F parameter, lower R&D ratios, relative to the black band, are present. In this case, firms cannot imitate foreign firms' R&D strategies, and thus in some countries, firms will not converge to the steady state. Due to the knowledge spillovers between countries (according to equation 8), however, these countries still show technical progress.

The case in which $F=0$ is also quite extraordinary with respect to the coefficient of variation of labour productivities. This variable does not show clear convergence to any particular value, due to the differences in R&D levels between countries that might arise. For $F>0$, the coefficient of variation converges to a band close to, although not equal to, zero.

Given these patterns in R&D and convergence, attention will be focused on the time series for convergence variables. From the many simulations carried out, two runs are selected in Figure 5. The top diagram in the figure gives the time series for the coefficient of variation and the log distance of country 2's productivity to the frontier, for the last 1000 'years' of a total of 5000, with F set to zero. The bottom diagram sets F to a positive value of 0.01, and depicts the same time series for the same variables

Although the time series for the two cases seem to differ somewhat in their nature and significantly in the amplitude of the fluctuations, neither of them displays a clear pattern in the sense of convergence to a unique long run equilibrium value. In both cases, long-run cyclical variations with a rather imprecise periodic nature seem to be present. Given the interpretation of the 'periods' in the model as 'years'[18], it is easily imaginable that these long-run variations, if studied within a limited 'window of observation', could be interpreted as trends, pointing to structural changes in the underlying economic process.

This interpretation, however, would be an incorrect one. The long run variations and apparent trend reversals were generated by the general, nonlinear structure of the model, which has remained constant over the whole simulation period. This impression of the time series generated by the model therefore leads us to put forward the hypothesis that the actual historical time series of convergence over the last one and a half century might be explained by general 'laws' underlying capitalist development, without having to invoke the uniqueness of one or the other period in history[19].

This hypothesis, however, is quite difficult to test, and we can therefore only explore the consequences of the model presented here a bit more, and leave further discussion of the idea to future research. Our strategy will therefore be to explore the time series in the above figures by means of spectral analysis, and compare the outcomes to the power spectra obtained in section 2. This will provide additional insight into the empirical relevance of our hypothesis.

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[18] The value of the capital-output ratio sets the time scale, while the *ex post* productivity growth rate can be adjusted to realistic values for these time units.

[19] This is not to say that we do not support the idea that there are general features of the postwar period that contributed to the observed patterns of economic growth.

In Figure 6 we have plotted the power spectrum of the coefficients of variation series shown in Figure 5 (note that the dataset now consists of 4000 rather than 1000 'years'). Both cases, although markedly different for the higher frequency spectral densities, clearly show a *1/f* noise pattern, corresponding to the empirical data presented in section 2. Obviously, there are differences between the empirical data and the simulation data, for example with regard to the actual slope of the lines. The case that the model makes should therefore not be overstated, but still the marked *1/f* noise pattern in both simulation and actual data is striking.

Figure 7 presents the power spectra for the other, individual country measures for convergence used in section 2. Again, the underlying data are the same as in Figure 5, but with 4000 'years' used for the spectral analysis. As in Figure 6, the power spectra show a clear *1/f* noise pattern, again supporting the interpretation the model provides about catching up or falling behind.

5. Conclusions

We have extended an evolutionary model of endogenous growth to the case of convergence between countries' productivity levels. The model makes very rudimentary assumptions in order to explore the boundaries of explanatory power of the broad class of formal evolutionary approaches to the topic of economic growth. Despite its simple nature, the model generates time series for convergence indicators that bear similarities to actual empirical time series for the core OECD countries over the last 120 years.

Based upon a comparison of the power spectra of the simulated and actual time series, we put forward an explanation of convergence based upon the idea that long-term variations in relative productivity levels are caused by the underlying general nature of the economic process, rather than specific structural breaks in, for example, institutional contexts. The latter hypothesis, in the form that the postwar period provided unusual circumstances facilitating rapid catching up by lagging countries, is one of the central themes of the recent literature on convergence.

We have admittedly dealt with social and organizational change in a very primitive way (i.e., by allowing for cross-country imitation mediated by the parameter F). If such imitation is introduced, long-term disparities in growth patterns nearly vanish. This is not surprising, given that behavioral imitation is modelled as possibly costless and instantaneous. We interpret this as indicating that country-specific rigidities, the delays and costs involved in accumulating the necessary human capital, and related factors do indeed need to be invoked to explain heterogeneous growth patterns and catching up (or falling behind) dynamics if a significant amount of behavioral imitation is admitted.

