MODEL MULTIREGIONAL LIFE TABLES AND STABLE POPULATIONS

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Preface

A central concern of the Human Settlements and Services research group at I.I.A.S.A. has been the analysis of the dynamics of multiregional population growth and distribution. Recently this activity has stimulated a concerted effort to extend and expand the applicability of mathematical demographic models in the study of such dynamics. This paper, the sixth, of a series addressing the general topic of spatial population dynamics, develops a family of model migration schedules and illustrates their potential application in studies of model multiregional stable populations. (This working paper is a preliminary draft of a forthcoming Research Report and is being reproduced to elicit comments and suggestions for possible incorporation into the final version.)

March 1976

1. Andrei Rogers and Frans Willekens, "Spatial Population Dynamics," RR-75-24, July, 1975, forthcoming in *Papers*, Regional Science Association, Vol. 36, 1976.
2. Andrei Rogers and Jacques Ledent, "Multiregional Population Projection," WP-75-106, August, 1975, forthcoming in *Proceedings*, 7th I.F.I.P. Conference, 1976.
3. Andrei Rogers and Jacques Ledent, "Increment-Decrement Life Tables: A Comment," WP-75-127, October, 1975, forthcoming in *Demography*, 1976.
4. Andrei Rogers, "Spatial Migration Expectancies," RM-75-57 , November 1975.
5. Andrei Rogers, "Aggregation and Decomposition in Population Projection," RM-76-00, January 1976.
6. Andrei Rogers and Luis J. Castro, "Model Multiregional Life Tables and Stable Populations," WP-76-00, March 1976.

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Abstract

This paper focuses on the construction of hypothetical "model" migration schedules and multiregional stable populations. It begins by identifying the persistent regularities that are exhibited by observed migration schedules and then summarizes these regularities by means of regression equations to establish a family of hypothetical migration schedules. These schedules are then combined with model fertility and mortality schedules to generate hypothetical stable populations which offer valuable insights into the dynamics of spatial population growth and change.

Summary

Model schedules have two important applications: 1.) they may be used to infer empirical schedules of populations for which the requisite data are lacking and 2.) they can be applied in analytical studies of human population dynamics.

The development of model fertility and mortality schedules and their use in studies of the evolution of human populations have received considerable attention. The construction of model migration schedules and their application in studies of the spatial evolution of human populations have not. This paper addresses the latter question and demonstrates how techniques that have been successfully applied to treat the former problem can be readily extended to deal with the latter.

Migration rates vary substantially by age. They are relatively high for the young but decline sharply with age. The basic age profiles of migration schedules may be summarized by means of regression equations that relate age-specific migration rates to indices of migration levels. These equations, together with comparable ones for mortality schedules, may be used to construct "model" multiregional life tables which describe the mortality-migration patterns of a multiregional population. Such tables, in turn, may be combined with model fertility schedules to create hypothetical "model" multiregional stable populations.

Model multiregional stable populations reveal the long-run consequences of particular changes in fertility, mortality, and

migration levels. They show, for example, that the stable shares of regional populations exposed to identical schedules of fertility and mortality will vary inversely with the ratio of their respective migration levels. They demonstrate that higher rates of growth lead to stable populations that taper more rapidly with age. And they reveal that regional age compositions and birth rates are relatively insensitive to changes in migration levels.

Model migration schedules and model multiregional stable populations illuminate important aspects of spatial population dynamics. To the extent that a workable understanding of spatial population dynamics is an important ingredient of informed human settlement policymaking, they constitute a useful and necessary component of the spatial planner's analytical apparatus.

Model Multiregional Life Tables and Stable Populations

Andrei Rogers and Luis J. Castro

1. Introduction

The evolution of a human population undisturbed by emigration or immigration is determined by the fertility and mortality schedules to which it has been subject. If such a "closed" population system is disaggregated by region of residence, then its spatial evolution is largely determined by the prevailing schedules of internal migration.

The age-specific fertility, mortality, and migration schedules of most human multiregional populations exhibit remarkably persistent regularities. The age profiles of such schedules seem to be repeated, with only minor differences, in virtually all developed and developing nations of the globe. Consequently, demographers have found it possible to summarize and codify such regularities by means of hypothetical schedules called model schedules.

Model schedules have two important applications: 1.) they may be used to infer (or "smooth") empirical schedules of populations for which the requisite data are lacking (or inaccurate) and 2.) they can be applied in analytical mathematical examinations of population dynamics.

Countries that lack accurate vital registration data with which to compute age-specific fertility and mortality rates have had to rely on schedules developed on the basis of census data alone.

> "Suppose that a closed population is enumerated in two censuses at an interval of exactly ten years, and that each census contains tabulations of males and females by age, in five-year intervals.... A sequence of life table values can be based on the sequence of calculated census survival ratios, and by well-tested actuarial procedures, a life table can be constructed for ages above five--provided that the two censuses achieved accurate coverage of the population, and that ages were accurately recorded." (Coale and Demeny, 1967, p. 7).

Census survival ratios derived from census-enumerated age distributions distorted by age-misreporting must be adjusted after calculation in order to "smooth" out those that are unreasonably low or that exceed unity. Model life tables offer a convenient solution to such problems of data smoothing.

Compare, for example, the empirical and model survival ratios in Figure 1. The female survival ratios calculated from Indian and Turkish censuses illustrate the highly erratic pattern that can be introduced by age misreporting. The survival ratios derived from the Korean censuses, however, generally fall inside of the range defined by model life tables with expectations of life at birth of 35 and 45 years, respectively. This is an indication that no serious misreporting of age probably occurred in those censuses.

The growth dynamics of empirical populations are often obscured by the influences that particular initialänditons have on future population size and composition. Moreover, the vast quantities of data and parameters that go into a description of such empirical dynamics make it somewhat difficult to maintain a focus on the broad general outlines of the underlying demographic process and instead often encourage a consideration of its more peculiar details. Finally, studies of empirical growth dynamics are constrained in scope to population dynamics that have been experienced and recorded; they cannot be extended readily to studies of population dynamics that have been experienced but not recorded or that have not yet been experienced at all. In consequence, demographers frequently have resorted to examinations of the dynamics exhibited by hypothetical model populations that have been exposed to hypothetical model schedules of growth and change. An illustration of such an approach appears in the work of Ansley Coale, from whose recent book (Coale, 1972) we have extracted Figure 2 below.

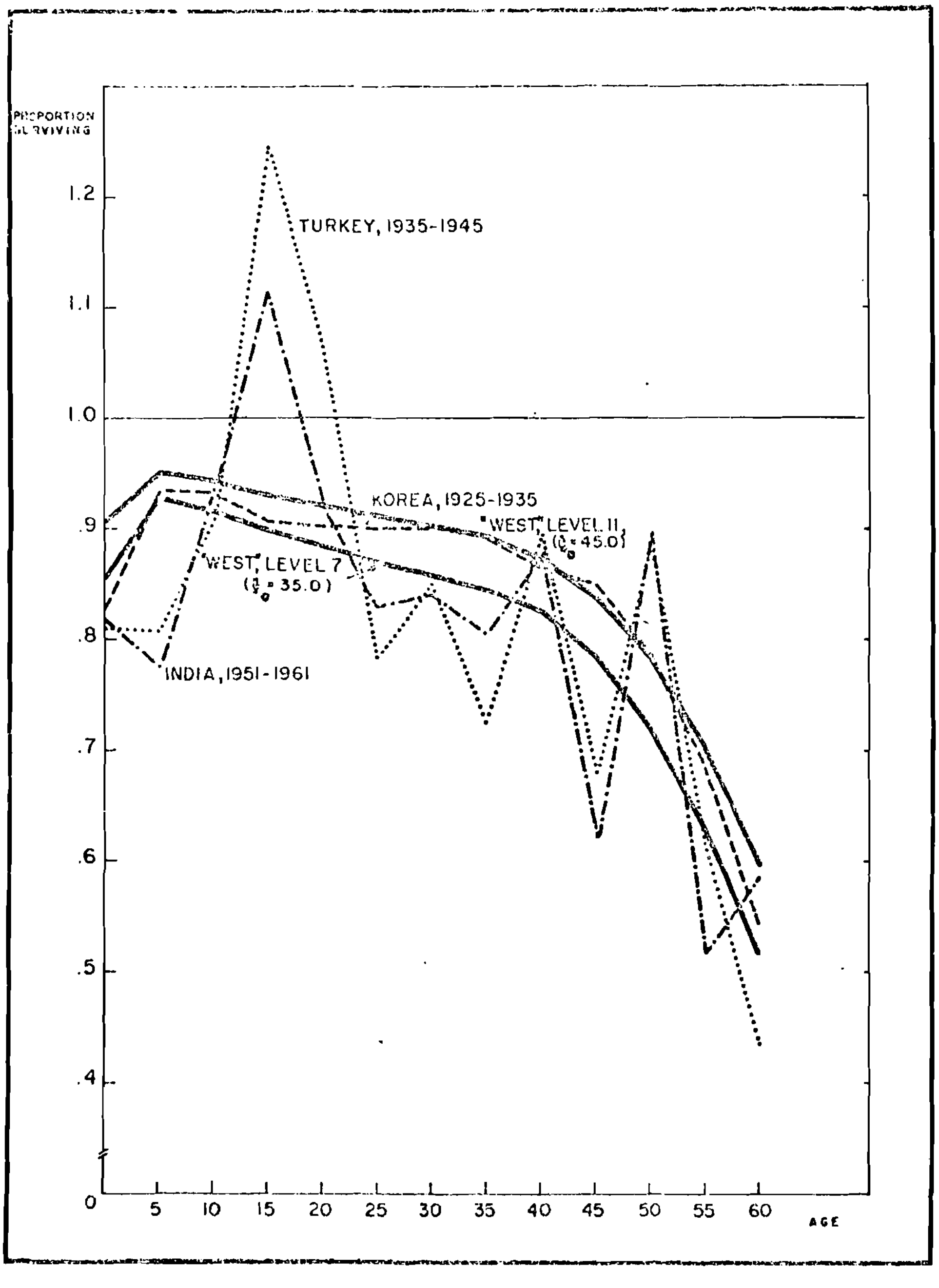


Figure 1: Census survivorship rates of females from age x to x + 5 at time t to age x + 10 to x + 15 at time t + 10, according to censuses of India, Korea and Turkey, and according to selected "West" model life tables

Source: Coale and Demeny, 1967, p. 9

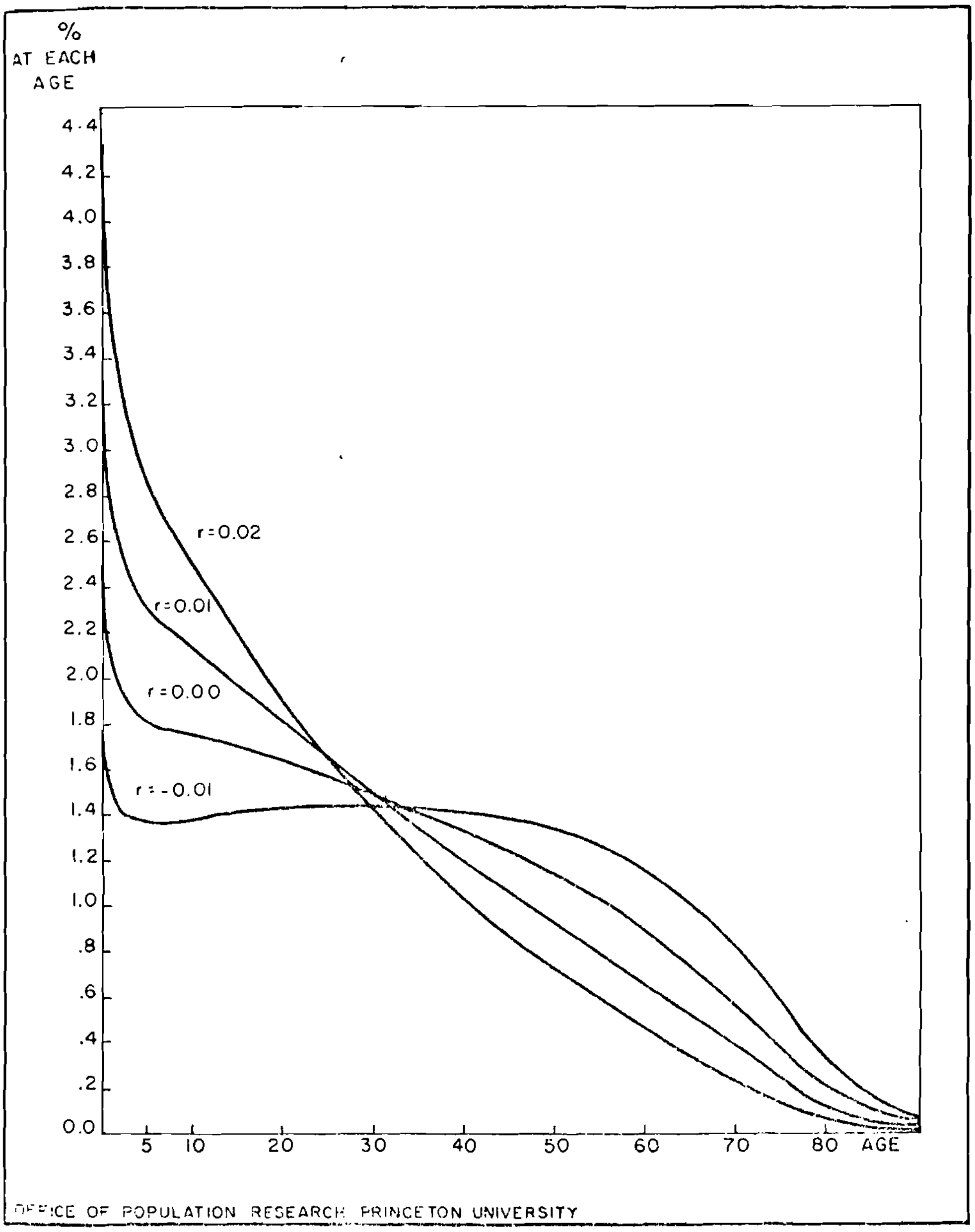


Figure 2: Age distribution of stable populations, same mortality schedule, different rates of increase

(e(0) = 40 years)

Source: Coale, 1972, p. 29

Figure 2 describes the age compositions of stable populations that have evolved from a very long exposure to the same constant mortality schedule and one of several different levels of unchanging fertility. Inherent in the interaction of every such pair of human fertility and mortality schedules is a unique age composition, called the stable population, that ultimately grows at a constant "intrinsic" rate of growth, r, and assumes a stable constant age composition, c(x). If r is zero, for example, the age composition is that of the stationary zero-growth population. In Figure 2 the shape of a stationary population is contrasted with those of growing and declining populations. Observe that higher values of r create stable age compositions that taper more rapidly with age, thereby causing such populations to have a lower mean age than low fertility populations.

