SOME REMARKS ON OLIGOPOLISTIC EQUILIBRIUM

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1. Introduction

This paper may be considered as a continuation of "Some Instruments of Agricultural Policies in a General Equilibrium Framework". In that paper, national governments were assumed to introduce tariffs and quota on international trade and possibly even a stock policy, for two purposes: 1) in order to keep the balance of trade of certain commodities within desired bounds, or 2) in order to reach a certain desired domestic price level. Both producers and consumers were assumed to take all prices as given and the governments to take the world market prices as given.

Neither of these papers deals with the specific goals which would determine these desired levels for domestic variables. A later paper will discuss possible optimum (tariff) strategies, but to do this in a relevant way, one first needs more information about the actual goals of the different societies and governments. Actually, this paper will not reach the subject of government policies. It should be considered a preparation for the discussion of imperfect competition between nations. In other words, the government of a country, especially of a large one, may try to influence variables other than domestic variables by means of tariffs, quota, etc. It may, for example, try to influence world market prices.

But in order to maintain a connection with economic theory, and in order to understand the basic features of oligopoly, this paper shall discuss only the interaction between producer and consumer, without governmental influence.

The usual approach to imperfect competition by economic theory looks somewhat casuistic. The aim of this paper is to present a somewhat unified picture of imperfect competition from which the special well-known cases may be derived. In the course of this paper we shall find some justifications for the basic assumptions made in connection with the theory of general economic equilibrium under perfect competition. When one thinks about the real world, each of these assumptions cries for relaxation. Perhaps the general equilibrium approach itself should be relaxed in order to describe the process of arriving at equilibrium. However, this paper will take the "classical" G.E. approach as a starting point. We shall remain within a static context and list the main assumption as:

1. numbers of actors, commodities, factor endowments are given,
2. actors take all prices as given,
3. actors react on prices only,
4. two kinds of actors: producers and consumers
5. producers maximize profits,
6. producers have full knowledge of prices and technology,
7. the technology has nonincreasing returns to scale,
8. consumers maximize utility given the budget constraint and the full knowledge of prices,
9. the consumers own all endowments,
10. every commodity has a market,
11. in equilibrium every commodity has one price in any market.

§ 2. Imperfect Competition

The relaxation of any assumption from the list above can be interpreted as the introduction of imperfect competition, but traditionally imperfect competition means the relaxation of assumption #3. The converse of #3 is "Not (all actors take all prices as given)", which is equivalent with "some actors take some prices as not given."

In general, one thinks of the producer as affecting the price. We shall also make that assumption, but first a description of what producers do, or better, what producers can do, will be given.

§ 3. Producer's Behaviour

We shall stick to assumptions #4,5. When given profit maximization and full knowledge of the technology, what other factors determine the behaviour of the producer? Two factors come to mind:

1) the knowledge of the producer,

2) the instruments available to him.

We shall assume

ad1) that the producer has full knowledge of the demand schedule of the consumer, of his income, and of all prices. (It will become clear during the discussion that the producer does not need to have all that information separately, he needs only to know the slope of "his" demand curve with respect to his price.)

ad2) that every producer can directly influence his own price and his own production, but takes the production of the other producers and their prices as given.

§4. Some Historical Perspective

In order to better understand the position taken in this paper, it may be helpful to present a very brief historical account of the theory of imperfect competition. The history of the theory of imperfect competition starts in 1839, when A. A. Cournot tried to show that all forms of competitive behaviour could be derived from monopolistic theory. The case of perfect competition was, according to him, a limiting case. His work in this field was left unnoticed for almost fifty years. Cournot assumed that the price and quantity of the competitors was given (there was no equation in the "think model" of the producer, that is, among the constraints of the maximization problem, for the reaction of competitors on the own action).

Cournot assumed one homogenous product. Launhardt (1885) and Hotelling (1929) have relaxed this assumption for the case of two producers.

Edgeworth (1897) rejected the "no reaction" assumption, thus introducing a more game theoretic approach into oligopolistic theory.

Cyert and March (1963) have strongly criticized the approach of describing producer behaviour as <u>centralized profit maximization</u> with <u>perfect knowledge</u>. They conceive of producers as organizations with <u>many decision makers</u> with <u>many goals</u>, with <u>aspiration levels</u>, instead of maximizing behaviour, and with <u>uncertainty</u>.

