Working Paper

From the Artificial to the Endogenous: Modelling Evolutionary Adaptation and Economic Growth

Gerald Silverberg and Bart Verspagen

WP-95-08
February 1995

IIASA
International Institute for Applied Systems Analysis ◻ A-2361 Laxenburg ◻ Austria
Telephone: +43 2236 807 ◻ Fax: +43 2236 71313 ◻ E-Mail: info@iiasa.ac.at

From the Artificial to the Endogenous: Modelling Evolutionary Adaptation and Economic Growth

Gerald Silverberg and Bart Verspagen

WP-95-08
February 1995

IIASA International Institute for Applied Systems Analysis □ A-2361 Laxenburg □ Austria
Telephone: +43 2236 807 □ Fax: +43 2236 71313 □ E-Mail: info@iiasa.ac.at

Preface

The research project on *Systems Analysis of Technological and Economic Dynamics* at IIASA is concerned with modeling technological and organisational change; the broader economic developments that are associated with technological change, both as cause and effect; the processes by which economic agents – first of all, business firms – acquire and develop the capabilities to generate, imitate and adopt technological and organisational innovations; and the aggregate dynamics – at the levels of single industries and whole economies – engendered by the interactions among agents which are heterogeneous in their innovative abilities, behavioural rules and expectations. The central purpose is to develop stronger theory and better modeling techniques. However, the basic philosophy is that such theoretical and modeling work is most fruitful when attention is paid to the known empirical details of the phenomena the work aims to address: therefore, a considerable effort is put into a better understanding of the 'stylized facts' concerning corporate organisation routines and strategy; industrial evolution and the 'demography' of firms; patterns of macroeconomic growth and trade.

From a modeling perspective, over the last decade considerable progress has been made on various techniques of dynamic modeling. Some of this work has employed ordinary differential and difference equations, and some of it stochastic equations. A number of efforts have taken advantage of the growing power of simulation techniques. Others have employed more traditional mathematics. As a result of this theoretical work, the toolkit for modeling technological and economic dynamics is significantly richer than it was a decade ago.

During the same period, there have been major advances in the empirical understanding. There are now many more detailed technological histories available. Much more is known about the similarities and differences of technical advance in different fields and industries and there is some understanding of the key variables that lie behind those differences. A number of studies have provided rich information about how industry structure co-evolves with technology. In addition to empirical work at the technology or sector level, the last decade has also seen a great deal of empirical research on productivity growth and measured technical advance at the level of whole economies. A considerable body of empirical research now exists on the facts that seem associated with different rates of productivity growth across the range of nations, with the dynamics of convergence and divergence in the levels and rates of growth of income, with the diverse national institutional arrangements in which technological change is embedded.

As a result of this recent empirical work, the questions that successful theory and useful modeling techniques ought to address now are much more clearly defined. The theoretical work has often been undertaken in appreciation of certain stylized facts that needed to be explained. The list of these 'facts' is indeed very long, ranging from the microeconomic evidence concerning for example dynamic increasing returns in learning activities or the persistence of particular sets of problem-solving routines within business firms; the industry-level evidence on entry, exit and size-distributions – approximately log-normal – all the way to the evidence regarding the time-series properties of major economic aggregates. However, the connection between the theoretical work and the empirical phenomena has so far not been very close. The philosophy of this project is that the chances of developing powerful new theory and useful new analytical techniques can be greatly enhanced by performing the work in an environment where scholars who understand the empirical phenomena provide questions and challenges for the theorists and their work.

In particular, the project is meant to pursue an 'evolutionary' interpretation of technological and economic dynamics modeling, first, the processes by which individual agents and organisations learn, search, adapt; second, the economic analogues of 'natural selection' by which interactive environments – often markets – winnow out a population whose members have different attributes and behavioural traits; and, third, the collective emergence of statistical patterns, regularities and higher-level structures as the aggregate outcomes of the two former processes.

Together with a group of researchers located permanently at IIASA, the project coordinates multiple research efforts undertaken in several institutions around the world, organises workshops and provides a venue of scientific discussion among scholars working on evolutionary modeling, computer simulation and non-linear dynamical systems.

The research focuses upon the following three major areas:

1. Learning Processes and Organisational Competence.

2. Technological and Industrial Dynamics

3. Innovation, Competition and Macrodynamics

From the Artificial to the Endogenous: Modelling Evolutionary Adaptation and Economic Growth

Gerald Silverberg and Bart Verspagen*

1 Introduction

The process of economic growth and development strikes us as messy and anything but a clean steady state. Firms evolve strategies over time and are heterogeneous within and across industries and national economies. Growth and cycles overlap and interact in ways that are still controversial. Structural shifts occur, often rather suddenly on a historical time scale, and without any apparent exogenous cause. The rate of technical change is not God given but rather results from the collective but largely uncoordinated decision of numerous economic agents to pursue and implement innovation, primarily motivated by the expectation of realizing profits. The conditions under which this can be reliably done are anything but transparent in terms of the directions in which to search, the probabilities of success, and the ability to appropriate a return in the face of competition, temporal lags and informational spillovers.

Yet the economy can often be characterized by certain regularities of behaviour, both over time and cross-sectionally. Firms may come to adopt similar behavioral rules, such as price markups, investment payback periods, or target levels for inventories and R&D spending. Such relatively simple rules have been the object of study of the behavioral theory of the firm first outlined by Cyert and March (the classic exposition is Cyert and March 1963), who very early on advocated empirical study of actual business procedures on the one hand, and computer-based simulation as a research tool on the other. In this tradition, Nelson and Winter (1982) introduced the concept of routines as a 'genetical' description of the gamut of behavioral traits internal to and integral to the firm determining in some sense its standard operating procedure and its very individuality. The inheritability or persistence of such rules may be motivated by limited information processing ability and 'menu costs', or Simon's notion of satisficing as a common denominator of boundedly rational behaviour: agents only change their strategies when the gap between aspirations and realizations exceeds a certain threshold.

However, the rejection of fully optimizing behaviour as an explanation of economic activity does not single out any precise alternative as a theory of boundedly rational behaviour. It is probably for this reason—the absence of an operationalizable alternative based on 'first principles'—that economists continue to cling so tenaciously to the standard paradigm.

In the model we present in the following, firms must determine how much to spend on R&D in relation to either their profits or their sales[1]. In contrast to the more neoclassical approaches, we investigate this decision problem in the context of *bounded rationality*, where agents can have only vague ideas about the relationship between their actions and outcomes. To provide some anecdotal evidence, we recall an interview with the director of R&D of the

* Maastricht Economic Research Institute on Innovation and Technology, University of Limburg, P.O. Box 616, NL-6200 MD Maastricht, The Netherlands. The research of Bart Verspagen has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences.

[1] This paper extends Silverberg and Verspagen 1994a,b, where the decision rule is based only on the R&D to profits ratio.

Japanese firm Canon published in *The Financial Times* a number of years ago. The director reported that the firm had some time before raised its R&D/turnover ratio from 11% to 11½%. This appeared to have been beneficial to firm, so that the directors were debating whether to cautiously raise it even further. There in fact did not seem to be any way to determine where an upper limit might lie, and what the optimum policy might be, short of actually trying it out, but the firm was set to continue in this direction. Given that firms may operate in a trial and error mode in some areas of their strategic behavior, and moreover that they interact, is there any modelling approach which promises to capture this form of collective 'learning' in a scientifically reproducible way?

The perspective we adopt to address this question simulates a model economy consisting of 'artificial' agents endowed with limited but nonnegligible abilities to update their behavior in interaction with each other on the basis of their performance. Their environment is such that the product of their efforts is a pattern of economy-wide growth as they learn to undertake the R&D necessary to induce innovation and generate profitable avenues of investment. Hence it is a model of endogenous growth in the sense of Romer (1986, 1990), but without an aggregate production function, a representative agent, and intertemporal optimization.

In the next section we discuss recent work on learning and evolutionary games as well as the more computationally based artificial worlds approach. We then outline the details of the model before introducing the methods we have developed to evaluate its behavior. The last section compares the implications of the modelling exercise with some casual observations about real economies and locates them within the general research agenda on growth theorizing and economic history.

2 Selection, Learning and the Artificial Worlds Modelling Philosophy

Selten (1989) presents a provocative discussion of the tensions between the boundedly rational perspective and perfect rationality as traditionally used in economics. The traditional 'as if' argument of Milton Friedman (1953) holds that regardless of the boundedness of individual agents in their decision making, market selection will ensure that the traditional optimization result will prevail. This seems to absolve the economist of any necessity of explaining in detail how an equilibrium comes about, given arbitrary and out-of-equilibrium initial conditions. And in the event of multiple equilibria, as is common in many interesting problems in game theory, it provides no help at all. A bridge between a theory based on causal mechanism and outcomes based on equilibrium has been sought in learning or evolutionary game models, often inspired by biological applications (cf. Friedman 1991 for a survey). The static notion of *evolutionarily stable strategy* is the biologist's refinement of a Nash equilibrium, while dynamic versions have usually been based on replicator dynamics (cf. Hofbauer and Sigmund 1988). The key biological stability concept is *uninvadability*, the inability of a small number of mutants to invade an equilibrium population. As in standard game theory, the interesting results concern possible existence of multiple equilibria, but in contrast to game theory the dynamic theory provides a means of equilibrium selection based on initial conditions. Most of this work has been in a deterministic framework in economics, although this has not been the case in biology. But evolutionary game theory reflects just one half of the evolutionary process, namely selection, and has focused on the equilibria as once-and-for-all asymptotic states.

Recently, attention has shifted to the role of mutation and stochasticity in further refining the notion of stability in evolutionary games (Foster and Young 1990, Young 1993, Kandori, Mailath and Rob 1993, Binmore and Samuelson 1993). In contrast to conventional evolutionary game theory, agents or species are allowed to mutate continually over time before an equilibrium is attained. In general, the equilibria that remain (usually in the limit as the mutation probability goes to zero) will be a significant further refinement of the limiting states. This framework has been proposed as a model of learning by ongoing trial and error with selection. Once again, though, the focus has been on asymptotic states and the refinement of equilibrium concepts.