The development of model fertility and model mortality schedules and their use in studies of the evolution of human populations have received a considerable amount of attention (Arriaga, 1970; Coale and Demeny, 1966 and 1967; Coale, 1972; Rele, 1967). The construction of model migration schedules and their application to studies of the spatial evolution of human populations disaggregated by region of residence, however, have not. This paper addresses the latter question and shows how techniques, that have been successfully applied to treat the former problem can readily be extended to deal with the latter. We begin, in Section 2, by considering the regularities and dynamics exhibited by a specific empirical population disaggregated into four regions of residence and observed at two points in time. We then follow this study of the regularities and dynamics of an empirical population with an examination, in Sections 3 and 4, respectively, of the regularities and dynamics of hypothetical model populations. The paper concludes with a brief consideration of directions for further research.

2. Regularities and Dynamics in Empirical Multiregional Populations

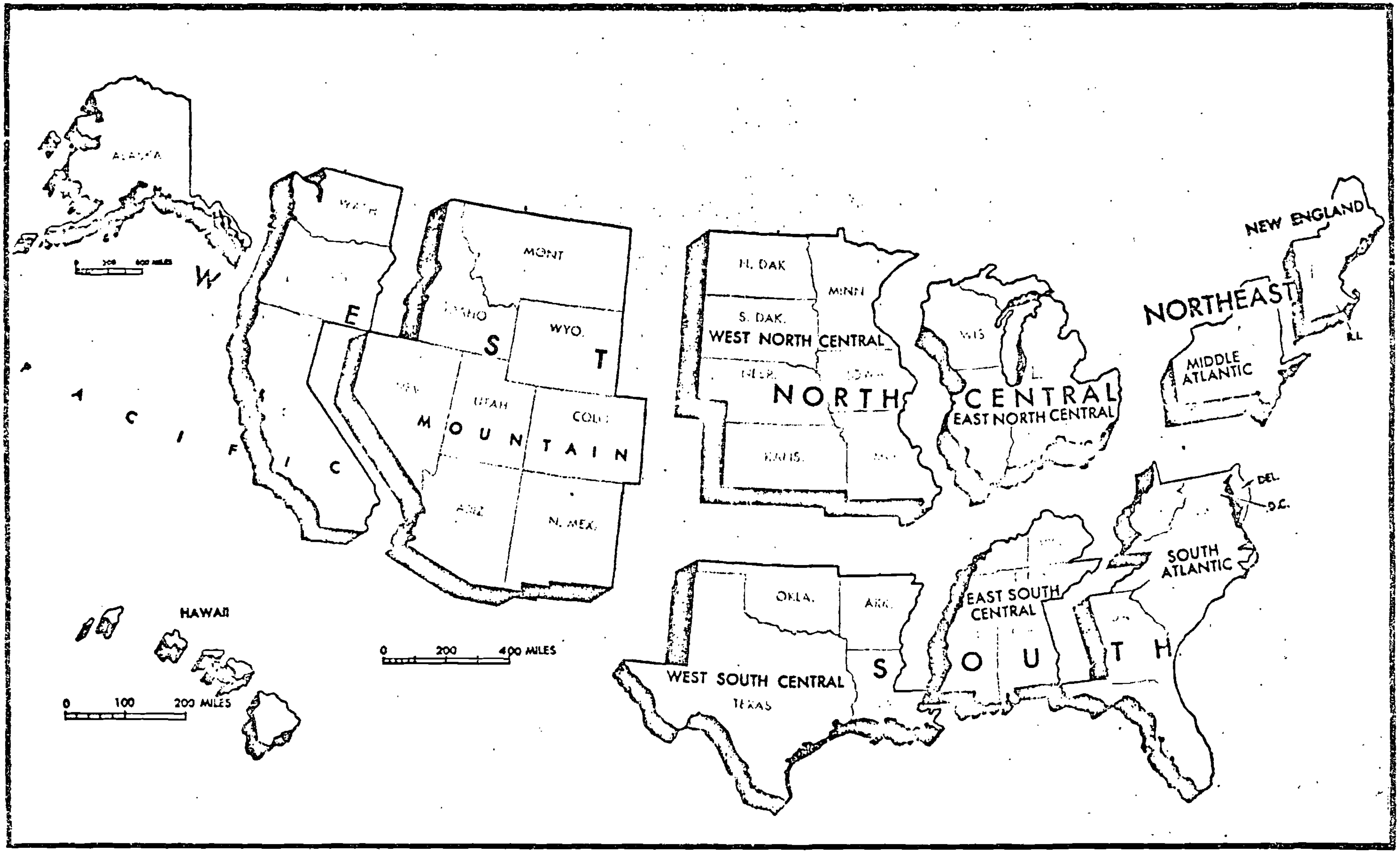
Our examination of the regularities and dynamics of an empirical population will focus on the evolution, over a decade, of the U.S. total population resident in the four Census Regions that collectively exhaust the national territory: 1.) the Northeast Region, 2.) the North Central Region, 3.) the South Region, and 4.) the West Region. Figure 3 illustrates this particular geographical division of the U.S. and also exhibits the finer spatial disaggregation of the four regions into the corresponding nine Census Divisions. Although most of this paper deals with the four-region system, we will briefly refer to the nine-region system in Section 3.4.

2.1 Regularities in Empirical Demographic Schedules of Growth and Change

The shape, or profile, of an age-specific schedule of fertility, mortality, or migration is a feature that usefully may be studied independently of its intensity, or level. This is because there is considerable evidence that although the latter tends to vary significantly from place to place, the former very often remains relatively constant between localities. We now shall consider the regularities in the profiles of such schedules in turn, starting with fertility.

Fertility. Age-specific rates of childbearing exhibit a fundamental pattern that persists over a remarkably wide range of human populations.

> "...age schedules of fertility in human populations have a number of general features in common. All rise smoothly from zero at an age in the teens to a single peak in the twenties or thirties, and then fall continuously to near zero in the forties and to zero not much above age 50." (Coale, 1972, p. 5.)



SOURCE: U.S. BUREAU OF THE CENSUS

FIGURE 3: Regions and Geographic Divisions of the United States

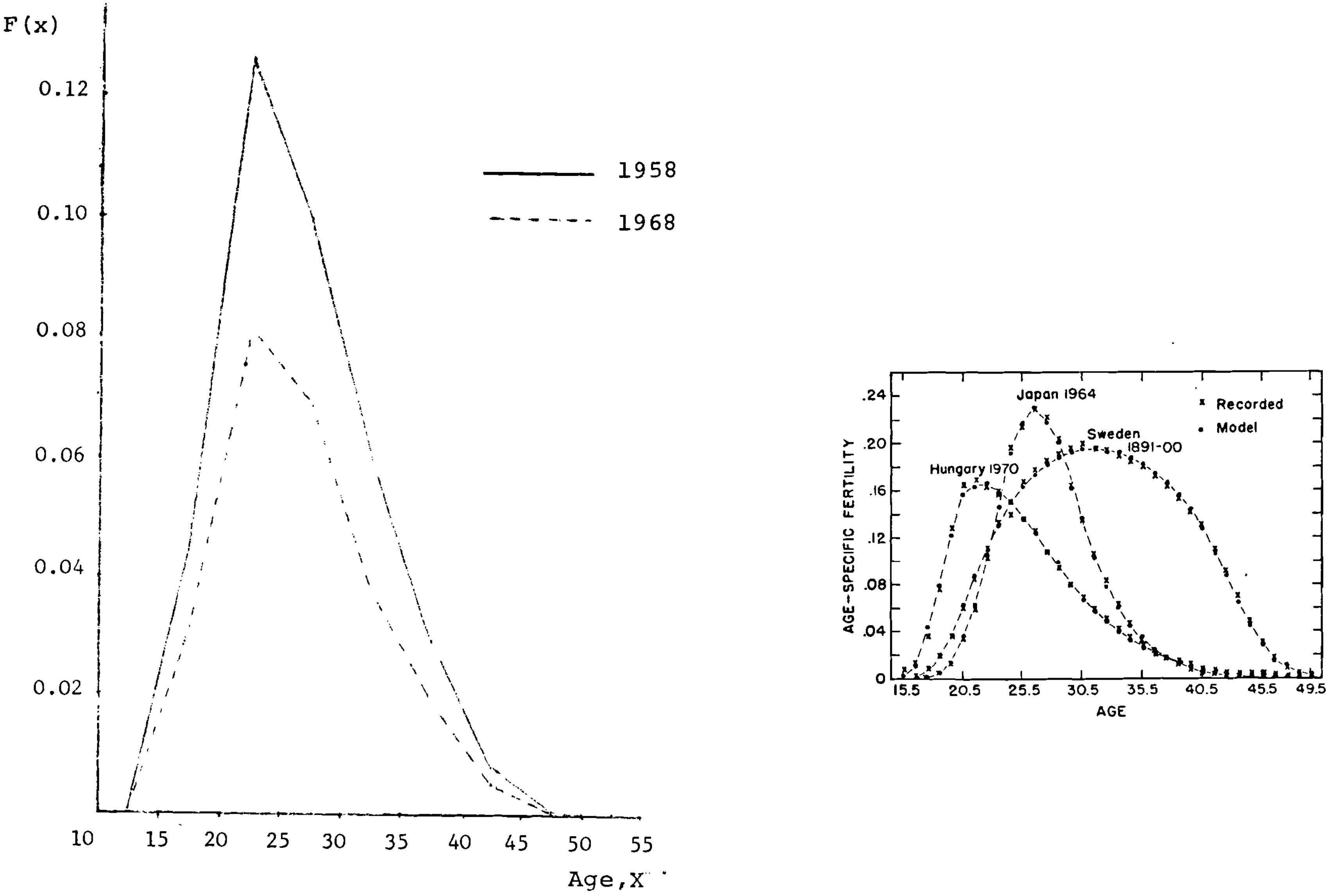
Figure 4 presents several schedules of fertility, all of which follow the general profile described above. In Figure 4A are outlined the fertility schedules of the U.S. total population in 1958 and 1968, respectively. Figure 4B gives the fertility schedules of Hungary in 1970, Japan in 1964 and Sweden in 1891-1900. All of the schedules exhibit the same general age profile but vary substantially in the mean age of this profile and its standard deviation. According to Coale and Trussell (1974), the age schedules in Figure 4B had the lowest and highest mean ages (Hungary and Sweden) and the lowest standard deviation (Japan) among those that they examined in their recent study of model fertility schedules.

Mortality. Observed schedules of mortality vary in a predictable way with age. They normally follow a U-shaped pattern in which rates are moderately high during infancy decrease thereafter to a low in the very early teens, and then rise monotonically to the last years of life.

> "In almost every accurately recorded schedule of death rates by age, mortality declines sharply during the first year from a high value immediately after birth, falls more moderately after age 1 to a minimum between age 10 and 15, increases gradually until about age 50, then increases ever more steeply until the highest age for which a rate is given." (Coale, 1972, p. 8.)

Figure 5 presents mortality schedules for the U.S., Japan, the U.S.S.R., and Poland. The fundamental age profile of mortality is evident in all. Mortality is high during infancy, ranging anywhere from 5 to 8 per thousand live births; it achieves its minimum between ages 10 and 15, dropping to a value between 0.3 to 0.5 per thousand; it then rises to values that in the late sixties vary between 16 to 38 per thousand.

Migration. Rates of migration vary substantially by age. They tend to be highest for people in their early twenties, after which time they generally decline sharply with age.

