

INTERIM REPORT IR-97-017 / March 1997

Learning Equilibria in Games Played by Heterogeneous Populations

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Preface

This new research project at IIASA is concerned with modeling technological and organisational change; the broader economic developments that are associated with technological change, both as cause and effect; the processes by which economic agents – first of all, business firms – acquire and develop the capabilities to generate, imitate and adopt technological and organisational innovations; and the aggregate dynamics – at the levels of single industries and whole economies – engendered by the interactions among agents which are heterogeneous in their innovative abilities, behavioural rules and expectations. The central purpose is to develop stronger theory and better modeling techniques. However, the basic philosophy is that such theoretical and modeling work is most fruitful when attention is paid to the known empirical details of the phenomena the work aims to address: therefore, a considerable effort is put into a better understanding of the ‘stylized facts’ concerning corporate organisation routines and strategy; industrial evolution and the ‘demography’ of firms; patterns of macroeconomic growth and trade.

From a modeling perspective, over the last decade considerable progress has been made on various techniques of dynamic modeling. Some of this work has employed ordinary differential and difference equations, and some of it stochastic equations. A number of efforts have taken advantage of the growing power of simulation techniques. Others have employed more traditional mathematics. As a result of this theoretical work, the toolkit for modeling technological and economic dynamics is significantly richer than it was a decade ago.

During the same period, there have been major advances in the empirical understanding. There are now many more detailed technological histories available. Much more is known about the similarities and differences of technical advance in different fields and industries and there is some understanding of the key variables that lie behind those differences. A number of studies have provided rich information about how industry structure co-evolves with technology. In addition to empirical work at the technology or sector level, the last decade has also seen a great deal of empirical research on productivity growth and measured technical advance at the level of whole economies. A considerable body of empirical research now exists on the facts that seem associated with different rates of productivity growth across the range of nations, with the dynamics of convergence and divergence in the levels and rates of growth of income in different countries, with the diverse national institutional arrangements in which technological change is embedded.

As a result of this recent empirical work, the questions that successful theory and useful modeling techniques ought to address now are much more clearly defined. The theoretical work described above often has been undertaken in appreciation of certain stylized facts that needed to be explained. The list of these ‘facts’ is indeed very long, ranging from the microeconomic evidence concerning for example dynamic increasing returns in learning activities or the persistence of particular sets of problem-solving routines within business firms; the industry-level evidence on entry, exit and size-distributions – approximately log-normal; all the way to the evidence regarding the time-series properties of major economic aggregates. However, the connection between the theoretical work and the empirical phenomena has so far not been very close. The philosophy of this project is that the chances of developing powerful new theory and useful new analytical techniques

can be greatly enhanced by performing the work in an environment where scholars who understand the empirical phenomena provide questions and challenges for the theorists and their work.

In particular, the project is meant to pursue an ‘evolutionary’ interpretation of technological and economic dynamics modeling, first, the processes by which individual agents and organisations learn, search, adapt; second, the economic analogues of ‘natural selection’ by which interactive environments – often markets – winnow out a population whose members have different attributes and behavioural traits; and, third, the collective emergence of statistical patterns, regularities and higher-level structures as the aggregate outcomes of the two former processes.

Together with a group of researchers located permanently at IIASA, the project coordinates multiple research efforts undertaken in several institutions around the world, organises workshops and provides a venue of scientific discussion among scholars working on evolutionary modeling, computer simulation and non-linear dynamical systems.

The research will focus upon the following three major areas:

1. Learning Processes and Organisational Competence.
2. Technological and Industrial Dynamics
3. Innovation, Competition and Macrodynamics

Summary

Consider two populations of agents who learn to play a game through repetition. In fictitious play, each agent chooses a best reply to the frequency distribution of actions taken by the other side. A natural variant of this model is to assume that agents are heterogeneous in their information and their behavioral response rules. Assume that each agent knows only a randomly drawn sample of past actions. Given their information, agents sometimes choose best replies, and sometimes they imitate behavior in their own population. In contrast to the stochastic best reply dynamics studied by Fudenberg and Kreps (1993), Kaniovski and Young (1995), and Benaïm and Hirsch (1994), such process can cycle in a 2×2 game even when the probability of imitators is arbitrarily small. We show how to characterize its asymptotic behavior through an extension of Bendixson's theory for excluding cycles combined with standard techniques from stochastic approximation. *Journal of Economic Literature* Classification Numbers: C44, C73, D83.

Key words: repeated game, best reply, fictitious play, sample, imitator, robustness, convergence with probability one, cycles, dynamically stable Nash equilibria.

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