Approaching learning from another extreme is work based on artificial intelligence and computer science such as neural nets, genetic algorithms, and classifier systems. While these approaches are also based on complex interdependence of interacting subunits as in game theory, selection, and stochastic perturbations, the complexity of the problems workers have addressed has made analytical results difficult to come by. Whereas in the beginning, most work focused on computing standard optimization problems and thus converging to a hopefully unique point, more recent efforts have been directed at using these tools open-endedly to simulate the self-organization and evolution of life-like systems, whether they be abstract ecosystems, microorganisms, individual behaviour or human economies (cf. the work in Langton 1989, Langton, Taylor, Farmer and Rasmussen 1992, Arthur 1991, and Lane 1993).

For systems exceeding a certain complexity in their organization, if not in the nature of their constituent parts and their interactions, the existence and uniqueness of an asymptotic steady state may be less interesting than two heuristic phenomena that have often been commented on without precise definition. The first is *emergent properties*, i.e., complex but identifiable patterns of behaviour that emerge spontaneously at some point in the history of the system and are not in any obvious way inherent in the constitution of its parts. The second is *punctuated equilibria*, periods of quasi-stable behaviour separated by usually very rapid periods of transition and disorder. Evolutionary game theory, by focusing on very low-dimensional game structures may well provide an explanation of the emergence and stability of a single such behaviour, but has deliberately excluded the complexity apparently necessary to generate an ongoing sequence of such states. There is obviously a tradeoff here between analytic tractability and behavioral richness as well as a philosophical difference of opinion on how science should proceed. Lane (1993) argues that the artificial world framework differs from conventional approaches in one or more of the following ways:

1) Transients are at least as important as steady states;
2) The concept of stability must be relativized to some notion of metastability, i.e., one which may contain the seeds of its own destruction;
3) New statistical methods will have to be developed to enable metastable states and emergent properties to be identified and characterized;
4) Computational methods will attain a scientific status equal to that reserved until now for analytical ones.

Most of the work in artificial worlds has been based on discrete genetic codings of strategies such as are called for by genetic algorithms or classifier systems. One may ask whether this perspective, borrowed from biological genetics, should not be made more congruent with the way agents may be interpreted to formulate and modify their strategies in reality. Thus, although a genetic algorithm may be used to solve an optimization problem in a continuous space by representing real values in binary form, in some cases it may be more natural to represent for example mutation as a local operation rather than the discrete and possibly very large jumps implicated by a binary coding in a genetic algorithm. In our case at hand—the determination of firms' R&D investment—it may make more sense to mutate locally around existing values than to allow for jumps across the entire parameter space. This 'realistic' approach to evolutionary modelling based on stylized behaviour is characteristic of the work initiated by Nelson and Winter (1982) and in the computer science realm to the real-valued evolutionary algorithms of Schwefel (1981). Thus mutation or trial and error will be represented in our model by a draw from a normal distribution centred around the current value of the R&D to investment ratio. Imitation, in contrast, does permit large discrete jumps in parameter space but will be modified here to reflect a satisficing principle—only firms with low profits will go out and imitate.

3 The Model

Our model has much in common with recent work on 'endogenous growth' (cf. Romer 1986 and 1990, Lucas 1988, Aghion and Howitt 1992, Helpman 1992), where technical change comes about as a result of the profit-seeking activities of individual agents, and increasing returns, spillovers and other phenomena known from the economics of innovation may be present. While recognizing that innovation is associated with many imponderables and uncertainties, these models still hinge on the standard tools of market clearing, classical rationality, intertemporal optimization, rational (technological) expectations and the identification of steady-state equilibria.

Moreover, their representation of technology has often been oversimplified to correspond to known and tractable special cases. Thus, in the model most closely related to the structure of our own, Aghion and Howitt (1992) assume that at any time only one technology prevails in the economy. When a new innovation is made, it instantly and costlessly (except for the sunk cost of the R&D which went to invent it) replaces its predecessor. The monopoly returns associated with it, for as long as it prevails, are completely recouped by the single innovator, with no imitation or spillovers. Thus the appropriate stochastic decision problem to determine the level of R&D spending, they argue, can be represented as a patent race with a commonly known probability of making an innovation as a function of R&D effort.

As any student of the history of technology knows, however, this is a far cry from anything that has ever prevailed economy-wide. And even in some particular industries where a monopoly of a truly key innovation has been defended for as long as possible (one thinks of the Boulton and Watt patent on the condensing steam engine), earlier technologies (such as the Newcomen engine and water power) retained a significant place in production for a considerable time (partly because of the monopoly, no doubt).

These cursory reflections suggest a number of requirements for an 'endogenous' growth model regarding both a model's representation of technology and of the decision problem confronting the innovator:

1) At any given time a number of technologies will be concurrently in use, particularly if these technologies are capital embodied (the vintage effect);
2) Even at the investment 'frontier', different technologies may be adopted simultaneously;
3) The aggregate rate of technical change will be a function of the rate of diffusion of new technologies and not of the rate of instantaneous innovation;
4) Even if one may accept that the return to innovative effort is representable as a draw from a stationary random process, the parameters of this process are not *a priori* known to agents, and their subjective priors may differ widely. Moreover, even if they are Bayesians, they may not live long enough to draw more precise conclusions. Their problem is not

dissimilar to that of the multiarmed bandit;

5) Technological knowledge, like information in general, can have a public and codifiable, a private, and a tacit character. It can only be imperfectly protected as private property (the mere knowledge that something can be done, which a patent discloses, can already be very useful information). This state of affairs will certainly influence innovative activity, but in ways that are difficult to anticipate.[2]

To deal with these different aspects of the problem, we have constructed the model around three basic blocks. The first block describes how the artificial economy evolves with a given set of technologies and firms, with selection taking place at both levels. This block consists of equations for the rate of capital accumulation, the diffusion of new technologies in the total capital stock of the firms, and the real wage rate. The second block describes a set of rules that is used to introduce new technologies and firms into the economy. This block takes the innovative behaviour of firms (to be explained below) as given, and then describes the probability that individual firms will make an innovation, as well as how this innovation is introduced. The third block describes how innovative behaviour changes under the influence of the evolution of the economy and firm learning. This block, in other words, describes a feedback from performance to innovative behaviour and thus a form of collective learning. The parameters of the model and the values used in the simulations are summarized in the Appendix.

a. The evolution of the artificial economy with a given set of firms and technologies

The basic framework of the model is taken from Silverberg and Lehnert (1993), which in turn draws on Silverberg (1984) and Goodwin (1967). Let hats above variables denote proportional growth rates, w be the (real) wage rate, v the employment rate (persons employed as a fraction of the labour force), and m and n parameters (both positive). Then the wage rate is determined by the following differential equation:

$$\hat{w} = -m + nv. \tag{1}$$

It is assumed that there is a fixed number q of firms in the economy, while each of these firms has a variable number p_q of different types of capital goods that it utilizes to produce a homogeneous product. New capital arises from the accumulation of profits, a process described by the following equation:

$$\hat{k}_{ij} = (1-\gamma_{1i})r_{ij} - \frac{\gamma_{2i}}{c} + \alpha(r_{ij}-r_i) - \sigma. \tag{2}$$

The capital stock is denoted by k, r stands for the profit rate, and σ is the exogenous rate of *physical* depreciation of capital (technological obsolescence is an endogenous component of the model itself). The subscript i $(1..q)$ denotes the firm, and j $(1..p_q)$ the type of capital (the

[2] In classical articles Nelson (1959) and Arrow (1962) underscored the disparity which may exist between the social and the private rates of return and incentives to innovation. Recent literature has uncovered the possibility of both insufficient as well as socially excessive, redundant R&D, depending on the precise assumptions made. Nelson 1990 brings the empirical literature to bear on these issues, and shows in particular that not insubstantial own R&D efforts are necessary even to imitate.

absence of any these indices indicates an aggregation over this particular dimension). Eq. (2) assumes that the principal source for type *ij*-capital accumulation is profits generated by *ij*-capital. This is modelled by the first term on the rhs of (2), i.e., $(1-\gamma_i)r_{ij}$. A firm-specific portion of profits (denoted by γ_{1i}) plus a firm-specific portion of total output (sales) (denoted by γ_{2i}) is used for the development of knowledge (R&D) (when r_i<0, γ_{1i} is set to zero).

However, profits may also be redistributed in such a way that more profitable types of capital accumulate even faster, less profitable even slower, than would otherwise be the case. The mechanism used to model this was first proposed by Soete and Turner (1984), and is represented by the second term on the rhs of eq. (2). By changing the value of α, redistribution of profits takes place faster (larger α) or slower (smaller α).

It is assumed that each type of capital is characterized by fixed technical coefficients, c and a (for capital coefficient and labour productivity, respectively). The capital coefficient is assumed to be fixed throughout the economy (and time), while labour productivity is assumed to change under the influence of technical progress. The profit rate of *ij*-capital is then given by $(1-w/a_{ij})/c$.

The principal variable used to describe firm dynamics is the share of the labour force employed on each capital stock. Production is assumed to be always equal to production capacity (the influence of effective demand is absent), so that the amount of labour employed by each capital stock is equal to $k_{ij}/(a_{ij}c)$. Dividing this by the labour force (assumed to grow at a fixed rate β) gives the share of labour employed, v_{ij} (called the employment share hereafter). The expression for the growth rate of this variable is

$$\hat{v}_{ij} = \hat{k}_{ij} - \beta = (1-\gamma_i)r_{ij} - \frac{\gamma_{2i}}{c} + \alpha(r_{ij} - r_i) - (\beta + \sigma). \tag{3}$$

R&D also has an employment effect. We assume that the ratio between R&D expenditures and R&D labour input is equal to a fraction δ of the economy-wide labour productivity. The employment rate v_q resulting from production is then found by summing v_{ij} over i and j. Under these assumptions, it can then be shown that the overall employment rate v is equal to $(1+\delta(\gamma_1 rc+\gamma_2))v_q$.

Eqs. (1) and (3) together create a selection mechanism in our artificial economy. Eq. (3) describes how more profitable (i.e., with above-average labour productivity) technologies tend to increase their employment share, whereas more backward (below-average) technologies tend to vanish. The real Phillips curve eq. (1) ensures that real wages tend to track labour productivity in the long run. In a situation in which new technologies are continually being introduced, this implies that all technologies, after an initial phase of market penetration and diffusion, will eventually vanish from the production system.

