SOME REMARKS ON ECONOMIC GROWTH RESILIENCE AND CATASTROPHES

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INTRODUCTION

It has long since been recognized by ecologists and biologists as being typical for living systems to posess several qualitatively different equilibria. Here equilibrium means a state which the system does not depart from by itself provided that the system's "law of motion" is not changed.[1]
Moreover, for living systems it is the exception, not the rule to remain for a very long time within the same domain of attraction, that is the set of states leading to a particular equilibrium. To be able to push oneself towards ever new equilibria can almost be taken as a synonym for being alive.
Hence, the weak point common to many theories of dynamical systems[2] is not so much the assumption that the systems is already in equilibrium when we start looking at it but the lack of explanation why the system has attained just this equilibrium and not another one, and whether it may be expected to change to a different domain of attraction in the future.[3]
It is precisely this aspect which has been somewhat neglected in economic theory namely in its mathematically most sophisticated branch, "General-Equilibrium Theory".

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[1] CHIPMAN (1965, p. 35) writes:
"Equilibrium-meaning a balance of opposing forces- is a concept as fundamental in economics as in physics. The reason why it is so fundamental is that the concept is much more complex than might at first be supposed".

[2] A classic reference is LOTKA (1956)

[3] As biological systems can, basically, be in a living as well as in a dead state, they can always be assumed to posess more than one equilibrium.

By that theory the existence of multiple equilibria in production and trading systems is well established but the qualitative differences between the equilibria and the reasons why the system might switch from one equilibrium to another are not in the center of interest.

A convenient tool to display the global behavior of one- and two-dimensional systems is the so called <u>phase-portrait</u>. Assume that for a two-dimensional system with variables (x_1, x_2) whose behavior or "law of motion" is governed by a differential equation

$$\frac{d}{dt}(x_1(t),\ x_2(t)) = f(x_1(t),\ x_2(t)),$$

the phase-portrait looks as sketched in Fig. 1.
The phase portrait which is determined by the form of the function f(.,.) tells us for any initial state $(x_1(0),\ x_2(0))$ how the system is going to evolve from thereon.
Assume, moreover, that the <u>viable states</u> of the systems are those lying to the left of the dashed line d-d.[1)]

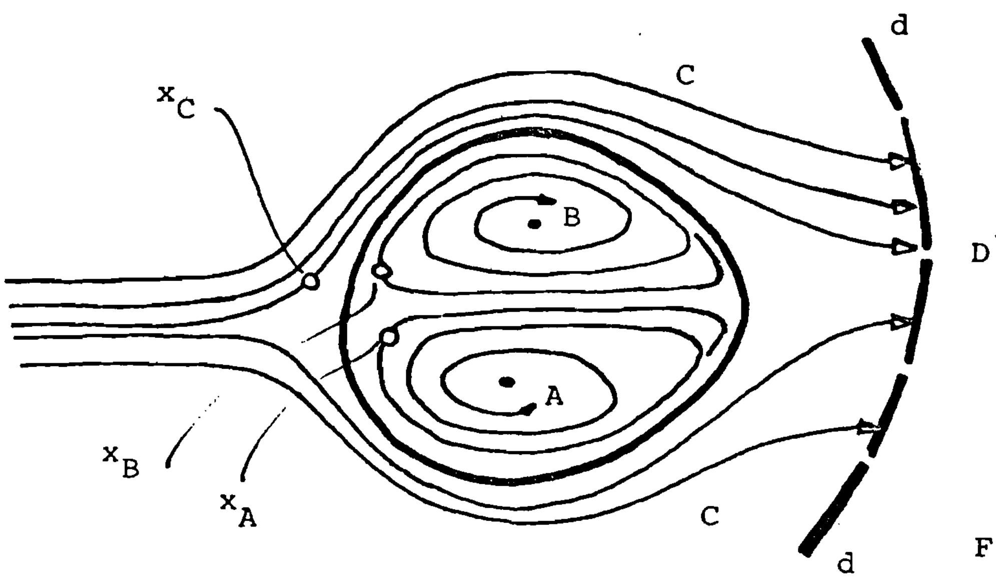

FIGURE 1

1) "viable states" are those for which the given differential equation is valid.

The phase portrait shows two stability regions A and B, fully within the viable domain and a further set of states C, outside the "circle", leading to the non-viable domain D to the right of the boundary d-d. The states on the line d-d are not stationary points of the differential equation which is valid in $A \cup B \cup C$ but it seems to be legitimate to call these states equilibria too because the system stops and remains there.

