Working Paper

Fuzzy Linear Programming in DSS for Energy System Planning

Tobias Canz

WP-96-132
November 1996

IIASA
International Institute for Applied Systems Analysis ◻ A-2361 Laxenburg ◻ Austria
Telephone: +43 2236 807 ◻ Fax: +43 2236 71313 ◻ E-Mail: info@iiasa.ac.at

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IIASA International Institute for Applied Systems Analysis ◻ A-2361 Laxenburg ◻ Austria
Telephone: +43 2236 807 ◻ Fax: +43 2236 71313 ◻ E-Mail: info@iiasa.ac.at

Foreword

This paper summarizes the results of the research conducted by the author during the Young Scientists Summer Program (YSSP) in IIASA's Methodology of Decision Analysis (MDA) Project.

The basic purpose of the research was to evaluate how the methodology of fuzzy linear programming (FLP) can support the decision-making process in energy system planning under uncertainty. Before presenting the more theoretical issues addressed in this context, the paper briefly introduces the general framework of the research conducted at the Institute for Energy Economics and the Rational Use of Energy at the University of Stuttgart, Germany. However, most application-oriented issues are not discussed in detail and should only be considered as illustrative examples for the reader.

FLP is one of the accepted methodologies for addressing parameter and decision uncertainties in model-based decision support. However, there are some difficulties with computing efficient solutions using FLP. In contrast to FLP, the multiple criteria model analysis (MCMA) methodology being developed and applied by the MDA Project provides robust ways to compute efficient solutions. Moreover, there are a number of similarities between these two methodologies.

The research reported in this paper provides a good overview of the methods and tools used for supporting decision making in energy system planning in Germany. It also shows how one can improve some elements of FLP applied to a real-world problem by learning from applications of MCMA for decision support and by using software tools developed for MCMA.

Abstract

Energy system planning requires the use of planning tools. The mathematical models of real-world energy systems are usually multiperiod linear optimization programs. In these models, the objective function describes the total discounted costs of covering the demand for final energy or energy services. The demand for various forms of energy or energy services is the driving force of the models. By using such linear programming (LP) formulations, decision makers can elaborate suitable strategies for solving their planning problems, such as the development of emission reduction strategies.

Uncertainties that affect the process of energy system planning can be divided into parameter and decision uncertainties. Data or parameter uncertainties can be addressed either by stochastic optimization or by the methodology of fuzzy linear programming (FLP). In addition, FLP allows explicit incorporation of decision uncertainties into a mathematical model.

This paper therefore aims at evaluating the methodology of FLP with respect to the support that it offers the decision-making process in energy system planning under uncertainty.

Employing the parallels between multi-objective linear programming (MOLP) and FLP, problems of FLP in decision support system applications are pointed out and solutions are offered. The proposed modifications are based on the methodology of aspiration-reservation based decision support and still enable modeling of uncertainties in a fuzzy sense. A case study is documented to show the application of the modified FLP approach.

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Fuzzy Linear Programming in DSS for Energy System Planning

*Tobias Canz**

1 Introduction

A variety of tools that employ the methodology of linear programming (LP) are available for energy system and power sector planning. Of the established models, MESSAGE [GGM95] and EFOM [OHR93] have been adapted to deal with uncertainties. Because energy studies in general are conducted based on uncertain data, further work should be directed not only toward the explicit incorporation of uncertainties into the models, but also to the support of the decision-making process. To show how various kinds of uncertainties influence the process of energy planning, a possible characterization of these uncertainties is given in the following paragraphs.

First, one can identify uncertainties in the values of parameters supplied by the energy planner. Some examples from energy system planning are the forecasted values for future prices of energy carriers, the cost development for new equipment, and the future demand the energy system under consideration will have to meet.

Second, further uncertainties are introduced when information is aggregated and the aggregated values are used in models. Examples include the use of aggregated generic power plants comprising individual power plants and all related data such as efficiencies, emission factors, and specific costs.

Uncertainties related to cost coefficients are especially critical, because LP models always select the cheapest option to the maximum degree possible. Therefore, minor changes in the cost assumptions can lead to completely different results. These flip-flop effects, also called penny switching, pose a significant problem in conventional LP models when robust solutions are sought using parameter variations [Erd80].

The third kind of uncertainty has to do with the fact that the decision-making process cannot be captured in mathematical models. For example, from a human point of view it does not make sense that a solution changes from being entirely feasible to being completely infeasible within very small ranges.

Although this taxonomy is by no means complete, it is used throughout Section 2 for the discussion of how fuzzy sets can be used to model uncertainties in LP problems in order to overcome the deficiency of point estimates in energy optimization. Section 2 contains a short introduction to fuzzy set theory and outlines possible ways to derive fuzzy sets from observed data.

The most commonly used principle of solving fuzzy linear programming (FLP) problems is described in Section 3. This section also contains a demonstration of how FLP problems are transformed into crisp equivalent programs that have the form of linear vector optimization problems.

*Institute for Energy Economics and the Rational Use of Energy, University of Stuttgart, Stuttgart, Germany.

The problems arising from the solution techniques are pointed out in Section 4. These problems range from the determination of membership functions for individual objectives to the aggregation of multiple criteria to just one criterion. In this context, parallels to the approaches of multi-objective linear programming (MOLP) become clear. Also, various solutions for the problems are offered, although only the most promising solutions are discussed in greater detail for the individual problems.

In Section 5, the implementation of the proposed methodology is described for a sample case study from the field of energy system planning. Also, the integration of FLP into an interactive decision support system (DSS) is described in this section. For the interactive DSS, it is shown how the sample planning problem can be separated in a core model, an extended core model, and an interactive model.

2 Fuzzy Sets and Modeling of Uncertainties in LP Models

Energy system models are generally conceived of as planning tools for the cost-efficient planning of energy supply for regions within a country, entire countries, or even areas comprising several countries. The models are driven by exogenously defined demands for final energy or energy services that must be met by the energy system under consideration. The corresponding linear program is of the following form:

$$\min_{x \in X_D} \quad \mathbf{c}^t \mathbf{x} \tag{1}$$

$$\begin{aligned} s.t. \quad \mathbf{A}\mathbf{x} &\geq \mathbf{b}\,, \\ \mathbf{x} &\geq \mathbf{0}\,. \end{aligned}$$

In (1), $\mathbf{c}^t\mathbf{x}$ describes a single objective (most often, costs) that is to be minimized subject to various constraints. The constraints guarantee (among many other relationships, such as the fulfillment of the balance equations) that the demand for final energy and energy services is covered.

The following subsections contain a brief introduction of fuzzy sets and examples of how the various kinds of uncertainties influence the coefficients of $\mathbf{c}$ and $\mathbf{A}$ as well as the right-hand side, $\mathbf{b}$.

2.1 Fuzzy Sets and Representation of Fuzzy Numbers

A *classical set* (or crisp set) is defined as a collection of elements $x \in X$. Each element can either belong or not belong to a set A, $A \subseteq X$. The membership of elements x to the subset A of X can be expressed by a characteristic function in which 1 indicates membership and 0, nonmembership. For a normalized fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set. A fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = [(x, \mu_A(x)) \mid x \in X]\,, \tag{2}$$

where $\mu_A(x)$ is called the membership function of x in $\tilde{A}$, which maps X to the membership space $[0,1]$. When the membership space is $\{0,1\}$, $\tilde{A}$ is nonfuzzy and $\mu_A(x)$ is identical to the characteristic function of a nonfuzzy set. If X is the set of all positive real numbers $\Re^+$, a fuzzy set $\tilde{A}$ can be used to describe fuzzy numbers.

Besides the extension of set-theoretic operations, the fuzzy set theory also permits for algebraic operations and hence allows the use of fuzzy numbers for optimization or simulation.

Dubois and Prade [DuP80a] suggest a special type of representation for fuzzy numbers. These so-called LR-type fuzzy sets determine nonincreasing shape functions L and R that map $\Re^+ \mapsto [0,1]$. L is such a decreasing shape function if $L(0) = 1$; $L(u) < 1 \; \forall u > 0$; $L(u) > 0 \; \forall u < 1$; $L(1) = 0$; or alternatively $L(u) > 0$, $\forall u$ and $L(+\infty) = 0$. The same properties must be fulfilled for R to be a valid shape function.

A flat fuzzy number or fuzzy interval $\tilde{A}$ is of LR-type if there exist shape functions L and R and four parameters such that (for $x \in \Re^+$)

$$\mu_A(x) = \begin{cases} L(\frac{a_1 - x}{\alpha_1}); & x \leq a_1, \quad \alpha_1 > 0 \\ 1; & a_1 \leq x \leq a_2 \\ R(\frac{x - a_2}{\alpha_2}); & x > a_2, \quad \alpha_2 > 0 \,. \end{cases} \tag{3}$$

Different functions can be chosen for $L(u)$ and $R(u)$. Possible examples from Dubois and Prade [DuP80a] include $L(u) = max(0, 1-u)^p$; $L(u) = max(0, 1-u^p)$, with $p > 0$; $L(u) = e^{(-u^2)}$, and $L(u) = e^{-u}$.

A fuzzy interval $\tilde{A}$ is denoted by $\tilde{A} = (\alpha_1, a_1, a_2, \alpha_2)_{LR}$. If $\tilde{A}$ is a trapezoidal flat fuzzy number (see Figure 1), it is implied that $L(u) = R(u) = max(0, 1-u)$.

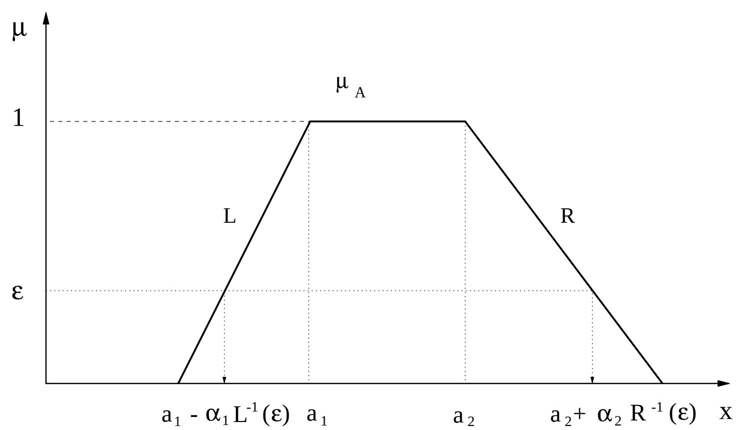

Figure 1: Fuzzy set in LR-notation.

For LR-fuzzy numbers, the computational effort for the basic algebraic operations such as addition and subtraction is greatly decreased ([DuP80a], [DuP80b]). Also, approximate expressions for the multiplication and the division of two fuzzy numbers are introduced by Dubois and Prade [DuP80a]. The expressions for multiplication and division are correct in cases where a fuzzy number is combined with a crisp number.

2.2 Uncertainties in Exogenously Defined Parameters

Figure 2 shows the development of the accumulated demand for final energy in Germany (old Länder, i.e., former West Germany) from 1953 until 1993. Possible forecasting results for the demand of energy that would have to be met by the supply system are also included. Although the forecasted values are given as ranges, it is unlikely that they include the

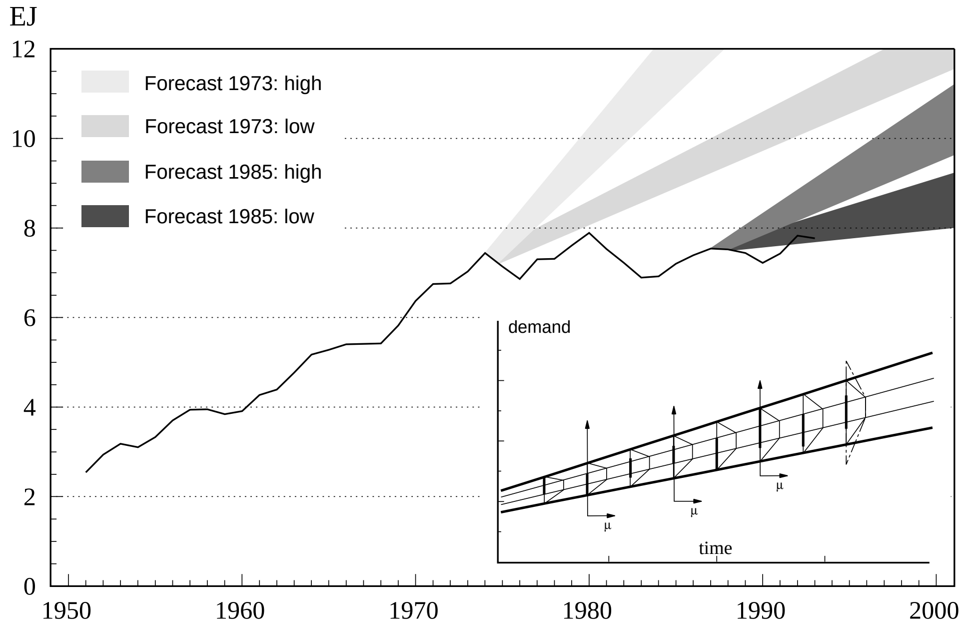

Figure 2: Development of final energy demand in Germany (old Länder, i.e., former West Germany).

correct future values. Obviously, it would have been even more unlikely to use the correct future values in an energy system model when describing the future energy demands using crisp numbers. The smaller subgraph shows the use of linear regression on the basis of interval arithmetics to determine fuzzy sets that model the future energy demand.

The general concept of a fuzzy linear regression model is shown in equation (4) for the case of triangular fuzzy sets. The fuzzy coefficients $\tilde{A}_i$ are determined from the central values (a_{ic}) and the symmetrical spreads (a_{iw}) of intervals A_i:

$$\begin{aligned} \tilde{Y}(\mathbf{x}_j) &= \tilde{A}_0 + \tilde{A}_1 x_{1j} + \ldots + \tilde{A}_n x_{nj} \ , \\ \text{where} \qquad & \\ \mu_{A_i}(t) &= \max[1 - \tfrac{|t-a_{ic}|}{a_{iw}}, 0] \ . \end{aligned} \tag{4}$$

The intervals A_i and the central values a_{ic} ($\forall i \in \{1, \ldots, n\}$) are determined such that the total width over all i intervals is minimized subject to the constraint that the central value minus the width of the interval must be smaller than any empirical point j. Similarly, the central value of the interval plus the width must be greater than any corresponding empirical point (see Figure 3).

This so-called possibility analysis for the determination of triangular fuzzy sets [TUA80] can be extended by a necessity analysis (in the case of interval-valued empirical data) to determine trapezoidal fuzzy sets ([TUA82], [TaI92], [SaP92]). Furthermore, uncertainties in both the explaining (i.e., $\mathbf{x}$) and in the explained variables can be incorporated into the fuzzy linear regression models introduced by Sakawa and Yana [SaY92]. The application of fuzzy regression analysis to extrapolate statistical data was first shown by Heshmaty and Kandel [HeK85] and was extended to a complete time series analysis including seasonal components by Watanda [Wat92].

Nonlinear projection methodologies such as econometric approaches (e.g., a Cobb-Douglas function) can also be fuzzified and used to determine fuzzy sets describing, for example, the demand for final energy [Ode94].

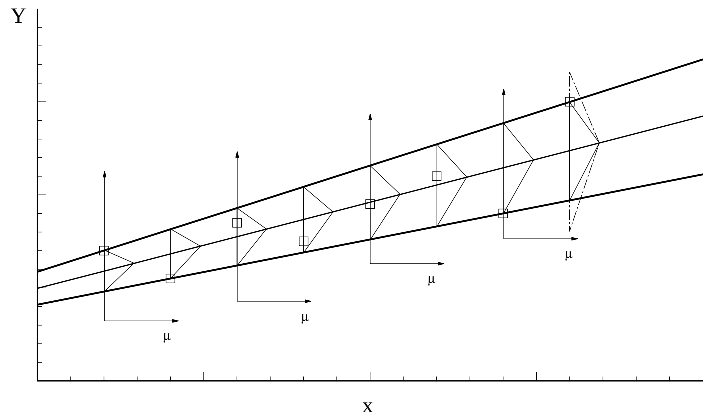

Figure 3: Fuzzy linear regression.

Of course, the forecasted demand values are not the only parameters that are subject to exogenous uncertainties. However, as the demand is the driving force of energy system models, it is necessary to pay special attention to modeling the uncertainties associated with this quantity.

2.3 Statistical Uncertainties

Uncertainties that are introduced into models due to the aggregation of data are called statistical uncertainties by Morgan and Henrion [MoH90]. Probabilistic approaches are best suited for modeling such uncertainties. Nevertheless, fuzzy sets can also be designed to capture statistical uncertainties.

The basic idea is to derive a trapezoidal probability density function, $f(x)$, from statistical data and to transform it into a possibility distribution.

