RR-76-15

THE IIASA WATER RESOURCES PROJECT: A STATUS REPORT

JUNE 1975

International Institute for Applied Systems Analysis
2361 Laxenburg, Austria

Preface

Water is a limiting factor to development in many regions of the world. The idea of including water resources problems in the IIASA research program was advanced by the late Professor Alexander M. Letov, the first leader of the Water Resources Project. In accordance with the discussions at the IIASA Water Planning Conference in June 1973, the Project focused attention on specific problems of universal methodology for water resources development and optimal operation. Results of these studies were presented orally to the Water Project Advisory Committee and to members of IIASA at a seminar held in Laxenburg, Austria, on June 3, 1975.

This report is the written version of the oral presentation. Section I presents the historical background and general framework ot the Project; the following two sections describe the major methodological results obtained from modeling water resources systems and from applying optimization techniques to river basin management, respectively. Section IV gives some results of the cooperative studies carried out by IIASA and by National Member Organizations.

Zdzislaw Kaczmarek
Leader, Water Resources Project

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I. Systems Analysis in Water Resources

Zdzislaw Kaczmarek

Systems analysis in water resources is a philosophical approach to studying the interdependent relations of hydrologic, economic and environmental elements which, as a whole, form a unified system. The ultimate goal is to improve the performance of the system, usually on the scale of the river basin. The need for integrated river basin analysis arises from the relationship of the various sectors of national and regional economy. Thus, various links exist between water resources and other natural, social and economic systems, as for example, water and energy, and water and food production.

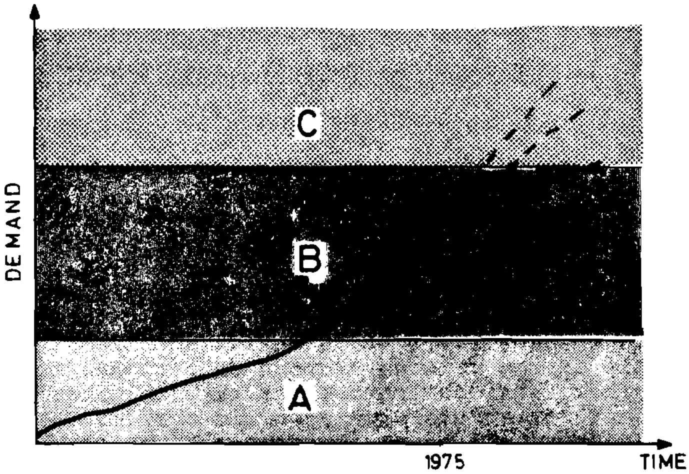
The goals of the system may differ substantially, mostly depending on the relation between resource availability and need. Referring to Figure 1, it is possible to distinguish at least three basic situations:

A. *There is sufficient water for meeting all needs at any moment of time;* the main objectives of water management are to prevent flood damage and to maintain environmental standards.

B. *The total amount of water resources is sufficient to meet the demands but, because of unequal distribution in time and space, local deficits may be expected;* the main objectives of water management are to liquidate the deficits by optimal redistribution of resources, to prevent flood damage and to maintain environmental standards.

C. *There is a total deficit of water in the basin considered;* to meet the demands it is necessary to find new sources of water (for example by desalination or long-distance transfer), and to use water in an optimal way. Of course, the demands also may be rationalized, using different technologies.

At present, most of the regions in the world seem to be in situation B, and this will continue for the next twenty to thirty years. Thus the Water Resources Project concentrates on methodological and applied problems of optimal river basin management. While each action undertaken for development and optimal operation of a water resources system has to be tailored to the physical, social and economic circumstances in the given area, there may exist some universal, generally applicable methods and procedures; it is in these that the Project is mainly interested.



A – Flood protection, quality control

B – Storage, local transfers, optimal management, flood protection, quality control

C – External supply + B

Figure 1. Water resources situation.

Table 1 gives information on present and future world water needs and their relation to the total mean world river runoff (R = 38,830 km^3). The estimated figures for the year 2000 may vary depending on technological changes, especially for industrial and energy demand.

It is not possible to find the relevant figures for all countries and continents; for the United States, for example, the following estimates can be made (U.S. National Water Commission, 1973):

$$100 \frac{T}{R} = 20.6\%/4.9\% \qquad \text{for the year 1970;}$$

and

$$100 \frac{T}{R} \doteq 44.8\%/7.1\% \qquad \text{for the year 2000.}$$

The relation between the demands and resources of any region depends on both physical (hydrologic) and economic factors. The information given in Table 2 shows the mean annual water resources per capita by continent and selected countries.

Table 1.

Users	Water Demands/Consumptive Use km^3/year	
	Year-1970	Year-2000
Population	150/75	1,070/280
Industry	200/40	3,000/600
Energy	250/15	3,100/270
Agriculture	2,800/2,100	3,950/3,550
Others	235/175	675/585
Total T	3,635/2,405	11,795/5,285
$\frac{T}{R}$%*	9.4/6.2	30.4/13.6

*Total mean world river runoff, R=38,830 km^3.

Source: U.S. National Water Commission (1973).

Table 2.

Continent/ Selected Countries	Water Resources Per Capita (m^3/year)		
	Mean	Maximum	Minimum
Africa	12,300	328,000 (Gabon)	120 (Egypt)
Asia	6,250	77,000 (Laos)	1,140 (Pakistan)
Australia	110,000	-	-
Europe	5,111	96,900 (Norway)	810 (Hungary)
North America	19,500	128,000 (Canada)	5,400 (USA, Western States)
South America	54,300	73,200 (Venezuela)	11,900 (Argentina)

Source: Lvovich (1974).

Among the IIASA National Member Organizations, we find very large differences in the amount of water available for population, industry, agriculture and other purposes. The mean resources per capita of these countries are shown in Table 3. Significant discrepancies are found within some large countries, as, for example, in the USSR and the USA.

Table 3.

Country	Water Resources Per Capita (m^3/year)
Canada	128,000
USSR	17,800
USA	11,400
Austria	7,700
France	4,570
Japan	3,820
Italy	2,980
UK	2,730
Bulgaria	2,100
Czechoslovakia	1,900
Poland	1,720
FRG	1,440
GDR	1,200
Hungary	810

Source: Lvovich (1974).

The emphasis of systems analysis today is mostly on describing physical, social and economic systems by means of mathematical models. The first step in any systems analysis study is the realistic formulation of the problem; the next phase involves construction of the models, and the last step could be described as the linkage of different models in one complex water resources system. Figure 2 gives an example of a typical scheme such as is often used for finding optimal operation rules for water resources systems. In general, the following types of models must be included in such a scheme:

- Hydrologic models;
- Hydrodynamic models;
- Models of transportation, dispersion and self-purification of wastes; and
- Models of water demands.

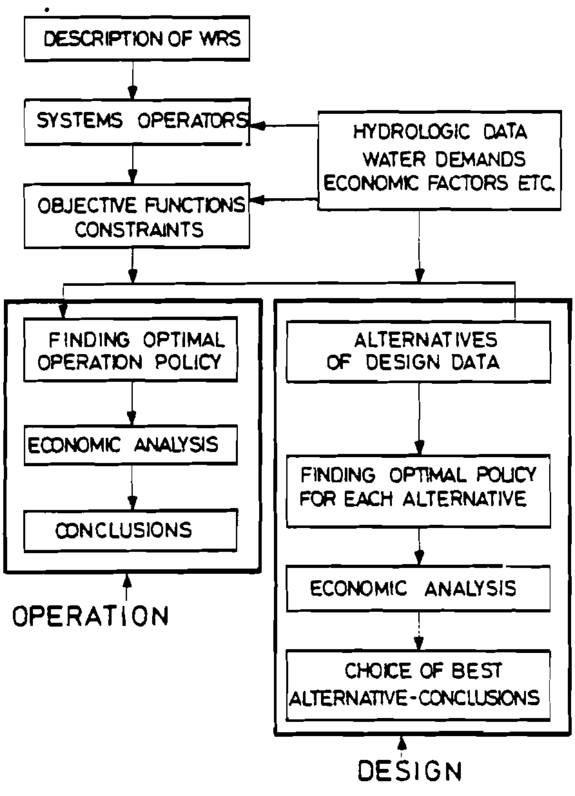


Figure 2. General scheme of river basin management (RBM).

Different optimization techniques are used both for planning and for day-to-day management of water resources systems.

The above-mentioned models and techniques, and IIASA's contribution to their development, is discussed in the Status Report by E. Wood and by Yu. Rozanov. Although modern methods are now available for practical application, there are still many unsolved problems. Let us consider three simple examples.

Example 1. Dikes, spillways, and the like are constructed on the basis of so-called "design floods" or "design peak discharges", usually described as a T-year flood or a flood that will be equalled or exceeded by a probability, $p = \frac{1}{T}$.

Different types of probability distribution are used for such analyses, as for example:

- Log-normal with 2 or 3 parameters;

- Gamma (Pearson, type III) with 2 or 3 parameters; and

- Extreme-value distribution (Gumbel).

The results differ for different distributions; for a particular profile at the Vistula River, the following values were obtained for T = 1000 years:

Q_{max} = 10,350 m^3/sec for log-normal distribution;

Q_{max} = 9,710 m^3/sec for gamma distribution; and

Q_{max} = 11,040 m^3/sec for extreme-value distribution.

The economic consequences of such discrepancies may be significant. Unfortunately, because of scarce hydrologic data no technique of mathematical statistics can help us to distinguish the hypothetical distributions. In some countries, the problem is "solved" by using government standards as the basis for choosing probability distribution of floods; in other cases, different consulting firms and research institutions use different distributions with no objective background for such practice. The problem is open and probably can be solved only on the basis of a joint physical-probabilistic approach to the flood phenomenon.

Example 2. It is well known that reservoirs are the main tools for optimal control of water resources. After about two decades of work in the field of modern stochastic storage theory, there are still many unsolved problems, and very limited practical applications of this theory can be cited. The storage theory for more than one reservoir is in the early stages of development. This lack of success indicates both the difficulties in finding exact analytical solutions in the field of water resources, and the need for a clear statement of direction for further research. Should we develop the more advanced analytical theory of water systems, or should we limit ourselves to the simulation technique, using the artificially generated hydrologic data? Have we sufficient knowledge of the structure of processes whose realization we would like to generate? Many similar questions can be asked in connection with storage problems.

Example 3. Several optimization techniques were used in water resources planning and operation during the last fifteen years. However, none can effectively account for stochastic inputs and the nonlinear objective function for a number of reservoirs and water users. Different approximations were proposed and the whole problem is open for discussion and investigation. While some existing techniques are suited to solving multidimensional problems, they cannot treat complicated descriptions of hydrologic phenomena. Others are well suited to decision problems involving multistage decision problems, but are restricted by present-day computers to optimizing no more than three or four variables. In general, the use of programming techniques requires a drastic simplification of complex water resources systems, the main question being: what are the consequences of such simplifications?

A FEW WORDS ABOUT THE IIASA WATER RESOURCES PROJECT

The idea of including water resources problems in the IIASA research program was put forward by the late Professor A.M. Letov, who was also the first leader of the Water Resources Project. In March 1974, Professor Yuri Rozanov assumed leadership of the Project; in November 1974, he was succeeded by Professor Z. Kaczmarek.

In accordance with the discussions of the IIASA Planning Conference held in June 1973 (*Proceedings*, 1973) and with subsequent discussions with IIASA National Member Organizations, during the period 1974 to 1976, the Project is concentrating on specific problems of universal methodology of water resources development and optimal operation. The research program for 1975 includes the following subtopics:

1. Intercomparison and improvement of existing stochastic models of multisite and multiseason stream-flow generation. Preparation of a research paper to be presented to the United Nations Water Conference in 1977;

2. Mathematical description of flood protection alternatives in conjunction with flood protection investment policy in the dynamic context. Adaptation and improvement of flood routing models. Multiple objective analysis for handling hydrologic and other types of uncertainty;

3. Water quality modeling. Development or improvement of mathematical models describing physical, biological and chemical interactions and changes in dispersion, and decomposition of different types of waste water discharges. Application of these models to the forecasting and optimal operation of water resources systems;

4. Mathematical models for the description and optimal operation of water storage systems. Development of stochastic storage theory, forecasting and control models. Computer implementation of management models using man-machine procedures;

5. Development of network models for optimal water allocation. Application of mathematical programming techniques to network models, taking into account water quality and quantity. Game theory analysis of interregional transfer problems. Computer implementation for test case studies;

6. Application of utility theory to problems of conflict resolution in water resources. Investigation of underlying preference structures and their effects on decision strategies. Possible application to intercity water quality conflict situations, reservoir release policy, and investment strategy.

The program for 1975 stresses research activities related to:

- Description and modeling of different water resources system operations (hydrologic and hydrodynamic models, water quality models, and so forth);

- Development and improvement of optimization techniques and models, in close cooperation with the IIASA Methodology Project.

Special attention is given to the cooperation with water-affiliated organizations in the NMO countries. We look for such cooperation in both the methodological and applied areas, especially in future large-scale demonstrations (case studies) of the practical significance of methods developed at IIASA. The present status of our cooperative research is discussed in the Status Report by Dr. I. Belyaev.

In conclusion, I would like to express the hope that this seminar on our Status Report will not only summarize the present activity of the Water Project, but will also help in formulating IIASA's future policy in this field and in ensuring its successful implementation.

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II. Project Work in Hydrologic and Water Quality Models

Eric F. Wood

(in collaboration with project colleagues)

INTRODUCTION

This section describes the types of hydrologic models used in the analysis of water resource systems, and reports on the Project work in this area.

We will follow the example of Dooge (1973) and Clarke (1973), and define a *hydrologic system* as a set of physical, chemical and/or biological processes acting upon an input variable set to convert it into an output set. Thus, we will describe work both on water quantity and quality. A *hydrologic model* is a simplified representation of the hydrologic system and can be a physical, analog or mathematical model. The research work of the Water Resources Project deals exclusively with mathematical models in which the behavior of the system is represented by a set of equations expressing relationships between variables and parameters of the hydrologic system.

