A CONCEPT OF COMPLEXITY FOR THE SOCIAL SCIENCES [*]

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Abstract

While most approaches to the idea of "complexity" attempt to handle this concept as an inherent property of the system under observation, this article considers complexity to be a property of the relationship between a system and its observer. It is argued that the concept of complexity must be used to measure the reliability attached by an individual to the classification model that he or she adopted in order to extract 'events' from empirical experiences. Complexity cannot be defined without previously modelling the individual's cognitive processes.

By assuming the knowledge of the mental categories by which an individual perceives reality, a quantitative measure of complexity is defined, and it is shown that its numerical value can be used to evaluate alternative patterns of behaviour. In particular, complexity is able to account for the features of decision-making in situations where empirical reality continuously generates facts of novel kind, as it is usual in the social sciences.

Finally, a comparison is drawn with theories of complexity diffused in physics, computer science and biology. Although most of these theories consider complexity an inherent property of the system under observation, it is argued that they can be seen as an approximation of the view on complexity presented here.

1. Self-Referentiality

The Cretan philosopher Epimenides (VI century B.C.) left, among others, a fragment where he claims that "All Cretans are liars". This statement originated what is known today as 'the Epimenides paradox': If what Epimenides claims is true, that is, if all Cretans lie, then Epimenides too, who is a Cretan, is lying. Thus, Cretans tell the truth. But if Cretans tell the truth, so does Epimenides. Hence, all Cretans lie.

That of Epimenides is only the first of a long series of self-referential paradoxes invented along the centuries. Alfred Whitehead and Bertrand Russell pointed to their common structure, which formally consists of a set containing some elements that are defined with reference to the set itself.

Whitehead and Russell (1910, p.37) note that, if one claims that a collection of propositions contains a proposition stating that 'all propositions are either true or false', such a statement could not be legitimate unless "all propositions" referred to some already definite collection; which is not possible, since a statement about "all propositions" is itself a new proposition, which can also be false.

This article explores the analogy between self-referentiality in formal systems and self-referentiality in human societies, and claims that self-referentiality is the core of the concept of complexity: Only when the difficulty to describe a system is due to the presence of a self-referential paradox we are allowed to speak of 'complexity', rather than of 'complication'. Clearly, this statement is not compatible with viewing complexity as an intrinsic property of a system, like mass or volume. It rather implies that complexity arises from the interaction between a system and its observer, when this interaction is such that it is in principle impossible for the observer to contemplate all the possible reactions of the system to the actions he can undertake on it.

As F.J. Varela pointed out, already at the level of molecular organisation self-referentiality is what distinguishes living organisms from inanimate objects (Varela 1979). In the aggregates of highly sophisticated living organisms that human societies are, self-reference arises in a straightforward way from the very fact that the members of a society have imaginative capabilities about its possible arrangements. The possibility of the emergence of new arrangements of society can generate a number of self-referential loops, and consequently logical indeterminacy. In order to keep the matter tractable, we will limit our analysis to the complexity which can arise between a generic ‘social system’ and its ‘observer’, where one single self-referential loop can occur.

Instead of defining the “complexity” the system should possess, let us focus on the adjective 'complex' the observer of a system may attach to it: Let us say that a social system is viewed as “complex” by its observer, when its reactions depend on what the system “imagines” the observer is imagining. A process of the kind "I think that you think that I think that ..." is clearly a self-referential loop. It also suggests that besides the complexity of a system seen by its observer, we might speak of the complexity of the observer “seen” by the system as well.

R.E. Lucas' famous statement about the impossibility to use econometric models to evaluate the effects of alternative economic policies (Lucas 1976) can be seen as deriving from the recognition of the complex nature of the interaction between a Government and the economy it is supposed to control. In fact, what Lucas claims is that the reaction of the economic system to the Government’s actions depends on what economic agents imagine about the possible actions of the Government, which in turn depends on what the Government imagines about the possible reactions of the other economic agents, and so on endlessly like in all self-referential loops.

The logical limits engendered by self-referential loops become clear when they are analysed within the framework of formal logic, where self-referential loops are exploited in Gödel's theorem, which states that in any formal system there are true propositions that cannot be proven. However, considering self-referentiality within the framework of logic is not only important because in this way we can stress the logical limits of decision-making, but also because it enables us to understand how a self-referential paradox can be overcome, and at what cost.

J.P. Dupuy observes that, in order to overcome the paradoxes caused in decision-making by self-referentiality, the crucial point to keep in mind is that any indecidability problem refers to a particular formal system, since for any proposition which cannot be proven within some formal system, there exists another formal system where it can be proven (Dupuy 1982).

Formal systems differ in the propositions they take as axioms, just like scientific theories differ about what should be regarded as invariant properties, and more generally about which are the relevant aspects of reality. Since a particular indecidability problem disappears if one frames reality in another way, the way out consists of re-defining what must be regarded as “axioms”, as well as the “rules of the game” they must obey.

Lucas takes just this way out of the self-referential paradox posed by economic policy when he wishes that the Government and the other economic agents take joint decisions in accordance to the principles of neoclassical economic theory, so that their effects are foreseeable, in the sense that there exists an agreement between all the actors regarding their behaviours (Lucas 1976). In fact, this corresponds to choose as “axioms” the neoclassical economic “laws”, and to fix the way they must be used in social interactions. However, Lucas’ choice is not unique, since in practice nothing prevents economic agents to establish agreements which do not strictly obey neoclassical prescriptions.

In any case, self-referential loops force decision-makers to acknowledge their limits and to reshape the decision process. In the framework which will be used in the subsequent sections, re-shaping the decision process will mean to re-define the mental categories by which the observer of a system perceives it.

This is a problem which will not be tackled in this article. Instead, we will concern ourselves with the perception of the limits to decision-making caused by self-referential loops, i.e. with the perception of the “complexity” of a system.

Section 2 contains the basic assumptions concerning the cognitive processes, which underlie the measure of complexity presented in section 3. Section 4 analyses the implications of this measure of complexity for decision theory, while sections 5 and 6 investigate the links with other concepts of complexity, used in disciplines other than the social ones.

