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EXAMINING CORPORATE POLICY USING MULTIATTRIBUTE UTILITY ANALYSIS

Ralph L. Keeney

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Abstract

This paper illustrates the formalization of preferences over the fundamental objectives of a corporation. Specifically a von Neumann-Morgenstern utility function with ten attributes is assessed for members of Woodward-Clyde Consultants. The objectives and their associated measures of effectiveness are first specified and structured in a hierarchy. The objectives concerned personal, professional, and financial goals. The assessment of one individual's utility function is presented in detail. Current uses and potential uses of the assessment procedure and the resulting utility function are discussed.

Every corporation periodically asks itself: " How should we run our business?" More specifically, this raises such questions as: Given the complex social, economic, technological, and political characteristics of our society, which management policies should we adopt now? Are these policies consistent with our personal objectives, with the desires of our shareholders, and with our social value structure? If we choose policy A, will it be possible to account for the contingencies which may arise in the near future and adapt accordingly? How can we best maintain the leadership position in our field and simultaneously, keep the vitality of our organization? All of these are crucial questions which deny the simple dollars and cents answers which are mythically supposed to be appropriate for almost all "business" decisions.

Since early 1972, Woodward-Clyde Consultants, a holding firm for several professional-service consulting firms has used some innovative approaches based on multiattribute utility theory [2,3,5,8] to help them examine questions such as those

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[1]We would like to thank the management of Woodward-Clyde Consultants for its permission to discuss this work. The assistance of Dr. Keshavan Nair of Woodward-Clyde in writing this paper is greatly appreciated.

raised above.[2] Although this effort is still in progress, it is sufficiently interesting and informative to discuss. Two aspects of this effort seem to be unique. First, multi-attribute utility functions over attributes measuring fundamental objectives of the corporation have been assessed for many executives at Woodward-Clyde. Second, this work was done not to evaluate a specific decision, but rather to aid communication among the decision makers: To grapple with fundamental issues of the firm, to determine and examine differences of opinion in a quantitative fashion, and to aid in generating creative alternatives in solving corporate problems.

The affiliate consulting firms of Woodward-Clyde Consultants operate mainly in the geotechnical engineering and environmental areas. Problems they examine include design of earth dams, siting and design of nuclear power plants, geotechnical and environmental studies associated with pipeline systems (e.g., the Trans-Alaska pipeline), and design of structures for earthquake prone regions. None of the affiliates build any products (e.g. roads, dams, power plants); they are exclusively professional-service consulting firms. Collectively, their fees received in 1973 were approximately twenty-five million dollars, and historically, this has increased at approximately twenty percent annually. All the shareholders of Woodward-Clyde must be senior professionals on the staff of one of the affiliates.

In 1972, Richard J. Woodward, the Chairman of the Board of Woodward-Clyde Consultants, appointed a long-range planning committee whose assignment included "the development of a long-range plan for Woodward-Clyde Consultants that includes quantified objectives and is responsive to the Statement of Purpose and Standing Policies." After this original committee reported, the 1973 and 1974 Long Range Planning Committees have successively updated the objectives of Woodward-Clyde and examined policy alternatives in terms of these objectives. Douglas C. Moorhouse was the chairman of each of these three committees. Dr. Keshavan Nair, a Vice-President of Woodward-Lundgren and Associates, one of the affiliates of Woodward-Clyde, was also a member of these committees.

Much of the work discussed here, specifically Sections 2 through 5, concerning the structuring of attributes and assessing the utility function, was done jointly by Dr. Nair and myself, working as a consultant to Woodward-Clyde. Section 1

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[2] In November, 1974, Woodward-Clyde made some very broad organizational changes. It is no longer a holding firm but rather one consulting firm with five regional divisions. The work described in this section was done from 1972 through October 1974, so the organizational structure which prevailed during that period is described. The subsequent organizational changes are briefly summarized at the end of the paper.

discusses the original Long-Range Planning Committee's work, which has served as an excellent basis on which to build. The final Section 6 surveys some of the specific uses being made of Woodward-Clyde's utility function. The Appendix summarizes the main technical terms, preferential independence and utility independence, and the main theoretical result used in the paper.

1. The 1972 Objectives and Measures of Effectiveness

The basic approach taken by the 1972 Long Range Planning Committee to fulfill its mission was 1) to establish the primary objective of the firm, 2) to divide this into subobjectives, and 3) to conduct a deficiency analysis indicating discrepancies between present state and desired state on each objective. By weighting the various objectives, the deficiencies were ranked in order of importance and policies recommended for eliminating these deficiencies.

The overall objective of Woodward-Clyde was provided by a sentence in their Statement of Purpose: "The combined efforts of Woodward-Clyde Consultants and its affiliates are directed toward the creation and maintenance of an environment in which their employees can realize their personal, professional, and financial goals." It was felt that growth was essential in the achievement of this objective.

The hierarchy of objectives developed by the 1972 Long Range Planning Committee is presented in Figure 1. The numbers in parenthesis in the box with each objective indicates the division of weight among subobjectives. More will be said about this later. In Table 1, the weights of each of the attributes associated with the lowest-level objectives and the range of each attribute are identified.

It was implicitly assumed that an additive value function [1,8]

$$v(x_1,x_2,\ldots,x_{12}) = \sum_{i=1}^{12} k_i\, v_i(x_i) \quad ,$$

where the x_i's represent levels of the attributes, each v_i is a value function over the i^{th} attribute, v and the v_i's are scaled zero to one, and the weights, that is the k_i's sum to one, was appropriate.[3] For each attribute, component value functions were constructed and present states and desired states, defined as the practical maximum felt to be achievable, were identified. Deficiency on each of these lowest-level objectives was then calculated by multiplying the weight of the objective times the difference in the value of its present and desired states. This indicated "areas" where approvement was needed.