There are many ways in which hydrologic models can be subdivided. Clark's paper gives an excellent review of hydrologic models and their current development and use. Figure 1 is an attempt to classify hydrologic models along lines similar to Clark's and to still display the Project's work.

Often hydrologists talk about *dynamic* and *non-dynamic* models. *Probabilistic* models are non-dynamic or static models. This class includes flood frequency analysis, regression analysis and other studies where the analysis is concerned with relationships between probabilities. For example, in flood reliability analysis, we are often concerned with the distribution of inflows.

If input variables, output variables, or error terms are random variables with certain time-dependence, then these models are regarded as *stochastic* models. This emphasizes the hydrologic modeling of the stochastic process, and all stochastic models are dynamic models. Examples of these models are: Markov streamflow models, input-output rainfall-runoff models, and many catchment models.

If all the variables of the model are regarded as free from random variation and do not have a probability density function associated with them, then these models would be

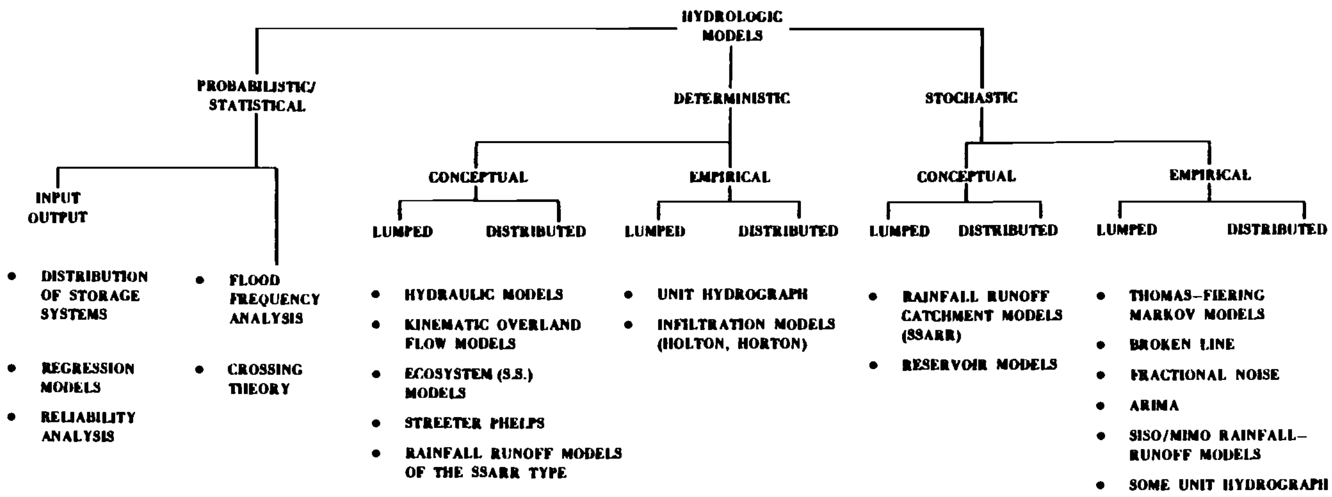


Figure 1. Hydrologic models.

deterministic hydrologic models. Deterministic models can be either dynamic or static. Examples of these models are: hydraulic models, kinematic overland flow models, and the Streeter Phelps BOD/DO model.

Hydrologic models are usually described as either *conceptual* or *empirical*. Conceptual models are based upon consideration of the physical processes acting upon the input variables, while empirical models are based upon the observation of input and output sets.

Another way of characterizing models, which is very useful to water resources systems analysis, is to consider their use in *operation* or *design* decisions. The consideration usually lies in the time scale of the analysis. Thus, virtually all static models are for design purposes, and the deterministic and stochastic models are for either operation or design.

The purpose of this general overview is to indicate both the range of models that have been applied to water resource analysis and the areas where hydrologic research of the Water Resources Project was concentrated.

INTERCOMPARISON OF STOCHASTIC MODELS

Over the last ten years the construction of empirical stochastic models for the analysis and generation of rainfall and streamflow events has been immense. Some models have been formulated on a purely statistical basis using only the observed discharge records, while others try to incorporate a physical understanding of the hydrologic process as well as a statistical basis. Because of the complexity of the hydrologic process and the relative scarcity of the data, it is not clear which models do and which do not perform well. Thus, there is a feeling that an intercomparison study of stochastic models is needed, and some initial work has begun. One immediate problem to be addressed is the establishment of adequate criteria for comparing the models. Initial discussions with water resource researchers around the world (for example, M. Fiering of the USA, N. Matalas of the USA, R. Clarke of the UK, S. Dyke of the GDR, and G.G. Svanidze of the USSR) indicate that the criteria should be closely connected to the application of the generating model. This implies that a set of criteria (or operating conditions) be set up and the stochastic models be analyzed for each of the criterion. As stated earlier, this study has just started so that adequate analysis of the problem has not been performed.

FLOOD FREQUENCY MODEL UNCERTAINTY

One study closely related to the intercomparison of stochastic (generating) models is that of flood frequency model uncertainty. In fact, certain facets of these results may give insights into the criteria for comparing the various generating models.

In flood frequency analysis, hydrologists are often confronted with the problem of choosing one statistical model from many contending models. The problem is often complicated by the fact that many models seem to fit well the available data, but each leads to different decisions. In recent years considerable progress has been made in developing statistical procedures for comparing alternative models. In 1974, these procedures were extended and applied to the problem of flood frequency analysis. The results have been reported in Wood (1974)

Consider a composite model, $\hat{f}(q|\underline{A},\underline{\Theta})$, made up from the competing statistical models, $f_i(q|A)$, of the form

$$\hat{f}(q|\underline{A},\underline{\Theta}) = \Theta_1 \cdot f_1(q|\underline{A}_1) + \cdots + \Theta_n \cdot f_n(q|\underline{A}_n) \ , \qquad (1)$$

where

$$\underline{A} = \bigcup_{i=1}^{n} \underline{A}_i$$

$$\underline{\Theta} = \sum \Theta_i = 1 \ .$$

The composite model, $\hat{f}(q|\underline{A},\underline{\Theta})$, is conditioned upon a set of unknown model parameters $\underline{A}$ and an unknown composite model parameter set $\underline{\Theta}$. $\Theta_1, \ldots,$ and Θ_n are parameters that take on a value of either 0 or 1; their value is uncertain. If $\Theta_1 = 1$, then model $f_i(q|\underline{A}_i)$ is the true model.

It can be shown that the Bayesian distribution for the composite model is

$$\tilde{f}(q) = p''(\Theta_1) \cdot \tilde{f}_1(q) + \cdots + p''(\Theta_n) \cdot \tilde{f}_n(q) \ , \qquad (2)$$

where

$\tilde{f}(q)$ = the composite Bayesian distribution for flood discharges;

$\tilde{f}_i(q)$ = the Bayesian distribution of model i (see Wood and Rodriquez-Iturbe, 1975); and

$P_i(\Theta_i)$ = the posterior probability that the probability model i is the true model.

The posterior model probabilities can be shown to equal

$$p''(\Theta_i) = \frac{K_i}{K^*} p'(\Theta_i), \qquad (3)$$

where

$p'(\Theta_i)$ = the prior model probability;

K_i = the marginal likelihood function of the observed data coming from the i-th model; and

K^* = a normalizing constant.

The functional form of K_i and K^* is derived analytically in Wood (1974) for three models.

Some Monte Carlo experiments were carried out with samples generated from known distributions. A sample length n was sampled m times, where n varied from 10 to 200. Because of computer limitations, $n \cdot m$ was held constant at 3000. The results presented in Figures 2 and 3 show that the technique developed here can distinguish between competing models, often with short record length. This is especially true when comparing Figures 2 and 3 since the generating model and the competing model are reversed.

The composite Bayesian distribution of Equation (3) is the probability model which should be used in making inferences about future flood discharges and in evaluating the expected utility of various flood protection decisions.

APPLICATION OF RELIABILITY ANALYSIS TO FLOOD LEVEE DESIGN

Another statistical analysis, which is an extension of flood frequency analysis, is the use of probabilistic procedures to analyze the reliability of flood levees. This extension expands the analysis of flood levees from one-dimensional consideration of flood stages to multidimensional considerations. The analysis, reported in Wood (1975), considers the load upon the levee owing to floods that have been generated by some stochastic process. The levee is defined by two decision variables, the height H, and the base width W. The flood discharge at which failure occurs, q_o, is considered a fixed but unknown quantity and is represented by a probability distribution function $f(q_o)$. The resistance of the levee therefore depends upon the occurrence of floods of a particular magnitude and upon the "strength" of the levee. It is conceptually convenient to consider such uncertainty within the framework of Bayesian risk analysis. Higher flood resistance levels lead to higher and stronger levees, but such levee systems are extremely expensive and, if extended far enough, lead to lower net benefits. Certain tradeoffs exist between the objectives of levee reliability and economic benefits. These tradeoffs are particularly significant when the resistance of the levee is considered a random variable.

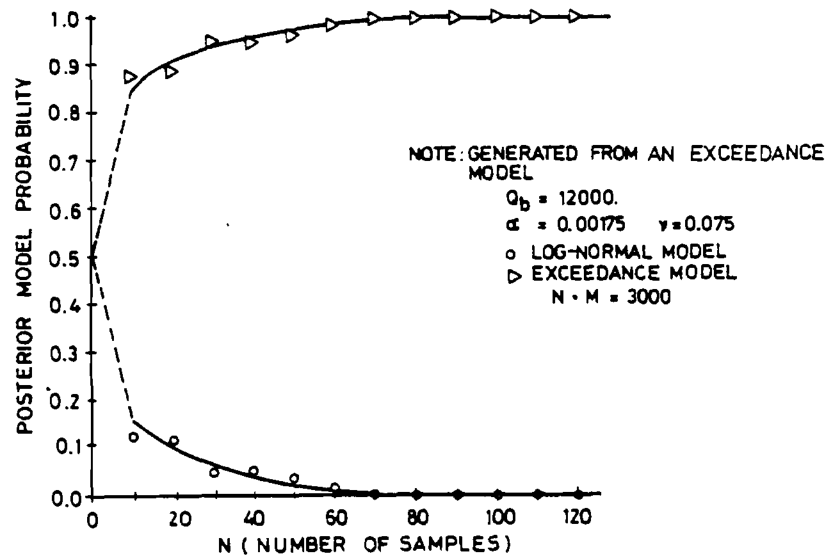


Figure 2. Posterior model probability versus sample size. exceedance model.

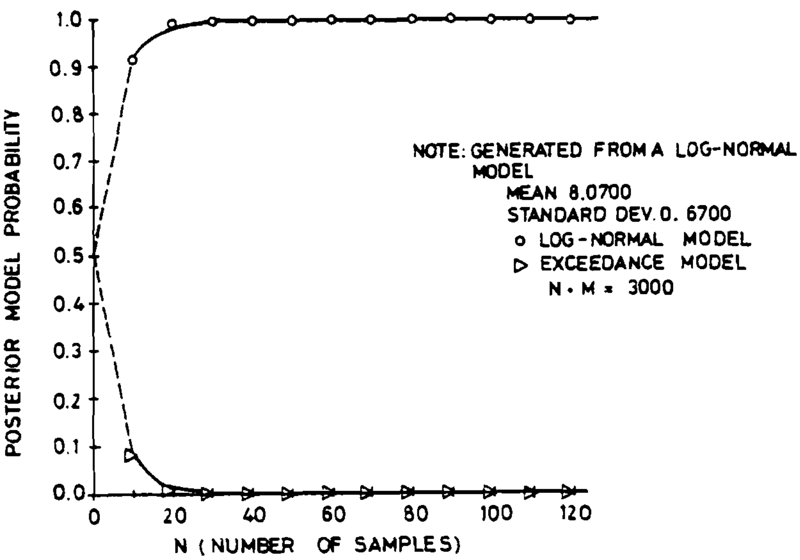


Figure 3. Posterior model probability versus sample size, log-normal model.

If the resistance of a levee system, q_o, is deterministic at a given flood discharge, q_d, then the reliability of the system against failure is easily found from the probability of failure:

$$p_f = \int_{q_d}^{\infty} f_Q(q)\ dq = 1 - F_Q(q_d) \quad , \tag{4}$$

where

p_f = probability of failure;

$d_Q(q)$ = probability density function of flood events; and

$F_Q(q_d)$ = cumulative distribution function at the design discharge q_d.

The reliability of the system is $1 - p_f$. If the resistance of the levee system is uncertain and if the level of resistance, q_o (maximum discharge before levee failure), is described by the density function, $f_{Q_o}(q_o)$, then the probability of failure $\tilde{p}_f$ is found from

$$\tilde{p}_f = \int_{q_o=0}^{\infty} f_{Q_o}(q_o) \cdot \int_{q=q_o}^{\infty} \tilde{f}_Q(q)\ dq\ dq_o$$

$$= \int_{q_o=0}^{\infty} f_{Q_o}(q_o) \cdot [1 - \tilde{F}_Q(q_o)]\ dq_o \quad . \tag{5}$$

It can be easily shown that the probability of failure (5) is the expected probability of failure, $E[p_f]$, of the density function for failure $f(p_f)$. The second moment of $f(p_f)$ is

$$E^2[p_f] = \int_{q_o=0}^{\infty} [1 - F_Q(q_o)^2] \cdot f_{Q_o}(q_o)\ d_{q_o} \quad . \tag{6}$$

In modeling flood events greater than a particular size as an exponential distribution and in modeling occurence of such floods as a Poisson process, it was possible to calculate the distribution of flood levee reliability. Two modes of failure were considered: structural failure of the levee before the flood stage exceeded the levee, and overtopping of the levee. The probability that failure would occur in each of the modes was determined a priori by considering the structural design variables of levee. The probability of structural failure was modeled with both a uniform probability density and a quadratic density function.