The model emphasizes the stochastic character of the innovation process, which leads to an uneven spread of innovations over time. Given the mechanism specified for diffusion of innovations, this uneven stream of innovation generates an uneven growth path for individual economies over time. When multiple economies are linked to each other by a simple catch-up mechanism, this leads to a rather typical pattern of convergence over the long run. In terms of spectral analysis, this pattern corresponds to the presence of *1/f* noise.

1/f noise is characterized by increasing importance of lower frequencies in explaining the overall motion of a times series. When the log of the frequency is plotted against the log of the spectral density, a linear, downward sloping curve emerges. This indicates that frequency components in the time series become more important as the frequency declines (i.e, fluctuations take place at all time scales). Following the interpretation of Silverberg and Lehnert (1993), who found the same property for individual countries, using a similar, but simpler model than ours, we interpret this as a long wave theory of convergence in the world economy. When confronted with these very long-term components of the underlying dynamics it is natural to be inclined to interpret short intervals of data as characterized by significant trends or structural breaks specific to the historical epoch observed. The alternative we have outlined here is to regard such phenomena as intrinsic components of a larger fundamental mechanism.

Although we do not want to stretch the similarities between our model and the actual data too far, we still think that the striking similarity between the empirical stylized facts and the stylized facts of our model are close enough to warrant further empirical and formal work examining our hypothesis about the general nature of observed convergence patterns.

Appendix - Parameter values used in the simulation

Number of Firms = 10
Number of countries = 5
$L = 5$
$c = 3$
$\alpha = 1$
$\beta = 0.01$
$m = 0.9$
$n = 1$
$\tau = 0.02$
γ endogenous, initialized randomly from a uniform distribution between 0 and 1
$\rho = 0.01$
$A = 10$
$\pi = 0.02$
$s = 0.02$
$\mu = 0.02$
$\delta = 1$
$E = 0.005$
$b = 0.1$
$\kappa = 4$
$F = 0$ through 0.05

References

Abramovitz M.A., 1979, "Rapid Growth Potential and its Realisation: The Experience of Capitalist Economies in the Postwar Period", in E. Malinvaud, (ed.), Economic Growth and Resources, vol. 1 The major Issues, Proceedings of the fifth World Congress of the International Economic Association, London: Macmillan, pp. 1-51.

Abramovitz M.A., 1994, "The Origins of the Postwar Catch-Up and Convergence Boom, in J. Fagerberg, B. Verspagen and N. Von Tunzelmann (eds), The Dynamics of Trade, Technology and Growth, Aldershot: Edward Elgar (in press).

Aghion P., Howitt, P., 1992, A Model of Growth Through Creative Destruction, Econometrica, 60 (2), pp. 323-351.

Amable B., 1993, "Catch-up and convergence: a model of cumulative growth", International Review of Applied Economics, 7(1), pp. 1-25.

Baumol W.J., 1986, "Productivity Growth, Convergence, and Welfare: What the Long Run Data Show", American Economic Review, 76, pp. 1072-1085.

Cheng, L. and Dinopoulos, E., 1992, "Schumpeterian Growth and International Business Cycles", American Economic Review. Papers and Proceedings, 82, pp. 409-414.

Cheng, L. and Dinopoulos, E., 1994, "Schumpeterian Growth and Stochastic Economic Fluctuations", paper presented at the International Schumpeter Society conference, Münster, August 1994.

Cornwall J., 1977, Modern Capitalism. Its Growth and Transformation, London: Martin Robertson.

DeLong J. B., 1988, Productivity Growth, Convergence and Welfare: Comment, American Economic Review, 78(5), pp. 1138-1154.

Dollar D., and Wolff, E.N., 1993, Competitiveness, Convergence and International Specialization, Cambridge MA: MIT Press.

Dosi G., 1988, "Sources, Procedures and Microeconomic Effects of Innovation", Journal of Economic Literature, 26, pp. 1120-1171.

Fagerberg J., 1988, "Why Growth Rates Differ", in G. Dosi, C. Freeman, R. Nelson, G. Silverberg and L. Soete (eds), Technical Change and Economic Theory, London: Pinter, pp. 432-457.

Fagerberg J., B. Verspagen and N. Von Tunzelmann, 1994, The Dynamics of Technology, Trade and Growth, Aldershot: Edward Elgar (forthcoming).

Freeman C., Clark, J. and Soete, L., 1982, Unemployment and Technical Innovation, London: Pinter.