All these criticisms seem justified, but they complicate the discussion considerably.

Hicks (1940) writes (p.85), "Personally I doubt if most of the problems we shall have to exclude for this reason [i.e. by excluding decreasing marginal costs and by assuming perfect competition] are capable of much useful analysis by the methods of economic theory."

The model of perfect competition is such a powerful device because of its simplicity. The more complex the behaviour of the different actors becomes, the more doubt is cast on the existence and stability of equilibrium. On the other hand, increasing the number of decision makers again could increase the "flexibility" and thus the stability. Joan Robinson (1974) supports the assumption of a given stable hierarchy between producers where 1) there is one price leader in each branch who sets the price as a mark-up over cost, and 2) where other producers take that price as given (this is a simplification of the theory by Von Stackelberg (1934)).

It must be clear to the reader by now that the approach to the producer's behaviour set out in the previous paragraph is very simple indeed. The argument for this is simply that there is not yet a formal economic theory which can take the "no reaction" criticism and the Cyert and March criticisms into account in a model with many commodities and many actors. For reference, see J.J. Laffont and G. Laroque, (1976).

In this paper, we shall work under some simplifying assumptions which are not made in the Laffont/Laroque article.

1) We shall disregard the case where the maximum profit is negative. That is, we shall assume homogeneity of the production function which corresponds to zero fixed costs.

This again implies that the integral of marginal costs is equal to the total cost. Under the assumptions of non increasing returns this guarantees that profit maximization implies non-negative profits.

2) Only consumption goods are considered.

3) Only one aggregate consumer is considered: the treatment of many consumers would not essentially change the approach, it would only make it more complex.

4) We assume that the income of the consumer is given, that is we neglect all feedback effects of prices on wages, income, etc. If these assumptions would only imply given factor prices, it would not be very serious. More serious problems arise, however, because of the implication the monopolistic profits are not redistributed to consumers. Only the relaxation of this assumption would yield a truly general equilibrium approach, but for the simplicity of exposition it is not attempted here.

§5. The Case of Full Monopoly: The Formula by Amoroso-Robinson.

We consider a monopolist to be one who knows exactly what the demand curve for his product is: $y = g(p)$. The monopolist can influence prices and quantities.

The behavioural model is

$$\max z = p \cdot y - F(y) \tag{5.1}$$

$$\text{S.T. } y = g(p)$$

$$\text{and } p,y \geqq 0$$

where y is the quantity, p is the price, F(y) is the cost function. The demand function is assumed to be monotonous so that we may write

$$p = g^{-1}(y) \quad .$$

Substituting this equation into the goal function and then taking the first order condition, we get the famous quality between marginal costs and marginal returns:

$$p(1+\eta) = \frac{\partial F}{\partial y} \quad , \tag{5,2}$$

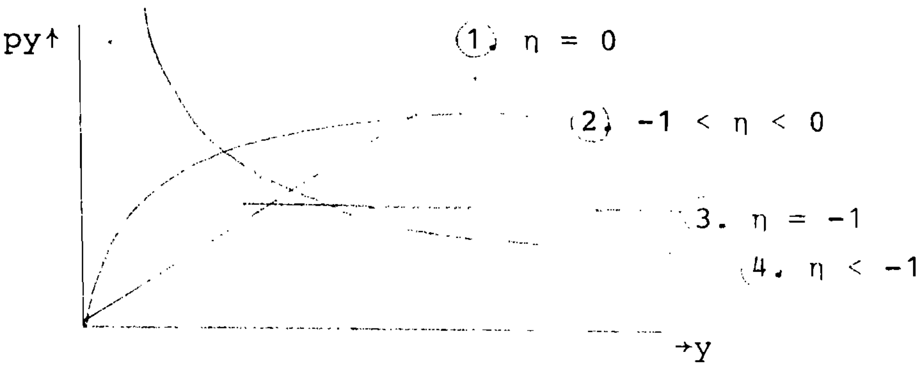
where

$$\eta = \frac{\partial \ln p}{\partial \ln y} \; (<0) \quad .$$

This formula is called the Amoroso-Robinson formula.
Thus a monopolist will produce less at a given price than would a producer under full competition. Consider a case with constant elasticity:

$$py = \alpha \; y^{(1+\eta)} \quad .$$

We can then draw the following graph.