The effect of levee strength on the flood frequency curve owing to varying levee strengths is illustrated in Figures 4 and 5.

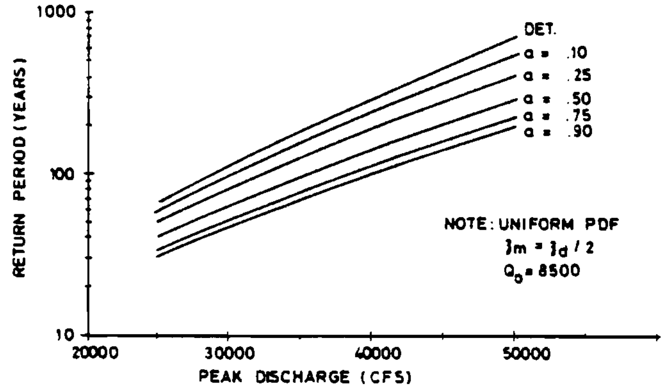


Figure 4. A flood frequency curve for uniform pdf.

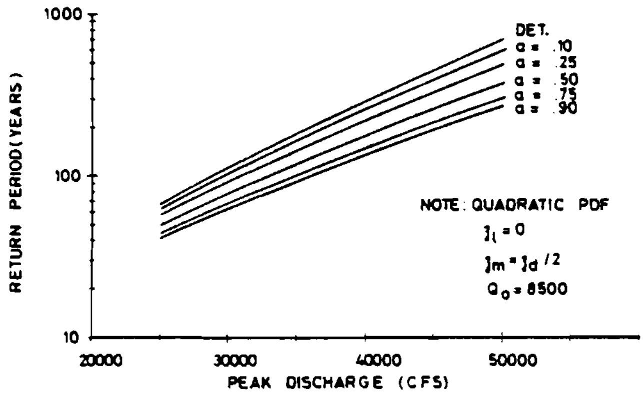


Figure 5. A flood frequency curve for uniform pdf.

In the figures, the coefficient "a" refers to the value of the cumulative distribution function for structural failure at the "design" discharge; that is, when overtopping occurs. Thus a value of a = .25 implies that the probability of structural failure of the levee before overtopping is .25.

It is interesting to note that for a design discharge of 35,000 cu ft/sec and a uniform failure probability density function, a levee that will fail only by overtopping (a = 0) has an expected return period of almost 200 years, while a levee that has a 90% chance of failing before overtopping has an expected return period of 70 years--only one third the value of the former. With a quadratic failure probability distribution, a levee of the same "strength" has an expected return period of 90 years, or about one half that of the deterministic levee.

In a manner similar to the analysis of the failure probability, the damage from levee failure was also considered. Assume that the damage function is of the form:

$$D(q,q_o | q_d) = cq_d^{.5}(q - q_o)^2 \quad ; \qquad (7)$$

then the expected damage for a known failure discharge, q_o, is

$$E[D|q_o,qd] = \int_{q_o}^{\infty} cq_d^{.5}(q - q_o)^2 f(q)\,dq \quad . \qquad (8)$$

Considering that q_o is a random variable, the expected damage for various levee strengths was calculated. Using a simple transformation, the expected net benefits can also be derived. These results are presented in Figures 6 and 7.

The procedure developed here can also be applied to the analysis of other systems, for example, the distribution of isotherms from thermal power plant outfall and the reliability of the large water resource systems for flood control or for water supply. Many of the extensions may require numerical analysis as opposed to the analytical derivations of this study, but that should not limit applications.

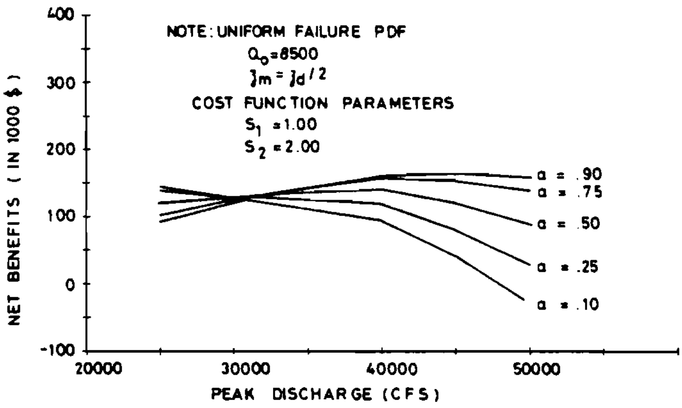


Figure 6. Net benefits versus design discharge.

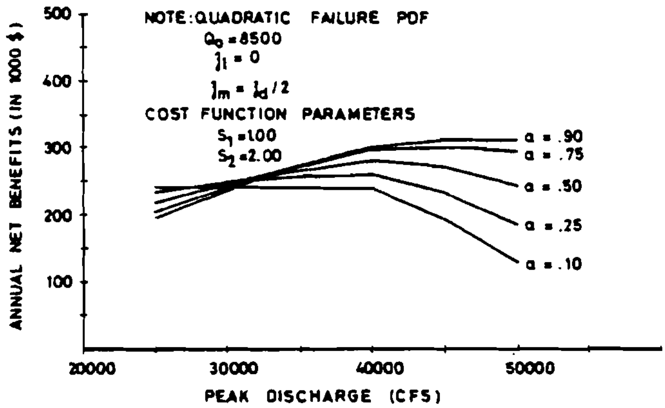


Figure 7. Net benefit versus design discharge.

ANALYZING UNCERTAINTY IN RAINFALL-RUNOFF MODELS

A third study involving flood frequency analysis considers uncertainty in the parameters of "deterministic" rainfall-runoff models used to obtain derived flood frequency curves. Flood frequency analysis using distribution theory has the basic assumption that the probability of a flood of given magnitude does not change with time. Thus, basins that are non-stationary (through urbanization, for example) are usually analyzed by considering the stochastic rainfall process and overland flow routing through simulation. This procedure is often referred to as "derived" flood frequency curves.

The basis of these derived flood frequency curves is the concept that there exist in the rainfall intensity--storm duration plane ($\bar{I}$ - t_r) lines of constant peak discharge from catchment area. This is shown in Figure 8. Thus to find the probability that the peak discharge will be less than some value q_m, $F(q_m)$, we must find the volume under the joint density function f $(\bar{i},t_r)$ of the intensity--storm duration and the boundary q_m equal to a constant. This is equal to solving the integral

$$F(q_m) = \int_{r_{q_m}} f(\bar{i},t_r)\, d\bar{i}\, dt_r \quad . \tag{9}$$

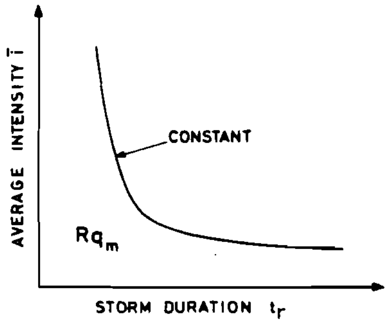


Figure 8. $\bar{i}, t_r$ plane showing peak discharge.

Of course, owing to uncertainty in the runoff model we do not know that the exact boundary of q_n is equal to a constant. Current practice is to put in average values for uncertain parameters and stochastic processes (for example, for infiltration). Our analysis, which considered the infiltration to be a stochastic process, shows that the flood frequency curve may be seriously underestimating peak discharge for the given risk level. This can be seen in Figures 9 and 10.

SOME PROCESSES IN DAM STORAGE

Rozanov's (1975) consideration of some stochastic reservoir storage properties may give insights into systems approaches to water resources. He develops properties of dam storage by considering physical arguments. For example, it is often observed that natural storage can be characterized by the property that the discharge kX_t is proportional to the reservoir level x_t. Furthermore, by considering long-term operation it may be reasonable to treat the reservoir inflow hydrograph, $\dot{\xi}_t$, as an impulse-type process with random peaks occurring independently because of rainfall. Moreover, if time bases of the hydrograph peaks are comparatively small then the reservoir inflow

$$\xi = \int_{t_o}^{t} \dot{\xi}_s d_s \quad , \quad t \geq t_o \tag{10}$$

may be considered a random process with independent increments.

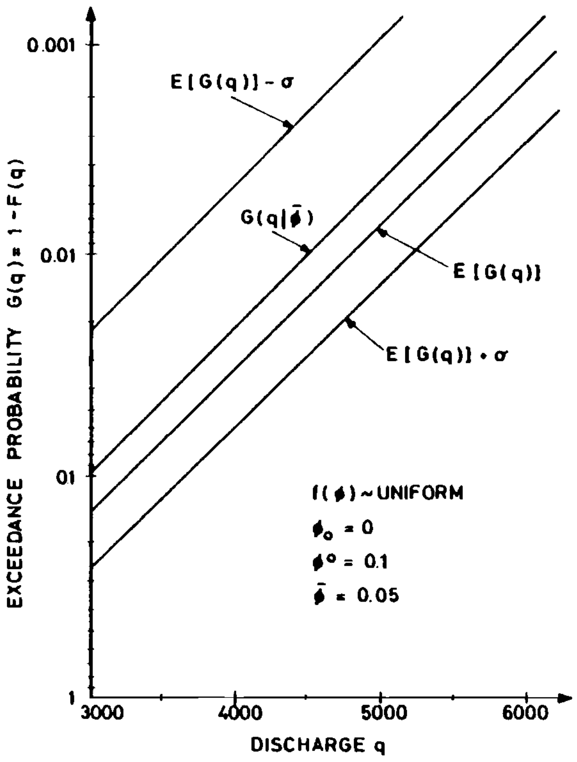


Figure 9. Frequency curves for $f(\phi)$, uniform with $\bar{\phi} = 0.05$ in/hr.

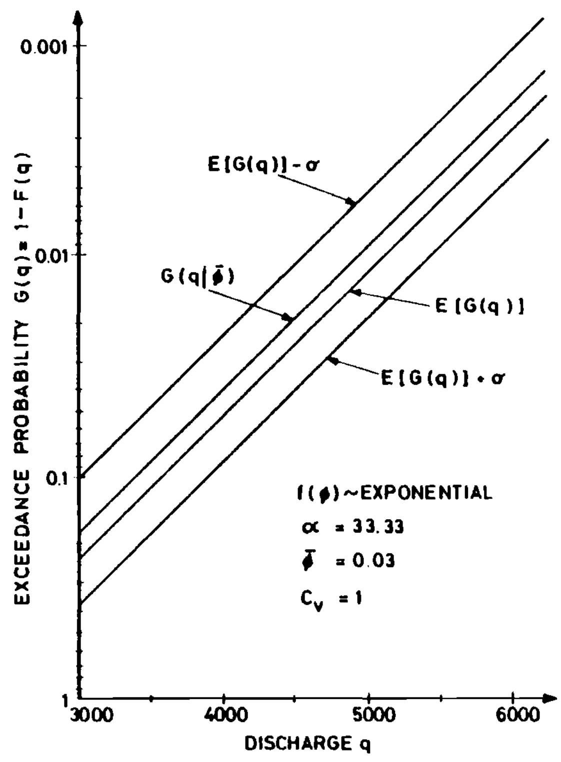


Figure 10. Frequency curves for $f(\phi)$, exponential with $\bar{\phi} = 0.03$.

Thus, the reservoir process x_t satisfies the stochastic integral equation

$$x_t = x_{t_o} + w_t - \int_{t_o}^{t} kx_s \, ds \quad , \; t \geq t_o \quad , \tag{11}$$

so the process x_t is Markovian.

The assumption that the inflow process has independent increments is not realistic in the case of continuous time, t, except after large time intervals, T, of reservoir operation. Namely, if random fluctuations in the reservoir inflow arise mainly because of short-time rainfall, and if transient time for the water to reach the reservoir from the catchment is small in comparison with the time T, then one can neglect not only the unlikely coincidence of random hydrograph peaks but also any influence of inflow variables on each other. In other words, one can treat the inflow process ξ_k; $k = 0,1,\ldots,$ as a series of independent variables. If the operation of a reservoir depends upon the current period only and is a function of x_k, the available amount of water, then the variables $\xi_o \xi_1 \ldots$ are independent, and the reservoir process x_k; $k = 0,1,\ldots,$ is Markovian.

HYDRODYNAMIC ASPECTS OF DETERMINING LEVEE HEIGHTS

The problem of finding the required dike height along a channel or river reach is an important element of the river basin project. An analysis should contain a system of models and algorithms for the design of the protection constructions. Now one of the simplest models of the type will be discussed. Koryavov (1975) developed the following procedure. Consider the situation where an investment in dike construction is limited and we must find the dike height D(X) which leads to minimum losses from floods, under the condition of a fixed investment, $\hat{L}$, and with the following assumptions:

- River flow in the channel system is described by stationary Saint-Venant equations;

- Stochastic features of the process are ignored.

Let S be flood lossed which is some function of the amount of the water Q overflowing the dike along an interval $0 \leq x \leq X$; thus

$$S - f(Q(X)) \quad , \tag{12}$$

where x[0,X] is the distance along the reach of the river or the channel from its beginning.

Denote

$$\dot{Q}(x) = q(x) \quad , \tag{13}$$

where q(x) is the discharge of the overflow over a dike in the cross-section of the river with coordinate x. This value depends on the height of a dike D(x) and the depth of water in the river h(x). Thus

$$q(x) = g(D,h) \quad ; \tag{14}$$

Koryavov used an empirical formula for this relationship.

The depth h satisfies the differential equation for steady flow in open channels. Let us now specify the functions in differential equations of the problem.