Comparing the evolution of the system originating from three alternative initial states x_A, x_B, x_C we see that the behavior of the system might be qualitatively different although the states x_A, x_B, x_C may be located arbitrarily close to each other. Now, obviously, one possibility for the system to change qualitatively its path of evolution is that by exogenous shock the initial state is changed from x_B to x_C, say, and the system then exhibits a tendency towards the non-viable domain. Another possibility is that by a sudden change of the 'law of motion' f(.,.) itself the circle-line seperating domains $A \cup B$ and $C \cup D$ could contract, thus leaving x_B and x_A out of $A \cup B$ and within C and, consequently, leading to a path of evolution of the system not predictable before.[1)]

RESILIENCE AND OPTIMALITY: THE BIG TRADEOFF

HOLLING (1973), abstracting from a rich menu of case studies of ecological systems introduced the concept of resilience of a multiple equilibria system which "is the ability of a system to absorb and even to benefit by unexpected finite changes in system variables and parameters, without detoriating irreversibly", HOLLING (1976).

[1)] This has some implications for systems, such as nuclear reactor safety systems, which cannot be designed by a trial (choose a function f) and error (accident happens and system swiches to non-viable equilibrium) procedure.

That is the reaction of a resilient system to sudden changes of the kind described above should not be to move towards a qualitatively different and possibly non-viable equilibrium, YORQUE (1975). We have to distinguish here resilience against exogenous changes of state ("resilience in the phase-plane") and resilience against change of the 'law of motion' (resilience of the phase plane", see GRÜMM (1976)). By changes of the second kind an etire stability region could collaps and vanish. In Fig. 1 this happens if, for example, domain A∪B is contracted to a point.
Imagine now that we know that the state of the system is at present somewhere within domain A∪B of Fig. 1. It is intuitively clear that, without any further knowledge, the likelihood that the state of the system is pushed out of the 'secure' domain A∪B, or that this domain of attraction collapses totally should be expected to be inversely related to the before-the-shock size of that region.
Interestingly enough, it has been demonstrated by PETERMAN (1976) that for reproductive ecosystems, like a population of fish in a lake, the size of the domains of attraction is substantually influenced by man's harvesting from the system. The same effect will be shown below for a simple model of economic growth.

It appears to be a principle of a very general kind that increased harvesting causes a shrinking of domains of attraction around natural, viable equilibria and, thus, diminishes the recuperative powers of such systems.
And, clearly, as the system becomes more likely to react to a small, sudden change of external conditions by a qualitative change of behavior - its resilience is on the decrease.

On the other hand, economic systems are in most cases designed[1)] for maximum yield, that is to maximise the harvest or sum of withdrawals from the reproduction cycle.
A system then, which yields maximum harvest either in

[1)] Or, at least, economic theory says they should be. Otherwise they are being called un-economical.

a static or dynamic sense is called an optimally designed and controlled system.
This points to a general tradeoff between the goals of resilience (in the sense of the likelihood of qualitative persistence of the system in case of perturbations) and optimality (in the sense of maximality of harvest from an unperturbed system).

ECONOMIC SYSTEMS

What can we, economists and engeneers learn from this ?
The economic theory of reproduction-harvest systems, the theory of economic growth, has tended to view the economic world as a globally viable system. Accepting this view there is, of course, no reson why one should follow a policy other than the on which maximizes "harvest", that is the consumption flow from the economic reproduction system (see KOOPMANS (1965)). We shall argue, however, by means of a simple growth-model that the presence of multiple equilibria deserves more attention, simply because alternative equilibria might be of qualitatively different nature.[1] And we plead that economists should stop seeing a virtue in having a globally stable model and to assume the global viability of their economic systems-just as ecologists had to give up the comfortable idea of infinitely forgiving Mother Nature.

A SIMPLE GROWTH MODEL

The following growth model is, in several components, very similar to the well-known neo-classical aggregate growth models

[1] Here the term 'qualitatively different' is not to mean only the difference between stable and unstable equilibria but, above all, points to the difference between equilibria in which the system could continue to exist and others in which it could not.

(see SOLOW (1956), SWAN (1956)). The reason why we picked this model is that it appeared to be the simplest one by the use of which we could still make our point.

Assume that the production system uses two homogeneous inputs called "capital stock" and "labour" in the respective amounts K(t) and L(t) at time t, and let the gross output flow of a universal good G(t) at time t be given by a COBB-DOUGLAS production function as[1)]

$$G(t) = A \cdot K(t)^{\alpha} \cdot L(t)^{1-\alpha} + Y_o$$

with $A, Y_o > 0$, $0 < \alpha < 1$.
Y_o is the part of output notproduced by capital and labour, or which is not produced"domestically".
After deducting depreciation of capital stock $\lambda \cdot K(t), \lambda > 0$, we arrive at net output flow[2)] Y(t)

$$Y = AK^{\alpha}L^{1-\alpha} + Y_o - \lambda K.$$

The part of output that is consumed is denoted by G, the rest is immediately reinvested leading to an increase in capital stock of

$$\left(\frac{dK}{dt} =:\right) \qquad \dot{K} = Y - C.$$

Writing

$$k = K/L$$
$$c = C/L$$
$$\omega = Y_o/L,$$

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1) It is not necessary to assume that G(t) has this special form, but this is not the kind of "generality" we are after.