Gomulka S., 1971, Inventive Activity, Diffusion and the Stages of Economic Growth, 24, Skrifter fra Aarhus Universitets Okonomiske Institut, Aarhus.

Hofman A.A., 1992, "Capital Accumulation in Latin-America: A Six Country Comparison for 1950-1989", Review of Income and Wealth, ??.

Kleinknecht A., 1987, Innovation Patterns in Crisis and Prosperity. Schumpeter's Long Cycle Reconsidered, London: Macmillan.

Lundvall B.A., 1989, "Long Waves and the Uneven Development of Capitalism", in M. Di Matteo, R. Goodwin and A. Vercelli (eds), Technological and Social Factors in Long Term Fluctuations, Berlin: Springer.

Maddison A., 1991, Dynamic Forces in Capitalist Development. A Long-Run Comparative View, Oxford and New York: Oxford University Press.

Mankiw N. G. and Romer, D.W.D., 1990, "A Contribution to the Empirics of Economic Growth", mimeo.

Mensch G., 1979, Stalemate in Technology. Innovations Overcome Depression, Cambridge, MA: Ballinger.

Metz R., 1992, "A Re-examination of Long Waves in Aggregate Production Series", in A. Kleinknecht, E. Mandel, I. Wallerstein (eds), New Findings in Long-Wave Research, New York: St. Martin's Press, pp. 80-119.

Reijnders J.P. G., 1990, Long Waves in Economic Development, Aldershot: Edward Elgar.

Romer P.M., 1986, "Increasing Returns and Long Run Growth", Journal of Political Economy, 94(5), pp. 1002-1037.

Rostow W.W., 1960, The Stage of Economic Growth: A Non-Communist Manifesto,

Cambridge: Cambridge University Press.
Schumpeter J.A., 1939, Business Cycles (vol I and II), New York: McGraw Hill.
Silverberg G. and Lehnert, D., 1993, "Long Waves and 'Evolutionary Chaos' in a Simple Schumpeterian Model of Embodied Technical Change", Structural Change and Economic Dynamics, 4(1), pp. 9-37.
Silverberg G. and Verspagen, B., 1994a, "Collective Learning, Innovation and Growth in a Boundedly Rational, Evolutionary World", Journal of Evolutionary Economics, 4 (in press).
Silverberg G. and Verspagen, B., 1994b, "Learning, Innovation and Economic Growth. A Long Run Model of Industrial Dynamics", Industrial and Corporate Change, 3 (in press).
Soete L. and Turner, R., 1984, "Technology Diffusion and the Rate of Technical Change", Economic Journal, 94, pp. 612-623.
Solomou, S., 1987, Phases of Economic Growth, 1850-1973. Kondratieff Waves and Kuznets Swings, Cambridge: Cambridge University Press.
Takayasu, H., 1990, Fractals in the Physical Sciences, Manchester and New York: Manchester University Press.
Verspagen B., 1991, "A New Empirical Approach to Catching Up or Falling Behind", Structural Change and Economic Dynamics, 2(2), pp. 359-380.