The steady-state flow in the channel could be described by Saint-Venant equations

$$\begin{aligned} u\frac{du}{dx} &= g(\Theta - \chi) - g\frac{dh}{dx} \\ u\frac{dh}{dx} + h\frac{dh}{dx} &= \frac{1}{B}q(x) \quad . \end{aligned} \tag{15}$$

Here

u = the average velocity of the inflow;

B = the width of the channel, which we assume constant;

g = the acceleration of gravity;

Θ = the slope of the channel bed;

χ = the "frictional slope" of the channel, a nonlinear function of the velocity u:

$$\chi = \frac{u\,u}{\gamma R^n} \quad ;$$

R = the hydraulic radius equal to the ratio of the cross-section area of the water to the wetted parameter; and

γ and n = positive empirical parameters.

Thus the differential, h, can be expressed as $\dot{h} = F(d,h,Q)$. Investment for dike construction can also be represented in a differential form, $\dot{L} = \ell(D,x)$. For this Koryavov used a form, $\dot{L} = aD^b$.

Koryavov reduced the problem of finding the height of the dike to an optimal control problem where the height of the dike plays the role of control variable.

The accurate statement of the problem considered is the following: find control function D(x) and phase variables Q(x), h(x), and L(x) related by differential equations so that S is a minimum. The solution procedure was to use Lagrange multipliers λ_Q, λ_h, and λ_L and to construct the Hamiltonian functions (see Bryson and Ho, 1969)

$$H = \lambda_Q(D,H) + \lambda_h F(D,h,Q) + \lambda_L \ell(D) \quad ; \qquad (16)$$

D(x) can be defined as a function of λ_Q, λ_h, λ_L, Q, h, L, and X.

To solve this problem a standard Newton method program was used. The problem presented could have different forms. In particular, it can be formulated as a dual problem: find the minimum investment for the dike construction for a given level of flood losses. By solving duel problems we could find functions $L(S,h_o)$. Such a function could serve as a base for making decisions on choosing the shape of the dike.

OPTIMAL PREDICTION SCHEME FOR MULTIPLE INPUT - MULTIPLE OUTPUT (MIMO) HYDROLOGIC MODELS

One class of hydrologic models that is finding greater application today than ever before is that of the stochastic empirical models of MIMO formulation, many of which have been developed in other systems fields, notably in aerospace applications. Such models are now being applied in water resources for particular problems such as rainfall-runoff models of very large catchments and real-time operation of reservoirs using on-line rain-gauge measurements. The results have been encouraging.

This work, reported by Szöllösi-Nagy (1975), uses the Kalman filter technique, which is extremely useful for describing stochastically excited dynamic systems. The procedure is based on the state-space time domain formulation of the process.

Consider a water resource system whose behavior evolves on the discrete time set $T = t_k$; $k = 0,1,2,...,$ can be described by

$$\underline{x}(t_{k+1}) = \mathcal{S}[\underline{x}(t_k),\ \underline{u}(t_k),\ \underline{w}(t_k)], \qquad (17)$$

$$\underline{z}(t_k) = T[\underline{x}(t_k,)\underline{v}(t_k)], \qquad (18)$$

where

$\underline{x}(t_k)$ = the n-vector of the states of the system at the discrete time $t_k \varepsilon I$;

$\underline{u}(t_k)$ = the s-vector of control variable or known system inputs;

$\underline{w}(t_k)$ = the r-vector of uncertain disturbances "driving" the system;

$\underline{z}(t_k)$ = the m-vector of measurements on the system;

$\underline{v}(t_k)$ = the m-vector of uncertain disturbances corrupting the observations; and

$\mathcal{J}$ and T are certain functionals, characterizing the properties of the particular system.

Equation (17) is called the state equation and Equation (18) is the measurement equation, as the measurement noise $\underline{v}(\cdot)$ is sometimes referred to as measurement uncertainty, while some components of $\underline{w}(\cdot)$, or the entire $\underline{w}(\cdot)$ itself, might be referred to as model uncertainty. Considering the simple example of a reservoir system consisting of n·reservoirs, $\underline{x}(t_k)$ might be sought as a vector composed of the values of the amount of stored water of each reservoir at time t_k; $\underline{u}(t_k)$ as a vector of water releases (control variables); $\underline{w}(t_k)$ as the vector natural (uncontrolled stochastic) inflows to the reservoirs; and $\underline{z}(t_k)$ as the vector of measured outflows from the reservoirs. In this case, the state vector $\underline{x}(\cdot)$ refers to actual physical states, namely to the amount of stored water in the system; but it is not at all necessary to associate the state vector with "physical" states. In other words, one can choose among different types of state variables to describe the same process.

One type of application may be as follows. It is well-known that a fairly large class of hydrologic systems (for example, rainfall excess/surface runoff, and runoff/transformations of flood routing) can be described by a consolution type of model

$$y(t) = h(t) * u(t) \quad , \tag{19}$$

where

u(t) = the input of the system (either controllable or non-controllable);

h(t) = the impulse response of the system;

y(t) = the output process;

The asterisk denotes the convolution.

In practice, however, we have only noise corrupted measurements

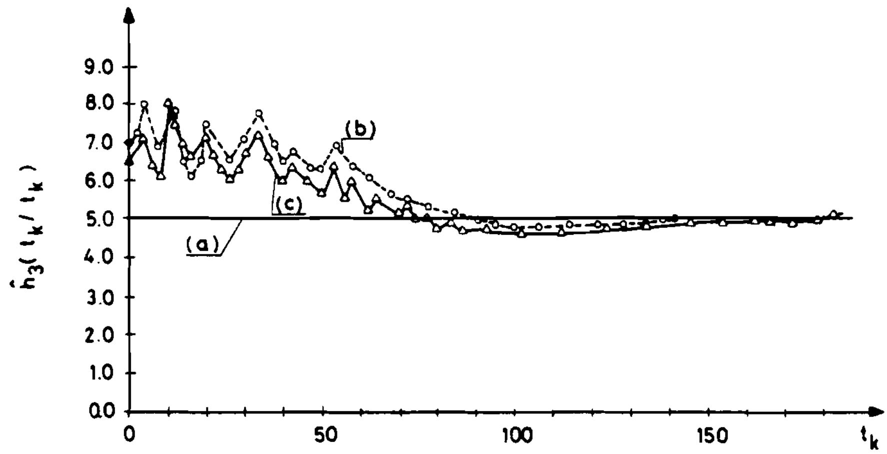
$$z(t) = y(t) * v(t) \quad , \tag{20}$$

where $v(t)$ is an unknown noise process. Hence, for linear time invariant lumped systems,

$$z(t) = \int_{-\infty}^{\infty} h(\tau)u(t - \tau)\, d\tau + v(t) \quad , \tag{21}$$

where in case of physically realizable systems the upper bound of the integration is t. Note that although the system was assumed linear, in case of slight non-linearities the noise process $v(\cdot)$ might be sought as a term including those "small" non-linear disturbances.

Using this model, an adaptive algorithm was developed. The procedure was tested with the following simple simulation exercise. A given impulse response was assumed; using that and an arbitrary input sequence, the output process was calculated through the simple discrete convolution. Then a Gaussian white noise sequence was generated with zero mean and variance 0.1. This sequence was added to the output process; the resulting noise corrupted sequence and the original input sequence were further analyzed to see whether the algorithm does or does not give "back" the impulse response assumed. As an example, Figure 11 shows the situation for a particular ordinate of the



Notes: (a) = true third ordinate of the impulse-response, h_3;
(b) = estimated values using prior knowledge;
(c) = shows how ordinate's estimated values evolve when there is no prior knowledge.

Figure 11. Sequential estimation of the third impulse response ordinate, h_3.

impulse response. The constant line (a) means the "true" third ordinate of the impulse-response, h_3; curve (b) shows its estimated values using the prior knowledge (if it is available) of the variance; curve (c) shows how its estimated values evolved when there is no prior knowledge, i.e. an initial guess for the variance had been considered and the adaptive noise variance estimation technique was used.

WATER QUALITY MODELS

A fair amount of work is also being done on water quality modeling. This work is mostly concerned with the quality of water as measured by biological oxygen demand (BOD), dissolved oxygen (DO), and concentration of pollutants of various types. As was reported earlier in the Status Report, hydrologic models play an important role within the systems analysis and are often enmeshed in the optimization procedure. This was true in Koryavov's dike problem and is also true in the work performed by Szöllösi-Nagy and by Ostrom in conjunction with the Methodology Project (see Gros and Ostrom, 1975).

Szöllösi-Nagy's study used a filtering algorithm, similar to the MIMO description given previously, to find the optimal control (treatment) policies for a polluted river reach. To describe the self-purification process the Streeter-Phelps model, modified by the mode/measurement uncertainties, was used. The control variables were as follows: timing of effluent discharge from a sewage treatment plant, and timing of artificial aeration facilities along the river, if there are any. The main difference from the previous hydrologic model is that the water quality model is a distributed parameter model.

In their study, Gros and Ostrom were concerned with the impact of waste discharges and various water quality standards of other users. Since preferences for water quality levels were considered explicitly, quality standards were not introduced as constraints; rather, they were represented as parameters of a utility objective function. They wanted to include in the model a representation of the decision process by a water authority; consequently, a simplification was made in expressing the relationship between waste water treatment and water quality. The self-purification and transportation phenomena were modeled using the Streeter-Phelps equations. Hydrologic uncertainty in the ambient river conditions (for example, flow rate or waste loads) was expressed by discretizing a cumulative probability function. The analysis using a control theory approach produced various operating trajectories which considered the preferences for water quality and costs of waste treatment by each user, the impacts of waste discharges on other users and the uncertainties in inputs.

Schmidt (personal communication) has started to develop a generalized description of water quality, as expressed in terms of physical, chemical and biological properties. The physical state is characterized by temperature, turbidity and suspended matter. The chemical state includes all inorganic and organic substances which are measurable as single components or as lumped groups (for example, BOD, electrical conductance), while the biological state is expressed by the saprobic indices and their relationship to the oxygen balance as well as to the consumers and producers in an aquatic community.

All models under development include river flow as an important factor. Depending on this condition, Schmidt's models are divided into models under steady and unsteady flow conditions. Models for steady flow conditions are involved in long-term strategies for decision making, while those for unsteady flow conditions are involved in simulation models for long- and short-term strategies for river management. In general, the steady flow models include the prediction of the distribution of the quality criterion in two dimensions of river bodies. This is important for the prediction of mixed lengths and for the reliable evaluation of reaction rates in river stretches.

The unsteady flow models include time and space dependence of water quality criteria. In general, the solutions are one-dimensional in space, but for temperature and salinity a two-dimensional solution is available. The solutions are mainly given as numerical results; only some of the steady flow models for physical criteria are given as closed analytical solutions.

It is planned that the models will be completed shortly and will be tested using data from various sources.

CONCLUSION

This report indicates the type of problems in hydrology in which the Water Resources Project is engaged. The report is not exhaustive; a list of papers published by the Project is given in Appendix I.

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III. Optimization Techniques in Systems Analysis of Water Resources

Yuri A. Rozanov
(in collaboration with project colleagues)

Because of continuously expanding urbanization and industrialization, one can observe water shortages and intense rivalries for water among users. In many countries there are complex problems of water resources distribution, quality management and proper development of river basins. Where a river basin involves several countries, the problems are international.

During the reported period, various aspects of these problems were considered from the point of view of systems analysis and general optimization techniques.

It should be noted that water engineers are increasingly interested in the systems analysis of water resources problems. There are a few interesting applied projects of complex use of water resources, apart from many "academic" approaches based on standards methods of linear and dynamic programming. Nevertheless, we believe that the systems analysis methodology needs further development in the field of water resources.

Our efforts have concentrated mainly on solving problems of optimal water distribution with application to dam storage, and on problems of water supply systems with water distribution in conflict situations involving a few integrated users, as, for example, different countries.

When applying any sensible procedure for water distribution over a comparatively long period, one has to take into account future situations. It is here that the main difficulties arise, because in general future water inflow is uncertain and some kind of forecast mechanism must be applied.

I. A general water supply scheme may be described in the following way. A sequence of time periods $(t_k, t_k + T)$; $k=0,1,\ldots,$ is considered, and the water supply during the k-period is determined by an operational graph, $z_k = a_k(x,\omega)$, as a function of the available water, $x = x_k$, and a record of the current river basin.

The variables x_k appear as

$$x_k = y_k + \xi_k \quad , \quad y_{k+1} = x_k - z_k \quad (k=0,1,\ldots,) \quad ,$$

where

y_{k+1} is the water surplus;

z_k is the water supply (y_o is some initial variable); and

ξ_k is the proper water inflow during the k-period.

The time periods $(t_k, t_k + T)$ are not necessarily such that $t_{k+1} = t_k + T$, as they are, for example, in a reservoir water release operation. The water supply along a river basin may be divided into sections $S_o, S_1, \ldots$; y_o is an initial amount of water going down the river during the time period $(t_o, t_o + T)$; its transformation between each of the sections, S_k, S_{k+1}, is represented by the corresponding variables y_k; $K=0,1,\ldots,$. In this case, $t_k \sum_{i=1}^{k} T_i$, where T_i is the time required for the water to reach section S_k from S_{k-1}; $k=1,2,\ldots,$.

Three different levels of water control can be distinguished. First, there is the problem of meeting the minimum water demands of human beings which is separate from other control problems; this gives us constraints of the type

$$z_k \geq a_k \quad ; \quad k=0,1,\ldots, \quad .$$

Secondly, there is a flood control problem when too much water goes through one or another river section during a short time period, thereby causing floods. Finally, there is a regular water flow that can be controlled for the purpose of optimizing the water supply operation. Let us associate with each water supply, z_k, a proper loss, $f_k(z_k)$, that occurs in the case of water deficit, $w_k - z_k$, where variable w_k represents water demands during the k-period; $k=0,1,\ldots,$. In this case, the optimal water supply problem may be that of minimizing the total loss

$$\Phi(z,\xi) = \sum_{k=1}^{\tau} f_k(z_k)$$

as a function of $z = \{z_k\}$, which depends on the inflow process $\xi = \{\xi_k\}$. Here variable τ means the first "wet" time period among $(t_k, t_k + T)$; $k=0,1,\ldots,$ after which the previous water supply process, $z_o, \ldots, z_{\tau-1}$, does not matter. (A wet period is not necessarily a flood.)

