Working Paper

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WP-96-68

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IIASA International Institute for Applied Systems Analysis □ A-2361 Laxenburg □ Austria

Telephone: +43 2236 807 □ Fax: +43 2236 71313 □ E-Mail: info@iiasa.ac.at

LOVE DYNAMICS: THE CASE OF LINEAR COUPLES

Sergio Rinaldi

Centro Teoria dei Sistemi, CNR, Politecnico di Milano, Milano, Italy

Running title: Linear love dynamics

Address for correspondence

Professor Sergio Rinaldi	phone - 39.2.23993563
Centro Teoria dei Sistemi, CNR	fax - 39.2.23993412
Politecnico di Milano	e-mail rinaldi @ elet.polimi.it
Via Ponzio 34/5	
20133 Milano, Italy	

Abstract

This paper proposes a minimal model composed of two ordinary differential equations to describe the dynamics of love between two individuals. The equations take into account three mechanisms of love growth and decay: the pleasure of being loved (return), the reaction to the partner's appeal (instinct), and the forgetting processes (oblivion). Under suitable assumptions on the behavior of the individuals, the model turns out to be a positive linear system enjoying, as such, a number of remarkable properties, which are in agreement with common wisdom on the argument. These properties are used to explore the consequences that individual behavior can have on the population structure. The main result along this line is that individual appeal is the driving force that creates order in the population. Possible extensions of this theory of linear love dynamics are briefly discussed.

1. INTRODUCTION

Ever since Newton introduced differential calculus, dynamic phenomena in physics, chemistry, economics and all other sciences have been extensively studied by means of differential equations. Surprisingly, one of the most important problems concerning our lives, namely the dynamics of love between two persons, has not yet been tackled in this way. The only exception is a one page paper in which Strogatz [St88] describes how the classical topic of harmonic oscillations can be taught to capture the attention of students. He suggests to make reference to "a topic that is already on the minds of many college students: the time evolution of a love affair between two people". The model proposed by Strogatz (discussed also in [Ra93] and [St94]) is definitely unrealistic because it does not take into account the appeal of the two individuals. Thus Strogatz's model, for example, does not explain why two persons who are initially completely indifferent one to each other can develop a love affair.

The model proposed in this paper is more realistic, although it is still a *minimal model*. Three aspects of love dynamics are taken into account: the forgetting process, the pleasure of being loved, and the reaction to the appeal of the partner. These three factors are assumed to be independent and are modelled by linear functions [St88]. The resulting model is a linear dynamical system, which turns out to be *positive* if the appeals of the two individuals are positive. The theory of positive linear systems [Be79, L79, Gra87, Be89, Ri95] can therefore be applied to this model and gives quite interesting results. Some of them describe the dynamic process of falling in love, i.e. the transformation of the feelings, starting from complete indifference (when two persons first meet) and tending toward a plateau. Other results are concerned with the influence that appeal and individual behavior have on the quality of romantic relationships.

Some of these properties are used to identify the consequences that individual appeal and behavior can have on partner choice and on population structure. Although the results are extreme, they explain to some extent facts observed in real life, such as the rarity of couples composed of individuals with very uneven appeal.

The conclusion is that the proposed model, besides being a good example for capturing student attention, is also an elegant tool for deriving general properties of love dynamics from purely conceptual arguments.

2. THE MODEL

The model analyzed in this paper is a dynamic system with only two state variables, one for each member of the couple. Such variables, indicated by x_1 and x_2, are a measure of the love of individual 1 and 2 for the partner. Positive values of x represent positive feelings, ranging from friendship to passion, while negative values are associated with antagonism and disdain. Complete indifference is identified by $x = 0$.

The model is a typical minimal model. Firstly, because love is a complex mixture of different feelings (esteem, friendship, sexual satisfaction, ...) and can be hardly captured by a single variable. Secondly, because the tensions and emotions involved in the social life of a person cannot be considered in such a simple model. In other words, only the interactions between the two individuals are modelled, while the rest of the world is kept frozen and does not participate explicitly in the formation of love dynamics. This means that the present theory cannot be easily related to the well known *attachment theory* [Bo69, 73, 80], which has been a main investigation tool in adult romantic relationships in the last decade (see, for instance, [H87, Co90, Fe90, Si90, Sh92, Ki94]).