2. Assumptions about the Cognitive Processes

The view of complexity as a property of the relationship between a system and its observer is originally due to R. Rosen (Rosen 1985), who argues that an observer sees a system as 'complex' when he has more than one single description of it, and these descriptions cannot be reduced to only one. The point is not to have some alternative models of the same system, each with a certain probability to turn out to be the right one;

rather, a system is seen as “complex” by its observer when, due to the presence of a self-referential loop, the observer can never compile a finite list of the behaviours the system will exhibit.

In this situation, it is not possible to neglect the distinction between the system’s behaviour, which can exhibit unpredictable and novel features, and the mental categories by which the observer perceives this very behaviour, categories which must be relatively invariant with respect to it. When dealing with complex systems, "events" cannot be considered as objects existing by themselves in the empirical reality; rather, they are the product of the classifying activity of the human mind. Individuals first have to decide which phenomena are repetitions of a smaller number of abstractly defined 'events', before applying inferential methods to discover statistical or deterministic laws; after that, the individuals might revise the adopted definition of events, depending on the extent to which these laws fit reality. For example, F.H. Knight pointed out that it is up to the entrepreneur to form categories of investment types, before calculating anything like a probability distribution of successes and failures (Knight 1921).

Complexity can be quantified upon description of the mental categories employed by an individual, of the relationships occurring between them, and of the extent to which they are appropriate to provide an orientation to the individual’s decision-making. In order to provide a clear framework for this task, let us first of all define a few key-concepts:

action: The behaviour of an individual towards the rest of the social system. An individual’s actions can undergo unpredictable qualitative changes; in this respect, they are basically different from Savage’s “acts” (Savage 1954).

result: The reaction of the social system to an individual’s action on it. Like actions, results can undergo unpredictable qualitative changes, and are thus basically different from Savage’s “consequences” (Savage 1954).

empirical fact: An ‘empirical fact’ is a pairing of an action and of a result. Being composed of actions and results, also empirical facts can undergo unpredictable qualitative changes (e.g. emergence of new ideas, new technologies, etc.).

mental category: Mental categories have the purpose to enable the individual to distinguish actions and results according to the features which are relevant to the problems he has to face; they can be conceived as “containers” appropriate to classify the facts of the real world. However, G. Lakoff points out that mental categories cannot be conceived as containers operating according to some similarity criteria fixed once and for all; rather, the categories the mind forms depend on the empirical facts it receives as input, and the shape of these mental categories may not even be precisely defined, since the boundaries between them can be fuzzy. Furthermore, although most mental categories are organised around a prototype, not all the members of a category need to be similar one another; rather, they sometimes have some similarity with only some of the other members of the category, not with all of them (Lakoff 1987).

event: An 'event' is a pairing of an action category and a result category: events are thus the product of the classifying activity of human mind, they are not givens in the empirical reality. We say that the same event occurred two times, if and only if two empirical facts occurred, which differed only in features which were felt to be irrelevant for the decision problems at hand, so that they could be classified in the same mental category.

mental model: P.N. Johnson-Laird showed that human behaviour can be more easily explained assuming that the mind constructs mental models of reality, rather than by assuming the existence of a "mental logic" (Johnson-Laird 1983). Such mental models do not produce the correct behaviour in any possible situation, but in most of the situations the individual usually encounters. If we refer to the basic ideas of connectionism (Hebb 1949; von Hayek 1952), we can think mental categories as implemented by paths in which information flows in closed loops, and mental models as connections between these categories (Hayek calls "map" the set of the mental categories, and "model" the set of the connections between them).

At this point, let us make the following assumptions about the social systems we will consider, and about the cognitive processes of their members:

A. 1

Let us consider the point of view of a single individual towards the rest of the social system. This is the simplest application of the system-observer scheme; eventual extensions to more detailed analyses would require: 1) Consideration of observers, like firms or governments, composed by many individuals; 2) Consideration of the agents composing the social system, and of their points of view as observers of what they define as "social system" in their turn.

Our individual-observer interacts with the social system by undertaking 'actions' and obtaining 'results' which can undergo unpredictable qualitative changes with time; therefore, no "set of the states of the world" can be specified. On the other hand, description of the individual's mental categories allows to specify the "set of the states of the mind" by which the world is understood.

A. 2

As time goes by, the individual who observes the social system may change the categories he uses to understand reality, creating sub-categories and re-defining the criteria by which actions and results are assigned to existing categories. However, it seems reasonable to assume that the categories by which the individual understands the world change on a slower time scale than the world itself: otherwise it would be hard to imagine how the individual could detect changes of the external reality. For simplifying purposes, let us make this assumption in its extreme form, assuming that the mental categories by which the individual perceive reality do not change at all.

Let $\mathcal{A} = \{A_1, A_2, \ldots A_I\}$ be the finite set of mental categories available to the individual-observer to classify the actions he undertakes; let us denote them by an index $i = 1, 2, \ldots I$. Let $\mathcal{R} = \{R_1, R_2, \ldots R_J\}$ be the finite set of mental categories available to the individual-observer to classify the results he obtains; let us denote them by an index $j = 1, 2, \ldots J$. The other individuals or organisations of individuals which constitute the social system may have different categories; however, when information is transferred from the social system to the individual who is observing it, this information is categorised according to the individual's criteria.

A. 3

The individual is endowed with a memory of finite length L where he stores the data relative to the actions he undertook and to the results he obtained during the last L interactions with the social system. The finiteness of memory dimension, as well as the finiteness of the number of categories of actions and of results, is an assumption of "bounded rationality" (Simon 1964, 1972).

The data contained in the memory can be used to measure the probabilities of obtaining a result belonging to category j, provided an action belonging to category i has been undertaken: $p\{A_i \to R_j\}$. Let us also assume that some utility function is defined on the sets $\mathcal{A}$ and $\mathcal{R}$. Then, as far as the individual deems that his mental categories appropriately describe the world, traditional decision theory applies, and some objective function (e.g. expected utility) can be calculated. Up to this point, the only difference with established decision theory (e.g. Savage 1954) is that these utilities are defined on mental categories, rather than on objects of the empirical reality.

However, at each interaction between the individual and the social system he observes, information relative to the undertaken action and the obtained result enters his memory; at the same time, the information referring to the oldest pair action-result exits it. In this way, the contents of the categories of actions and results change with time, even if the categories themselves do not change.

