

**Solving Multiple Objective Programming Problems  
Using Feed-forward Artificial Neural Networks:  
The Interactive FFANN Procedure**

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# Solving Multiple Objective Programming Problems Using Feed-forward Artificial Neural Networks: The Interactive FFANN Procedure\*

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In this paper, we propose a new interactive procedure for solving multiple objective programming problems. Based upon feed-forward artificial neural networks (FFANNs), the method is called the Interactive FFANN Procedure. In the procedure, the decision maker articulates preference information over representative samples from the nondominated set either by assigning preference "values" to the sample solutions or by making pairwise comparisons in a fashion similar to that in the Analytic Hierarchy Process. With this information, a FFANN is trained to represent the decision maker's preference structure. Then, using the FFANN, an optimization problem is solved to search for improved solutions. An example is given to illustrate the Interactive FFANN Procedure. Also, the procedure is compared computationally with the Tchebycheff Method (Steuer and Choo 1983). The computational results indicate that the Interactive FFANN Procedure produces good solutions and is robust with regard to the neural network architecture.

*(Multiple Objective Programming; Feed-forward Artificial Neural Networks; Multiple Criteria Decision Making; Analytic Hierarchy Process; Interactive Procedures)*

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## 1. Introduction

We propose a new procedure for solving multiple objective programming problems. Called the Interactive FFANN Procedure, it focuses on the elicitation, representation, and utilization of preference information obtained from a decision maker (DM) in a feed-forward artificial neural network (FFANN) framework. One advantage of the Interactive FFANN Procedure over ex-

isting procedures is that it takes the initiative in searching for improved solutions, generalizing preference information provided by the DM, rather than merely judging the discrete solutions generated by some sampling method. Another advantage is that the FFANN within the procedure makes it possible to represent various types of nonlinear preference structures.

During the last two decades, much progress has been made in the modeling of multiple objective programming problems. However, although many solution procedures have been proposed, these methods have generally not been fully satisfactory. The most effective

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methods have been interactive procedures, which typically include alternating phases of analysis—the solution generation phase and the solution evaluation phase. Examples of interactive multiple objective programming procedures include STEM (Benayoun et al. 1971), the Geoffrion-Dyer-Feinberg Procedure (Geoffrion et al. 1972), the Visual Interactive Approach (Korhonen 1987b, Korhonen and Wallenius 1988), the Tchebycheff Method (Steuer and Choo 1983, Steuer 1986), the Zionts-Wallenius Method (Zionts and Wallenius 1983), the Reference Point Method (Wierzbicki 1982), and others as summarized in Gardiner and Steuer (1994).

Whenever a multiple objective programming problem is solved interactively in practice, three issues must be addressed: (i) how to elicit preference information from the DM over the set of feasible solutions, (ii) how to represent the DM's preference structure in a systematic manner, and (iii) how to use the DM's preference structure to guide the search for improved solutions. Many methods have been developed for eliciting preference information from the DM, but finding an effective device to capture preference information and use it effectively in the search for improved solutions has been problematic.

This research addresses these three issues as follows. The DM has the choice of articulating his or her preference information either by assigning "values" to trial solutions or by making comparisons between pairs of trial solutions. The preference information elicited is then used to train a FFANN so as to "store" the preference information. The trained FFANN then serves as an approximate representation of the DM's preference structure and is combined with nonlinear programming techniques to search for improved solutions.

The contribution of this research is twofold. From the perspective of those working in management science, this research may be viewed as a new paradigm for solving multiple objective optimization problems using artificial intelligence methods. From the perspective of those working in artificial intelligence, this research can be seen as a new application of artificial neural networks to problems in constrained optimization.

The remainder of this paper is organized as follows. We briefly review the topology and dynamics of a FFANN in §2. In §3, we introduce notation and discuss

issues related to preference information elicitation and representation. The Interactive FFANN Procedure is detailed in §4, followed by an illustrative example in §5. Computational results are reported in §6, and concluding remarks are given in §7. The algorithm for training the FFANNs employed in this paper is presented in Appendix A.

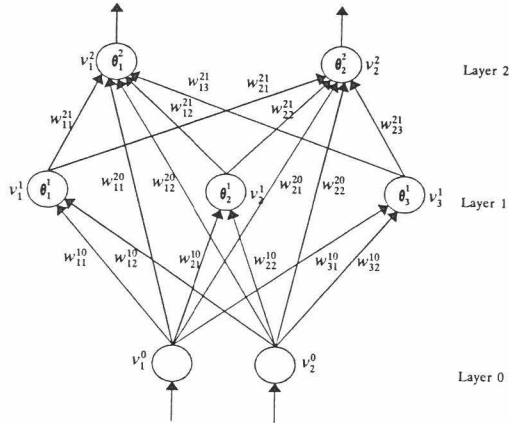
## 2. Feed-forward Artificial Neural Networks

An artificial neural network consists of a set of processing units, called nodes, connected by weighted arcs, where the weights represent the strength of connections. A FFANN is an artificial neural network where the nodes are organized into layers, and the weighted arcs only link nodes in lower layers to nodes in higher layers (Rumelhart et al. 1986; Wasserman 1989). Nodes in the input layer, called input nodes, accept input from the outside world and nodes in the output layer, called output nodes, generate output to the outside world. Nodes in the input layer are used to distribute inputs only and do not serve any processing or computational function. Nodes in layers between the input layer and the output layer are called hidden nodes, and these layers are called hidden layers.

Let the input layer also be known as layer 0 and let the number of layers aside from the input layer be  $m$ . Denote node  $k$  in layer  $i$  by  $v_k^i$ ; the number of nodes in layer  $i$  by  $n_i$ ; and the connectivity weight from  $v_r^i$  to  $v_k^j$  by  $w_{kr}^j$ . If two nodes are not connected, the connectivity weight between them is 0. Associated with  $v_k^i$  is a node bias or threshold  $\theta_k^i$ . Further, denote the set of connectivity weights and node biases by  $W = \{w_{kr}^j, \theta_k^i\}$ . An example of a FFANN, with direct connections from the input layer to the output layer, is given in Figure 1.

Mapping vectors from the input space  $\mathfrak{R}^n$  to the output space  $\mathfrak{R}^m$ , a FFANN can be expressed as FFANN:  $\mathfrak{R}^n \rightarrow \mathfrak{R}^m$ . The mapping of an input vector to an output vector is a dynamic process, in which node inputs and outputs are updated sequentially from the input layer to the output layer. For  $i > 0$ , the input to  $v_k^i$ , denoted by  $z_k^i$ , is the weighted sum of the outputs of all nodes directly connected to it from all other lower layers plus  $\theta_k^i$ , i.e.,

Figure 1 A Fully Connected FFANN



$$z_k^i = \sum_{j=0}^{i-1} \sum_{r=1}^{n_i} w_{kr}^{ij} u_r^j + \theta_k^i \quad (2.1)$$

where  $u_r^j$  is the output of  $v_r^j$ .