Note that a flood wave can be diminished by a corresponding "water supply" to some reservoir system; however, the problem of the proper flood catchment capacity of the reservoir system should be considered separately from that of the regular water supply, because, as we believe, flood damage is normally incomparably high with respect to the regular water deficit loss represented by the loss functions $f_k(z_k)$.

The loss, $f_k(z_k)$, associated for various reasons (that are not all technical) with the water deficit $w_k - z_k$, is thought to be convex as is shown in Figure 1. One can easily recognize the main difficulty of the minimization problem

$$\Phi(z,\xi) \longrightarrow \min_z$$

namely, that at the current time, $(t_k, t_k + T)$, the future inflow, ξ_j, $j > k$, forms a random process.

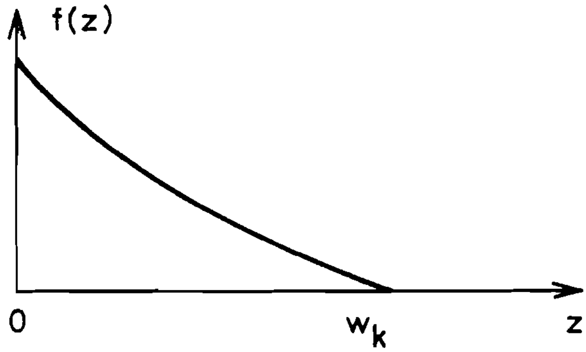

Figure 1.

In the case of expected loss minimization, the corresponding future loss is determined as follows:

$$F_k(y) = \min_z \{E \sum_{i=k+1}^{T} f_i(z_i)/\omega_k\} \quad ,$$

where ω_k is the river basin record up to the current k-period, including the observed inflow ξ_k.

Because of the usual lack of proper statistical data, a future deficit loss may be estimated as follows:

$$F_k(y) = \min_z \max_{\xi \geq \underline{\xi}} E\{ \sum_{i=k+1}^{T} f_i(z_i)/\omega_k\} \quad .$$

Here $\xi = \{\xi_k\}$ represents some reliable low estimate of future inflow

$$P\{\xi \geq \xi/_{\omega_k}\} \sim 1 - \alpha \quad ,$$

where $1 - \alpha$ is a proper level of reliability. This approach to minimizing the stochastic loss function gives us the operational strategy $z^o = \{z^o{}_k\}$, which is optimal in the sense of the minimax principle in our operational game against nature, with possible inflow $\xi \geq \underline{\xi}$. That is, based on general (physical) assumptions, we have

$$\min_z \max_\xi \Phi(z,\xi) = \max_\xi \min_z \Phi(z,\xi) = \Phi(z^o,\underline{\xi}) \quad ,$$

where the game solution z^o is the minimizing point of the loss $(z,\xi) = \sum_{k=0}^{\tau} f_k(z_k)$ with respect to the inflow process $\underline{\xi} = \{\underline{\xi}_k\}$ (Rozanov, 1975):

$$z^o: \quad \Phi(z,\xi) \longrightarrow \min_z \quad .$$

The computation program for each of the minimization steps mentioned above deals with

$$f(z) + F(x - z) \longrightarrow \min_z \quad ,$$

and, in the case of convex functions f, F can be based on the following property.

Let $z^o = z^o(x)$ be a minimizing point with respect to parameter x which represents a resource distributed in some units Δx. Then

$$z^o(x + x) = z^o(x) \text{ or } z^o(x) + x \quad ,$$

according to $f(z^o + \Delta x) - f(z^o)$ being more or less than $F(y^o + \Delta x) - F(y^o)$, where $y^o(x) = x_o - z^o(x)$. (Such a recurrent equation lets us easily determine $z^o(x)$, $x = 0, \Delta x, 2\Delta x, \ldots, .$)

Note that this very simple minimization procedure is not valid in the case of non-convex functions. For example, suppose that the future loss F can be reduced only by a significant water supply $y > \Delta x$, but that the current loss f decreases even with a minor water supply Δx. Then according to our recurrent equation, one has to meet current water demands using all available water: $z^o(x) = x$, and $y^o(x) = 0$ for any x. Obviously this procedure is wrong in a case such as that shown in Figure 2 below where $z^o(x) = 0$, and $y^o(x) = x$ for $x = 4\Delta x$.

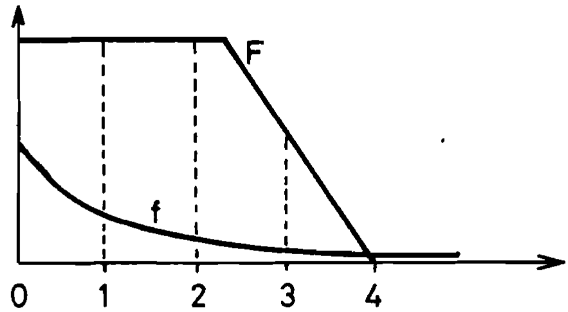

Figure 2.

With respect to reservoir operations, one does not release at once the corresponding amount of water, $z = z(x)$, as is often supposed in mathematical models; also, the water distribution during time depends particularly on water demands per time unit and canal capacity. If these river basin characteristics are constant during the time period considered, $(t_k, t_k + T)$, then a local policy may be of the following type. The amount of water Δz_t per time unit Δt released with constant discharge at the current time interval, $(t, t + \Delta t)$, is

$$\Delta z_t = \min \{c, z(x_t - z_t\} \quad ,$$

where

x_t is the water available during the period (t_o, t);

z_t is the water already released from the reservoir during the period (t_o, t); and

c is the operational constant limited by the canals capacity.

Note that this procedure, and any other type of continuous reservoir operation, implies that operation graph, $z = z(x)$, must be a monotone function. This property holds true for optimal operational graph, $z = z^{o}(x)$, in the case of the convex loss functions f_x introduced above. (The typical operational graph, $z = z^{o}(x)$, is shown in Figure 1.)

II. Approaches to solving water distribution problems based on a concept of total water deficit loss may be irrelevant in the case of a conflict situation in an international river basin (when there is no direct interest in the total loss reduction). In this case, the following principle may apply.

Suppose that in an attempt to reach a compromise, some bargain-level $2 = \{w_k\}$ has already been achieved. (Here w_k means the water demand claimed by the k^{th} partner.) Let z be a set of all feasible distributions, $z = \{z_k\}$, considered as points in a proper metric space; $z = \{z_k^o\}$ is a point at minimal distance from the ideal feasible point $w = \{w_k\}$:

$$\rho(z^{o},w) = \min_{z \in Z} \rho(z,w) \quad ,$$

The point $z^{o} = \{z_k^o\}$ is suggested for resuming negotiations (Figure 3).

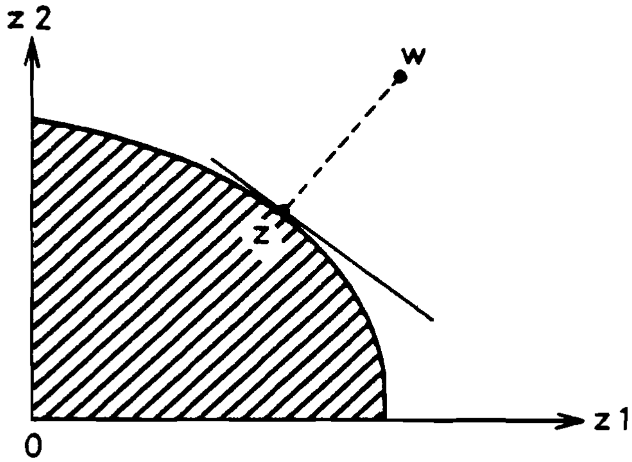

Figure 3.

This minimization problem and its stochastic versions (involving random parameters ξ_0, $\xi_1,\ldots,$) can be reduced to the loss-function scheme considered earlier in a quadratic metric case of

$$\rho(z,w) = \sqrt{\sum_{k=0}^{n} C_k(z_k - w_k)^2}$$

by using the loss functions

$$f_k(z) = C_k(z_k - w_k)^2 \quad ; \quad k=0,1,\ldots, \quad .$$

With respect to our knowledge of the random inflow process, $\xi = \{\xi_k\}$, several different minimization procedures have been applied, in particular minimization of expected total loss (see Letov and Rozanov, 1974).

The problem of conflicting water distribution is typical for decision theory; in the IIASA Water Resources Project, many aspects of general decision theory were carefully analyzed with a view to applying them to multi-objective water resources problems. The choice of the utility function is the major problem when we are dealing with groups having different interests. Particular attention was paid to the multi-objective utility function of a structure that was consistent with given single-objective functions representing separate interests. It was suggested that for many important cases, including reservoir control problems and water quality management, the multi-objective utility function could be chosen in additive or multiplicative form.

A special study was made of a river basin water quality system, the river being divided into several reaches with single-expected utilities. Using a weighted sum of these utilities, subject to some constraints, a technique for obtaining Pareto-admissible decisions was suggested. In the case of the Rhine River, important observations were made about the sensitivity of the results to the parameters of the utility function used (see Ostrom and Gros, 1975).

III. Some methodological aspects of optimal water control do not touch upon the actual interactions of water management with other (ambient) systems in the river basin. We now consider such problems with respect to the optimization models of multi-reservoir system WR and its environment, called ambient system A.

All processes in both systems are considered in a period of integer time $k=1,2,\ldots,$. System WR includes m reservoirs $i=1,\ldots,m$, and M water users $j=1,\ldots,M$. If the j^{th} user at the k^{th} time stage is supplied from the i^{th} reservoir with the amount of water z_k^{ij}, he will suffer a loss $f_k^{ij}(z_k^{ij})$ which is zero if there is no actual water deficit. Apart from the water deficit loss $f_k^{ij}(\cdot)$, there are other loss functions: function $c_k^{ij}(\cdot)$ associated with the cost of transferring an amount of water z_k^{ij}, and function $f^i(y^i)$ associated with future loss in the case of having water surplus y^i at reservoir i as a result of our operation during the multi-stage period considered.

Using some economical units i=1,...,n in the ambient system A, we want to compensate the loss in system WR by producing additional (over-planned) outputs in A. The income of the ith controlled unit in A at the k-stage is denoted by u_k^i. Function $g_k^i(u_k^i)$, which measures the expenditure needed to increase the outputs in A, is associated with each of these variables.

A possible way to put such compensation into practice is to consider an optimization model in which, as an objective function, the total loss in WR, and the total expenditure associated with increasing the outputs in A, are considered. That is,

$$\sum_{i=1}^{m} \sum_{k=1}^{\tau} \sum_{j=1}^{M} \left[f_k^{ij}(z_k^{ij}) + c_k^{ij}(z_k^{ij}) \right] + \sum_{i=1}^{m} f^i(y^i)$$

$$+ \sum_{i=1}^{n} \sum_{k} g_k^i(u_k^i) \longrightarrow \min \quad .$$

Apart from some physical and technological constraints, the minimization is carried out subject to the condition where the loss in WR is equal to the additional income obtained in A.

To overcome the difficulties connected with high dimensionality of the optimization problem, the following hierarchical approach, based on the separable property of the objective function, may be proposed.

At the first step, the total amount of water, $z_k^i = \sum_i z_k^{ij}$, released from the ith reservoir at the kth time stage considered as a parameter is distributed, thus solving the following problem:

$$\sum_{j=1}^{M} f_k^{ij}(z_k^{ij}) + c_k^{ij}(z_k^{ij}) = \underset{\sum_j z_k^{ij} = z_k^i}{\longrightarrow \min} = F_k^i(z_k^i) \quad ,$$

subject to other constraints associated with canal capacities, etc. For every value of the total loss $F_k^i\ (Z_k^i)$ in WR, the optimal compensation output u_k^j, $j=1,\ldots,n$, in A, is proposed as a solution to the second optimization problem:

$$\sum_{j=1}^{n} g_k^j(u_k^j) \longrightarrow \min \quad ,$$

subject to

$$\sum_{j=1}^{n} u_k^j = F_k^i(z) \quad ,$$

apart from other technical constraints. Let us denote the minimum value of $\sum_j g_k^j(u_k^j)$ as $G_k^i(F_k^i(Z_k^i)$.

At the second step, allocating water among the reservoirs and determining the optimal value of released water Z_k^i from each reservoir is considered. The main problem has to be solved: replacing $\sum_{j=1}^{M} f_k^{ij}(Z_k^i)$ and $\sum_{j=1}^{n} g_k^j(u_k^j)$ by $G_k^i(F_k^i(Z_k^i))$. A further computation procedure is based on the standard separable programming technique (see Gouevsky, 1975, and Gouevsky and Popchev, 1975).

IV. We have discussed optimization mainly for the operation of water resources systems, which obviously has to be taken into account when one considers any planned development of a river basin.

River basin development, from the point of view of general loss-function economical analysis, is not something very specific, and a number of well-elaborated economical models may be applied. In collaboration with the IIASA Methodology Project, we suggest the following model.

A development program is supposed to have a finite set of stages, $1 = \{1,\ldots,N\}$, during which particular jobs have to be implemented. Let x_t^i be a proportional part of the i^{th} job already accomplished by the current time t. Each i^{th} job can be intensified at time moment t by the corresponding control parameter u in such a way that

$$\frac{dx_t^i}{dt} = f_t^i(u) \quad ; \quad i=1,2,\ldots, \quad .$$

Here $u = u_t$ is a vector function reflecting the amount of various resources needed for completing the jobs. There are some constraints:

$$r(t)u(t) \leq R(t)$$

for resources that must be used instantly; and

$$\int_0^t q(s)u(s)ds \leq Q(t)$$

for resources that can be stored for a while. (Here r(t), q(t) are matrices and R(t),Q(t) are vector functions.)