A. 4

Mental categories are shaped by the empirical experience along with the mental model connecting them, in order to provide the individual with an orientation in decision-making. This orientation consists of specifying which are the categories of results expected from actions belonging to a certain category. In more formal terms, we can say that mental categories are such that some connections between them is expected to occur much more often than the others, and that these connections are what we call ‘model of the world’: $\forall A_i, \quad \exists \mathcal{R}_i \in \mathcal{R}: \quad p^e\{A_i \rightarrow R_j : R_j \in \mathcal{R}_i\} \gg p^e\{A_i \rightarrow R_j : R_j \notin \mathcal{R}_i\}$, where p^e are probabilities the individual is confident will hold in the future, while they do not necessarily coincide with the probabilities measured in the past.

A. 5

For simplicity, let us assume that the number of categories of actions equals the number of categories of results, and that the model of the world consists of one-to-one correspondences between them. Fig.(1) shows such a model of the world:

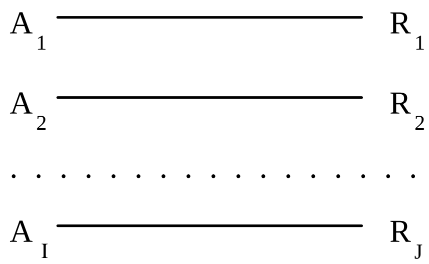


Fig. 1

This model of the world says to the individual that if he undertakes an action belonging to category A_2, a result belonging to category R_2 is the “normal” outcome (e.g. the expected return of an investment belonging to some category). Let us call such a simple model of the world the ‘fundamental description’.

A. 6

When correspondences other than those of the fundamental description occur (e.g. A_2 causes R_1), the individual may think that this is just a case which seldom occurs, but already considered in the distribution of the probabilities $p\{A_i \rightarrow R_j\}$. This would indeed be the only possibility if elementary events were objectively defined in a context where the identification of possible empirical facts is self-evident, as traditional decision theory does.

But in the context we are considering, there is also the possibility that our individual thinks that the mental categories and the model of the world he is using, are no longer appropriate to discriminate the relevant features which are emerging in the empirical reality. That is, the individual may think that actions and results underwent such deep qualitative changes in their features, that the categories used to classify them are no longer able to lay stress on the relevant features and to neglect the irrelevant ones. In this situation, the occurrence of correspondences between categories of actions and of results other than those of the fundamental description can be interpreted as a consequence of the generation of qualitatively new empirical facts (e.g. in the case of investment decision-making, new goods and new technologies may be such that the usual return prospects do not hold). Consequently, a measure of the tightness of the correspondences between categories of actions and categories of results can be taken as a measure of complexity.

A. 7

Individuals calculate the probabilities $p\{A_i \rightarrow R_j\}$ using all the data at disposal in their memories, because in this part of the evaluation of empirical information they lay stress on the similarities among empirical facts. But as far as they cast doubts on the appropriateness of the classification criteria underlying these probability distribution, they use the most recent data only, since only these data contain the novel features that emerge in the empirical reality.

Let us assume that, if M is the number of the most recent memory positions the individual uses to detect eventual novel features of reality, it is $M << L$. Looking at the most recent empirical facts, the individual observes whether any correspondences between action categories and result categories occurred, which are not those specified by the fundamental description. If this happened, he measures a complexity greater than zero.

3. Computation of Complexity

Let us now expose the details of the computation of complexity from the most recent M action-result pairs the individual stored in his memory. It is worth to note that, although the computation of complexity will take the form of an algorithm, Church thesis is not implied: as a number of researchers already pointed out (Smolensky 1986; Harnad 1990; Mitchell and Hofstadter 1990) algorithmic computation on abstract symbols defined by the human mind in order to perform high level tasks is perfectly compatible with categorisation, generalisation and induction at the basic level.

Let us assume that the fundamental description is always at hand in the mind of the individual, no matter whether the pairs (A,R) constituting it are present in his memory, or not. If, in the mind of the individual, some connections between action categories and result categories are recorded, which are not those of the fundamental description, then the correspondences between action categories and result categories which the individual knows might be for example those depicted in fig.(2):

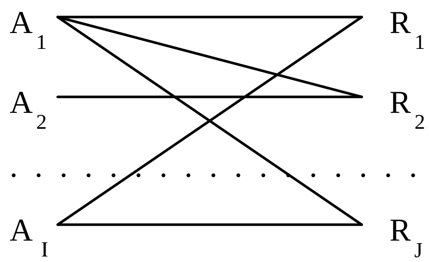


Fig. 2

Let us recall Rosen's definition of complexity reported at the beginning of the previous section: an observer sees a system as 'complex' when he has more than one single description of it. Let us define as 'description of reality' any set of correspondences between action categories and result categories, so that the "fundamental description" we chose as "model of the world" is only one of the many possible descriptions of reality (for simplicity, we assumed that the model of the world coincides with the simplest possible description). Then, complexity is greater than zero when, by selecting some of the correspondences between mental categories that are present in the first M locations of the individual's memory, more than one single description can be constructed. Thus, complexity can be measured by how tightly the A-R relations contained in the first M memory locations are intertwined.

Note incidentally that what matters for the computation of complexity, is whether a pair (A,R) is present at least one time among the last M data of the individual's memory, while it does not matter whether a pair is present one, two, three times, or more. This is a very important point, because it shows that what matters is whether the individual has been surprised by the emergence of some novel feature of reality, while it does not matter how many times this very same novel feature occurred.

In order to measure the tightness of weaves of the kind shown in fig. (2) we will resort to a work of R. Atkin (1975, 1981), which uses algebraic topology to represent the hierarchical relations between the individuals of a society. Since algebraic topology is also a fundament of Category Theory (Rosen 1991), it makes sense to apply Atkin's studies to the context of the hierarchies of action categories and of result categories in an individual's mind. The hierarchies investigated herein actually border triviality, having just a single level of categories of actions and a single level of categories of results.