For the determination of the probability density function, it is sufficient to characterize $f(x)$ by its moments. All probability density functions that fit these moments can be considered equally well-suited to correctly describing the underlying distribution. The first-order moment describes the mean value (M_1) of a distribution and is given by

$$M_1 = \int_{-\infty}^{+\infty} x f(x)\, dx \; . \tag{5}$$

Moments of higher orders, r, are usually given by the following equation:

$$M_r = \int_{-\infty}^{+\infty} (x - M_1)^r f(x)\, dx \; . \tag{6}$$

The second (central) moment ($r = 2$) is used to describe the variance σ^2; the third and fourth central moments describe the skew and the kurtosis of a distribution $f(x)$, respectively.

In the case of a trapezoidal probability density function with a lower bound, l ($l = a_1 - \alpha_1$), and an upper bound, h ($h = a_2 + \alpha_2$, and $l \leq x \leq h$), on x, the analytical expressions for the first and second moment can be derived as

$$M_1 = \int_{l}^{a_1} x f(x)\, dx + \int_{a_1}^{a_2} x f(x)\, dx + \int_{a_2}^{h} x f(x) dx \; , \tag{7}$$

$$M_2 = \sigma^2 = \int_l^{a_1} (x - M_1)^2 f(x)\, dx + \int_{a_1}^{a_2} (x - M_1)^2 f(x)\, dx + \int_{a_2}^{h} (x - M_1)^2 f(x)\, dx. \quad (8)$$

Because a trapezoidal probability density function is fully described by l, a_1, a_2, and h, and because l and h are already known from the empirical data, the problem now is to solve (7) and (8) for two unknowns. This can be accomplished without major difficulties using the estimation of M_1 and σ^2 from the statistical data.

In this approach, the estimation of the skew and the kurtosis are not used to determine the shape of the probability density function. The estimation of these higher moments from empirical data requires around 100 data points in the sample [Sac78]. Because this amount of information will be available in very few cases, it is felt that neglecting the skew and the kurtosis is only a minor drawback to the approach.

Having determined the trapezoidal probability density function, the membership function of the fuzzy set can be derived according to the methods described by Dubois and Prade [DuP83] and Civanlar and Trussel [CiT86]. The latter of these approaches determines an optimal (normalized) membership function from the known $f(x)$ by minimizing the area under the squared membership function. The approach by Dubois and Prade is based on the definition of a bijective mapping that transforms a probability measure into a possibility measure. Both methods ensure that the newly generated membership function fulfills the possibility-probability consistency principle [Zad78], according to which the degree of possibility of an event is greater than or equal to its degree of probability.

Figure 4 shows the result of determining the probability density function as described above and its transformation into a fuzzy set using bijective mapping. The resulting fuzzy set describes the efficiency of a group of conventional coal-fired power plants with 200 to 400 MW output. The same procedure can be used to determine fuzzy coefficients for the vector **c** (i.e., specific investment costs, specific variable costs, etc.) as well as for other coefficients of the matrix **A** (e.g., specific emissions).

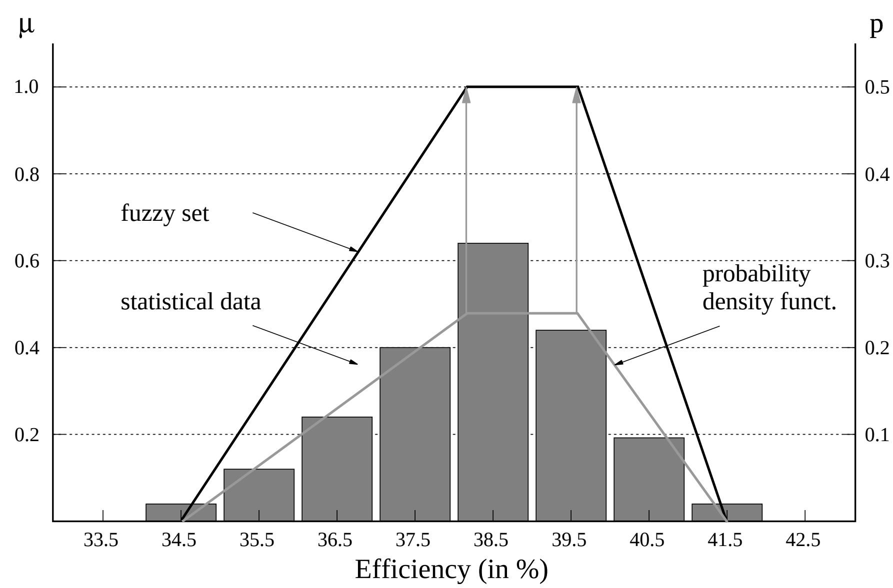

Figure 4: Statistical data with resulting fuzzy set superimposed.

2.4 Uncertainties in the Decision-Making Process

Turning to the right-hand sides of the constraints, the energy planner must model the consistency of the decision space with the strategies/goals pursued in an energy planning study. In this context, it is especially difficult to capture the human decision-making process using mathematical models.

For example, from a human standpoint it does not make sense that a solution changes from being perfectly feasible to being entirely infeasible within very small ranges. Gerking identifies the resulting problems as "decision uncertainties" [Ger89]. One can use Germany's goals for carbon dioxide (CO_2) emission reductions by the year 2005 as an example. In 1990 these emissions amounted to approximately 1000 megatonnes per year. Hence, the goal of reducing annual emissions by 25% until 2005 requires a constraint that keeps all solutions with more than 750 megatonnes of CO_2 emissions from being considered.

However, this is not the way human beings make decisions. A decision maker might also allow solutions that have emissions of approximately 800 megatonnes if these solutions are financially more attractive than solutions with lower emissions. This fact can be captured by modeling right-hand sides of constraints in LP problems, as shown in Figure 5. The degree of satisfaction with the emission values achieved decreases as emission levels increase from 750 megatonnes to 800 megatonnes. By using piecewise linear membership functions, many concave functions can be approximated to express the subjective degree of "acceptability" for different solutions. Hereby, the decision maker can interactively adjust the levels, ϵ_{Ai}, such that the increased understanding for the decision problem can be taken into account.

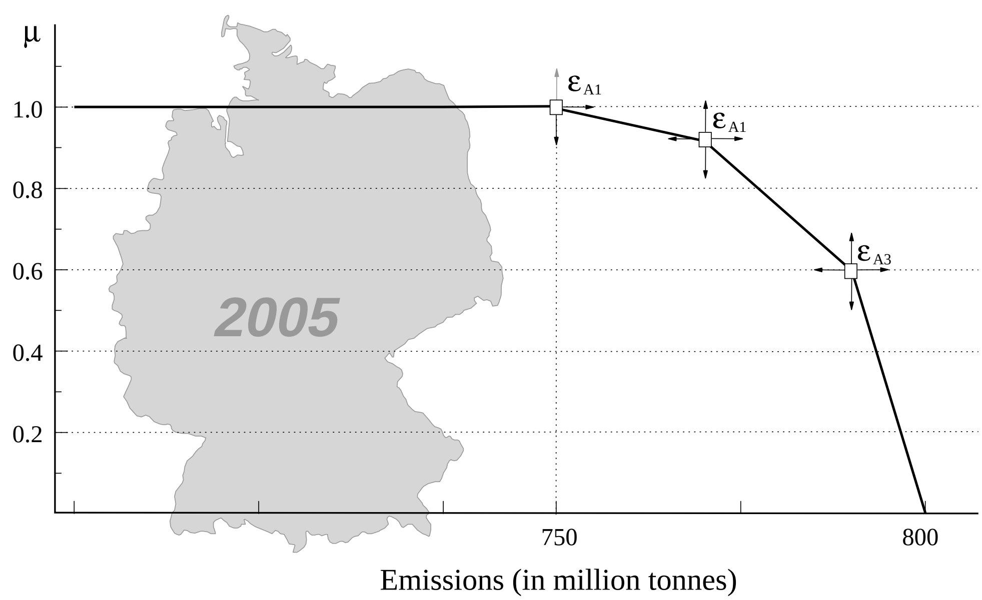

Figure 5: Fuzzy set describing Germany's CO_2-emission goal 2005.

3 Fuzzy Linear Programming

Using fuzzy sets as coefficient values in the objective function and the constraints, as well as in the right-hand sides of the constraints, the following single-objective LP problem can be derived:

$$\begin{aligned} \widetilde{min}\, [\, \tilde{z}(\mathbf{x}) &= \tilde{\mathbf{c}}^t\mathbf{x}\,] \\ s.t.\quad \tilde{\mathbf{A}}\mathbf{x} &\tilde{\geq} \tilde{\mathbf{b}}, \\ \mathbf{x} &\in X_D\, . \end{aligned} \tag{9}$$

where $\tilde{\mathbf{c}}^t$ is the transpose of the n-dimensional fuzzy objective vector; $\tilde{\mathbf{A}}$ represents the $m \times n$ fuzzy constraint coefficient matrix; $\tilde{\mathbf{b}}$ is the m-dimensional vector of fuzzy right-hand sides; $\tilde{z}(\mathbf{x})$ is the fuzzy objective value; $\tilde{\geq}$ represents fuzzy inequality; and X_D is the set of admissable activity vectors $\mathbf{x}$, with $\mathbf{x} \in \Re_n^+$, that fulfill all crisp constraints.

Various concepts for the solution of the above problem are described in the pertinent literature (see [RaR85], [Rom88], [Sak93], [Slo86], [Wer87b], [Zim78], [Zim87], [Zim91]). Only two of the possible concepts for modeling fuzzy inequalities with crisp equivalent inequality constraints are described in the following text. It is also shown under which circumstances the combination of the crisp equivalent constraints with the fuzzy minimization of the objective value results in MOLP problems and how these problems are solved.

3.1 Modeling of Fuzzy Inequality Constraints

Modeling linear fuzzy inequality constraints in an LP formulation requires the comparison of two fuzzy sets, namely, $\tilde{\mathbf{A}}\mathbf{x}$ and $\tilde{\mathbf{b}}$. The graphical representation of a fuzzy inequality for LR-type fuzzy numbers is shown in Figure 6. The term $\tilde{\mathbf{A}}\mathbf{x}$ can be computed according to Dubois and Prade's fuzzy operations (see Section 2.1).

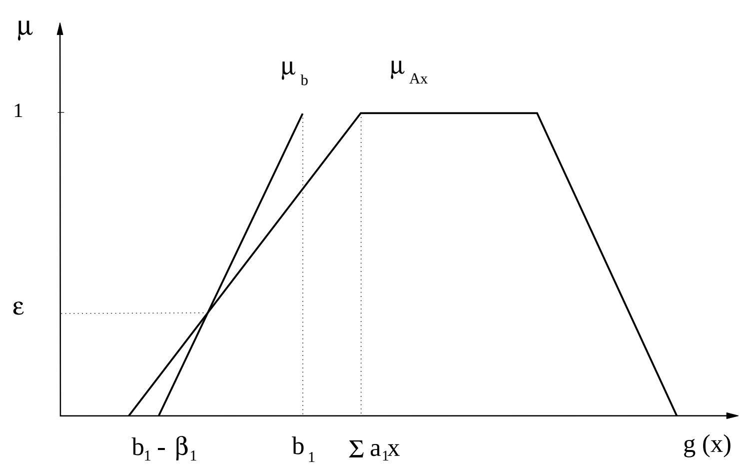

Figure 6: Fuzzy inequality.

Two notions of such fuzzy inequality constraints can be conceived. The first has to do with the fact that the constraints in LP formulations describe real-world technical systems. Under this assumption, no compensation between the original objective and the fulfillment of fuzzy constraints is recognized. Hence, the transformation of a fuzzy constraint into a crisp equivalent formulation does not add an additional objective function to the model. For example, the uncertain market penetration of one kind of power plant cannot exceed a fuzzy upper bound that is imposed by a given technology mix. To model the concept in an LP formulation, m fuzzy inequalities are replaced by the following set of $2m$ crisp inequalities (for purely linear membership functions):

$$\tilde{\mathbf{A}}\mathbf{x} \tilde{\geq}_\epsilon \tilde{\mathbf{b}} \Leftrightarrow \begin{cases} \mathbf{a}_{1i}\mathbf{x} & \geq & b_{1i} & i = 1, \ldots, m \\ (\mathbf{a}_{1i} - \alpha_{1i}\, L^{-1}(\epsilon))\,\mathbf{x} & \geq & b_{1i} - \beta_{1i}\, L^{-1}(\epsilon) & i = 1, \ldots, m\,. \end{cases} \tag{10}$$

In (10), $\mathbf{a}_{1i}$ describe the second fuzzy components of the ith row vector of the fuzzy constraint coefficient matrix. The fuzzy constraint coefficient matrix hereby consists of the elements $(\alpha_{1ij}, a_{1ij}, a_{2ij}, \alpha_{2ij})_{\mathrm{LR}}$, $i \in \{1, \ldots, m\}$, and $j \in \{1, \ldots, n\}$. This kind of substitution guarantees that the constraint is fulfilled for all membership values greater than ϵ. The parameter ϵ can be varied according to the decision maker's preference structure.

A different concept applies for the second perception of fuzzy inequality constraints. Here, the interaction of constraints and objectives is taken into consideration. The basic idea is that transforming of a fuzzy constraint into a linear equivalent problem guarantees that the inequality relation is at least fulfilled on an ϵ level. In contrast to the approach described in (10), an additional objective is introduced that maximizes the degree of fulfillment of the respective constraint. In the case of a fuzzy constraint that aims at achieving a certain target value (e.g., for emission reduction), this means that the degree of satisfaction with the achievement of the value is maximized. This concept of substitution by Rommelfanger [Rom88] is modeled by the following terms:

$$\tilde{\mathbf{A}}\mathbf{x} \tilde{\geq}_R \tilde{\mathbf{b}} \Leftrightarrow \begin{cases} (\mathbf{a}_{1i} - \alpha_{1i}\, L^{-1}(\epsilon))\mathbf{x} & \geq & b_{1i} - \beta_{1i} L^{-1}(\epsilon) & i = 1, .., m \\ \mu_i(\mathbf{a}_{1i}\mathbf{x}) & \rightarrow & Max & i = 1, .., m\,, \end{cases} \tag{11}$$

where

$$\mu_i(\mathbf{a}_{1i}x) = \begin{cases} 1 & \text{if } \mathbf{a}_{1i}\mathbf{x} \geq b_{1i} & i = 1, \ldots, m \\ L(\frac{\mathbf{b}_{1i} - \mathbf{a}_{1i}\mathbf{x}}{\beta_{1i}}) & \text{if } b_{1i} > \mathbf{a}_{1i}\mathbf{x} & i = 1, \ldots, m\,. \end{cases} \tag{12}$$

Using this so-called R relation for the first m_1 constraints together with the ϵ relation for the next m_2 constraints, the following MOLP problem is equivalent to the FLP problem (9):

$$\begin{pmatrix} \tilde{z}(\mathbf{x}) \\ \mu_1(\mathbf{x}) \\ \ldots \\ \mu_{m_1}(\mathbf{x}) \end{pmatrix} \rightarrow V\text{-}Opt \tag{13}$$

$$\begin{array}{lrcll} s.t. & \mathbf{a}_{1i}\mathbf{x} & \geq & b_{1i} & i = m_1 + 1, \ldots, m_1 + m_2 \\ & (\mathbf{a}_{1i} - \alpha_{1i} L^{-1}(\epsilon))\mathbf{x} & \geq & b_{1i} - \beta_{1i} L^{-1}(\epsilon) & i = 1, \ldots, m_1 + m_2 \\ & \mathbf{x} & \in & X_D\,. & \end{array} \tag{14}$$

3.2 Modeling of Fuzzy Objective Functions

Using fuzzy algebraic operations, the objective function can be rewritten as follows:

$$
\begin{aligned}
\widetilde{min}\,[\tilde{z}(\mathbf{x}) &= \tilde{\mathbf{c}}^t\mathbf{x}]\ ,\ \text{or} \\
\widetilde{min}\,[\tilde{z}(\mathbf{x}) &= (\sum_{j=1}^{n}\gamma_{1j}x_j;\ \sum_{j=1}^{n}c_{1j}x_j;\ \sum_{j=1}^{n}c_{2j}x_j;\ \sum_{j=1}^{n}\gamma_{2j}x_j)] \\
s.t.\ \mathbf{x} &\in X_D\ ,
\end{aligned}
\tag{15}
$$

where X_D represents the set of feasible solutions that satisfy all substitute restrictions resulting from the fuzzy constraints (on an ϵ level), all crisp constraints, and the requirement that $\mathbf{x} \geq 0$.