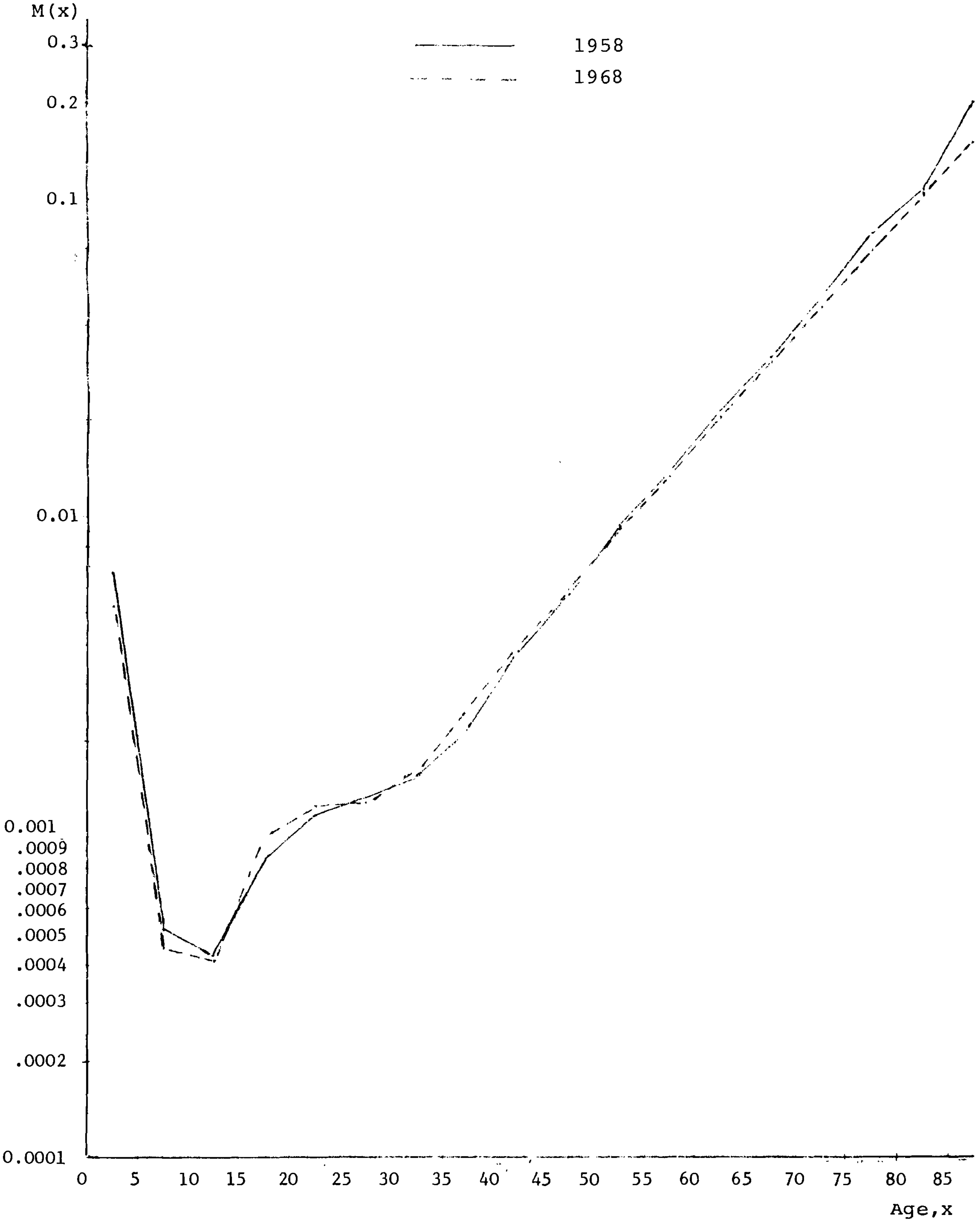


A. Age-Specific Fertility Rates of the Total United States Population: 1958 and 1968

B. Age-Specific Fertility Rates of Three Populations

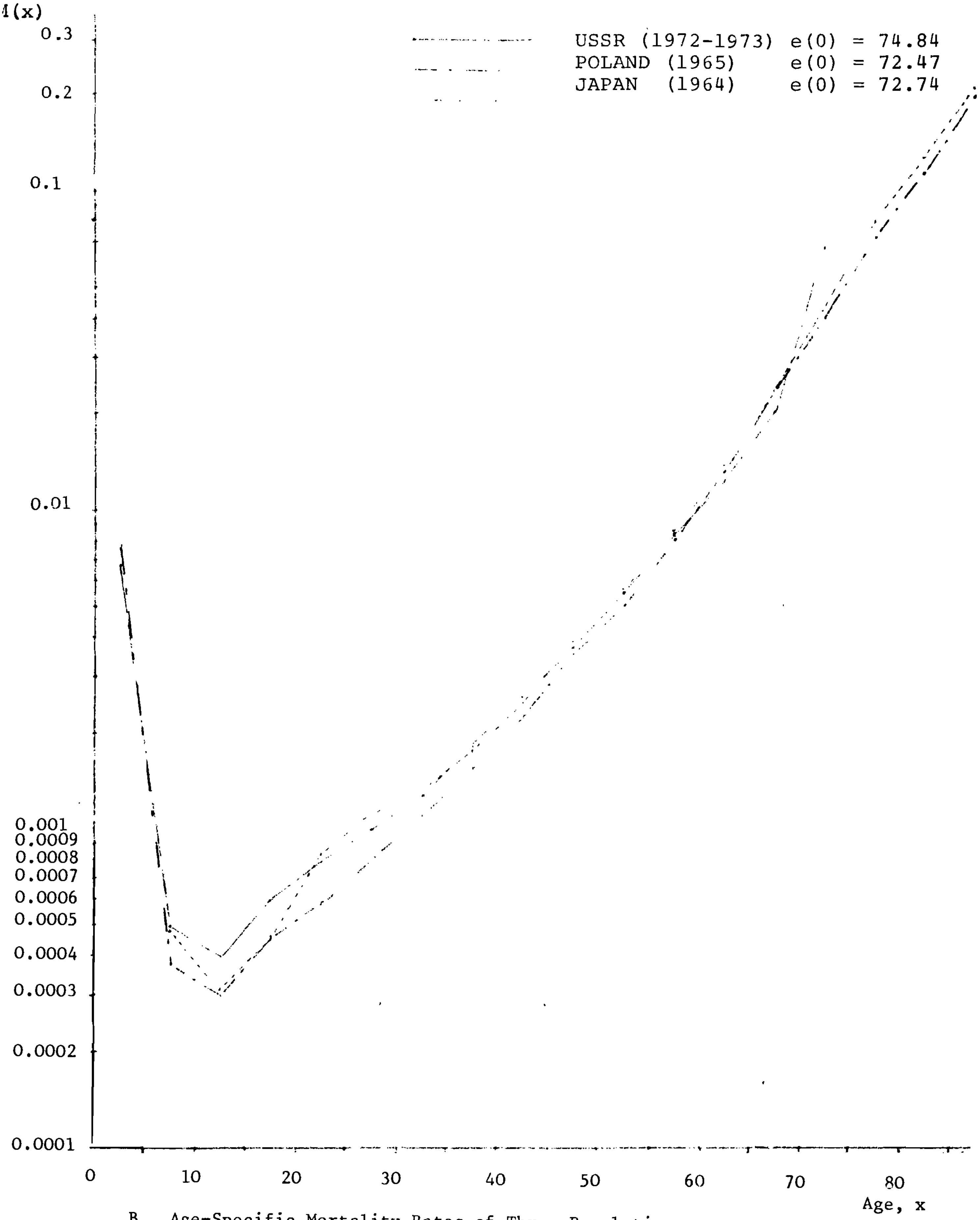
Source: Coale and Trussell (1974), p. 191

Figure 4: Observed Age-Specific Fertility Rates for Several Populations



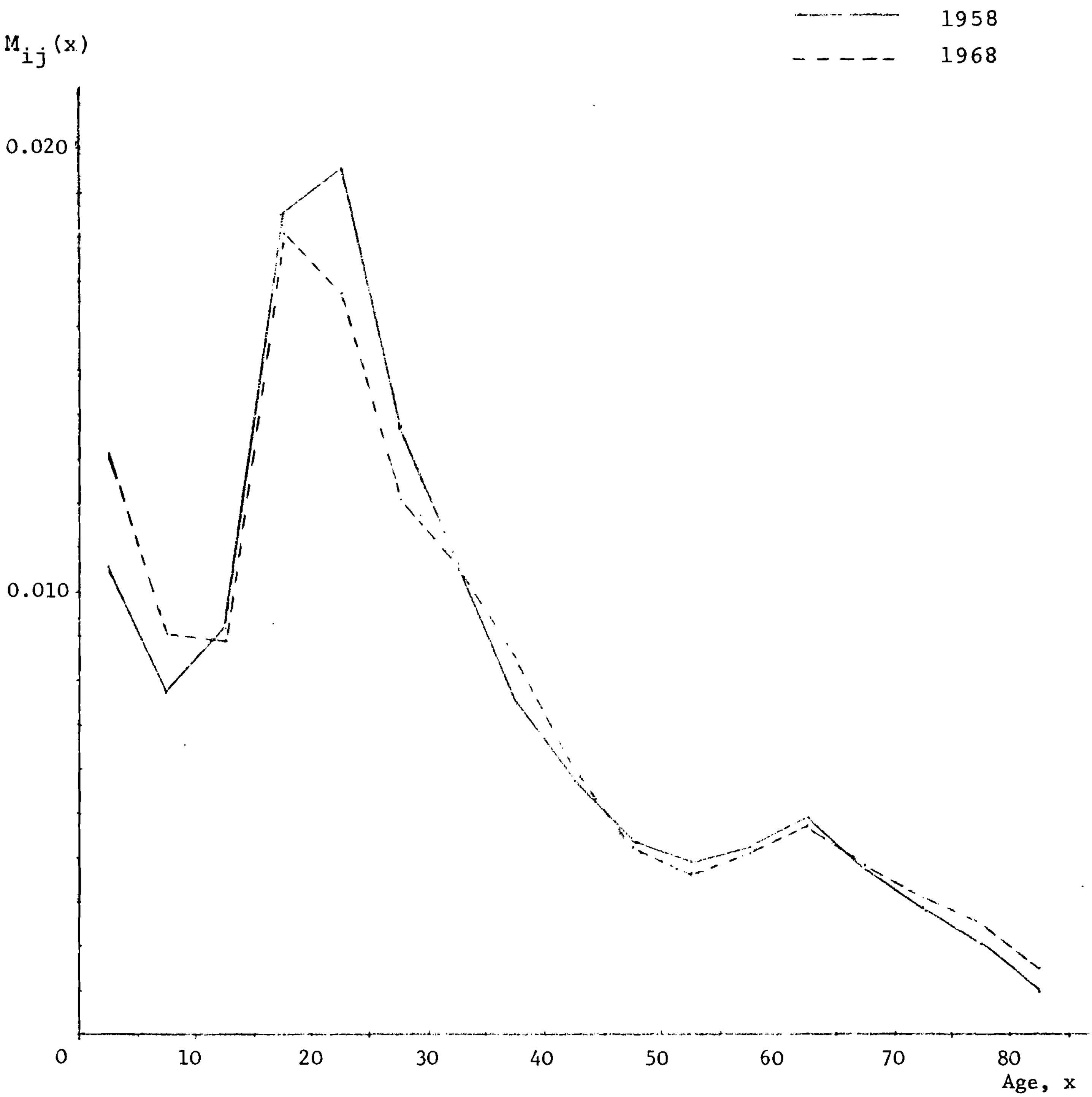
A. Age-Specific Mortality Rates of the Total United States Population: 1958 and 1968

Figure 5: Observed Age-Specific Mortality Rates for Several Populations



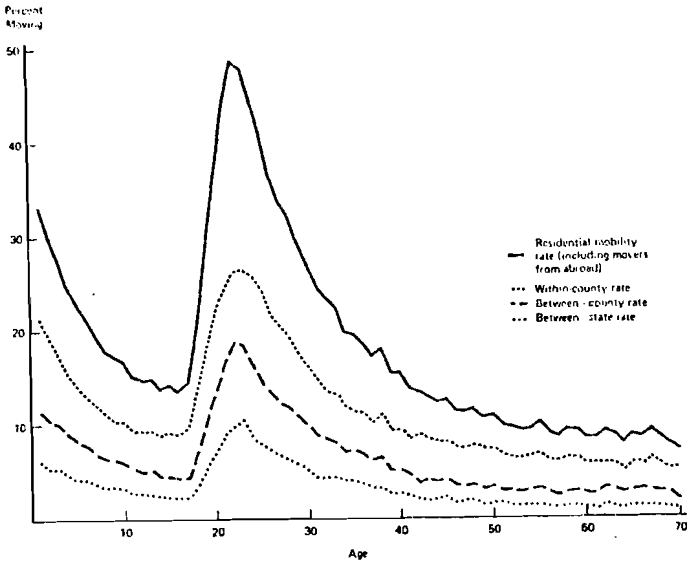
B. Age-Specific Mortality Rates of Three Populations

Figure 5 (Continued): Observed Age-Specific Mortality Rates for Several Populations



A. Age-Specific Migration Rates of the Total United States Population: 1958 and 1968

Figure 6: Observed Age-Specific Migration Rates for Several Populations



Source: Long (1973), p.38

B. Age-Specific Migration Rates of the Total United States Population By Category of Move: Average of 1966-1971

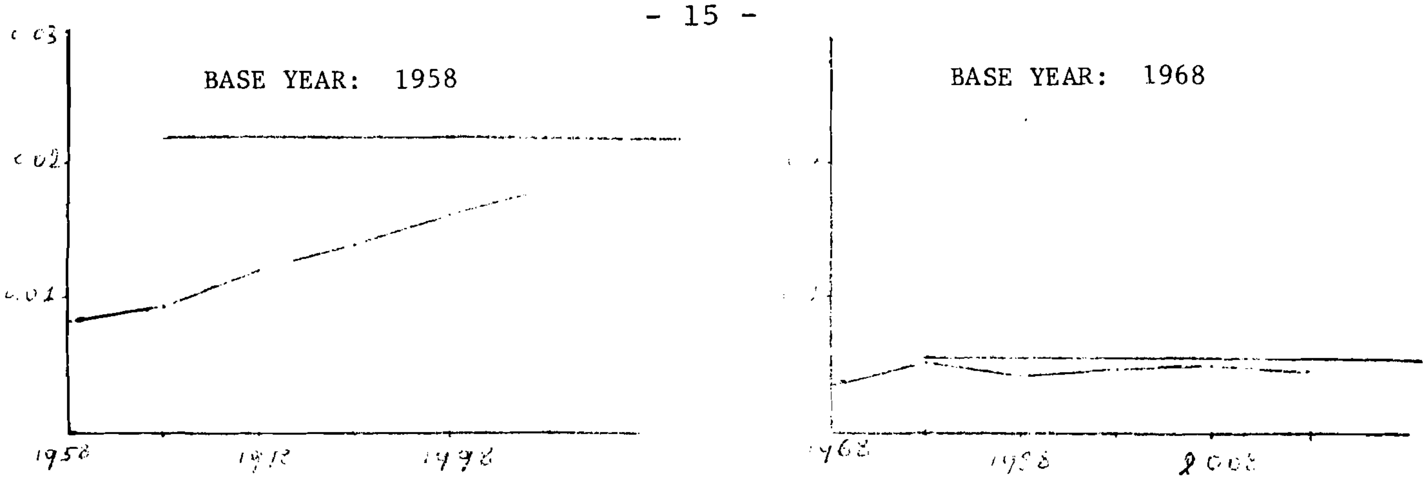
Figure 6 (continued): Observed Age-Specific Migration Rates for Several Populations

> "...research on migration generally corroborates the proposition that persons in their late teens, twenties, and early thirties are more migratory than their counterparts. The interpretation is that the young are able to adapt more easily to new situations. Also,...they are envisioned as being more readily disposed to taking advantage of new opportunities...." (Shaw, 1975, p. 18.)