To see the world as complex, that is, to consider other descriptions apart from the fundamental one as equally possible, means that at least one action category A_i has correspondences with more than one result category. Let us denote by $\mathcal{A}$ and $\mathcal{R}$ the sets of action categories and respectively of result categories, and let us denote by $\mathcal{R}_i$ the subset of $\mathcal{R}$ constituted by the categories of results that are in correspondence with A_i, and by $\mathcal{A}_j$ the subset of $\mathcal{A}$ constituted by the categories of actions that are in correspondence with R_j. In more formal terms, the individual introduces a relation λ between $\mathcal{A}$ and $\mathcal{R}$ such that $(A_i, R_j) \in \lambda$ when A_i is in correspondence with a subset of $\mathcal{R}$ which contains R_j.

A 'simplicial complex' $K_{\mathcal{A}}(\mathcal{R}, \lambda)$ is an object we introduce with the purpose to represent geometrically this relation λ between sets $\mathcal{A}$ and $\mathcal{R}$. It is made of so many simplices as there are elements of $\mathcal{A}$; vertices of simplex A_i are the elements of $\mathcal{R}$ which belong to subset $\mathcal{R}_i$ (see appendix A). If the only connections between categories of actions and categories of results are those of the fundamental description, simplices are not connected to one another. In contrast, if there are also other connections, at least two of the corresponding simplices have at least one vertex in common.

Furthermore, let us represent relation λ also by an incidence matrix $\mathbf{\Lambda}$ of dimensions $I \times J$, whose generic element λ_{ij} can take the values:

$$\lambda_{ij} = \begin{cases} 1 & \text{if } A_i \in \mathcal{A}_j \text{ and } R_j \in \mathcal{R}_i \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Let us now consider the dual relation λ^*, defined inverting the roles of sets $\mathcal{A}$ and $\mathcal{R}$: now $(R_j, A_i) \in \lambda^*$ when to R_j corresponds a subset of $\mathcal{A}$ containing A_i. Dual relation λ^* can be represented by the dual simplicial complex $K_{\mathcal{R}}(\mathcal{A}, \lambda^*)$, which has incidence matrix $\mathbf{\Lambda}^T$. The reason for introducing the dual simplicial complex is that element (i,i') of matrix $\mathbf{\Lambda}\mathbf{\Lambda}^T$ equals the number of vertices that simplices A_i and $A_{i'}$, of simplicial complex $K_{\mathcal{A}}(\mathcal{R}, \lambda)$, have in common. Then, element (i,i') of matrix $\mathbf{\Lambda}\mathbf{\Lambda}^T$ minus one, is the dimension of the eventual common face between simplices A_i and $A_{i'}$. If such a number is negative, then simplices A_i and $A_{i'}$ have no common point.

Let us define the following $I \times I$ matrix:

$$\mathbf{L} := \mathbf{\Lambda}\mathbf{\Lambda}^T - \mathbf{1}\mathbf{1}^T \tag{2}$$

where $\mathbf{1}$ denotes a $I \times 1$ vector of ones. The dimension of the eventual common face between simplices A_i and $A_{i'}$ is then equal to element $l_{ii'}$ of matrix $\mathbf{L}$.

Two simplices having no point in common may nonetheless have common points with other simplices, which in turn do have common points. We say that simplices A_i and $A_{i'}$ are 'q-connected' in $K_{\mathcal{A}}(\mathcal{R}, \lambda)$ if there exists a chain of simplices $\{A_{h_1}, A_{h_2}, \ldots A_{h_r}\}$ such that $q = \min\{l_{i h_1}, l_{h_1 h_2}, \ldots, l_{h_r i'}\} \geq 0$.

Let Q be the dimension of the simplex of $K_{\mathcal{A}}(\mathcal{R}, \lambda)$ which has the largest dimension. We can partition the set of simplices composing $K_{\mathcal{A}}(\mathcal{R}, \lambda)$ in the following way: for any q, $0 \leq q \leq Q$, by inspection of matrix $\mathbf{L}$ one obtains which simplices are connected at any level q.

Let us introduce a 'structure vector' $\mathbf{q}$, of dimensions $(Q+1)\times 1$, and let us denote its q-th component by $\mathbf{q}_q$. In general, for any level of connection q there is some number of disjoint classes of simplices, such that the simplices belonging to a class are connected at that level: the q-th component of structure vector $\mathbf{q}$ is the number of disjoint classes of simplices connected at level q.

Simplices connected at level q are obviously connected at level $q-1$, too. However, in order to avoid repetitions, we do not consider a class of simplices connected at level q to be also a class of simplices connected at levels $q-1$, $q-2$, etc. For example, let simplices A_1 and A_2 be connected at level $q=2$, and let simplex A_3 be connected at level $q=1$ with A_2. Then, $\{A_1, A_2\}$ is a class of simplices connected at $q=2$ and $\{A_1, A_2, A_3\}$ is a class of simplices connected at $q=1$; however, $\{A_1, A_2\}$ is not a class of simplices connected at level $q=0$.

The measure of complexity of a simplicial complex is a measure of the extent to which it is connected. Thus, let us consider only the case where the simplices of a simplicial complex are at least connected at level $q=0$, i.e. each of them has at least a common point with another simplex. Otherwise, i.e. if $K_{\mathcal{A}}(\mathcal{R},\lambda)$ is made of parts having no point in common, let us agree to take as complexity of $K_{\mathcal{A}}(\mathcal{R},\lambda)$ the largest among the complexities of its parts.

An evaluation of the complexity of Atkin's simplicial complexes can be found in J. Casti (Casti 1989, p.410). Here I propose a slight modification of this measure since Casti's formula, for low values of complexity, can attach the same value to very different simplicial complexes:

$$c(K) = \begin{cases} 0 & \text{if } \boldsymbol{\Lambda} \equiv \mathbf{I} \\ \displaystyle\sum_{q=0}^{Q} \left.\frac{(1+q)}{\mathbf{q}_q}\right|_{\forall \mathbf{q}_q \neq 0} & \text{otherwise} \end{cases} \tag{3}$$

This measure of complexity is zero when the simplicial complex reduces to single simplices, each of them being just a point. On the other hand, the largest value which complexity can attain depends on the number of categories of actions and of results, which is also the largest possible number of simplices, and respectively of their vertices. Thus we can write

$$0 \leq c(K) \leq c_{MAX\,(I,J)} \tag{4}$$

In order to calculate the contribution to complexity of a single action classified in category A_i, let us calculate the difference between the complexity of the original simplicial complex and the complexity of the simplicial complex obtained by removing the simplex corresponding to A_i. Denoting these two simplicial complexes by $K_{\mathcal{A}}$ and $K_{\mathcal{A} - \{A_i\}}$, the contribution to complexity of an action classified in A_i is

$$c(A_i) := c\left(K_{\mathcal{A}}\right) - c\left(K_{\mathcal{A} - \{A_i\}}\right) \tag{5}$$

Note that, while for simplicial complex $K_{\mathcal{A}}$ it is $I = J$, for simplicial complex $K_{\mathcal{A} - \{A_i\}}$ it is $I = (J - 1)$. Note also that the sum of the $c(A_i)$, in general, is not equal to the complexity of the original simplicial complex.