Three phenomena are considered, namely, *oblivion*, *return*, and *instinct*. The first gives rise to a loss of interest for the partner, and explains, for example, the typical decay of x_i, which takes place after the death of j, $i \neq j$. On the contrary, the second and the third are sources of interest. Moreover, the return increases with the love of the partner, while the instinct is sensitive only to appeal (physical, intellectual, financial,...).

The following simplifying assumptions are also made. First, the appeals and the personalities of the two individuals do not vary in time: this rules out aging, learning and adaptation processes which are often important over a long range of time [Ko91, Sc94]. Thus, the model can only be used for short periods of time (months/years), for example in predicting if a love story will be characterized by regular or stormy feelings. Second, synergism is assumed to be negligeable, i.e. oblivion and return depend only upon one state variable. Finally, all mechanisms are supposed to be linear. The result is the following model

$$\dot{x}_1(t) = -\alpha_1 \, x_1(t) + \beta_1 \, x_2(t) + \gamma_1 \, A_2$$

$$\dot{x}_2(t) = -\alpha_2 \, x_2(t) + \beta_2 \, x_1(t) + \gamma_2 \, A_1$$

(1)

where α_i, β_i and γ_i, as well as the appeals A_i, are constant and positive parameters. The negative term $-\alpha_i \, x_i(t)$, due to oblivion, says that the love of i, in the absence of the partner j, decays exponentially $(x_i(t) = x_i(0) \exp(-\alpha_i(t)))$. The second term $\beta_i \, x_j(t)$ is the return, and the third $\gamma_i \, A_j$ is the reaction to the partner's appeal. Thus, each person is identified by four parameters: the appeal A_i, the forgetting coefficient α_i and the reactiveness β_i and γ_i to the love and appeal of the partner. The estimate of the behavioral parameters is undoubtedly a difficult task, although some studies on

attachment styles [Ba91, Ca92, Gri94] might suggest ways for identifying categories of individuals with high or low reactivenesses or forgetting coefficients. This identification problem will not be considered in the present paper, which is only centered on the derivation of the properties of the model.

Model (1) is a linear system which can be written in the standard form $\dot{x} = Ax + bu$ with $u=1$, and

$$A = \begin{vmatrix} -\alpha_1 & \beta_1 \\ \beta_2 & -\alpha_2 \end{vmatrix} \qquad b = \begin{vmatrix} \gamma_1 A_2 \\ \gamma_2 A_1 \end{vmatrix}$$

Such a system is positive because the matrix A is a Metzler matrix (non negative off-diagonal elements) and the vector b has positive components [L79]. Thus, $x(0) \geq 0$ implies $x(t) \geq 0 \; \forall \; t$. This means that our assumptions imply that two persons will never become antagonist, because they are completely indifferent one to each other when they meet for the first time (i.e. $x(0)=0$).

Positive linear systems enjoy a number of remarkable properties, in particular if they are asymptotically stable. In the present case, the necessary and sufficient condition for asymptotic stability is

$$\beta_1 \beta_2 < \alpha_1 \alpha_2 \tag{2}$$

i.e. the system is asymptotically stable if and only if the (geometric) mean reactiveness to love ($\sqrt{\beta_1 \beta_2}$) is smaller than the (geometric) mean forgetting coefficient ($\sqrt{\alpha_1 \alpha_2}$). In the following, condition (2) is assumed to hold. When this is not the case, namely when the two individuals are quite reactive to the love of the partner, the instability of the model gives rise to unbounded feelings, a feature which is obviously unrealistic. In

that case (i.e. when $\beta_1 \beta_2 > \alpha_1 \alpha_2$) one must model the couple more carefully by assuming, for example, that the reaction function is increasing but bounded with respect to the partner's love. This extension, already proposed in [St88], is not considered in this paper.