There are also constraints of a different type; namely, a set of all jobs is semi-ordered, and for any job i one knows that it cannot be started before some other group $\tilde{I}_i$, $\tilde{I}_i \subseteq \tilde{I}$ is completed.

The optimization problem is to minimize the time T = T(u) of the development program performance:

$$T(u) \longrightarrow \min_u \quad .$$

Some kind of generalization of the Pontriagin maximum principle may be applied here. The corresponding computer program for the optimal identification vector function $u = u_t$, written in FORTRAN, was developed (see Belyaev and Zimin, 1975).

V. The most practical applications of stochastic systems analysis and optimization techniques in planning development are based on the assumption that the system process will reach so-called statistical equilibrium. This phenomenon lets us neglect a variety of different initial conditions and systems processes, and to take into account the (stationary) process that represents the statistical equilibrium of evolution.

Let us consider, for example, a reservoir process

$$x_k, y_k, z_k \quad ; \quad k=0,1,\ldots,$$

representing at each k^{th} time period the available water x_k, the initial reservoir volume y_k, and the water release z_k. Here $x_k = y_k + \xi_k$, $y_{k+1} = x_k - z_k$ and $\xi_k; k=0,1,\ldots,$ defines the inflow process.

The statistical equilibrium phenomenon can be described as follows. Probability distribution P_n of the "future" process

$$(x_{k+n}, y_{k+n}, z_{k+n}) \quad ; \quad k=0,1,\ldots,$$

tends to some limit

$$\lim_{n\to\infty} P_n = P \quad ;$$

this limit probability distribution P is invariant with respect to the annual time shift transformation

$$(x_k, y_k, z_k) \longrightarrow (x_{k+\Delta}, y_{k+\Delta}, z_{k+\Delta}) \quad ,$$

where Δ denotes the whole-year period. Moreover, the frequency of any annual event A during a series of years N also tends to the corresponding probability P(A):

$$\lim_{N\to\infty} \frac{\nu_N(A)}{N} = P(A) \quad ,$$

where $\nu_N(A)$ is a number of years in which event A occurs.

As we know, the existence of such a statistical equilibrium phenomenon in reservoir processes was proved only in a case of Markovian inflow ξ_k; $k=0,1,\ldots$,by using classical ergodic theorems for finite Markovian chains.

Actually this phenomenon occurs under very general physical conditions (see Rozanov, 1975a, 1975b). Let us consider an arbitrary water release policy with the only assumption that the current release z_k does not exceed the water demands w_k if there is no water excess. We believe that the inflow process ξ_k, current water demands w_k and operational reservoir level r_k, established so as to have a flood catchment capacity $(k=0,1,\ldots,)$, may be considered stationary with respect to the annual time-shift transformation.

Naturally we can expect statistical equilibrium only under some ergodic conditions for the process (ξ_k, w_k, r_k), and under such conditions the following result holds true.

Suppose that during a long-term operation the total reservoir inflow, $\sum_{k=0}^{n} \xi_k$, sometimes becomes comparatively high with respect to the total water demands, $\sum_{k=0}^{n} w_k$; $n=0,1,\ldots,$ and moreover that the sequence

$$\eta_n = \sum_{k=0}^{n} (\xi_k - w_k) \quad ; \quad n=0,1,\ldots,$$

with non-zero probability exceeds the reservoir level R (or at least the operational level $r = r_n$; $n=0,1,\ldots,$) at some (random) time period $n=\tau$. Then the statistical equilibrium phenomenon (x_k^*, y_k^*, z_k^*) holds true. Moreover, we find the ergodic process stationary with respect to the annual cycle, such that

$$(x_k, u_k, z_k) = (x_k^*, y_k^*, z_k^*) \quad , \quad k \geq \tau \quad .$$

The statistical equilibrium probability distribution, $P = \lim P_n$, coincides with the probability distribution of the process

$$(x_k^*, y_k^*, z_k^*) \quad ; \quad k=0,1,\ldots,$$

and with the variance of $P_n - P \leq 2P\{\tau > n\}$.

Note that in most cases of interest the τ-distribution is of an exponential type:

$$P\{\tau > n\} \leq C_e^{-an} \quad ,$$

so that the convergence rate of $P_n \longrightarrow P$ is very high.

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IV. Cooperative Studies

Igor Belyaev
(in collaboration with project colleagues)

INTRODUCTION

The application of systems analysis to "real-world" water resources studies at IIASA has encountered a number of difficulties of the following type:

- The relative lack of model systems that can be used from beginning to end to solve a comprehensive water management problem;

- The need to collect, process and analyze a large amount of data on "real-world" applications, which is beyond the resources of the Water Resources Project.

Irrespective of these difficulties, the Project considers the development of applied case studies an important activity. The work is accomplished through cooperation with water resources groups in IIASA National Member Organizations. Cooperation allows us to amplify the systems of existing models and methods in different countries with additional data obtained from exchanges of information. We hope that in the near future the methods of systems analysis and the systems of models will be routinely applied to solving water resources problems.

Accordingly, Project scientists worked on establishing links with national organizations in the USSR (A. Letov, I. Belyaev, Z. Kaczmarek, P. Koryavov); in Hungary (I. Belyaev, Z. Kaczmarek, P. Koryavov, A. Szöllösi-Nagy); in Poland (W. Spofford, Z. Kaczmarek, J. Kindler); and at the University of Arizona in the USA (A. Szöllösi-Nagy). This led to the signing of the following documents of cooperative studies:

1. Minutes of discussions on scientific cooperation between the IIASA Water Resources Project and the Hungarian National Water Authority, held in Budapest on January 2 and 3, 1975.

2. Minutes of discussions on possible cooperation between IIASA and the University of Arizona, held in Tucson, Arizona, January 6-10, 1975.

3. Minutes of discussion between Project scientists and representatives of the Polish Water Resource affiliate organizations (Institute of Environmental Engineering, Warsaw Polytechnical University; "Hydroprojekt", Consulting Engineers) held on the occasion of the Workshop on the Vistula and Tisza River Basins, held in Laxenburg, Austria, on February 11-13, 1975.

4. Report on the visit of the Water Resources Project delegation to the USSR and minutes of the discussions on possible cooperation with the USSR subcommittee on complex use of water resources, held on February 17-20, 1975.

5. General agreement between the Institute for Organizational Management and Control Sciences, Warsaw, and IIASA, concerning cooperation within the framework of the IIASA Water Resources Project, reached on July 4, 1974.

6. Agreement between the Hungarian National Water Authority and IIASA on scientific cooperation in water resources management, reached on January 3, 1975.

COLLABORATION WITH USSR NATIONAL MEMBER ORGANIZATIONS

Recently, cooperation between IIASA and Soviet institutions has developed very successfully. An important step was the establishment in the USSR of the Council for the Application of Systems Analysis to the Problems of Complex Use and Protection of Water Resources. The Council carries out its activities on public efforts within the Systems Analysis Committee of the USSR Academy of Sciences. Collaboration of the two groups led to real accomplishments in our research.

Another important result of collaboration with organizations in the USSR is the report *Systems Analysis Methods for Problems of Rational Use of Water Resources* (Volume I, December 1974, in Russian).

The report contains the following chapters:

1. The Soviet Variant of the Water Resources Project Concept and Regional Project Characteristics;

2. Models of River Runoff and Water Environment (including Quality Aspects);

3. A Programmed Approach to Regional Development Planning;

4. Irrigation Models and the Problems of Rational Water Distribution for Agricultural Needs;

5. A Dynamic Model of the Sea of Azov Water Community Activity;

6. A Simplified Model of a River Basin;

7. Some Aspects of Decision Making in Modeling Rational Water Use; and

8. Draft of the Software System for Optimization Blocks.

The main goal of the Soviet variant of the Water Resources Project concept is to create a simulation system for describing a wide range of objectives for using possible alternatives of regional development, under conditions of water deficiency. Part of this work is devoted to analyzing three regional based projects: water usage in Moldavia, in Armenia and in the Sea of Azov Basin.

The Soviet approach to water economy development is based on a programmed method of planning and control. It is assumed for simplicity that regional development occurs within the bounds of a single state. Two main aspects are emphasized: 1) regional planning and management hierarchy; and 2) the fact that a region is not a closed production system. The resources for state and regional programs are provided by industrial enterprises. The state operates with a greater amount of resources than does a region. Consequently, state goals (such as programs) are primary points in decision-making procedures. Regional programs have to take into account information from other regional programs and from state programs.

Program formulation determines both the bases for building a model system and the procedures for its application. A model for use by high-level management (that is, a network model of a program) is needed to make the preliminary program selection and evaluation.

The resources needed to carry out a program are supplied by industry. Natural resources are considered restrictive conditions to development. The highly aggregated industrial processes are described by a linear multibranch dynamic industrial model, the π-model, developed at the Computing Center of the USSR Academy of Sciences. The model is based on dynamic balance equations of the Leontiev kind.

In the simplest model of this type, a basic balance equation is written,

$$q(t+1) = q(t) + x(t) - A(t)x(t) - y(t) - w(t) - Q(t) \quad ,$$

where

$x(t) = (x^1(t)),\ldots,x^L(t);$

$x^i(t)$ = (gross) output of the i^{th} industry within the period $t,t+1$. (The duration of the production cycle is considered equal to 1.);

$q(t)$ = vector of resource reserves within the same period;

$A(t)$ = L x L matrix consisting of input coefficient $a_{ij}(t)$ of the product going from sector i to sector j (that is, the quantity of the output of sector i absorbed by sector j per unit of its total output);

$w(t)$ = vector of consumer goods;

$y(t)$ = investment vector;

$Q(t)$ = portion of the final product which goes into the development program.

The model describes multibranch industry and allows one to take into account expansion, conversion, creation of new industrial capacity and its management. It is assumed that production funds and labor resources can be completely or partially used. The controls include stocks, consumption of the final products, investments and acting capacities themselves. One may achieve purposeful economic development by manipulating these controls.

One of the main parts of simulation is a system of models describing water resources formation and functioning. The peculiarity of this kind of model is that it combines industry, agriculture and public demand into one entity. The basis of the runoff description is a so-called cell model which allows one to describe river run-off water reservoirs, underground water and the like. An interesting aspect here is the application of the numerical method and its comparison with the Saint-Venant model for an arbitrary branching river basin network. The latter model is similar to the water quality hydraulic models described in the above mentioned report; special attention should be paid to Chapters 7 and 8.

COLLABORATION WITH THE HUNGARIAN NATIONAL WATER AUTHORITY

Collaboration between the IIASA Water Resources Project and institutions in Hungary began in 1973 with the formulation of the Project's program. During 1974, a number of meetings were held in Budapest and Laxenburg between Project staff and experts from the Hungarian National Water Authority. The water economy problems of the Tisza River basin and the mathematical models being developed in Hungary and at IIASA were discussed at these meetings; as a result, valuable initial data were obtained for use in the work of the Project staff.

These contacts led to a cooperative study of the Tisza River basin. It is known that the main water resources problems in the Tisza River basin are related to agriculture and flood protection, and include flooding, water deficits during certain months and years, inadequate drainage of the flooded part of the basin area and water pollution. The international character of the Tisza River basin and the complexity of its hydrological regime makes it particularly attractive as a cooperative study.

IIASA's contribution to the study is described by P. Koryavov and I. Belyaev (1975).

Methods of network analysis and modeling have been applied in water conservation. Results have been obtained at the Texas Water Development Board (1970). However, the development of the Vistula River Project (United Nations, 1972) can be considered more successful. Of particular interest is the improved version of the Out-of-Kilter Algorithm which was elaborated at the Rand Corporation (Durbin and Kroenke, 1967). The IIASA approach differs from those mentioned above in some details.

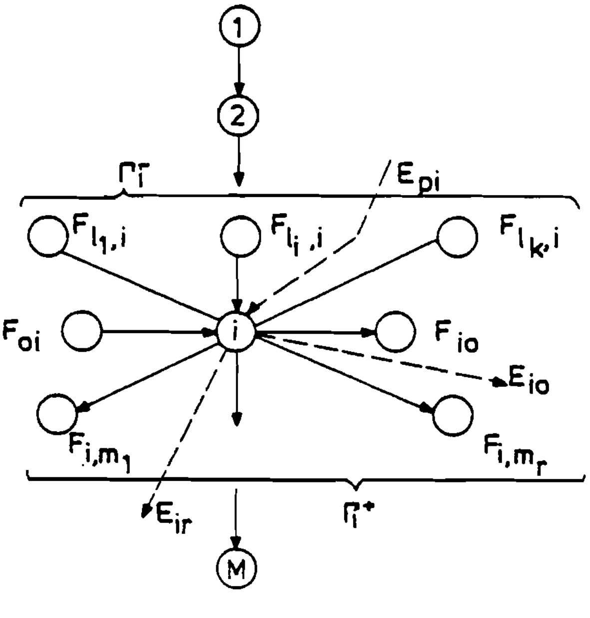

Figure 1. River basin network diagram.

Figure 1 shows the Tisza River system as a network without loops. The nodes of the network are separate cross-sections of the river, its tributaries and engineering constructions on which water-intake units (such as towers, irrigation canals and reservoirs) are situated. The arcs connecting the nodes are marked by arrows that indicate the direction of water flow. The system may be expressed as follows:

$$w_i(t+1) = w_i(t) + \sum_{\ell \in \Gamma_i^-} F_{\ell i}(t) - \sum_{k \in \Gamma_i^+} F_{ik}(t) + F_{oi}(t) - F_{io}(t) ,$$

where

$w_i(t)$ = total volume of water in element i (phase variables in the model);

$F_{ik}(t)$ = volume of water running from element i (the network node) to element j during the time, t,t + 1 (controls in the model);

$F_{io}(t)$ = water withdrawal in element i within the interval t,t + 1 of the transmission of water to another industry (control variable);

$F_{oi}(t)$ = water inflow into element i from without-- surface and underground inflow, precipitation (uncontrolled factors);

Γ_i = set of elements (cross-sections, reservoirs, channels) which precede (lie upstream from) element i; and

Γ_i = set of elements into which water comes from element i.