Appendix A contains a numeric example of calculation of the complexity of a simplicial complex.

4. Influence of Complexity Evaluation on Decision-Making

The purpose of this section is to connect the theory of complexity exposed above to the established decision theory, and to point to some possible fields of application.

Complexity measures the confidence of an individual in the way he decided to define events by neglecting some features of empirical facts, while stressing others. Once events are defined, usual decision theory applies, and some objective function must be maximised. Let us assume that the most simple objective function, subjective expected utility, is appropriate to evaluate the decision problem as it is defined by the existing mental categories, and let us examine how complexity influences decision-making.

To this aim, let us derive from the complexity attached to a single action category (5) a 'degree of confidence' in the appropriateness of a single action category to the decision problem at hand.

We want this degree of confidence to increase when complexity decreases, and *vice versa*. Furthermore, we require that it ranges within $[0,1]$, so that we can take account of it by simply multiplying it for the expected utility. Consequently, let us define the degree of confidence ω attached to action category A_i as follows:

$$\omega\left(A_i\right) \ := \ 1 - \frac{c\left(A_i\right)}{c(K)} \tag{6}$$

The objective function the individual attempts to maximise thus becomes

$$J(A) \ := \ [\omega(A)]^{k(A)} \sum_{j=1}^{J} p\left(R_j \middle| A\right) u\left(R_j\right) - u(A) \tag{7}$$

where exponents $k(A)$ have a twofold meaning:

1) The importance that an individual attaches to the degree of confidence depends on his personal attitudes, as well as on the type of decision problem. For example, investment decisions are much more risky than purchase decisions at the supermarket; consequently, economists speak of the "animal spirits" of entrepreneurs, not of consumers (Keynes 1936).
2) The degree of confidence should have opposite effects on the individual's decisions when it refers to a result worse than the one foreseen by the fundamental description, and when it refers to a result better than the one foreseen by the fundamental description. In the first case, when the degree of confidence is <1 the objective function should decrease, while in the second case, when the degree of confidence is <1 the objective function should increase.

Let us denote by R_F the category of results which is associated to category of actions A in the fundamental description. Given the disutility associated with A, if utility function has on R_F a value above (under) the average, the degree of confidence should be such that the value of objective function is lower (higher) than without taking account of it.

From these considerations it is immediate to derive the following expression for the exponent of the degree of confidence:

$$k(A) \; = \; \zeta(A)\,\xi(A) \; = \; \zeta(A)\,\frac{u\left(R_F\right) \; - \; u(A)}{\sum_{R_j \in \mathcal{R}} \left[u\left(R_j\right) - u(A)\right] \Big/ card\ \mathcal{R}} \tag{8}$$

where $\zeta(A)$ takes account of the individual's attitude towards the degree of confidence on action category A, while $\xi(A)$ takes account of the possibility that $\omega(A)$ can be referred to a result with utility above or under the average.

It is important to remark that objective function (7) differs from the ones usually used in decision theory, since its arguments are mental categories of actions and of results instead of "acts" and "consequences" given once and for all (like in Savage 1954). This means that preferences are supposed to be defined on the mental structures by which individuals perceive objects, rather than on the objects themselves.

As long as the model of the world and the underlying mental categories are not questioned, the effect of the degree of confidence is that of introducing sudden jumps in the individual's behaviour, which occur at the moments where correspondences between categories, which are different from those of the fundamental description, enter or exit the first M memory locations. In these moments the degrees of confidence in the action categories can change abruptly to different degrees, making suddenly some action category preferable to the others.

However, an even more important consequence for decision theory is the effect of large values of the overall complexity (3), which refers to the whole representation of reality of an individual. When complexity is high, the mental model and the underlying mental categories do not provide anymore an orientation in a world whose qualitative features continuously change. The individual must re-formulate his mental categories, as well as the model of the world defined upon them; this can lead to the decision to postpone decision-making until the problem will be framed in a more proper way. However, this article does not deal with the modellisation of how mental categories and mental models of the world evolve over time, but only with the recognition of the need to re-formulate them.

A comparison can be drawn with classifier systems (Holland 1975, 1986), if we accept a reductive view of mental categories as collections of objects that share some common properties. Then, classifiers can be taken as rudimentary mental categories, the connections between classifiers induced by their “strengths” as connections between the mental categories which constitute the model of the world, and the genetic algorithm as the process of category re-formulation. As long as this metaphor holds, we can say that complexity evaluation has the purpose to decide when the genetic algorithm must be activated. However, it is important to stress that, unlike classifier system, the model presented in this article neither needed to specify the set of the input characters which carry information from the empirical reality, nor the length of the strings of these characters. This is an important point, since if the set of possible input characters and the length of their strings is fixed once and for all, the set of the possible “novelties” which can be discovered is fixed once and for all, too.

Another comparison can be drawn with R. Heiner’s remark that an individual may prefer to exclude some action “a” from his repertoire, if he is not always able to discriminate the information which should trigger “a” from the rest of the information, and if undertaking “a” on the wrong occasion causes a substantial loss (Heiner 1983). Even if only in the most recent work Heiner indirectly refers to mental categories, instead of to actions specified once and for all (Heiner 1986), it is evident that excluding “a” from the set of the possible actions is a particular case of reformulation of the mental categories underlying the decision process. However, Heiner himself (Heiner 1986, p.67) points out that the probability that the individual is able to process correctly the information he receives, which has a crucial importance in his theory, can only be evaluated *ex post* by the experimenter which observes decision-making; it cannot be evaluated by the decision-maker himself while he is taking a decision. On the contrary, the complexity and the degree of confidence defined in this article aim to describe the decision process from the point of view of the decision-maker himself.

