ON A METHOD FOR ANALYZING
INTERCONNECTIONS WITHIN COMPLEX SYSTEMS

Wladyslaw Olshansky

August 1976 WP-76-23

2361
Laxenburg
Austria

International Institute for Applied Systems Analysis

On a Method for Analyzing Interconnections within Complex Systems

One of the problems that usually arises in dealing with complex systems is what to do in a complicated network of informational, organizational, and other interconnections among the elements of these systems.

I can refer the reader to the STAC Bulletin Number 3 of July 16, 1976 where a hypothetical organizational structure of IIASA is presented and where the Organization Task Force of the Uncoordinated Management Committee advises us to use it as often as possible [1].

An approach to facilitate using such complex network diagrams may be suggested. This approach is to aggregate a complex multi-verticed graph of interconnections and then reduce it to a simpler graph that will adequately describe the structure of the whole complex graph. This simpler graph should have as many vertices as is reasonable for the purposes of the users, this number usually being sufficiently less than the original number of vertices.

The vertices of this structural graph will actually be subsets of the set of all vertices of the original graph, and the arcs will represent bunches of arcs of the original graph.

The problem here is in the adequateness of the aggregated representation. In [2], a criterion for measuring this adequateness and a method for maximizing it is introduced.

In that article [2], interesting examples of application of this method--in psychology (testing compatibility of crew members, attitudes in kindergartens), in economy (analyzing product flows)--are given.

In [3] this method is applied to the organization of information exchange in a large-scale scientific project.

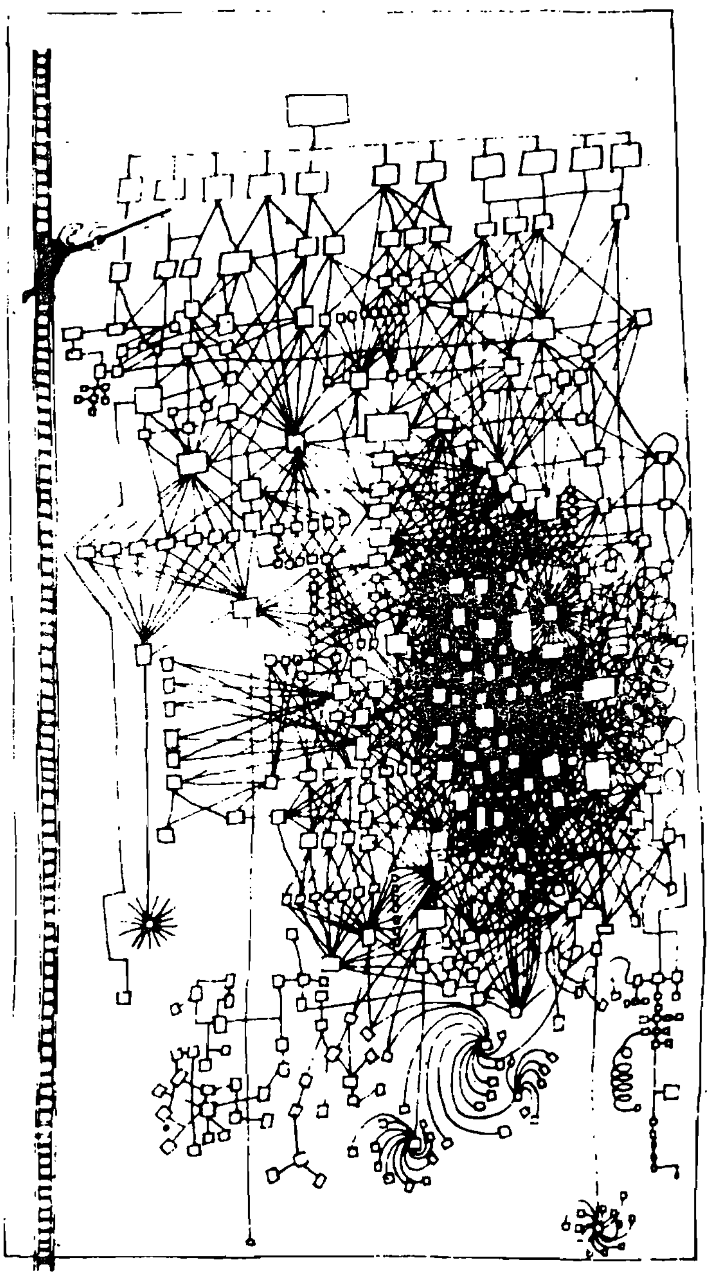
In [4], an application of this method for the analysis of information flows in cancer research using references in scientific publications is presented.

Further results in this direction may be found in [5].

* * *

As the graph of interrelations can be represented by its matrix of adjacency, we shall study this adjacency matrix, A.

The essence of the method may be described easiest by an example. Let us consider the system of interconnections from [1]. This is the following graph:



If we enumerate the vertices of this graph, we can represent it as its adjacency matrix having those elements which correspond to rows and columns representing vertices linked together equal to one.