If we want to find a positive production and a bounded price the marginal returns must be positive: that is $-1 < \eta \leqq 0$. This implies $\varepsilon = \frac{1}{\eta} < -1$; the demand must be quite elastic. As a side product, we may note that if $\varepsilon < -1$, there may be equilibrium even under decreasing or constant unit costs (nondecreasing returns to scale). This is standard economic theory. In the following, we will, however, assume nonincreasing returns to scale.

The case of full monopoly is quite unrealistic. Classical demand theory does not consider independent demand schedules for different commodities, but starts from utility maximization under the budget constraint. In doing so, it allows for substitution and substitution limits any monopoly.

Before we discuss the demand side, a slight generalization of 5.1 has to be introduced. As returns equal price multiplied by the quantity sold, we may write

$$x = y + z \tag{5.3}$$

demand = supply + stock mutation.

Considering in nonstorable commodity, we may write

$$\max w = p(y-z) - F(y) \tag{5.4}$$

$$\text{S.T. } y = g(p) + z$$

$$p, y, z \geqq 0 \quad .$$

§6. Oligopoly and a Two-Level Utility Function.

6.1 Two-level utility functions

Sato (1967) has introduced the notion of a two-level production function. That is, a function which may be written in a hierarchical way as

$$y_1 = F_1(y_2)$$

$$y_2 = F_2(y_3) \quad ,$$

in which y_1 is a scalar, y_2 is a vector valued function of the structure

$$y_{2,1} = F_{2,1}\ (y_{3,1}, \ldots, y_{3,n_1})$$

$$y_{2,2} = F_{2,2}\ (y_{3,n_1+1}, \ldots, y_{3,n_1+n_2})$$

$$y_{2,m} = F_{2,m}\ (Y_{3,n_{m-1}+1}, \ldots, Y_{3,n_{m-1}+n_m}) \quad .$$

Strotz (1957) has, on the other hand, developed the concept of the utility tree, also a hierarchical structure. Brown and Heien (1972) have integrated both approaches and studied a two-level utility function. We shall use this concept in this paper in order to differentiate between commodities and goods. In this we follow Cramer (1973):

> "We consider consumer demand and distinguish between homogenous goods on the one hand and composite commodities on the other. By a good, we mean a specific variety or brand sold at a single price; its representatives are indistinguishable in use. If, from the consumers' point of view the quantities of several goods can sensibly be added together, such goods belong to the same commodity."

Although we do not know precisely what "sensibly adding together" means we interpret this as "contributing to the same branch of the utility function".

We write the model of consumer behaviour as being:

$$\max u(x)$$

$$\text{S.T. } x_i = f_i\,(q_{i,1},\ldots,q_{in_i}) \qquad i = 1,\ldots,m \qquad (6.1)$$

and

$$\sum_{i=1}^{m} \sum_{j_i=1}^{n_i} p_{ij_i}\, q_{ij_i} = M \ .$$

For simplicity of notation we write:

$$\max u(x) \qquad (6.2)$$

$$\text{S.T. } x = F(q)$$

and $$p \cdot q = M \ .$$

- We assume u(x) and F(q) to be linear homogenous (in the short run only, the parameters may be adapted between years).
- We assume that every brand corresponds to one producer and that one producer produces only one brand.
- The consumer is assumed to take his income and all prices as given.