2) From hereon, the argument "t" is omitted for ease of notation

k being the capital stock per capita, etc., we find a system equation which involves only the variables k and ω,

$$\dot{k} = Ak^{\alpha} + \omega - (\lambda + \frac{\dot{L}}{L})k - c.$$

Moreover, if labor grows exponentially, i.e. $\frac{\dot{L}}{L} = \eta$ and c = constant[1)] then the behavior of the resulting growth of capital stock per capita can be visualized by the following Figures 2 and 3.

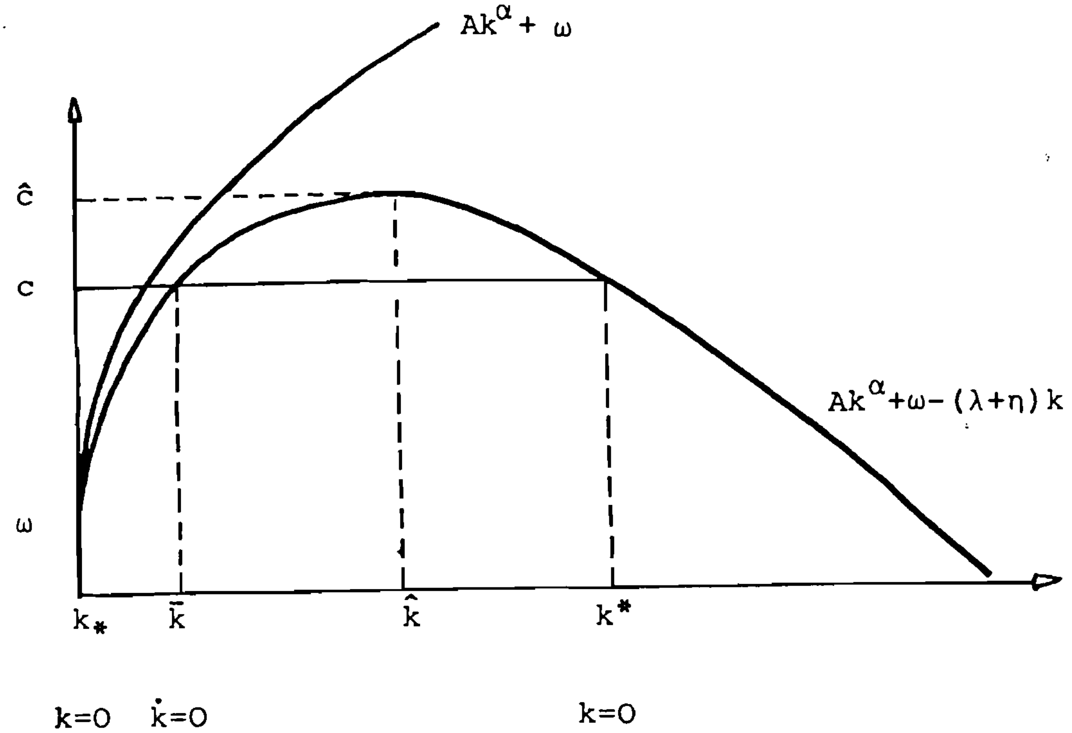

FIGURE 2

1) In reality, both the consumption behavior of highly developed and of very underdeveloped economies tend to be determined by factors other than aggregate production possibilities. For the latter economies it is the subsistence minimum which dictates consumption per capita, for the former it is, among other things, the reluctance to realize that the "Empire" does not exist any more.