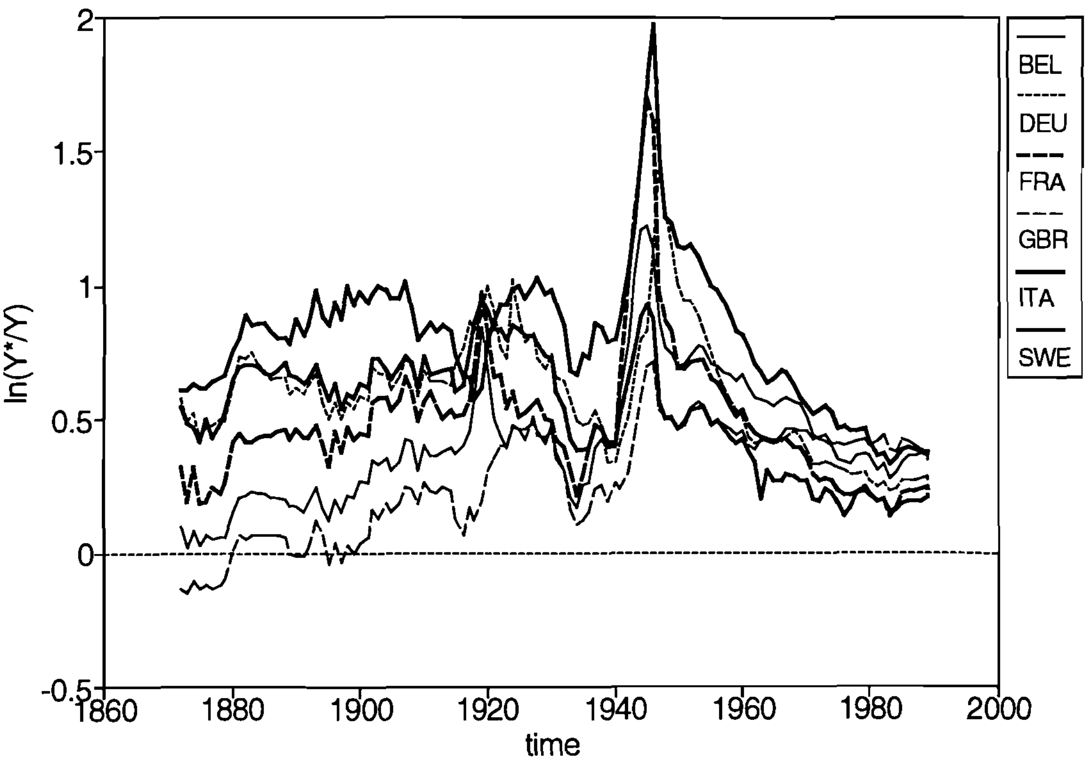


Figure 1. The log of the ratio of GDP per capita in the United States (Y^*) and six European countries (Y), 1870-1989

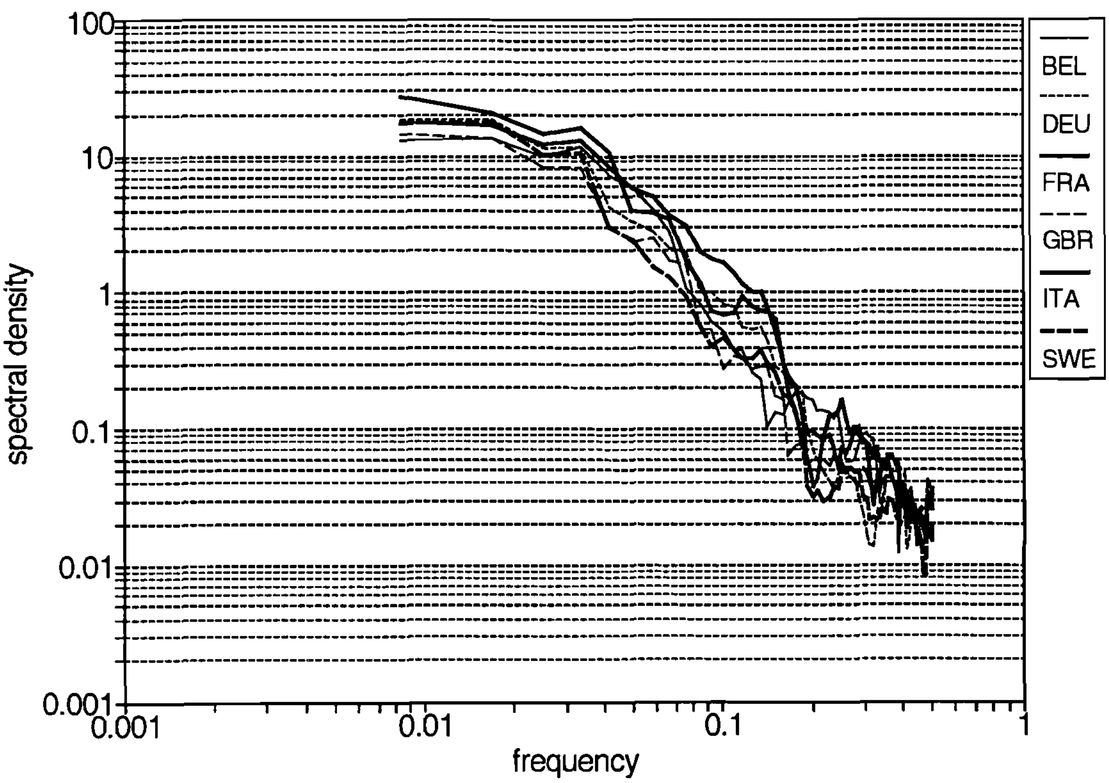


Figure 2. Power spectrum of the log distance to the USA-frontier of per capita GDP, six countries