However, for a given set of technologies, long-run per capita growth is no longer possible once all firms converge to exclusive employment of the highest productivity technology. The next section outlines how new technologies can enter the system and thereby open up the possibility of long-run growth.

b. The introduction of new technologies and firms into the economy

It is assumed that in each time period, firms devote resources (R&D) to the systematic search for new production possibilities (i.e., new types of capital). The outcome of this search process is assumed to be stochastic. The structure of the *technological space* is assumed to be a simple directed graph (Figure 1). More complicated graphs could well be imagined with branching and even merging nodes; this remains a subject for future research.

Each time an innovation occurs, the firm creates a new type of capital. The labour productivity of this type of capital is given by the following process:

$$a_{i,t}^{*} = (1+\tau)a_{i,t-1}^{*}, \tag{4}$$

where τ is the fixed proportional increase in labour productivity between innovations and $a_{i,t}^{*}$ is the firm-specific best practice labour productivity. The new type of capital is seeded with a small employment share (say 0.0001). In order to keep the total employment rate constant, this seed value is (proportionally) removed from the other types of capital of the innovating firm. The number of technologies employed by any given firm may vary in time.

As real wages rise over time, every technology will generate negative profits at some stage (because of its fixed labour productivity). It is assumed that these losses are financed by an equivalent decrease of the capital stock. In other words, losses imply that capital will be scrapped, and scrapped capital can be 'consumed' to cover the losses. Note that for individual capital stocks, the point at which scrapping occurs lies *prior* to the point where profits are negative, due to the α-related diffusion term in eq. (2). When a technology's employment share falls below a specified (very small) value E, it is scrapped completely.

A firm's R&D activities as well as possibly those of its rivals enter an innovation potential function T_i. This in turn determines the firm's probability of making an innovation according to a Poisson process with arrival rate ρ_i. The simplest relation is simply linear:

$$\rho_i = AT_i + \rho_{\min}, \tag{5}$$

where ρ_{min} is the (small) autonomous probability of making a fortuitous innovation without doing formal R&D, and A is the innovation function slope. One can also posit a nonlinear relationship with both increasing and decreasing returns to R&D, such as a logistic:

$$\rho_i = \frac{\rho_{\min}\rho_{\max}}{\rho_{\min}+(\rho_{\max}-\rho_{\min})\,e^{-AT_i}}. \tag{6}$$

This logistic function has intercept ρ_{min} and (asymptotic) saturation level ρ_{max}. In this case, the parameter A determines the speed at which the saturation level is approached.

T_i, the innovation potential, is determined both by the firm's own R&D level (h_i, to be defined below) and its ability to profit from other firms' R&D (technological spillovers):

$$T_i = h_i+\phi_1 h+\phi_2 hh_i. \tag{7}$$

These spillovers can take two forms. First, there is a term related to the economy-wide value of h (written without subscript). The economy-wide R&D level h is defined to be the market share weighted average of firm-specific R&D levels. Second, there is a term related to the product of the economy-wide and the firm-specific value h_i. This latter term takes into account the argument that in order to assimilate spillovers, a firm has to have some technology-generating proficiency itself (see Cohen and Levinthal 1989, Nelson 1990). The parameters ϕ_1 and ϕ_2 determine the importance of each spillover mode[3].

The firm-specific R&D level h_i is defined to be the ratio of a moving average of firm R&D investment to its total physical capital stock. A ratio is used to normalize for firm size, since otherwise such a strong positive feedback between R&D and firm growth exists that monopoly becomes inevitable. While *a priori* it is by no means clear why the size of individual R&D effort should not directly relate to innovative success, a pure scale effect must be ruled out by the continuing existence of competition and the ability of small countries to remain or even advance in the technology race. The exponential moving average RD_i on R&D for a lag of L (or a depreciation rate of $1/L$) is given by the following differential equation:

$$\frac{d}{dt}RD_i = ((\gamma_1 r_i + \gamma_2/c)\, k_i - RD_i)/L. \tag{8}$$

Hence the firm-specific R&D level is

$$h_i = RD_i/k_i. \tag{9}$$

An innovation can be defined in a narrow or a wide sense. In the wide sense, the adoption of any technology not yet employed by a firm (or a country) is an innovation to that unit. In the narrow sense, only technologies that have never been employed before anywhere are considered innovations at their time of introduction. If firms innovate according to the above Poisson arrival rates in the narrow sense, however, a very considerable intertemporal externality is created, because firms' innovations always build on each other. Thus there can be no duplication of effort and, as long as firms maintain a minimal level of R&D, no cumulative falling behind. On the other hand, once an innovation has been introduced somewhere into the economy, it should be progressively easier for other firms to imitate or duplicate it; it should not be necessary to reinvent the wheel. We capture this by introducing a catching-up effect. Let the labour productivity of the economy-wide best practice technology be a^*, and the best practice technology of firm i a^*_i. Then firm i's innovation potential T_i will be augmented by a measure of its distance from the best practice frontier:

$$T_i' = T_i\,(1 + \kappa \ln(a^*/a_i^*)). \tag{10}$$

Thus, adopting an old innovation is facilitated for backward firms, but they are still required to invest in their technological capacity to reap these catchup benefits. Here, however, R&D efforts should be interpreted in the larger sense of technological training and licensing, reverse engineering, or even industrial espionage (all costly activities, if not as costly as doing state-of-the-art R&D).

We have also experimented with innovations in the narrow sense, but the results on

[3] Spillovers will not be dealt with in this paper. They are examined within the context of one-parameter strategies in Silverberg and Verspagen (1994a,b).

strategic selection are rather ambiguous. This is not surprising, since the import of the intertemporal externality is indeed quite large. We consider the Ansatz in eq. 10 therefore to be a justifiable first formulation, since technology adoption decisions are never passive, but rather require technological efforts of the adopting firm. However, it does place too much of the burden of catching up onto R&D.

In the artificial economy modelled here, entry of a new firm occurs only as a result of exit of an incumbent firm. Exit occurs whenever a firm's employment share (excluding its R&D employment) falls below a fixed level E. While exit of incumbent firms is completely endogenous, entry only occurs in case of exit, so that the total number of firms is constant. Naturally, this feature of the model is not very realistic, as in reality entry may be independent of exit and the total population of firms may vary. However, it is not the aim of this model to describe the phenomena of entry and exit as such. Instead, the main function of entry and exit is to maintain potential variety in the population of firms while providing for firm elimination.

Whenever entry occurs, the entrant is assigned a single technology with an amount of capital corresponding to an employment share of $2E$ (the remaining employment is proportionally removed from other firms so that total employment remains constant). The labour productivity of this technology is drawn uniformly from the range $[(1-b)A,(1+b)A]$, where A is the unweighted mean value of labour productivity of all the firms in the economy, and b is a parameter. The values for h and γ are (uniformly) drawn from the range existing in the economy at the time of entry.

c. Firm-strategies for innovation

In Sections (a) and (b) we have outlined the system whereby innovating firms generate technical change and undergo selection in a closed economy model as a function of their R&D strategy parameters γ. Learning now enters the picture in the form of two 'genetic' operators, mutation and imitation. Thus *behavioral evolution* takes place in a two-dimensional continuous space, where the economy at any given point in time is represented by a cloud of points (Figure 2). With probability Π each decision period, which is set exogenously and is equal for all firms, a firm will draw from a normal distribution and alter one or both of its strategy-parameters γ within the admissible range [0,1] (mutation). Given that mutation occurs, each possibility (change either one or both parameters) occurs with equal probability:

$$\begin{gathered}\gamma_{1it}=Min(1,Max(\gamma_{1it-1}+\varepsilon,0)),\quad \varepsilon\sim N(0,s),\ or\\ \gamma_{2it}=Min(1,Max(\gamma_{2it-1}+\varepsilon,0)),\quad \varepsilon\sim N(0,s),\ or\\ \gamma_{lit}=Min(1,Max(\gamma_{lit-1}+\varepsilon_l,0)),\quad \varepsilon_l\sim N(0,s),\quad l=1,2.\end{gathered}\tag{11}$$

With variable probability Π^c_i the firm simply imitates the strategy of another firm. Again, given that imitation occurs, each possibility (imitate either one or both parameters) occurs with equal probability:

$$\begin{gathered}\gamma_{1it}=\gamma_{1jt-1},\ j(\neq i)\in[1..q]_i,\ or\\ \gamma_{2it}=\gamma_{2jt-1},\ j(\neq i)\in[1..q]_i,\ or\\ \gamma_{lit}=\gamma_{ljt-1},\ j(\neq i)\in[1..q]_i,\ l=1,2.\end{gathered}\tag{12}$$

The imitation probability is partly endogenous to reflect satisficing behaviour. Only firms with unsatisfactory rates of profit with respect to economy leaders will choose or be forced (for example by their stockholders or by hostile takeovers) to adopt the strategy of a competitor:

$$\Pi_i^c = \mu\left(1-\frac{y_i-y_{\min}}{y_{\max}-y_{\min}}\right). \tag{13}$$

y_i is the firm's rate of expansion of physical capital (defined as $min(r_i\text{-}\gamma_2/c,\ (1\text{-}\gamma_i)r_i\text{-}\gamma_2/c))$, $y_{\max}$ and y_{min} are the maximum and minimum values of y in the sample, and μ is the (exogenously determined) maximum imitation probability. Thus, the more profitable a firm is, the less likely it will change its strategy by imitating another firm. The most profitable firm has an imitation probability equal to zero; the least profitable the maximum probability μ. Once a firm has decided to imitate, it selects a firm to imitate randomly from the industry with weight equal to the target firm's market share in output. If neither imitation nor mutation occur, the firm simply retains its strategy from the previous period.

4 Identification of Steady States by Random and Spontaneous Generation

The model has been implemented to run on both MS-DOS and Unix computers. To make the solution as time-step invariant as possible, the selection mechanism, which is basically a system of differential equations, is solved using a fixed-step, fourth-order Runge-Kutta algorithm. The innovation decisions are executed during each computational step (using Poisson arrival rates scaled by the computation time step), and when an innovation is made, the corresponding changes in initial conditions, number of equations, and coefficients are made for the next step. Mutation and imitation are only performed at fixed intervals of one 'year', which may be many times the step employed in the Runge-Kutta algorithm.