Each node, except for the ones in the input layer, has an activation function which computes the node's output based upon its input. The most frequently used activation function, which is also used in this paper, is the logistic function, defined as

$$u_k^i = [1 + \exp(-z_k^i/T)]^{-1}, \quad (2.2)$$

where the "temperature"  $T$ , a user-selected scalar, determines the steepness of the activation function.

A FFANN is usually trained to represent an unknown mapping by employing a training set (a collection of paired input and desired output vectors observed from the unknown mapping). The purpose in training a FFANN is to determine the values of the elements in  $W$  so that the FFANN can closely represent the unknown mapping.

The training of a FFANN is accomplished by (1) mapping input vectors from the training set by the current version of the FFANN to their computed output vectors, (2) comparing the computed output vectors with their respective desired output vectors in the training set, and then (3) adjusting the values of the components of  $W$  so as to reduce any differences between the computed and

desired output vectors. After a number of training iterations, the connectivity weights and node biases of the FFANN will converge to a set of values that minimizes the differences between the computed and desired output vectors, and the FFANN will organize itself internally, constructing a model to represent the unknown mapping from the input space to the output space. Thus any new input vector presented to an appropriately trained FFANN will yield an output vector similar to the one that would have been given by the actual mapping. The training algorithm that we used in this paper is based upon the *error back-propagation* algorithm (see Rumelhart et al. 1986) as described in Sun (1992) and is presented in Appendix A.

Artificial neural networks have been applied to many real world problems, especially in classification and pattern recognition (Másson and Wang 1990, Zahedi 1991). Also, artificial neural networks have been applied to problems in combinatorial optimization (Hopfield and Tank 1985, Aarts and Korst 1989) and linear programming (Tank and Hopfield 1986, Wang and Chankong 1992). Recently, Wang and Malakooti (1992) and Malakooti and Zhou (1994) have used FFANNs to solve discrete multiple criteria decision making problems. Burke and Ignizio (1992) provide an overview of connections between artificial neural networks and operations research.

### 3. Notation and Preference Information Elicitation

As for notation and terminology, a multiple objective programming problem is written as

$$\begin{aligned} &\max\{f_1(\mathbf{x}) = z_1\} \\ &\quad \vdots \\ &\max\{f_k(\mathbf{x}) = z_k\} \\ &\text{s.t. } \mathbf{x} \in S, \end{aligned}$$

or equivalently as

$$\begin{aligned} &\max\{f(\mathbf{x}) = \mathbf{z}\} \\ &\text{s.t. } \mathbf{x} \in S, \end{aligned}$$

where  $k$  is the number of objectives, the  $z_i$  are criterion values, and  $S \subset \mathcal{R}^n$  is the feasible region in decision

space. Let  $Z \subset \mathfrak{R}^k$  be the feasible region in criterion space where  $\mathbf{z} \in Z$  if and only if there exists an  $\mathbf{x} \in S$  such that  $\mathbf{z} = (f_1(\mathbf{x}), \dots, f_k(\mathbf{x}))$ . Criterion vector  $\bar{\mathbf{z}} \in Z$  is *nondominated* if and only if there does not exist another  $\mathbf{z} \in Z$  such that  $z_i \geq \bar{z}_i$  for all  $i$  and  $z_i > \bar{z}_i$  for at least one  $i$ . The set of all nondominated criterion vectors is designated  $N$  and is called the *nondominated set*. A point  $\bar{\mathbf{x}} \in S$  is *efficient* if and only if its criterion vector  $\bar{\mathbf{z}} = (f_1(\bar{\mathbf{x}}), \dots, f_k(\bar{\mathbf{x}}))$  is nondominated. The set of all efficient points is designated  $E$  and is called the *efficient set*. If a multiple objective program is all linear, it will be referred to as a MOLP (multiple objective linear program).

Let  $V: \mathfrak{R}^k \rightarrow \mathfrak{R}$  be a DM's value function. A  $\mathbf{z}^{\text{opt}} \in Z$  that maximizes  $V$  over  $Z$  is an *optimal criterion vector* and any  $\mathbf{x}^{\text{opt}} \in S$  such that  $(f_1(\mathbf{x}^{\text{opt}}), \dots, f_k(\mathbf{x}^{\text{opt}})) = \mathbf{z}^{\text{opt}}$  is an *optimal solution* of the multiple objective program. Our interest in the efficient set  $E$  and the nondominated set  $N$  stems from the fact that if  $V$  is *coordinatewise increasing* (that is, more is always better than less of each criterion),  $\mathbf{x}^{\text{opt}} \in E$  and  $\mathbf{z}^{\text{opt}} \in N$ . However, in interactive multiple objective programming, because of the difficulty in precisely locating the best nondominated criterion vector, we typically conclude the search for an optimal solution with a *final solution*  $\mathbf{z}^{\text{fn}} \in Z$  (a solution that is either optimal, or close enough to being optimal to satisfactorily terminate the decision process).

A DM's value function may be difficult, if not impossible, to assess. In fact, the mere existence of a value function is not guaranteed (Yu 1985). Moreover, a DM's preference structure may not be stable over time. See Farquhar (1984), Fishburn (1974, 1984), Keeney and Raiffa (1976), and Yu (1985) for some excellent research on value functions. Because of the potential difficulties in assessing a DM's value function, we have been intrigued by artificial neural networks because of their ability to represent complex mappings (linear or nonlinear, convex or nonconvex, continuous or discontinuous, differentiable or nondifferentiable). For instance, Hecht-Nielsen (1987) has shown that a FFANN with three layers can represent any continuous mapping from  $\mathfrak{R}^{n_0}$  to  $\mathfrak{R}^{n_m}$ , and others have shown that FFANNs with two hidden layers can represent any set in  $\mathfrak{R}^n$  (Cybenko 1989; Zwietering et al. 1991). With this kind of potential, the strategy of this paper becomes clear—to develop a FFANN approach that can capture a DM's

preference structure well enough to enable the Interactive FFANN Procedure to quickly locate final solutions of top quality. Hence, in this paper, we are interested in employing a FFANN:  $\mathfrak{R}^{n_0} \rightarrow \mathfrak{R}^{n_m}$  with  $n_0 = k$  and  $n_m = 1$  such that FFANN:  $\mathfrak{R}^k \rightarrow \mathfrak{R}$ . In other words, in this case, the FFANN input is a  $k$ -dimensional (rescaled) criterion vector, and the FFANN output is a single (rescaled) preference value.