Setting aside that the optimization of the fuzzy objective function will have to be accomplished over a fuzzy decision space, the minimization can still be accomplished according to a definition proposed by Zimmermann [Zim78]. The approach recognizes that the optimization of r fuzzy objective functions is equivalent to optimizing $p = 4r$ possibly conflicting goals and hence maximizes the degree of satisfaction with the solution values achieved for the four characteristic points of $\tilde{\mathbf{c}}^t\mathbf{x}$ in LR-notation (in the case of $r = 1$). In order to give the decision maker an impression of the attainable objective values, the individual objectives are optimized over the set of feasible solutions X_D (i.e., for a specific value $\epsilon \in [0, 1]$) as follows:

$$
\begin{aligned}
z_1^u &= \min_{\mathbf{x}\in X_D}(\mathbf{c}_1 - \gamma_1 L^{-1}(\epsilon))^t\mathbf{x} && \text{resulting in } \mathbf{x}_1^*, \\
z_2^u &= \min_{\mathbf{x}\in X_D}\mathbf{c}_1^t\mathbf{x} && \text{resulting in } \mathbf{x}_2^*, \\
z_3^u &= \min_{\mathbf{x}\in X_D}\mathbf{c}_2^t\mathbf{x} && \text{resulting in } \mathbf{x}_3^*, \\
z_4^u &= \min_{\mathbf{x}\in X_D}(\mathbf{c}_2 + \gamma_2 R^{-1}(\epsilon))^t\mathbf{x} && \text{resulting in } \mathbf{x}_4^*.
\end{aligned}
\tag{16}
$$

Using standard notation of multicriteria optimization (c.f., [Van90]), these four objective values, z_1^u, z_2^u, z_3^u, and z_4^u, can be found on the diagonal of the payoff matrix, and the vector $\mathbf{z}^u = (z_1^u, z_2^u, z_3^u, z_4^u)$ is called the ideal point (or Utopia). In order to determine an approximation of the elements z_k^n of the Nadir point for all p objectives, standard procedures require the computation of the maximum over $z_k(\mathbf{x}_l^*)$, $l = (1, ..., p)$, $k = (1, ..., p)$:

$$
\begin{aligned}
z_1^n &= \max_{k=1,\dots,p}\{(\mathbf{c}_1 - \gamma_1 L^{-1}(\epsilon))^t\mathbf{x}_k{}^*\}, \\
z_2^n &= \max_{k=1,\dots,p}\{\mathbf{c}_1^t\mathbf{x}_k{}^*\}, \\
z_3^n &= \max_{k=1,\dots,p}\{\mathbf{c}_2^t\mathbf{x}_k{}^*\}, \\
z_4^n &= \max_{k=1,\dots,p}\{(\mathbf{c}_2 + \gamma_2 R^{-1}(\epsilon))^t\mathbf{x}_k{}^*\}.
\end{aligned}
\tag{17}
$$

However, Isermann and Steuer [IsS87] show that the computation of the Nadir point by payoff tables frequently produces incorrect approximations of the most pessimistic solution. These problems are aggravated by the fact that in (16) and (17) the impact of different levels of ϵ (for the fulfillment of the substituting restrictions) on the set of feasible solutions, X_D, and on the objective values is not considered.

Nevertheless, it is assumed that the decision maker does not accept a solution whose objective value is worse than the components of $\mathbf{z}^n$. Similarly, a decision maker is considered completely satisfied with solutions where $\mathbf{z}$ is better than $\mathbf{z}^u$ (of course, such solutions do not exist for a given level ϵ). Hence, the membership functions expressing the satisfaction with the objective values achieved can be formulated for the four objectives as follows:

$$
\begin{aligned}
\mu_{z_1}((\mathbf{c}_1 - \gamma_1 L^{-1}(\epsilon))^t \mathbf{x}) &= \frac{z_1^n - (\mathbf{c}_1 - \gamma_1 L^{-1}(\epsilon))^t \mathbf{x}}{z_1^n - z_1^u}, \\
\mu_{z_2}(\mathbf{c}_1^t \mathbf{x}) &= \frac{z_2^n - \mathbf{c}_1^t \mathbf{x}}{z_2^n - z_2^u}, \\
\mu_{z_3}(\mathbf{c}_1^t \mathbf{x}) &= \frac{z_3^n - \mathbf{c}_2^t \mathbf{x}}{z_3^n - z_3^u}, \\
\mu_{z_4}((\mathbf{c}_2 + \gamma_2 R^{-1}(\epsilon))^t \mathbf{x}) &= \frac{z_4^n - (\mathbf{c}_2 + \gamma_2 R^{-1}(\epsilon))^t \mathbf{x}}{z_4^n - z_4^u}.
\end{aligned} \tag{18}
$$

Independent of the procedure used for soliciting the exact shape of the membership functions, the decision maker will pursue the goal of maximizing each of the four membership functions (instead of minimizing the original fuzzy objective):

$$
\begin{aligned}
\mu_{z_1}((\mathbf{c}_1 - \gamma_1 L^{-1}(\epsilon))^t \mathbf{x}) &\rightarrow Max, \\
\mu_{z_2}(\mathbf{c}_1^t \mathbf{x}) &\rightarrow Max, \\
\mu_{z_3}(\mathbf{c}_2^t \mathbf{x}) &\rightarrow Max, \\
\mu_{z_4}((\mathbf{c}_2 + \gamma_2 R^{-1}(\epsilon))^t \mathbf{x}) &\rightarrow Max.
\end{aligned} \tag{19}
$$

Let $\mu_Z(\mathbf{x})$ be the vector expressing the satisfaction of the decision maker with the objective functions for a solution $\mathbf{x}$, and let $\mu_R(\mathbf{x})$ be the equivalent vector for the constraints, then the complete multi-objective substitute problem for problem (9) is

$$
\begin{pmatrix} \mu_Z(\mathbf{x}) \\ \mu_R(\mathbf{x}) \end{pmatrix} \rightarrow V\_Max \tag{20}
$$

$$
\begin{aligned}
s.t. \qquad \mathbf{a}_{1i}\mathbf{x} &\geq b_{1i} && i = m_1 + 1, ..., m_1 + m_2 \\
(\mathbf{a}_{1i} - \alpha_{1i} L^{-1}(\epsilon))\mathbf{x} &\geq b_{1i} - \beta_{1i} L^{-1}(\epsilon) && i = 1, ..., m_1 + m_2 \\
\mathbf{x} &\in X_D \ .
\end{aligned}
$$

3.3 Solving MOLP Problems Resulting from Fuzzy Linear Programming

The fuzzy set theory provides various logical operators that allow the aggregation of several criteria to just one criterion. These operators can be evaluated with respect to axiomatic requirements (e.g., monotony), numerical efficiency, robustness, degree of compensation among the criteria, and ability to model expert behavior. All of the evaluation criteria are important in a sense, and taking them into consideration the so-called Min-Bounded-Sum is considered to be well-suited to solving the multi-objective programming problems that result from fuzzy problem descriptions (c.f., [Luh82], [Ode94]). The mathematical form of the Min-Bounded-Sum is given in (21):

$$
\mu_{D_{max}}(\mathbf{x}) = \max_{\mathbf{x} \in X_D} \{\delta \min_k \{\mu_k(\mathbf{x})\} + (1-\delta) \min\{1, \sum_{k=1}^{p} \mu_k(\mathbf{x})\}\} \ . \tag{21}
$$

This operator makes it possible to model compensation among the criteria and has a formal appearance that is similar to the Chebyshev norm. For $\delta = 1$, the Min-Bounded-Sum becomes the non-compensatory Min-operator that can be interpreted as the logical AND-aggregation of several objectives.

An FLP problem with the Min-Bounded-Sum as the aggregation operator is modeled correctly by the following substitute problem [Luh82]:

$$
\max_{\mathbf{x} \in X_D} \delta \lambda_1 + (1-\delta)\lambda_2 \tag{22}
$$

$$
\begin{array}{llll}
s.t. & \lambda_1 & \leq & \mu_k(\mathbf{x}) \qquad \forall k = 1, ..., p \\
& \lambda_2 & \leq & 1 \\
& \lambda_2 & \leq & \sum_{k=1}^{p} \mu_k(\mathbf{x}) \ .
\end{array}
$$

For problem (22), $p = r4 + m_1$, for r fuzzy objective functions.

Another possible aggregation operator is the Fuzzy-AND operator. In comparison with the Min-Bounded-Sum, this operator has the advantage of being a strongly monotonically increasing function:

$$
\mu_{D_{max}}(\mathbf{x}) = \max_{\mathbf{x} \in X_D} \{\delta \min_k \{\mu_k(\mathbf{x})\} + (1 - \delta)\frac{1}{p} \sum_{k=1}^{p} \mu_k(\mathbf{x})\} \ . \tag{23}
$$

The Fuzzy-AND aggregation can be used in the following way to model the aggregation of several criteria into just one criterion as an LP problem [Wer87a]:

$$
\max_{\mathbf{x} \in X_D} \lambda_1 + (1 - \delta)\frac{1}{p} \sum_{k=1}^{p} \alpha_k(\mathbf{x}) \tag{24}
$$

$$
\begin{array}{lrcll}
s.t. & \lambda_1 + \alpha_k & \leq & \mu_k(\mathbf{x}) & \forall k = 1, ..., p \\
& \lambda_1 + \alpha_k & \leq & 1 & \forall k = 1, ..., p \\
& \lambda_1, \alpha_k & \geq & 0 & \forall k = 1, ..., p \\
& \mathbf{x} & \in & X_D \ . &
\end{array}
$$

Hereby the parameter δ can be determined from empirical observation of human decision making, as described by Zimmermann [Zim87], [Zim91].

This method requires the gathering of membership values for multiple criteria, k, and their empirical aggregation into a single membership value, μ_{emp}. Using the criteria evaluations μ_k and their empirical aggregation μ_{emp}, it is possible to minimize the mean square error of the approximation of the empirical aggregation ($\mu_{emp} - \mu_{cal}$) by adjusting the parameter δ of the aggregation operator.

It can then be tested for the optimal value of the parameter δ ($0 < \delta < 1$) if the differences between the empirically aggregated values μ_{emp} and the computed values μ_{cal} are significant.

Assuming that the differences are normally distributed with a mean value of zero and an unknown variance, the hypothesis of random error can be tested with the Students-t-distribution. The results for the Fuzzy-AND operator show that possible values for δ range between 0.35 and 0.85, depending on the application chosen [Wer87a].

The next section describes problems that can arise when solving the kinds of models introduced here. Solutions for these problems are also offered in Section 4.

4 Problems in Fuzzy Linear Programming

For the methodology described in Section 3, problems can arise with respect to the aggregation operator used to determine a single-criterion optimization problem. Problems can also be related to the determination of the membership functions, $\mu_k(\mathbf{x})$, expressing the decision maker's degree of satisfaction with the individual objective values, $z_k(\mathbf{x})$.

The remainder of this section will employ the notation of multicriteria optimization and FLP to outline the problems of FLP. In particular, parallels to the aspiration-reservation based decision support (ARBDS) methodology will be used to show that solutions for these problems can be found. In this context, it is important to acknowledge that the component achievement functions of ARBDS correspond to fuzzy sets that describe the degree of satisfaction with individual objectives, whereas the achievement scalarizing function is equivalent to the fuzzy aggregation operator used to derive a single-criterion optimization problem from the converted fuzzy problem (20).

The notation of the mathematical concept will be explained prior to the discussion of the problems with FLP. The modeling concept starts with a set of admissible solutions X_D in the decision space X. The actual decisions, $\mathbf{x} \in X$, are represented by elements of $\Re^n$. The outcomes of the decisions are modeled by outcome variables $\mathbf{y} \in Y$, where Y is the outcome space. The transformation is denoted by $h : X_D \mapsto Y$ (or $\mathbf{y} = h(\mathbf{x})$). The set Y_0 of attainable outcomes contains the outcomes that can result for the admissible solutions, $Y_0 = h(X_D)$. From the outcome variables $\mathbf{y}$, it is possible to select a vector of objectives (or criteria) $\mathbf{z}$ that determines the objective space Z. Z is a subspace of Y, $Z \subseteq Y$, $\mathbf{z} = g(\mathbf{x})$, where $g : X_D \mapsto Z$ is the restriction of the function $h : X_D \mapsto Y$. In the following, it is assumed that $g : X_D \mapsto \Re^p$ and the outcome variables $\mathbf{y}$ are not used to discuss the problems.

4.1 Selection of the Aggregation Operator

The choice of a suitable aggregation operator for multicriteria optimization is guided by the requirement that every optimal solution of (20) is a Pareto-optimal solution for the original problem (i.e., the problem given in (9)).

4.1.1 Criteria for the Selection of Aggregation Operators

For conventional multicriteria optimization problems, the definition of a Pareto-optimal solution is as follows:

Definition 1 *A solution $\mathbf{x}^0 \in X_D$ is called a Pareto-optimal solution if there is no other feasible solution that can improve the value of one criterion without worsening the value of at least one other criterion. Pareto-optimal solutions, also called efficient or non-dominated solutions, are defined as follows (for criteria z_k that are to be minimized):*

$$\neg\exists \mathbf{x} \in X_D \neq \mathbf{x}^0 : \{z_k(\mathbf{x}) \leq z_k(\mathbf{x}^0)\ \forall k \in \{1, ..., p\} \quad \mathit{and} \quad \exists l \in \{1, ..., p\} : z_l(\mathbf{x}) < z_l(\mathbf{x}^0)\} \ .$$

In accordance with this definition, the pertinent literature (c.f. [Van90], [GrW94]) shows that if there is a scalarizing function $f : \Re^p \mapsto \Re^1$ that is strongly monotone; that is, if $\mathbf{z}(\mathbf{x}^*) < \mathbf{z}(\mathbf{x}) \Rightarrow f(\mathbf{z}(\mathbf{x}^*)) > f(\mathbf{z}(\mathbf{x}))$, where $\mathbf{z}(\mathbf{x}^*) < \mathbf{z}(\mathbf{x})$ if and only if $z_k(\mathbf{x}^*) \leq z_k(\mathbf{x})\ \forall k$ and $\mathbf{z}(\mathbf{x}^*) \neq \mathbf{z}(\mathbf{x})$, the solution identified by maximizing $f(\mathbf{z})$ is an efficient solution for the original (non-fuzzy) MOLP problem. For fuzzy problems, $z_k(\mathbf{x})$ can be replaced with $\mu_k(z_k(\mathbf{x}))$.

The definition of Pareto optimality can be directly applied to MOLP problems that result from fuzzy LP formulations if the following requirements are fulfilled:

- The definition is valid for FLP only if minimizing and maximizing objectives are considered (i.e., no fuzzy equality objectives can be included).
- The membership functions $\mu_k(z_k(\mathbf{x}))$, $k = 1, ..., p$ must be strictly monotone over the ranges of possible values $z_k(\mathbf{x})$.

The latter of these is required for the identification of Pareto-optimal solutions (c.f., definition (1)) when using $\mu_k(z_k(\mathbf{x}))$ instead of $z_k(\mathbf{x})$ and a continuous vector valued objective function $\mu : X_D \mapsto \Re^p$, where $X_D \subseteq \Re^n$ is the set of admissible or feasible solutions [GrW94].

The definition of Pareto-optimality will have to be reconsidered in cases, where fuzzy equality is also to be expressed in the objective functions (meaning that some objective values $z_k(\mathbf{x})$ should be as close as possible to a desired value r_k or a range of values $[r_k^l, r_k^u]$). In such cases, Sakawa [Sak93] proposes the following definition, which holds for fuzzy minimizing, fuzzy maximizing, and fuzzy equality objectives.

Definition 2 *A solution $\mathbf{x}^0 \in X_D$ is called an M-Pareto-optimal (or membership-Pareto-optimal) solution for the generalized multi-objective linear programming problem if and only if there is no other feasible solution $\mathbf{x} \in X_D$ such that $\mu_k(z(\mathbf{x})) \geq \mu_k(z(\mathbf{x}^0))\ \forall k$ and $\mu_l(z(\mathbf{x})) \neq \mu_l(z(\mathbf{x}^0))$ for at least one l.*

The use of definition (2) guarantees that the dominance of a solution over all other solutions in the membership space is reflected in the optimal result. However, situations may still arise where the individual membership functions have constant values over ranges of $z_k(\mathbf{x})$, if all $\mu_k(z_k(\mathbf{x})) = 0$ or all $\mu_k(z_k(\mathbf{x})) = 1$. In such cases, definition (2) does not alleviate the problem in the objective space Z.

For the discussion of the efficiency of solutions identified with the Min-Bounded-Sum aggregation and the Fuzzy-AND operator, only fuzzy minimizing and fuzzy maximizing objectives are considered. Furthermore, it is assumed that all $\mu_k(z_k(\mathbf{x}))$ are strictly monotone increasing or decreasing functions over the entire ranges of possible values for $z_k(\mathbf{x})$. These assumptions are relaxed in Sections 4.2 and 4.3.

4.1.2 Properties of the Min-Bounded-Sum

In terms of multi-objective programming, the Min-Bounded-Sum is a function that maps $[0,1]^p \mapsto [0,1]$. However, because of its upper bound, this operator is a monotonically increasing function and not a strongly monotonically increasing function. For such monotonically increasing functions, it can only only be guaranteed that the solutions found are weakly non-dominated. The solutions are non-dominated in the sense of definition (1) only in the case of unique optimal solutions [Van90].

