Working Paper

A decomposition technique for equilibrium programming under uncertainty

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Abstract

We consider a decomposition technique for solving monotone stochastic Nash equilibrium models based on scenarios and policy aggregation. The algorithm works by splitting the large multi-scenario equilibrium programming problem into separable scenario equilibrium subproblems that are amenable to solution via mixed complementarity problem solvers. We will consider preliminary numerical experience on a small stochastic trade model with two agents, two goods, and two scenarios.

Key words: equilibrium programming, decomposition, proximal point methods, stochastic programming

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1 Introduction

Equilibrium programming models[1] [2] have been used for years to analyze a variety of important problems. Examples abound in the literature, and range from the well known computable general equilibrium and Nash equilibrium formulations of economic games, to the modeling of multi-body mechanical systems via frictional contact problems. Many of these problems can be represented as mixed complementarity problems. Recently, a great deal of effort has been put forth to develop efficient algorithms to solve problems of this sort ([BiF95],[ChM95b],[ChM95a],[DiF94], [DiF95a],[DiF95b],[FMR95],[GaP92], [HaP90],[HaX90],[Jos79], [KMN91], [KMN89],[Man76], [Man79],[MaS93], [Mat87],[Mor94],[Pan91],[PaG93],[PaQ93], [Ral94],[Rob94],[WrR93],[XiH94]). A variety of different formulations have been developed that model the underlying problem in different ways and result in unique algorithms with differing convergence behavior that are applicable to complementarity problems having different sorts of structure. Because of this effort, it is now possible to solve a wide assortment of moderately sized equilibrium problems.

In this paper, we will consider equilibrium problems that incorporate uncertainty. In particular, an economic equilibrium model where multiple agents make decisions sequentially over time in order to maximize their own expected utilities, with expectation taken across a finite (but possibly large) number of future states of the system, and using an appropriately defined measure to describe the likelihood of the different futures. We will also assume that each agent recognizes that his decisions, which may vary across scenarios, can only be functions of the information available at the time he makes his decision. This is called the principle of nonanticipativity. Finally, we assume each agent is attempting independently to do this for himself and that a perfect commodity market exists (where a complete commodity description includes the state of the world), so that, across any one scenario, the agent's decisions will always be in equilibrium.

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The problem of optimal decision making under uncertainty by a single agent has already been dealt with extensively. A small sample of the references from the stochastic programming literature follows ([BiW91], [Bir82], [ErW88], [MuR92], [MuR91], [Rus92], [HiS91], [Fra92], [Wet89],[RoW86],[Wet83]). We highlight especially the work of Spingarn [Spi85], and Rockafellar and Wets [RoW91a], and their work, respectively, with the method of partial inverses, and the principle of progressive hedging. These ideas result in powerful decomposition schemes that enable finite scenario stochastic programming problems of potentially enormous size to be solved via the solution of a large number of separable scenario optimization subproblems.

We will aim to do a similar thing with stochastic equilibrium problems. In particular, we will see that the method of partial inverses can also be applied in the context of monotone stochastic equilibrium programming. In this case, though, rather than separable scenario optimization subproblems as was the case with stochastic programming, we will see that it is necessary to solve separable scenario equilibrium subproblems.

The remainder of this paper will be organized as follows. In §2 we will formally define the stochastic multistage equilibrium programming problem. In section §3 we will review the work of Spingarn and discuss the applicability of his findings to this papers problem. In section §4 we will state the decomposition algorithm that will be used to solve our problem. In section §5 we will review the structure of the model that we use for the numerical tests. We will then present results that illustrate how effectively this decomposition algorithm solves the described model. Finally, we will consider some illustrative examples from the model of key parameters that show the significantly different results one can expect to obtain when one undertakes an explicit stochastic equilibrium analysis rather than a scenario sensitivity analysis.

2 Problem statement

We consider an important class of complementarity models known as multistage stochastic equilibrium programming problems.

We first discuss the methodology we use to describe the stochasticity of the model (developed in [RoW91b]), and show how this fits within the structure of a representative agent's decision problem. We then write out the conditions that describe the equilibrium solution we wish to find. The set of all such agents will be denoted as $\Omega = \{1, 2, \ldots, A\}$. The basic object for modeling the passage of time and the flow of information is the *scenario tree*. The trees levels $1, \ldots, T$ correspond to time stages and each path from the root to the leaves corresponds to a *scenario* from the set $S = \{j | j = 1, \ldots, N\}$. With each scenario path j we associate the following objects from the a^{th} agent's decision problem: the decision subvector

$$x_a^j = (x_a^j(1), x_a^j(2), \ldots, x_a^j(T)) \in \mathbb{R}^{q_a^1} \times \cdots \times \mathbb{R}^{q_a^T},$$

the closed concave utility function $u_a^j(x_a^j, x_{\Omega_{-a}}^j) : \mathbb{R}^{q_a^1} \times \cdots \times \mathbb{R}^{q_a^T} \times \prod_{b \in \Omega_{-a}} (\mathbb{R}^{q_b^1} \times \cdots \times \mathbb{R}^{q_b^T}) \rightarrow (-\infty, +\infty]$ where Ω_{-a} represents the set of all agents other than a, and the probability p_j. Let $q_a = q_a^1 + \ldots + q_a^T$, $n_a = Nq_a$ and $n = n_1 + \ldots + n_A$. The a^{th} agent

must make decisions that are a function only of information available at the time of the decision. In particular, the agent's entire decision vector $x_a = (x_a^1, \ldots, x_a^N) \in \mathbb{R}^{n_a}$ must satisfy the *nonanticipativity* constraint: for all $t = 1, \ldots, T-1$ and for all pairs (i, j) of scenarios indistinguishable through the first t time stages, one must have