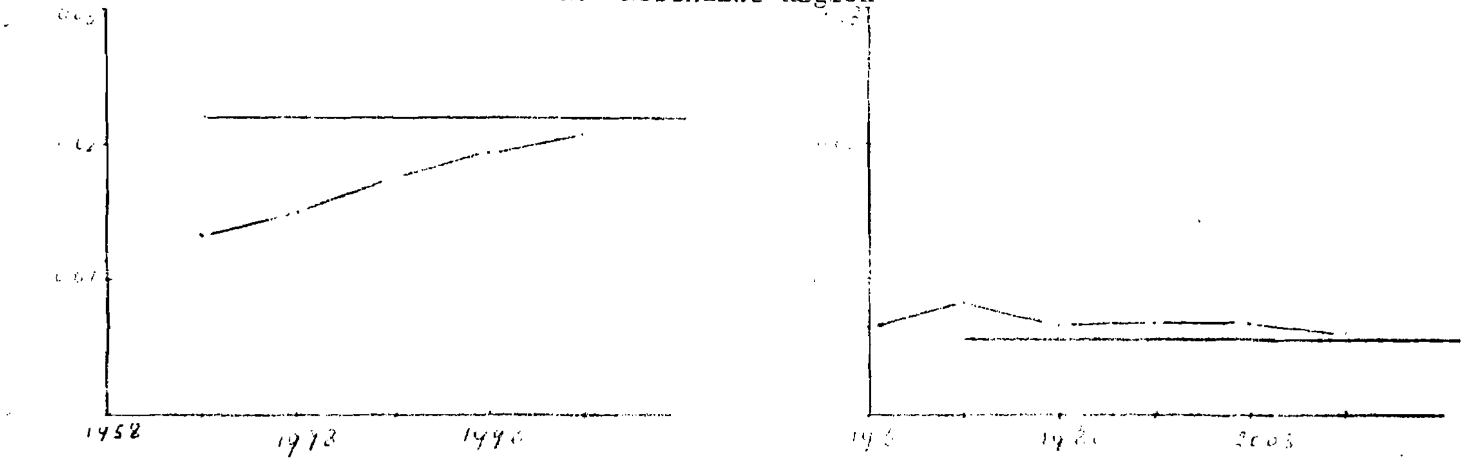
Figure 5 sets out several migration schedules for the U.S. total population. Those in Figure 6A refer to migration between Census Regions in 1958 and 1968, respectively. The age schedules in Figure 6B describe the geographical mobility of the population with respect to finer spatial disaggregations. From this graph we see, for example, that rates of residential mobility exceed those of intra-country and inter-county movements which, in turn, are greater than migration rates for between-state moves. Yet the same fundamental age profile is repeated in all of the schedules.

2.2 Dynamics of Empirical Multiregional Populations

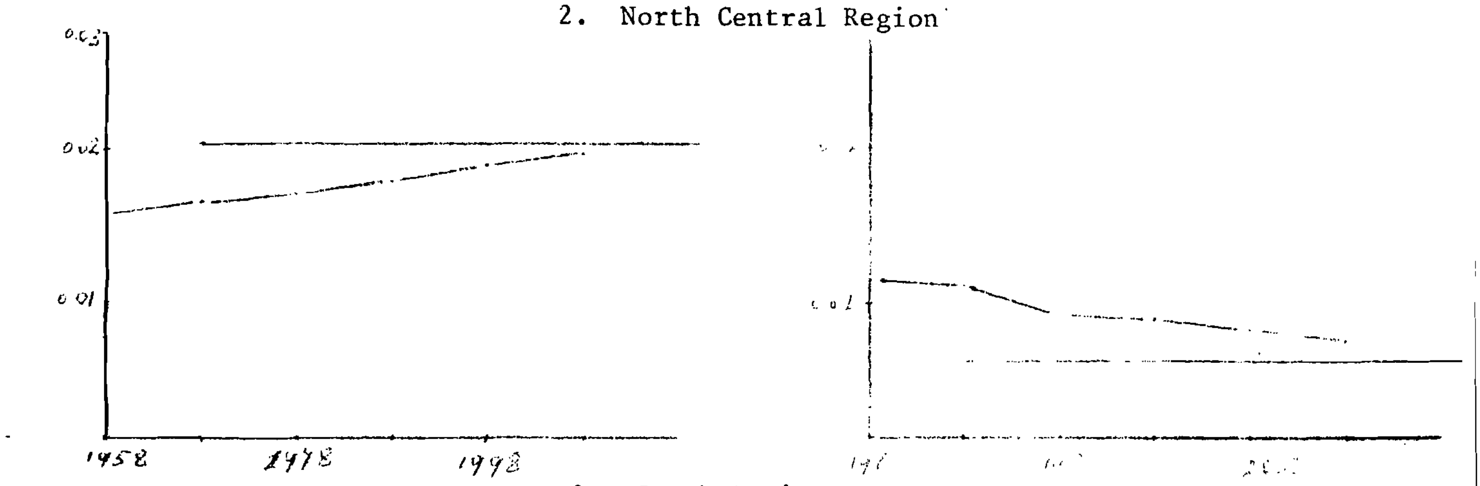
The growth, spatial distribution, and regional age compositions of a "closed" multiregional population are completely determined by the recent history of fertility, mortality, and internal migration to which it has been subject. Its current crude regional birth, death, migration, and growth rates are all governed by the interaction of the prevailing regime of growth with the current regional age compositions and regional shares of the total population. The dynamics of such growth and change are clearly illustrated, for example, by the four-region population system exhibited in Figure 3. Holding the prevailing regime of growth constant, one may derive the two sets of spatial population projections summarized in Appendix A and graphed in Figures 7 through 10 below. These offer interesting insights into the growth rates, regional shares,



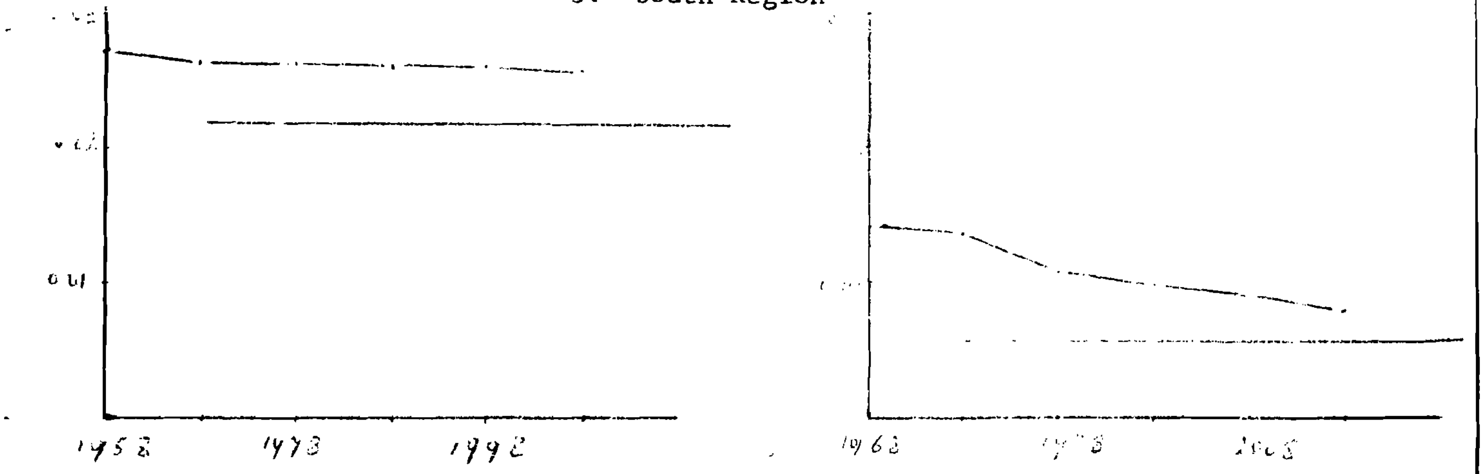
1. Northeast Region



2. North Central Region

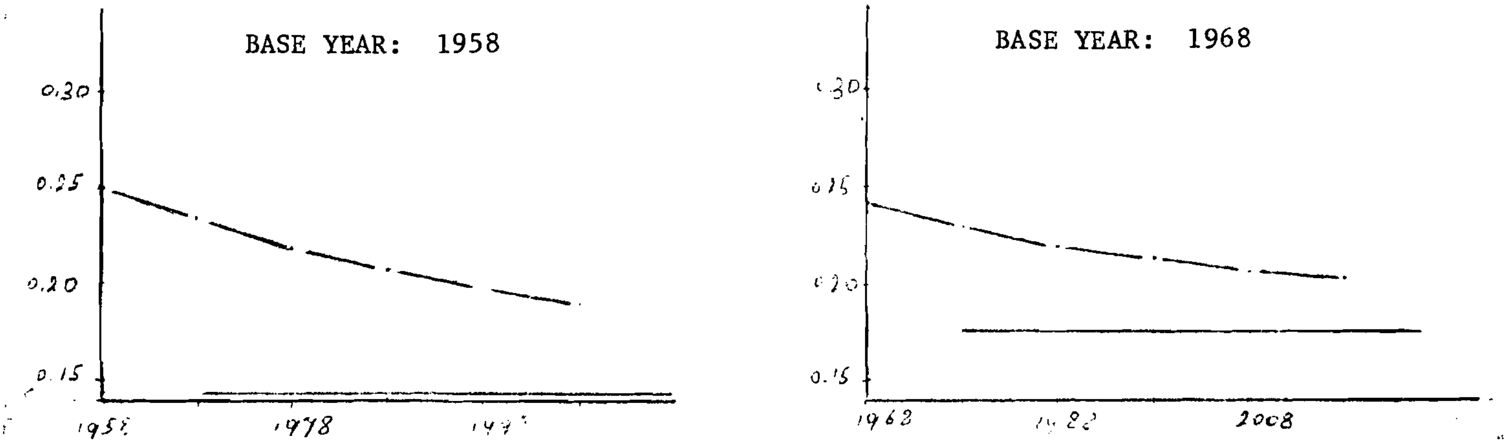


3. South Region

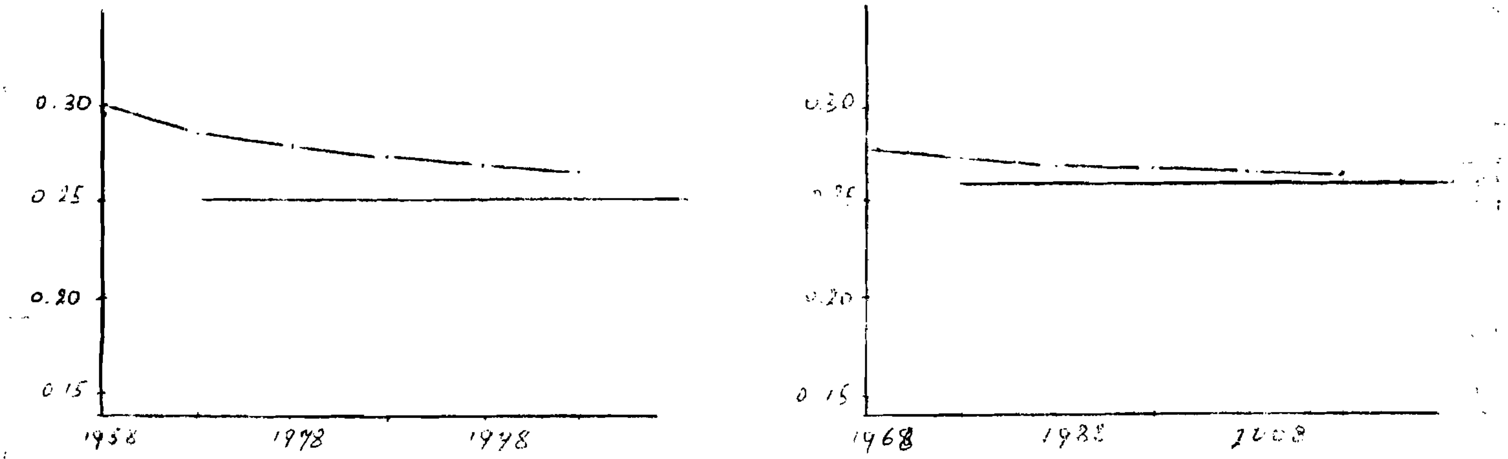


4. West Region

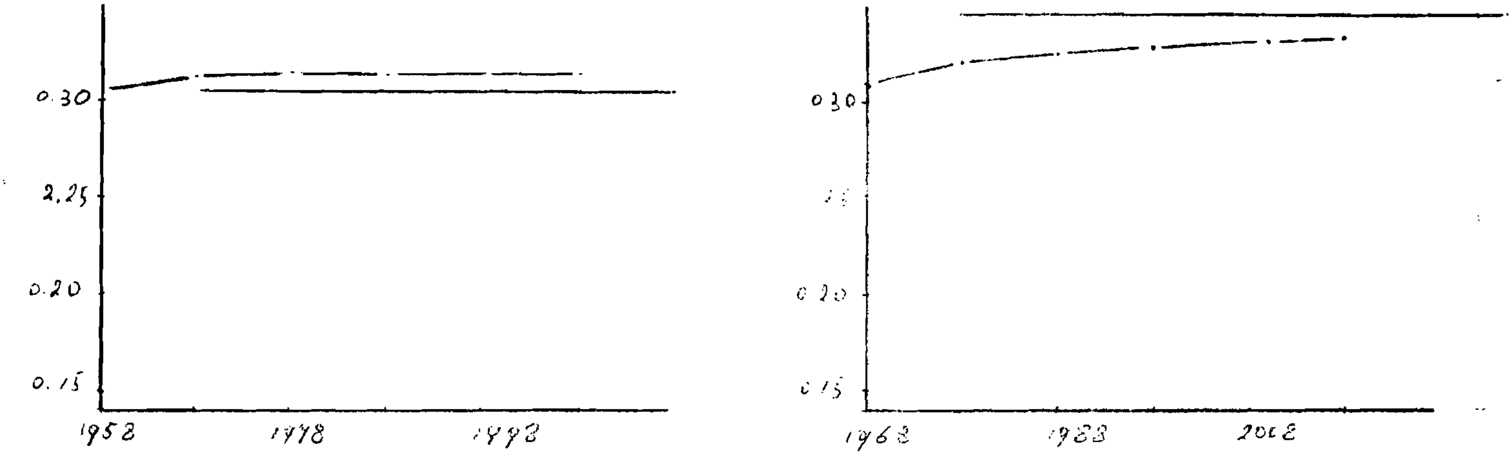
Figure 7: Projected Annual Rates of Growth: Total United States Population