The efficiency problem of the Min-Bounded-Sum is due to its boundedness which makes it a monotonically increasing function for which

$$\delta \min_k\{\mu_k(\mathbf{x}^0)\} + (1-\delta)\min\{1, \sum_{k=1}^{p} \mu_k(\mathbf{x}^0)\} \geq \delta \min_k\{\mu_k(\mathbf{x})\} + (1-\delta)\min\{1, \sum_{k=1}^{p} \mu_k(\mathbf{x})\}. \tag{25}$$

Let the possible solutions be $(\mathbf{x}, \lambda_1, \lambda_2)$ and $(\mathbf{x}^0, \lambda_1^0, \lambda_2^0)$, where $(\mathbf{x}^0, \lambda_1^0, \lambda_2^0)$ is the non-dominated solution to (20). Such a solution is characterized by $\mu(\mathbf{x}^0) > \mu(\mathbf{x})$, meaning that $\mu_k(\mathbf{x}^0) \geq \mu_k(\mathbf{x}), \forall k \in \{1, ..., p\}$, and $\mu(\mathbf{x}^0) \neq \mu(\mathbf{x})$ [Van90].

When the two minimum terms on both sides of the inequality are equal, the (undesirable) case where the left and right sides in (25) are equal can occur. This might happen for several goals when the upper bound becomes active for the second minimum term. In this case, only the first minimum term can differentiate among the solution vectors. In the substitute problem (22), the dominance of $\mathbf{x}^0$ over $\mathbf{x}$ is then only reflected in λ_1 if this value is determined by a $\mu_k(\mathbf{x}^0)$ that is greater than the smallest $\mu_k(\mathbf{x})$. Thus, the following equation must be fulfilled in order to determine a non-dominated (Pareto-optimal) solution:

$$\delta \min_k \{(\mu_k(\mathbf{x}^0) + (1-\delta)\} > \delta \min_k \{(\mu_k(\mathbf{x}) + (1-\delta)\} \;, \tag{26}$$

or, because δ is a constant;

$$\min_k \{\mu_k(\mathbf{x}^0)\} > \min_k \{\mu_k(\mathbf{x})\} \;. \tag{27}$$

This is exactly the problem that arises in a maximin optimization of multi-objective programming problems. For linear maximin problems, Behringer [Beh83] proposes the use of a lexicographical optimization algorithm to resolve the problem.

Yet another possibility for addressing this problem is introduced by Sakawa [Sak93]. The author proposes a two-stage approach in which the second-stage optimization problem is as follows:

$$\max_{\mathbf{x}\in X_D} \sum_k e_k$$

$$\begin{aligned} s.t. \quad \mu_k(\mathbf{x}) - e_k &\geq \mu_k(\mathbf{x}^*) \; \forall k \;, \\ \mathbf{x} &\in X_D \;, \\ e_k &\geq 0 \;. \end{aligned} \tag{28}$$

Using $\mathbf{x}^*$ as the result of the first stage, the two-stage approach guarantees the identification of Pareto-optimal solutions $\mathbf{x}$.

The Fuzzy-AND aggregation (23) does not lead to this problem. Every solution of (24) is an efficient solution to the problem described in (20) for every δ such that $0 < \delta < 1$. The proof of this is given by Werners [Wer87b] under the assumption that all $\mu_k(z_k(\mathbf{x}))$ are strictly monotone functions over the entire ranges of possible objective values. Therefore, in the following discussion, only the Fuzzy-AND operator will be considered for the aggregation of several criteria.

4.1.3 Properties of the Fuzzy-AND Aggregation

An aggregation operator's ability to generate all solutions of the Pareto frontier (or Pareto-optimal or non-dominated solutions) is as interesting as the efficiency of the solutions. For the evaluation of this property, it is necessary to reconsider the definition of Pareto optimality given in definition (1). It should be pointed out that defintions (1), (3), and (4) consider only problems with minimizing objectives; where minimizing and maximizing objectives are considered simultaneously the terminology will have to be changed.

The following definitions of properly and weakly Pareto-optimal solutions are used for the discussion of the constructiveness of the Fuzzy-AND aggregation.

Definition 3 *A solution* $\mathbf{x}^0 \in X_D$ *is called a properly Pareto-optimal solution if and only if it is Pareto-optimal and there exists a scalar* $M > 0$ *such that* $\forall k \in \{1, ..., p\}$ *with* $z_k(\mathbf{x}) < z_k(\mathbf{x}^0)$ $\exists\, l \in \{1, ..., p\} :$ $z_l(\mathbf{x}^0) < z_l(\mathbf{x})$ *and* $\frac{z_k(\mathbf{x}) - z_k(\mathbf{x}^0)}{z_l(\mathbf{x}^0) - z_l(\mathbf{x})} \leq M$.

The value of M is the trade-off coefficient between objectives k and l. It should be noted that in MOLP the set of Pareto-optimal solutions Q_0 corresponds to the set of properly Pareto-optimal solutions with $M \to \infty$.

A relaxed version of definition (1) describes weakly Pareto-optimal solutions as follows.

Definition 4 *A solution* $\mathbf{x}^0 \in X_D$ *is a called weakly Pareto-optimal solution if there is no other feasible solution that has uniformly better criteria values. Weakly Pareto-optimal solutions are defined as (for criteria* z_k *that are to be minimized)*

$$\neg\exists \mathbf{x} \in X_D \neq \mathbf{x}^0 : z_k(\mathbf{x}) < z_k(\mathbf{x}^0)\ \forall k \in \{1, ..., p\} .$$

Weakly Pareto-optimal solutions are mathematically easier to generate than properly Pareto-optimal solutions. However, because weakly Pareto-optimal solutions are irrelevant from a decision-making point of view, any method that is used to solve an MOLP problem should avoid such solutions. In terms of definition (3), this can be accomplished by only accepting properly Pareto-optimal solutions that have a finite prior bound on the trade-off coefficients M.

As has been pointed out, the Fuzzy-AND operator identifies Pareto-optimal solutions. It will now be shown which parameters of the operator contribute to the trade-off coefficients M. Also, the possible consequences of limiting the trade-off coefficients M will be pointed out.

Using μ_D as the Fuzzy-AND aggregation of the objectives ($\mu_D(\mathbf{x}) = \delta \min_k\{\mu_k(\mathbf{x})\} + (1-\delta)1/p \sum_{k=1}^{p} \mu_k(\mathbf{x})$ for $k = \{1, 2\}$ and $\min_k\{\mu_k\} = \mu_1$), the following trade-off coefficients between the objectives can be derived:

$$|\frac{\Delta\mu_2}{\Delta\mu_1}| = 1 + \frac{\delta p}{1-\delta} = M . \tag{29}$$

Wierzbicki [Wie90] shows the effects of limiting trade-off coefficients for augmented Chebyshev norms using a different notation. In this work, the trade-off coefficient $M = 1+1/\epsilon$ (where ϵ corresponds to $(1-\delta)/(\delta p)$ in the notation of the Fuzzy-AND aggregator) is derived for an achievement scalarizing function that is composed of l_1 and l_∞ norms. The value ϵ is interpreted in terms of the opening angle of strictly positive cones. Its importance becomes clear in combination with the definition of the principle of Pareto optimality in terms of such cones. The only points of the Pareto frontier that are non-dominated are those for which the intersection of the set Q_0 with the conical set (shifted to the corresponding point) forms an empty set.

Figure 7 depicts this principle. It is apparent that from the set Q_0 only the points with trade-off coefficients of less than $1 + 1/\epsilon = M$ can be identified as Pareto-optimal solutions (i.e., the points in segment **CD**). Points with larger trade-off coefficients (e.g., from **B** to **C**) are treated as if they are only weakly Pareto-optimal solutions, such as the ones in segments **AB** and **DE**.

Obviously, M should be made large enough to ensure that all interesting solutions (i.e., all points of segment **BD**) can be explored in a decision-making process. This requirement corresponds to making ϵ or $(1-\delta)/(\delta p)$ a sufficiently small number. However, ϵ cannot be too small, because it must be computationally significant. The recommendations for the choice of ϵ are in the range of 10^{-3} to 10^{-5} (e.g., [SaY86], [Mak94]).

Comparing these findings with the results of Section 3.3, where $0.35 \leq \delta \leq 0.85$ (for 10 objectives) was found to model human decision-making behavior, it follows that $6.4 \leq M \leq 57.7$ (or $1.76 \cdot 10^{-2} \leq \epsilon \leq 1.85 \cdot 10^{-1}$). Thus, depending on the choice of δ, solutions with trade-off coefficients larger than relatively small numbers cannot be found when using the Fuzzy-AND aggregation. For two rescaled objectives μ_k with a nominal range in outcome values of 1, a decision maker would not be able to find solutions that require worsening μ_2 by 1 unit in order to improve μ_1 by 1/6.4 (for $M = 6.4$).

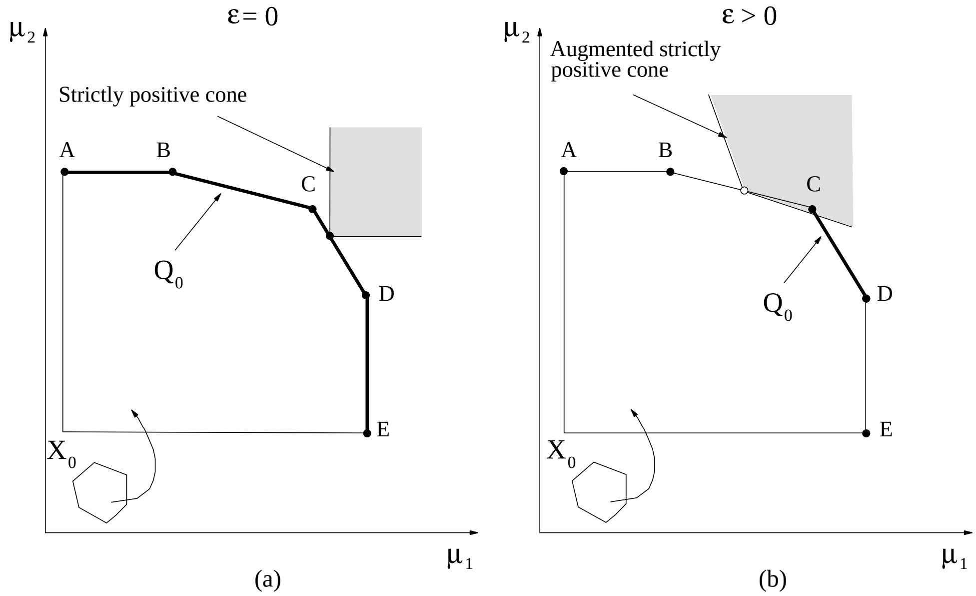

Figure 7: Examples of weakly Pareto-optimal (a) and properly Pareto-optimal (b) sets Q_0 [WiM92]

It is important to keep this limitation in mind when applying the Fuzzy-AND aggregation in an interactive procedure where the decision maker is supposed to explore the set of (properly) Pareto-optimal solutions. If a decision maker is not content with the solutions found after various trials, it might be necessary to change δ in order to explore solutions from other parts of the set of efficient solutions.

The strict monotonicity of the membership functions $\mu_k(z_k(\mathbf{x}))$ plays a major role for the efficiency of solutions $\mathbf{x}$ identified with the Fuzzy-AND aggregation operator. Therefore, the next section will deal with the determination of appropriate membership functions.

4.2 Selection of Membership Functions

The membership function for a fuzzy objective can be interpreted as a function that specifies the preference of the user and implies an ordering of the solutions in the decision or objective space. Assuming that the strict monotonicity of the membership functions μ_k is not given, the dominance of a solution $\mathbf{x}^0$ over $\mathbf{x}$ (i.e., $z_k(\mathbf{x}^0) \leq z_k(\mathbf{x})\ \forall k$ and $z_l(\mathbf{x}^0) < z_l(\mathbf{x})$ for at least one l) does not have to show in the vector of membership values.

In this context, it is important to recall that each μ_k is defined between the Nadir point component z_k^n (for which $\mu_k(z_k^n) = 0$) and the Utopia point component z_k^u (with $\mu_k(z_k^u) = 1$). Iserman and Steuer [IsS87] show that Nadir point components determined by the "payoff matrix" approach (see Section 3.2) can be subject to either overestimation or underestimation of the corresponding values.

For minimizing objectives, the overestimation of some components of $\mathbf{z}^n$ causes the decision maker to expresses his or her preference structure in terms of fuzzy sets whose support is too large. The decision maker might therefore get the wrong perception of the decision problem under consideration. Different problems occur if Nadir point components are underestimated. Here, situations can arise in which the fuzzy sets for the achievement of some objectives do not differentiate among the solutions that lie between the correct Nadir point $\mathbf{z}^n$ and the wrongly determined point $\mathbf{z}^{n'}$. Thus, there might be multiple solution values $z_k(\mathbf{x})$ for which $\mu_k(z_k(\mathbf{x})) = 0$. Consequently, it is not no longer guaranteed that the dominance of an optimal solution $\mathbf{x}^0$ over all other $\mathbf{x}$ in the criterion space is also evident in the objective value of the substitute problem.

One possible solution to both of these problems would be calculating the correct Nadir point according to the methodology described by Isermann and Steuer [IsS87]. However, for large problems the computational burden of this methodology is prohibitive. Instead, an application-oriented approach is introduced to avoid overestimating Nadir point components for minimizing objectives.[1]

Once the Utopia point has been determined, the components of $\mathbf{z}^n$ describing the Nadir point are computed as the maximum over $z_k(\mathbf{x}_l^{**})$, $k = (1, ..., p)$, $l = (1, ..., p)$, and $k \neq l$, where $\mathbf{x}_l^{**}$ result from the p optimization runs given in (30) with a small positive number ϵ. The normalization is conducted in order to scale all objectives to the same dimension:

$$\begin{aligned} \min[z_k/z_k^u \quad &+ \quad \epsilon \sum_{l\neq k} z_l/z_l^u] \\ s.t. \quad \mathbf{x} \quad &\in \quad X_D \;. \end{aligned} \tag{30}$$

Finally, the components of the Nadir point $\mathbf{z}^n$ are again computed according to (17).

To avoid a Nadir point that is determining an element of the shaded area in Figure 8, $\mathbf{x}_l^{**}$ is used instead of $\mathbf{x}_l^{*}$. In terms of Figure 8, it is guaranteed that the objective that is not minimized takes on the smallest of the range of possible values (hatched parts of the axes).

In addition, Granat and Wierzbicki's [GrW94] idea of so-called extended membership functions η are used to address the efficiency problem when underestimating the Nadir point (for minimizing objectives).

The basic idea is to provide piecewise linear extended membership functions that allow the ordering of alternatives with membership values outside $[0, 1]$. The procedure assumes that decision makers can specify at least two values for every objective function expressing the range of acceptable solutions (based on the knowledge of the Utopia point $\mathbf{z}^u$ and the approximation of the Nadir point $\mathbf{z}^n$). The first value describes the reservation point (or pessimistic value) z_k^R, for an objective with which the decision maker is still content and for which $\eta_k = 0$. The second, optimistic value, z_k^A, is called the aspiration point and gives the level at which the decision maker is fully satisfied with a solution, i.e., $\eta_k = 1$. Between these two values, the extended membership function η is equivalent to the conventional membership function μ. However, for values better than the optimistic value z_k^A and for values worse than the pessimistic value z_k^R, the extended membership function can take on values greater than 1 and less than 0, respectively.

With this definition, the extended membership functions are guaranteed to be strictly monotone functions over the range of possible values for every objective function.

The definition of the extended membership function requires the use of piecewise linear functions to describe the decision maker's degree of satisfaction. Allowing several segments in the range $[z_k^A, z_k^R]$ (for minimizing objectives), a more general description of

[1]However, even using the proposed methodology does not guarantee that the Nadir point will be computed correctly. Therefore, when referring to the Nadir point, the correct expression is "an (improved) approximation of the Nadir point."

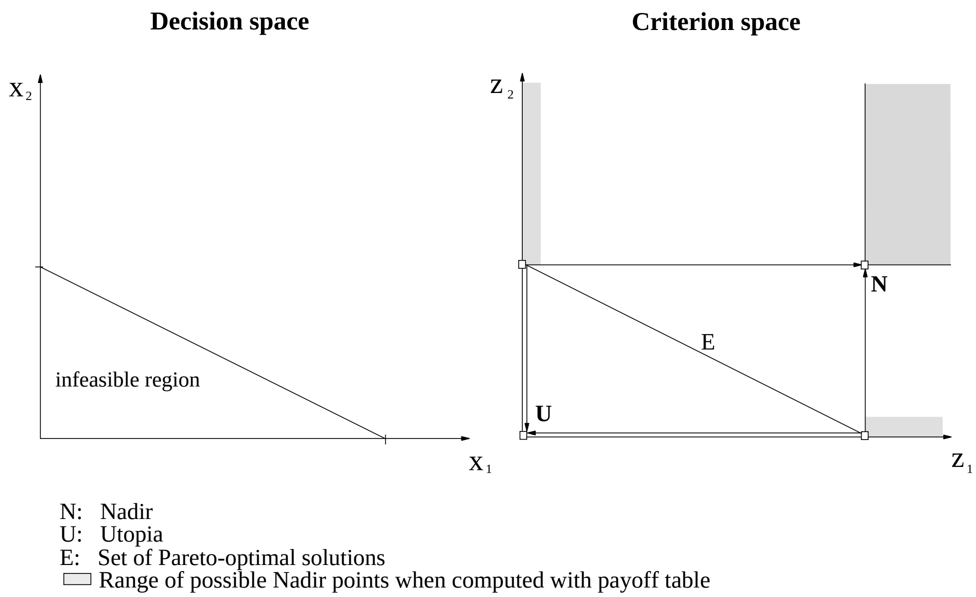

Figure 8: Approximation of the Nadir point in the criterion space.

the extended membership functions is given in the following equations. The different segments s of an extended membership function η_{ks} for objective k are defined as

$$\eta_{ks} = \alpha_{ks} z_k + \beta_{ks} \ , \quad \text{where} \quad z_{k,s-1} \leq z_k \leq z_{ks} \quad s = 1, ..., p_k \ , \tag{31}$$

where p_k is the number of segments for the k-th objective,

$$\alpha_{ks} = \frac{\eta_{k,s} - \eta_{k,s-1}}{z_{k,s} - z_{k,s-1}} \quad \text{and} \quad \beta_{ks} = \eta_{k,s-1} - \alpha_{ks} z_{k,s-1} \ . \tag{32}$$

The concavity of these piecewise linear membership functions (c.f., Figure 9) can be ensured by the following condition:

$$\alpha_{k1} \geq \alpha_{k2} \geq \ \ ... \ \ \geq \alpha_{kp_k} \ . \tag{33}$$

The use of such extended membership functions in combination with the Fuzzy-AND aggregation of the fuzzy objectives guarantees that the dominance of a criterion vector $\mathbf{z}(\mathbf{x}^0)$ over $\mathbf{z}(\mathbf{x})$ will also be reflected in the objective value of the substitute problem.