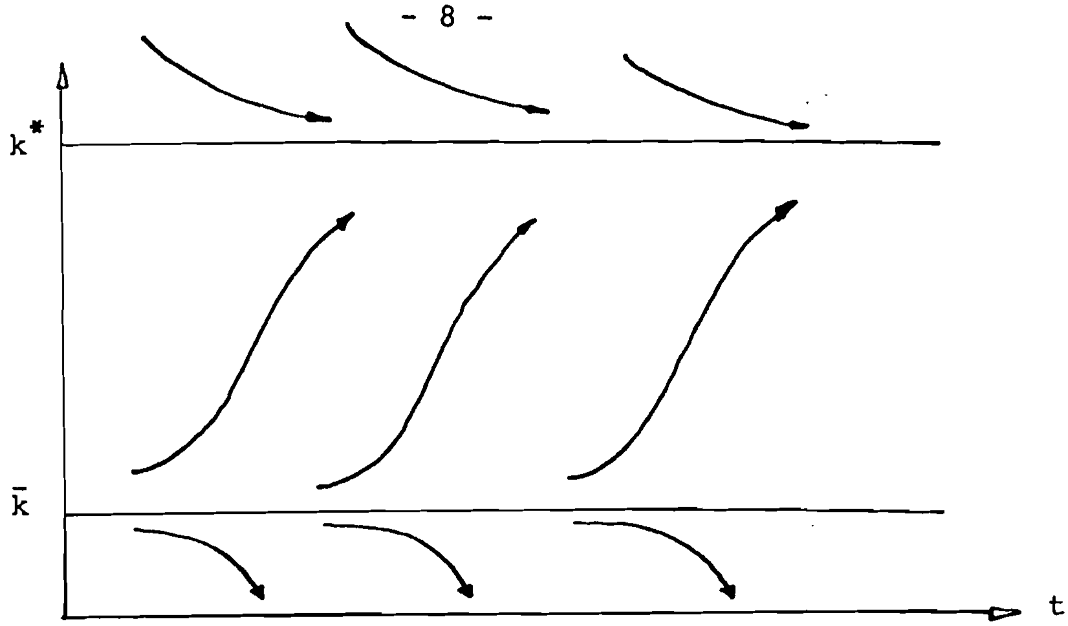

FIGURE 3

It is clear from FIGURE 2 that for each $c \varepsilon \langle \omega, \hat{c} \rangle$ the system has two stable equilibria, of which the upper one, k^*, can be thought of as the viable one, because the lower one, k_*, corresponds to zero capital stock. Note also that the lower equilibrium does <u>not</u> correspond to a stationary point of the system equation ($\dot{k} = 0$) but is similar to the boundary d-d in FIG.1. FIGURE 3 shows the phase portrait expanded by the time variable t. The seperatrix, that is the set of points seperating the domains of attraction $[0, \bar{k} \rangle$ and $\langle \bar{k}, \infty \rangle$ of the respective stable equilibria k_* and k^*, consists, in this case of a one-variable system[1), of the real number $\bar{k}$, which corresponds to an unstable equilibrium.

Hence, if an economy starts with a capital stock per capita below $\bar{k}$, this capital stock will tend to decline even more, if it is above $\bar{k}$, it will approach the upper stable equilibrium k^*. The capital stock per capita $\bar{k}$ is the critical one for a

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1) forgetting about ω for the moment, or ω = const.

take-off into self-sustained growth. We shall, in fact, propose below[1] to distinguish between developed and underdeveloped regions by checking whether they operate around one of their unstable or around one of their stable and viable equilibria. Note here, that a cut in unproduced income per capita (or foreign aid or whatever you call it) could swith a system from the upper to the lower domain of attraction, if it is operating around the unstable equilibrium $\bar{k}$.

CATASTROPHES

Let us now examine how our economic system reacts to variations in the consumption per capita c. Suppose, thus, that in FIGURE 2 the system is at the upper equilibrium k^* and c is slowly increasing. Then k^* moves smoothly to the left until it reaches the point $\hat{k}$, with the corresponding consumption per capita of $\hat{c}$. Once we got to this point, only a slight increase in c[2] causes the equilibrium to drop to k_*. Or, to put it differently, $\hat{k}$ finds itself in the domain of attraction of the equilibrium k_*. We can also follow this process in FIGURE 3 and observe that increasing c leads to a decrease in the size of the stability region of the upper equilibrium, measurable by the length of the interval $[\bar{k}, k^*]$ until, finally, at $\hat{c}$ this interval collapses to a point.
This demonstrates that for our economic model in parallel to PETERMAN's ecological case, the size of the upper stability region, corresponding to the viable equilibrium of the system is inversely related to the harvest from the system.

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[1] as a sort of "stylized fact"

[2] or decrease in ω , or another change of the $Ak^{\alpha}+\omega-(\lambda+\eta)k$ curve downwards.

Note that, after having increased consumption beyond $\hat{c}$, the corresponding "catastrophic jump" of the equilibrium cannot be reversed by decreasing c to $\hat{c}$ again. A much larger decrease of c will be needed to "catch the system" on its way into disaster.

Looking at the simple model many would probably argue that the model is not correct because the assumptions of the model are not realistic because, for example, it is obvious that, want it or not, consumption has to be decreased as output is dropping to zero. Also, as everybody knows, the technological coefficient α would fall with rising capital stock per capita.