In the Interactive FFANN Procedure, two different approaches are employed for evaluating the criterion vectors generated at each iteration so that they can be used for initially training and then retraining the FFANN for use on each iteration. One approach is for the DM to assign an interval-scale preference "value" to each criterion vector, higher "values" representing higher degrees of satisfaction. So as to anchor the scale, the *nadir* criterion vector  $\mathbf{z}^{\text{nad}}(z_i^{\text{nad}} = \min\{f_i(\mathbf{x}) | \mathbf{x} \in E\})$  could be given a preference value of 0, and the *ideal* criterion vector  $\mathbf{z}^{\text{max}}(z_i^{\text{max}} = \max\{f_i(\mathbf{x}) | \mathbf{x} \in S\})$  could be given a preference value of 100. One way to obtain  $\mathbf{z}^{\text{nad}}$  would be to examine the criterion vectors of all efficient extreme points. If  $\mathbf{z}^{\text{nad}}$  cannot be obtained in this way, for instance if the problem is too large to enumerate all efficient extreme points,  $\mathbf{z}^{\text{nad}}$  can be estimated from the minimum values in the columns of a payoff table (Iserrmann and Steuer 1988; Korhonen et al. 1996). In this way, the preference value of every nondominated criterion vector should fall within the range of 0 to 100. Actually, the scale is not important. What really matters is the order of the preference values and the differences between them.

The other approach is to make pairwise comparisons between trial solutions. In this approach, the DM is asked questions similar to those posed in the Analytic Hierarchy Process (AHP) (Saaty 1988) and in its software implementation Expert Choice (Expert Choice 1992). The advantage of eliciting preference information by pairwise comparisons is that it is easier for many DMs to provide relative than absolute preference information. The pairwise comparisons result in a reciprocal comparison matrix. Saaty (1988) has shown that the principal eigenvector components of this matrix can be viewed as the priorities of the alternative solutions. In the Interactive FFANN Procedure, the components of this priority vector are used as the desired outputs when training the FFANN.

The AHP appears to be an easy and convenient methodology for eliciting preference information from the DM. In fact, it has been used to solve different types of real world discrete multiple criteria decision making problems. Recently, some authors have used the AHP to elicit preference information from the DM in multiple objective programming and have incorporated the AHP into interactive solution procedures. In their interactive method, Arbel and Oren (1987) use the AHP to assess the relative preference of the current solution and adjacent solutions. Gass (1986) used the AHP to determine goal priorities and objective function weights in a linear goal programming formulation. Korhonen (1987a) discusses the use of the AHP to find reference directions, which are then used as search directions in his visual interactive approach. Korhonen and Wallenius (1990) use the AHP to determine objective coefficients and parameter values for an MOLP problem which is subsequently solved using their visual interactive package VIG. Kok and Lootsma (1985) propose using the AHP within the framework of the Reference Point Method (Wierzbicki 1982) to find the weighting vector for use in an achievement scalarizing program that projects reference points onto  $N$ . Barzilai and Golany (1990) derive weights for additive value functions from a reciprocal comparison matrix.

A caveat of the AHP is that it has been criticized for several theoretical shortcomings (Dyer 1990), in spite of many successful applications. One problem is the phenomenon of rank reversal when new alternatives are added to, or old alternatives are removed from, the current set of alternative solutions. Another problem is that the questions DMs are asked about the pairwise relative importance of the criteria may be viewed as ambiguous. To date, proponents and critics have not yet fully resolved these issues (Winkler 1990; Schoner et al. 1992). However, in the Interactive FFANN Procedure rank reversal problems can be handled through the interactive nature of the procedure in that at any iteration previous solutions can be re-ranked to correct for any errors that may have been made earlier in the solution process.

#### 4. Interactive FFANN Procedure

In this section we specify the Interactive FFANN Procedure followed by comments about its different steps.

Step 0: Determine  $\mathbf{z}^{\max}$  and  $\mathbf{z}^{\text{nad}}$  (if nadir values are not available, use the minimum values in the columns of a payoff table). Specify the number of criterion vectors  $P$  to be presented to the DM at each iteration and the number of iterations  $t$  the procedure is to run. Select a particular FFANN architecture to use. Generate  $P$  dispersed criterion vectors from the nondominated set.

Repeat for  $h = 1, \dots, t$ :

Step 1: After presenting the  $P$  criterion vectors along with  $\mathbf{z}^{\text{nad}}$  and  $\mathbf{z}^{\max}$  to the DM, identify the best criterion vector seen so far. If  $h = t$ , or if the DM feels that the best criterion vector obviates the need for additional iterations, designate this criterion vector as the final criterion vector  $\mathbf{z}^{\text{fn}}$  and stop. Otherwise, let the DM articulate his/her preference information either by directly assigning values to the criterion vectors or by making pairwise comparisons.

Step 2: Rescale the components of each of the  $P$  criterion vectors using the transformation

$$z_i^f = \frac{z_i - z_i^{\text{nad}}}{z_i^{\max} - z_i^{\text{nad}}}.$$

Step 3: If pairwise comparisons are made, compute and normalize the principal eigenvector of the reciprocal comparison matrix so that its largest component is one. If preference values are assigned, let  $V(\mathbf{z})$  be the value assigned to  $\mathbf{z}$ . Then, for each of the  $P$  criterion vectors, compute a rescaled preference value using

$$v(\mathbf{z}) = \frac{V(\mathbf{z}) - V(\mathbf{z}^{\text{nad}})}{V(\mathbf{z}^{\max}) - V(\mathbf{z}^{\text{nad}})}. \quad (4.1)$$

Step 4: Use the rescaled criterion vectors (from Step 2) with either their rescaled assigned preference values or the components of the principal eigenvector of the reciprocal comparison matrix (from Step 3) to train (if  $h = 1$ ) or retrain (if  $h > 1$ ) the FFANN.

Step 5: With the most recently trained or retrained FFANN as the objective function, solve the optimization problem

$$\max \text{FFANN}(\mathbf{z})$$

$$\text{s.t. } \mathbf{z} = f(\mathbf{x})$$

$$\mathbf{x} \in S$$

to obtain a new solution  $(\mathbf{z}^{(h)}, \mathbf{x}^{(h)})$ .

Step 6: If  $z^{(h)}$  is different from any criterion vector previously presented to the DM, generate  $P - 1$  new dispersed criterion vectors. If  $z^{(h)}$  duplicates a previously seen criterion vector, generate  $P$  new dispersed criterion vectors for presentation to the DM on the next iteration.  
**End Repeat.**

In Step 0 there are no specific guidelines as to what FFANN architecture to use, in terms of the numbers of hidden layers and hidden nodes. Fortunately, as evidenced by the computational tests in §6, the particular FFANN representation of the DM's preference structure is not very sensitive to the particular FFANN structure employed. In Step 0 (and also in Step 6), we use the augmented weighted Tchebycheff program (Steuer and Choo 1983, and Steuer 1986) to generate the dispersed criterion vectors required at each iteration.