The main feature of the model we will investigate is the existence and nature of an 'evolutionary attractor' in the dynamics, i.e., whether a 'stable' configuration of firm R&D strategies exists to which our artificial economy will converge from particular classes of initial conditions. We have initialized the system in two ways: first in a 'grapeshot' mode we term random generation in which the initial γ's are drawn from a uniform distribution over [0,1] and [0,0.2] (respectively for γ_1 and γ_2); and second, in a 'spontaneous generation' mode in which all initial γ's are set to zero. We present the results of these experiments by means of density plots made on the basis of 3-dimensional histograms of the two γ's. On the horizontal axis we plot the experimental parameter that is being varied through different simulation runs (A or μ). The (market share weighted) mean value of the strategy parameter over the firms in each run is shown on the vertical axis. The data are pooled from the last 1000 years of five simulation runs for each value of the experimental parameter, each generated with a different random seed. Darker shading indicates higher frequencies.

In Figure 3, we plot the results for the runs initialized by random generation. The figure shows clearly that the converging behaviour of the two strategy parameters is quite different. Parameter 1 (the targeted R&D to profits ratio) does not converge very clearly, except for higher values of the innovation slope, when relatively high frequencies are found at values near zero. Parameter 2 targeting R&D to sales shows a more tight convergence, although there are still high frequencies near zero (the white band between the horizontal axis and the attractor is quite narrow). Thus, firms show a tendency to select a relatively tight range of values for parameter 2, while parameter 1 tends to drift or, if anything, go to zero. Summarizing, firms seem to display a tendency to select tightly defined strategies based upon parameter 2 and indifference to parameter 1.

Figure 4 shows corresponding results for the case of spontaneous generation, i.e., a situation in which firms have to discover R&D as an activity. In this case, the economy starts out in a stagnant phase, with no intentional technical progress. As firms explore the strategy space by mutation and imitation, they may (or may not) find R&D a useful activity. The density plots show that in this case, the evolutionary attractor is much more clearly defined. The type 1 strategy parameter remains at values near zero (in our interpretation, the positive values found are largely attributable to random 'evolutionary' noise). The type 2 strategy parameter, however, shows a well-defined peak significantly distant from zero, as indicated by the white space bordering it from below.

This behaviour of the system can be interpreted as a particular form of lock-in or path dependency. When the system is started 'clean', without any form of 'commitment' to any type of R&D-strategy, it will select a much more unambiguous evolutionary attractor than in the random generation case. This is not the case when only the type 1 strategy parameter is employed (compare Silverberg and Verspagen 1994a,b). There the asymptotic steady states of the two initializations are identical.

For a logistic innovation function (eq. 6) we obtain similar patterns from the histograms over innovation opportunity (Figure 5). In contrast to the linear case, however, the steady-state value of parameter 2 does seem to decline somewhat with increasing technological opportunity. What is also remarkable is the sudden collapse of the technological regime below values of A of about 40. The steady-state values of parameter 2, in the range 20—30%, are also higher than in the linear case.

Why is strategy parameter 2 subject to positive selection, while parameter 1 displays either drift or is constrained to zero? Our interpretation is that R&D comes to be regarded as a 'core' business activity in the model, for, due to the 'Goodwin' business cycles of the underlying economy, profits are more variable than sales. Thus, firms which base their R&D expenditures upon profits will have more fluctuating R&D stocks than firms which base their R&D on sales. The selection environment seems to favour the latter firms because, in the long run, their R&D behaviour provides a more reliable stream of innovations.

Whereas the steady-state values of the strategy parameters do not appear to depend on technological opportunity, as represented by the value of A[4], the rate of technical change is a simple linearly increasing function of it (Figure 6). The transient time paths in the spontaneous generation case on the way to a growth steady-state are also of interest in themselves. Figure 7 displays the market-share-weighted values of the two strategy parameters for a single run over 8000 years. Convergence is relatively rapid in comparison with the single parameter case. Figure 8 shows the time paths of the rate of technical change and the Herfindahl concentration index[5]. Viewed as a process in historical time and not just as out-of-equilibrium transient, this figure recapitulates a piece of virtual economic history. The economy starts off with no R&D and an essentially vanishing rate of technical change. Within this regime the rate of market concentration is quite high, although the identity of the near monopolist changes at almost regular intervals, as indicated by the breaks in the level of concentration. As the γ's rise with time and with them the overall rate of technical change, this market regime breaks down. It is replaced by low levels of concentration and

[4] This is not true when only parameter 1 is used. In this case, the value gradually *falls* with increasing A.

[5] This is defined as $H = \Sigma f_i^2$, where f_i is the market share of the ith firm. It ranges from $1/n$, for n equally sized firms, to 1, for complete monopoly.

considerable market turnover.

We have also begun to investigate how the structure of the evolutionary learning process affects the outcomes of these experiments. Recall that the variables Π and μ, representing the probability of mutation and imitation, are exogenously imposed. We have compiled histograms for the two strategy parameters by varying each of these rates separately. Figure 9 shows the results for mutation, Figure 10 for imitation, for runs with $A = 10$ and random generation initial conditions. Varying the mutation probability does not change the picture in any essential way. In contrast, increasing the ceiling on the imitation probability leads to a progressive collapse of γ_2 selection. If firms imitate each other too strongly, they become involved in an evolutionary game of musical chairs, and no nontrivial strategy is able to establish itself.

5 Discussion and Conclusions

In this paper we have developed an evolutionary model describing the relation between endogenous technological change and economic growth along the lines of an 'artificial world' modelling philosophy. By this we mean that the economy is disaggregated into diverse individual behavioral subunits (instead of the representative agent so prevalent in most macroeconomic modelling) connected by nontrivial nonlinear dynamic interactions based on plausible notions of disequilibrium competition and investment. Rather than search for a strategic equilibrium based on a concept of rationality, we have assumed that these agents use boundedly rational behavioral procedures. In the present case this is an extremely simple rule for the R&D/profits (or gross investment) and R&D/sales ratios, which is parameterized by two real numbers between 0 and 1. Learning is modelled by allowing for mutation and imitation rules operating on the agents' strategy parameters. An element of behavioral realism is injected into the model by insisting that mutations are local in the strategic 'genotype' space, and that imitation is only prompted by less than satisfactory performance.

Using both a linear and a logistic innovation function we were able to show that evolutionary steady states exist that are attractive in the behavioral space, but may differ depending on 'history'. Thus the model does establish a case for endogenous growth in the sense of demonstrating that economic competition, even with very relaxed assumptions about individual goal-seeking behaviour and profit maximization, leads to an approximately steady-state growth path with a positive rate of technical change and R&D investment.

However, the spontaneous generation experiments underline the fact that the mere existence of such a steady state does not mean that history does not matter. Quite the contrary. A society starting with no or low rates of R&D will pass through a phase of very high market concentration, but with periodic upheavals or 'palace revolts' of market leadership on a time scale of centuries. Eventually such an economy will 'bootstrap' itself to higher rates of R&D and technical change[6].

To demonstrate that the levels of R&D to which the model points are by no means unrealistic, Figures 11 and 12 present OECD data on the share of R&D expenditures in value added[7] for five industries in the USA and Japan, respectively. These data are representative

[6] This should be compared with the one-parameter strategy case, where the evolution is slower and passes through at least three distinct historical phases.

[7] We employ value added instead of sales to reflect more accurately the proper interpretation of our model, where firms are assumed to be completely vertically integrated, and thus sales and value added are identical.

of the extreme cases of a mature and a catch-up economy. The evidence for convergence to a high and stable level of R&D is striking, both over time and between these countries. Even more remarkable is the fact that these values correspond very closely with those predicted by our model. The range of 10—25% of R&D to value added, independent of technological opportunity, thus seems to be both a long-period invariant of capitalist development, as captured in our model, and an empirically verifiable phenomenon.

While we do not wish to overburden such a simple model with historical interpretations, the point must still be made that it would be unfortunate to restrict the concept of endogenous growth to steady-state growth paths with no real structural development, social learning, and historical contingency. For this reason an evolutionary approach appears to offer an attractive alternative explanation of how an economy can 'bootstrap' itself in historical time through a succession of growth phases and market structures. The concepts of growth or development stages, takeoffs and changes of regime were prominent in a classical line of thought associated with Marx, Schumpeter and later Rostow. The criticism that these theories smacked of rigid mechanistic determinism is overcome by the artificial worlds methodology, which demonstrates that reversions, variable delays, and path dependence resulting from underlying stochasticity and nonlinearity cannot be excluded. Needless to say, such a broad and differentiated perspective on economic growth has mostly fallen by the wayside in the postwar literature on growth and development (cf. Rostow 1990).

Our model also demonstrates that a bounded rationality approach to the theory of the firm, coupled with an evolutionary framework for analyzing market selection and collective learning, does yield dividends both in terms of explaining how identifiable patterns of behaviour emerge from *profit-seeking* rather than completely rational *profit-maximizing* assumptions, and how market structures and growth regimes may be simultaneously endogenized. We believe that the apparent inconvenience of bounded rationality and an artificial worlds, computer-based modelling strategy, is more than outweighed by the ability to go far beyond the mere reproduction of conventional wisdom and open up a range of phenomena and relationships to theoretical and quantitative empirical study that historians have repeatedly emphasized but economists have for the most part ignored.

Appendix

A summary of the parameters and the values employed in the runs analyzed in the paper is presented below.

$q = 10$	number of firms
$m = 0.9$	parameters of the Phillips curve eq. 1
$n = 1$	
$\alpha = 1$	Soete-Turner coefficient eq. 2
γ (endogenous)	R&D/investment ratio in eq. 2
$c = 3$	capital-output ratio
$\sigma = 0$	rate of physical depreciation in eqs. 2 and 3
$\beta = 0.01$	rate of growth of labour force eq. 3
$\delta = 1$	ratio of productivity in goods and R&D sectors
$\tau = 0.06$	proportional jump in labour productivity eq. 4
A (variable)	innovation slope in eq. 5
$\rho_{min} = 0.01$	autonomous rate of innovation eq. 5
ϕ_1 (variable)	type 1 spillover coefficient in eq. 7
ϕ_2 (variable)	type 2 spillover coefficient in eq. 7
$L = 5$	lag for R&D moving average eq. 8
$\kappa = 4$	catch-up parameter eq. 10
$\Pi = 0.02$	mutation probability eq. 11
$s = 0.02$	standard deviation of mutation step size eq. 11
$\mu = 0.02$	maximum imitation probability eq. 12
$E = 0.005$	exit level in employment share
$b = 0.1$	labour productivity bandwidth for entrants

References

Aghion, P. and Howitt, P., 1992, "A Model of Growth through Creative Destruction", *Econometrica,* **60:** 323-351.