6.2 Heterogenous oligopoly

The first order conditions of 6.2 are then

$$\frac{\partial u}{\partial q_{ij_i}} = \lambda \, p_{ij_i} \quad ; \tag{6.3}$$

and

$$p \cdot y = M \quad .$$

Due to linear homogeneity we may write

$$\lambda = \frac{u}{M} \quad ,$$

and 6.3 becomes

$$p_{ij_i} = \frac{\partial u}{\partial q_{ij_i}} \cdot \frac{M}{u} \quad . \tag{6.4}$$

Differentiating with respect to the price of an arbitrary good i,ji and arranging all prices and quantities along one vector $[1_1, \ldots, n_i, \ldots; 1_m, \ldots, n_m]$, we can write

$$\hat{u} \cdot \begin{bmatrix} \frac{q_h}{M} \\ \frac{\partial q_1}{\partial p_h} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial q_r}{\partial p_h} \end{bmatrix} = \begin{bmatrix} -\frac{u}{M} \cdot q_h \\ 0 \\ | \\ 0 \\ \frac{u}{M} \\ 0 \\ | \\ 0 \end{bmatrix} \tag{6.5}$$

where $\hat{u}$ is the bordered Hessian of $u\ [F(q)]$:

$$\hat{u} = \begin{bmatrix} 0 & u_1 & \cdots & u_r \\ u_1 & u_{11} & \cdots & u_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ u_r & u_{r1} & \cdots & u_{rr} \end{bmatrix}$$

applying Cramer's rule we write

$$\frac{\partial q_h}{\partial p_h} = \frac{D^h}{|\hat{u}|} \tag{6.6}$$

where D^h is the determinant of the matrix $\hat{u}$ with the hth colunm replaced by the right-hand side vector.

We now recall the Amoroso-Robinson equation (5.2):

$$p(1+\eta) = \frac{\partial F}{\partial y} \quad , \qquad \text{where } \eta = \frac{\partial p}{\partial x}\,\frac{x}{p} \tag{5.2}$$

and rewrite it in the present context as

$$p_h + q_h\left(\frac{\partial p_h}{\partial q_h}\right)^* = \frac{\partial F_h(y_h)}{\partial y_h} \quad , \qquad h = 1,\ldots,r \quad , \tag{6.7}$$

where

$$\left(\frac{\partial p_h}{\partial q_h}\right)^*$$

is the inverse of the slope of the demand curve as perceived by the producer.

It is a fundamental assumption of the present approach that we assume that (at least in equilibrium):

$$\left(\frac{\partial p_h}{\partial q_h}\right)^* = \frac{\partial p_h}{\partial q_h} \quad .$$

It is not so much the assumption that the producer has imperfect perception which would complicate matters, as it is the assumption that the producer expects reactions by his competitors on his own moves. In equilibrium we may write $q_h = y_h$. Substituting 6.4 and 6.6 in 6.7, we get a set of r simultaneous nonlinear equations in y. Substituting the results in the right hand side of 6.4 yields the prices.

The system discussed here may or may not have a solution but we skip this problem here because the food scene seems to correspond to a more restricted case: that of homogenous oligopoly.

§7. From Heterogenous to Homogenous Oligopoly

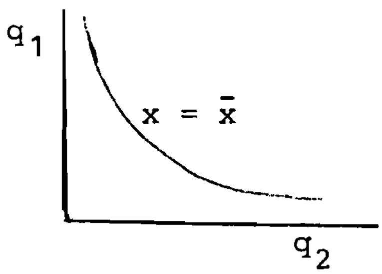
Moving from heterogenous oligopoly to homogenous oligopoly implies that

$$x_i = f(q_{ij_i}, \ldots, q_{in_i})$$

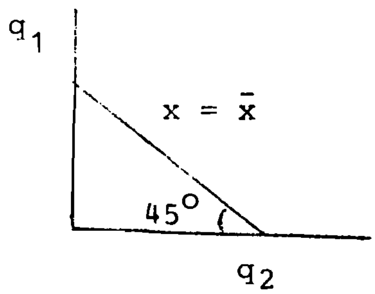
more and more approximates

$$x_i = \sum_{j_i=1}^{n_i} q_{ij_i} \quad ,$$

which implies that the substitution elasticities become larger and larger and that the marginal utility of the good more and more approaches the marginal utility of the commodity. In terms of a two-dimensional iso-commodity curve, it means that



approaches to



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It is clear that the tangent to the iso-commodity curve will come closer and closer to a slope of (-)45° as long as both q_1 and q_2 are positive, which implies that the prices of two goods must move toward each other as the two goods become better substitutes for each other, as long as demand for both goods is positive.

In order to demonstrate that this will be the case under homogenous oligopoly and nonincreasing returns, we reconsider the equations of §5 and 6. (From now on we will omit the subscript i of j.)