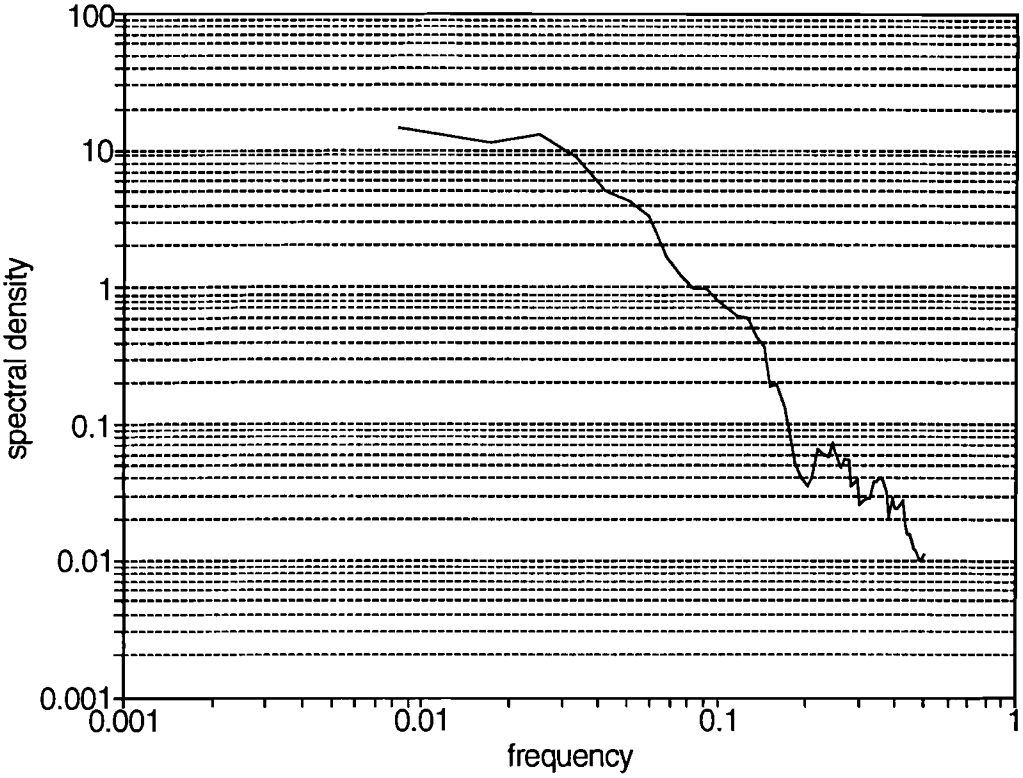


Figure 3. Power spectrum of the coefficient of variation of per capita GDP in six OECD countries