3. PROPERTIES OF THE MODEL

We now point out five simple but interesting properties of model (1), under the assumption that condition (2) holds. Thus the system is asymptotically stable and the love of each individual is bounded. Moreover the positivity of α_1 and α_2 rules out the possibility of cyclic behavior (Bendixon's criterion [St94]), so that one can conclude that $x_i(t)$ tends toward an equilibrium value $\bar{x}_i$, which must be non-negative because the system is positive. More can be said about this equilibrium, however, as specified by the following remark.

Remark **1**

The equilibrium $\bar{x} = (\bar{x}_1, \bar{x}_2)$ of system (1) is strictly positive, i.e. $\bar{x}_i > 0$, $i=1,2$.

Proof

The proof follows immediately as a result of a general property of positive systems [M91] which states that asymptotically stable and excitable systems have strictly positive non trivial equilibria. (It may be recalled that a positive system $\dot{x} = Ax + bu$ is excitable if and only if each state variable can be made positive by applying a suitable positive input starting from $x(0) = 0$). In the present case, the system is excitable because the components of the vector b are positive.

An alternative proof consists in explicitly computing the equilibrium $\bar{x}$, which turns out to be given by

$$\bar{x}_1 = \frac{\alpha_2 \, \gamma_1 \, A_2 + \beta_1 \, \gamma_2 \, A_1}{\alpha_1 \, \alpha_2 - \beta_1 \, \beta_2} \qquad \bar{x}_2 = \frac{\alpha_1 \, \gamma_2 \, A_1 + \beta_2 \, \gamma_1 \, A_2}{\alpha_1 \, \alpha_2 - \beta_1 \, \beta_2} \tag{3}$$

∎

Thus, if two individuals meet for the first time at $t=0$ ($x(0) = 0$) they will develop positive feelings $x_i(t)$ tending toward the positive equilibrium value $\bar{x}_i$. Since positive systems have at least one real eigenvalue (the so-called Frobenius eigenvalue λ_F, which is the dominant eigenvalue of the system), a second order system cannot have complex eigenvalues, i.e., the equilibrium of system (1) cannot be a focus. In other words, the transients of $x_i(t)$ cannot be damped oscillations characterized by an infinite number of minima and maxima. But even the possibility of a single maximum (minimum) can be excluded, as specified in the following remark.

Remark 2

The function $x_i(t)$, corresponding to the initial condition $x(0) = 0$, is strictly increasing, i.e. $\dot{x}_i(t) > 0 \quad \forall t, \ i = 1,2$.

Proof

The isoclines $\dot{x}_i = 0$ are straight lines given by

$$x_2 = \frac{\alpha_1}{\beta_1} \, x_1 - \frac{\gamma_1}{\beta_1} A_2 \qquad (\dot{x}_1 = 0)$$

$$x_2 = \frac{\beta_2}{\alpha_2} \, x_1 + \frac{\gamma_2}{\alpha_2} A_1 \qquad (\dot{x}_2 = 0)$$

These isoclines (see dotted lines in Fig. 1) intersect at point E (representing the strictly positive equilibrium $\bar{x} = (\bar{x}_1, \bar{x}_2)$), thus partitioning the state space in four regions. In the region containing the origin, $\dot{x}_i > 0$, i=1,2, which proves the stated result.

■

It should be noticed that for non zero initial conditions, one of the two variables $x_i(t)$ can first decrease and then increase (see trajectory $A\,E$ in Fig. 1) or viceversa (see trajectory $B\,E$). This can be easily interpreted as follows. Suppose, that a couple is at equilibrium and that individual 2 has, for some reason, a sudden drop in interest for the partner. The consequence (see trajectory $A\,E$ in Fig. 1) is that individual 1 will suffer during the whole transient bringing the couple back to equilibrium.