Being aware of all that[1] we make the point here that c and α, as well as A and λ, are varying qualitatively more slowly than production and capital stock.
Moreover, for the qualitative analysis presented it turns out to be fruitful to consider relatively slow variables as constants first and to examine the equilibrium behavior of the relatively fast variables. Thereafter the reactions of the equilibria and domains of attraction on variations of the slow variables can be traced out.
Fortunately, the myriads of different equilibrium configurations which one might expect can - at least for models with few variables - all be categorized into a finite, and even small, number of "elementary catastrophes"[2]. This is the main implication for dynamical systems analysis of the deeply rooted but often easy to apply results of "catastrophe theory", see THOM (1972). The resulting methodology has been used extensively (e.g.JONES (1975)),

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[1] Modelling in greater detail could, for example, reveal an additional lower stable equilibrium k_*, $0 < k_* < \bar{k}$, corresponding to a primitive prehistorical society. But, eventueally, it also leads to economic models like "cembalo playing automatons", Marchetti (1976), where everything depends on everything.

[2] The terminology will become apparent below.

ISNARD & ZEEMAN (1974), ZEEMAN (1976) to explain the qualitative evolution of multiple equilibria systems[1].
Thus, keeping all slow variables constant but one - in our case c is the one - and then tracing out the equilibria with respect to variations in c we arrive at the simplest "elementary catastrophe" configuration, the "fold catastrophe" displayed by the following FIGURE.
FIGURE 4 shows the "equilibrium pairs of consumption per capita c and capital stock per capita k that can occur in the model. The equilibrium manifold is given by the line A-B-C-D and the stable (unstable) equilibria correspond to full (dashed) line segments.

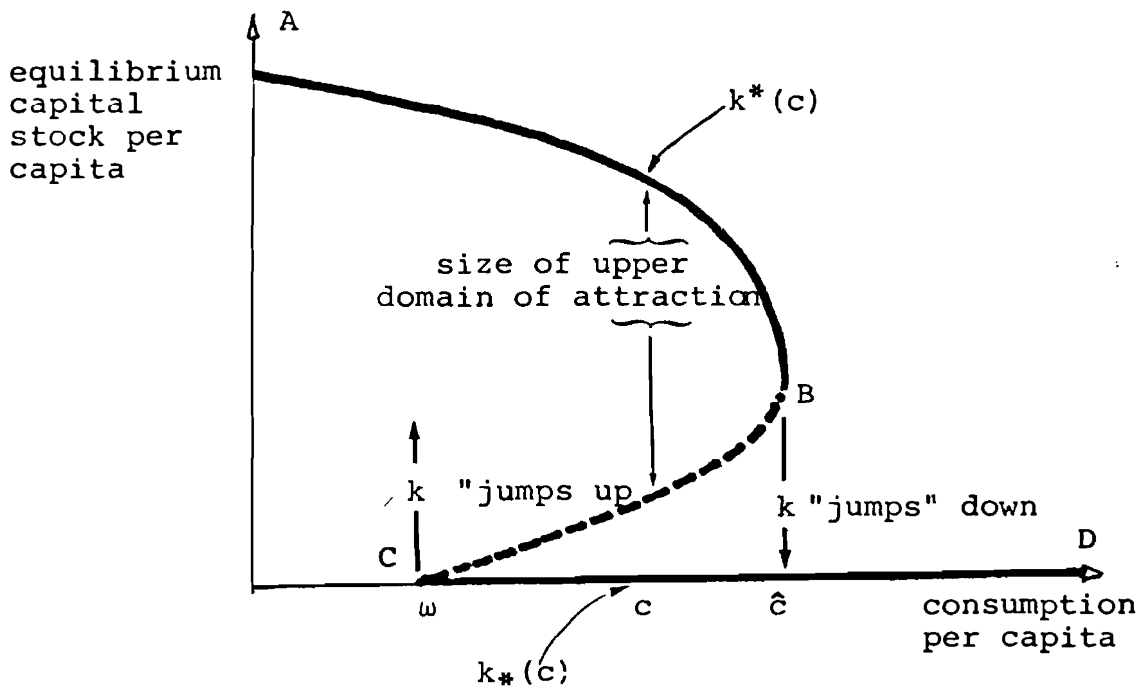

FIGURE 4

[1] The only application in economics, so far, seems to be ZEEMAN (1974) on a subject where the presence of multiple equilibria is more obvious.

The idea behind drawing the equilibrium manifold is that because k is assumed to be a fast variable it can always be thought of as being at its equilibrium value with respect to c. Clearly the position and the "Gestalt" of the manifold depend on the, presently constant, values of the other slow variables as well, and we return to that shortly.