Although a FFANN can be easily modified to automatically scale the input vectors and outputs, by introducing more layers, we always recommend in Steps 2 and 3 that all input vectors and outputs be rescaled prior to their presentation to the FFANN. In this way, the number of elements in  $W$  is kept at its minimum so as to reduce the time needed to train the FFANN.

Saaty (1988) has shown that if the priorities of all trial solutions are known exactly and each pairwise comparison is made based on these priorities, the components of the principal eigenvector of the reciprocal comparison matrix are identical to these priorities. Saaty (1988) suggests several alternative methods for estimating the principal eigenvector of the comparison matrix. In Step 3, we use the *power* method (Burden and Faires 1989) for this purpose. The dimensions of the reciprocal comparison matrix are  $(P + 2) \times (P + 2)$  because of the  $P$  trial solutions plus  $z^{\max}$  and  $z^{\text{nad}}$ .

The training algorithms developed based on error back-propagation (Rumelhart et al. 1986) in Sun (1992) are used to train the FFANNs in Step 4. The details of one of these algorithms are provided in Appendix A. As we will see below, for multiple objective programming problems with a reasonable number of objectives the structure of the FFANN needs not to be very complicated and the training time is typically only a few seconds. After the first iteration, the procedure offers the option to continue the training in subsequent iterations with either a warm start, using the connectivity

weights of the previous iteration as initial weights, or a cold start, using random initial weights. In the computational experiments, each iteration is started with random initial weights.

The objective function of the optimization problem in Step 5, in this case the trained FFANN, may be complicated, requiring nonlinear programming solution techniques. In the implementation, the GRG2 package (Lasdon and Waren 1989) is used for this purpose. The gradient of the trained FFANN with respect to the  $k$  criterion values at a specific solution is determined numerically. The following *three point formula* (Burden and Faires 1989) is used to estimate the partial derivative of the trained FFANN:

$$\frac{\partial \text{FFANN}(z)}{\partial z'_i} \approx \frac{\text{FFANN}(z'_1, \dots, z'_i + \epsilon_i, \dots, z'_k) - \text{FFANN}(z'_1, \dots, z'_i - \epsilon_i, \dots, z'_k)}{2\epsilon_i},$$

where  $\epsilon_i$  is a small positive scalar. In the computational tests, we obtained similar results for various  $\epsilon_i$ -values in the range from 0.001 to 0.01.

## 5. An Example

To illustrate how the Interactive FFANN Procedure works step-by-step, consider the following MOLP problem:

$$\begin{aligned} \max \quad & 2x_2 + 5x_3 + 5x_4 - 2x_5 + 5x_6 = z_1 \\ \max \quad & -x_1 - 2x_2 + 4x_5 - x_6 = z_2 \\ \max \quad & 5x_1 + 3x_2 - 2x_3 - x_5 - x_6 = z_3 \\ \text{s.t.} \quad & 7x_4 + 2x_5 + 6x_6 \leq 28 \\ & 3x_1 + 4x_6 \leq 23 \\ & 4x_1 + 4x_3 + x_4 \leq 23 \\ & x_2 + 6x_3 + 7x_4 + 4x_6 \leq 23 \\ & 2x_1 + 5x_2 + 5x_3 + 5x_4 + 8x_5 \leq 29 \\ & x_j \geq 0, \quad 1 \leq j \leq 6. \end{aligned}$$

Let us assume a hypothetical DM has the following value function

$$V_4(z) = 50 - \left( \sum_{i=1}^3 [\lambda_i (z_i^{\max} - z_i)]^4 \right)^{1/4},$$



Table 1 Criterion Vectors of the First Iteration for the Example Problem

Solution	$z_1$	$z_2$	$z_3$	$V_4(z)$
1	24.35460	-11.54862	27.64540	39.13516
2	-5.69318	14.18864	-3.93636	35.80484
3	22.86093	2.25756	-7.88655	37.39273
4	-4.61749	7.45757	14.19598	37.69865
5	29.56935	-9.20832	6.82426	38.94388
6	2.32488	-6.27676	34.03545	38.96402
7	-3.17575	1.33950	27.90013	38.28441
$z^{max}$	33.10000	14.00000	39.25000	50.00000
$z^{nad}$	-7.25000	-16.41200	-9.20700	33.07733

Table 2 Rescaled Criterion Vectors of the First Iteration for the Example Problem

Solution	$z'_1$	$z'_2$	$z'_3$	$v_4(z)$
1	0.78326	0.15733	0.76052	0.35797
2	0.03858	0.98993	0.10877	0.16117
3	0.74624	0.60396	0.02725	0.25501
4	0.06524	0.77218	0.48296	0.27308
5	0.91250	0.23304	0.33083	0.34667
6	0.23730	0.32787	0.89239	0.34786
7	0.10097	0.57426	0.76577	0.30770
$z^{max}$	1.00000	1.00000	1.00000	1.00000
$z^{nad}$	0.00000	0.00000	0.00000	0.00000

with  $\lambda = (0.319, 0.416, 0.265)$ . Using GRG2 (Lasdon and Waren 1989), the optimal solution is found to be  $z^{opt} = (16.517, -0.886, 18.970)$  with a hypothetical value function value  $V_4(z^{opt}) = 42.42288$ .

The vector-maximum code ADBASE (Steuer 1992) was used to compute all efficient extreme points, from which the ideal criterion vector was found to be  $z^{max} = (33.100, 14.500, 39.250)$  with  $V_4(z^{max}) = 50.00000$  and the nadir criterion vector was found to be  $z^{nad} = (-7.250, -16.412, -9.207)$  with  $V_4(z^{nad}) = 33.07733$ . Furthermore, the worst criterion vector in the nondominated set was found to be  $z^{worst} = (-7.250, 14.500, -3.625)$  with  $V_4(z^{worst}) = 35.50926$ . The worst nondom-

inated criterion vector, of course, is used as a benchmark to measure the quality of solutions only and is not used in the Interactive FFANN Procedure. Now let the number of solutions that are to be presented to the DM at each iteration be  $P = 7$  and the number of iterations the procedure is to run be  $t = 5$ .

The augmented weighted Tchebycheff program (Steuer and Choo 1983, Steuer 1986) was then used to generate the seven dispersed nondominated solutions in the first iteration, as shown in Table 1. Together with  $z^{max}$  and  $z^{nad}$ , the seven nondominated solutions are presented to the DM, who then evaluates them, either by directly assigning preference values or by making pairwise comparisons. In the example, the  $V_4(z)$  values in Table 1 represent the preference information elicited from the hypothetical DM through the value function  $V_4(z)$ .