Moreover, for very particular initial conditions (straight trajectories in Fig. 1) the two functions $(x_i(t)\text{-}\bar{x}_i)$, i=1,2 decay exponentially at the same rate (equal to an eigenvalue of A). The slowest decay occurs along a trajectory which has a positive inclination and is identified by the dominant eigenvector. On the contrary, the fastest decay occurs along the other straight trajectory which has a negative inclination. The result is a direct consequence of the well-known Frobenius theory [Fr12] which says that in a positive and irreducible system the dominant eigenvector is strictly positive and there are no other positive eigenvectors (it may be recalled that a system is irreducible when it cannot be decomposed in the cascade or parallel connection of two subsystems, a property which is guaranteed in the present case by $\beta_1\beta_2 > 0$). Applied to a second order system, the *Frobenius theory* states that the dominant eigenvector has components with the same sign, while the other eigenvector has components of opposite sign.

We can now focus on the influence of various parameters on the equilibrium and dynamics of the system, starting with the reactiveness to love and appeal.

Remark 3

An increase of the reactiveness to love [appeal] β_i [γ_i] of individual i gives rise to an increase of the love of both individuals at equilibrium. Moreover, the relative increase $\Delta x / \bar{x}$ is higher for individual i.

Proof

The result can be obtained directly from (3) by deriving $\bar{x}_i$ with respect to β_i [γ_i] and then dividing by $\bar{x}_i$. Condition (2), of course, must be taken into account.

Nevertheless, this derivation is not needed. Indeed, the first part of the remark is a direct consequence of the famous *law of comparative dynamics* [L79]. This law states that in a positive system the increase of a positive parameter gives rise to an increase of the components of the state vector at any time, and hence also at equilibrium. The second part of the remark is the consequence of a general theorem concerning positive systems, known as *theorem of maximum relative variation* [Ri95]. Such a theorem states that if the i-th component of the vector b or one element of the i-th row of the matrix A of an asymptotically stable and excitable positive system is slightly increased (in such a way that the system remains asymptotically stable and excitable), the i-th component $\bar{x}_i$ of the state vector at equilibrium is the most sensitive of all in relative terms.

■

The following, somehow intriguing, remark specifies the influence of appeal on the equilibrium.

Remark 4

An increase of the appeal A_i of individual i gives rise to an increase of the love of both individuals at equilibrium. Moreover, the relative increase $\Delta x / \bar{x}$ is higher for the partner

of individual i.

Proof

The proof results from Remark 3, with notice to eq. (3) in which γ_i is multiplied by A_j,

$i \neq j$.

As in the proof of Remark 3, we can notice that the result is the direct consequence of the law of comparative dynamics and of the theorem of maximum relative variation, because A_1 [A_2] enters only in the second [first] state equation.

■

The last is a remark concerning the influence of the reactiveness to love on the dynamics of the system.

Remark 5

An increase of the reactivenesses to love gives rise to an increase of the dominant time constant of the system, which tends to infinity when $\beta_1 \beta_2$ approaches $\alpha_1 \alpha_2$. On the contrary, the other time constant decreases and tends to $1/(\alpha_1+\alpha_2)$.

Proof

Since the trace of the matrix A is equal to $-(\alpha_1+\alpha_2)$, the sum of the two eigenvalues remains constant and equal to $-(\alpha_1+\alpha_2)$ when β_i varies. On the other hand, the two eigenvalues remain real (because the system is a second order positive system) and one of them (the dominant one λ_F) tends to zero because the system looses stability when β_1 β_2 appoaches $\alpha_1 \alpha_2$. This means that the dominant time constant $T'=-1/\lambda_F$ increases with β_i and tends to infinity when $\beta_1 \beta_2$ tends to $\alpha_1\alpha_2$. On the contrary, the other time constant T'' decreases, because $1/T'+1/T''=\alpha_1+\alpha_2$. For $\beta_1 \beta_2$ tending to $\alpha_1 \alpha_2$ the time

constant T'' tends to $1/(\alpha_1+\alpha_2)$ because T' tends to infinity.