FIGURE 4 illustrates again how the size of the upper domain of attraction drops when consumption per capita moves towards $\hat{c}$ from lower levels. Arriving at $\hat{c}$, obviously, the resilience of the system is at a minimum, because even the smallest "shock" for example a decrease in output by the agricultural sector caused by bad weather or a decrease in non-domestic product ω caused by a sudden currency revaluation, etc., could lead to a "catastrophic" jump of the equilibrium to a non-viable domain, represented here by a zero capital stock. Moreover FIGURE 4 shows that once capital stock got close to zero, in order to attain the upper domain of attraction c has to be decreased to below ω or, in turn, ω has to be increased to above c.

THE REAL WORLD: WESTERN EUROPE AND MIDDLE-SOUTH ASIA

Naturally the question arises whether real economies behave as if they maximized consumption, or resilience or a combination of both. To arrive at a rough indication the following aggregate data for the regions Europe (excluding Eastern Europe) and Middle and South East Asia have been used.[1)]

1) The data have been compiled and aggregated from the following sources:
Mesarovic and Pestel, eds., Multilevel Computer Model of World Development System, IIASA SP-74-2, vol. II, 1974, p. B50; W. Stroebele, Untersuchungen zum Wachstum der Weltwirtschaft mit Hilfe eines regionalisierten Weltmodells, Dissertation, TU Hannover, 1975, pp. 137, 174; UN Demographic Yearbook, 1973, p. 81; UN Yearbook of National Account Statistics, pp.6, 7 and others.

	Developed Region (Europe)	Less Developed Region (Middle and South Asia)
	All in 1970 $ for 1970	
output elasticity of capital α	0.12	0.80
capital per capita k	7240 $	250 $
income per capita y	2270 $	120 $
coefficient A (calculated from α,k,y)	783	1.44
population growth rate* η	.008	0.028
depreciation rate λ (weighted average)	.105	0.06
consumption/cap. $\bar{c}$ (incl.gov.exp.)	1691 $	105 $
consumption/cap. $\bar{c}$ (excl.gov.exp.)	1336 $	91 $

*The difference between "labor growth" and "population growth" has been neglected.

The situations of the different regions as mapped into our simple growth model are now illustrated by FIGURES 5 and 6,[1] which are similar to FIGURE 2.

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[1] We assume for the moment that ω, the unproduced income, is small compared to produced income and set $\omega = 0$. Furthermore we assume that the data given above correspond to equilibria states of the respective economies. This is a weak point in the argument but it is also the only way how static data can be used in a dynamic model.