Four shortcomings of the 1972 "quantification of objectives" might be categorized as follows:

1) the weights were assigned to each objective without explicitly considering the range of the associated attributes,

2) the component value functions were estimated by a direct value estimation technique independent of each other,

3) the overall objective function, being a value function, was not appropriate for examining policies with uncertain consequences, and

4) the additive value structure did not lend itself to investigating overlap among the objectives.

Even with these weaknesses, the Long Range Planning Committee and the Board of Directors felt this quantification of objectives was a big improvement over informally articulated objectives. This set of objectives and measures has proven to be an excellent basis for modification and improvement, the substance of which we begin to describe in the next section.

Before proceeding, let us briefly remark on aspects of the attributes and their measurement units which may not be clear from Table 1. For the first attribute, using the number of shares requested divided by fees implicitly assumes the cost of a share is known in order to make the measure readily interpretable. The measure of the scope of services offered is an index meant to indicate breadth in handling the interdisciplinary projects increasingly requested by society. With relevant experience, the idea is to have the staff available to do quality work on those projects which the Woodward-Clyde affiliates would like to do. For formal training, the number of degrees per professional staff member is defined as follows:

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[3]A value function provides a ranking of the consequences $(x_1, x_2, \ldots, x_{12})$. It is not necessarily a von Neumann-Morgenstern utility function since its expected value cannot be used to indicate preference in situations involving uncertainty.

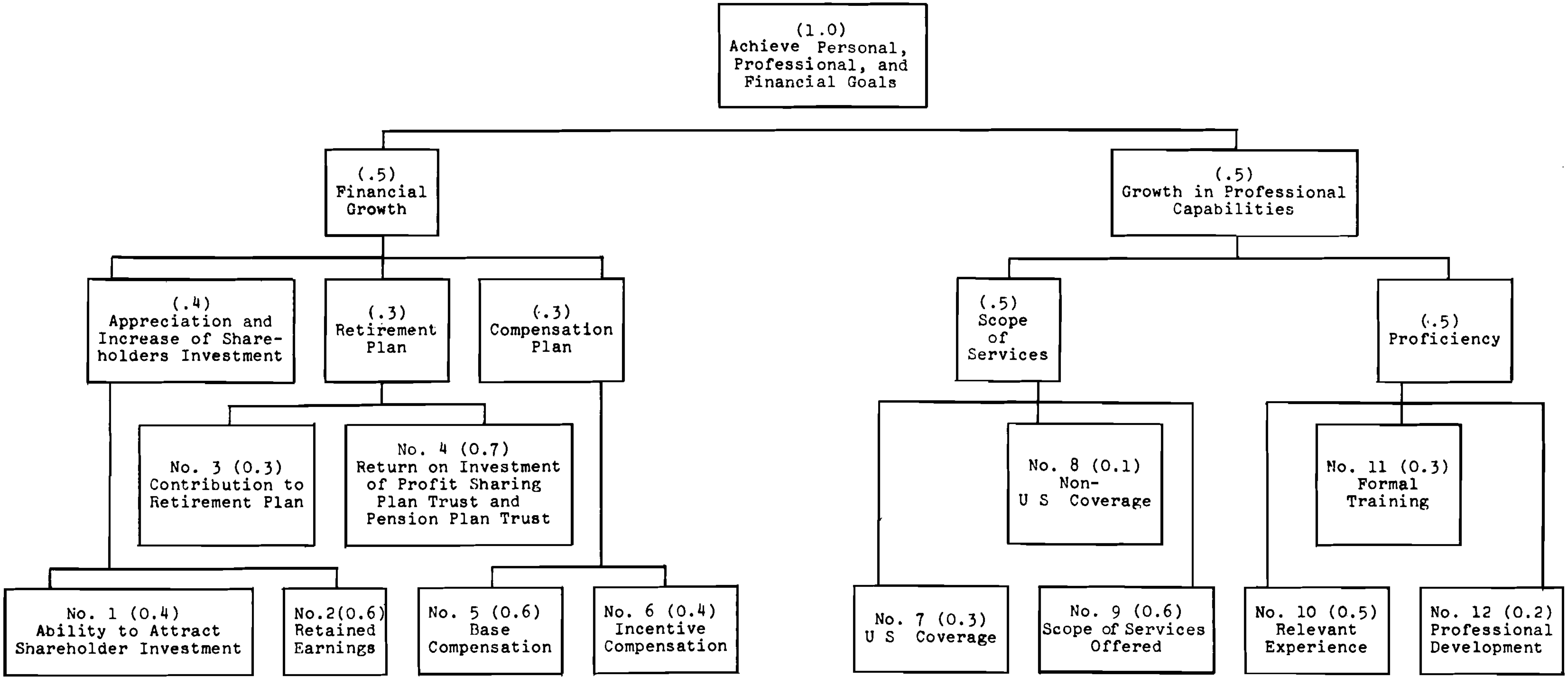

Figure 1. 1972 objectives hierarchy of Woodward-Clyde Consultants.

Table 1. 1972 attributes for Woodward-Clyde Consultants.