■

The above five remarks can be easily interpreted. The first states that individuals with positive appeal are capable of establishing a steady romantic relationship. The emotional pattern of two persons falling in love is very regular - beginning with complete indifference, then growing continuously until a plateau is reached (Remark 2). The level of passion characterizing this plateau is higher in couples with higher reactiveness and appeal (Remarks 3, 4). Moreover, an increase in the reactiveness of one of the two individuals is more rewarding for the same individual, while an increase of the appeal is more rewarding for the partner. Thus, there is a touch of altruism in a woman [man] who tries to improve her [his] appeal. Finally, couples with very high reactiveness respond promptly during the first phase of their romantic relationship, but are very slow in reaching their plateau (Remark 5). Together with eqs. (3), this means that there is a positive correlation between the time needed to reach the equilibrium and the final quality ($\bar{x}_1$ and $\bar{x}_2$) of the relationship. Thus, passions that develop too quickly should be expected to be associated with poor romantic relationships.

4. CONSEQUENCES AT POPULATION LEVEL

We can now try to identify the consequences, at population level, of the dynamics of love discussed in the previous section. Let us consider a population composed by N women and N man structured in N couples $[A_1^n, \alpha_1^n, \beta_1^n, \gamma_1^n; A_2^n, \alpha_2^n, \beta_2^n, \gamma_2^n]$, $n = 1, 2, \ldots, N$ and suppose that 1 is a woman and 2 is a man. For simplicity, suppose that there are no women [men] with the same appeal, i.e. $A_i^h \neq A_i^k$, $\forall (h,k)$ with $h \neq k$. Such

a population structure is considered *unstable* if a woman and a man of two distinct

couples believe they could be personally advantaged by forming a new couple together.

In the opposite case the population structure is *stable*. Thus, practically speaking,

unstable populations are those in which the separation and the formation of couples are

quite frequent. Obviously, this definition must be further specified. The most natural

way is to assume that individual i would have a real advantage in changing the partner,

if this change is accompanied by an increase of $\bar{x}_i$. However, in order to forecast the

value $\bar{x}_1$ [$\bar{x}_2$] that a woman [man] will reach by forming a couple with a new partner,

she [he] should know everything about him [her] (in mathematical terms, she [he]

should know his [her] appeal A_2 [A_1] and his [her] behavioral parameters α_2, β_2, and γ_2

[α_1, β_1, and γ_1]). Generally, this is not the case and the forecast is performed with

limited information. In this case it is assumed that the only available information is the

appeal of the potential future partner and that the forecast is performed by imagining

that the behavioral parameters of the future partner are the same as those of the present

partner. This choice obviously emphasizes the role of appeal, quite reasonably, however,

because appeal is the only easily identifiable parameter in real life.

The above discussion is formally summarized by the following definition.

Definition 1

A population structure $\left[A_1^n, \alpha_1^n, \beta_1^n, \gamma_1^n; A_2^n, \alpha_2^n, \beta_2^n, \gamma_2^n\right]$, $n = 1, 2, \ldots, N$ is unstable if and

only if there exists at least one pair (h, k) of couples such that

$$\bar{x}_1\left(A_1^h, \alpha_1^h, \beta_1^h, \gamma_1^h; A_2^k, \alpha_2^k, \beta_2^k, \gamma_2^k\right) > \bar{x}_1\left(A_1^h, \alpha_1^h, \beta_1^h, \gamma_1^h; A_2^h, \alpha_2^h, \beta_2^h, \gamma_2^h\right)$$

$$\bar{x}_2\left(A_1^h, \alpha_1^k, \beta_1^k, \gamma_1^k; A_2^k, \alpha_2^k, \beta_2^k, \gamma_2^k\right) > \bar{x}_2\left(A_1^k, \alpha_1^k, \beta_1^k, \gamma_1^k; A_2^k, \alpha_2^k, \beta_2^k, \gamma_2^k\right)$$

(4)

where the functions $\bar{x}_1(\cdot)$ and $\bar{x}_2(\cdot)$ are given by eqs. (3). A population structure which is not unstable is called stable.