$$x_a^i(\tau) - x_a^j(\tau) = 0 \quad \text{for } \tau = 1, \ldots, t.$$

This constraint can be expressed as $A_a x_a = 0$ for a suitable matrix $A_a \in \mathbb{R}^{m_{A_a} \times n_a}$ and so all nonanticipative vectors x_a form a linear subspace $\mathcal{L}_a = \{x_a : A_a x_a = 0\}$ of $\mathbb{R}^{n_a}$. The stochastic equilibrium programming problem can be formed as follows:

$$\text{Find that set } \{x_1^*, x_2^*, \ldots, x_A^*\} \in \mathbb{R}^n \text{ where} \tag{2.1}$$

$$x_1^* = \arg \max_{x_1 \in \mathcal{L}_1} \sum_{j=1}^{N} p_j u_1^j(x_1^j, (x_{\Omega_{-1}}^j)^*)$$

$$\vdots$$

$$x_A^* = \arg \max_{x_A \in \mathcal{L}_A} \sum_{j=1}^{N} p_j u_A^j(x_A^j, (x_{\Omega_{-A}}^j)^*)$$

It is clear that this form of equilibrium is a special case of the classical Nash equilibrium. It is special for a number of reasons. One reason is that each agent's utility function is a sum of separate scenario components. They are separable because each scenario component of each agent's utility function maps the set of decisions (both his and his competitors decisions) from **the corresponding scenario** into the real line (i.e., they are not functions of decisions from other scenarios). These separable components of each agent's utility function are then linked by the need for decisions to be nonanticipative. This structure will prove to be very useful and certainly preferable, at least as regards its decomposability along scenario lines, to a more convoluted structure in which an agent's utility in one scenario might be a function of a decision from a different scenario. Another way in which this structure is special has to do with the way in which we define a solution to the stochastic equilibrium programming problem. In an ordinary intertemporal deterministic game with multiple agents, an equilibrium set of sequential decisions would be defined as those decisions from which, given the sequential decisions of all other players, no one player would choose to move. Therefore, the decisions that an agent makes at any particular time period depend freely on his and all other agent's decisions throughout the study horizon. That is, they depend on both the past and future. A set of decisions is in nonanticipative equilibrium when each set of sequential decisions for each separate scenario game meets the above requirements for a standard deterministic game, and, additionally, is nonanticipative with regards to the information structure described by the scenarios. Thus, a scenario's decisions no longer depend freely on that scenarios past and future. They depend on the past and the future of that scenario only so far as they not utilize information about the future before it becomes available (i.e., they must be nonanticipative).

We will find it useful in later sections of this paper to use operators that project vectors of decisions into the nonanticipative subspaces discussed above. The orthogonal projection

$u = P_{\mathcal{L}_a} x_a$ of x_a onto $\mathcal{L}_a$ can be calculated as follows (see [RoW91b]). For each $a = 1, \ldots, A$,$j = 1, \ldots, N$ and $t = 1, \ldots, T$, determine the set of scenarios indistinguishable from scenario j at time t:

$$I_a^j(t) = \{i : \ \nu_\tau^a(i) = \nu_\tau^a(j), \ \tau = 1, \ldots, t\},$$

and average $x_a^i(t)$ over this subset:

$$u_a^j(t) = \frac{1}{|I_a^j(t)|} \sum_{i \in I_a^j(t)} x_a^i(t).$$

3 Method of Partial Inverses

Spingarn [Spi85] works within a general Hilbert space H with given inner product $\langle x, y \rangle$. He considers methods for solving the following complementarity problem:

$$\text{Find } x \in A \text{ and } y \in B \text{ such that } y \in T(x) \tag{3.1}$$

where A, B are complementary subspaces of H. He starts by considering methods for solving the problem of finding the *zero* of a maximally monotone operator T ($x \in H$ such that $0 \in T(x)$) where T is monotone if $\langle x - x', y - y' \rangle \geq 0$ for all $y \in T(x)$ and $y' \in T(x')$, and maximally monotone if for every $z \in H$ and $c > 0$ there exists a unique $x \in H$ such that $z \in x + cT(x)$. In particular, he considers the proximal point methods [Roc76] that generate a sequence (z_k) from an arbitrary $z_0 \in H$ by using a sequence $c_k > 0$, and the following iteration:

$$z_{k+1} = (I + c_k T)^{-1}(z_k).$$

These methods are known to converge to a zero of T given that (c_k) is bounded away from zero and that T has such a zero. To use these methods for complementarity problems, he develops the concept of the *partial inverse of* T *with respect to* A where A is, again, a subspace of H. He denotes the partial inverse operator as T_A and defines its graph as:

$$Gr(T_A) = \{(x_A + y_B, y_A + x_B) : y \in T(x)\}$$

where x_A, y_A and x_B, y_B are, respectively, the unique components of x, y that lie in A and B. It is clear that the solution of 3.1 corresponds to the

$$x \in A \text{ and } y \in B \text{ such that } 0 \in T_A(x + y). \tag{3.2}$$

Thus, the complementarity problem can be solved by applying the proximal point iteration to the operator T_A. Spingarn shows that the proximal point iteration applied to T_A becomes the following algorithm when written in terms of T:

Algorithm 3.1.
Step 0: Select $x_0 \in A$ and $y_0 \in B$. Set $k = 1$.