ATTRIBUTE	MEASUREMENT UNIT	RANGE	ATTRIBUTE WEIGHT
Ability to attract shareholders investment	(Number of shares requested / fees) %	0-5	.08
Retained earnings	% of fees	0-8	.12
Contribution to retirement plan	% of fees	0-10	.045
Return on investment for retirement plan	% of investment	0-20	.105
Base compensation	% annual increase	0-20	.09
Incentive compensation	% of fees	0-8	.06
US coverage	(Geographic centers adequately covered / Centers where relevant work can be generated) %	25-100	.075
Non-US coverage	(Geographic centers adequately covered / Centers where relevant work can be generated) %	0-50	.025
Scope of services offered	(Number of disciplines having threshold capability / Number of synergistic disciplines required by society) %	25-100	.15
Relevant experience	(Existing man-years experience / Required man-years experience) %	25-100	.125
Formal training	Number of degrees per professional staff member	1-3	.075
Professional development	% of fees	0-2	.05

a doctorate is three, a masters degree two, and a bachelors one. Professional development includes attending management or technical seminars, holding in-house study sessions, etc.

2. Clarifying the Measures of Effectiveness

One of the first issues Dr. Nair and I jointly considered was whether the measures of effectiveness communicated the desired information and could be used in practice. For each objective, the question "Can a better attribute be found?" was asked. In several cases, the answer was yes. Let us discuss some examples.

a) Ability to Attract Shareholders Investment. The measurement unit for this attribute was changed to the dollar value of shares requested divided by the fees. Thus in interpreting trends, and simply in evaluating various levels of the attributes, one does not need to keep the value of the shares implicitly in mind.

b) Scope of Non-US Coverage. The 1974 Long Range Planning Committee changed this measure to percentage of the United States business in terms of fees received. It was the Committee's viewpoint that the major reason for expanding overseas was to reduce the consequences of a possible recession in the United States and to take advantage of current foreign opportunities. Since Woodward-Clyde will remain primarily a US operation in the forseeable future, the new measure both is more easily quantifiable than the previous one and also more directly indicates vulnerability to domestic recessions.

c) Relevant Experience and Professional Development. As demand for Woodward-Clyde services increases, the need to increase their relevant experience grows. The 1972 measure of relevant experience indicated the level at any given time, as opposed to focusing on the increase of relevant experience. Increased relevant experience is funded out of the Professional Development budget and usually consists of opportunities for employees to work on projects under experienced personnel at company expense and to take specialized courses in areas of their practice. Because it is the increase in relevant experience which is currently important at Woodward-Clyde, the measure was changed to percent of fees committed to the relevant experience program.

This change of the relevant experience measure required a redefinition of the components of the professional development measure. In 1972, the latter measure included fees used for obtaining relevant experience. However, with the new relevant experience measure, the professional development measure must explicitly exclude the fees used for acquiring relevant experience.

d) Formal Training. The measure remained the same for formal training but the desirability of particular levels has greatly changed. The value function in this case is interesting in that it is not monotonic. It is low at a level of 1, since all professionals then only have a bachelors degree, and increases to a peak and then falls rapidly as the level of degrees increases. With a level of 3, the firm would consist entirely of professionals with doctorates. In 1972, the desired state was identified as 2.25, the peak of the value function. On further examination, this level seemed high. If just twenty-five percent of the professionals of Woodward-Clyde had only a bachelors, a minimum of fifty percent would have to have a doctorate to get the average level to the "desired state" 2.25.

As an aid to thinking about the implications of different levels of "degrees per professional," Table 2 was constructed. For evaluating preferences over average degree levels, an individual is meant to select the best distribution of degrees for each average level, and then compare these "best" distributions.

3. Checking for Independence Conditions

To structure a utility function over the twelve attributes of Table 1, modified as indicated in the previous section, the process began by examining whether pairs of attributes were preferentially independent of their complements.[4] In most cases it seemed appropriate to assume preferential independence, but let us indicate three situations where this was not so.

In examining preferential independence assumptions involving the attribute "ability to attract shareholder investment," the Long Range Planning Committee came to the agreement that it was redundant based on present policy. This attribute was meant to indicate the ability and desirability for principals to invest in the corporation. The Committee felt the desirability aspect was adequately captured by retained earnings. On the other hand, the ability to invest was measured by both incentive compensation and base compensation. For these reasons, the "ability to attract shareholder investment" was dropped from the list of attributes.

In another case it at first seemed advantageous to subdivide the objective concerning base compensation into three groups: senior principals, junior principals and associates, and associate candidates. In effect, the current attribute "base compensation" would have been replaced by three attributes,

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[4]Initial assessments were done using Dr. Nair's preferences. Subsequently, Dr. Nair has assessed the preferences of other members of the Long Range Planning Committee. See the Appendix for a definition of preferential independence and other technical terms.

Table 2. Formal training--percent distribution of degrees.

DEGREES PER PROFESSIONAL STAFF MEMBER	5% PhD BS MS PhD	10% PhD BS MS PhD	15% PhD BS MS PhD	20% PhD BS MS PhD	25% PhD BS MS PhD	30% PhD BS MS PhD	35% PhD BS MS PhD	40% PhD BS MS PhD	45% PhD BS MS PhD	50% PhD BS MS PhD
1.5	55, 40, 5	60, 30, 10	65, 20, 15	70, 10, 20	75, 0, 25	Not possible	Not possible	Not possible	Not possible	Not possible
1.6	45, 50, 5	50, 40, 10	55, 30, 15	60, 20, 20	65, 10, 25	70, 0, 30	"	"	"	"
1.7	35, 60, 5	40, 50, 10	45, 40, 15	50, 30, 20	55, 20, 25	60, 10, 30	65, 0, 35	"	"	"
1.8	25, 70, 5	30, 60, 10	35, 50, 15	40, 40, 20	45, 30, 25	50, 20, 30	55, 10, 35	60, 0, 40	"	"
1.9	15, 80, 5	20, 70, 10	25, 60, 15	30, 50, 20	35, 40, 25	40, 30, 30	45, 20, 35	50, 10, 40	55, 0, 45	"
2.0	5, 90, 5	10, 80, 10	15, 70, 15	20, 60, 20	25, 50, 25	20, 40, 30	35, 30, 35	40, 20, 40	45, 10, 45	50, 0, 50
2.1	Not possible	0, 90, 10	5, 80, 15	10, 70, 20,	15, 60, 25	20, 50, 30	25, 40, 35	30, 30, 40	35, 20, 45	40, 10, 50
2.2	"	Not possible	Not possible	0, 80, 20,	5, 70, 25	10, 40, 30	15, 50, 35	20, 40, 40	25, 30, 45	30, 20, 50
2.3	"	"	"	Not possible	Not possible	70, 30	5, 60, 35	10, 50, 40	15, 40, 45	20, 30, 50
2.4	"	"	"	"	"	Not possible	Not possible	0, 60, 40	5, 50, 45	10, 40, 50
2.5	"	"	"	"	"	"	"	Not possible	Not possible	0, 50, 50