Arrow, K., 1962, "Economic welfare and allocation of resources for invention", in *The Rate and Direction of Inventive Activity,* ed. R. Nelson, Princeton: NBER/Princeton University Press.

Arthur, W.B., 1991, "Designing Economic Agents that Act Like Human Agents: A Behavioral Approach to Bounded Rationality", *American Economic Review. Papers and Proceedings,* **81:** 353-359.

Binmore, K.G. and Samuelson, L., 1993, "Muddling Through: Noisy Equilibrium Selection", Alicante: paper presented at the workshop "Learning and Industrial Dynamics", May 31-June 2, 1993.

Cohen, W.M. and Levinthal, D.A., 1989, "Innovation and Learning: The Two Faces of R&D", *Economic Journal,* **99:** 569-596.

Cyert, R.M. and March, J.G., 1963, *A Behavioral Theory of the Firm*, Englewood Cliff, NJ: Prentice-Hall.

Foster, D. and Young, P., 1990, "Stochastic Evolutionary Game Dynamics", *Theoretical Population Biology,* **38:** 219-232.

Friedman, M., 1953, "The Methodology of Positive Economics", in *Essays in Positive Economics,* Chicago: Chicago University Press.

Friedman, D., 1991, "Evolutionary Games in Economics", *Econometrica,* **59:** 637-666.

Goodwin, R.M., 1967, "A Growth Cycle", in *Socialism, Capitalism and Economic Growth,* ed. C.H. Feinstein, London: Macmillan.

Helpman, E., 1992, "Endogenous Macroeconomic Growth Theory", *European Economic Review,* **36:** 237-267.

Hofbauer, J. and Sigmund, K., 1988, *The Theory of Evolution and Dynamical Systems,* Cambridge: Cambridge University Press.

Kamien, M.I. and Schwartz, N.L., 1982, *Market Structure and Innovation*, Cambridge: Cambridge University Press.

Kandori, M., Mailath, G.J. and Rob, R., 1993, "Learning, Mutations, and Long Run Equilibrium in Games", *Econometrica,* **61:** 29-56.

Lane, D.A., 1993, "Artificial Worlds and Economics. Parts 1 and 2", *Journal of Evolutionary Economics,* **3:** 89-108, 177-197.

Langton, C.G., (ed.), 1989, *Artificial Life,* Redwood City, CA: Addison-Wesley.

Langton, C.G., Taylor, C., Farmer, J. D. and Rasmussen, S., (eds), 1992, *Artificial Life II,* Redwood City, CA: Addison-Wesley.

Lucas, R., 1988, "On the Mechanisms of Economic Development", *Journal of Monetary Economics,* **22:** 3-42.

Nelson, R.R., 1959, "The Simple Economics of Basic Scientific Research", *Journal of Political Economy,* **67:** 297-306.

Nelson, R.R., 1990, "What is Public and What is Private about Technology?", Berkeley: CCC Working Paper No. 90-9.

Nelson, R.R. and Winter, S.G., 1982, *An Evolutionary Theory of Economic Change,* Cambridge MA: The Belknap Press of Harvard University Press.

Romer, P.M., 1986, "Increasing Returns and Long-Run Growth", *Journal of Political Economy,* **94:** 1002-1037.

Romer, P.M., 1990, "Endogenous Technological Change", *Journal of Political Economy,* **98:** 71-102.

Rostow, W.W., 1990, *Theorists of Economic Growth,* Oxford: Oxford University Press.

Scherer, F.M and Ross, D., 1990, *Industrial Market Structure and Economic Performance,* Third Edition, Boston: Houghton Mifflin Company.

Schwefel, H.P., 1981, *Numerical Optimization of Computer Models,* Chichester: John Wiley.

Selten, R., 1989, "Evolution, Learning, and Economic Behaviour", *Games and Economic Behaviour,* **3:** 3-24.

Silverberg, G., 1984, "Embodied Technical Progress in a Dynamic Economic Model: the Self-Organization Paradigm", in *Nonlinear Models of Fluctuating Growth,* ed. R. Goodwin, M. Krüger, and A. Vercelli, Berlin, Heidelberg, New York: Springer Verlag.

Silverberg, G. and Lehnert, D., 1993, "Long Waves and 'Evolutionary Chaos' in a Simple Schumpeterian Model of Embodied Technical Change", *Structural Change and Economic Dynamics,* **4**: 9-37.

Silverberg, G. and Verspagen, B., 1994a, "Learning, Innovation and Economic Growth: A Long-Run Model of Industrial Dynamics", *Industrial and Corporate Change,* **3**: 199-223.

Silverberg, G. and Verspagen, B., 1994b, "Collective Learning, Innovation and Growth in a Boundedly Rational, Evolutionary World", *Journal of Evolutionary Economics,* **4**: 207-226.

Simon, H.A., 1986a, "Rationality in Psychology and Economics", in *The Behavioral Foundations of Economic Theory,* ed. R.M. Hogarth and M.W. Reder, special issue of *The Journal of Economics,* **59**: 209-224.

Simon, H.A., 1986b, "On the Behavioral and Rational Foundations of Economic Dynamics", in *The Dynamics of Market Economies,* ed. R.H. Day and G. Eliasson, Amsterdam: North Holland.

Soete, L. and Turner, R., 1984, "Technology Diffusion and the Rate of Technical Change", *Economic Journal,* **94**: 612-623.

Young, H.P., 1993, "The Evolution of Conventions", *Econometrica,* **61:** 57-84.

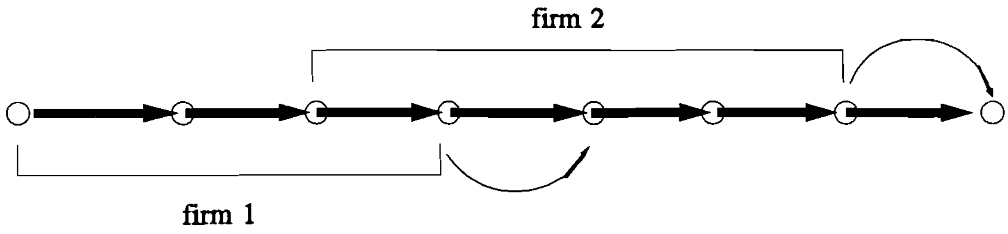

Figure 1. The technology space is a simple directed graph. Firms' current capital stocks are bracketed, and their next innovations are shown by the circular arrows.

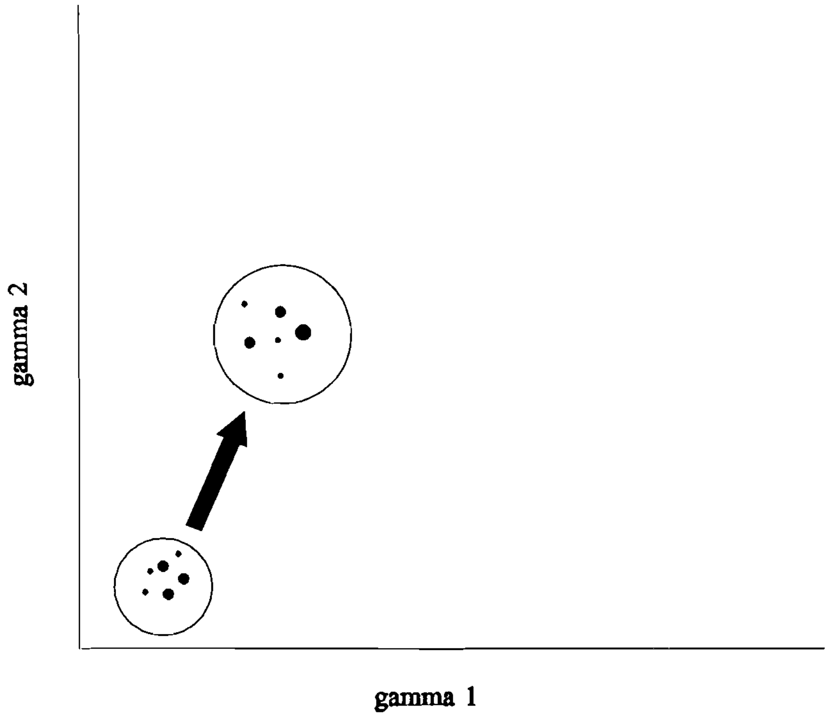

Figure 2. The space of behavioral evolution. A firm's strategy is represented as a point in a two-dimensional space whose diameter corresponds to the firm's market size. In the course of time the economy 'cloud' shifts through this space.