■

We can now prove that stable population structures are characterized by a very simple but peculiar property involving only appeal.

Remark 6

A population structure is stable if and only if the partner of the n-th most attractive woman of the population ($n = 1, 2, ..., N$) is the n-th most attractive man.

Proof

First notice that Remark 4 implies that condition (4) is equivalent to

$$A_2^k > A_2^h \qquad\qquad A_1^h > A_1^k \qquad\qquad (5)$$

i.e. a population is unstable if and only if there exists at least one pair (h, k) of couples satisfying (5). Condition (5) is illustrated in Fig. 2a in the appeal space, where each couple is represented by a point.

Consider a population structure in which the partner of the n-th most attractive woman is the n-th most attractive man. Such a population is represented in Fig. 2b, which clearly shows that there is no pair (h, k) of couples satisfying inequalities (5). Thus, the population structure is stable.

On the other hand, consider a stable population structure and assume that the couples have been numbered in order of increasing appeal of the women, i.e.

$$A_1^1 < A_1^2 < ... < A_1^N \qquad\qquad (6)$$

Then, connect the first point $\left(A_1^1, A_2^1\right)$ to the second point $\left(A_1^2, A_2^2\right)$ with a segment of a straight-line, and the second to the third, and so on until the last point $\left(A_1^N, A_2^N\right)$ is reached. Obviously, all connecting segments have a positive inclination because, otherwise, there would be a pair of couples satisfying condition (5) and the population would be unstable (which would contradict the assumption). Thus, $A_1^1 < A_1^2 < \ldots < A_1^N$. This, together with (6), states that the partner of the n-th most attractive woman is the n-th most attractive man.

■

On the basis of Remark 6, higher tensions and frictions should be expected in populations with couples in conflict with the appeal ranking. This result, derived from purely theoretical arguments, is certainly in agreement with empirical evidence. Indeed, partners with very uneven appeals are rarely observed in relatively stable communities. Of course, in making these observations one must keep in mind that appeal is an aggregated measure of many different factors (physical, financial, intellectual, ...). Thus, for example, the existence of couples composed of a beautiful lady and an unpleasant but rich man does not contradict the theory, but, instead, confirms a classical stereotype.

5. CONCLUDING REMARKS

A minimal model of love dynamics composed of two ordinary differential equations has been presented and discussed in this paper. The equations take into account three mechanisms of love growth and decay: the forgetting process, the pleasure of being loved and the reaction to the partner's appeal. For suitable, but reasonable, assumptions on the behavioral parameters of the individuals, the model turns out to be an

asymptotically stable, positive, linear system, and enjoys a number of remarkable properties. The model predicts, that the feelings of the two partners vary monotonically, growing from zero (complete indifference) to a maximum. The value of this maximum, i.e. the quality of the romantic relationship at equilibrium, is higher if the reactivenesses to love and appeal are higher. The same is true, if the time needed to reach the maximum is longer. All these properties are in agreement with common wisdom on the dynamics of love between two persons.

These properties have been used to derive the characteristics under which the couples of a given population have no tendency to separate (stability). The main result along this line is that the driving force that creates order in the population is the appeal of the individuals. In other words, couples with uneven appeals should be expected to have higher chances to brake off. These results are somehow complementary to those predicted by attachment theory, where appeal has a very limited role.

As for any minimal model, the extensions one could propose are innumerable. Aging, learning and adaptation processes could be taken into account allowing for some behavioral parameters to slowly vary in time, in accordance with the most recent developments of attachment theory. Particular nonlinearities, as well as synergism, could be introduced in order to develop theories for classes of individuals with personalities different from those considered in this paper. Men and women could be distinguished by using two structurally different state equations. The dimension of the model could also be enlarged in order to consider individuals with more complex personalities or the dynamics of love in larger groups of individuals. Moreover, the process followed by each individual in forecasting the quality of the relationship with a potential new partner could be modelled more realistically, in order to attenuate the role

of appeal, which has been somehow overemphasized in this paper. This could be done quite naturally by formulating a suitable differential game problem. Undoubtedly, all these problems deserve further attention.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Fig. 1   Trajectories (continuous lines) and isoclines (dotted lines) of the system. The straight trajectories are identified by the two eigenvectors. Single and double arrows indicate slow and fast motion.