The values of $w_i(t)$ are limited by the feasible capacity of water reservoirs

$$\underline{w}_i(t) \leq w_i(t) \leq \overline{w}_i(t) \quad .$$

If element i corresponds to a river reach or a canal, then

$$w_i(t) = 0 \quad .$$

Besides, as river reaches have ultimate capacities, there are necessarily corresponding constraints in the model

$$0 \leq F_{ij}(t) \leq \overline{F}_{ij}(t) \quad ,$$

where $\overline{F}_{ij}(t)$ is the maximum feasible capacity of canal or river reach ij. Overflow occurs in river reach i at moment t when the water inflow into reach i exceeds the maximum feasible water outflow. That is, the following condition holds

$$\sum_{\ell \varepsilon \Gamma_i^-} F_{\ell i}(t) + F_{oi}(t) > \sum_{k \varepsilon \Gamma_i^+} \bar{F}_{ik}(t) + \bar{F}_{io}(t) \quad .$$

This model was used first for solving short-term problems, assigning such values of control variables (relationship of reservoirs and water supply to water users) so as to minimize the damage from overflow and from shortages of water for users. The objective function used can be represented in the following generalized form:

$$\sum_{t=i}^{T} \sum_{j=i}^{N} P_{1j}^{t}(d_{ij}(t) - F_{ij}(t)) + \sum_{i=1}^{M} P_{2j}^{t}(F_{ij}(t) - \bar{F}_{ij}(t))_{+} \quad ,$$

where p^t represents a penalty for irrational use of water, and $d_{ij}(t)$ represents city or irrigation-area demand. There is a function $(X)_+$ which means that

$$(X)_+ = \begin{cases} X, & \text{if} \quad X \geq 0 \ ; \\ 0, & \text{if} \quad X \leq 0 \ . \end{cases}$$

A simplified sketch of the complex Tisza River basin is shown in Figure 2. The network includes:

Tributaries - Viseu, Iza, Teresva, Borzava, Szamos, Bodrog, Sajo, Körös, Maros;

Cities - Fehergyarmat, Debrecen, Polgar, Szolnok, Czongrad, Cseged;

Irrigation Areas - Tiszalök and Kisköre; and

Seventeen reaches of the Tisza River.

For simplicity, the sources of the Tisza and the Danube Rivers were treated as tributaries; however, this does not change the physical picture of the flow.

The computed results suggest that the useful capacities of Tisza River reservoirs should be substantially increased.

Figure 3 shows the flows and overflows along the Tisza River, assuming that the useful storage capacity will be six times greater than at present. The Kisköre storage, with its expected useful capacity after construction, can accumulate the overflow in April-May of the mean wet year and release water downstream in August-September to cover the water supply together with levee system fortification in regions subjected to floods.

Figure 4 shows a reasonable operating rule for storage. In April-May of the mean wet year, barrages Besha and Tiszalök

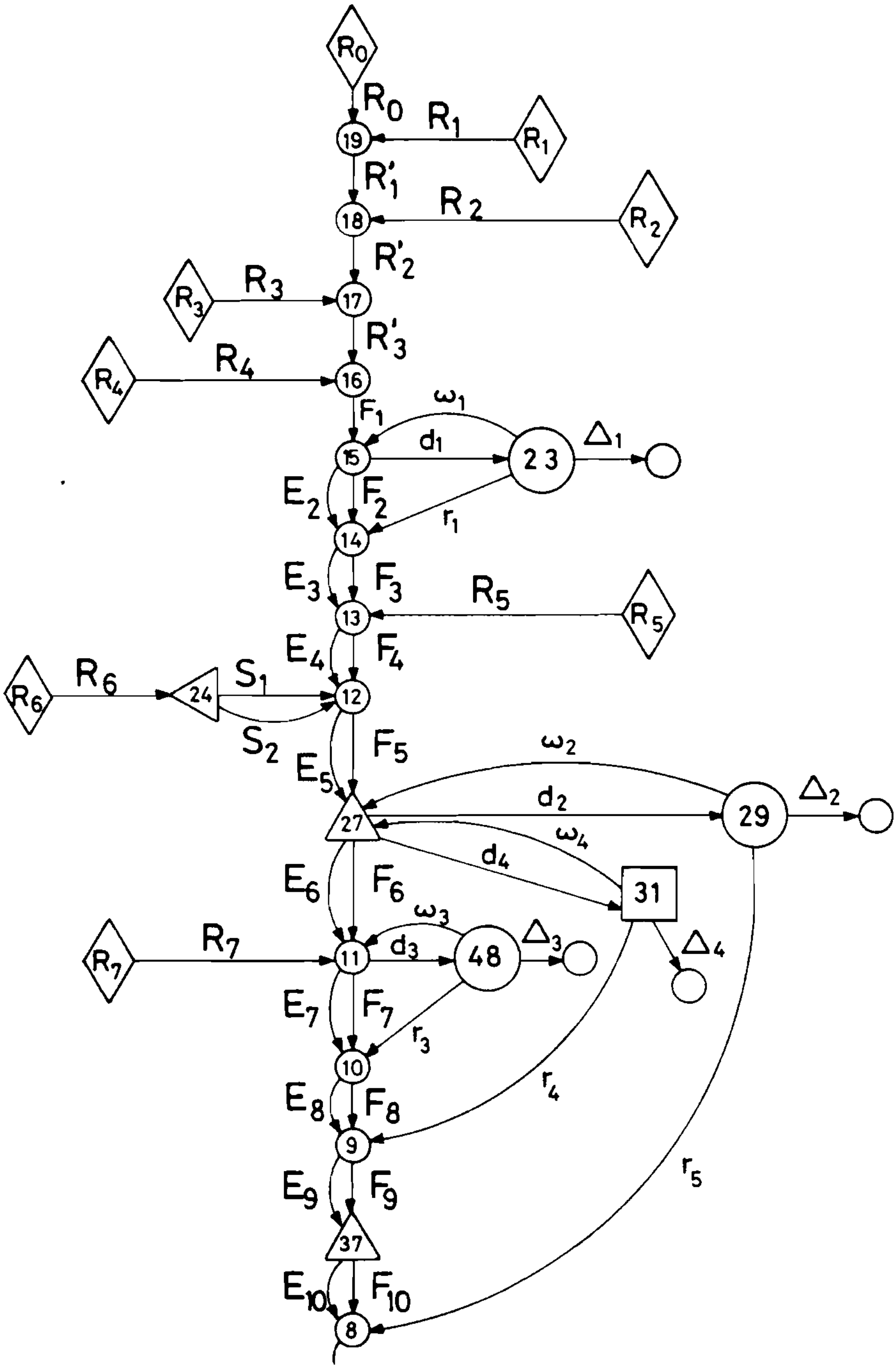

Figure 2. Tisza River basin. network flow diagram.

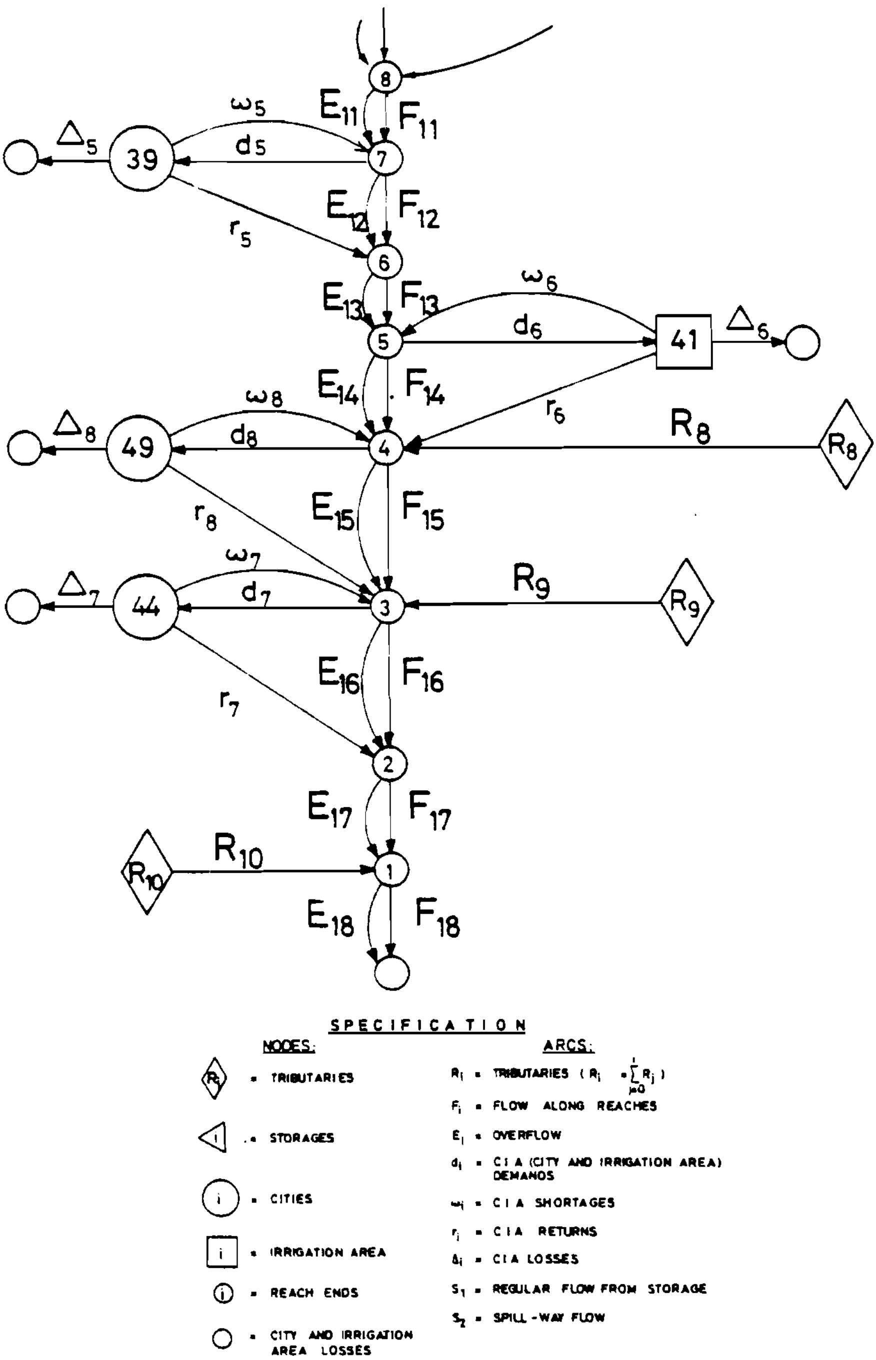

Figure 2. Tisza River basin: network flow diagram (concluded).

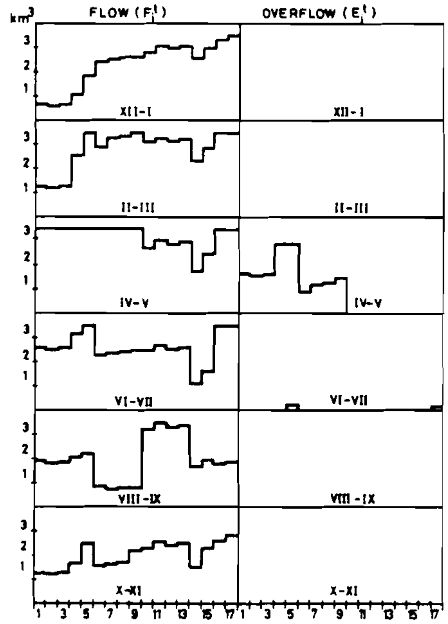

Figure 3. Flows and overflows along the Tisza River.

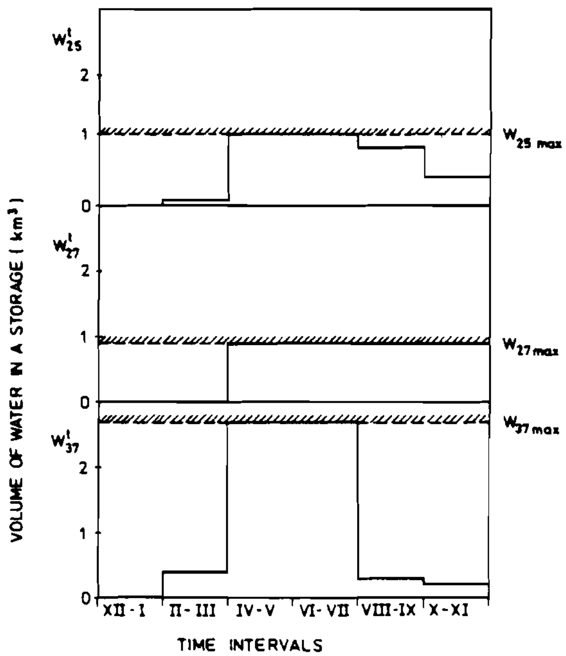

Figure 4. Variation with time of the volume of stored water.

cannot accumulate flood water from upstream when there is an overflow. This result illustrates the fact that for solving the overflow problem in the Tisza River basin, one needs to build in the upper part of the Tisza River and its main tributaries (the Soviet and Romanian parts).

To solve this problem, a matrix generator (MAWJWN) and a linear programming package (APEX) were used. The program was prepared by D. Bell of the IIASA Methodology Project.

Information for the use of some mathematical models in existing geographical, hydrological, and economic sources is rarely available in a convenient form for application. Thus, it is necessary to make a preliminary analysis and handling of initial data, which cannot be accomplished with some formalized methods.