7.1 The Consumer

Under homogenous oligopoly, the bordered hessian $(\hat{u})$ becomes singular. We may, however, derive what the consumer will do:

$$\max u(x)$$

$$\text{S.T. } x_i = \sum_{j=1}^{n_i} q_{ij} \qquad (7.1)$$

and

$$\sum p_{ij} q_{ij} = M \quad .$$

x_{ij_i} is the quantity of i demanded from the j_ith producer. The consumer will demand everything from the cheapest producer. For simplicity's sake, we take the prices of all other commodities as given, and write

$$x_i = x_i(p_{ij}{}^*) \qquad (7.2)$$

where

$$p_{ij}{}^* = \min_j (p_{ij}) \quad , \qquad (7.3)$$

write $h^* = i,j^*$ and $h = i,j$.

Can this be an equilibrium situation? To find that this is not the case, consider the producer model 5.4.

7.2 The Producer

First we rewrite 5.4:

$$\max w_h = p_h(y_h - z_h) - F_h(y_h) \quad ,$$

$$p_h, y_h, z_h \geqq 0 \quad .$$

The first order condition now becomes:

$$\frac{\partial w_h}{\partial y_h} = \frac{\partial p_h}{\partial (y_h - z_h)} (y_h - z_h) + p_h - \frac{\partial F_h(y_h)}{\partial y_h} \quad ,$$

$$\frac{\partial w_h}{\partial z_h} = p_h - \frac{\partial p_h}{\partial (y_h - z_h)} \cdot (y_h - z_h) \quad ,$$

$$\frac{\partial w_h}{\partial z_h} = - p_h(1 + \eta_h) \quad ,$$

we can write

$$\frac{\partial w_h}{\partial y_h} = - \left(\frac{\partial w_h}{\partial z_h} + \frac{\partial F_h(y_h)}{\partial y_h} \right) .$$

Thus the condition $\frac{\partial w}{\partial y_h} = 0$ can only be realized if $\frac{\partial w}{\partial z_h} < 0$,

as $\frac{\partial F_h(y_h)}{\partial y_h} > 0$ (not necessarily increasing);

$\frac{\partial w}{\partial z_h} < 0$ implies that z_h will be minimized, that is $z_h = 0$.

To formulate it in a more economic sense: the producer will always prefer to adapt his price and quantity than to have unsold production. In this model, <u>market equilibrium is desired by the producer</u>.

- if one producer is the cheapest, he will get all the demand:

$$\eta_h = \frac{x_i}{p_h^* \cdot \frac{\partial x_i}{\partial p_h^*}}$$

- if two producers have the same lowest price, we get by defining $q_h^* \equiv y_h^* - z_h^*$; $h^* = h_1 , h_2$,

$$\eta_h^* = \eta_h \cdot \frac{\partial x_i}{\partial q_h^*} \cdot \frac{q_{h^*}}{x_i} .$$

As mentioned before, an essential assumption of the oligopoly theory discussed here is that $\frac{\partial x_i}{\partial q_h^*}$ for the producer equals the real $\frac{\partial x_i}{\partial q_h}$ for the consumer. In this case, this implies $\frac{\partial x_i}{\partial q_{h^*}} = 1$, which yields, $\eta_h^* = \eta_h \cdot S_h^*$ by defining the share in demand as $S_{h^*} \equiv \frac{q_h^*}{x_i}$.

This means that under homogenous oligopoly the flexibility of the demand curve with which the producer is confronted equals the flexibility of the market demand schedule multiplied by the share in that demand:

- the Amoroso-Robinson condition under homogenous oligopoly may be written as

$$p_h \cdot (1+\eta_h^* \frac{q_h}{x_i}) = \frac{\partial F_n}{\partial y_n} .$$

If $p_h > p_h^*$ then $q_h = 0 \rightarrow p_h = \frac{\partial F}{\partial y_h} \rightarrow y_h > 0 \rightarrow z_h > 0$.

So this situation cannot be consistent with desired producer behaviour. This again implies that in equilibrium, every commodity will have only one price. This is a generalization of the traditional models of competitive equilibrium. The equality of supply and demand is now a direct consequence of producer behaviour. The uniqueness of price is a condition for market equilibrium instead of an independent assumption.