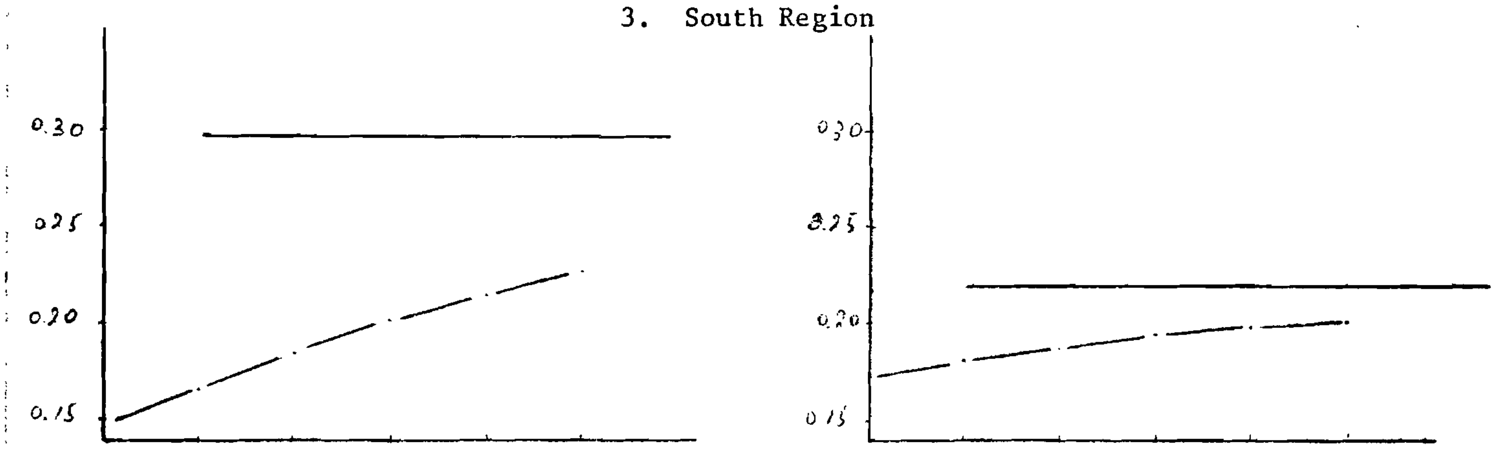
1. Northeast Region



2. North Central Region

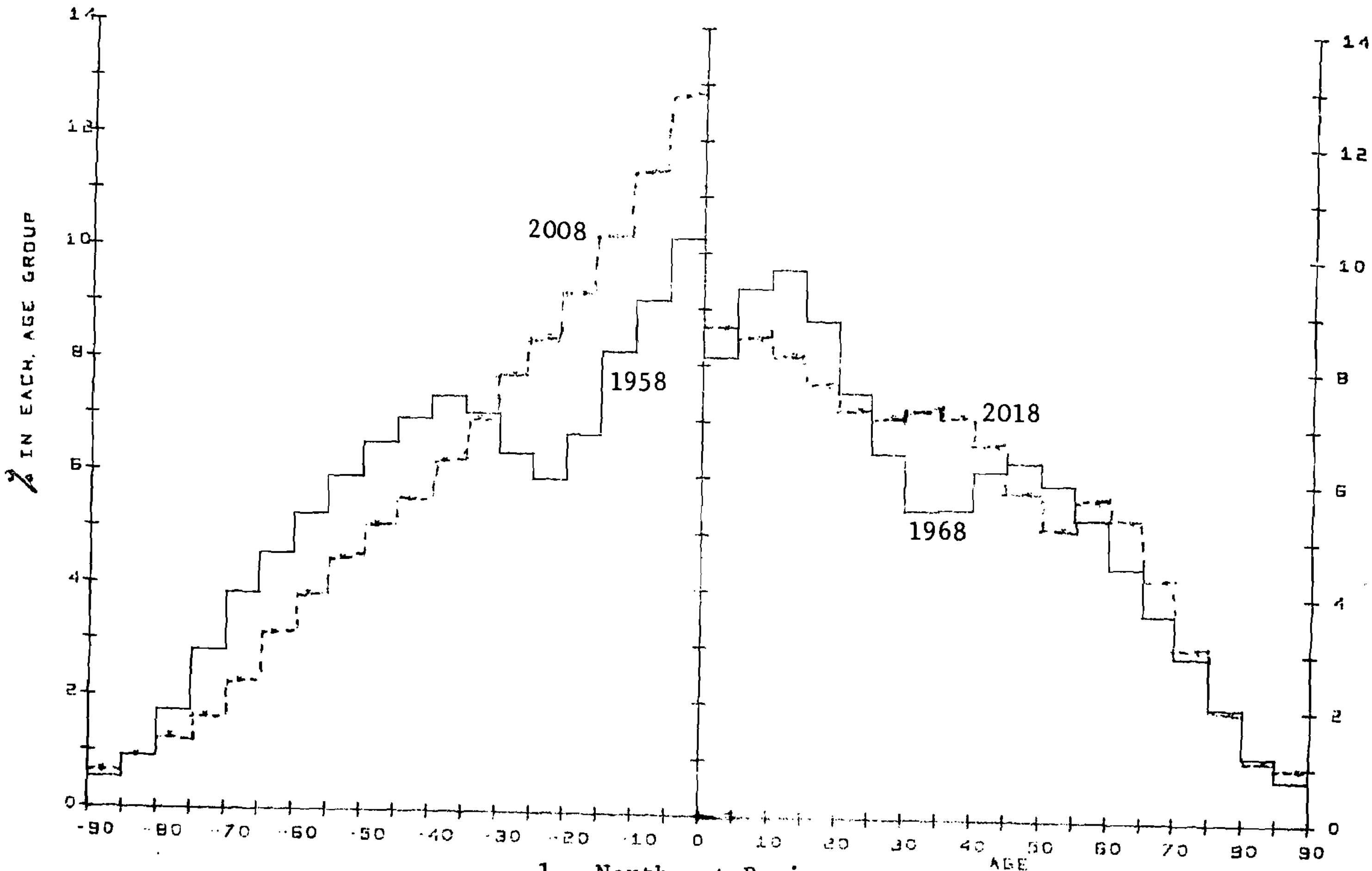


3. South Region

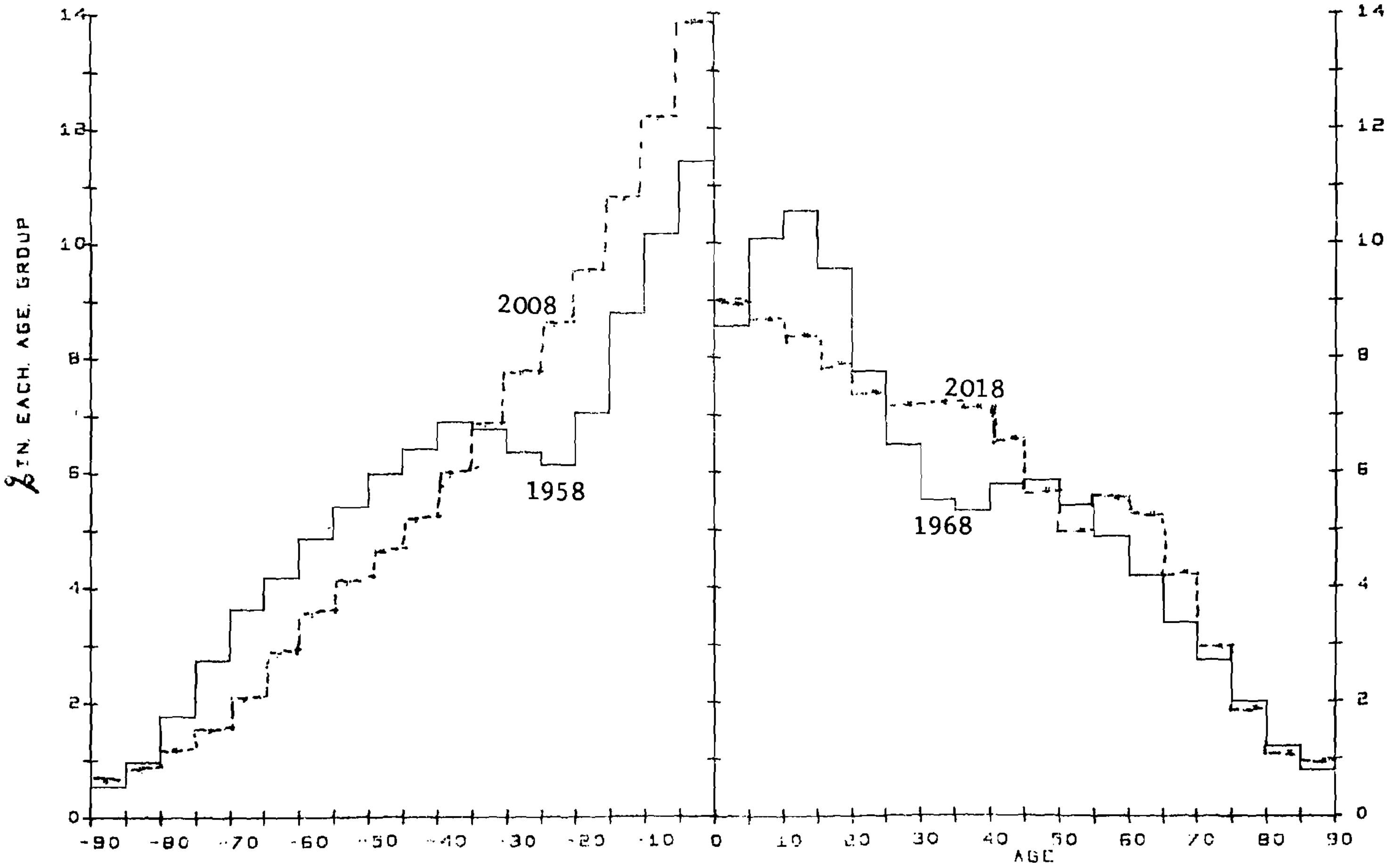


4. West Region

Figure 8: Observed and Projected Regional Shares: Total United States Population

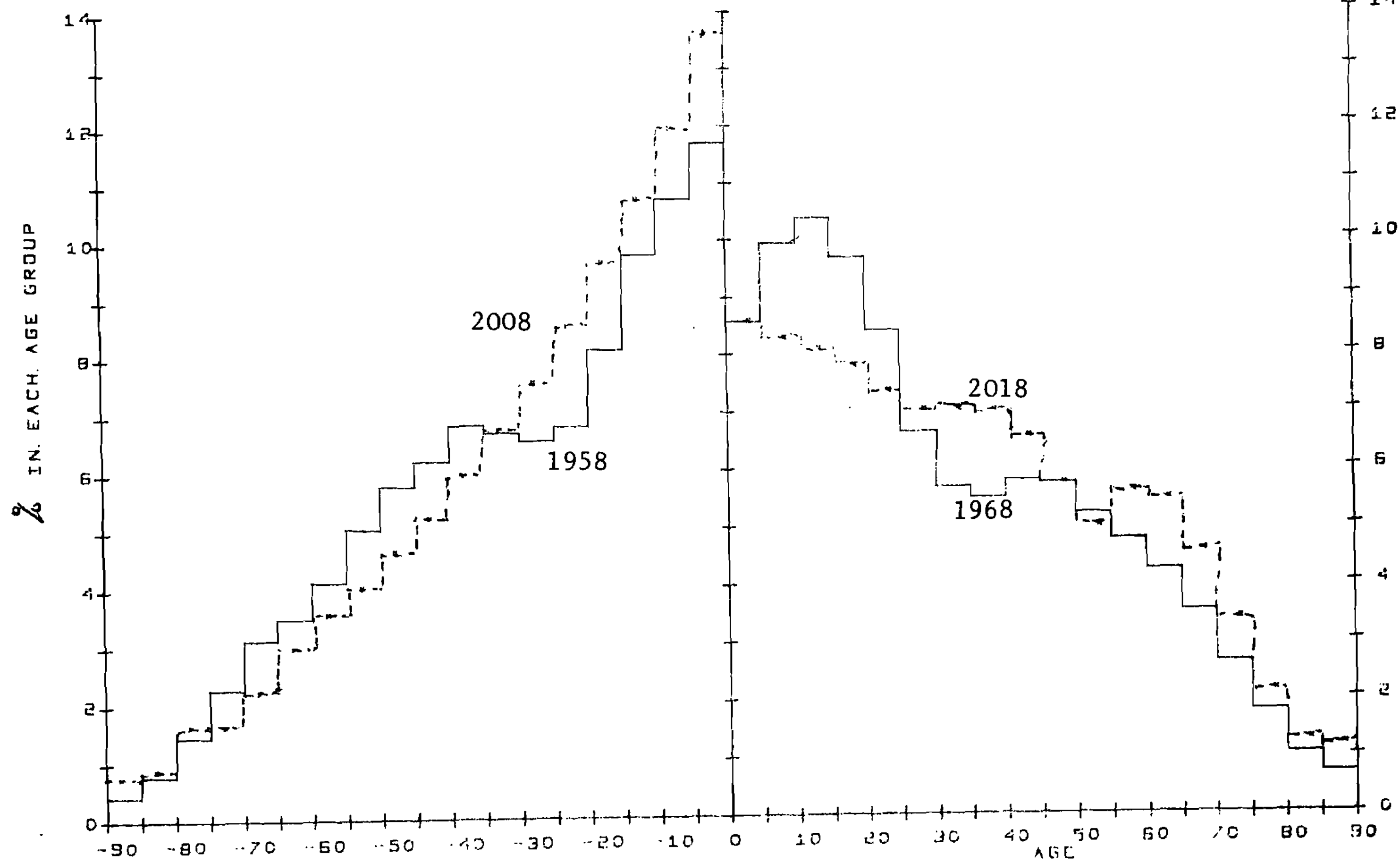


1. Northeast Region