It should be pointed out here that the methodology was only described for minimizing objectives and that it can easily be extended for problems with minimizing and maximizing objectives. These modifications will be applied in Section 5.

Still, from an application point of view the integration of fuzzy equality constraints and the aggregation of several criteria into one, more general criterion remain unsolved. These problems are addressed in the following subsection.

4.3 Hierarchical Aggregation of Multiple Fuzzy Criteria

The problems that still remain to be addressed are the integration of fuzzy goal objective functions and the aggregation of several criteria into one, more general criterion. Both problems can be resolved by the same principle.

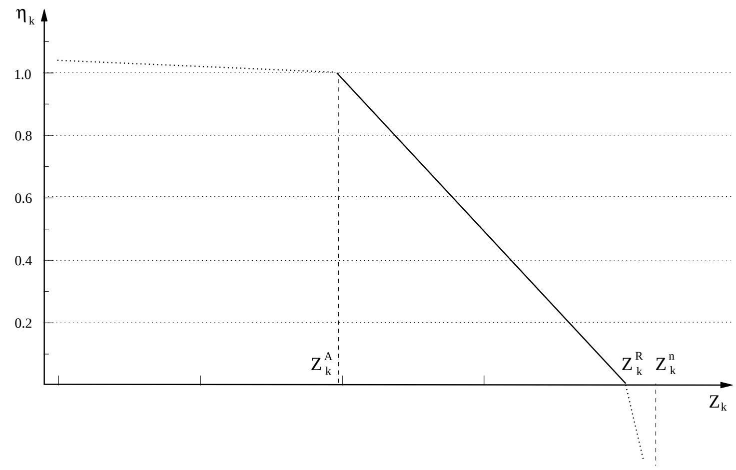

Figure 9: Piecewise linear membership function η for a minimizing objective.

First, the incorporation of fuzzy goal (or fuzzy equality) objectives will be addressed. By using fuzzy goal objectives, it is possible to ensure that some objectives $z_k(\mathbf{x})$ take on values that equal r_k in a fuzzy sense. The following procedure for the implementation of such objectives uses the principles described in the previous subsection:

1. A fuzzy equality objective is replaced by one fuzzy minimizing objective with aspiration level r_k and one fuzzy maximizing objective with the same aspiration level r_k.
2. The aggregation of the two corresponding membership functions is accomplished using the logical "AND" aggregation, which is equivalent to to the non-compensatory Min-aggregation in fuzzy set theory.
3. The result of the Min-aggregation of the corresponding membership functions is then used in the compensatory Fuzzy-AND aggregation.

If an objective must be in the range $[r_k^l, r_k^u]$, an additional constraint can be introduced after the first step to ensure that $\eta_k \leq 1$. As such, it is possible to model the maximizing objective with an aspiration point r_k^l and the minimizing objective with an aspiration level r_k^u. The procedure guarantees that all solutions with $z_k(\mathbf{x}) \in [r_k^l, r_k^u]$ have membership values of 1 and hence are "fuzzy equal" on an α-level of 1.

Before turning to the aggregation of several criteria, it is important to point out that using the extended membership functions η is crucial for preserving the efficiency of the solution in cases where decision makers (or model users) specify attainable aspiration points or unattainable reservation points to describe fuzzy goal criteria [Wie92].

The same procedure as used for the fuzzy goal criteria can be used to aggregate the elements k of the set of all objectives K. Hereby, logically connected objective functions are aggregated into new sets of objectives H_c. Each of these new sets then comprises elements (i.e., objectives) of K. It is assumed that within the individual sets H_c, compensation among the objectives is not modeled. However, compensatory effects are allowed between the different sets of objectives H_c. The set C consisting of elements c is introduced to model the compensation between the different classes H_c.

Example: From a toxicological point of view, it can be argued that sulfur dioxide (SO_2), nitrous oxide (NO_x) and carbon dioxide (CO_2) emissions will have to be reduced in order to achieve better living conditions. It is also clear that the costs of the required emission reductions will be a decision criterion. The set K then contains the following objectives: $\{\text{costs}, SO_2, NO_x, CO_2\}$.

Implementing the hierarchical concept, the aggregate of the membership functions of the emissions will be used as a new objective, so that $C = \{costs, emissions\}$. The resulting H_c are

$$\begin{aligned} \mathrm{H}_{emissions} &= \{SO_2, NO_x, CO_2\} \\ \mathrm{H}_{costs} &= \{\text{costs}\}. \end{aligned}$$

As for the fuzzy equality constraints, the aggregation of the membership values within the sets H_c is accomplished using the fuzzy Min-operator.

The hierarchical version of the Fuzzy-AND operator is shown in (34):

$$\max_{x \in X_D} [\delta \min_{c \in C} \{ \min_{h \in H_c} \{\eta_h(\mathbf{x})\}\} + (1-\delta) \frac{1}{|C|} \sum_{c \in H_c} \min_{h \in H_c} \{\eta_h(\mathbf{x})\}] \, , \tag{34}$$

where $|C|$ is the cardinality of set C.

Using a simple counterexample, it can be shown that the accordingly modified crisp equivalent problem of (20) with the aggregation (34) does not necessarily identify optimal solutions that are also efficient for the initial problem. Therefore, the following modification of the Fuzzy-AND operator is introduced:

$$\eta_D(\mathbf{x}) = \delta \min_{c \in C} \{ \min_{h \in H_c} \{\eta_h(\mathbf{x})\}\} + (1-\delta) \frac{1}{|C|} \sum_{c \in C} \min_{h \in H_c} \{\eta_h(\mathbf{x})\} + \nu \frac{1}{|K|} \sum_{c \in C} \sum_{h \in H_c} \eta_h(\mathbf{x}) \, , \tag{35}$$

where ν is a small constant (in the order of 10^{-4}).

This slightly modified version of the Fuzzy-AND aggregation with $\delta < 1$ allows the use of the proposed hierarchical aggregation and still guarantees efficient solutions. Also, fuzzy equality constraints (or, in terms of MOLP, stabilized criteria) can now be modeled with the new aggregation operator. It is assumed that the cardinality of K and C are m_k and m_c, respectively. Also, an additional index set S for the segments s of every piecewise membership function has been introduced so that the final formulation of the objective function is

$$\begin{aligned} &\max_{x \in X_D} [\delta \min_{c \in C} \{ \min_{h \in H_c} \{ \min_{s \in S} \{\eta_{h,s}(\mathbf{x})\}\}\} \quad + \\ &(1-\delta) \tfrac{1}{m_c} \sum_{c \in C} \min_{h \in H_c} \{ \min_{s \in S} \{\eta_{h,s}(\mathbf{x})\}\} \quad + \quad \nu \tfrac{1}{m_k} \sum_{c \in C} \sum_{h \in H_c} \min_{s \in S} \{\eta_{h,s}(\mathbf{x})\}] \, . \end{aligned} \tag{36}$$

The crisp equivalent problem for the above aggregation operator is shown in (37).

$$\max_{\lambda, \lambda_c, \gamma_h, \mathbf{x} \in X_D} \{ \lambda + (1-\delta) \frac{1}{m_c} \sum_{c \in C} \lambda_c + \nu \frac{1}{m_k} \sum_{c \in C} \sum_{h \in H_c} \gamma_h \} \tag{37}$$

$$\begin{array}{llll} s.t. & \lambda + \lambda_c & \leq \gamma_h & \forall c \in C, \quad \forall h \in H_c \, , \\ & \gamma_h & \leq \eta_{h,s}(\mathbf{x}) & \forall h \in K, \quad \forall s \in S \, , \\ & \lambda_c & \geq 0 & \forall c \in C \, , \\ & \mathbf{x} & \in X_D \, . & \end{array}$$

Now it will be shown that the aggregation operator shown in (36) is modeled correctly by the transformation (37). The proof will be conducted in analogy to Werners' ideas [Wer87b] validating the transformation of the Fuzzy-AND operator without hierarchical aggregation and without the regularization term (i.e., only the correctness of the transformation of (23) into (24) is proved). In the proof, the following will be used: $H_c \subseteq K \;\; \forall c \in C$ and that $\bigcup_{c\in C} H_c = K$, as well as $\bigcap_{c\in C} H_c = 0$.

Proposition 1 *If* $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$ *is optimal for (37), then*

$$\begin{array}{lll} & \lambda^0 & = \min_{c\in C}\{\min_{h\in H_c}\{\min_{s\in S}\{\eta_{h,s}(\mathbf{x}^0)\}\}\}, \\ \forall c \in C & \lambda^0 + \lambda_c^0 & = \min_{h\in H_c}\{\gamma_h^0\} = \eta_c(\mathbf{x}^0), \\ \forall k \in K & \gamma_h^0 & = \min_{s\in S}\{\eta_{h,s}(\mathbf{x}^0)\} = \eta_k^0(\mathbf{x}^0), \end{array}$$

if $\delta > 0$ *and* $\mathbf{x}^0$ *is optimal for problem (36).*

Proof Suppose $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$ is optimal for (37) and $\gamma_h^0 \neq \min_{s\in S}\{\eta_{h,s}(\mathbf{x}^0)\}$. Let $\gamma_h^* = \min_{s\in S}\{\eta_{h,s}(\mathbf{x}^0)\}$; then, because $\forall h \in K,\ \forall s \in S\ \gamma_h^0 \le \gamma_{h,s}(\mathbf{x}^0),\ \exists h_0 \in K : \gamma_{h_0}^0 < \gamma_{h_0}^* = \min_{s\in S}\{\eta_{h_0,s}\}$. The solution with

$$\gamma_h^0 = \begin{cases} \gamma_h^0, & h \neq h_0 \\ \min_{s\in S}\{\eta_{h,s}\} & h = h_0, \end{cases}$$

is feasible and the following relation contradicts the optimality of $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$:

$$\lambda^0 + (1-\delta)\frac{1}{m_c}\sum_{c\in C}\lambda_c^0 + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0 < \lambda^0 + (1-\delta)\frac{1}{m_c}\sum_{c\in C}\lambda_c^0 + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^* .$$

Suppose that $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$ is optimal for (37) and $\lambda^0 + \lambda_c^0 \neq \min_{h\in H_c}\{\gamma_h^0\}$. Let $\lambda^0 + \lambda_c^* = \min_{h\in H_c}\{\gamma_h^0\}$; then, because $\forall c \in C,\ \forall h \in H_c\ \lambda + \lambda_c \le \gamma_h,\ \exists c_0 \in C : \lambda^0 + \lambda_{c_0}^0 < \min_{h\in H_{c_0}}\{\gamma_h^0\} = \eta_c(\mathbf{x}^0) = \lambda^0 + \lambda_{c_0}^* \Leftrightarrow \lambda_{c_0}^0 < \eta_c(\mathbf{x}^0) - \lambda^0 = \lambda_{c_0}^*$. With

$$\lambda_c^* = \begin{cases} \lambda_c^0, & c \neq c_0 \\ \eta_c(\mathbf{x}^0) - \lambda^0 & c = c_0 , \end{cases}$$

$(\mathbf{x}^0, \lambda^0, \lambda_c^*, \gamma_h^0)$ is feasible and

$$\lambda^0 + (1-\delta)\frac{1}{m_c}\sum_{c\in C}\lambda_c^0 + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0 < \lambda^0 + (1-\delta)\frac{1}{m_c}\sum_{c\in C}\lambda_c^* + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0$$

contradicts the optimality of $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$ (for any $\delta < 1$).

Using the properties of the Min-operator and the previous results, the following transformations can be made: $\lambda^0 = \min_{c\in C}\{\min_{h\in H_c}\{\min_{s\in S}\{\eta_{h,s}(\mathbf{x}^0)\}\}\} = \min_{c\in C}\{\min_{h\in H_c}\{\gamma_h^0\}\}$ $= \min_{c\in C}\{\eta_c^0(\mathbf{x}^0\} = \min_{h\in K}\{\gamma_h^0\}$. Suppose that $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$ is optimal for (37) (with $\delta > 0$) and $\lambda^0 \neq \min_{c\in C}\{\min_{h\in H_c}\{\min_{s\in S}\{\eta_{h,s}(\mathbf{x}^0)\}\}\}$. This is equivalent to $\lambda^0 \neq \min_{h\in K}\{\gamma_h^0\}$. From the constraints of (37) (i.e., from $\lambda_c^0 \ge 0$), it follows that $\lambda^0 + \lambda_c^0 \le \gamma_h^0 \Rightarrow \lambda^0 \le \gamma_h^0$ $\forall h \in K$ and, therefore, $\lambda^0 < \min_{h\in K}\{\gamma_h^0\}$. Let $\lambda^* = \min_{h\in K}\{\gamma_h^0\}$; then, because $\forall c \in C$ $\min_{h\in H_c}\{\gamma_h^0\} = \eta_c(\mathbf{x}^0)$, there must also be some $\lambda_c^* \ge 0$ so that $\forall c \in C\ \lambda_c^* = \eta_c(\mathbf{x}^0) - \lambda^*$. $(\mathbf{x}^0, \lambda^*, \lambda_c^*, \gamma_h^0)$ is feasible and

$$\lambda^* + (1-\delta)\frac{1}{m_c}\sum_{c\in C}\lambda_c^* + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0 = \delta\lambda^* + (1-\delta)\frac{1}{m_c}\sum_{c\in C}(\lambda_c^* + \lambda^*) + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0 =$$

$$\delta\lambda^* + (1-\delta)\frac{1}{m_c}\sum_{c\in C}\eta_c(\mathbf{x}^0) + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0 > \delta\lambda^0 + (1-\delta)\frac{1}{m_c}\sum_{c\in C}(\lambda_c^* + \lambda^*) + \nu\frac{1}{m_k}\sum_{c\in C}\sum_{h\in H_c}\gamma_h^0$$

contradicts the optimality of $(\mathbf{x}^0, \lambda^0, \lambda_c^0, \gamma_h^0)$. This concludes the proof of the proposition.

The optimal values of the variables γ_h ($\forall c \in C$ and $\forall h \in H_c$) have an important interpretation in terms of the interactive procedure. Negative values for γ_h^0 indicate that the user specified unattainable reservation values for the corresponding objectives. Similarly, values greater than one show that attainable aspiration levels were specified. Hereby, membership values greater than one (i.e., $\gamma_h^0 > 1$) are entirely consistent with the fuzzy set theory. However, negative membership values are not defined for fuzzy sets. Therefore, the user of the model should be made aware of the fact that the interactive model in such cases does not produce results that are consistent with the fuzzy set theory (nevertheless, such solutions are still properly efficient solutions in terms of the underlying multicriteria optimization problem (20)). The required information regarding the minimum value over all γ_h^0 can easily be extracted from the model by checking if λ^0 is negative. To obtain results that justify the way uncertainties were incorporated into the model, the user is advised to continue the interactive decision procedure until the optimal value of λ fulfills the condition $\lambda^0 \geq 0$.

5 Application to Energy System Planning

This section shows the application to energy system planning of the methodology developed in Sections 2 and 3 with the modifications discussed in Section 4.

5.1 The Reference Energy System of the Case Study

In energy planning, a convenient way to describe entire energy systems is to use the so-called reference energy system (RES). An RES is a network that consists of commodities that are linked via links to processes. In RES notation, energy and material flows occur in links and are balanced at the commodities. Processes are used to transform one or several inflows into one or several outflows. In addition, each process has the ability to add generating capacity to its existing capacity throughout one time period and, at the end of one period, to pass on the remainder of the capacity to the consecutive period. In this way the spatial flows of energy and material are augmented by intertemporal flows. In Figure 10 the spatial flows are shown for a small RES with five sectors (for one time period). The two sectors on the left model the import, mining, and transformation of primary energy carriers that are made available in the central electricity producing sector and in the two sectors on the right. The two right-hand sectors cover the exogenously given demand for transportation and iron and steel products. Uncertain cost coefficients and uncertain demand values for the transport sector and the iron and steel sector were considered in the sample case study.