Now, in equilibrium the Amoroso-Robinson equation is for every producer:

$$p_i + x_i \frac{\partial p_i}{\partial x_i} \cdot \frac{y_{ij}}{x_i} = \frac{\partial F_j\ (y_{ij})}{\partial y_{ij}} \quad , \text{ while } q_{ij} = y_{ij}$$

given p_i, x_i and $\frac{\partial p_i}{\partial x_i}$ may be computed from the demand side; then every y_{ij} may be calculated from the corresponding Amoroso-Robinson equation. If the production function is homogenous of a degree smaller than one, this implies that all producers will produce something. $\frac{y_{ij}}{x_i}$ approximates 0 as the number of producers increases. In the limit we are in the case of perfect competition with given prices, $p_i = \frac{\partial F_j y_{ij})}{\partial y_{ij}}$.

If there is only one producer we are again back to the case of monopoly. A coalition of producers could make more profit than the individual producers separately. Consider the profit of a coalition

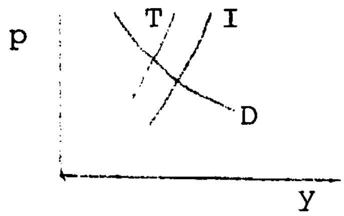
$$\max p \ \Sigma \ y_i - \Sigma \ F_i(y_i) \ .$$

which yields as F.O.C.

$$p + x \frac{\partial p}{\partial x} = \frac{dF_i}{dy_i} \ .$$

The trust can exploit its monopolistic

power and thus make more profit. The production will be lower, due to the negative slope of the demand schedule (D). The supply schedule of the trust (T) will be to the left of the aggregate supply schedule of independent producers (I).



A well-known problem for trusts is that a small producer will make more profit outside the trust than within.

§8. Relevance of Oligopoly for Food and Agriculture

The foregoing exposition of "old" theory may seem somewhat irrelevant for the field of food and agriculture because government behaviour was left out, and because production and consumption of agricultural commodities are in the hands of an immense multitude of actors producing fairly homogenous commodities such as rice, wheat, etc. Where then, is there such a centralization of decisions that an oligopolistic treatment may be relevant? Two fields come immediately to mind.

1) The field of marketing and processing of agricultural commodities,

2) the field of governmental policy at least concerning imports and exports.

The next paper will therefore treat some optimal agricultural policies in an oligopolistic (-oligopsonistic) context, thus attempting to endogenize government policies.

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APPENDIX

Two-Level Utility Function and the Activity Approach to Consumption Theory

Many people have criticized the utility function for being a tautological construct, devoid of any psychological content. This criticism has induced research on the structure of the utility function. An explicitly structured utility function could also help in solving some decomposition problems, so that an optimal solution could be reached in a decentralized way.

A utility function could also help for interpersonal comparison of welfare. Direct interpersonal utility comparison is believed to be inappropriate, however, and a commodity-by-commodity comparison of personal consumption would not lead to a consistent ordering. It seemed that some intermediate cardinal concept could be fruitful. In that context, Lancaster developed his linear activity analysis of consumption.

Basically, the classical problem is reformulated as

$$\max u(z)$$

$$\text{S.T } z = Bx$$

and $$px \leq M \quad .$$

B is the matrix of consumption technology, and every commodity contributes to one or more characteristics. It can be held that the characteristics may be defined in an interpersonal way. Interpersonally inefficient consumption activities may be defined as the boundary of the "characteristic possibility curve" when one thinks in terms of oligopoly, the linearity does not seem appropriate for obvious reasons.

A more detailed formulation would then be:

$$\max u(z)$$
$$\text{S.T. } z = G(x)$$
$$\text{and } px \leqq M \quad .$$

This formulation would also be more general than the one presented in the paper because one good would contribute to many characteristics: the tree would be a network. The oligopolistic producer could spend money on advertising in order to change the structure of G(x) in an optimal fashion, that is in a way in which the price elasticity of the demand of the good would become very small.

He could also study z = G(x) for given parameter values. in order to find out with which producers he has the highest rate of substitutability and with which he should merge.

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