We train the FFANN shown in Figure 2, with one hidden layer comprised of two hidden nodes, using the

Figure 2 FFANN Architecture and Connectivity Weights for the Example Problem

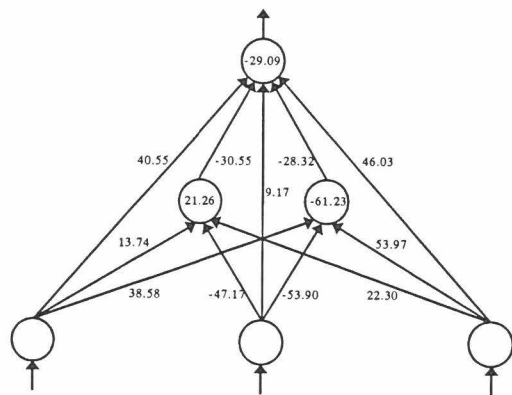


Table 3 Iteration by Iteration Solutions for the Example Problem

Iteration (h)	$z_1^{(h)}$	$z_2^{(h)}$	$z_3^{(h)}$	$V_4(z^{(h)})$
1	19.16292	-4.44382	24.18538	41.80951
2	18.81381	-4.06534	23.91694	41.92090
3	18.96599	-2.52878	20.36740	42.27592
4	18.70442	-3.13536	22.33424	42.17731
5	18.70442	-3.13536	22.33424	42.17731
$z^{opt}$	<b>16.51700</b>	<b>-0.88555</b>	<b>18.97000</b>	<b>42.42288</b>

**Table 4** Number of Efficient Extreme Points over the Ten Test Problems for Each Problem Size

Problem Size	Number of Efficient Extreme Points		
	Minimum	Maximum	Average
3 × 5 × 6	5	19	11.9
5 × 5 × 10	6	83	27.7
5 × 8 × 15	29	554	158.0
5 × 10 × 20	226	1131	417.7

rescaled criterion vectors in Table 2 as the inputs and the rescaled preference values as the desired outputs. Thus, each line of Table 2 corresponds to one training pattern. The connectivity weights and node biases of the trained FFANN are shown in Figure 2. The connectivity weights between the nodes are given by the values on the arcs, while the node biases are indicated inside each node in the hidden and output layers. The temperature used in training this FFANN was  $T = 10$ .

To demonstrate the mapping of the FFANN in Figure 2, let  $\mathbf{z}' = (0.78, 0.16, 0.76)$  be a given input vector of rescaled criterion values. It follows from (2.1) that

$$\begin{aligned} z_1^1 &= 13.74(0.78) - 47.17(0.16) + 22.30(0.76) \\ &\quad + 21.26 = 41.38, \quad \text{and} \\ z_2^1 &= 38.58(0.78) - 53.90(0.16) + 53.97(0.76) \\ &\quad - 61.23 = 1.26. \end{aligned}$$

Hence, from (2.2) it follows that

$$\begin{aligned} u_1^1 &= [1 + \exp(-41.38/10)]^{-1} = 0.98 \quad \text{and} \\ u_2^1 &= [1 + \exp(-1.26/10)]^{-1} = 0.53, \end{aligned}$$

respectively. Similarly,

$$\begin{aligned} z_1^2 &= 40.55(0.78) + 9.17(0.16) + 46.03(0.76) \\ &\quad - 30.55(0.98) - 28.32(0.53) - 29.09 = -5.96, \quad \text{and} \\ u_1^2 &= [1 + \exp(-(-5.96/10))]^{-1} = 0.36. \end{aligned}$$

Solving the optimization problem in Step 5 of the procedure, with the trained FFANN as the objective function, yields  $\mathbf{z}^{(1)} = (19.16292, -4.44382, 24.18538)$  with  $V_4(\mathbf{z}^{(1)}) = 41.80951$ . At this point, one iteration has been

completed. Solution  $\mathbf{z}^{(1)}$  is different from any of the solutions previously presented to the DM.

Along with  $\mathbf{z}^{(1)}$ , in the second iteration six new non-dominated criterion vectors are presented to the DM for evaluation. The rescaled nondominated criterion vectors and the DM's rescaled preference values are then used to retrain the FFANN. The retrained FFANN is then used to search for improved solutions. This process is repeated four more times. Table 3 lists the solutions obtained at each iteration.

As seen, the best solution was found in Iteration 3. Thus, the final solution is  $\mathbf{z}^{\text{fin}} = (18.96599, -2.52878, 20.36740)$ . This represents a 98.43%

$$\left( \frac{42.27592 - 33.07733}{42.42288 - 33.07733} \times 100\% \right)$$

achievement of the DM's value function value from that of the nadir point  $\mathbf{z}^{\text{nad}}$  to that of the optimal point  $\mathbf{z}^{\text{opt}}$ , and a 97.93%

$$\left( \frac{42.27592 - 35.50926}{42.42288 - 35.50926} \times 100\% \right)$$

achievement from that of the worst nondominated point  $\mathbf{z}^{\text{worst}}$ .

## 6. Computational Experiments

In this section, we conduct computational experiments in order to test the Interactive FFANN Procedure against the Tchebycheff Method, which has tested well in a previous study (Buchanan and Daellenbach 1987).

Similar to the previous section, for each problem we assumed a hypothetical DM with a particular value function. This is useful for test purposes because it enables us to determine an optimal solution for each problem ahead of time and helps us in providing preference information by acting as the DM. This is especially useful when two procedures are compared computationally because it provides the same preference information required by both of the procedures. In the tests, the value function, of course, is only used in the preference elicitation phase and not in the search for improved solutions. With this experimental design, the performance of the Interactive FFANN Procedure is measured along four dimensions: (1) solution quality, (2) problem size, (3) type of value function, and (4) FFANN architecture.