namely base compensation for senior principals, base compensation for junior principals and associates, and base compensation for associate candidates. It was found that one of these attributes taken together with a different attribute, say retained earnings, was not preferentially independent of its complement. The reason was that the rate at which one would substitute retained earnings for base compensation for associate candidates depended on the level of base compensation increases to the principals and associates. If these latter groups received large increases in base compensation, it seemed reasonable to give up more retained earnings to bring increases in base compensation for associate candidates up to some comparable level, than one would give up to make the same increase for associate candidates if in fact the other groups received low increases in base compensation. The concept of equity among the three groups made it inappropriate to assume preferential independence in this case.

There were two other possibilities investigated. Each pair of the three base compensation attributes was found to be conditionally preferentially independent of the third, given all other attributes are fixed at an arbitrary level. This implies there exists an additive value function, which we could have assessed, over the three attributes. The alternative was to use the original aggregated base compensation attribute. It was felt that members of the Long Range Planning Committee could keep the equity considerations in mind when using the aggregated attribute. Therefore, since it is simpler to use one attribute than the three component attributes, the former was chosen.

Base compensation and incentive compensation do have some overlap in purpose and, because of this, the latter paired with, for instance, retained earnings is not exactly preferentially independent of its complement. However, the overlap is not great since the function of the former is to provide a solid salary for competent work within the "normal" call of duty, whereas the function of the later is to provide motivation and reward for efforts "beyond" the call of duty. Hence after considerable checking, it was decided that it was a reasonable approximation to assume the preferential independence condition. This "appropriateness" decision was taken in conjunction with the decision to eliminate the attribute "ability to attract shareholder investment" from the list in Table 1.

It was decided that the two attributes concerning the retirement plan should be aggregated into one called "growth in retirement plan," since in fact both seemed to meet the same fundamental objective. Woodward-Clyde desires that any participant in their retirement plan receive a combined amount from the plan and social security equal to fifty percent of his or her last five years' average salary. The new measure for "growth of retirement plan" is the annual increase of assets in the retirement plan. Its range is zero to thirty percent, and it should be clear that this excludes the social security benefits. In effect, this change is simply moving up the objectives hierarchy of Figure 1 for a quantitative assessment of retirement plan consequences.

4. The 1974 Objectives and Measures of Effectiveness

The objectives and attributes updated from the original 1972 list are given in Table 3. After considerable examination, Dr. Nair felt that it was appropriate to assume that for the ranges given in the table, each pair of attributes was preferentially independent of its complement. The reasonableness of this assumption has been preliminarily accepted by each of the other members on the 1974 Long Range Planning Committee.

5. Assessing the Utility Function

The preferential independence conditions imply [4,6] that an additive value function exists over the ten attributes in Table 3. From the theorem stated in the Appendix, by verifying that just one attribute is utility independent of its complement, either a multiplicative or additive utility function is appropriate to quantify preferences. It was verified that retained earnings was in fact utility independent of its complement, and utility independence was also verified for other attributes to serve as consistency checks. For future reference, it turned out, the final utility function over the attributes in Table 3 was multiplicative, and thus expressible in the form

$$1 + ku(x) = \prod_{i=1}^{10} [1 + kk_i u_i(x_i)] \quad , \qquad (1)$$

where u and the u_i's are scaled zero to one, $0 < k_i < 1$, and k is a non-zero scaling constant greater than minus one which can be evaluated from the k_i's.

The task remaining was to assess the component utility functions, assess their scaling factors, and then evaluate the k-value for the multiplicative form.

5.1 Assessing the Component Utility Functions

All the ten utility functions were assessed on a zero to one scale using the techniques discussed in Schlaifer [9]. Let us briefly consider those for retained earnings and formal training, attributes X_1 and X_9 in Table 3.

Table 3. 1974 attributes for Woodward-Clyde Consultants.