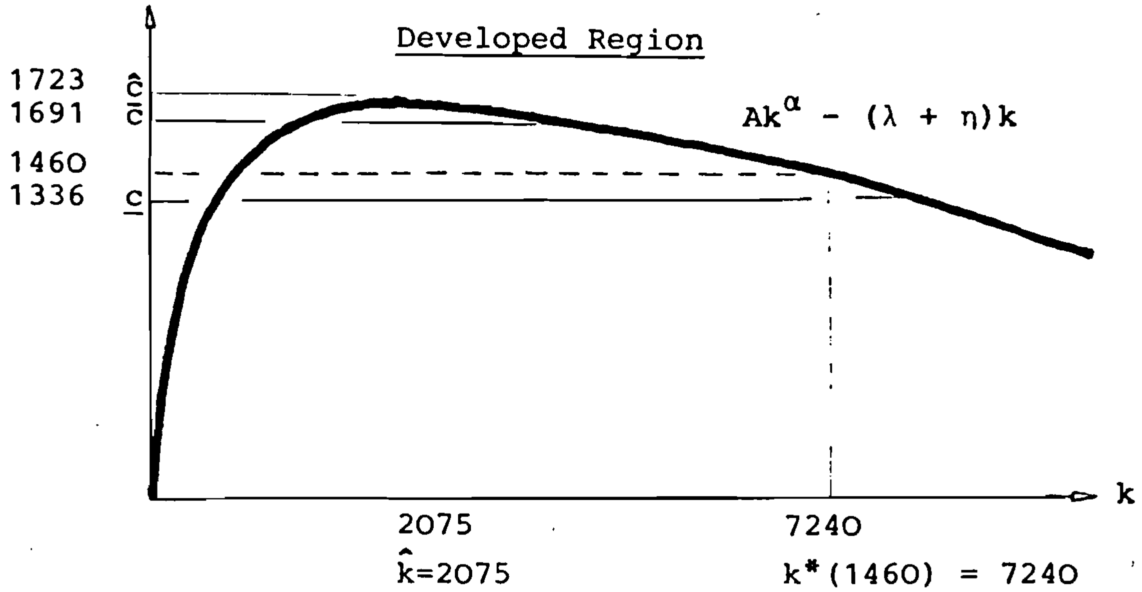

FIGURE 5

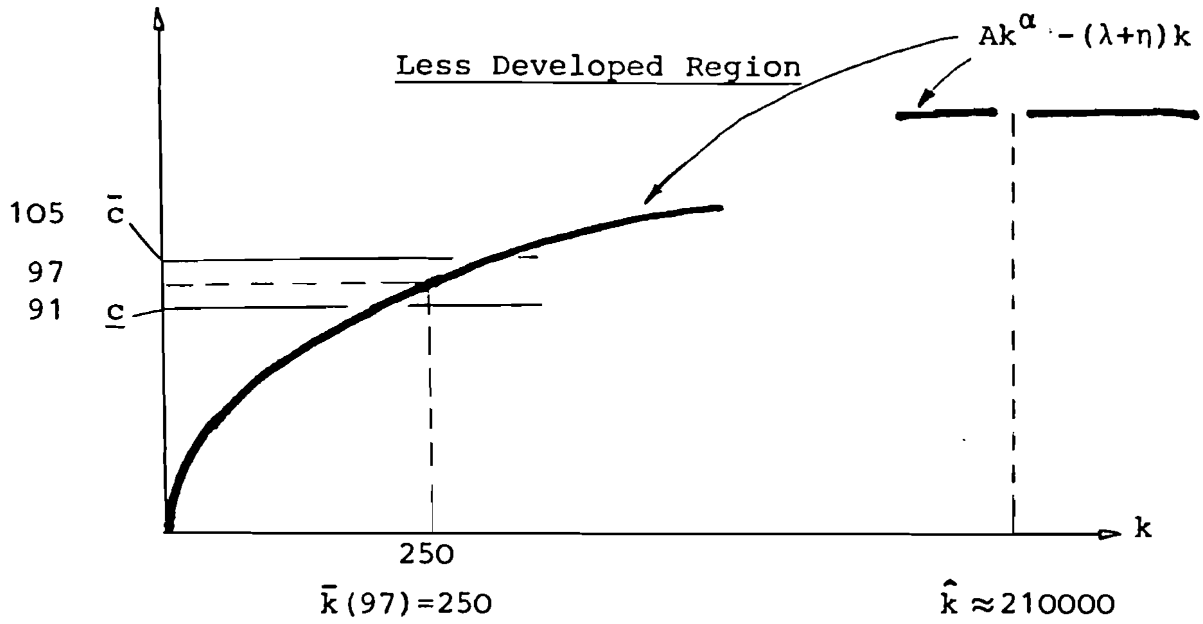

FIGURE 6

What springs to eye is that the LDR operates at the lower, unstable equilibrium $\bar{k}$ of its economic system. According to the share of goverment expenditure that leads to capital formation the capital stock of the LDR either declines to k_* or begins to grow towards the upper equilibrium k^*. This underlines the crucial importance of goverment policy for LDRs.

The situation for the DR is qualitatively different because this region operates at the upper, stable equilibrium of its economic system and this is so, irrespective of the consumptive share in goverment expenditure. Apparently the reason is that the DR has accumulated a capital stock per capita considerably larger than the one necessary to sustain actual consumption (with "perfect foresight"). On the other hand, maximum sustainable consumption would be $\hat{c}$= 1723 \$, while actual consumption is only 1460 \$.

But this means that the DR behaves as if it followed a composite objective, including the harvest from the system but also attach weight to the resilience of the system, that is in this case the "distance" from the critical consumption $\hat{c}$.[1]

Note that the above statements are made for each region with respect to "its economic system", i.e. it s particular set of values for the parameters A, α, λ and η. But, to say that again, these are slowly changing variables, too, and if we were to model the transition of a LDR to a DR, we would have to investigate how these changes take place.

The following FIGURE 7 shows the equilibrium capital stock as depending on the values of the slow variables c and α.