5.2 Modeling Concepts and Mathematical Model Formulation

In general, a model analyzed in a DSS is composed of a core model and additional constraints and variables generated during the interactive decision-making process. The reasons for using a model structure that utilizes the concepts of a core model and a set of additional formulations are described by Makowski [Mak94], who also offers guidelines for the formulation of core models. Here, the concept of core models and additional formulations for the interactive decision-making process is briefly reviewed and is then augmented

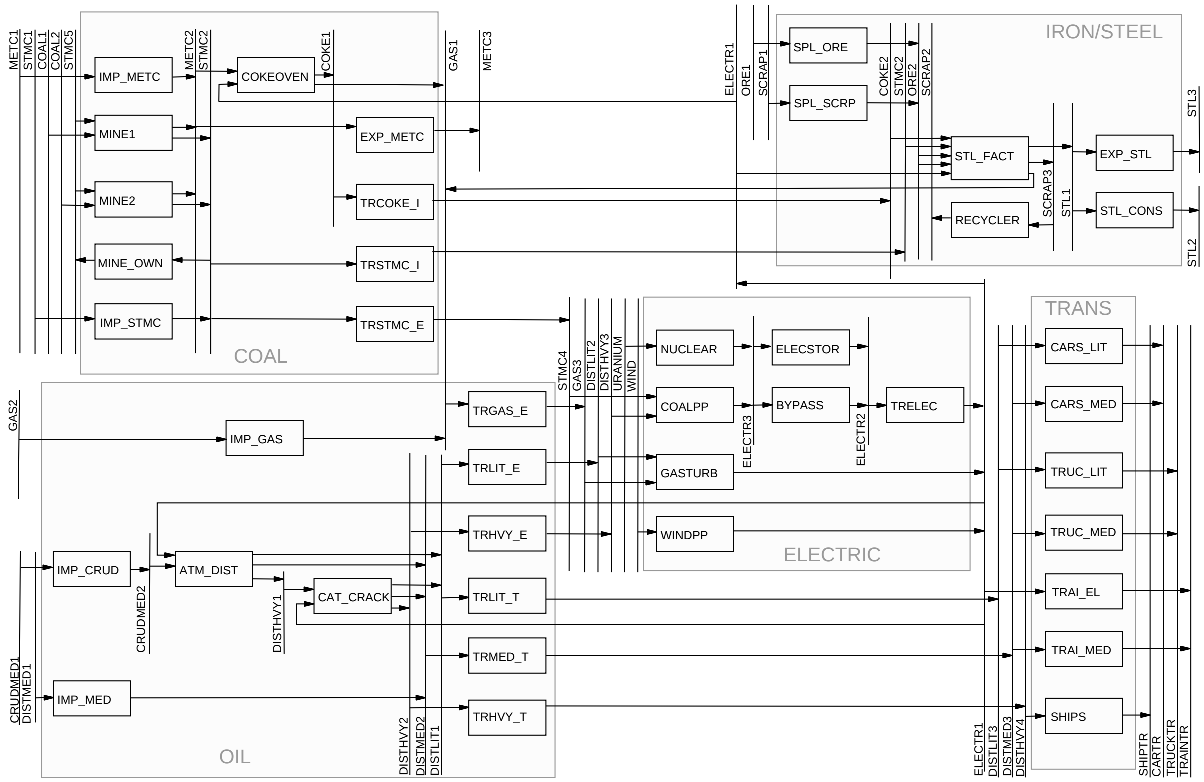

Figure 10: RES of the sample energy system.

by the concept of a so-called *extended core model.* In order to illustrate this concept, the roles of decision variables, constraints, and the objective function in the core model of energy system models are described. It also is shown how concepts with a core model and an interactive model part can be applied to energy system planning.

In a conventional DSS, the purpose of the core model is to describe the physical and logical relations of a real-world system without a representation of the decision-makers preferential structure. For energy system models, this means that the decision variables model the flow of mass, energy, and currencies across intertemporal and interspatial borders. Examples for such variables are the spatial flows of energy carriers (i.e., the flow of energy in one time period) into a power plant and the corresponding flows of energy out of a conversion technology. The concept of intertemporal flow variables can be used to model the discounting of yearly costs to the base year. The related process can be conceived of as a conversion technology with an efficiency that corresponds to the appropriate present worth factor. Similarly, the capacities of existing power plants can be transferred to consecutive model periods and are thus modeled by intertemporal flows.

Among the decision variables, those describing capacity additions are usually of the most interest, because they model the decisions that should be made. They are therefore potential candidates for selection as decision criteria. However, any convolution of the variables that describe intertemporal and interspatial flows can be declared as decision criteria in order to evaluate the complex decision problem. For energy system planning tools such as EFOM, MARKAL, and MESSAGE, costs are usually the only criterion defined. In these models, the objective function is the sum of all discounted costs over the entire modeling horizon. Costs taken into consideration are the variable and fixed costs of operation and the investment costs for the processes.

Constraints are used to model each process according to its characteristic properties, such as costs, emissions, efficiencies, ratios among the input flows, ratios among the output flows, and capacity. Besides modeling the processes, inequalities model the requirement that the sum of the inflows into each commodity equals the sum of the outflows from that commodity. In this context, they can also be used to impose upper and lower bounds on the amount of a commodity produced and consumed as well as on individual flows if these relationships are of purely physical/logical character. Moreover, using inequality constraints, it is possible to establish ratios among inflows and outflows for specific commodities in order to describe market and product allocation. The driving force of energy system models is the requirement that the sum of all inflows in a demand commodity (commodity without outflow) be at least as great as the exogenously defined demand for this commodity.

The variables and the constraint set defined in a core model remain the same for the entire decision-making process. In contrast, the set of additional constraints is used to model all information that pertains to the decision process. The idea is that even with the user-specified preference, no solution that is infeasible for the core model can be identified in the later stages of the procedure. This means that the additional definition of user preferences narrows the feasible set to the set of "acceptable solutions." When implementing these additional constraints as soft constraints, the DSS with a nonempty feasible set for the core model never produces infeasible solutions. As a consequence, the same core model can be given to different decision makers so that they can explore their own "optimal" solutions. Finally, after the core model is defined and the additional constraints have been added (which usually enlarges the original problem by only a small fraction), the possibility to use the results of previous optimization runs significantly speeds up computation and the decision process.

Once the core model is defined and the decision criteria have been selected, the solution procedure is as follows: first, the Utopia and Nadir point are computed according to the methodology described in Section 4. Using these two points, a compromise (or neutral) solution is automatically determined, assuming linear membership functions between the Nadir and Utopia components. This step concludes the preparatory phase of the analysis, and control is passed onto the user, who can then interactively modify aspiration and reservation points for the various objectives, as well as the shape of membership functions [GrM95]. After each interactive modification of the membership functions, the problem is solved again and the new Pareto-optimal solution is presented to the user.

Subsections 5.2.1, 5.2.2, and 5.2.3 describe the basic constraints that make up the core model, the extended core model, and the interactive part for a fuzzy model, respectively. Subsection 5.2.2 shows why an extended core model becomes necessary when applying the concepts of Section 3 to energy system planning.

5.2.1 Formulation of the Core Model

The inequality constraints that form the core model can be divided into two classes. The first class is associated with every process modeled in the system and basically consists of formulas (38) and (39). The second class of constraints contains three types of formulations that are generated for the commodities in the system. These inequalities are referred to as *commodity balances* (40), *commodity bounds* (41), and *commodity relations* (42). Using the two classes of constraints as defined in formulas (38) through (44), RES constructs like the one shown in Figure 11 can be modeled. For the sake of simplicity and understandability, the time index is omitted whenever it is not explicitly needed.

Notation

X	The set of all decision variables describing the network flow, $X = X^{in} \cup X^{out}$.
X^{in}	Inflows into processes, where $X^{in} = X^{in,is} \cup X^{in,it}$, with $X^{in,is}$ representing interspatial inflows and $X^{in,it}$ representing intertemporal inflows.
X^{out}	Outflows from processes, where $X^{out} = X^{out,is} \cup X^{out,it}$, with $X^{out,is}$ representing interspatial outflows and $X^{out,it}$ representing intertemporal outflows.
Z^{costs,T_0}	Decision criterion: total costs in base year T_0.
Z^{dem}_{co}	Decision criterion: coverage of the demand of a commodity co.
CO	The set of all commodities co, co', $CO \supseteq CO^{in} \cup CO^{out}$.
CO^{in}	Commodities that have emerging flows into processes.
CO^{out}	Commodities that have incoming flows from processes.
PR	The set of all processes pr, pr', $PR \supseteq PR^{in} \cup PR^{out}$.
PR^{in}	Processes that have incoming flows.
PR^{out}	Processes that have flows emerging toward the commodities.
B	The set of bounds imposed on commodities and processes $B = B^{up}_{co} \cup B^{up}_{pr} \cup B^{lo}_{co} \cup B^{lo}_{pr}$.
$A_{co,pr}$	The set of all parameters.
D_{co}	Demand for a commodity co.
R_{co}	Available resources of a commodity co.
T	The set of all modeling periods t with base year T_0.

Constraints that Model Processes

The first kind of inequality constraints associated with the processes models the flow balances for energy and material for each process, as well as the inflow of currencies needed to keep the process running. A more general formulation of this constraint can also be used to separately model ratios among the inflows and outflows:

$$\sum_{co \in CO^{in}} X^{in,is}_{co,pr} A_{co,pr} + \sum_{co \in CO^{in}} X^{in,it}_{co,pr} \geq \sum_{co \in CO^{out}} X^{out,is}_{co,pr} A_{co,pr} + \sum_{co \in CO^{out}} X^{out,it}_{co,pr} \quad \forall t \in T \; . \quad (38)$$

The second type of inequality is used to impose upper or lower bounds on the sum of incoming or outgoing flows. As an example for the four possible constraints, the one imposing an upper bound on the (weighted) sum of the inflows is shown in (39):

$$\sum_{co \in CO^{in}} X^{in,is}_{co,pr} A_{co,pr} \leq B^{up}_{pr} \quad \forall t \in T \; . \quad (39)$$

Constraints for Commodities

Three different kinds of linear inequality constraints are associated with the commodities. The first of these guarantees that the flow balances for each commodity are maintained:

$$\sum_{pr \in PR^{in}} X^{in,is}_{co,pr} \; + \; R_{co} \geq \sum_{pr \in PR^{out}} X^{out,is}_{co,pr} \; + \; D_{co} \quad \forall t \in T \; . \quad (40)$$

The second kind can be used to impose upper bounds on the inflows or outflows of the commodities. As an example, the constraint that places an upper bound on the sum of the inflows is shown in (41):

$$\sum_{pr \in PR^{out}} (X^{out,is}_{co,pr} + X^{out,it}_{co,pr}) A_{co,pr} \leq B_{co} \quad \forall t \in T \; . \quad (41)$$

The last of the relations associated with the commodities are those that model the inflow/outflow relations for the commodities:

$$\sum_{pr \in PR^{out}} X^{out,is}_{co,pr} A_{co,pr} \geq \sum_{pr \in PR^{in}} X^{in,is}_{co,pr} A_{co,pr} \quad \forall t \in T \; . \tag{42}$$

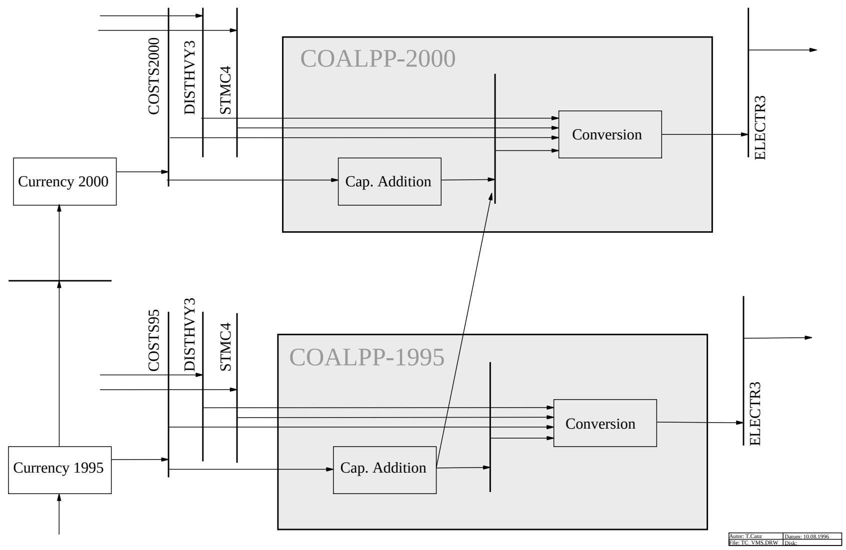

Figure 11: RES concept with capacity addition [Küh96].

Objective Functions

The cost objective that is used almost exclusively in the established energy system models is modeled as

$$Z^{costs,T_0} = \sum_{t \in T} \psi^t \Big(\sum_{pr \in PR^{in}} X^{in,is}_{co=costs,pr} - \sum_{pr \in PR^{out}} X^{out,is}_{co=costs,pr} \Big) \; , \tag{43}$$

where $X^{in,is}_{co=costs,pr}$ represents costs occurring in process pr; $X^{out,is}_{co=costs,pr}$ is revenue generated in process pr; and ψ^t is the present worth factor.

Because the inflows of currency occur for the capacity-adding processes as well as for the conversion itself (see Figure 11), it is possible to model variable and fixed costs of operation using the internal representation of the processes given in (38).

Assuming that the coverage of the demand values in the various sectors can also be seen as decision criteria, the following decision variables can be derived from the flow balances (40):

$$Z^{dem}_{co} = \sum_{pr \in PR^{in}} X^{in,is}_{co,pr} \geq D_{co} \quad \forall t \in T \; , \tag{44}$$

where Z^{dem}_{co} is the level of the criterion “coverage of the demand.”

For the two different kinds of criteria, the latter is to be maximized (with a lower bound D_{co}), whereas costs are to be minimized.

5.2.2 Formulation of the Extended Core Model

Looking at an implementation of the case study with fuzzy cost coefficients in the objective function and with fuzzy demand vectors, the concept with a core model and an additional interactive component is augmented by a so-called *extended core model.* The use of this additional module allows the definition of the core model to be retained when fuzzy formulations are used. The two reasons for using the extended core model are introduced in the following paragraphs. As an example, objectives that aim at "maximizing" the coverage of exogenously given demands are considered.

The first reason for the introduction of an extended core model has to do with the computation of the Utopia and Nadir point before the start of the interactive procedure. Bearing in mind that energy system models are *demand driven*, it is clear that if there is no demand, the optimal solution for the cost objective (i.e., the solution from the selfish optimization of the cost objective) is close to zero. This is because in such cases the only costs are the fixed costs of maintaining the installed capacities. Similarly, when computing the Nadir point for the cost objective from the activity vectors that maximize the coverage of the (unbounded) demands, the cost objective is also unbounded (if no other constraint of the system description renders the problem bounded). Setting aside the problems that are encountered in the unbounded case and assuming instead the coverage of a very high demand, the combination of the resulting Utopia and Nadir point components for the cost objective clearly does not make sense. In such cases, the compromise solution would not correspond to the actual decision problem.

To avoid such problems, the extended core model contains two additional types of constraints for every fuzzy demand $\tilde{D}_{co}$. The first of the additional constraints guarantees that the demand covered is greater than the minimal expected load $d_{co,1} - \delta_{co,1} L^{-1}(\epsilon)$; the second class of constraints does not allow demand values greater than the upper bound $d_{co,2} + \delta_{co,2} R^{-1}(\epsilon)$. These constraints can be interpreted as the inequality parts of two fuzzy constraints that model goal decision criteria according to Rommelfanger's inequality relations $\tilde{\geq}_R$ and $\tilde{\leq}_R$ [Rom88].

In the case study, $\tilde{\mathbf{A}}\mathbf{x}$ is non-fuzzy, because availability factors, efficiencies, etc., of conversion technologies are modeled by crisp numbers.

The constraints added for every fuzzy demand commodity and every modeling period can be seen as special instances of the *commodity balance* (40) and the *commodity bounds* (41), the latter of which put upper bounds on the commodities produced by the processes PR^{out}(see Figure 12). In these equations $R_{co} = X_{co \in CO^{out}} = 0$, so that

$$\begin{aligned} \sum_{pr \in PR^{out}} X_{co,pr}^{out,is} &\geq d_{co,1} - \delta_{co,1} L^{-1}(\epsilon) \;, \\ \sum_{pr \in PR^{out}} X_{co,pr}^{out,is} &\leq d_{co,2} + \delta_{co,2} R^{-1}(\epsilon) \;. \end{aligned}$$

The second reason for introducing an extended core model is obvious from the above inequalities: the formulations that replace D_{co} and B_{co}^{up} in the corresponding constraints depend on the value ϵ. These levels are chosen according to the preference structure of the model user (i.e., the decision maker) and therefore by definition are not allowed to enter the core model. Furthermore, it is recommended that $\epsilon = 0$ be used at the beginning of the model evaluation in order to limit the feasible set of the core model description as little as possible by the additional constraints. Also, the value of the parameter ϵ should not be changed during the interactive process of exploring the efficient set. Changing the parameter value would result in different Nadir and the Utopia point components for at least some of the objectives.