Table 5 Final Solution Quality with  $L_1$ -Metric Value Function

Problem Size	Interactive FFANN Procedure (No Hidden Nodes in the FFANN)			Tchebycheff Method		
	Worst	Best	Average	Worst	Best	Average
$3 \times 5 \times 6$	98.18	100.00	99.82	94.95	99.95	98.48
$5 \times 5 \times 10$	100.00	100.00	100.00	90.50	99.98	97.81
$5 \times 8 \times 15$	97.09	100.00	99.71	84.89	100.00	97.19
$5 \times 10 \times 20$	95.97	100.00	99.26	91.12	99.59	97.69
$6 \times 50 \times 100$	99.62	99.97	99.81	93.40	100.00	96.98

### 6.1 Test Problems

The MOLP test problems used in the experiments were generated using the problem generation capability in ADBASE (Steuer 1992), the same capability used in other interactive multiple objective programming computational studies such as those reported in Reeves and Franz (1985), Steuer (1986), and Buchanan and Daellenbach (1987). The type of dense test problems generated is representative of a number of application areas, for instance that of resource allocation. The problem sizes, defined by  $k \times m \times n$  ( $m$  is the number of linear constraints), used in the experiments are  $3 \times 5 \times 6$ ,  $5 \times 5 \times 10$ ,  $5 \times 8 \times 15$ ,  $5 \times 10 \times 20$  and  $6 \times 50 \times 100$ . ADBASE was used to find the criterion vectors of all efficient extreme points (see Table 4) for all problems except for those in the  $6 \times 50 \times 100$  category. The difficulty with the  $6 \times 50 \times 100$  category is that the tens of thousands of efficient extreme points that such problems are likely to have is beyond the capability of any currently existing code. For all problems except those in the  $6 \times 50 \times 100$  category,  $z^{\max}$ ,  $z^{\text{nad}}$  and  $z^{\text{worst}}$  were obtained from the generation of all efficient extreme points. For the  $6 \times 50 \times 100$  problems, the  $z^{\text{nad}}$  criterion vectors were estimated from their payoff tables.

### 6.2. Value Functions

In the experiments we used four different value functions of  $L_p$ -metric form with  $p = 1$ ,  $p = 2$ ,  $p = 4$  and  $p = \infty$ ,

$$V_p(z) = K - \left[ \sum_{i=1}^k [\lambda_i (z_i^{\max} - z_i)]^p \right]^{1/p}, \quad (6.1)$$

where the  $\lambda_i$  are given by

$$\lambda_i = \frac{1}{z_i^{\max} - z_i^{\text{nad}}} \left[ \sum_{j=1}^k \frac{1}{z_j^{\max} - z_j^{\text{nad}}} \right]^{-1},$$

and  $K$  is a constant to ensure that all value function values are positive. When  $p = \infty$  we note that (6.1) reduces to

$$V_\infty(z) = K - \max_{1 \leq i \leq k} \{\lambda_i (z_i^{\max} - z_i)\},$$

in which case the value function is nondifferentiable. Thus, it will be interesting to study the performance of the Interactive FFANN Procedure when dealing with this potentially difficult mapping.

In the computational experiments, we set the number of iterations to  $t = 5$ , (except for with the  $6 \times 50 \times 100$  problems in which case we used  $t = 6$ ), and evaluated  $P = 7$  nondominated criterion vectors at each iteration. Note that in the experiments the principal eigenvector components of the reciprocal comparison matrix are identical to the rescaled preference values since the hypothetical DM makes each preference judgment according to the pre-specified value function. Therefore, the test results will be the same regardless of which method is used for eliciting preference information (pairwise comparisons or direct assessment).

### 6.3. Solution Quality

In the computational tests, except for the  $6 \times 50 \times 100$  problems, the solution quality of a  $z \in Z$  is measured by comparing its preference value against that of  $z^{\text{opt}}$  relative to that of  $z^{\text{worst}}$  by means of

**Table 6** Final Solution Quality with  $L_2$ -Metric Value Function

Problem Size	Interactive FFANN Procedure											
	Number of Hidden Nodes in the FFANN									Tchebycheff Method		
	0			1			2					
	Worst	Best	Average	Worst	Best	Average	Worst	Best	Average	Worst	Best	Average
3 × 5 × 6	61.42	100.00	94.14	90.23	100.00	98.19	97.68	100.00	99.48	92.35	99.95	97.52
5 × 5 × 10	83.35	100.00	92.94	85.98	100.00	98.67	89.39	100.00	99.00	93.46	100.00	98.64
5 × 8 × 15	96.46	99.56	98.27	97.10	99.96	98.76	97.60	99.90	99.01	93.57	99.98	97.89
5 × 10 × 20	92.32	99.99	97.59	95.41	100.00	98.63	95.41	100.00	98.77	92.05	98.87	97.00
6 × 50 × 100	98.29	99.84	99.27	98.28	99.84	99.27	97.29	99.84	99.30	89.03	98.87	96.39

$$\frac{V(\mathbf{z}) - V(\mathbf{z}^{\text{worst}})}{V(\mathbf{z}^{\text{opt}}) - V(\mathbf{z}^{\text{worst}})} \times 100. \quad (6.2)$$

The convenience of this measure is that the quality of  $\mathbf{z}^{\text{worst}}$  is 0 and the quality of  $\mathbf{z}^{\text{opt}}$  is 100. For the  $6 \times 50 \times 100$  problems, the quality of a  $\mathbf{z} \in Z$  is measured by comparing its preference value against that of  $\mathbf{z}^{\text{opt}}$  relative to that of the estimated nadir point  $\mathbf{z}^{\text{nad}}$  by means of

$$\frac{V(\mathbf{z}) - V(\mathbf{z}^{\text{nad}})}{V(\mathbf{z}^{\text{opt}}) - V(\mathbf{z}^{\text{nad}})} \times 100,$$

because  $\mathbf{z}^{\text{worst}}$  is not available.

#### 6.4 Experimental Results

In the experiments, fifty (ten in each problem size category) MOLP test problems were employed. In testing the Interactive FFANN Procedure, different FFANN structures with different numbers of hidden nodes were employed. We report the computational results for four different FFANN structures with no, one, two, or six hidden nodes in one hidden layer respectively. In Tables 5–8, we summarize the quality of the final solutions obtained from the Interactive FFANN Procedure as compared against those obtained from the Tchebycheff Method when using the  $L_1$ -,  $L_2$ -,  $L_4$ - and  $L_\infty$ -metric value functions. More computational results with different quality measures are reported in Sun (1992).

In Table 5, the Interactive FFANN Procedure was run for the  $L_1$ -metric value function, with a neural network structure without any hidden nodes, resulting in higher

quality solutions than with the Tchebycheff Method, in terms of average as well as best and worst qualities. In this case, the output node is the only node which performs a computational function. If this node were to have a linear activation function, the FFANN would reduce to a linear regression model and be able to represent the DM's linear value function exactly. However, in this case the nonlinear nature of this activation function introduces "imprecision" into the FFANN. Nevertheless, this imprecision hardly has an impact on the performance, as the Interactive FFANN Procedure correctly identifies the optimal solution, within five iterations, for 35 out of the 50 test problems, and approximates the optimal solution closely (within one to four percent) for the remaining problems.