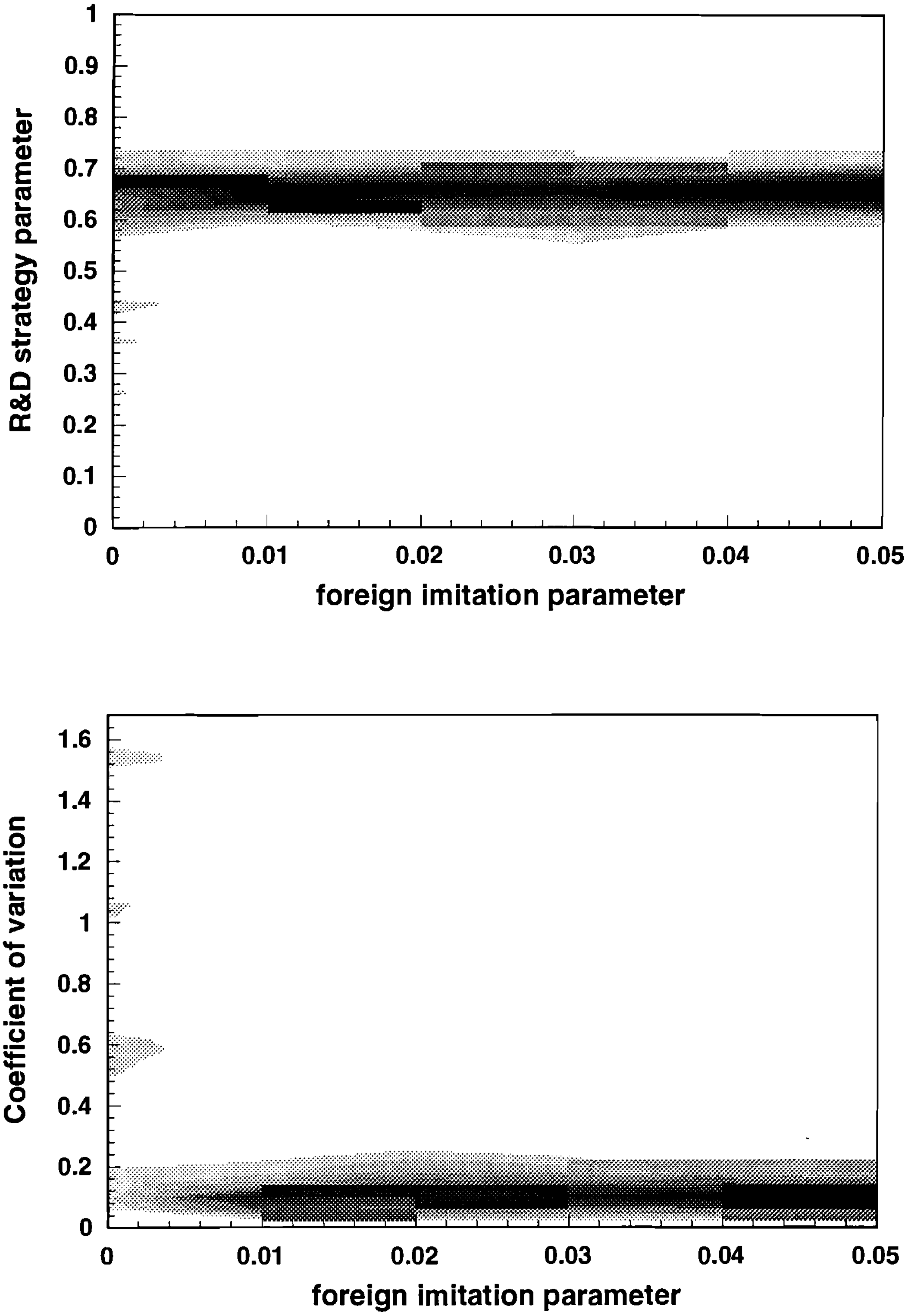
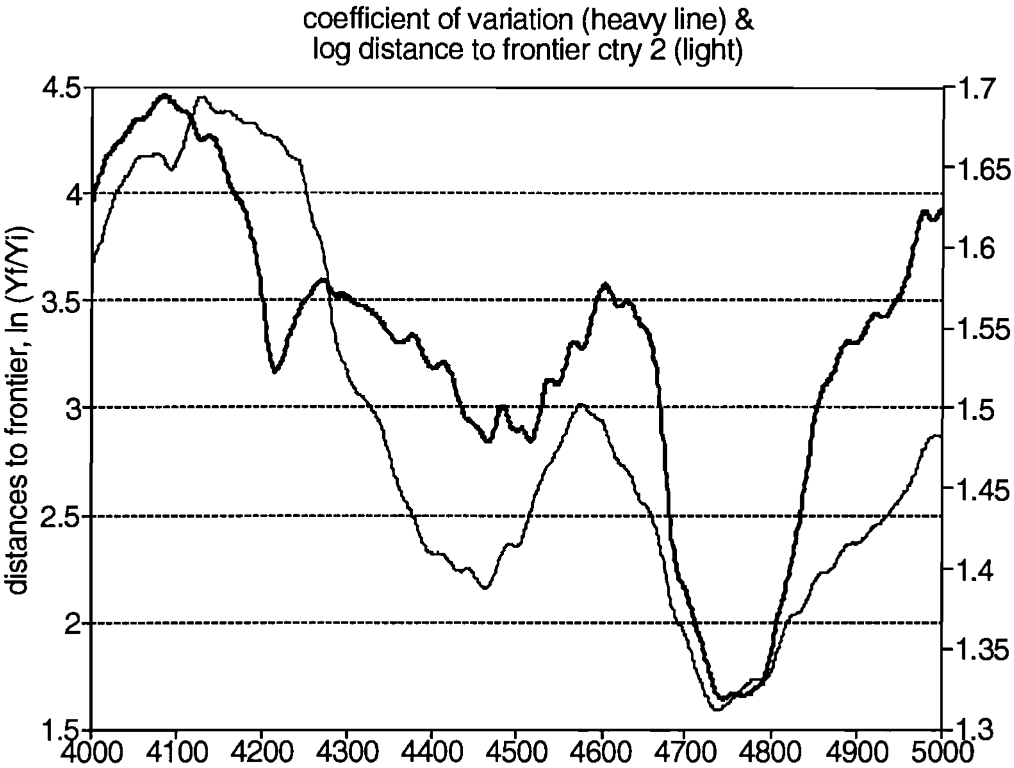