ATTRIBUTE	MEASUREMENT UNIT	RANGE
X_1 ≡ Retained earnings	% of fees	0-8
X_2 ≡ Growth in Retirement Plan	% of existing assets	0-30
X_3 ≡ Base Compensation	% annual increase	0-30
X_4 ≡ Incentive Compensation	% of fees	0-8
X_5 ≡ Scope--Geographic (US)	(Geographic centers adequately covered / Centers where relevant work can be generated)	25-100%
X_6 ≡ Scope--Geographic (Outside US)	% of U. S. business	0-50
X_7 ≡ Scope--Services Offered	(No. of disciplines having threshold capability / No. of synergistic disciplines required by society)	25-100%
X_8 ≡ Relevant Experience (annual increment)	% of fees	0-1
X_9 ≡ Formal Training	No. of degrees per professional staff member	1.5-2.5
X_{10} ≡ Professional Development (excluding relevant experience)	% of fees	0-1

The range of retained earnings is zero to eight percent, so since preferences are monotonically increasing, we set

$$u_1(0) = 0 \quad , \qquad u_1(8) = 1 \quad ,$$

where u_1 is the utility function for retained earnings. Next, by checking certainty equivalents[5] for a number of lotteries, it was verified that Dr. Nair was risk averse in terms of retained earnings. It was found that 2 ~ <0,8>, 0.75 ~ <0,2>, 4 ~ <2,8>, 5.5 ~ <4,8>, and for a check, that 4 for certain was indifferent to a 0.75 chance at 8 and a 0.25 chance at zero. The utility function consistent with these assessments is shown in Figure 2.

The assessment of the utility function for formal training led to some surprises. What was not a surprise was that preferences for levels of this attribute are not monotonic; they increase up to a maximum point and then decrease. Originally, it was the thought to assess preferences from 1 to 3 degrees per professional staff member. However, once we began this task, it became clear that with levels between 1 and 1.3 and 2.7 and 3, Woodward-Clyde could not exist in a form similar to the present. Hence our viable range was changed from 1.5 to 2.5, which were practical limits for the foreseeable future.

Next, by using the Table 2, it became clear that the previously felt optimum level of 2.25 was too high and 2.1 was chosen as an alternative after some consideration. It was also felt that the undesirability of 1.5 or 2.5 degrees per professional was about equally as bad so u_9, the utility function for formal training was scaled by

$$u_9(1.5) = u_9(2.5) = 0 \quad , \qquad u_9(2.1) = 1 \quad .$$

Again with the aid of Table 2, it was concluded that 1.7 ~ <1.5,2.1>, 1.8 ~ <1.7,2.1>, and 2.3 ~ 1.8. The resulting utility function is shown in Figure 2.

5.2 Assessing the Relative Scaling Factors

The ranking of the ten attribute scaling constants of the multiplicative utility function--that is, the k_i's in (1)--is

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[5]If 2 is indifferent to the lottery written <0,8>, yielding a one-half chance at 0 and a one-half chance at 8, then 2 is referred to as the certainty equivalent of <0,8>. The symbol "~" reads is indifferent to.

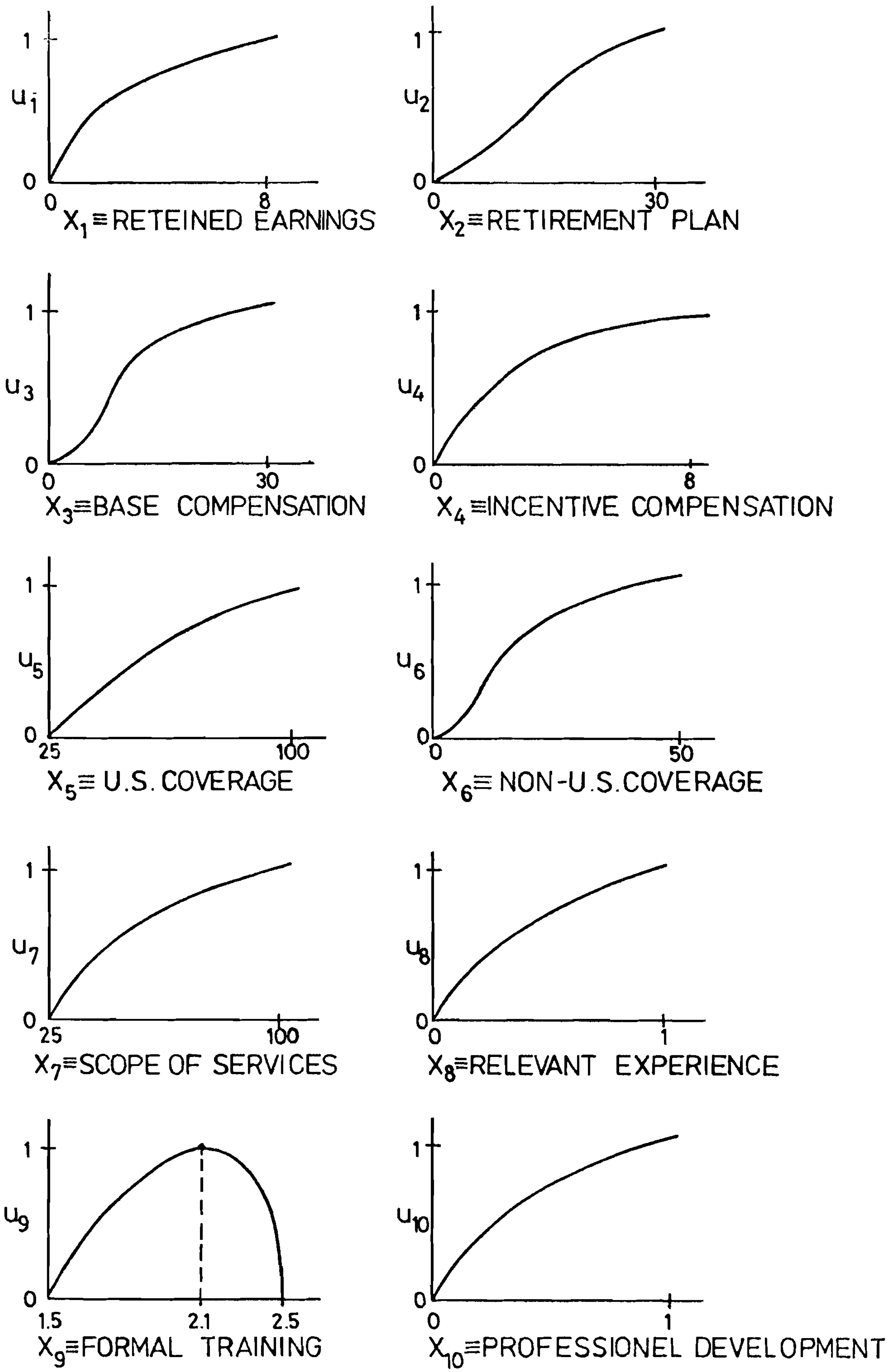

FIGURE 2: WOODWARD-CLYDE'S COMPONENT UTILITY FUNOTIONS

Table 4.

