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The Emergence of Network Organizations in Processes of Technological Choice: a Viability Approach

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Introduction

Traditionally, the analysis of emerging structures (technologies, conventions, etc.) starts from the specification of micro-behaviors through local rules (such as "agent i chooses a technology x if the majority of agents has chosen this technology") in order to study the macroscopic evolution of the system. In this context, the network organization is given. The general purpose of this approach is to derive the collective consequences which cannot be extrapolated from any kind of representative individual behavior. See for instance [12, David, Foray & Dalle] and its bibliography.

In this paper, we shall follow the opposite track: instead of deriving some indeterminism from deterministic system, we rather detect some regularities from indeterministic micro-mechanisms.

In other words, our goal is to allow the system to discover all the network structures — described by influence matrices among the agents — which are dictated by the state of the system whenever viability constraints are imposed on the system. To fix the ideas, we divide the constraints into two classes: the first one includes individual constraints while the second one describes interacting proximity effects through a loss function decreasing along the solutions.

Once the network structures known, we shall provides two classes of selection mechanisms of network structures, one mechanism being of a static nature, and the other one, dynamic.

The dynamic mechanism provides the variations of the influence matrices. Among them, we shall choose the one with maximal inertia, called "heavy evolution". We provide a definition of an "organizational niche", which is a network structure which regulates a nonempty set of states. We prove that heavy evolutions enjoy the property of locking any network organizational niche as soon as a trajectory enters it.

In order to avoid mathematical technicalities, we just describe the model we propose and state the results which can be obtained without specifying mathematical assumptions nor the exact formulas, which go beyond this introduction to the viability approach to the analysis of emerging structures. We refer to [4, Aubin] for more informations on these mathematical techniques.

1 The Network Constraints

We start the description of the model with

1. n firms, labelled $i = 1, \ldots, n$,

2. a finite dimensional vector space Y of technologies described by $x \in Y$. We denote by $X := Y^n$ the space of technological configurations $x := (x_1, \ldots, x_n)$ implemented by the n firms. A technological configuration $x := (x_1, \ldots, x_n)$ is an assignment of a technology x_j to each firm j.

The constraints are defined at

1. *the level of individual agents*, by cost or loss functions $h_i : Y \mapsto Z_i$ of the form
$$\{x_i \in Y \mid h_i(x_i) \in M_i \subset Z_i\}$$
describing the individual constraints of the firm.

2. *the level of agent interactions*, through a "proximity" function $h_0 : X \mapsto Z_0$. These proximity effects are due to Marshallian externalities that affect for example the costs of screening and hiring workers: Concretely, we postulate that for every firm, the relative wage costs of a worker of a given technological type is decreasing if the number

of workers of that type currently employed by the ensemble of firms in the immediate neighborhood of that firm is increasing (see [12, David, Foray & Dalle] for a modelisation of a system with marshallian externalities using stochastic Ising models).

For instance, one can take $Z_0 := \mathbf{R}^n$, so that the jth component $h_{0,j}(x)$ describes the cost for firm j of the technological configuration $x := (x_1, \ldots, x_n)$.

In summary, a viable evolution of technological configurations $t \mapsto x(t)$ is a time-dependent technological configuration satisfying

$$\begin{cases} i) & \forall\, t \geq 0, \ \forall\, i = 1, \ldots, n, \ \ h_i(x_i(t)) \in M_i \\ ii) & \forall\, t \geq 0, \ \ h_0(x(t)) \ \leq \ h_0(x(0))e^{-at} \end{cases}$$

so that the technological configuration should decrease exponentially to the maximal proximity $\overline{x}$ satisfying $h_0(\overline{x}) = 0$.

2 Influence Matrices Describing Network Organization

We further assume known the dynamical behavior of each firm j independently of the one of the other firms: It modifies the state of technology $x_i(t)$ at time t according to the differential equation

$$\forall\, i = 1, \ldots n, \ \ x_i'(t) \ = \ g_i(x_i(t)) \tag{1}$$

This is the dynamical analogue of the classical static description of the behavior of agents through utility or cost functions.

Now we assume that, due to the interaction constraints (the Marshallian effect), the solutions to the decentralized system (1) do not necessarily satisfy the above constraints and satisfycing property.

Therefore, some regulation mechanism should be designed. We propose to investigate a network organization described through a graph matrix W of influence weights w_i^j of firm j on firm i. The case when $w_i^j = 0$ describes the situation where firm i does not take into consideration the behavior of firm j. When $w_i^j > 0$, firm i displays an apish behavior towards firm j. The case when $w_i^j < 0$ denotes an antithetical behavior of firm i toward firm j.

We underline that these weights are not *a priori* predetermined probabilities of interactions. Our aim is precisely to let them emerge *a posteriori* from the confrontation of the dynamics and the constraints.

In this context, a **network organization** is described by an influence matrix (which, by the way, can be regarded as the matrix of a graph)[1].