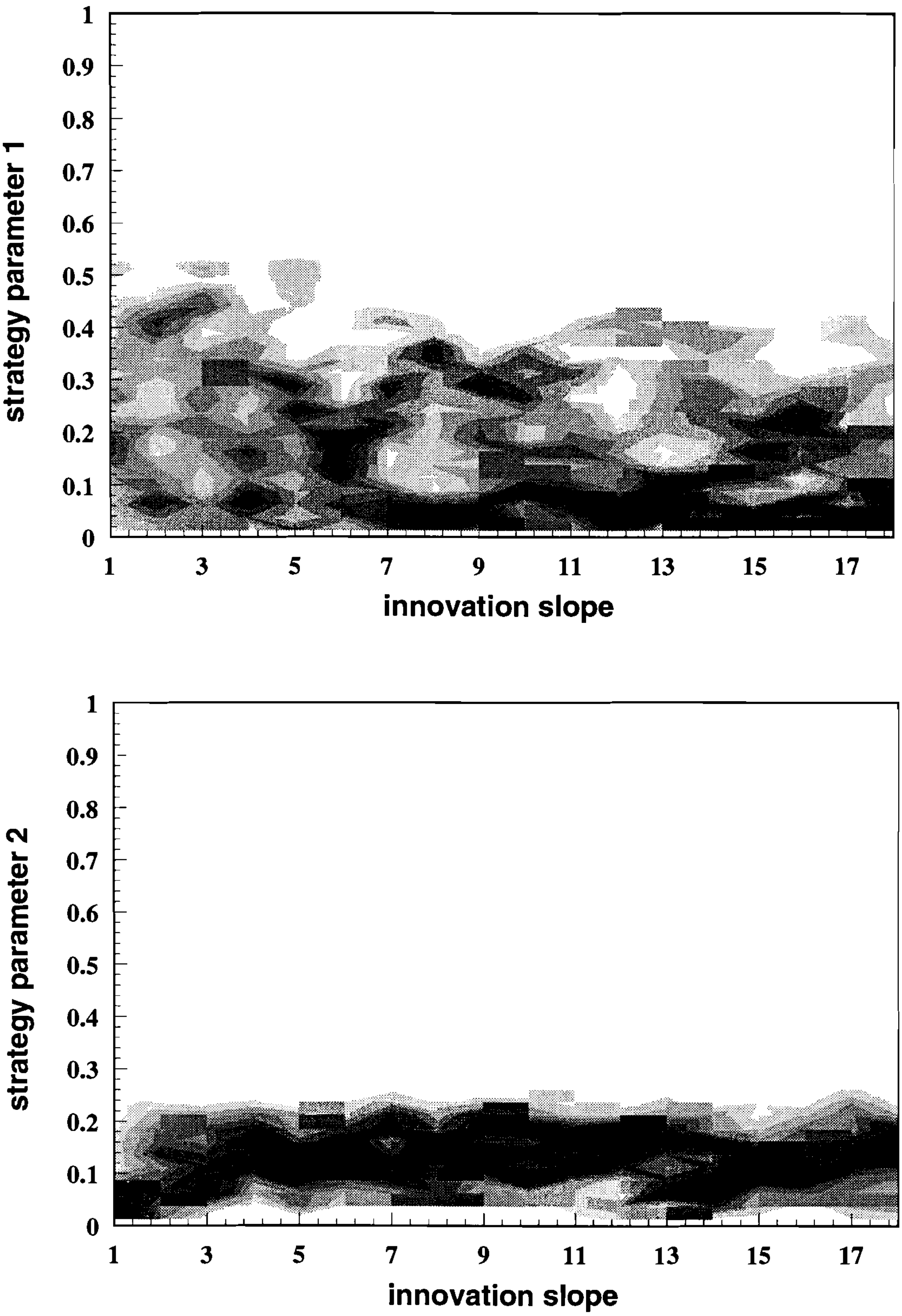

Figure 3. Histograms of strategy parameters from pooled data of five runs per value, last 1000 years of 8000 year runs (random generation), for a linear innovation function.

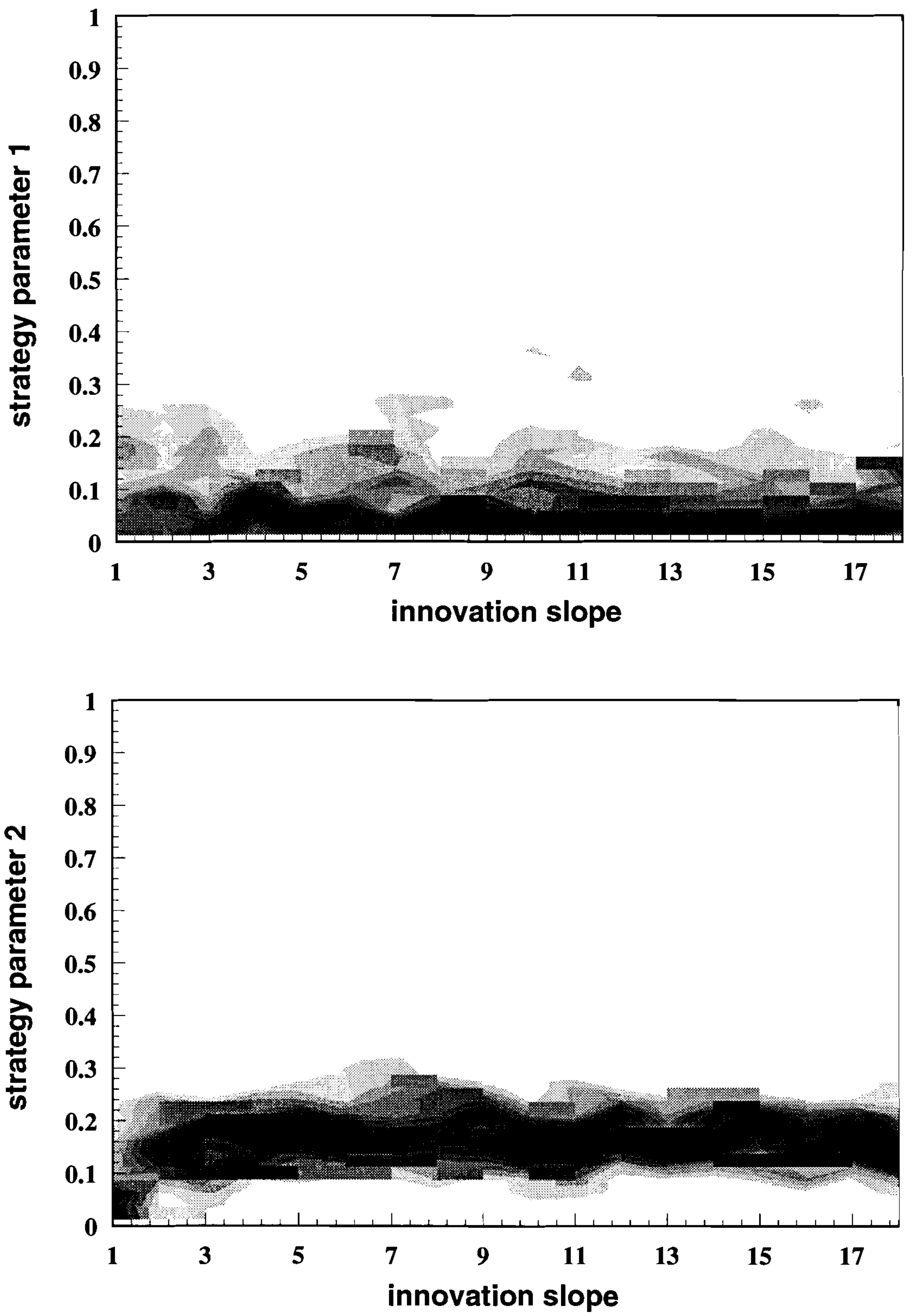

Figure 4. Histograms of strategy parameters from pooled data of five runs per value, last 1000 years of 8000 year runs (spontaneous generation), for a linear innovation function.

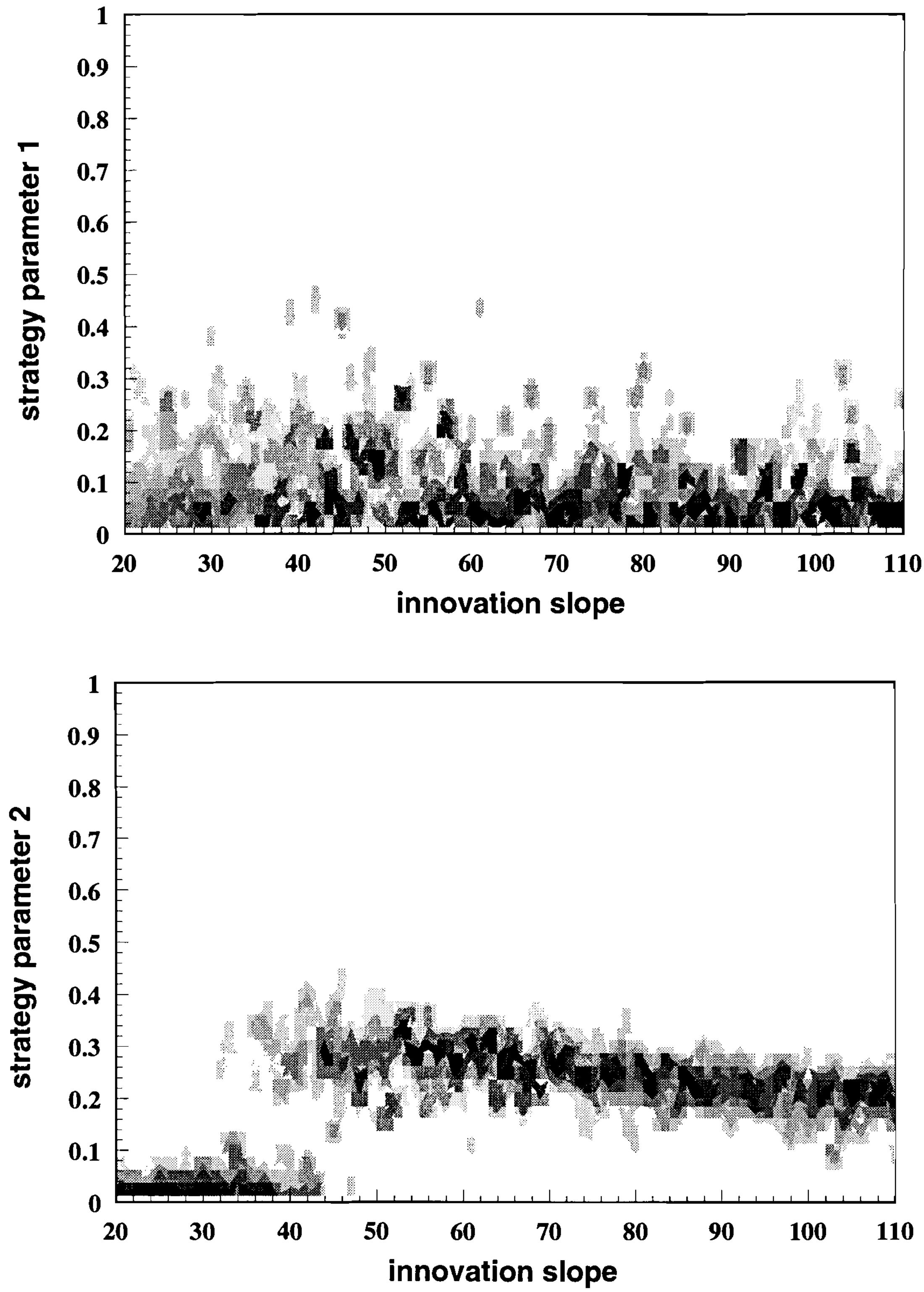

Figure 5. Histograms of strategy parameters from pooled data of five runs per value, last 1000 years of 8000 year runs (spontaneous generation), for a logistic innovation function.