Step 1: Find $x_k^{'}, y_k^{'} \in H$ such that

$$x_k + y_k = x_k^{'} + y_k^{'} \text{ and } \frac{1}{c_k}(y_k^{'})_A + (y_k^{'})_B \in T((x_k^{'})_A + \frac{1}{c_k}(x_k^{'})_B).$$

Step 2: Let $x_{k+1} = (x_k^{'})_A$ and $y_{k+1} = (y_k^{'})_B$[3]. Increase k by 1 and go to Step 1.

Rockafellar and Wets combined the general ideas found in Spingarn's notion of the partial inverse, with their unique interpretation of stochastic programming as optimization over a nonanticipative subspace to form the progressive hedging decomposition algorithm. Using similar ideas, we can create a decomposition algorithm suitable for the stochastic equillibrium program introduced in §2.

In particular, let's rewrite problem 2.1 as a variational inequality.

$$\text{Find that set } \{x_1^*, x_2^*, \ldots, x_A^*\} \in \mathbb{R}^n \text{ where} \tag{3.3}$$

$$\begin{Bmatrix} -p_1 \partial_{x_1^1} u_1^1((x_1^1)^*, (x_{\Omega_{-1}}^1)^*) \\ \vdots \\ -p_N \partial_{x_1^N} u_1^N((x_1^N)^*, (x_{\Omega_{-1}}^N)^*) \\ \vdots \\ -p_1 \partial_{x_A^1} u_A^1((x_A^1)^*, (x_{\Omega_{-A}}^1)^*) \\ \vdots \\ -p_N \partial_{x_A^N} u_A^N((x_A^N)^*, (x_{\Omega_{-A}}^N)^*) \end{Bmatrix}^T \begin{Bmatrix} x_1^1 - (x_1^1)^* \\ \vdots \\ x_1^N - (x_1^N)^* \\ \vdots \\ x_A^1 - (x_A^1)^* \\ \vdots \\ x_A^N - (x_A^N)^* \end{Bmatrix} \geq 0$$

$$\text{for all } x_1 \in \mathcal{L}_1, x_2 \in \mathcal{L}_2, \ldots, x_A \in \mathcal{L}_A.$$

If we set

$$T((x_1^1, \ldots, x_1^N), \ldots, (x_A^1, \ldots, x_A^N)) = \begin{Bmatrix} -p_1 \partial_{x_1^1} u_1^1(x_1^1, x_{\Omega_{-1}}^1) \\ \vdots \\ -p_N \partial_{x_1^N} u_1^N(x_1^N, x_{\Omega_{-1}}^N) \\ \vdots \\ -p_1 \partial_{x_A^1} u_A^1(x_A^1, x_{\Omega_{-A}}^1) \\ \vdots \\ -p_N \partial_{x_A^N} u_A^N(x_A^N, x_{\Omega_{-A}}^N) \end{Bmatrix}, \tag{3.4}$$

then our variational inequality becomes like problem 3.1 with $A = \{(x_1, \ldots, x_A) | x_1 \in \mathcal{L}_1, \ldots, x_A \in \mathcal{L}_A\}$. Hence, if we assume that the functions $\{u_1^1(\cdot), \ldots, u_1^N(\cdot), \ldots, u_A^1(\cdot), \ldots, u_A^N(\cdot)\}$ have the appropriate structure to make T maximally monotone, then we can apply the method of partial inverses directly to this problem. We will leave off discussion of just what structure ensures this property until another paper as, at present, we are primarily interested in the modeling and algorithmic aspects of equilibrium programming under uncertainty.

[3] $(\cdot)_A$ and $(\cdot)_B$ are the projection operators on the subspaces A and B respectively.

4 Decomposition via the Method of Partial Inverses

In order to ensure that our method results in an algorithm that decomposes along scenario lines, we must require that $(c_k) = 1$. This is exactly as has already been noted within the operator splitting literature by both [Spi85] and [EcB92]. To see this, note that when $(c_k) = 1$, performing Step 1 of algorithm 3.1 with T defined as in 3.4 reduces to finding the $x_k^{'} \in H$ and $y_k^{'} \in H$ such that $y_k^{'} = T(x_k^{'})$. This is an operation in which neither the structure of A nor B plays a part. Since in our case A corresponds to the nonanticipative subspace that links one scenario with another, this operation can be performed separably across scenarios, which is exactly what we'd like. When $(c_k) \neq 1$ the operation performed in Step 1 of algorithm 3.1 becomes significantly more difficult and requires explicit consideration of both A and B. Fortunately, the resulting algorithm will still converge when $(c_k) = 1$, since Spingarn's results only require that $c_k > 0$. Unfortunately, the fact that we can't allow c_k to grow towards ∞ as the algorithm progresses does remove the possibility of superlinear convergence. We feel, though, that the benefits associated with decomposability far outweigh this slight set back.