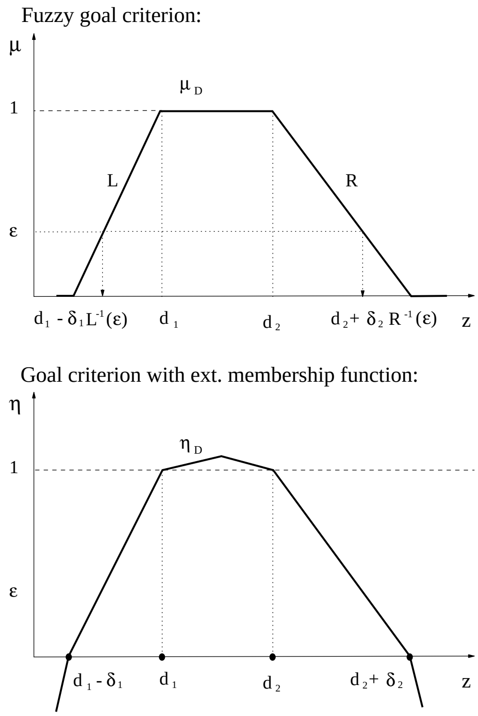

Figure 12: Fuzzy goal criterion that models the coverage of the demand.

As a generalization of the concept of the extended core model, all constraints that are required for the transformation of fuzzy constraints into crisp constraints should be entered as part of the extended core model.

In the case study, it is assumed that the following processes have uncertain cost coefficients for investment costs and the fixed costs of operation:

- Thermal power plants (COALPP, GASTURB),
- Power plants that use renewable energy (WINDPP, HYDROPP),
- Nuclear power plants (NUCLEAR),
- Pumped storage hydroplants (ELECSTOR),
- Transmission equipment (TRELEC, BYPASS).

In order to include uncertainties regarding the future costs of energy carriers, the following processes have fuzzy cost coefficients for the variable costs of operation:

- The importing processes in the oil sector (IMP_CRUD, IMP_MED, IMP_GAS);
- The importing/mining processes in the coal sector (IMP_STMC, IMP_METC, MINE1, and MINE2).

The resulting overall fuzzy costs (Z^{cost,T_0}) are computed in LR-notation as shown in Section 2.1 and are denoted by (Z_1^{costs,T_0}, Z_2^{costs,T_0}, Z_3^{costs,T_0}, Z_4^{costs,T_0}), where Z_1^{costs,T_0} is the lower mean value minus the lower spread of the overall fuzzy costs. Similarly, Z_4^{cost,T_0} is the maximum value of the support of the fuzzy set describing the cost objective. Furthermore, uncertain demand values are taken into consideration for all the demands in

- The transport sector;
- The iron and steel sector.

Table 1 shows the results of the selfish optimization (i.e., the Utopia point components) and the Nadir point components for each objective. The table shows only 20 pairs of Nadir and Utopia point components, because for some of the commodities no demand is defined for specific periods. It must be pointed out that for the determination of the Utopia and (an approximation of) the Nadir point, the objectives aiming at the coverage of the demand are treated as maximizing objectives that have an upper and a lower bound imposed on the criterion values. Therefore, the column entries are marked with an asterisk in Table 1. The corresponding objectives are treated as fuzzy goal criteria in the interactive decision process.

Table 1: Utopia and Nadir point components

	2000		2010		2020	
Demand	Nadir*	Utopia*	Nadir*	Utopia*	Nadir*	Utopia*
$Z^{dem}_{TRAINTR}$ [PJ]	37.0	39.0	36.0	40.0	36.0	41.5
$Z^{dem}_{TRUCKTR}$ [PJ]	710.0	1150.0	870.0	1410.0	900.0	1590.0
Z^{dem}_{CARTR} [PJ]	570.0	605.0	410.0	450.0	320.0	363.0
Z^{dem}_{SHIPTR} [PJ]	110.0	120.0	120.0	135.0	120.0	140.0
Z^{dem}_{STL2} [kt]	13000	14000	–	–	–	–
Z^{dem}_{STL3} [kt]	29000	30800	31000	33000	31000	33000
Costs[2]	Nadir			Utopia		
Z_1^{costs,T_0} $[10^8 \cdot DM^{90}]$	6.670E+4			5.015E+4		
Z_2^{costs,T_0} $[10^8 \cdot DM^{90}]$	6.705E+4			5.050E+4		
Z_3^{costs,T_0} $[10^8 \cdot DM^{90}]$	6.749E+4			5.090E+4		
Z_4^{costs,T_0} $[10^8 \cdot DM^{90}]$	6.801E+4			5.137E+4		

5.2.3 Integration of User Preferences Using Fuzzy Sets

This section documents the fuzzy aggregation operator, the fuzzy membership functions, and the aggregation principle chosen to determine the final solution.

Fuzzy Aggregation Operator

The Fuzzy-AND aggregation was chosen for the derivation of the single-criterion optimization problem according to (37). Initially, the parameter $\delta = 0.6$ was selected, meaning that for 6 aggregated criteria (see below), solutions with trade-off coefficients greater than 10 could not be found without changing δ. The use of the modified Fuzzy-AND aggregation guarantees that only properly Pareto-optimal solutions are identified by the method. The value ν is chosen to be 10^{-4}, therefore it does not limit the Pareto-optimal set significantly compared with the trade-off coefficient that results from the selection of δ.

[2] DM^{90}: "Deutsche Mark", 1990 value.

Fuzzy Membership Function

The extended fuzzy membership functions for each of the 20 non-aggregated objectives were modeled by two segments between the user-defined aspiration point (z_k^A) and reservation point (z_k^R). For purely minimizing objectives (i.e., the four cost objectives), an example of the membership functions is displayed in Figure 13.

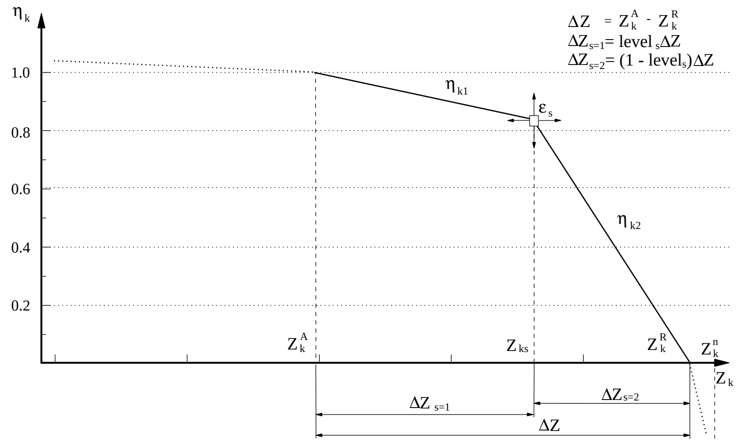

Figure 13: Extended membership function η describing the degree of satisfaction with the characteristic cost functions Z_k^{costs,T_0}.

The fuzzy goal criteria for the coverage of the various demands were modeled as shown in the lower part of Figure 12. Therefore, the Min-aggregation of two extended membership functions was required to model the coverage of every fuzzy demand (for every modeling period).

Table 2: Piecewise linear extended membership functions

	Minimizing objectives				Maximizing objectives			
	Segment 1		Segment 2		Segment 1		Segment 2	
	$\Delta z_{k,s=1}$	$\Delta\eta_k$	$\Delta z_{k,s=2}$	$\Delta\eta_k$	$\Delta z_{k,s=1}$	$\Delta\eta_k$	$\Delta z_{k,s=2}$	$\Delta\eta_k$
$Z_{TRAINTR}^{dem}$	0.5	0.5	0.5	0.5	0.2	0.8	0.8	0.2
$Z_{TRUCKTR}^{dem}$	0.5	0.5	0.5	0.5	0.2	0.8	0.8	0.2
Z_{CARTR}^{dem}	0.5	0.5	0.5	0.5	0.2	0.8	0.8	0.2
Z_{SHIPTR}^{dem}	0.5	0.5	0.5	0.5	0.2	0.8	0.8	0.2
Z_{STL2}^{dem}	0.5	0.5	0.5	0.5	0.2	0.8	0.8	0.2
Z_{STL3}^{dem}	0.5	0.5	0.5	0.5	0.2	0.8	0.8	0.2
Z_1^{costs,T_0}	0.3	0.2	0.7	0.8				
Z_2^{costs,T_0}	0.3	0.2	0.7	0.8				
Z_3^{costs,T_0}	0.3	0.2	0.7	0.8				
Z_4^{costs,T_0}	0.3	0.2	0.7	0.8				

Table 2 lists the values that describe the two segments for all objectives. For the fuzzy goal objectives, it is assumed that the shapes of the extended membership functions remain the same for all modeling periods. Hereby, $\Delta z_{k,s}$ is the difference between the reservation and aspiration points defined by the user. The reservation and aspiration levels that are depicted in Table 3 can be worse or better than the Nadir and Utopia point components given in Table 1.

Table 3: User-defined aspiration and reservation points

	2000		2010		2020	
Demand	Reserv.	Aspira.	Reserv.	Aspira.	Reserv.	Aspira.
$Z^{dem}_{TRAINTR}$ [PJ]	38.0	39.5	38.0	41.0	38.0	42.0
$Z^{dem}_{TRUCKTR}$ [PJ]	700.0	1030.0	800.0	1205.0	800.0	1318.0
Z^{dem}_{CARTR} [PJ]	570.0	592.0	400.0	440.0	300.0	352.4
Z^{dem}_{SHIPTR} [PJ]	115.0	122.5	133.0	144.3	134.0	149.0
Z^{dem}_{STL2} [kt]	13000	13500	–	–	–	–
Z^{dem}_{STL3} [kt]	29360	30260	31400	32400	31440	32540

Costs	Aspiration	Reservation
Z_1^{costs,T_0} $[10^8 \cdot DM^{90}]$	4.345E+4	6.000E+4
Z_2^{costs,T_0} $[10^8 \cdot DM^{90}]$	4.415E+4	6.070E+4
Z_3^{costs,T_0} $[10^8 \cdot DM^{90}]$	4.415E+4	6.080E+4
Z_4^{costs,T_0} $[10^8 \cdot DM^{90}]$	4.436E+4	6.100E+4

For the goal objectives, the reservation and aspiration levels refer to the left-hand shape function only, because this shape function represents the original character of the constraints. The graphical representation of a complete fuzzy goal criterion is shown in Figure 14.

Hierarchical Goal Aggregation

It was assumed that four of the sets H_c contain only one of the characteristic fuzzy cost values each. Two additional sets were used to aggregate the objectives that describe the coverage of the demand in the transportation sector and in the iron and steel sector. This means that the 20 criteria contained in set K are aggregated to just 6 criteria described in set C. Each of these criteria models the minimal achievement over all of the criteria contained in H_c. The individual sets H_c are listed below:

$$
\begin{aligned}
H_{Z_1^{costs,T_0}} &= \{\eta(Z_1^{costs,T_0})\}; \\
H_{Z_2^{costs,T_0}} &= \{\eta(Z_2^{costs,T_0})\}; \\
H_{Z_3^{costs,T_0}} &= \{\eta(Z_3^{costs,T_0})\}; \\
H_{Z_4^{costs,T_0}} &= \{\eta(Z_4^{costs,T_0})\}; \\
H_{Z^{dem}_{trans}} &= \{\min\{\eta(Z^{dem}_{co}(d_1)),\ \eta(Z^{dem}_{co}(d_2))\}\ \mid co \in CO^{out}_{trans}\}; \\
H_{Z^{dem}_{iron}} &= \{\min\{\eta(Z^{dem}_{co}(d_1)),\ \eta(Z^{dem}_{co}(d_2))\}\ \mid co \in CO^{out}_{iron}\}.
\end{aligned}
$$

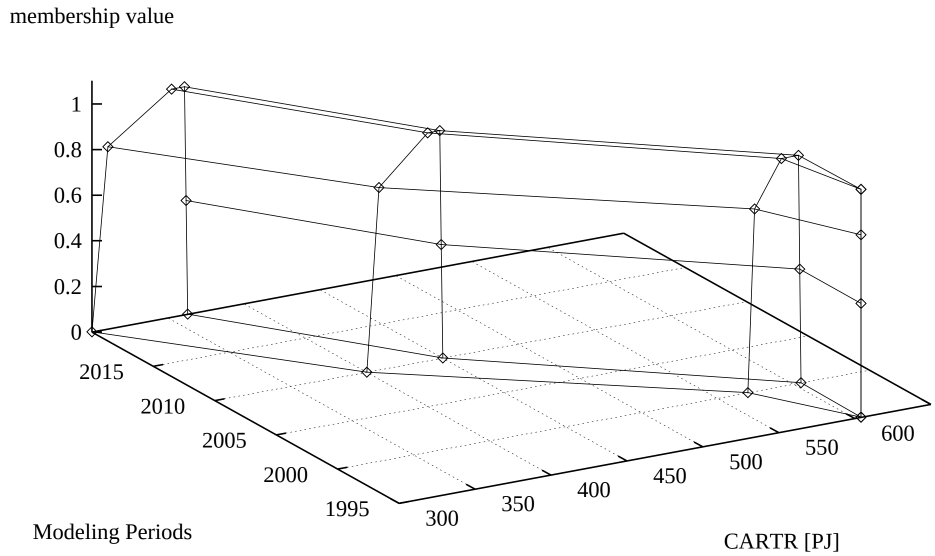

Figure 14: User-defined membership function describing the coverage of the energy service "transportation by car."

5.3 Results

The results presented in this section are only discussed in terms of the criterion space. First, the compromise (or first neutral) solution is shown; an example for the results of the interactive procedure is then given.

Compromise Solution

The compromise solution in Figure 15 shows the effects of the parameter δ on the trade-off behavior of the algorithm. Lines marked by squares represent the individual membership values for all 20 objectives; lines marked with circles indicate the minimum membership value within the classes of aggregated criteria (for the compromise solution, these lines overlap). For the entirely "symmetrical" problem that is used for the determination of the compromise solution, membership values of 0.5 can be expected for all objectives. However, in the solution on the left (using $\delta = 0.6$), the membership value for the coverage of the demands in the iron and steel sector equals 1, whereas the degree of satisfaction with all other objectives equals 0.498. The expected result was only achieved in the solution shown on the right. To explain this behavior, it is necessary to know that in the sample case study the coverage of the demand in the iron and steel sector has a minor impact on the four cost objectives (which are among the objectives with minimum membership values of 0.498). The identification of a solution with $\eta = 0.5$ for all objectives therefore required a large δ that allowed the generation of solutions with correspondingly large trade-off coefficients. For the example shown, it was necessary to have trade-off coefficients of more than $1/((2 \cdot 10^{-3})/0.5) = 250$ in order to generate the expected result. The improvement of the minimum value from 0.498 to 0.5 for all other objectives was achieved by using $\delta = 0.99$, for which the prior bound on the trade-off coefficients is 595.

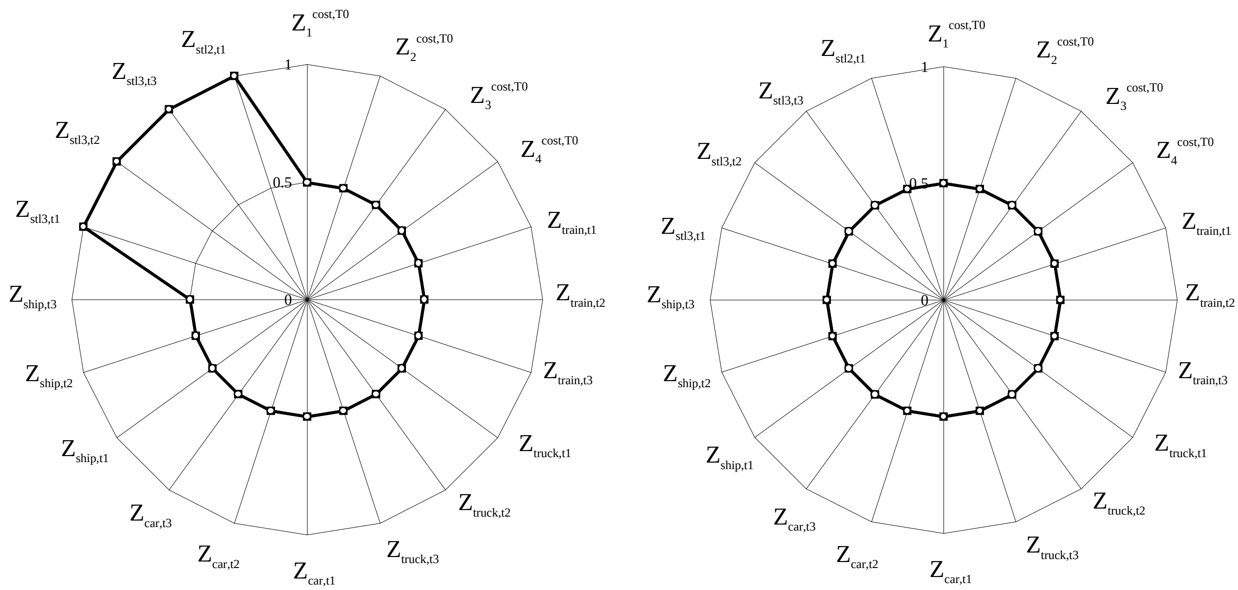

Figure 15: Neutral solutions (left: $\delta = 0.6$, right: $\delta = 0.99$).