Figure 4 3D histograms for the *ex post* R&D to investment ratio (top diagram) and the coefficient of variation of country productivity levels (bottom diagram)



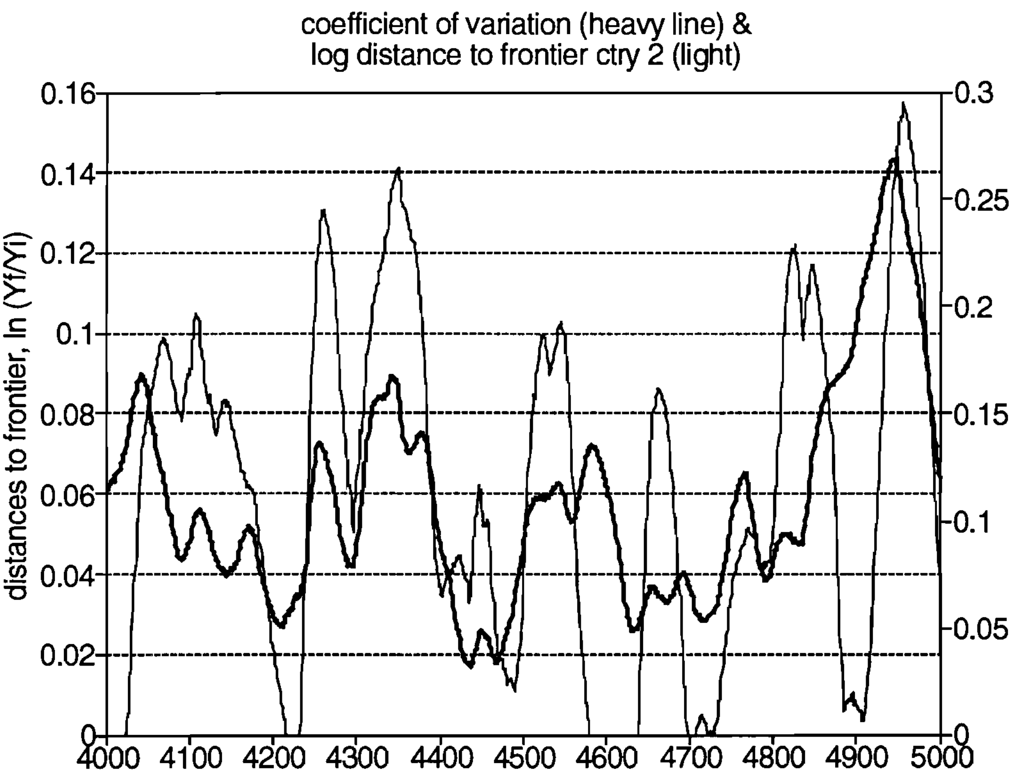


Figure 5. Time series of selected convergence variables in two runs, with f=0 (top diagram) and f=0.01 (bottom diagram)