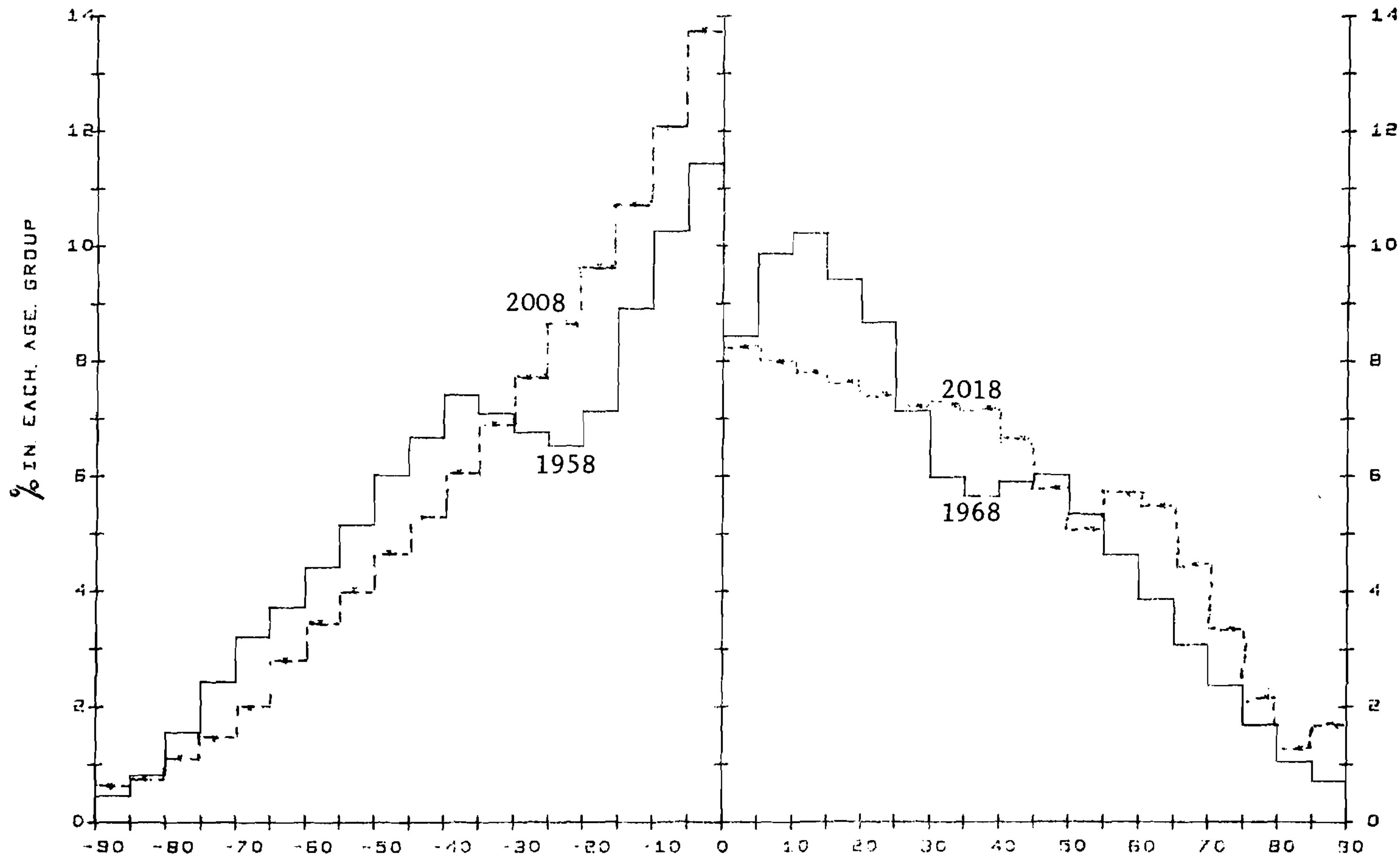


2. North Central Region

Figure 9: Observed and Projected Regional Age Compositions: Total United States Populations

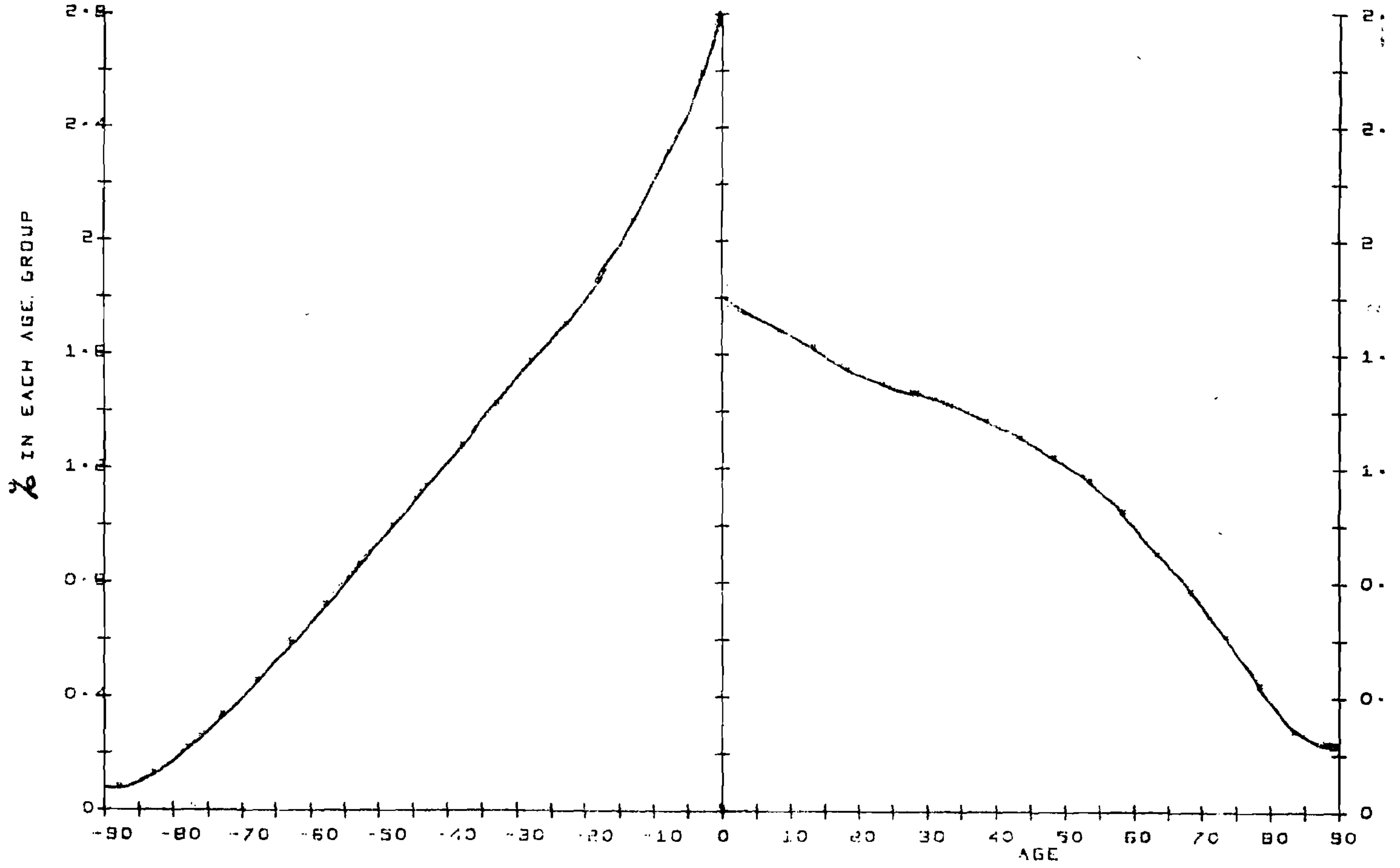


3. South Region

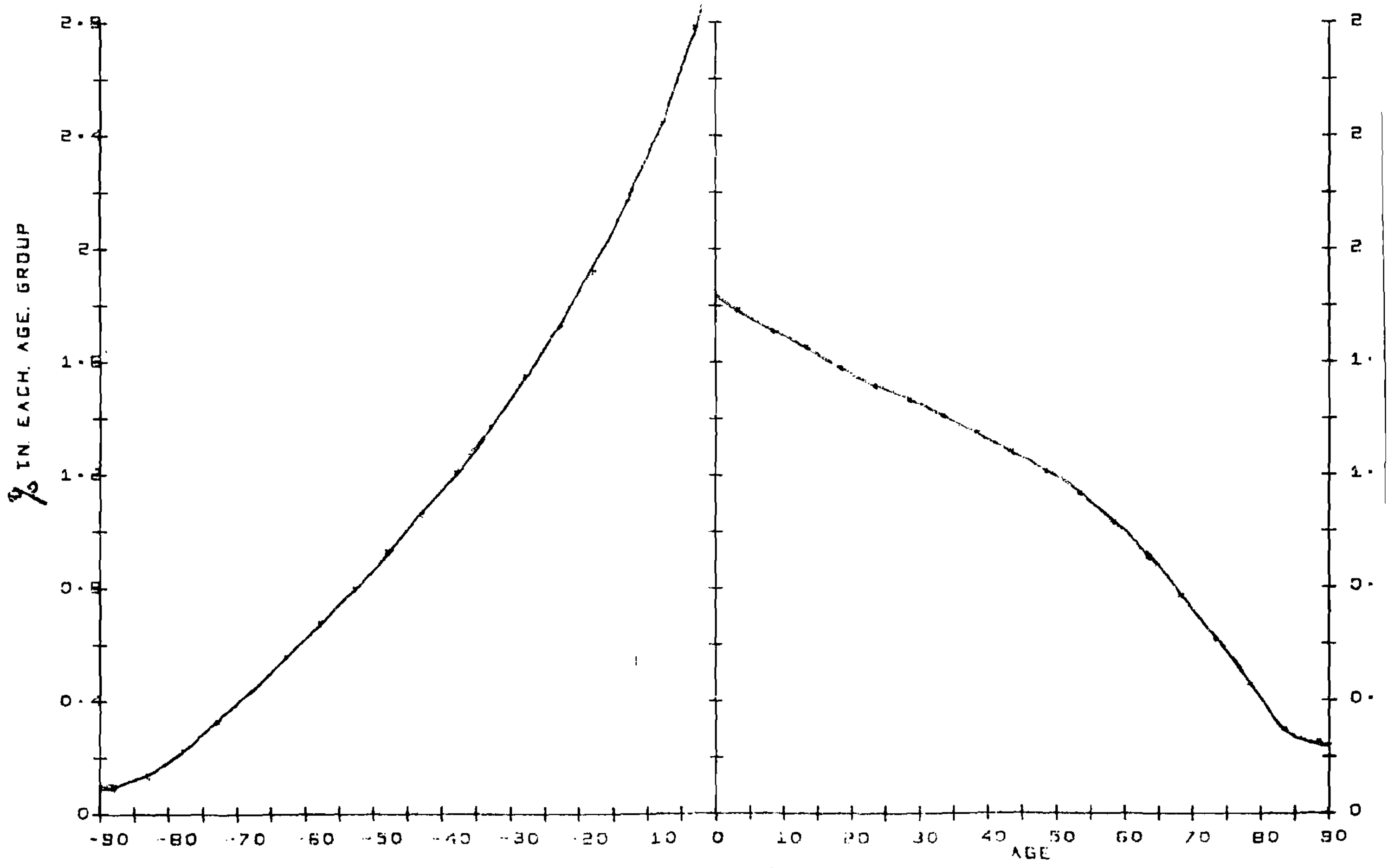


4. West Region

Figure 9 (Continued): Observed and Projected Regional Age Compositions: Total United States Populations