Fig. 2   Population structures in the appeal space: (a) two points corresponding to two couples (h, k) belonging to an unstable population structure (see (5)); (b) an example of a stable population structure (each dot represents a couple).
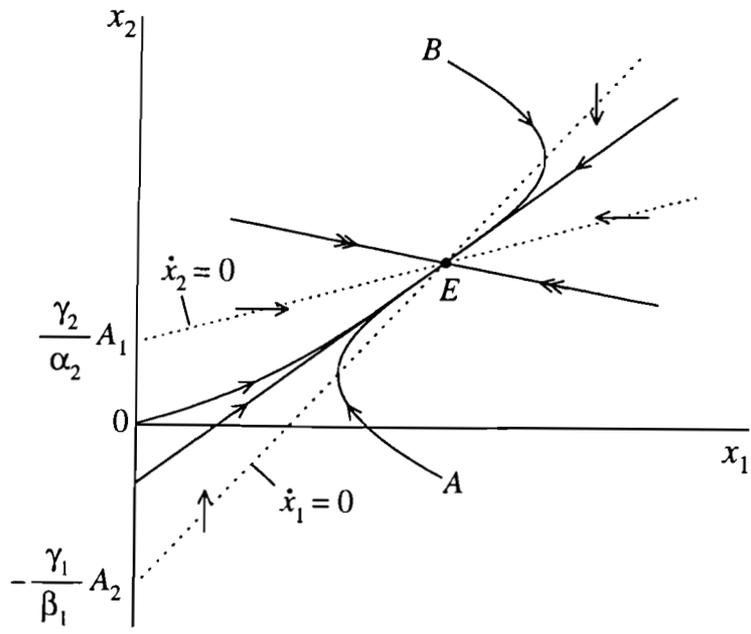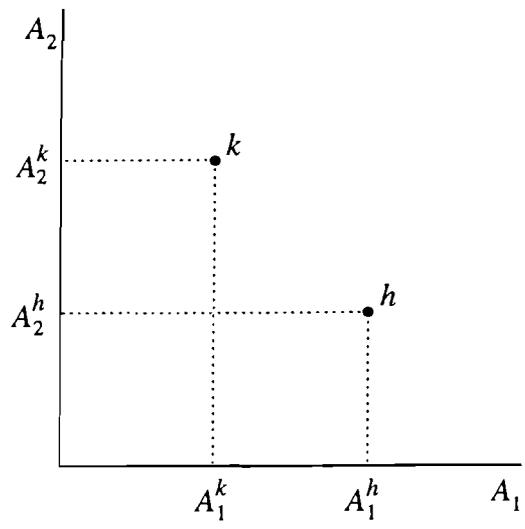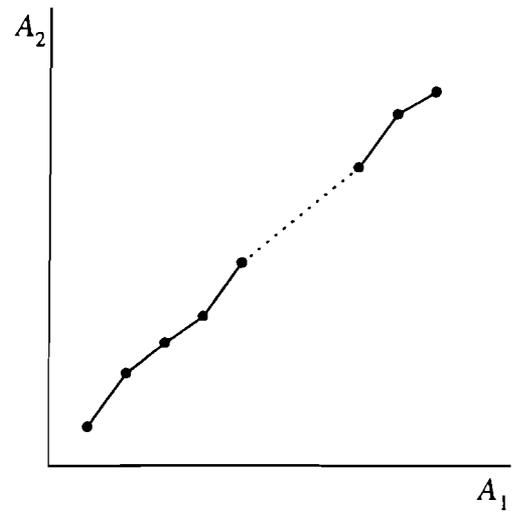
Figure 1

(a)   (b)

Figure 2