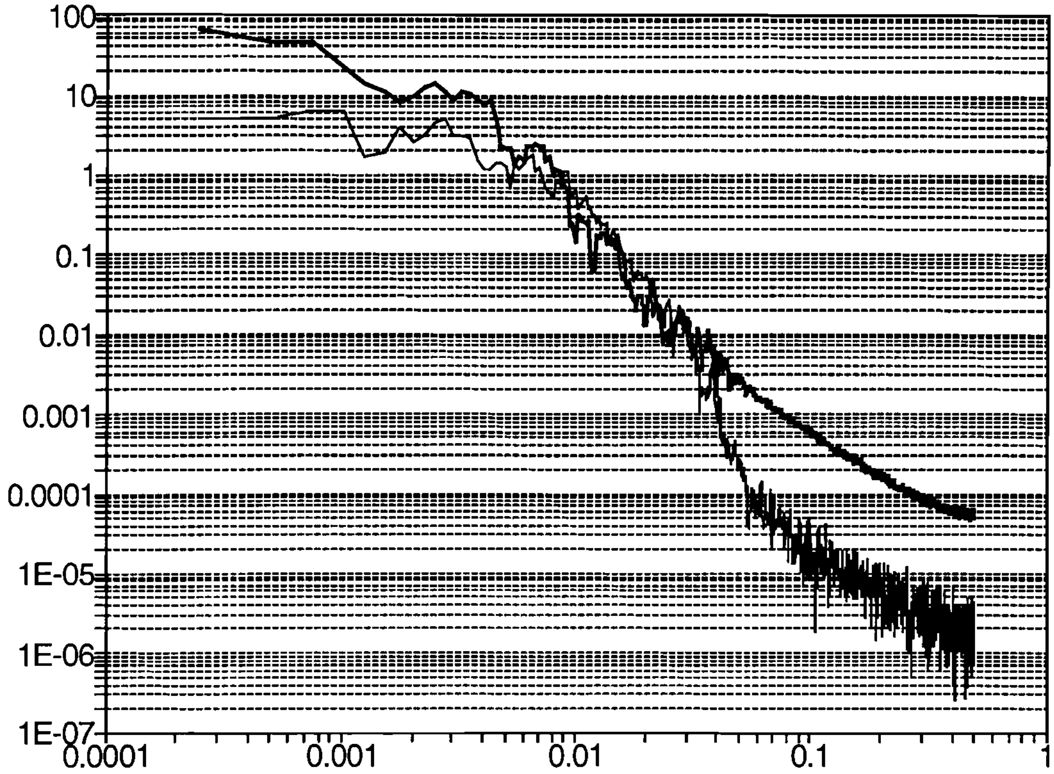


Figure 6. Power spectrum of the coefficients of variation for two simulation runs, one run with f=0 (heavy line) and one run with f=0.01 (light line)

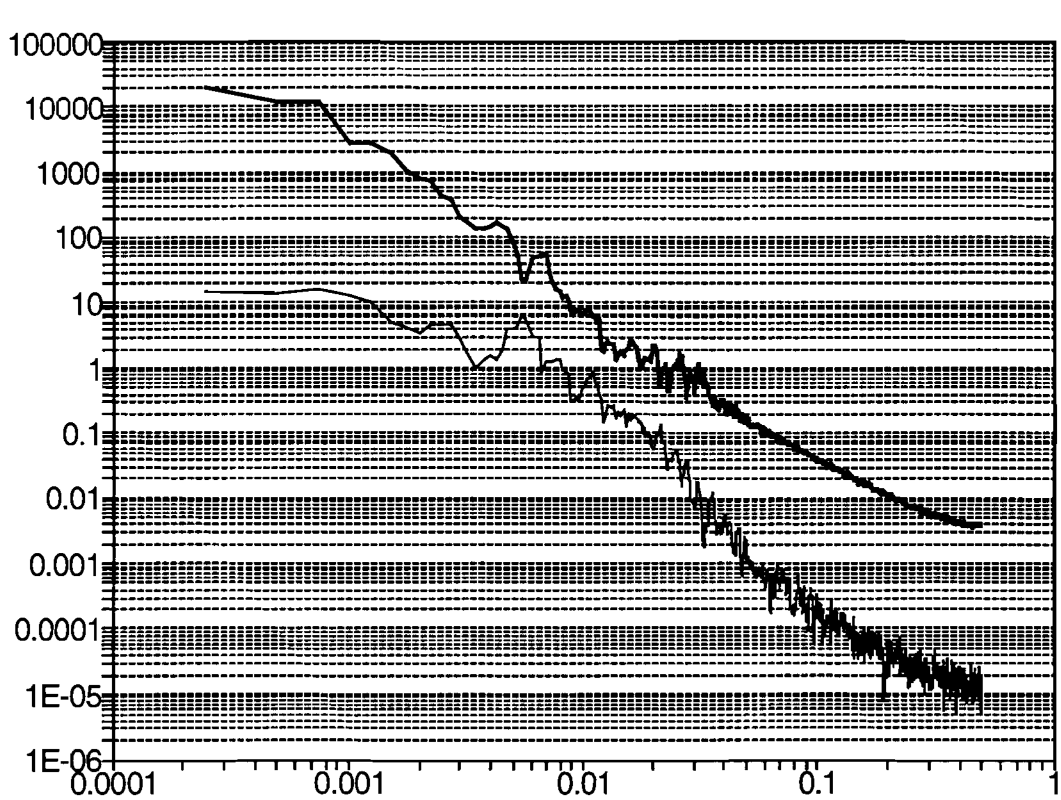


Figure 7. Power spectra for the log distances to the frontier for two simulation runs, one run with f=0, and one run with f=0.01