Let us consider now how the method of partial inverses will appear when cast in terms of our particular operator T and subspaces A and B. Using 3.4, we recall that Step 1 of algorithm 3.1 requires that we find $(x)^{k^{'}}, (y)^{k^{'}} \in H$ such that

$$0 = \quad -(y)^k + ((x)^{k^{'}} - (x)^k) + \left\{ \begin{array}{c} -p_1 \partial_{x_1^1} u_1^1((x_1^1)^{k^{'}}, (x_{\Omega_{-1}}^1)^{k^{'}}) \\ \vdots \\ -p_N \partial_{x_1^N} u_1^N((x_1^N)^{k^{'}}, (x_{\Omega_{-1}}^N)^{k^{'}}) \\ \vdots \\ -p_1 \partial_{x_A^1} u_A^1((x_A^1)^{k^{'}}, (x_{\Omega_{-A}}^1)^{k^{'}}) \\ \vdots \\ -p_N \partial_{x_A^N} u_A^N((x_A^N)^{k^{'}}, (x_{\Omega_{-A}}^N)^{k^{'}}) \end{array} \right\}.$$

But this is the same as solving the following problem:

$$\text{Find that set } \{(x_1)^{k^{'}}, (x_2)^{k^{'}}, \ldots, (x_A)^{k^{'}}\} \in \mathbb{R}^n \text{ where} \tag{4.1}$$

$$(x_1)^{k^{'}} = \arg\max\{\sum_{j=1}^{N} p_j u_1^j(x_1^j, (x_{\Omega_{-1}}^j)^{k^{'}}) + \langle y_1^k, x_1 \rangle - \frac{1}{2}||x_1 - (x_1)^k||^2\}$$

$$\vdots$$

$$(x_A)^{k^{'}} = \arg\max\{\sum_{j=1}^{N} p_j u_A^j(x_A^j, (x_{\Omega_{-A}}^j)^{k^{'}}) + \langle y_A^k, x_A \rangle - \frac{1}{2}||x_A - (x_A)^k||^2\}$$

which is a deterministic equilibrium problem that is completely separable across scenarios. That is, solvable as N separate equilibrium problems. We, thus, arrive at the decomposition algorithm that we will use to solve stochastic equilibrium programs.

Algorithm 4.1.
Step 0: Select $(x)^0 \in A$, $(y)^0 \in B$ and $\epsilon > 0$. Set $k = 1$.

Step 1: Solve problem 4.1.

Step 2: Let $(x_a)^{k+1} = P_{\mathcal{L}_a}(x_a)^{k'}$ for all $a = 1, \ldots, A$. and $(y_a)^{k+1} = P_{\mathcal{L}_a^{\perp}}(y_a)^{k'} = (y_a)^k - P_{\mathcal{L}_a^{\perp}}(x_a)^{k'}$.
If $||(x_a)^{k+1} - (x_a)^k|| > \epsilon$ or $||(y_a)^{k+1} - (y_a)^k|| > \epsilon$, increase k by 1 and go to Step 1.
Otherwise stop.

We note that we can easily initialize our algorithm by first relaxing the nonanticipativity constraints of our stochastic equilibrium program, solving each of the scenario subproblems separately, and then projecting the resulting vector of scenario solutions into the nonanticipative subspace using the projection operator discussed at the end of §2.

We can interpret algorithm 4.1 as an iterative means of locating for each agent the implicit costs (multipliers) y_a, $a \in \Omega$ of his uncertainty regarding the future. That is, y_a can be thought of as the price for violating nonanticipativity that must be added to the objective functions of all agents in each of the scenario equilibrium subproblems in order for the entire equilibrium solution that we seek to correspond exactly to a partial equilibrium solution for each of the equilibrium subproblems.

5 Numerical Results

In order to analyze the behavior of algorithm 4.1, we consider a two agent multi-stage macroeconomic model formulated as a two scenario stochastic equilibrium program with general structure like that discussed in section §2. This model is based on an earlier multi-agent stochastic equilibrium model called JMU that was developed by Manne and Olsen [Ols94], and used to predict the path of economic development during the next two centuries given that competing economic regions hedge against the current uncertainty over the eventual costs associated with carbon emission induced environmental change.

The JMU model spans the two century period from 1990 to 2200 via twenty two 10 year time stages. Five multiple agents who represent different economic regions in the world produce a generic numeraire economic commodity, trade this commodity amongst themselves, and accumulate capital through interperiod investment. Associated with each regions production are carbon emissions which are not allowed to exceed that regions fixed portion of the total global emissions for the period[4] . Regions can ensure that their emissions are not excessive through the use of abatement technologies that are effective at an increasing marginal cost. They can also trade in carbon rights with neighboring regions. In addition to the required compliance with the international treaty, regions are induced to reduce their emissions through the incorporation in the model of a penalty term that sharply reduces productive output at a critical cumulative level of global carbon. The level at which this curtailment takes place is uncertain and constitutes the stochastic component of the model. In order to reduce end of horizon effects, Manne and Olsen have included constraints in the last ten periods that force investment to be greater than or equal to a fixed percentage of the current capital stock. They also limit the period to

[4]It is assumed that these percentages are agreed upon at the start of the horizon through an international treaty.

Columns	532
Rows	532
Multipliers	78

Table 5.1: Scenario model characteristics

period flexibility of the overall economy by requiring that each periods global emissions remain within a fixed percentage of the previous periods emissions. This models the historically observed putty clay behavior of most large economic systems. A more detailed description of this model can be found in [Ols94].