Attribute	Ranking of Scaling Factor	Range	Indifference Equivalent	Relative Scaling Factor	Scaling Factor
$X_1 \equiv$ retained earnings	1	0-8	—	k_1	.67
$X_2 \equiv$ retirement plan	7	0-30	30 of $X_2 \sim$ 3 of X_1	$k_2 = .66k_1$	.44
$X_3 \equiv$ base compensation	5	0-30	30 of $X_3 \sim$ 4 of X_1	$k_3 = .77k_1$	.517
$X_4 \equiv$ incentive compensation	9	0-8	8 of $X_4 \sim$ 2.5 of X_1	$k_4 = .58k_1$	.391
$X_5 \equiv$ U.S. coverage	6	25-100	100 of $X_5 \sim$ 3.5 of X_1	$k_5 = .72k_1$	.482
$X_6 \equiv$ non-U.S. coverage	10	0-50	50 of $X_6 \sim$ 50 of X_5	$k_6 = .5k_5$	.241
$X_7 \equiv$ scope of services	3	25-100	55 of $X_7 \sim$ 100 of X_5	$k_5 = .75k_7$	.634
$X_8 \equiv$ relevant experience	4	0-1	1 of $X_8 \sim$ 50 of X_5	$k_8 = .5k_5$	.241
$X_9 \equiv$ formal training	2	1.5-2.5	2.1 of $X_9 \sim$ 7 of X_1	$k_9 = .97k_1$	.647
$X_{10} \equiv$ professional development	8	0-1	1 of $X_{10} \sim$ 50 of X_5	$k_{10} = .5k_5$	.241
					4.505

given in Table 4. To specify their relative magnitude, Dr. Nair considered the relative desirability of consequences with one attribute at its most preferred level and all other attributes at their worst levels. He decided that the one he would most like to have at its best level was retained earnings. Thus the scaling factor associated with retained earnings is the largest. The attribute he would next prefer to have alone at its most desirable level was formal training so its scaling factor is second largest. Repeating this procedure led to the ranking of the scaling factors indicated in Table 4.

To quantitatively establish the relative values of the scaling factors, trade-offs between pairs of attributes were explicitly assessed. Dr. Nair was asked, for nine pairs of attributes, questions such as:

> Assume all attributes other than retained earnings and retirement plan are fixed at convenient levels. Now, how high would retained earnings have to be, given the retirement plan is at its lowest level, in order for you to be indifferent between this option and an alternative option with the retirement plan at its most desirable level of 30 and retained earnings fixed at its lowest level?

The responses are shown in Table 4 in the column labeled "indifference equivalent." Thus if we designate the scaling factor of X_1 as k_1, the scaling factor for X_2, for instance, must be $.66k_1$ since, using u_1 in Figure 2, the utility of a retained earnings of three percent is 0.66. This follows since the utility of three percent retained earnings, with the growth in retirement plan at its least desirable level, must equal the utility of thirty percent growth in retirement plan, with retained earnings at its minimum level. Because of the preferential independence assumptions, the levels of the attributes other than retained earnings and retirement plan do not matter. The relative values of the scaling constants are also shown in Table 4.

5.3 Selecting a Utility Function

We felt fairly confident about the relative values of the scaling constants, but to get their absolute magnitudes requires the answer to a difficult question. Dr. Nair was asked:

> What probability π_1 would you select such that you would be indifferent between option 1 with retained earnings at 8 percent and all other attributes at their least desirable levels and an alternative option 2 consisting of a lottery yielding all attributes at their most desirable level with probability π_1 or otherwise all attributes at their least desirable level?