In the case of Tisza network modeling, the initial data from the Hungarian National Water Authority and from other sources were prepared by I. Belyaev. A number of difficulties were encountered in water economy data determination, particularly in obtaining values of demand and consumption uses through time and space. The main data were taken from the Hungarian National Water Authority (1973), the *Atlas der Donau Länder* (1973), and the *Yearbook of the Hungarian Hydrographical Service*.

Calculations have been done for a one-year horizon with two-month time steps.

The Hungarian approach to solving the Tisza valley problems is based on the natural conditions of the Hungarian side of the Tisza River*. The flood control subsystem consists predominantly of the flood levee. Consequently, in this system, flood control and other aspects of water management can be considered separately with respect to both control and development. Obviously, this treatment is not possible if the entire river basin, or a subcatchment thereof, is analyzed where multi-purpose valley storage can be realized.

First, optimum control of the present water management system will be treated by outlining the models for the optimal operation of water management, excluding flood control and water damage aversion. Secondly, the analogous development model of the system will be considered.

1. Optimum Control of the Water Management System

The principal elements of the water management system are reservoirs, river sections and the related water supply and irrigation districts, and drainage sections. The control variables of the reservoirs and river sections are water transfer, industrial withdrawal, and irrigation withdrawal. The state variable is the natural inflow.

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*See Hungarian National Water Authority (1975).

The objective function minimizes the expected value of the sum of economic losses resulting from damages to all reservoirs, all river sections and to all related districts over the entire year.

2. Model of the Water Management System Development

This model concerns an economic region of about 1,000 km in the northern part of Hungary. It is assumed that models for the individual subsystems and their combination for studying the complete system will provide the planning data for developing the complete system. The need for long-term water management development is considered in the plans for water management development.

Planned development in settlements, industry, agriculture and environment protection must be entered by region as input data distributed over time. Water demands and water production must be predicted by sector and region. Possible alternatives must be evaluated to formulate the optimal water management strategy for the system, under complex physical, economic and social conditions.

The objective of the system model is to meet the requirements of regional development for water management at the minimum possible cost. The model suggested is a simple linear programming model that our Hungarian colleagues developed into a stochastic linear programming model.

COLLABORATION WITH WATER-ECONOMY ORGANIZATIONS IN POLAND

Collaboration between the IIASA Water Resources Project and management organizations in Poland began at the end of 1974, and consists of joint studies of the Vistula River basin.

Unlike the Tisza River Basin, where the application of systems analysis to water economy development is in the initial phase (with respect to the river basin as a whole), the system approach to the study of the Vistula Basin has a history: the so-called Vistula River Project. The goal of this project is to formulate a water resources development (investment) program capable of meeting the demands projected for the period 1985 to 2000. A starting point for the methodological studies is a proposed spatial and problem-oriented decomposition of the system. It has been decided to divide the basin spatially into 14 sub-systems (see Figure 5); each system represents an area whose economic structure is as uniform as possible, has a similar hydrological nature and similar hydraulic engineering problems. This proposal conformed to the generally recognized specific character of water management in Poland, which concentrates on the problems of water supply for pollution control, and the independent investigation of rational solutions for flood control.

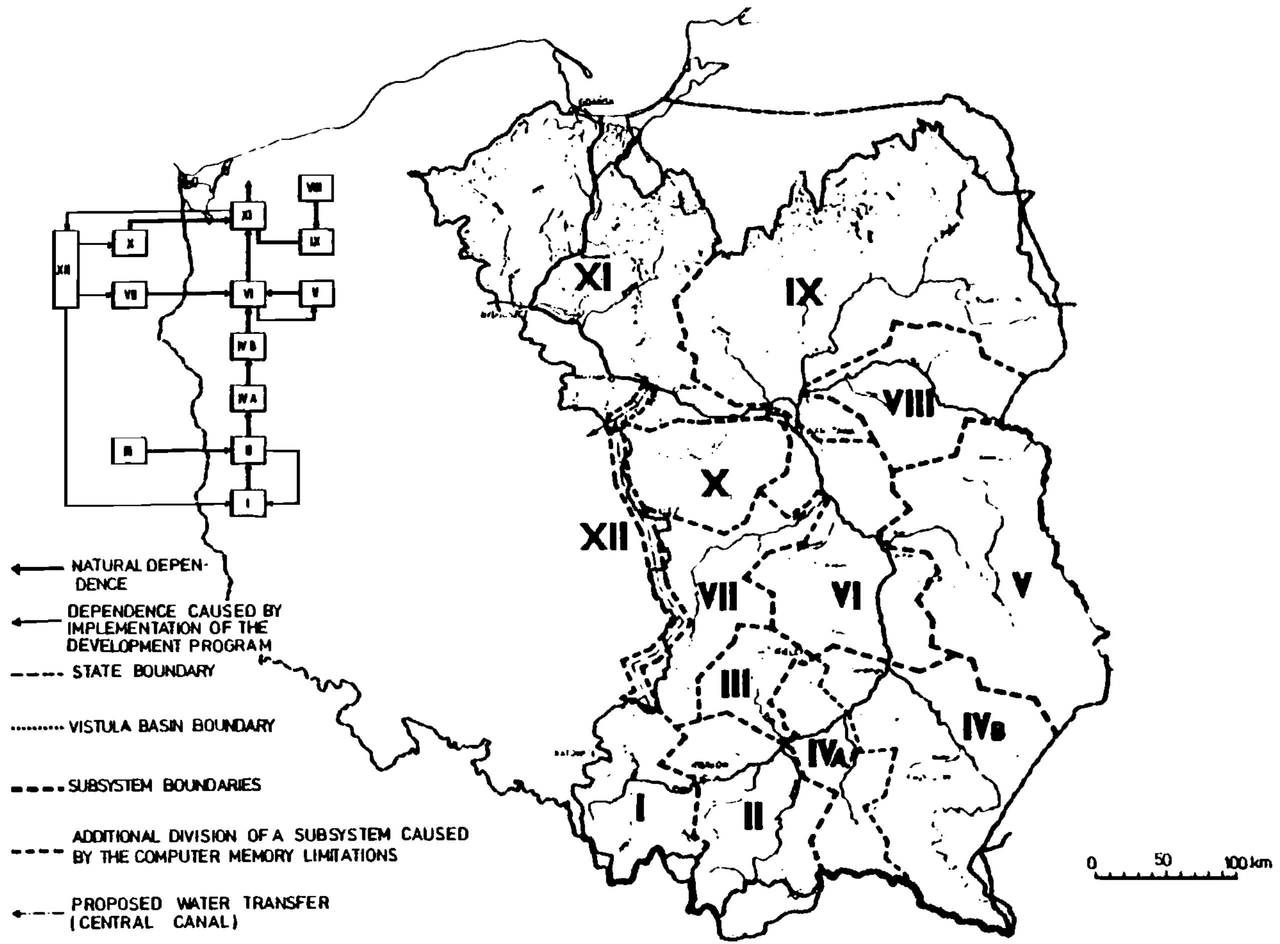

Figure 5. Topology of subsystems.

The fundamental assumption of the planning strategy is that the investment program alternatives, including reservoir sites, water transfer rates, diversions and sizes of all hydraulic engineering undertakings, are reduced to a finite set D_{ij} by subsystems i and variants j.

The primary thrust of modeling activities was directed toward developing a water resources management model. Once each of the investment variants was defined for a given subsystem, its performance was evaluated by a simulation-optimization procedure (WRM model) to allocate the 180 monthly flows on record to each water user; the target values for the period 1985 to 2000 correspond to the following control objectives:

- Water supply to the population, agriculture and industry;
- Maintenance of minimum acceptable flows; and
- Development of hydropower production.

When an allocation fell short of the target, it was weighted by a penalty factor reflecting the relative priority of each water user to all water users. The system of weights was based on the relationship of unit production costs of a given commodity and volume of water needed per production unit; production costs were based on economic studies carried out by each of the branches of the national economy. Projections of technological changes were taken into account in determining the indispensable volume of water.

The model ensured minimization of the sum of weighted deficit allocations. This set of allocations was defined as the optimum for a given investment variant.

The WRM model uses the Out-of-Kilter Algorithm to solve problems of water resources allocation in the complex multi-reservoir system. Figure 6 illustrates an example of a network derived from Figure 5. Nodes portray the reservoirs, the non-storage junction control and balance profiles.

RHINE RIVER STUDY

A decision analytic approach to the study of water quality management has been proposed by Gros and Ostrom (1975). Utility functions were used to express the preferences of several interest groups for multiple objectives in an uncertain environment. Preferences for quality levels were represented as parameters of a utility objective function,

$$u(,x^*,z,q) \quad ,$$

where

x = the water quality level,
x^* = the standard,
z = the waste water treatment cost, and
q = a random variable (e.g. streamflow).

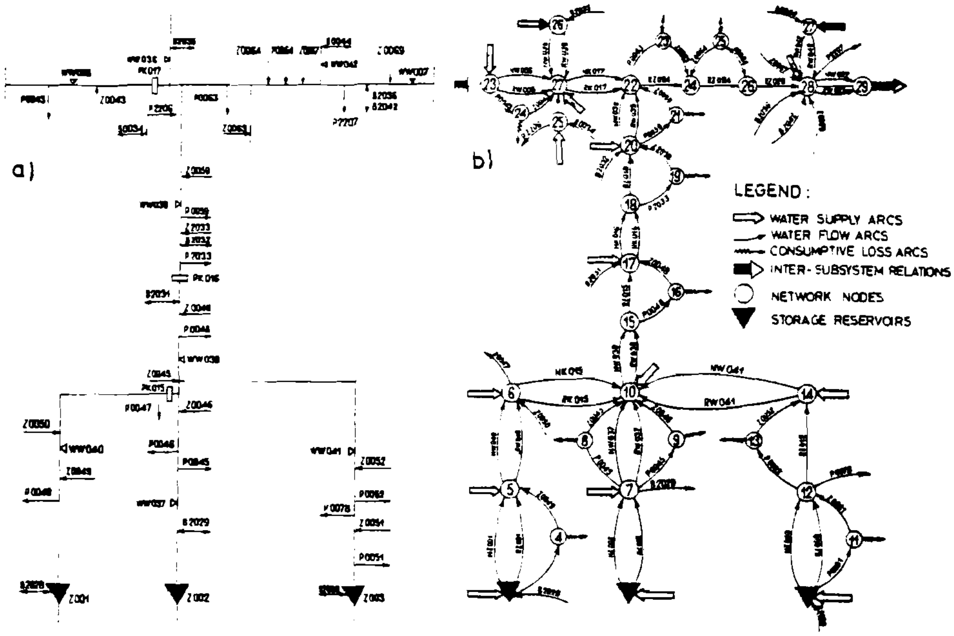

Figure 6. Node-arc network.

An application was then carried out using water quality data, supplied by Stehfest et al (1975), on the Rhine River in the Federal Republic of Germany. We used Stehfest's estimated point sources of chemical-oxygen-demand (COD) and relative population levels in 1985 for each of the ten reaches of the Rhine River between Mannheim and the German border with the Netherlands. A utility function was obtained for each of the expressed tradeoffs between COD concentration and operating costs for waste water treatment plants, and the uncertain streamflow was presented by a discrete probability function. An optimal trajectory of waste treatment strategies was found by maximizing the expected value of the objective function repeatedly for each of the discrete values of the uncertain streamflow. The purpose of this first-order analysis, which has been described by Gros and Ostrom (1975), was to test the sensitivity of optimal treatment strategies to parameters of the utility objective function, and to check computational feasibility of the model. Preliminary results indicate that, although a decision analytic approach can provide a useful and workable framework for incorporating much relevant information in a single model, more work is needed on the questions of utility assessment, values of scaling weights and the choice of an appropriate objective function. Work is continuing on testing different objective functions and on interpreting the results.

CONCLUSION

One aspect of the activities of the IIASA Water Resources Project should be emphasized.

Water economy development, as a whole, in a region or state is a problem requiring long-term planning. The top authority dealing with the water economy must select the most rational development strategy or program that includes the expansion and creation of new structures such as reservoirs, canals and dams, in order to satisfy the water demands of the urban sector, industry, and agriculture and the demand for flood protection. Realization of a program requires time and resources (for example, energy, building materials, and labor), the latter being always limited at the regional and state levels.

Some mathematical models and computer-assisted algorithms are now available and in use. Such approaches are being intensively developed in the USSR by the computing Center of the USSR Academy of Sciences and at IIASA by the Water Resources and Methodology Projects.

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Appendix I

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Appendix II

Water Resources Project

Members

Zdzislaw Kaczmarek, Project Leader
Igor Belyaev
Ilya Gouevsky
Pavel Koryavov
Anthony Ostrom
Yuri Rozanov
Juergen Schmidt
Andras Szöllösi-Nagy
Kuniyoshi Takeuchi
Eric F. Wood

Alumni

Alexander M. Letov
Franco Del Monte
Myron Fiering
Janusz Kindler
Allen Kneese
Nikta Moiseev
Walter Spofford
Valerie Tokhadze
Hirofumi Uzawa

Appendix III

Agenda

June 3, 1975 Laxenburg, Austria

9:00 Session I

SYSTEMS ANALYSIS IN WATER RESOURCES

General description of the activities of the Water Resources Project (followed by discussion)

Professor Zdzislaw Kaczmarek

10:30 Session II

HYDROLOGY AND WATER RESOURCES MODELING

Dr. Eric Wood in collaboration with project colleagues (followed by discussion)

13:00 Session III

OPTIMIZATION TECHNIQUES IN SYSTEMS ANALYSIS OF WATER RESOURCES

Professor Yuri Rozanov in collaboration with project colleagues (followed by discussion)

15:15 Session IV

COLLABORATIVE STUDIES

Dr. Igor Belyaev in collaboration with project colleagues (followed by discussion)

16:45 Closing Remarks