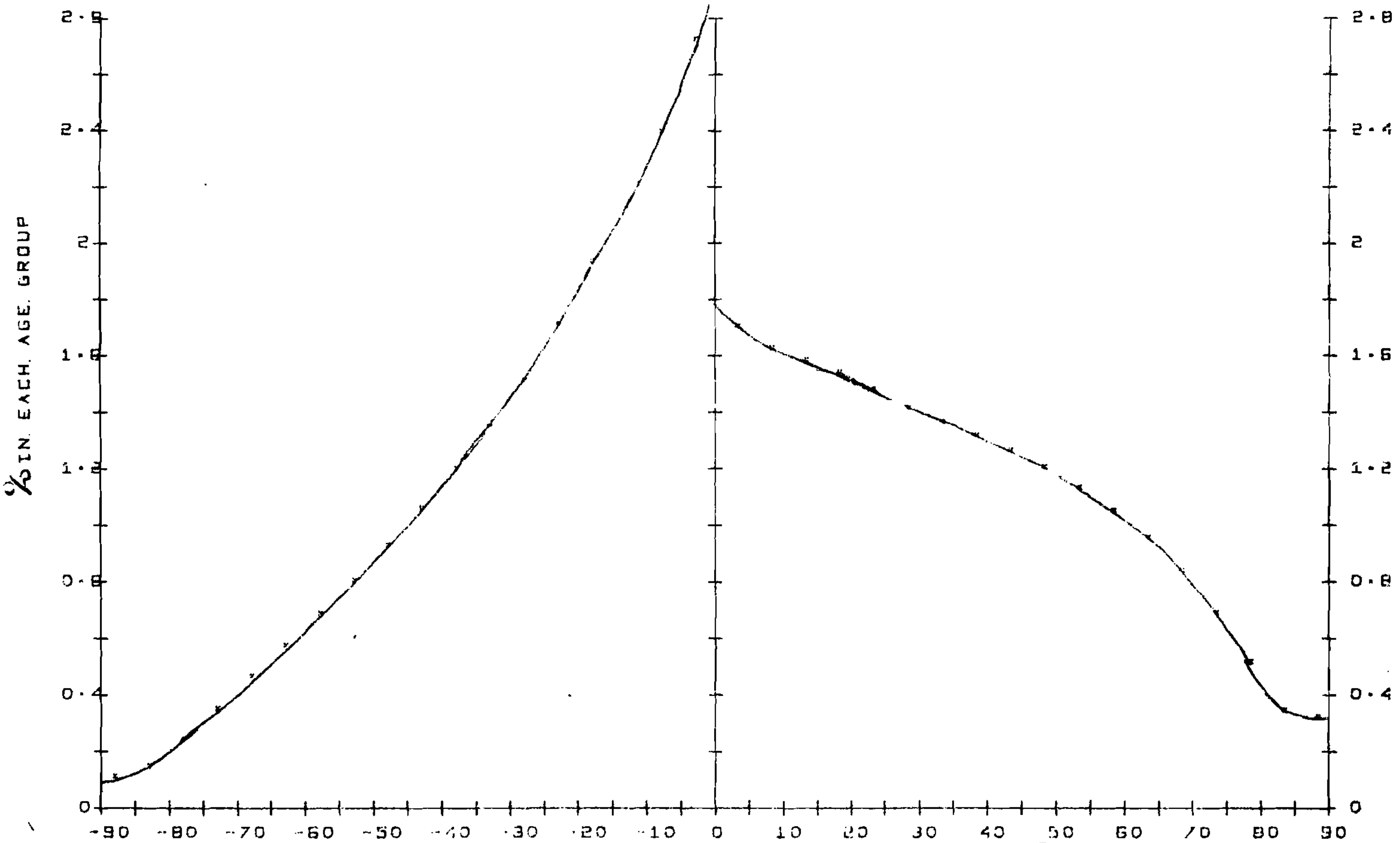


1. Northeast Region

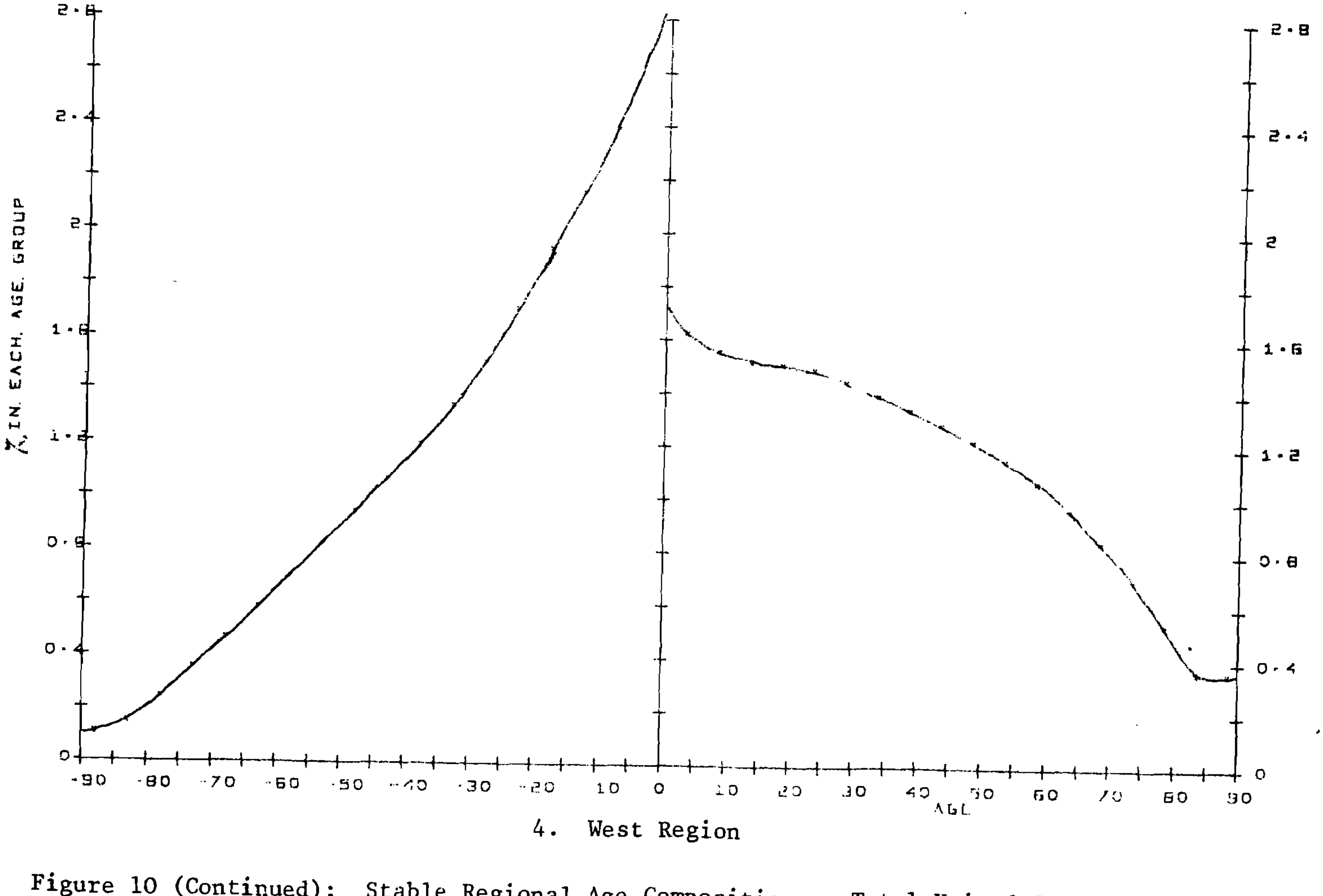


2. North Central Region

Figure 10: Stable Regional Age Compositions: Total United States Population



3. South Region



4. West Region

Figure 10 (Continued): Stable Regional Age Compositions: Total United States Population and regional age compositions that evolve out of a projection of current trends into the indefinite future, taking 1958 and 1968 as alternative base years from which to initiate the projections.

Regional Growth Rates. Table 1 in Appendix A, shows that between the two base years (1958 and 1968) the regional growth rates of the South and West regions were higher than the national average whereas those of the Northeast and North Central regions were lower. By virtue of the assumption of a linear model and a constant regime of growth, all four regional growth rates ultimately converge to the same intrinsic rate of increase: 0.021810 in the case of the 1958 regime of growth and 0.005699 in the case of the 1968 growth regime. However, what is interesting is that the trajectories converging toward these two intrinsic rates are quite different. Only in the case of the West region is a decline in the long-run growth rate projected under either of the two observed growth regimes. Also of interest is the substantial difference between the two intrinsic growth rates themselves. This difference clearly documents the dramatic drop in fertility levels that occurred during the decade in question.

Regional Shares. Both in 1958 and in 1968 approximately 31 percent of the U.S. population resided in the South. This regional share remains relatively unchanged in the projection under the 1958 regime of growth but increases to over 34 percent under the 1968 growth regime. Thus the ultimate spatial allocation of the national population changed in favor of the South during the decade between 1958 and 1968. According to Figure 8, a large part of this change came at the expense of the West's regional share, which declined from roughly 30 percent to about 22 percent. Note, however, that despite this decline the West's projected share of the national population nonetheless shows a substantial increase over the base year allocation. This increase and that of the South's matches the decrease in the regional shares of the Northeast and North Central regions. Thus, under either projection, the "North's" share of the U.S. population is headed for a decline while that of the "Southwest" is due to increase.

Regional Age Compositions. Figure 9 vividly illustrates the impact that a high growth rate has on age composition. The age compositions in the four regional graphs depict both the age compositions observed at the time of the base year and those projected 50 years forward on the assumption of an unchanging regime of growth. Since the regional growth regimes in 1958 produced a relatively high time series of growth rates after a period of 50 years, the age compositions on the left-hand side of the age composition in Figure 9 show a relatively steep slope. Because the 1968 growth regimes, on the other hand, produced relatively low regional growth rates after 50 years, the regional age compositions on the right-hand side of the graphs show a relatively shallow slope. This contrast is perhaps more readily apparent in Figure 10 which exhibits the age compositions that would arise at stability. These in fact do not differ much those that evolve after 50 years and are drawn here in continuous form for ease of comprehension.

The age compositions in Figure 10 suggest a comparison with those of Figure 2. Although the latter describe populations exposed to much higher levels of mortality, the general outlines of the high growth rate and low growth rate age compositions are remarkably similar. We shall consider such age profiles in greater detail in Section 4 of this paper, after first examining the regularities that are exhibited by observed schedules of migration in Section 3 below.

3. Model Multiregional Life Tables

3.1 Life Tables

Conventional life tables describe the evolution of a hypothetical cohort of babies born at a given moment and exposed to an unchanging age-specific schedule of mortality. For this cohort of babies, they exhibit a number of probabilities of dying and surviving and develop the corresponding expectations of life at various ages.

Life table calculations normally are initiated by estimating a set of age-specific probabilities of dying within each interval of age, $q(x)$ say, from observed data on age-specific death rates, $M(x)$ say. The conventional calculation that is made for an age interval five years wide, is (Rogers, 1975, p. 12):

$$q(x) = \frac{5M(x)}{1 + \frac{5}{2} M(x)}$$

or, alternatively,

$$p(x) = 1 - q(x) = [1 + \tfrac{5}{2} M(x)]^{-1} [1 - \tfrac{5}{2} M(x)] \quad , \qquad (1)$$

where $p(x)$ is the age-specific probability of surviving from <u>exact age x</u> to <u>exact age x + 5</u>. These latter probabilities, in turn, may be used to define the corresponding probabilities of survival from one <u>age group</u> to the next (Rogers, 1975, pp. 16 and 85):

$$s(x) = [1 + p(x + 5)] \, p(x) [1 + p(x)]^{-1} \quad . \qquad (2)$$

To avoid any possible confusion between the two sets of probabilities, we shall hereafter refer to $s(x)$ as a survivorship proportion, i.e., the proportion of individuals surviving from *age group x to $x + 5$* to *age group $x + 5$ to $x + 10$*. A common alternative designation for this demographic measure is *survival ratio* (see for example, Section 1).

One of the most useful statistics provided by a life table is the average expectation of life at age x, $e(x)$ say. Such expectations of life are calculated by applying the probabilities of survival $p(x)$ to a hypothetical cohort of babies and then observing their average length of life beyond each age. Expectations of life at birth [$e(0)$] are particularly useful as indicators of the level of mortality in various regions and countries of the world. By way of example, Table 1 presents such expectations for several developing and developed countries in the 1960s.

A wide range of variation in mortality levels is illustrated in Table 1. At one extreme are Cameroon and Togo, with average expectations of life at birth of about 40 years; at the other extreme is Sweden, whose baby girls born in 1967 could expect to live over 76 years on the average. In between are Guatemala and Mexico, with average life expectancies of about 50 years.

Conventional life tables deal with mortality, focus on a single regional population, and ignore the effects of migration. To incorporate the latter and, at the same time, to extend the life table concept to a spatial population comprised of several regions requires the notion of a multiregional life table (Rogers, 1973). Such life tables describe the evolution of several regional cohorts of babies, all born at a given moment and exposed to an unchanging *multiregional* age-specific schedule of mortality and migration. For each regional birth cohort, they provide various probabilities of dying, surviving, and migrating, while simultaneously deriving regional expectations of life at various ages. These expectations of life are disaggregated both by place of birth and by place of residence and will be

TABLE 1

Expectations of Life at Birth for Six Countries

Stage in the Demographic Transition	Country	Expectation of Life at Birth, e(0)	
		Males	Females
High birth rate High death rate	Cameroon (1964) Togo (1961)	34.27 33.57	38.09 40.27
High birth rate Low death rate	Guatemala (1964) Mexico (1966)	49.25 46.26	50.87 50.43
Low birth rate Low death rate	Sweden (1967) USSR (1959)	71.87 67.73	76.58 72.87

Source: Keyfitz and Flieger, 1971, Part II: Summary Tables, pp. 60-123.

denoted by ${}_ie_j(x)$, where i is the region of birth and j is the region of residence.

Multiregional life table calculations are greatly facilitated by the adoption of matrix algebra. This leads to a compact notation and an efficient computational procedure; it also very clearly demonstrates a simple correspondence between the single-region and the multiregional formulas. For example, Equations 1 and 2 may be shown to have the following multiregional counterparts (Rogers and Ledent, 1976; Rogers, 1975, p. 85):

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x)]^{-1}[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)] \tag{3}$$

and

$$\underset{\sim}{S}(x) = [\underset{\sim}{I} + \underset{\sim}{P}(x+5)]\underset{\sim}{P}(x)[\underset{\sim}{I} + \underset{\sim}{P}(x)]^{-1} \quad . \tag{4}$$

The diagonal elements of $\underset{\sim}{P}(x)$ and $\underset{\sim}{S}(x)$ are probabilities of survival and survivorship proportions, respectively; the off-diagonal elements will be called probabilities of migrating and migration proportions, respectively.

Expectations of life in the multiregional life table reflect the influences of mortality and migration. Thus they may be used as indicators of levels of internal migration, in addition to carrying out their traditional role as indicators of levels of mortality. For example, consider the regional expectations of life at birth that are set out in Table 2 below for the U.S. population with both sexes combined. A baby born in the West, and exposed to the multiregional schedule of mortality and migration that prevailed in 1958, could expect to live an average of 69.94 years, out of which total an average of 8.95 years would be lived in the South. Taking the latter as a fraction of the former, we have in ${}_4\theta_3 = 0.1279$ an indicator

TABLE 2

Expectations of Life at Birth and Migration Levels by Region of Residence and Region of Birth: Total United States Population, 1958 and 1968.

A. 1958

Region of Birth	Region of Residence				Total
	1	2	3	4	
1. Northeast	50.90 (0.7295)	4.49 (0.0643)	8.88 (0.1273)	5.50 (0.0788)	69.76 (1.00)
2. North Central	3.18 (0.0452)	48.45 (0.6889)	9.10 (0.1294)	9.60 (0.1365)	70.32 (1.00)
3. South	4.58 (0.0664)	7.52 (0.1091)	49.21 (0.7134)	7.67 (0.1111)	68.98 (1.00)
4. West	3.18 (0.0454)	6.60 (0.0944)	8.95 (0.1279)	51.22 (0.7322)	69.94 (1.00)

B. 1968

Region of Birth	Region of Residence				Total
	1	2	3	4	
1. Northeast	50.61 (0.7146)	5.06 (0.0714)	10.00 (0.1412)	5.15 (0.0738)	70.83 (1.00)
2. North Central	3.69 (0.0519)	49.19 (0.6929)	10.37 (0.1460)	7.75 (0.1092)	70.99 (1.00)
3. South	4.81 (0.0685)	7.45 (0.1060)	51.39 (0.7313)	6.63 (0.0942)	70.28 (1.00)
4. West	3.87 (0.0543)	7.71 (0.1081)	11.20 (0.1570)	48.53 (0.6806)	71.31 (1.00)

of the (lifetime) migration level from the West to the South that is implied by the 1958 multiregional schedule. Note, however, that as a consequence of changing socioeconomic conditions, this same indicator increases to 0.1570 a decade later.