In Table 6, three different neural network structures were used for the  $L_2$ -metric value function. From this table, we see that superior results were obtained using the Interactive FFANN Procedure, as long as at least one hidden node is used. In Tables 7 and 8, generally better results were obtained with the Interactive FFANN Procedure than with the Tchebycheff Method, but at least two hidden nodes were required because of the more difficult  $L_4$ - and  $L_\infty$ -metric value functions. Particularly for the  $L_\infty$ -metric value function, the difference in performance becomes more favorable for the Interactive FFANN Procedure as the problem size increases.

Keeping in mind the fact that the test results were generated by the progenitors of the new procedure, we nevertheless feel that the results are very encouraging

**Table 7** Final Solution Quality with  $L_1$ -Metric Value Function

Problem Size	Interactive FFANN Procedure								
	Number of Hidden Nodes in the FFANN						Tchebycheff Method		
	2			6			Worst	Best	Average
	Worst	Best	Average	Worst	Best	Average			
$3 \times 5 \times 6$	93.88	99.98	98.13	93.71	99.98	98.13	95.00	100.00	98.78
$5 \times 5 \times 10$	86.63	100.00	97.70	84.05	100.00	97.21	74.16	99.67	94.15
$5 \times 8 \times 15$	94.45	99.96	97.61	93.06	99.96	97.15	74.44	99.70	95.15
$5 \times 10 \times 20$	95.59	99.77	98.35	92.89	99.91	98.11	81.39	98.87	95.36
$6 \times 50 \times 100$	93.53	99.06	97.13	97.07	99.78	99.02	93.19	98.86	96.24

and that the possibilities for embedding artificial neural network technology in the interactive procedures of multiple objective programming are promising.

The time required to train a FFANN depends on several factors, such as the number of patterns in the training set, the number of inputs (i.e.  $k$ , the number of objectives), the number of hidden nodes in the FFANN, the stopping criteria, and the complexity of the mapping the FFANN is to represent. In order to fully assess the usefulness of the Interactive FFANN procedure, we report the average computational effort required to train FFANNs for ten  $6 \times 50 \times 100$  MOLP problems in Table 9. FFANNs for smaller problems were trained within a few seconds. All of the computations performed in this

paper were conducted on the University of Georgia's IBM ES 9000 Model 720 computer.

From Table 9 we see that, as expected, the average computational effort increases as the number of nodes in the hidden layer and the number of patterns in the training set increase. Nevertheless, even for FFANN configurations with six hidden nodes and for training sets with 44 patterns the training times are reasonable.

## 7. Concluding Remarks

In this paper, we present an Interactive FFANN Procedure for solving multiple objective programming problems using feed-forward artificial neural networks. In

**Table 8** Final Solution Quality with  $L_1$ -Metric Value Function

Problem Size	Interactive FFANN Procedure								
	Number of Hidden Nodes in the FFANN						Tchebycheff Method		
	2			6			Worst	Best	Average
	Worst	Best	Average	Worst	Best	Average			
$3 \times 5 \times 6$	88.35	100.00	96.28	84.21	100.00	92.83	92.08	99.00	95.94
$5 \times 5 \times 10$	85.65	97.55	92.13	83.16	96.70	92.55	52.27	95.34	86.75
$5 \times 8 \times 15$	72.42	98.26	91.63	79.81	99.71	92.28	68.34	98.49	87.98
$5 \times 10 \times 20$	88.59	98.20	94.84	85.16	98.42	91.94	75.75	98.64	87.14
$6 \times 50 \times 100$	72.09	93.09	76.63	69.23	96.68	84.88	49.39	91.19	71.47

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**Table 9** Average Time to Train Six-input Node FFANNs for  $6 \times 50 \times 100$  MOLP

Number of Hidden Nodes	Number of Training Patterns					
	9	16	23	30	37	44
<i>L</i> <sub>1</sub> -Metric Value Function						
0	2.09	0.56	0.75	1.00	1.21	1.55
2	4.91	3.39	12.06	22.50	67.32	105.67
<i>L</i> <sub>2</sub> -Metric Value Function						
0	2.14	0.72	0.90	1.15	1.39	1.77
1	2.34	2.24	8.86	13.25	26.85	31.29
2	3.72	9.76	23.95	48.10	60.65	77.76
4	8.27	15.64	48.74	71.71	139.10	164.23
6	18.20	52.26	80.95	167.13	233.43	208.33
<i>L</i> <sub>3</sub> -Metric Value Function						
0	2.16	0.94	1.02	1.33	1.76	1.95
2	5.33	15.54	45.20	78.60	94.58	110.81
4	8.25	49.83	81.02	123.87	164.09	187.17
6	9.10	66.28	138.97	188.52	228.70	271.43
<i>L</i> <sub>4</sub> -Metric Value Function						
0	2.16	0.81	0.93	1.19	1.47	1.69
2	14.77	47.27	66.10	74.61	99.32	94.28
4	20.39	76.30	89.59	118.05	169.86	199.23
6	29.27	105.59	131.77	183.46	221.38	239.69

the procedure, the DM has the option of articulating his or her preference information either by directly assigning a preference value to each new solution or by making pairwise comparisons in a way similar to the AHP. Since preference structures may be very complex, a FFANN is used in the procedure because of its ability to capture and represent complicated mappings. Because the DM's aspirations may evolve over the course of the solution process, the FFANN has the chance to adapt to any such changes as the FFANN is retrained at each iteration.

From the computational results, it is evident that good solutions have been obtained, at least for the test problems and value functions used. Also, the procedure

is relatively robust in that similar solutions are obtained when different FFANN structures are employed. Computer-time-wise, because of the retraining of the FFANN at each iteration, the Interactive FFANN Procedure can be expected to take more time than other interactive procedures. However, in an era of rapidly decreasing computer costs, solution quality may be the most important issue for many users. Typically, FFANNs used in the Interactive FFANN Procedure can be trained within a few seconds.<sup>1</sup>

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**Appendix A: A FFANN Training Algorithm**

This appendix presents an algorithm for training FFANNs with multiple layers, which we use in our Interactive FFANN Procedure. The algorithm is developed based on the error back-propagation algorithm (Rumelhart, et al. 1986), and uses unconstrained nonlinear optimization techniques. Specifically, the algorithm uses a combination of the Golden Section Method and a "doubling and halving" line search strategy, and the Polak and Ribiere conjugate gradient direction. In the following, we discuss the mathematical details of the training algorithm, and outline the training algorithm.

**A1. Mathematical Details**

In the training process, the node biases,  $\theta_k$ , are treated the same as other connectivity weights. Actually, by adding a single node  $v_{n_0+1}^0$  to the input layer, connecting it to all nodes in all other layers, and assigning  $v_{n_0+1}^0$  an input value of 1, the connectivity weight  $w_{k,n_0+1}^0$  is the bias  $\theta_k$  of node  $v_k$ , i.e.,

$$\theta_k = w_{k,n_0+1}^0.$$

Suppose that  $\mathbf{z}_q \in \mathfrak{R}^{n_0}$  is the  $q$ th input vector and  $\mathbf{t}_q \in \mathfrak{R}^{n_m}$  is the associated desired output vector in the training set. The compound vector  $(\mathbf{z}_q, \mathbf{t}_q) \in \mathfrak{R}^{n_0+n_m}$  is called a training pattern. Let the number of patterns in the training set be denoted by  $Q$ .