Results Including User Preferences

The results for the user-defined membership functions are shown in Figure 16. Again, the effect of an increased value of δ can be seen from the polar graphs. In general, it can be said that an increased value of δ leads to a more uniform distribution of the results for the hierarchically aggregated values (marked by circles). Also, none of the resulting membership values was smaller than zero for the given user preferences, so that the results were entirely consistent with the fuzzy set theory.

6 Final Remarks

This paper aimed at evaluating the methodology of FLP with respect to the support that it can give to the decision-making process in energy system planning under uncertainty. In this context, the discussion was extended to the use of FLP as the underlying methodology for a DSS. The problems of FLP were addressed by known methods from the aspiration-reservation based decision support (ARBDS) methodology, and the modified FLP approach (implemented in GAMS) was applied to a sample case study.

The results identified with the proposed methodology can be interpreted as the outcome of a fuzzy LP model if the conditions given in Section 4 are fulfilled. If these conditions are not fulfilled, the results are not fully consistent with the fuzzy set theory. However, the described methodology defines easy-to-test conditions under which conformity with the fuzzy set theory is achieved.

The interpretation of the results – assuming consistency with the fuzzy set theory – is that the outcomes of the LP programs now explicitly incorporate the uncertainty associated with the coefficients of the objective function and the right-hand side of the restrictions. Because the outcome of the modeling process is determined in an interactive procedure, it can be assumed that it also includes the decision maker's preference

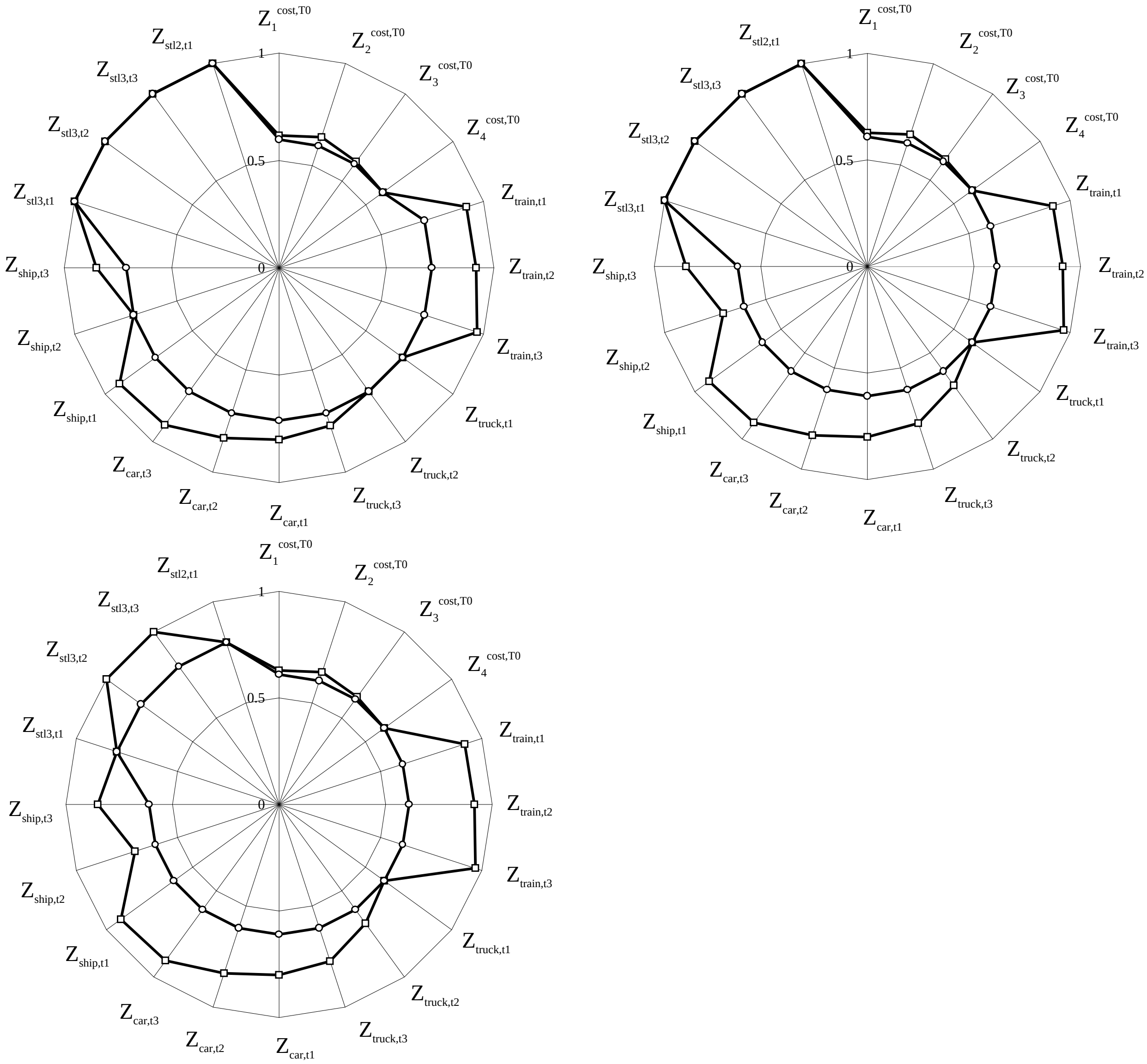

Figure 16: Results of the interactive procedure (top left: $\delta = 0.1$, top right: $\delta = 0.6$, bottom: $\delta = 0.99$).

structure in an uncertain environment. In this context, the use of a modified (continuously controllable) Fuzzy-AND aggregator, permits modeling of the decision maker's varying preference structures for different scenario environments. Furthermore, the approach allows the integration of various other kinds of uncertainties (e.g.,in the constraint coefficient matrix **A**) via fuzzy sets. Because within scenarios parameter variations are no longer required to estimate the effects of uncertain inputs, the fuzzy model alleviates undesireable flip-flop effects, or *penny switching*. However, *penny switching* can still occur across multiple scenarios.

Further work should be directed toward the development of a consistent scenario structure. Among the many possible questions arising in this context, a prominent problem seems to be the discussion of whether the fuzzy sets describing the input values must overlap across the scenarios. Another point of interest might be the improvement of the hierarchical structure proposed in this work. Although the proposed hierarchical structure helps the user to select the group of criteria to be improved, further work should focus on the control exercised over the criteria. With respect to this problem, it is necessary to discuss whether changing one membership function should update all other related criteria in the same group.

Finally, models can be conceived that employ FLP with fuzzy decision variables. An appealing aspect of this idea is that the increasing uncertainty with respect to the future could also be reflected in the fuzzy outcome of the decision process. Preliminary work regarding the integration in energy system models and a possible interpretation of the results were presented in [Can96].

7 Acknowledgments

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References

[Beh83] F. Behringer, *A simplex based algorithm for the lexicographically extended linear maxmin problem*, European Journal of Operational Research **7** (1983) 247–283.

[Can96] T. Canz, *On the application of fuzzy linear programming in energy system planning*, Conference Proceedings, European Workshop on Fuzzy Decision Analysis for Management, Planning and Optimization (1996) 66–73.

[CiT86] M. Civanlar and H. Trussel, *Constructing membership functions using statistical data*, Fuzzy Sets and Systems **18** (1986) 1–13.

[DuP80a] D. Dubois and H. Prade, *Fuzzy Sets and Systems, Theory and Applications*, Academic Press, London, 1980.

[DuP80b] ______, *Systems of linear fuzzy constraints*, Fuzzy Sets and Systems **3** (1980) 37–48.

[DuP83] ______, *Unfair coins and necessity measures: Towards a possibilistic interpretation of histograms*, Fuzzy Sets and Systems **10** (1983) 15–20.

[Erd80] G. Erdmann, *Energieökonomik*, Teubner, Stuttgart, 1980.

[Ger89] H. Gerking, *Die Ermittlung flexibler Entscheidungselemente in der strategischen Langfristplanung in der Energiewirtschaft – Dissertation*, Universität Stuttgart, Stuttgart, 1989.

[GGM95] A. Golodnikov, A. Gritsevskii and S. Messner, *A stochastic version of the dynamic linear programming model MESSAGE III*, Working Paper WP-95-94, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1995.

[GrM95] J. Granat and M. Makowski, *SAP–modular tool for specification and analysis of user preferences in muliple–criteria model analysis*, Working Paper WP-95-73, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1995.

[GrW94] J. Granat and A. Wierzbicki, *Interactive specification of DSS user preferences in terms of fuzzy sets*, Working Paper WP-94-24, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1994.

[HeK85] B. Heshmaty and A. Kandel, *Fuzzy linear regression and its application to forecasting in uncertain environments*, Fuzzy Sets and Systems **15** (1985) 159–191.

[IsS87] H. Isermann and R. Steuer, *Computational experience concerning payoff tables and minimum criterion values over the efficient set*, European Journal of Operational Research **33** (1987) 91–95.

[Küh96] R. Kühner, *Ein verallgemeinertes Schema zur Bildung mathematischer Energieplanungsmodelle*, Conference Proceedings, DGOR **1** (1996) 167.

[Luh82] M. Luhandjula, *Compensatory operators in fuzzy linear programming with multiple objectives*, Fuzzy Sets and Systems **8** (1982) 245–252.

[Mak94] M. Makowski, *Methodology and a modular tool for multiple criteria analysis of LP models*, Working Paper WP-94-102, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1994.

[MoH90] G. Morgan and M. Henrion, *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*, Cambridge University Press, New York, 1990.

[Ode94] C. Oder, *Entwicklung eines auf der Theorie der unscharfen Mengen basierenden Energie-Emissions-Modells – Dissertation*, Universität Karlsruhe, Karlsruhe, 1994.

[OHR93] C. Oder, H.-D. Haasis and O. Rentz, *Analysis of the Lithuanian final energy consumption using fuzzy sets*, The International Journal of Energy Research **17**, no. 1 (1993) 245–252.

[RaR85] J. Ramik and J. Rimanek, *Inequality relation between fuzzy numbers and its use in fuzzy optimization*, Fuzzy Sets and Systems **16** (1985) 123–138.

[Rom88] H. Rommelfanger, *Entscheiden bei Unschärfe*, Springer, Berlin, 1988.

[Sac78] L. Sachs, *Angewandte Statistik*, Springer, Berlin, 1978.

[Sak93] M. Sakawa, *Fuzzy Sets and Interactive Multiobjective Decision Making*, Plenum Press, New York, 1993.

[SaP92] D. Savic and W. Pedrycz, *Fuzzy linear regression models: Construction and evaluation*, in Fuzzy Regression Analysis, J. Kacprzyk and M. Fedrizzi, eds., Physica–Verlag, Heidelberg, 1992, pp. 91–101.

[SaY86] M. Sakawa and H. Yano, *Interactive fuzzy decision making for multiobjective nonlinear programming using augmented minimax problems*, Fuzzy Sets and Systems **20** (1986) 31–43.

[SaY92] ______, *Fuzzy linear regression and its applications*, in Fuzzy Regression Analysis, J. Kacprzyk and M. Fedrizzi, eds., Physica–Verlag, Heidelberg, 1992, pp. 61–81.

[Slo86] R. Slowinski, *A multicriteria fuzzy linear programming method for water supply system development planning*, Fuzzy Sets and Systems **19** (1986) 217–237.

[TaI92] H. Tanaka and H. Isibuchi, *Possibilistic regression based on linear programming*, in Fuzzy Regression Analysis, J. Kacprzyk and M. Fedrizzi, eds., Physica–Verlag, Heidelberg, 1992, pp. 47–61.

[TUA80] H. Tanaka, S. Uejima and K. Asai, *Fuzzy linear regression model*, IEEE Transactions on Systems, Man, and Cybernetics **10** (1980) 2933–2938.

[TUA82] ______, *Linear regression analysis with fuzzy models*, IEEE Transactions on on Systems, Man, and Cybernetics **12** (1982) 903–907.

[Van90] D. Vanderpooten, *Multiobjective programming: Basic concepts and approaches*, in Stochastic Versus Fuzzy Approaches to Multiobjective Mathematical Programming under Uncertainty, R. Slowinski and J. Teghem, eds., Kluwer Academic Publishers, Dordrecht, Netherlands, 1990, pp. 7–22.

[Wat92] J. Watanda, *Fuzzy time series analysis and forecasting of sales volume*, in Fuzzy Regression Analysis, J. Kacprzyk and M. Fedrizzi, eds., Physica–Verlag, Heidelberg, 1992, pp. 211–217.

[Wer87a] B. Werners, *Interactive Entscheidungsunterstützung durch ein flexibles mathematisches Programmierungssystem – Dissertation*, Technische Hochschule Aachen, Aachen, 1987.

[Wer87b] ______, *Interactive multiple objective programming subject to flexible constraints*, European Journal of Operational Research **31** (1987) 342–349.

[Wie90] A. Wierzbicki, *Multiple criteria solutions in noncooperative game theory, part III theoretical foundations*, Discussion Paper, Kyoto Institute of Economic Research (1990).

[Wie92] ______, *Multiple criteria games – theory and applications*, Working Paper WP-92-079, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1992.

[WiM92] A. Wierzbicki and M. Makowski, *Multi-objective optimization in negotiation support*, Working Paper WP-92-007, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1992.

[Zad78] L. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems **1** (1978) 3–28.

[Zim78] H.-J. Zimmermann, *Fuzzy programming and linear programming with several objectives*, Fuzzy Sets and Systems **1** (1978) 45–55.

[Zim87] ______, *Fuzzy Sets, Decision Making and Expert Systems*, Kluwer Academic Publishers, Dordrecht, 1987.

[Zim91] ______, *Fuzzy Set Theory and Its Applications*, Kluwer Academic Publishers, Dordrecht, 1991.

A Notation

A.1 Notation: Sets and Indices

i	$\in$	$\{1, ..., m\}$	index of fuzzy inequality constraints with $m = m_1 + m_2$
j	$\in$	$\{1, ..., n\}$	index of decision variables in fuzzy LP formulations
k	$\in$	$\{1, ..., p\}$	index of criteria
l	$\in$	$\{1, ..., p\}$	index of criteria
s	$\in$	S	index of segments for piece-wise linear membership functions μ or extended linear membership functions η
K			set of all criteria k (for hierarchical aggregation)
c	$\in$	C	index of hierarchically aggregated criteria
h	$\in$	H_c	index of criteria within sets H_c
H_c			set of aggregated criteria without compensation
X			decision space $\Re^n$ with X_D set of admissible solutions
Y			outcome space with outcome criteria $\mathbf{y}$; attainable outcome space Y_0
Z			objective space with restricted set of criteria $\mathbf{z} \in \Re^p$ and set of attainable objectives Z_0

A.2 Notation: Simplified Energy System Model

X	set of all decision variables with $X \supseteq X^{in} \cup X^{out}$
X^{in}	inflows into processes, where $X^{in} = X^{in,is} \cup X^{in,it}$, with $X^{in,is}$ representing interspatial inflows and $X^{in,it}$ representing intertemporal inflows
X^{out}	outflows from processes, where $X^{out} = X^{out,is} \cup X^{out,it}$, with $X^{out,is}$ representing interspatial outflows and $X^{out,it}$ representing intertemporal outflows
Z^{costs,T_0}	decision criterion: total costs in base year T_0
Z^{dem}_{co}	decision criterion: coverage of the demand of a commodity co
CO	set of all commodities co, co', $CO \supseteq CO^{in} \cup CO^{out}$
CO^{in}	commodities that have emerging flows into processes
CO^{out}	commodities that have incoming flows from processes
PR	set of all processes pr, pr', $PR \supseteq PR^{in} \cup PR^{out}$
PR^{in}	processes that have incoming flows
PR^{out}	processes that have flows emerging toward the commodities
B	set of bounds imposed on commodities and processes $B = B^{up}_{co} \cup B^{up}_{pr} \cup B^{lo}_{co} \cup B^{lo}_{pr}$
$A_{co,pr}$	set of all parameters
D_{co}	demand for a commodity co
R_{co}	available resources of a commodity co
T	set of all modeling periods t with base year T_0

A.3 Notation: Miscellaneous

r	number of fuzzy objective functions
r_k^l	aspiration level for goal-type maximizing objective k
r_k^u	aspiration level for goal-type minimizing objective k
m_c	cardinality of set C describing sets of hierarchically aggregated data
m_k	cardinality of set K containing all objectives
m_1	number of fuzzy restrictions that are replaced by an additional objective function
m_2	number of fuzzy restrictions that are replaced without using an additional objective function