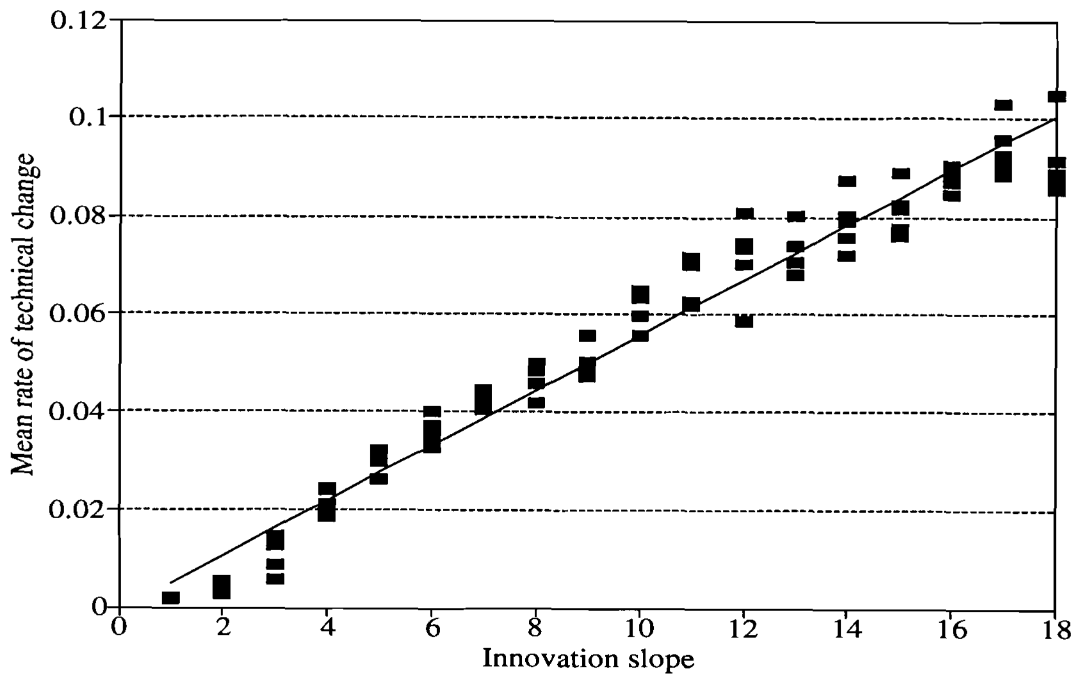

Figure 6. The rate of technical change is an increasing function of the innovation slope (A = 10, 50 firms, spontaneous generation).

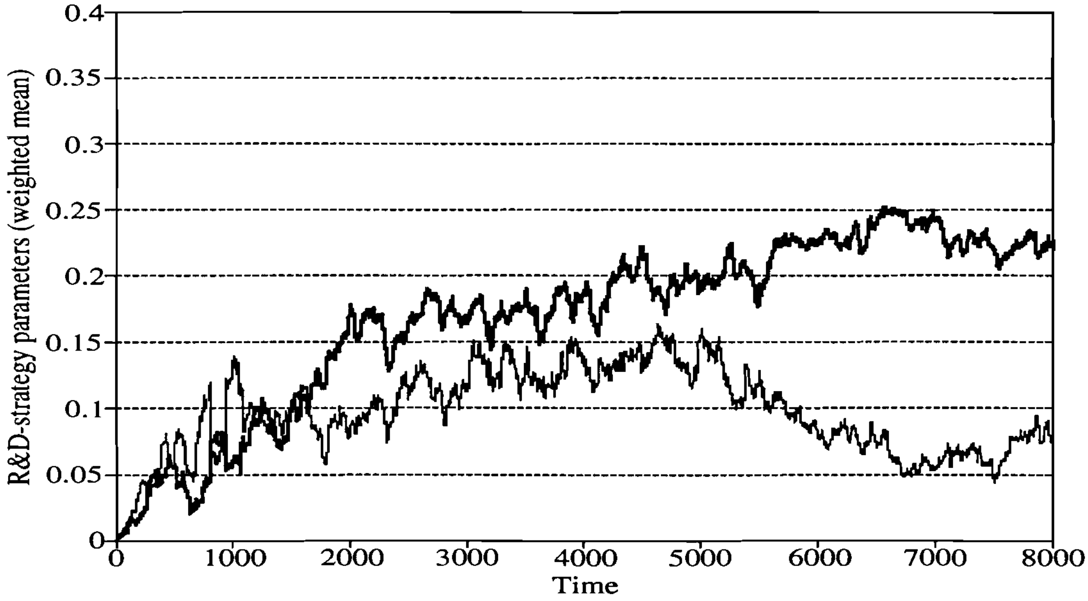

Figure 7. Time paths of market share-weighted-strategy parameters in a spontaneous generation run (50 firms, τ = 0.04). The light line is γ_1, the heavy line is γ_2.

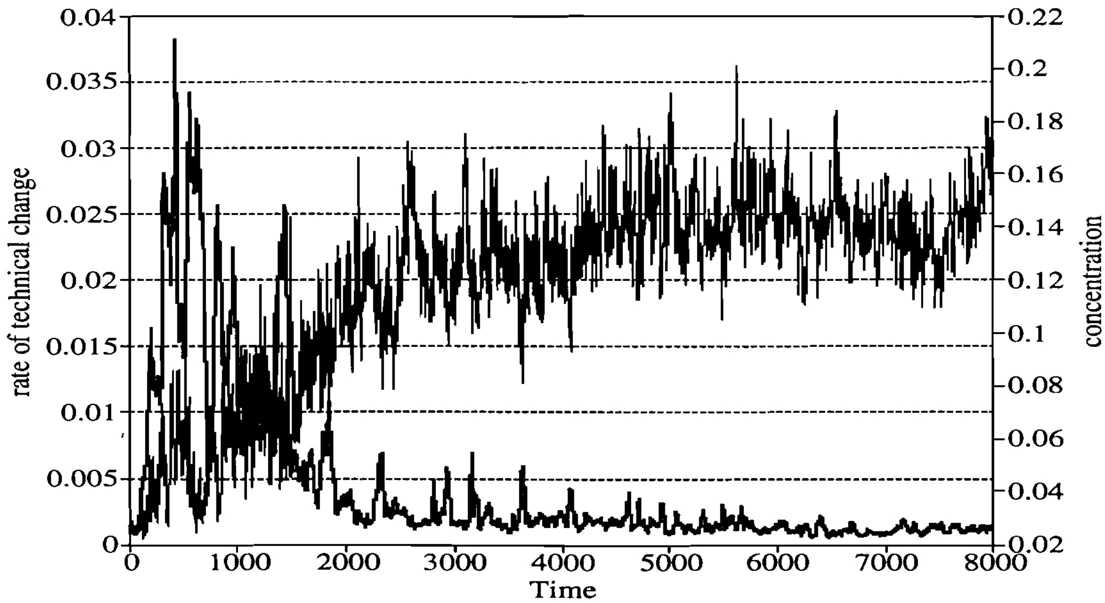

Figure 8. Time paths of rate of technical change (light line) and Herfindahl index of concentration (heavy line) for the same run as in Figure 5.

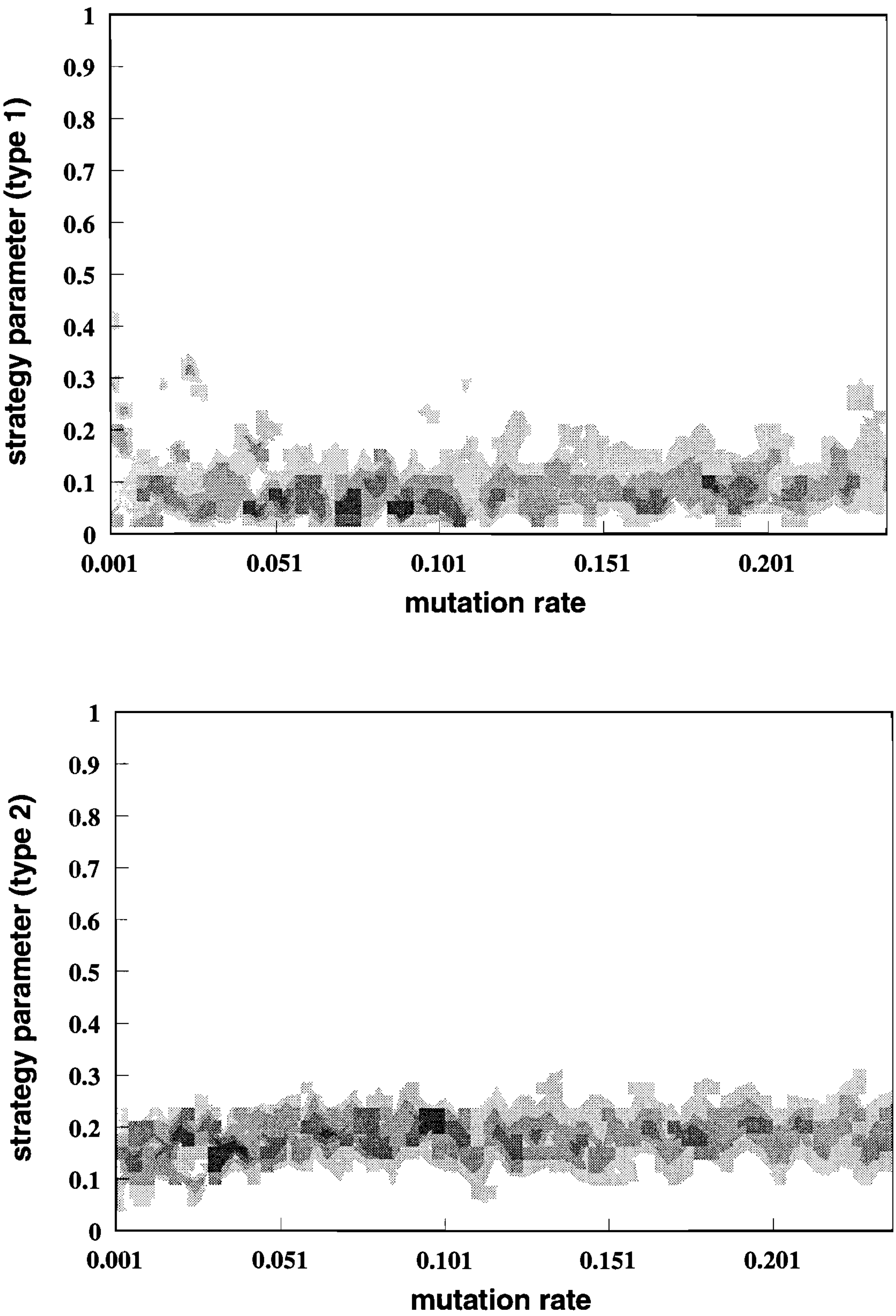

Figure 9. Histograms of strategy parameters for varying rates of the mutation probability (A = 10).