When  $\mathbf{z}_q$  is presented to the network, the FFANN maps it to an output vector  $\mathbf{u}_q$  based on (2.1-2.2). The error measure  $E_q$  for the  $q$ th training pattern is defined as

$$E_q = \frac{1}{2} \sum_{j=1}^{n_m} (t_{qj} - u_{qj})^2. \tag{A.1}$$

Thus,  $E_q$  is the sum over all output nodes of the squared differences between the computed and desired outputs. In our application,  $n_m = 1$ . For a given topology of the FFANN and a given set of training patterns,  $E_q$  is a function of the connectivity weights in  $W$  and can be written as  $E_q(W)$ . Summing over all training patterns, the overall error measure over all  $Q$  training patterns is given by

$$E(W) = \sum_{q=1}^Q E_q(W) = \frac{1}{2} \sum_{q=1}^Q \sum_{j=1}^{n_m} (t_{qj} - u_{qj})^2. \tag{A.2}$$

When a FFANN is trained, we try to adjust the values of the components of  $W$  so as to minimize  $E(W)$ . The partial derivative of  $E_q$  with respect to the connectivity weight  $w_{ik}^q$ , is given by

$$\frac{\partial E_q(W)}{\partial w_{ik}^q} = -\delta_{ik}^q u_{qr}^q, \quad 0 < i < m, \tag{A.3}$$

where  $\delta_{ik}^q$  is the error signal of node  $v_i^q$  and  $u_{qr}^q$  is the computed output of node  $v_r^q$  for the  $q$ th training pattern, respectively. If  $i = m$ ,  $\delta_{ik}^q$  is determined by

$$\delta_{ik}^q = (t_{qk} - u_{qk}^q) f'(z_{ik}^q), \tag{A.4}$$

and, if  $0 < i < m$ ,  $\delta_{ik}^q$  is computed recursively in terms of the error signals of all the nodes to which it directly connects as shown in

$$\delta_{ik}^q = f'(z_{ik}^q) \sum_{s=i+1}^m \sum_{t=1}^{n_s} \delta_{st}^q w_{ik}^{sq}, \quad 0 < i < m, \tag{A.5}$$

where  $f'(z_{ik}^q)$  is the first derivative of the node activation function of  $v_i^q$  evaluated at  $z_{ik}^q$  and  $z_{ik}^q$  is determined by (2.1) for the  $q$ th training pattern. The first derivative of the logistic node activation function in (2.2) is given by

$$f'(z_{ik}^q) = \frac{1}{4} u_{ik}^q (1 - u_{ik}^q). \tag{A.6}$$

Denote the gradient of  $E(W)$  with respect to  $W$  by  $G$ , i.e.  $G = \nabla E(W) = \{g_{ik}^r\}$ , for  $i = 1, \dots, m; j = 0, \dots, m - 1; k = 1, \dots, n_i$ , and  $r = 1, \dots, n_i$ , then  $g_{ik}^r$  is given by

$$g_{ik}^r = \frac{\partial E(W)}{\partial w_{ik}^r} = \sum_{q=1}^Q \frac{\partial E_q(W)}{\partial w_{ik}^r} = -\sum_{q=1}^Q \delta_{ik}^q u_{qr}^q. \tag{A.7}$$

The connectivity weights are updated according to the following rule

$$W_{h+1} = W_h + \eta D_h, \tag{A.8}$$

where  $h$  is the iteration counter, sometimes called learning time,  $\eta$  is the learning rate,  $D_h$  is the search direction at iteration  $h$ , and  $W_h$  is the set of connectivity weights at the beginning of iteration  $h$ . Letting the set of values of  $G$  at iteration  $h$  be denoted by  $G_h$ , the search direction  $D_h$  is determined by

$$D_h = -G_h + \alpha_h D_{h-1}. \tag{A.9}$$

In our training algorithm,  $\alpha_h$  is determined by a combination of the Polak and Ribiere gradient conjugate direction (Polak 1971, Luenberger 1984) and a momentum factor. In the Polak and Ribiere gradient conjugate direction,  $\alpha_h$  is determined by

$$\alpha_h = \frac{(G_h - G_{h-1}) \cdot G_{h-1}}{\|G_{h-1}\|^2}. \tag{A.10}$$

For a given training set and  $W_h$ , the error measure  $E$  at iteration  $h$  is a function of the search direction  $D_h$  and the learning rate  $\eta$ . For a given search direction  $D_h$ ,  $E$  becomes a function of the learning rate  $\eta$ . Let us denote this error measure by  $E(W_h + \eta D_h)$ . There are many line search methods to determine a value  $\eta^*$  for  $\eta$  at which  $E(W_h + \eta D_h)$  is approximately minimized along  $D_h$ . In this training algorithm, we use a "doubling and halving" strategy to locate the initial interval of uncertainty, and the Golden Section method (Bazaraa and Shetty 1979, Luenberger 1984) to find  $\eta^*$ .

**A2. The Training Algorithm**

Step 0. Initialize the connectivity weights  $W_1$  to small values. Let  $\epsilon_1 > 0$  and  $\epsilon_2 > 0$  be small. Let  $\alpha_0 > 0$  be a pre-determined constant. Set the iteration counter to  $h = 1$ .

Step 1. Compute  $G_h$  according to (A.1-A.7), let the search direction be  $D_h = -G_h$ .

Step 2. Perform a line search to minimize  $E(W_h + \eta D_h)$  along the direction  $D_h$ . Let  $\eta^*$  be the value of  $\eta$  corresponding to the minimum of  $E(W_h + \eta D_h)$ . Update the connectivity weights by setting  $W_{h+1} = W_h$

+  $\eta^* D_h$ . If  $E(W_h) - E(W_{h+1}) < \epsilon_1$ , then Stop. Let  $h = h + 1$ . If  $(h \bmod |W|) = 0$ , where  $|W|$  is the cardinality of  $W$ , go to Step 1.  
Step 3. Compute  $G_h$  according to (A.1–A.7). If  $\|G_h\| < \epsilon_2$ , then Stop. Otherwise, compute the value of  $\alpha$  according to (A.10). If  $\alpha > \alpha_0$ , then let  $\alpha = \alpha_0$ . Let the new search direction be  $D_h = -G_h + \alpha D_{h-1}$ . Go to Step 2.

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