Situations where self-referential loops generate novel features occur very often in the social sciences; whenever novel features emerge, it makes sense to attach a degree of confidence to the probability distribution of the categories of the results which can be obtained from the action which one undertakes.

The theory developed above was referred to the rather abstract situation of a social system observed by a single individual; in reality, agents that observe and attempt to control social systems are more often organisations of individuals. Many scholars in cognitive science however (Bateson 1972; Hutchins 1995), stress the similarity between the mind of a single individual and the "collective mind" of an organisation of individuals. We point now to situations where it is useful to consider the complexity and the degree of confidence defined above, with the understanding that the theory applies to collective agents as well:

1) Self-referentiality in the relationship between a Government and the rest of society has been already discussed in §(1), although the argument was limited to the economic domain. The case is actually much more general, 'power' being according to N. Luhmann the capability to reduce complexity by restricting the set of alternatives of the subordinates (Luhmann 1975). In the terms of this essay, the power of a Government resides in its ability to overcome decision impasses by casting the problem in terms such that a unique solution appears feasible. If decisions are taken before some consensus is reached, they are likely to be ineffective; thus, on some occasions a Government might prefer to postpone decision-making.
2) The evaluation of the environmental impact of human activities is open to very many interpretations, since it depends on a huge number of variables, many of them not directly observable. A self-referential loop arises between scientists and the society they are part of, with the consequence that scientists tend to stick on a consensus about the values of some critical magnitudes as they had been estimated at the beginning of environmental research (van der Sluijs, van Eijndhoven, Wynne and Shackley 1997). Practical users often attach little confidence to the environmental predictions scientists produce; for example, insurance companies often refuse to stipulate policies against catastrophic environmental risks, on the ground that scientifical estimates are not reliable enough (Kunreuther 1996). This kind of behaviour can be explained by the need to reformulate the mental categories the decision problem is cast into, postponing any decision until this task has been accomplished.
3) A firm's behaviour depends on what competing firms do. Due to the self-referential loops between a firm and the industrial system it is part of, strategic decision-making often cannot be univocally determined. If a firm is uncertain about the categories by which it interprets information about current technologies and customers' needs, the sudden jumps of the degree of confidence defined above produce a high variability in

decision-making that reminds of the “animal spirits” J.M. Keynes’ spoke of (Keynes 1936); in extreme situations, the lack of confidence in the model of the world may lead to the decision to postpone any decision. Appendix B discusses this example in detail.

5. Relationship to Complexity in Classical Physics and in Computer Science

The objects studied by classical physics belong to a class of particularly simple systems; essentially, macroscopic bodies moving according to the laws of Newtonian mechanics and thermodynamic systems. Scientific tradition established a univocal identification of elements which, for specific aims, can be considered as the ultimate constituents of these systems. What these constitutive elements do, are the 'elementary events' one is concerned with (e.g. positions and velocities of moving bodies); in this context, consideration of the observer's cognitive processes is usually omitted.

Consequently, the relationship between system and observer is trivial; scientific inquiry can focus upon properties of the system which can be considered to be "objective". Computer science also finds itself in a similar situation, since the definition of the set of symbols a computer will manipulate is equivalent to the definition of some “elementary events” all the others are constructed by.

Since in this situation the observer considers a set of exhaustive events the system can produce, the intuitive idea of complexity as "capacity of a system to surprise its observer" (or similar statements), can be expressed in probabilistic terms. It turns out that, in this context, the traditional concept of 'entropy' is a good starting point for the meaning one would intuitively attach to the term 'complexity', although these two concepts cannot be taken as equivalent.

In fact, let us recall that the immediate meaning of thermodynamic entropy is to be a measure of the “disorder” of a system: "disorder", in intuitive terms, is supposed to be more "complex" than "order", at least up to the point where disorder is so large that one gives up the hope to understand its laws. Information entropy, in its turn, measures the “disorder” of the information arriving at the end of a communication channel; it is thus a measure of “lack of information”, which is another meaning one would intuitively attach to “complexity”. Furthermore, L. Brillouin showed that thermodynamic entropy and information entropy are closely linked, since it is not possible to decrease one without increasing the other: in order to obtain less “disordered” information about a system, we must increase the “disorder” of the system itself (Brillouin 1956).

Entropy is also linked to the concept of 'computational complexity' of a sequence of characters, introduced by R.J. Solomonoff (1964), A.N. Kolmogorov (1965) and G.C. Chaitin (1966). The computational complexity of a sequence of characters Y, given a sequence of characters U, is the minimal length of the program *h* which computes Y knowing U: computational complexity is maximal when there is no simpler way to describe Y than listing all its characters, because there are no regular patterns in it; which is equivalent to say that the sequence is “disordered”, or that its entropy is maximal.

On the other hand, even if we limit our concern to the systems studied by classical physics, entropy is not a completely satisfactory measure of the meaning we would like to attach to the term “complexity”, since some completely disordered systems we can only describe by means of statistical averages, like e.g. the molecules of a gas, are not intuitively perceived to be “complex”. Rather, it seems that maximal complexity should lie somewhere between zero and maximal entropy, in the zone where a system is neither so completely ordered that it can be described by simple deterministic laws, nor so completely disordered that it can be described by simple stochastic laws. In this zone, the system does exhibit some structure, which an observer finds very “complex” to describe.

Some measures of complexity have been proposed, that take minimum value when entropy is either minimal or maximal, and take maximal value in between; most widely accepted magnitudes are ‘logical depth’ proposed by C.H. Bennett (1988) and ‘statistical complexity’ proposed by J.P Crutchfield and K. Young (1989) (see also Crutchfield 1994).

The point I make in this article is that these measures of complexity, although correct for the context they have been thought for, are of no use in the social sciences, for the basic reason that they assume the knowledge of the “elementary events” the system can exhibit. In the social sciences this circumstance is negated, and it is necessary to consider the mental categories by which information is perceived, and the different “events” which are defined when categories are shaped.