The firms modify their autonomous dynamical behavior by integrating the behavior of the other firms through their influence weights. We choose for simplicity a linear interaction of the form

$$\forall\, i = 1, \ldots n, \;\; x_i'(t) \;=\; \sum_{j=1}^{n} w_i^j(t) g_j(x_j(t)) \tag{2}$$

In a more compact form, it can be written in the form

$$x'(t) \;=\; W(t)g(x(t)) \tag{3}$$

where $W(t)$ denotes the time-dependent influence matrix.

3 Organizational Niches

One can impose a given network organization, described by a given influence matrix W, and study the properties of the dynamical system:

$$x'(t) \;=\; Wg(x(t))$$

For instance, we can look for the set $E(W)$ of equilibria $\overline{x}$ of the above systems, solutions to the equation $Wg(\overline{x}) = 0$, their stability property, their basin of attraction, and the dependence of these items with respect to W, using for instance bifurcation theory.

We shall not follow this course in this paper. However, for studying later lock-in properties, we introduce the concept of "organizational niche"

[1]The mathematical techniques used in this study have been devised in [3, Aubin] in the framework of neural networks and cognitive systems. They have been adapted to an economist context in [4, Aubin] in the framework of "connectionist complexity".

$N(W)$ of the influence matrix W: It is the viability kernel of the differential equation $x'(t) = Wg(x(t))$, i.e., the largest subset of states satisfying the constraints which is viable under the differential equation $x'(t) = Wg(x(t))$. It is also equal to the set of initial states x_0 from which the solution to differential equation $x'(t) = Wg(x(t))$ is viable.

In other words, starting from a state in the organizational niche, the solution to the system organized according the influence matrix W satisfies the above constraints forever.

4 How Network Organization Evolves

But *one can reverse the questioning* and, instead of studying the properties of a given network organization, *look for all network organizations compatible with the constraints* in the following sense: find (time-dependent) influence matrices $W(t)$ such that, from any initial state satisfying the constraints, there exists a viable solution $(x(\cdot), W(\cdot)$ to the parametrized differential equation

$$x'(t) \; = \; W(t)g(x(t)) \tag{4}$$

i.e., a solution such that $x(t)$ satisfies the constraints for ever.

The basic viability theorem (see for instance [2, Aubin]) applied to this situation provides the feedback map R associating with each technological configuration x a subset $R(x)$ of influence matrices W. *The system is viable if and only if $R(x)$ is not empty for every technological configuration x satisfying the constraints. In this case, the evolution of viable solutions $x(t)$ obeys the* regulation law

$$\forall\, t \geq 0, \;\; W(t) \; \in \; R(x(t)) \tag{5}$$

In other words, the feedback map R assigns to every technological configuration the set of network organizations "viable" with respect to the constraints.

In the favorable case, the set $R(x)$ of viable influence matrices may contain more than one matrix. Actually, the larger this set, the more robust, since it allows for errors.

So, the question of selecting influence matrices arises, and many scenarii can be considered.

We shall describe two prototypes of selection mechanisms, one "static", and the other one, "dynamic".

5 Minimizing a Static Complexity Index

The static one involves a complexity index[2] of a network organization described by an influence matrix W. It is defined by the distance between the influence matrix W and the unit matrix $\mathbf{1}$, which describes the decentralized situation. The idea is to regard the decentralized situation as the simplest one, and thus, to regard a network organization as complex as it is far from this simplest situation. One can then compute for each x the viable matrix $W^0 \in R(x)$ which is the closest to the unit matrix — hence the simplest — and to show that despite its lack of continuity, a solution to the differential equation

$$x'(t) = W^0(x(t))g(x(t))$$

still exists.

6 Minimizing a Dynamic Complexity Index

The dynamical one consists in differentiating the regulation law. Appealing to set-valued analysis (see for instance [6, Aubin & Frankowska]), one can

[2] Physicists have attempted to measure "complexity" in various ways, through the concept of Clausius's entropy, Shannon's information, the degree of regularity instead of randomness, "hierarchical complexity" in the display of level of interactions, "grammatical complexity" measuring the language to describe it, temporal or spatial computational, measuring the computer time or the amount of computer memory needed to describe a system, etc.

One can also measure other features of connectionist complexity through the sparsity of the connection matrix, i.e., the number — or the position — of entries which are equal to zero or "small". The sparser such a connection matrix, the less complex the system.

Each component of a system which can evolve independently in the absence of constraints, must interact each other in order to maintain the viability of the system imposed by its environment. Is not complexity meaning in the day-to-day language the labyrinth of connections between the components of a living organism or organization or system ? Is not the purpose of complexity to sustain the constraints set by the environment and its growth parallel to the increase of the web of constraints ?

derive from the regulation law (5) a differential inclusion of the form

$$W'(t) \in R'(x(t), W(t)) \tag{6}$$

which, together with the original system (4), specifies the evolution of both the technological configurations $x(t)$ and the influence matrix $W(t)$.