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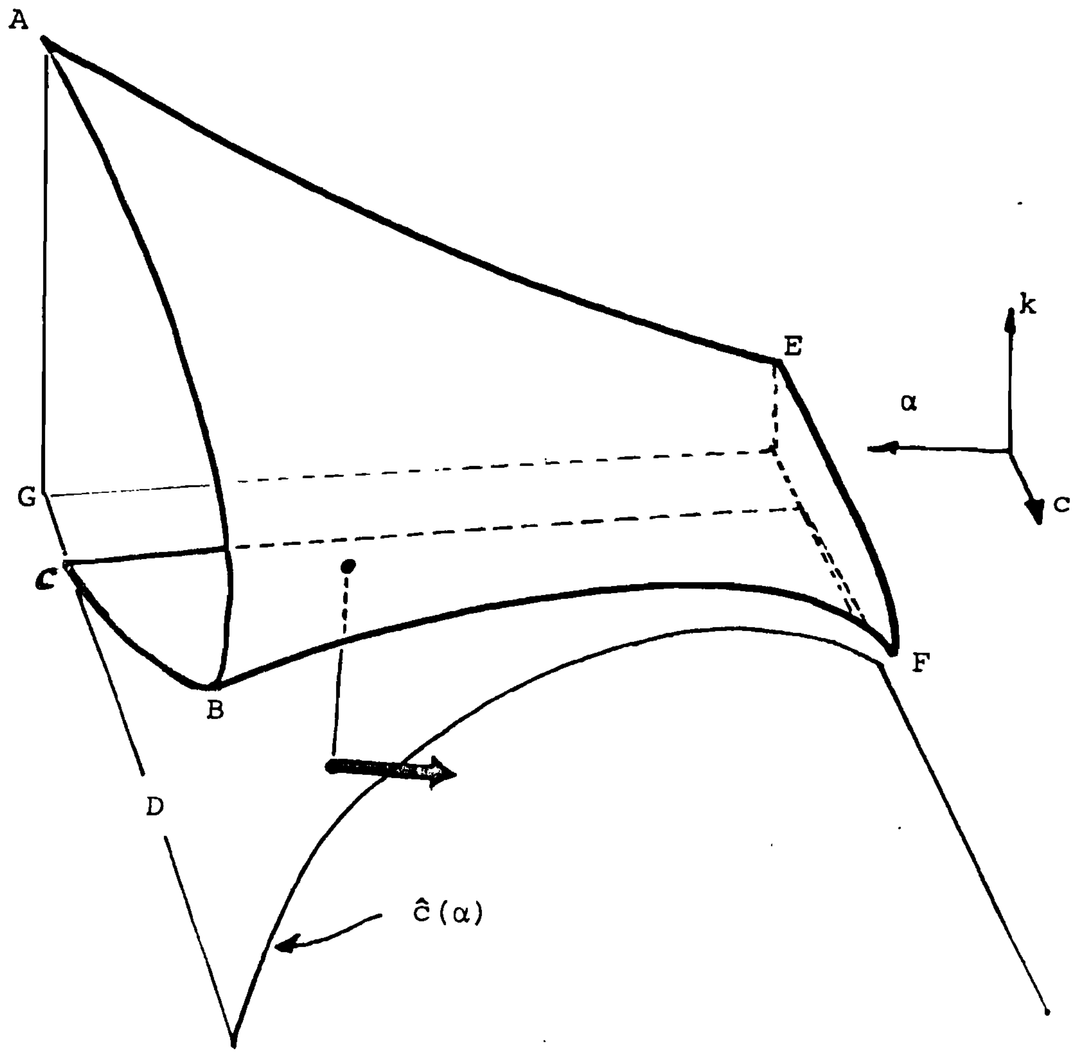
[1] If, reversing the argument, we categorize a region as developed or underdeveloped according to whether it operated at the stable or unstable equilibrium, then we would probably call "underdeveloping" (a term which has been used in connection with Britain and Italy) a region where consumption dangerously approaches or has just surpassed $\hat{c}$.

FIGURE 7



The upper equilibrium is represented by the upper sheet AEFB of the equilibrium manifold. For a particular fixed value of α, again, we obtain the submanifold ABCD of FIGURE 4. If an economy is in or near the position marked in the FIGURE by a dot in the front-left corner of the surface then - other things being unchanged - an increase in c and a decrease in α move the economy towards the edge BF of the manifold. Both α decreasing and c increasing are realistic assumptions supported by empirical evidence but, of course, other things are not unchanged. There is technical progress, decreases in population growth but also increases in the rate at which capital goods become obsolete and the superposition of all these external disturbances might shift the frontier BF to either side - with all the implactions mentioned in connection with FIGURE 4.

CONCLUSIONS

The purpose of the present paper was to indicate by means of a simple example how qualitative transitions in economic multi-equilibria systems could be stimulated, the purpose was not to show how these transitions would end up.
The first conclusion is that models like the ones from catastrophe theory should, in the social - sciences, be used to obtain qualitiative information about possible structural changes that are ahead. If the system does in fact change, in the form of a catastrophic jump to a new equilibrium,then the inner structure of the system is likely to undergo rapid change, which is certainly not predictable within the model itself.
Social sciences are, in this respect, in a somewhat different situation to (non-human) ecology, where "Mutter Natur" decides what the system is going to be like after a structural change. But who would dare to predict the year 1800 by using a model of the French economy and society in the year 1788 or the political structure in South-Africa in five years time ?

The second conclusion is that economic science should think twice about the conventional equivalence between being "rational" and being a "consumption maximizer". This does not mean that rational man does not try to optimize his situation but it means that rational man cares also about the maintenance (or change) of the qualitative structure of the economic system within which he operates.

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