In order to expedite computation, we altered Manne and Olsen's original formulation by removing that portion of their model that associated a given level of cumulative carbon emissions with an uncertain economic cost. We replaced this linkage with an explicit upper bound on cumulative emissions of carbon [5], and replaced the stochasticity with uncertainty regarding the future productivity of aggregate capital with resolution of that uncertainty occurring in the fifth period. We also shortened the horizon of the problem to 120 years and reduced the number of agents to two[6]. We included two different scenarios of future capital productivity (high[7] and low), and, using our decomposition strategy, this resulted in two nonlinear mixed complementary subproblems with relevant information concerning the size and structure of each problem shown in table 5.1 This numerical exercise was conducted wholly within the confines of the GAMS [BKM92] modeling language using the PATH [DiF95b] solver as needed for subproblem solution. Because of this, we report no solution times since the current implementation takes no advantage of the obvious parallelism across scenarios. We do report the number of iterations required to reach a sufficiently small violation of nonanticipativity, though, in figure 5.1. Our measure of this violation is: $\sum_{a\in\Omega} \frac{1}{2}||(x_a)^{k'} - (x_a)^k||^2$ where the vector $(x_a)^{k'}$ solves problem 4.1. This value gives us the orthogonal distance between our current solution and the nonanticipative subspace. This curve exhibits the characteristic behavior of proximal point methods with bounded sequences (c_k): Swift progress at the start followed by slower convergence in the tail. We see that, indeed, this method has delivered in practice no more than the linear convergence that we could expect from the theory. We reiterate, though, that this price is small in light of our ability now to decompose large stochastic equilibrium problems into smaller pieces sure to be solvable by algorithms like that used by PATH, and other complementarity solvers.

Having successfully solved the stochastic equilibrium problem using the above algorithm, we now compare the model results with those obtained when a simpler scenario

[5] As with the previous study, we assume that the upper bound on cumulative carbon deposits, along with each regions share of yearly emissions, would be agreed upon as part of an international treaty. In this exercise, we set the cumulative upper bound at 1000 billion tons of atmospheric carbon, with yearly global emissions of carbon split equally amongst the agents.

[6] The USA and ROW regions from Manne and Olsen's original model.

[7] After the resolution of uncertainty, capital becomes 20% more productive in scenario 1 than in scenario 2.

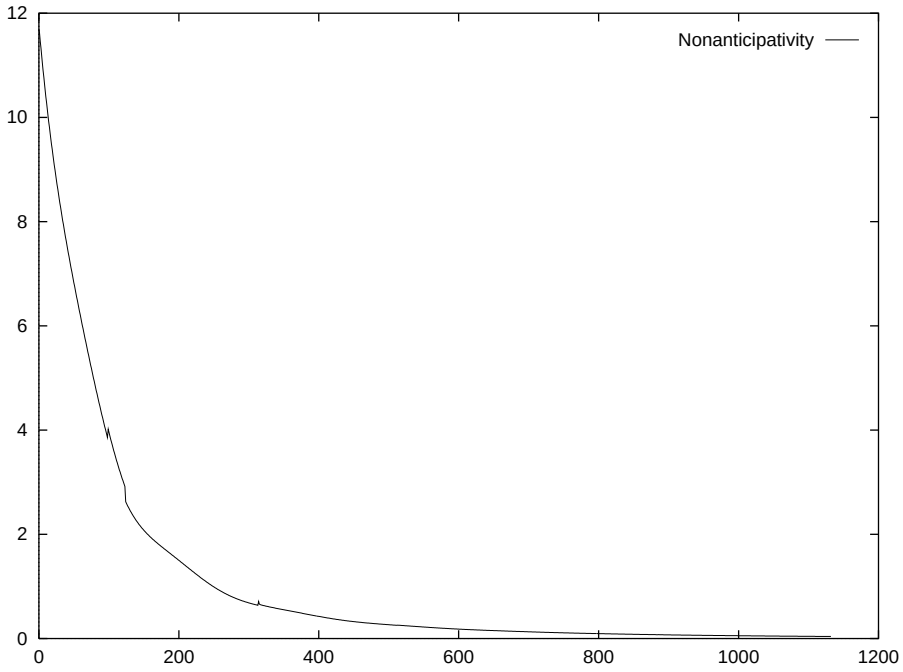

Figure 5.1: Nonanticipativity

analysis is performed that disregards the question of nonanticipativity. What we find is that several of the key decision variables that make up the hedging strategy returned by the stochastic equilibrium problem are significantly different from the corresponding decision variables of the strategy returned by the scenario analysis. This suggests that it is worthwhile to consider explicit uncertainty in equilibrium models because qualitatively different predictions of economic behavior can be obtained.

The first difference that is apparent is the rate of economic output[8] for the two agents. The results are pictured in figures 5.2 and 5.3. They indicate that the USA will produce more output during the years prior to 2020 (the year at which uncertainty is resolved) in the presence of uncertainty than when there is no uncertainty. Similarly, the ROW will produce less. The way in which this is accomplished is readily apparent when one looks at the rates of investment pictured in figures 5.4 and 5.5 and the resulting rates of capital accumulation pictured in figures 5.6 and 5.7. Accelerated investment in the USA during the period of uncertainty leads to higher levels of capital formation. This, in turn, leads to higher economic output. The opposite effect occurs in the ROW. Accompanying this switch in economic output, we see that consumption in the USA turns out to be higher during the period prior to the resolution of uncertainty than that predicted by either of the two deterministic scenarios. Likewise, the predicted consumption in the ROW is less during this same period. This is pictured in figures 5.8 and 5.9.

All these examples suggest that incorporating uncertainty into an equilibrium program can have an impact on the predicted paths of key variables. The exact nature of the impact will, of course, differ from model to model. To determine what that impact will be, there

[8]All pictured variables are in Trillions of US dollars.