One can regard a norm $\|W'(t)\|$ of the velocity $W'(t)$ of the influence matrix as a **dynamical complexity index**. The larger this dynamical complexity index, the fastest the connectionist complexification of the network organization. Hence, the question arises to select the velocity $v^0(x, W)$ with minimal norm in the subset $R'(x, W)$ of viable velocities of the influence matrices. One can prove that the system of differential equations

$$\begin{cases} i) & x'(t) = W(t)g(x(t)) \\ ii) & W'(t) = v^0(x(t), W(t)) \end{cases}$$

has solutions $(x(t), W(t))$, which are naturally viable. They are called "heavy solutions".

7 The Lock-In Property

"Heavy solutions" have the property of locking-in organizational niches: If, for some time T, the solution enters a organizational niche, i.e., if $x(T) \in N(W(T))$, then for all $t \geq T$, the technological configurations can be regulated by the constant influence matrix $W(T)$, i.e., according to the differential equation

$$\forall\, t \geq T,\;\; x'(t) = W(T)g(x(t))$$

and the solution will remain in the organizational niche of $W(T)$:

$$\forall\, t \geq T,\;\; x'(t) \in N(W(T))$$

This is another metaphor of the lock-in property of organizational niches.

8 Conclusions

In this paper, we have tried to propose a modeling strategy, the purpose of which is to allow a dynamic system of technological choices to discover and select the network structures (that is to say the particular influence matrix characterizing the set of interacting agents), which are compatible with the viability constraints generated by a particular technological configuration. As very well described in [8, Cohendet], the stream of works focussing on emergent structures within the context of stochastic interactions among agents continuously increases the complexity which is assigned to the micro-behaviors, in order to describe more complex trajectories of macroscopic evolution. Agents are allowed to deviate from the normative rules ([9, Dalle]; percolation probabilities are introduced in order to allow some subsystems to keep isolated and so not infected by the choices of the majority ([11, David & Foray]); a super-agent, providing to each one the same information, can be considered ([9, Dalle]; the parameter describing the strength of interactions can be changed ([12, David, Foray & Dalle]); last but not least some kinds of learning capacities are attributed to the agents, allowing them to adjust their behaviors with respect to what they learn in the course of their recurrent decisions ([1, Arthur]). Of course, we want not to claim that such exercises are evolving towards a deadlock. However, such a complexity increase on the side of individual and collective behaviors by no means allow this approach to escape from a deterministic logic: each decision center or agent possess ex ante a program of actions/reactions, certainly complex but ultimately invariable along the life of the collective system (see [10, Dalle & Foray], for a discussion of the status of individual rationality in stochastic models of interactions).

In this paper, we have proposed a clearly opposed vision. The individual programs of actions/reactions are not known ex ante. They are rather the object of inquiries, the emerging and lock-in structures, derived from the viability constraints and the selection mechanisms. This paper only provides a first step of this research program which will be continued in the near future.

References

[1] ARTHUR B. (1990) *On designing economic agents that behave like human agents*, Journal of Evolutionary Economics

[2] AUBIN J.-P. (1991) VIABILITY THEORY Birkhäuser, Boston, Basel

[3] AUBIN J.-P. (1996) NEURAL NETWORKS AND QUALITATIVE PHYSICS: A VIABILITY APPROACH CAMBRIDGE, University Press, Cambridge

[4] AUBIN J.-P. (1997) DYNAMIC ECONOMIC THEORY: A VIABILITY APPROACH, Springer-Verlag

[5] AUBIN J.-P. (1997) MUTATIONAL AND MORPHOLOGICAL ANALYSIS: TOOLS FOR SHAPE REGULATION AND OPTIMIZATION, Birkhäuser, Boston, Basel

[6] AUBIN J.-P. & FRANKOWSKA H. (1990) SET-VALUED ANALYSIS, Birkhäuser, Boston, Basel

[7] CARTELIER J. & MULLERS K. (1994) *An elementary Keynesian model: A preliminary approach*, IIASA WP 94-095

[8] COHENDET, P. (1995) *Networks, diffusion of technologies and irreversibilities*, Annals of telecommunications, 50(2)

[9] DALLE J.-M. (1995) *Dynamiques d'adoption, coordination et diversité: la diffusion des standards technologiques*, Revue Economique, 46.

[10] DALLE, J.M. & FORAY, D. (1995) *Des fourmis et des hommes: modèles stochastiques d'interactions et rationalité individuelle active en économie du changement technique*, Cahiers d'économie et sociologie rurales, 37

[11] DAVID P. & FORAY D. (1994) *Percolation structures, Markov random fields and the economics of EDI standard diffusion*, MERIT Working Paper

[12] DAVID P., FORAY D. & DALLE J.-M. (1996) *Marshallian externalities and the emergence and spatial stability of technological enclaves*, Economics of Innovation and New Technologies