The model presented in the previous sections does not contrast with the ideas underlying Bennett’s or Crutchfield’s measures of complexity. Rather, it can be used to re-interpret the idea that complexity is maximal when entropy is neither zero, nor maximal.

In fact, the crucial point of the whole theory of complexity presented in this article is that the individual who is observing a system needs a simple and reliable model of it; when entropy increases over a certain limit, he will re-define the categories by which he is trying to understand the system, switching e.g. from a deterministic microscopic description to a stochastic macroscopic description. Using the new categories, e.g. the macroscopic magnitudes thermodynamics defines, it is easy to describe the system by a simple and reliable model; in other words, the system does not appear "complex" anymore, once the mental categories to understand it have been changed. In this way, we have zero complexity at the two extremes of zero entropy and maximal entropy; on the contrary, complexity is maximal when entropy has already increased up to quite a high level, while the mental categories are still the initial ones.

Note that a distinctive feature of the "subjective" concept of complexity presented in this article is that it implies that complexity varies through discontinuous jumps. Minor jumps occur whenever correspondences between categories, which are different from those of the fundamental description, enter or exit the first M locations of the individual's memory. Moreover, much more dramatic jumps occur when new mental categories are defined (e.g. at the passage from a deterministic microscopic description to a stochastic macroscopic description of a physical system). In any case, whereas "objective" measures of complexity, like logical depth or statistical complexity, vary with continuity, the "subjective" approach presented in this article accounts for abrupt changes in an individual's attitude towards the system he is observing.

6. Relationship to Complexity in Biology

The first attempt to define complexity in biology is that of H. Atlan (1972), who proposed an extension of information theory (Shannon 1949; Khinchin 1957) which rests on the non conventional assumption that, in a transmission system, the receiver does not know all the words that the source can emit. This assumption has non-trivial implications for a transmission system which uses a redundant codification: if random disturbances modify the original words transforming them into some of the words that in a redundant codification are not used, the receiver is no longer able to distinguish between the words generated by the random disturbances, and the words actually emitted by the source.

In this way, Atlan can interpret evolution within the framework of information theory, with DNA’s nucleic acids as the characters emitted by the source of some transmission system, the living species as the "words", or sequences of these characters (i.e. sequences of nucleic acids of a given length), genetic mutations as the effect of a noisy channel between source and receiver and, finally, the natural environment as the receiver of such a transmission system. Of course, this is a rough schematisation which does not take any account of the very many environmental constraints that influence the passage from phenotype to genotype; however, in principle it would be possible to take account of these constraints by assuming that some “words” cannot be produced, without any need to change the conceptual framework.

The interesting remark is that, given a certain word length (length of the amino acid sequences), random mutations can create new “species” only at the expense of the words the redundant codification had left unused. For a word length fixed once and for all, the more new words (new species) are generated, the more difficult evolution becomes.

Atlan captures this property of evolutionary dynamics by means of the mean mutual information between source and receiver. If any two distant parts of a system are taken as ‘source’ and ‘receiver’ and their behaviour is analysed by means of information theory, mean mutual information measures how related these parts are, in the sense that it measures how similarly they behave. It is thus somehow a measure of the structures that are present in the system, and for this reason many scholars working in information theory take mean mutual information as a measure of complexity (see Bennett 1990 for a thorough discussion of this topic).

In the context of Atlan’s theory, mean mutual information first increases with time, because of the generation of useful species that diffuse quickly over the whole population; then it decreases, due to the random generation of useless mutants, which is the most that can happen after the redundancy of DNA codification has been exploited. Maximal mean mutual information corresponds to an optimal balance between replication of existing phenotypes, and experimentation with new mutants.

This is consistent with Stuart Kauffman's studies of the shape of the "fitness landscapes" where evolution takes place (Kauffman 1993). In fact, Kauffman argues that evolution is most effective when the frequency of mutations and the shape of the fitness landscape are balanced in such a way as to avoid on the one hand that the population spreads over all the possible species, no matter how fit they are, and on the other hand to avoid that the whole population concentrates on some species that are far from being the fittest ones. Such a balanced situation, bordering chaos on one side and order on the other, is denoted by Kauffman as a 'complex' one. In other words a "complex" situation, according to Kauffman, is one where mutations do occur, but they are not too many, which corresponds to the maximum of mean mutual information in Atlan.

The concept of complexity defined in this article can be connected to these theories of complexity.

From the point of view of an individual observing the biological system, mutations are novelties that may cause the model of the world not to function anymore, thereby increasing the complexity by which the world is seen. Complexity, as it has been defined in this article, increases by discrete amounts whenever emerging new traits make the old mental model useless, and decreases abruptly whenever the mental categories are re-formulated, and a new model of the world is constructed.

In Atlan’s theory, mean mutual information reproduces this pattern, although without the abrupt upward and downward jumps that characterise the measure of complexity presented in this article. However, in Atlan’s theory mean mutual information actually increases by discrete amounts any time a new word is created, even if this aspect has not been emphasised by Atlan himself.

The difference derives from the circumstance that in biology, like in classical physics but unlike in the social sciences, it is possible to identify some ultimate particles (nucleic acids), from whose combination all possible novelties can be generated. This allows to construct a theory of complexity which appears as “objective” as the ones used in classical physics and in computer science, and which measures complexity by means of a smooth function that does not take account of the obsrver’s cognitive processes.

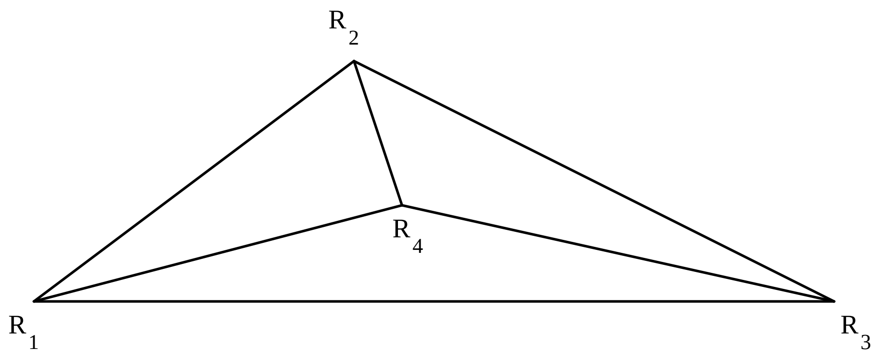
On the other hand, Atlan’s hypothesis actually introduces a subjective aspect in the theory, by allowing the receiver of a transmission system to evaluate the information carried by “new” words; the surprise caused by new words reminds to the misfunctioning of the model of the world caused by novel features of reality. Like in the social sciences, in Atlan’s theory “complexity” is the observer’s difficulty to understand the system, but like in classical physics, this complexity can be quantified without resorting to the description of what happens in the observer’s mind. Under this respect, to biology can be ascribed a status which is somehow intermediate between that of classical physics and that of computer science on one hand, and the status of the social sciences on the other.