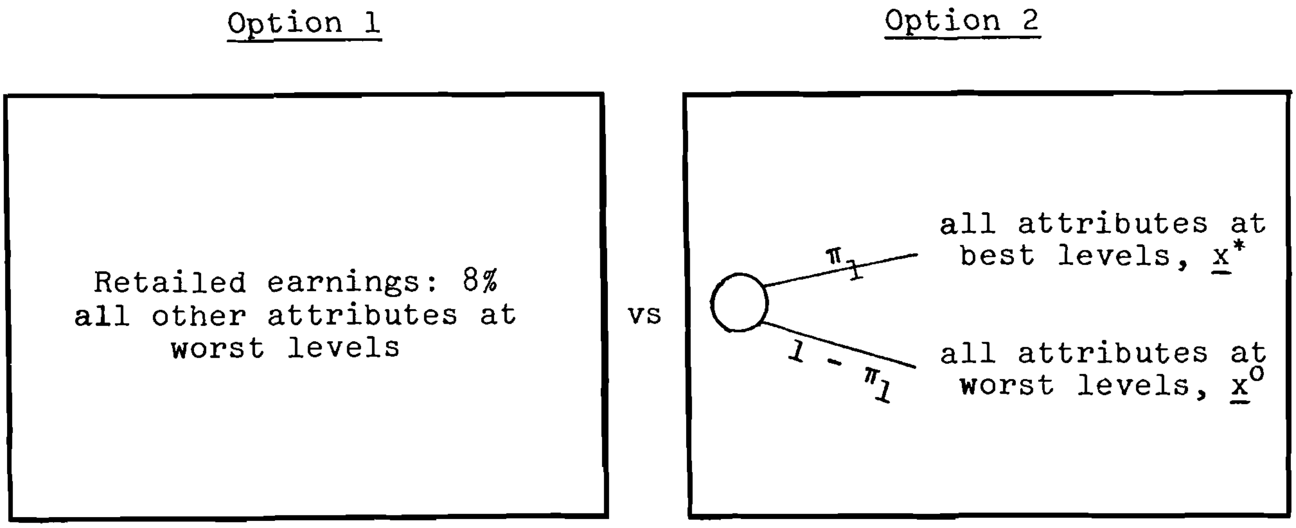

Figure 3. Adjust π_1 to get indifference.

Those two options are illustrated in Figure 3. Using a trial and error method to converge to indifference, $\pi_1 = 0.67$ was selected. This implied that the scaling factor k_1 should be 0.67, from which the values of the other scaling factors indicated in Table 4 follow.

Since the sum of the scaling factors was 4.505, we knew the multiplicative utility function (1) was appropriate to express Dr. Nair's preferences. Evaluating (1) for the most desirable consequence one finds

$$1 + k = \prod_{i=1}^{10} (1 + kk_i) \quad , \tag{2}$$

which was solved to yield

$$k = -.998 \quad .$$

Such a low level for k (it must be greater than -1) indicates a high level of complimentarity among preferences for the attributes. It is the general feeling of the Long Range Planning Committee that if retained earnings are at a high level, one can "take care of" the other attributes if proper policies are implemented. However, this feeling weakens as the time frame of reference increases. That is if our attributes represent one-year levels, Woodward-Clyde could stand a bad year with most attributes and make it up in the next year. On the other hand, if the attributes of Table 4 designate five-year averages, the desirability of waiting five years to "redistribute" high retained earnings to attributes at their lowest levels is understandibly much less. This situation, which became apparent during the assessment process, is clearly important to recognize in discussions of options affecting the future vitality of Woodward-Clyde. The original preference assessments were made using a one-year period. The results reported here are made using annual averages over a three-year period.[6]

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[6]For reference, the indifference probability π_1 for the options in Figure 3 was 0.75 when a one-year period was considered, whereas it was 0.67 for the three-year period.

5.4 Sensitivity Analysis

Because of the importance of the probability π_1 assessed to specify k_1, a small sensitivity analysis was made of this parameter using the same relative values of the scaling constants assumed in Table 4. Recall that $\underline{x}^*$ defines the consequence with all attributes at their best levels and $\underline{x}^o$ the consequence with all attributes at their worst levels. To assist in examining the implications of the various π_1 values, let us make two definitions:

$\pi' \equiv$ the probability such that a lottery with a π' chance at $\underline{x}^*$ and a $(1-\pi')$ chance at $\underline{x}^o$ is indifferent a consequence with retained earnings and formal training at their best levels and all other attributes at their worst levels,

$\hat{\pi} \equiv$ the probability such that $\langle \underline{x}^*, \hat{\pi}, \underline{x}^o \rangle$ is indifferent to the sure consequence with each attribute at its level of 0.5 utility.

The results, which were calculated using a computer program (see Keeney and Sicherman [7]) are shown in Table 5, where π_1 is first specified. Then, using the relative scaling factors from Table 4, the individual k_i's are fixed. Using these, k, π', and $\hat{\pi}$ were calculated. Further reflection and examination of Table 5 led Dr. Nair to stay with his original estimate of $\pi_1 = 0.67$ for the three-year period. Thus, the final scaling constants are those shown in the last column of Table 4.

Table 5.

π_1	Σk_i	k	π'	$\hat{\pi}$
.87	5.86	-.999	.98	.973
.74	4.96	-.999	.925	.947
.67	4.5	-.998	.884	.928
.60	4.06	-.996	.836	.903
.47	3.15	-.979	.714	.835
.34	2.25	-.900	.561	.733

6. Uses of Woodward-Clyde's Utility Function

Since the original assessments, Dr. Nair has essentially repeated the assessment procedure just described with each of the members of the 1974 Long Range Planning Committee. These assessments included verification of assumptions, assessing single-attribute utility functions, and specifying scaling constants. This resulted in some minor changes to Dr. Nair's utility function (already integrated into the previous sections) to achieve what may be referred to as a concensus corporate utility function. This obviously does not mean the Board of Woodward-Clyde will blindly make decisions with this utility function. It is being used to facilitate communication among officers of Woodward-Clyde and to help professional intuition.

The assessment process forced individuals to be a bit more precise in deciding why they felt certain levels of specific attributes were important. As previously mentioned, it also served to indicate how trade-offs among attributes depended on the time frame of reference. The general feeling of those involved in the utility function assessment may be summed up by the comment of one individual, "I've had to make trade-off decisions like this all my life, but until now the process has always been somewhat fuzzy and left me with the feeling that I didn't completely comprehend all the implications of my subjective judgments. The use of utility theory and explicit trade-offs helps considerably." With a better understanding of one's own trade-offs and preferences, it is a small wonder that it becomes easier to communicate these and discuss the issues with one's colleagues.

The process of assessing a utility function has also led to minor, but important, modifications in the overall evaluation process for long-range plans. Some objectives have been deleted or aggregated, and in other cases, several attributes have been altered to better indicate the concerns of Woodward-Clyde. Changing the attribute measure for relevant experience to reflect the yearly increase in experience is one such example.