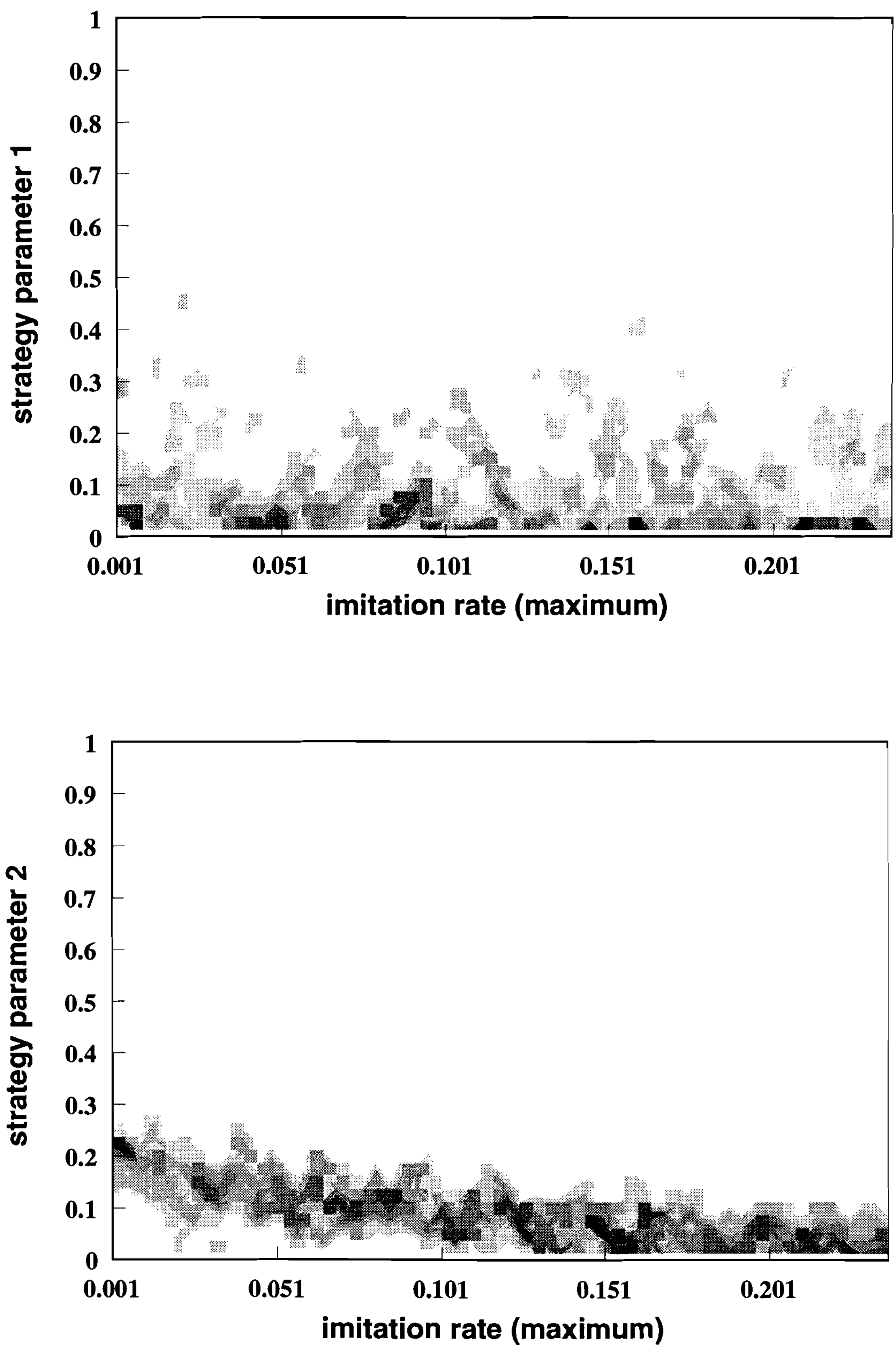

Figure 10. Histogram of strategy parameter for varying rates of the maximum imitation parameter (A = 10).

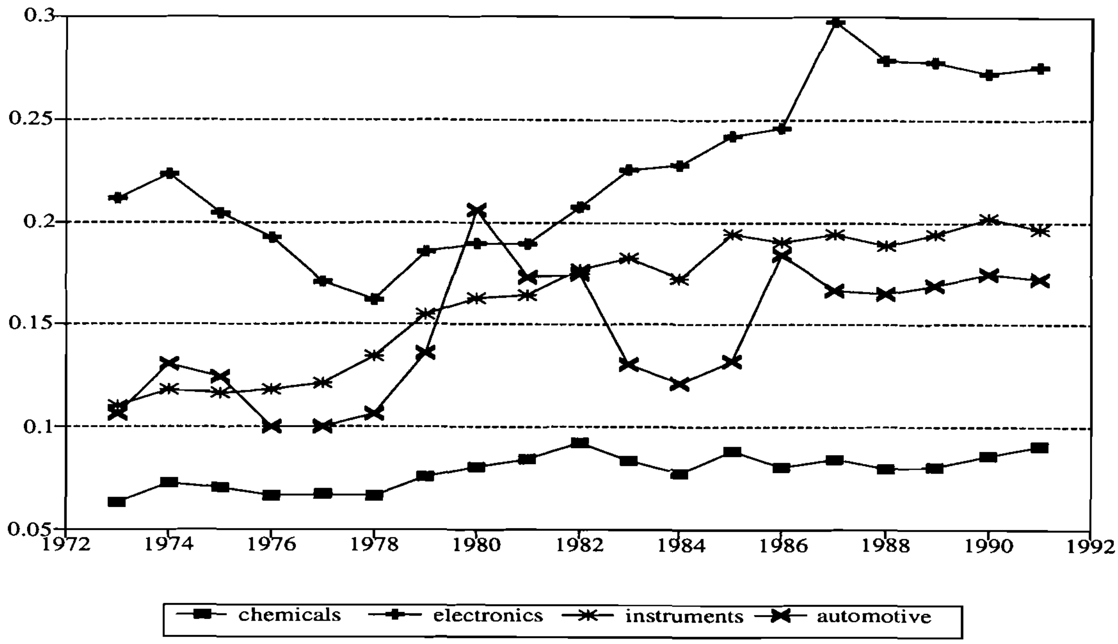

Figure 11. Ratio of R&D to value added for five industries in the USA. (Note: chemicals does not include pharmaceuticals.)

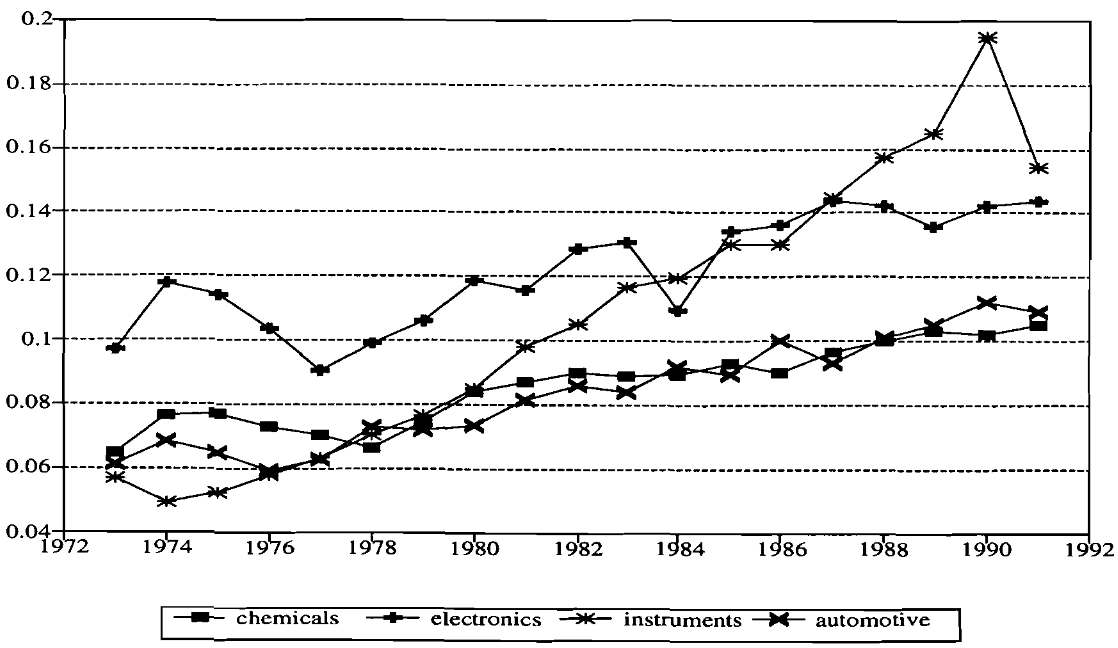

Figure 12. Ratio of R&D to value added for five industries in Japan. (Note: chemical does not include pharmaceuticals.)