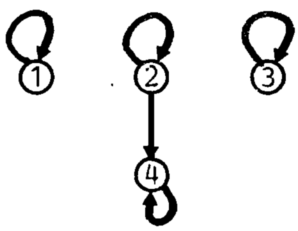
So we have the main matrix, A. For simplicity, let all the vertices of the original graph be divided into four lists, B, that do not intersect, i.e. list 1 consists of elements $1,\ldots,n_1$; list 2 - of elements $n_1+1,\ldots,n_2$; ...; list 4 - of elements $n_3+1,\ldots,n_4$.

Suppose that the lists have the linkage shown by the matrix of structure Γ:

Γ =

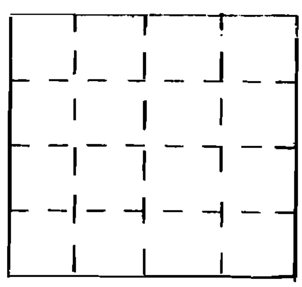
1	0	0	0
0	1	0	1
0	0	1	0
0	0	0	1

or by the following graph:



In matrix Γ, "one" means that the elements of the corresponding lists are strongly interconnected.

The ideal matrix, M, of interconnections among the elements should have horizontal and vertical strips corresponding to the lists. The squares appearing as the result of intersections of these strips correspond to the elements of the matrix Γ:



In the ideal matrix, M, there naturally should be ones in those squares that correspond to the elements of matrix Γ equal to one, and there should be zeroes in those squares that correspond to zero elements of matrix Γ:

all ones	all zeroes	all zeroes	all zeroes
all zeroes	all ones	all zeroes	all ones
all zeroes	all zeroes	all ones	all zeroes
all zeroes	all zeroes	all zeroes	all ones

Now, if the elements have interconnections described with matrix M, the aggregated structure of these elements (in list B) ideally would be the matrix Γ.

When list B and the structure Γ are specified and the ideal matrix M is reconstructed, this matrix M usually will not coincide with the main matrix A of really existing interconnections among the elements.

We should introduce some measure of discrepancy between real and ideal matrices of interconnections, A and M, correspondingly.

It is only natural to penalize for those ones in matrix A that appear in squares where matrix M has all zeroes, or for those zeroes in matrix A that appear in squares where matrix M has all ones.

If a unit of such a penalty is one, then the sum of these ones over the whole matrix A represents the measure of unlikeliness of --the discrepancy between--the real matrix A and the ideal matrix M.

Now, there are two ways to decrease this discrepancy:

(a) change the lists B;

(b) change the structure Γ.

One way to change the lists is to recompile them so that they intersect, i.e. to allow the elements to appear in more than just one list.

In doing this, the algorithms for the reconstruction of the ideal matrix M (given lists B and the structure Γ), and for the calculation of the amount of discrepancy become more complicated.

To find the optimal distribution of the elements in the lists and the optimal structure Γ, which correspond to the minimal unlikeness of matrices A and M, we should seek minima in two "dimensions"--(a) and (b).

Using different strategies in this search, we may arrive at different local minima, which also depend on the initial structure Γ_o and lists B_o.

This example deals with the simplest graphs of interconnections --without weighted arcs--which can be described by a binary adjacency matrix. When the interconnections have different strengths (the arcs are weighted), the method should be modified. This modified method is shortly described in [2].

* * *

The method described may be used in the analysis of various problems in various research areas.

For example, this methodology may be helpful in analyzing:

a) various kinds of migration flows among different geographic regions;

b) the energy exchange among subsystems of large electric power networks;

c) the flows of production and trade on the global or regional scale;

d) the information flows among the variety of directions in scientific research, and especially links among individual scientists;

e) scheduling sections and participant lists for conferences and workshops;

f) the psychology of grouping individual relations.

Of course this list may easily be expanded, and I imagine that some points in it could be of interest to IIASA: this methodology might provide an approach to the required interface between the research areas within IIASA's matrix framework.

References

[1] Intercepted Memos Dept. IIASA Stac Bulletin, 3, 4, (July 16, 1976), 5.

[2] Muchnik, I.B. Analysis of Structure of Experimental Graphs, Avtomatika i Telemekhanika, 9, 1974. Eng. trans. in Automation and Remote Control, 35, 9, (February, 1975), 1,432-1,447.

[3] Venediktov, D.D., et al. On International Scientific Project Implementation, with Special Reference to Cancer Research, Avtomatika i Telemekhanika, 1, 1976. Eng. trans. forthcoming in Automation and Remote Control.

[4] IIASA Bio-Medical Conference, Moscow, Laxenburg, December, 1975.

[5] Brodkin, L.I., I.B. Muchnik, A New Criterion for Aggregating Large Linkage Matrices, Problemy Analiza Diskretnoy Informatsii (Problems of Discrete Information Analysis), Part II, Institut ekonomiki i organizatsii promyshlennogo proizvodstva SO AN SSSR (Institute for Economics and Organization of Industrial Production, Siberian Branch of USSR Academy of Sciences), Novosibirsk, 1976.