Appendix A

Let relation λ be given by the following incidence matrix:

λ	R_1	R_2	R_3	R_4
A_1	1	0	1	1
A_2	0	1	1	0
A_3	1	0	1	0
A_4	1	1	1	1

Here A_1 is the two-dimensional simplex of vertices R_1, R_3, R_4; A_2 is the one-dimensional simplex of vertices R_1 and R_2, and so on. The simplicial complex can be drawn as follows:



The tethraedron shown in this picture is simplex A_4, its faces being simplices A_1, A_2 and A_3. Matrix **L** is:

$$\mathbf{L} = \Lambda\Lambda^T - \mathbf{1}\mathbf{1}^T =$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

In this case it is $Q = 2$. In order to build structure vector **q** we note that:
at $q = 2$ there is a single class of simplices $\{A_1, A_3\}$, so that $\mathbf{q}_2 = 1$;
at $q = 1$ there is a single class of simplices $\{A_1, A_2, A_3, A_4\}$, so that $\mathbf{q}_1 = 1$;
at $q = 0$ there is a single class of simplices $\{A_1, A_2, A_4\}$, so that $\mathbf{q}_0 = 1$.

Thus, structure vector is $\mathbf{q}^T = [1 \;\; 1 \;\; 1]$. At this point it is trivial to calculate complexity, obtaining $c(K) = 6$.

Appendix B

Let us consider a firm's investment decision in a situation where new technologies continuously emerge, and let us model the firm's cognitive apparatus by two categories of actions and two categories of results:

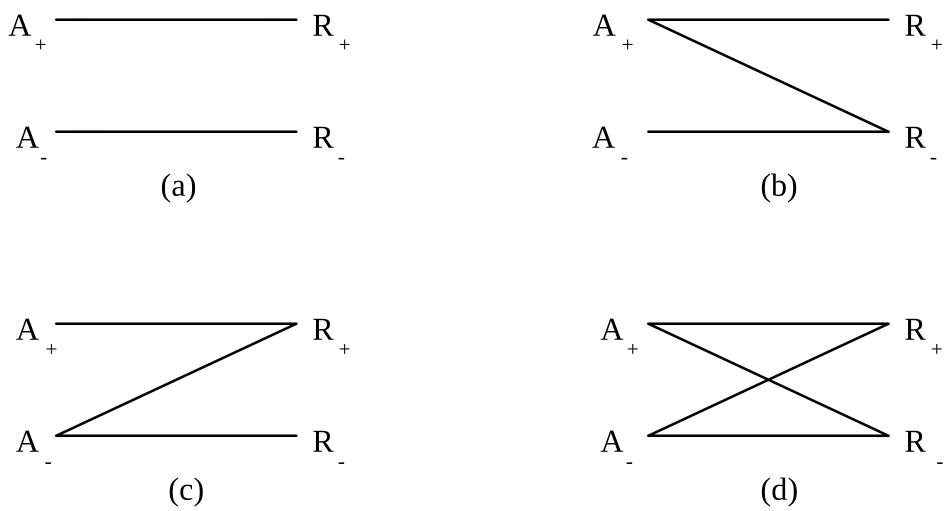
A_- is the category of the actions implying low investments, generally carried out by the firm with its own means; these actions usually do not lead to production and profit expansion, but rather to their stagnation or fall.

A_+ is the category of those actions implying large investments, generally possible only through bank financing; actions of this kind are undertaken because one expects from them a much better result than from actions of category A_-.

R_- is the category of mediocre results normally expected from actions of type A_-, while obtaining a result of type R_- from an action of type A_+ means the failure of the investment.

R_+ is the category of good results one expects from actions of type A_+, while receiving a result of category R_+ from an action of type A_- is a particularly favourable and unexpected event.

Let us assume that past experience be such that it is reasonable to classify actions into "actions implying low investments" and "actions implying high investments", because this leads to a simple model of the world: low investments caused low gain, high investments caused high gain. If this does not happen anymore, managers may think the cause are some relevant qualitative changes of the production technologies, which are not captured by the extremely rough categories "high investments" and "low investments". Four configurations are possible:



In case (a), only the connections foreseen by the fundamental description occurred, and the firm casts no doubt upon it. In the other cases, the model of the world is not verified: in case (b) our firm has doubts about the usefulness of the probability distribution of success and failures when it undertakes an action of category A_+, in case (c) when it undertakes an action of category A_-, while in case (d) doubts are present whatever the firm does.

Degrees of confidence, in these four cases, take the following values:

$$\text{case (a):} \quad \begin{cases} \omega(A_-) = 1 \\ \omega(A_+) = 1 \end{cases} \qquad \text{case (b):} \quad \begin{cases} \omega(A_-) = 1 \\ \omega(A_+) = 0 \end{cases}$$

$$\text{case (c):} \quad \begin{cases} \omega(A_-) = 0 \\ \omega(A_+) = 1 \end{cases} \qquad \text{case (d):} \quad \begin{cases} \omega(A_-) = 0{,}5 \\ \omega(A_+) = 0{,}5 \end{cases}$$

Case (a) is trivial: the firm has full confidence in the classification criteria of empirical facts it is using, only the probability distribution of successes and failures influences its decision, traditional decision theory applies.

In cases (b) and (c) the firm chooses an action of a kind which is very much influenced by the degree of confidence, rather than by the probability distribution.

Case (d) is apparently like (a), because in both cases all what matters the probability distribution of successes and failures. Yet the difference is that in case (d) a process of category re-formulation is likely to set in, possibly together with the decision to postpone any decision.

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