Since several of the attributes concern distribution of income available (i.e., percent of fees), it is a simple task to use the utility function to help select the best distribution among salaries, retained earnings, incentive compensation, professional development, relevant experience, and contribution to retirement plan. With any fixed percentage of fees available, the technically feasible surface of fee distribution, as well as the distribution with maximum utility, is easily specified.

As before, the component utility functions can still be used to conduct a deficiency analysis by indicating the difference between the present state and a desired state, representing what is technically feasible in a specified time span. A bit more broadly, by calculating the gradient of the utility function in each attribute for the present state position and combining this with subjectively assessed changes in the state of each attribute for an equivalent amount of effort (time and money), one gets an indicator of policies which may be particularly fruitful to pursue.

The utility function discussed here will no doubt go through additional metamorphoses in the future years, as needs and preferences of individuals at Woodward-Clyde adjust to better reflect their position in society, the external environment, and so on. For example, the Pension Reform Act of 1974, because of certain provisions with regard to the ability of Pension and Profit Sharing Plan Trusts to invest in company stock, is likely to alter the present relative value of the attribute "growth in retirement plan" among the attributes. Woodward-Clyde Consultants is presently examining the effect of this and other external changes on the utility functions for the various individual attributes and the trade-offs between the attributes. This will be a continuing activity.

The current function does overcome the original shortcomings of the 1972 quantification of objectives outlined in Section 1. It is being used to examine present decisions which effect the future existence of the company. In addition, the Woodward-Clyde objectives hierarchy partially provides an underlying and unifying basis for evaluating long-range plans and operational activities of the affiliated firms. Several individuals at Woodward-Clyde find the multiattribute utility concept interesting and helpful. Perhaps more importantly, they are enthusiastic about potential future uses. In this regard, partially as a result of the work discussed here, a special group within Woodward-Clyde Consultants has been set up and funded to begin to transfer the concepts and techniques of decision analysis into their professional practice.

As an interesting anecdote, in late 1974 Woodward-Clyde Consultants reorganized its operations from that of a holding company subsidiary relationship to an operating company with five regional divisions, each division having geotechnical and environmental capabilities. The more significant reasons given for this reorganization were to better serve its clients in terms of providing integrated geotechnical and environmental capability, establish a one-company image for improved marketing, and increase efficiencies by eliminating various subsidiary management structures. In evaluating the desirability of the organizational changes, many members of the Board of Directors made a subjective determination as to whether the changes would increase the companies ability to improve their level of performance over the various attributes. The explicit statement of attributes made it possible to make this evaluation.

APPENDIX

This appendix summarizes the technical terms and the theoretical result used in this paper. Let $X \equiv X_1 \times X_2 \times \cdots \times X_n$ be a consequence space, where X_i is the i^{th} attribute. A specific consequence will be designated by x or $(x_1, x_2, \cdots, x_n)$. We are interested in assessing the utility function over X, denoted by $u(x_1, x_2, \cdots, x_n)$ or $u(x)$, which is valid in the von Neumann-Morgenstern [10] sense. Let us define $\bar{X}_{ij}$ to mean $X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_{j-1} \times X_{j+1} \times \cdots \times X_n$ and $\bar{x}_{ij}$ to be a specific level of $\bar{X}_{ij}$. Similarly, the notation $\bar{X}_i$ is defined as $X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$, and $\bar{x}_i$ is a level of $\bar{X}_i$.

The main assumptions used in the paper concern the concepts preferential independence and utility independence. We will say $\{X_i, X_j\}$ is <u>preferentially independent</u> of $\bar{X}_{ij}$ if one's preference order for consequences $(x_i, x_j, \bar{x}_{ij})$, with $\bar{X}_{ij}$ held fixed does not depend on the fixed amount $\bar{x}_{ij}$. This is equivalent to assuming trade-offs under certainty between various amounts of X_i and X_j do not depend on $\bar{X}_{ij}$. The preferential independence assumption implies that the indifference curves over $X_i \times X_j$ are the same regardless of the value of $\bar{X}_{ij}$.

In a similar fashion, we say X_i is <u>utility independent</u> of $\bar{X}_i$ if one's preference order over lotteries on X_i, written $(\tilde{x}_i, \bar{x}_i)$, with $\bar{X}_i$ held fixed does not depend on the fixed amount $\bar{x}_i$. This implies the conditional utility function over X_i, given $\bar{X}_i$ is fixed at any value, will be a positive linear transformation of the conditional utility function over X_i, given $\bar{X}_i$ is fixed at any other value.

The main result used in this paper is the following.

THEOREM. Let $X \equiv X_1 \times X_2 \times \cdots \times X_n$, $n \geq 3$. If for some X_j, $\{X_i, X_j\}$ is preferentially independent of $\bar{X}_{ij}$ for all $i \neq j$ and X_j is utility independent of $\bar{X}_j$, then either

$$u(x) = \sum_{i=1}^{n} k_i u_i(x_i) \quad , \quad \text{if } \Sigma k_i = 1 \quad , \qquad \text{(A)}$$

or

$$1 + ku(x) = \prod_{i=1}^{n} [1 + kk_i u_i(x_i)], \text{ if } \Sigma k_i \neq 1, \qquad \text{(B)}$$

where u and the u_i are utility functions scaled from zero to one, the k_i are scaling constants with $0 < k_i < 1$, and $k > -1$ is a scaling constant.

Equation (A) is the additive utility function and (B) is the multiplicative utility function. More details about these, including suggestions for assessment, are found in Keeney [5].

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