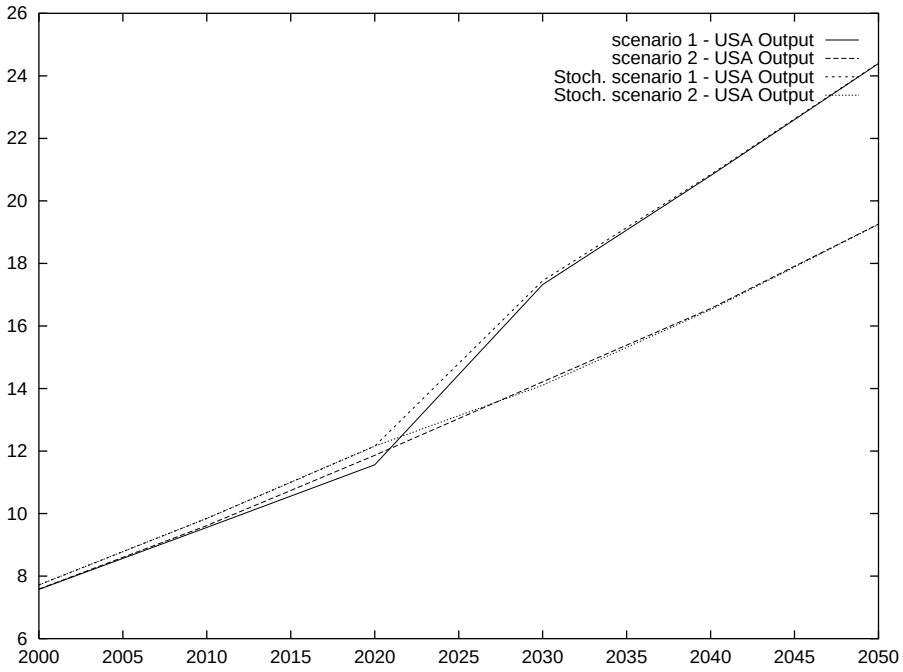

Figure 5.2: Economic output - USA

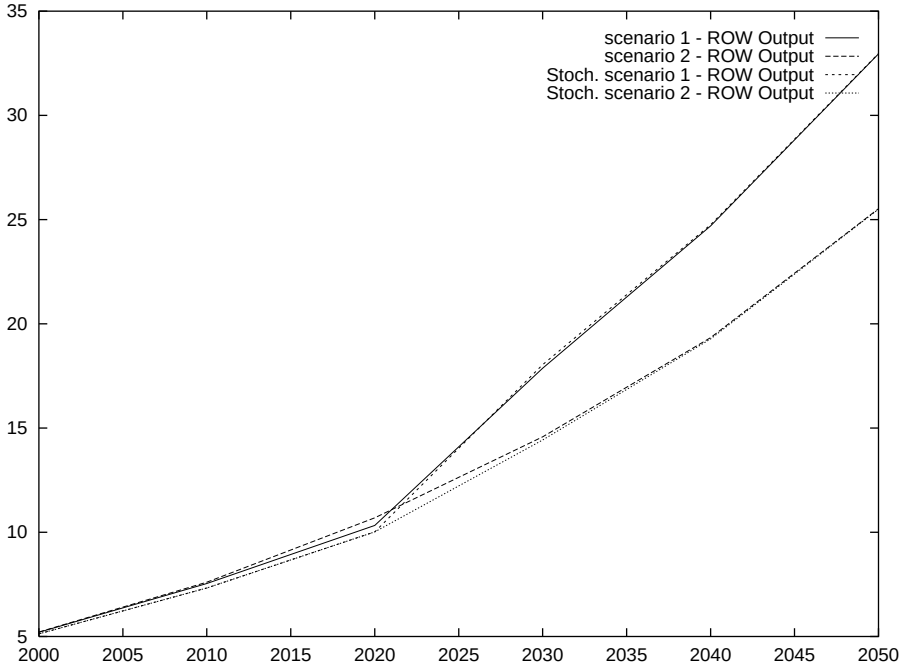

Figure 5.3: Economic output - ROW

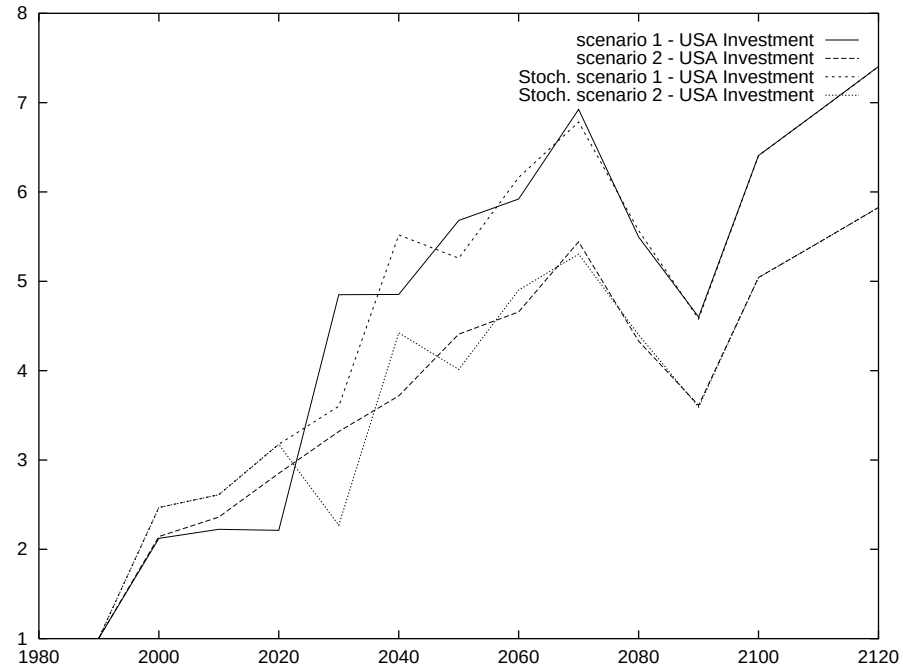

Figure 5.4: Investment - USA

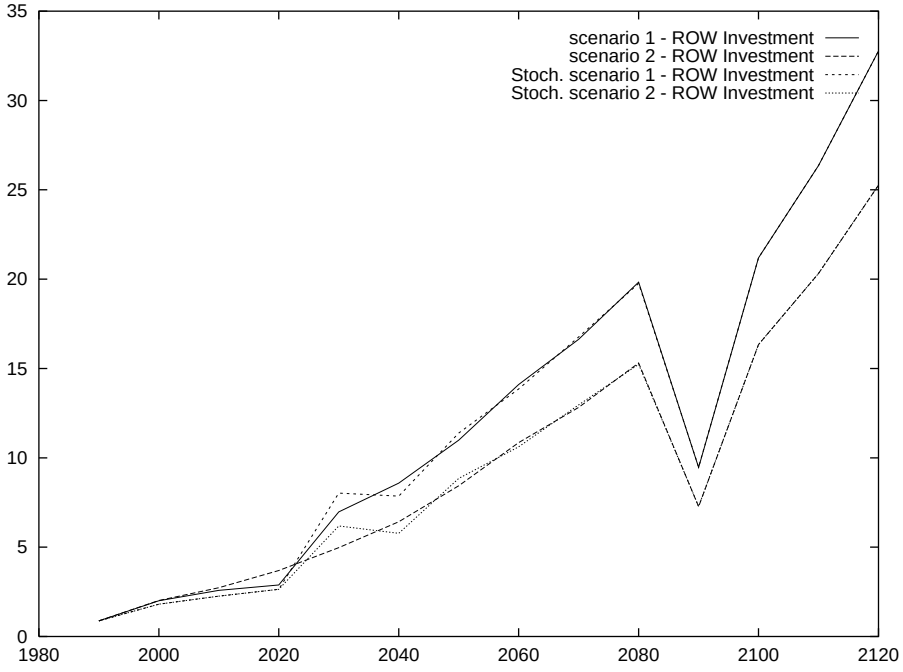

Figure 5.5: Investment - ROW

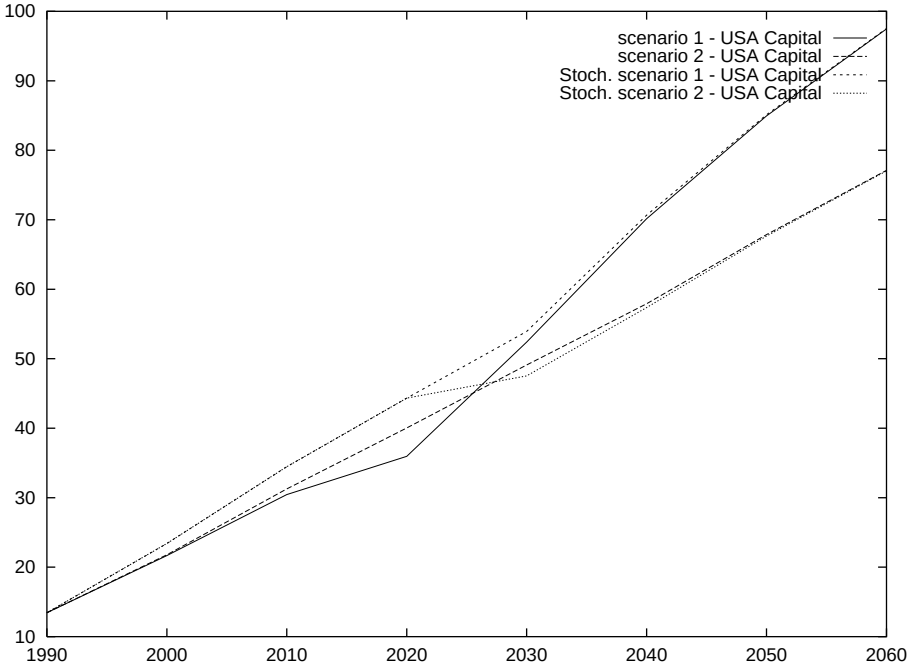

Figure 5.6: Capital - USA

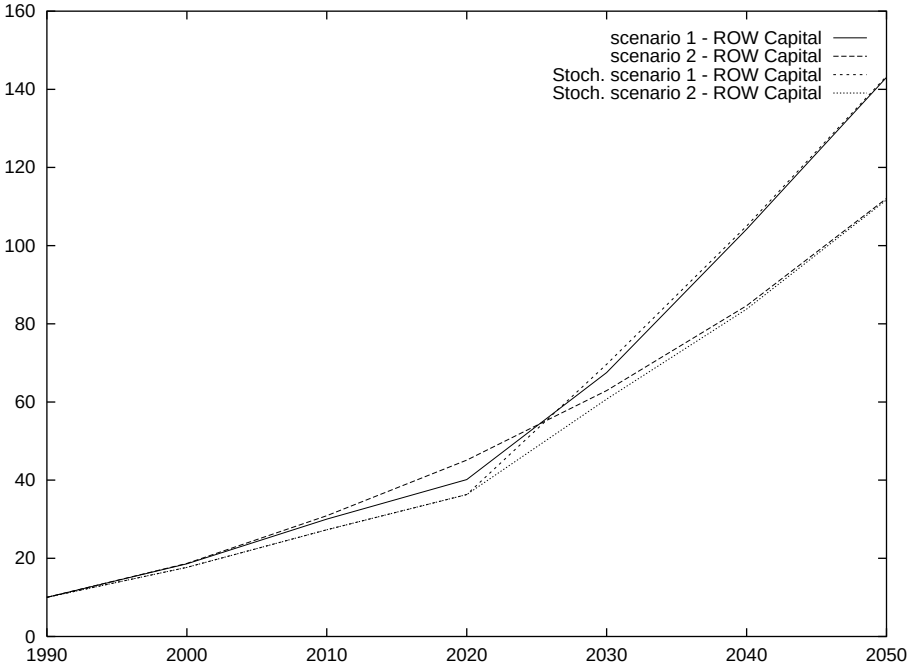

Figure 5.7: Capital - ROW

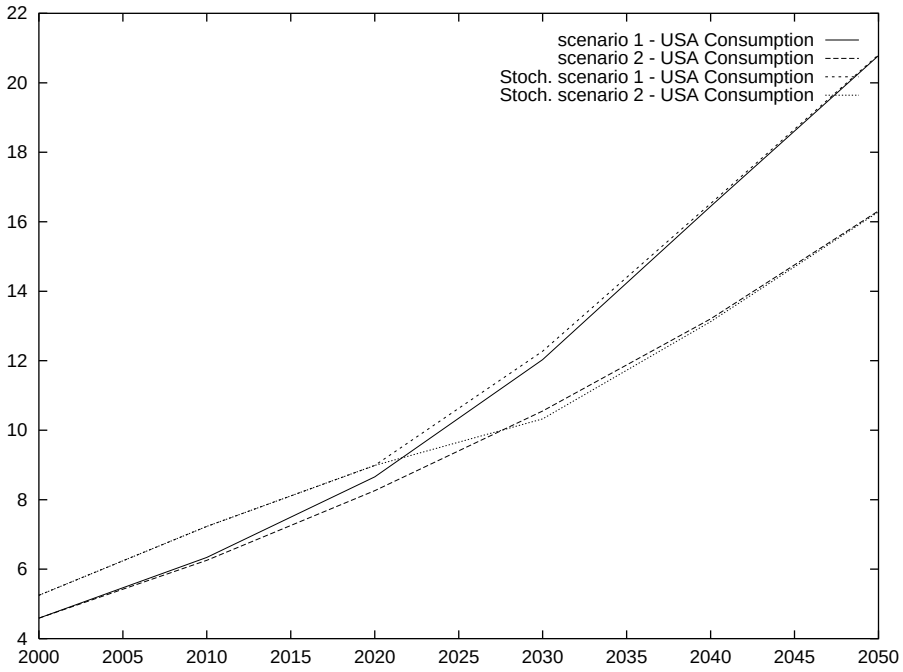

Figure 5.8: Consumption - USA

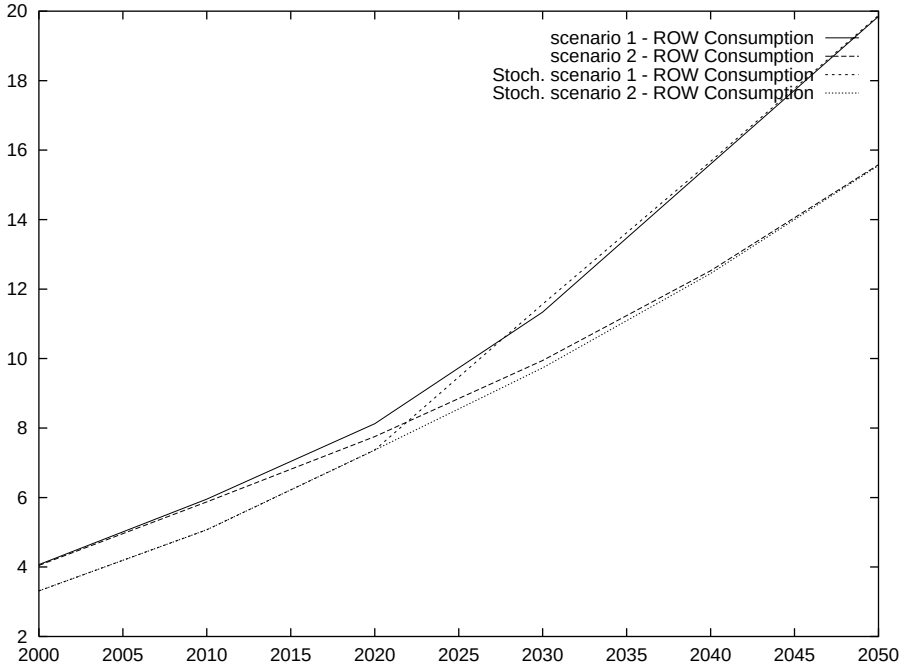

Figure 5.9: Consumption - ROW

must exist easy to implement methods that are capable of solving large stochastic equilibrium problems. Decomposition via the method of partial inverses is just such a method because it breaks the stochastic problem into its component scenario pieces and solves these separably, passing back and forth only the necessary limited information needed to reach the desired nonanticipative solution. Because one is required to solve only smaller deterministic subproblems to reach the intended goal, existing complementarity solvers prove very adequate. The algorithm is also desirable from a modeling standpoint. Most equilibrium models start out deterministic. Later, it might become useful to ascertain how the models results will change with the incorporation of uncertainty. Typically, this question is approached via scenario analysis by creating multiple copies of the underlying deterministic model having different data sets that correspond to the desired scenarios. Any method that can use these separable scenario deterministic subproblems to derive the nonanticipative stochastic solution is helpful because it means that the model need not be rewritten. Decomposition via the method of partial inverses does precisely this.

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