We have noted earlier that single-region life tables normally are computed using observed data on age-specific death rates. In countries lacking reliable data on death rates, however, recourse is often made to inferential methods that rely on model life tables such as those published by the United Nations (Coale and Demeny, 1967). These tables are entered with empirically determined survivorship proportions to obtain the particular expectation of life at birth (and corresponding life table) that best matches the levels of mortality implied by the observed proportions.

The inferential procedures of the single-region model may be extended to the multiregional case (Rogers, 1975, Ch. 6). Such an extension begins with the notion of model multiregional life tables and uses a set of initial estimates of survivorship and migration proportions to identify the particular combination of regional expectations of life, disaggregated by region of birth and region of residence, that best match the levels of mortality and migration implied by these observed proportions.

Model multiregional life tables approximate the mortality and migration schedules of a multiregional population system by drawing on the regularities observed in the mortality and migration experiences of comparable populations. That is, regularities exhibited by mortality and migration data collected in regions where these data are accurate and available are used to systematically approximate the mortality and migration patterns of populations lacking such data. We now turn to an examination of some of the regularities in observed migration schedules.

3.2 Regularities in Migration Schedules

Demographers have long recognized that persisting regularities appear in empirical age-specific migration schedules (e.g., Lowry, 1966; Long, 1973). Migration, viewed as an event, is highly selective with regard to age, with young adults generally being the most mobile group in any population. Levels of migration also are high among children, varying from a peak during the first year of age (the initial peak) to a low point around age 16. The migration age profile then turns sharply upward until it reaches a second peak (the high peak) in the neighborhood of 22 years, after which it declines regularly with age, except for a slight hump (the retirement peak), around ages 62 through 65.

The regularities in observed migration schedules are not surprising:

> "Young adults exhibit the highest migration rates because they are less constrained by ties to their community. Their children generally are not in school, they are more likely to be renters rather than home owners, and job seniority is not yet an important consideration. Since children move only as members of a family, their migration pattern mirrors that of their parents. Consequently, because younger children generally have younger parents, the geographical mobility of infants is higher than that of adolescents. Finally, the small hump in the age profile between ages 62 to 65 describes migration after retirement and reflects, for example, moves made to the sunnier and milder climates of states such as Arizona, California, and Florida." (Rogers, 1975, pp. 146-147).

Figures 11 and 12 illustrate the fundamental age profile of most migration schedules but focus on probabilities instead of rates and deal with five-year age groups instead of one-year age intervals[1]. The aggregation into broader age groups consolidates the low migration level at age 16 with the significantly higher levels that follow it, shifting the low point among teenagers to a lower age group.

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[1] No loss of generality is incurred by focusing on probabilities instead of rates since the former are simply linear transformations of the latter (see, for example, Equation 3).

Figure 11A: Probability of Migration from the North Central Region: Total Population, 1958

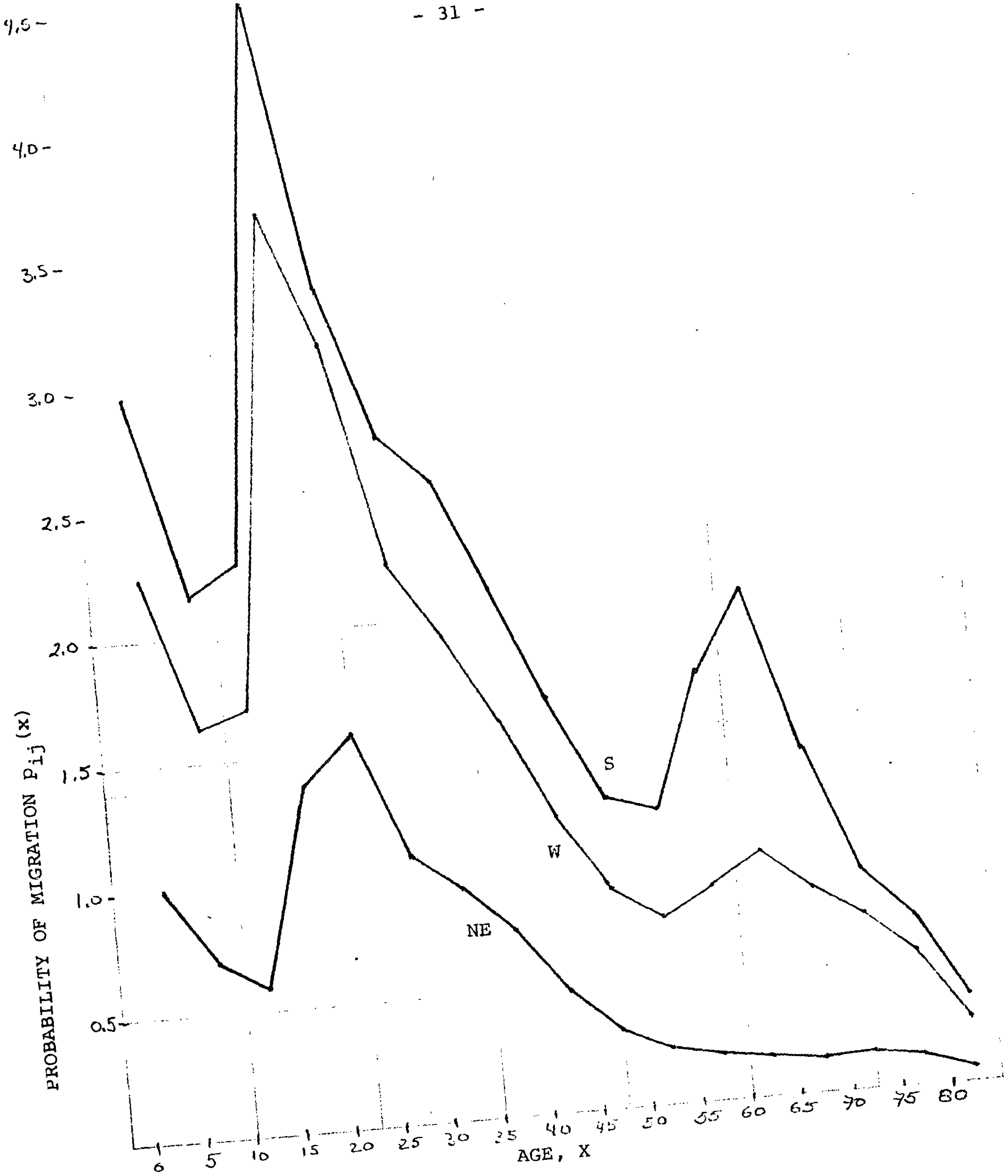

Figure 11B: Probability of Migration from the North Central Region: Total Population, 1968

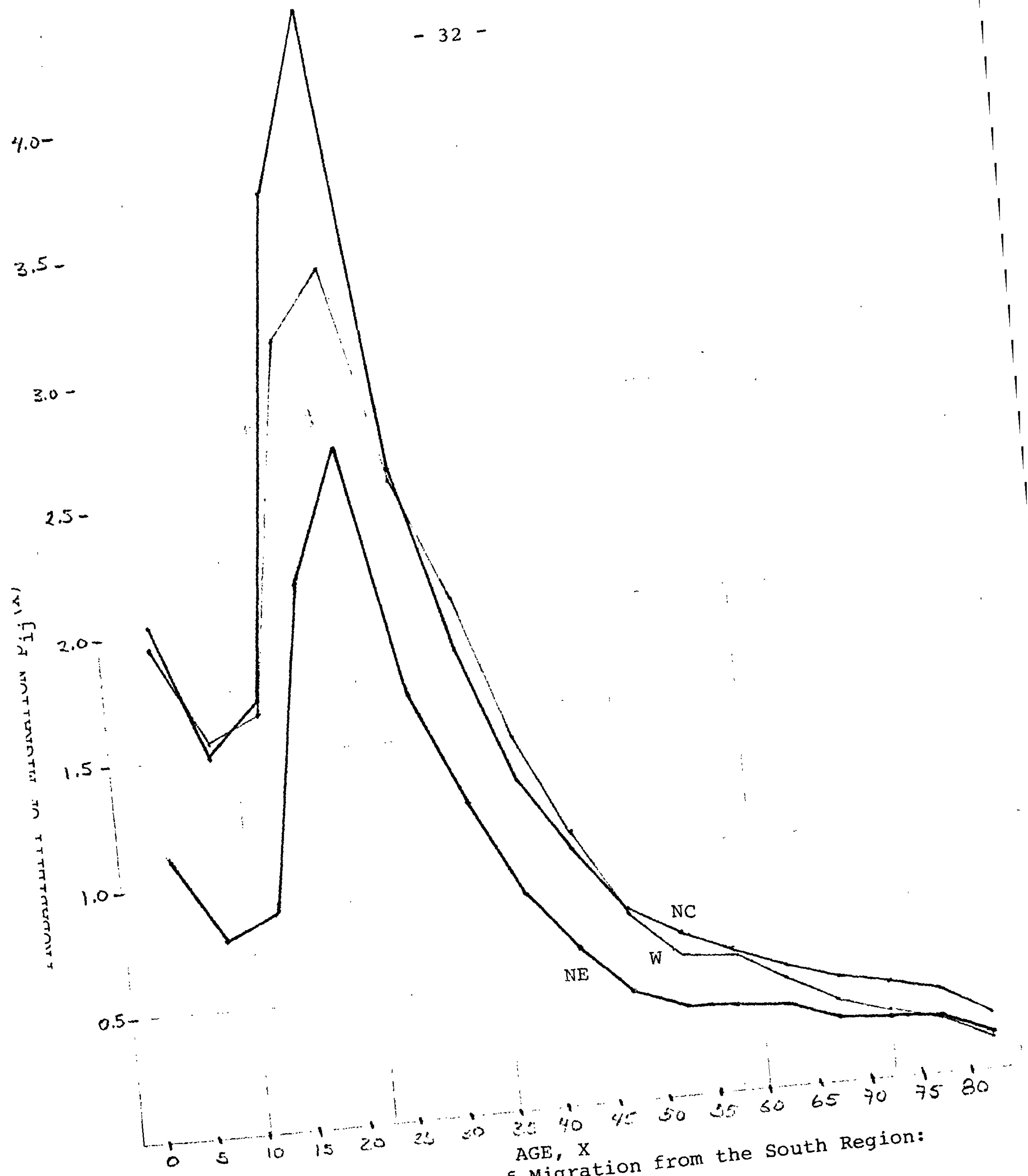

Figure 11C: Probability of Migration from the South Region: Total Population, 1958

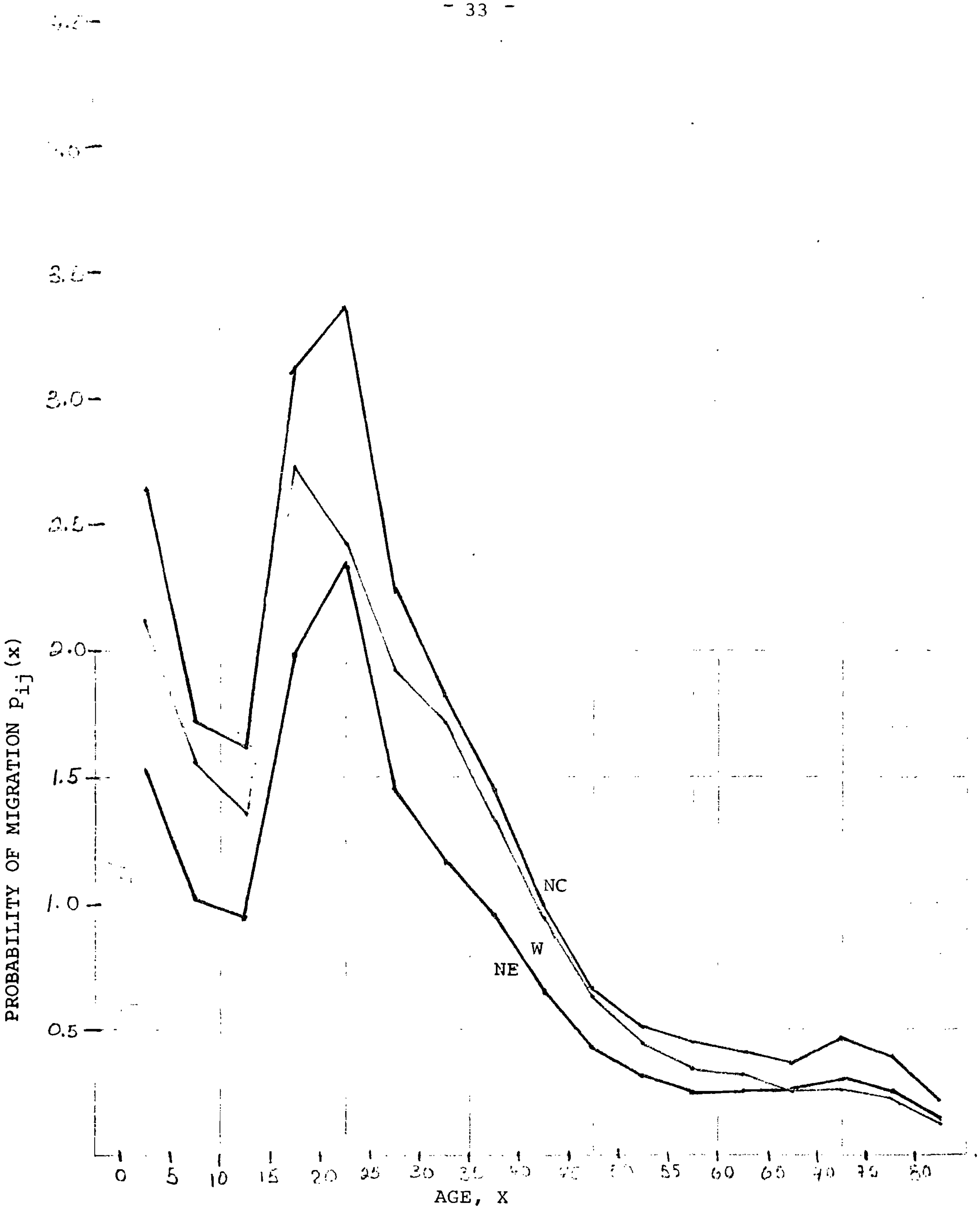

Figure 11D: Probability of Migration from the South Region: Total Population, 1968

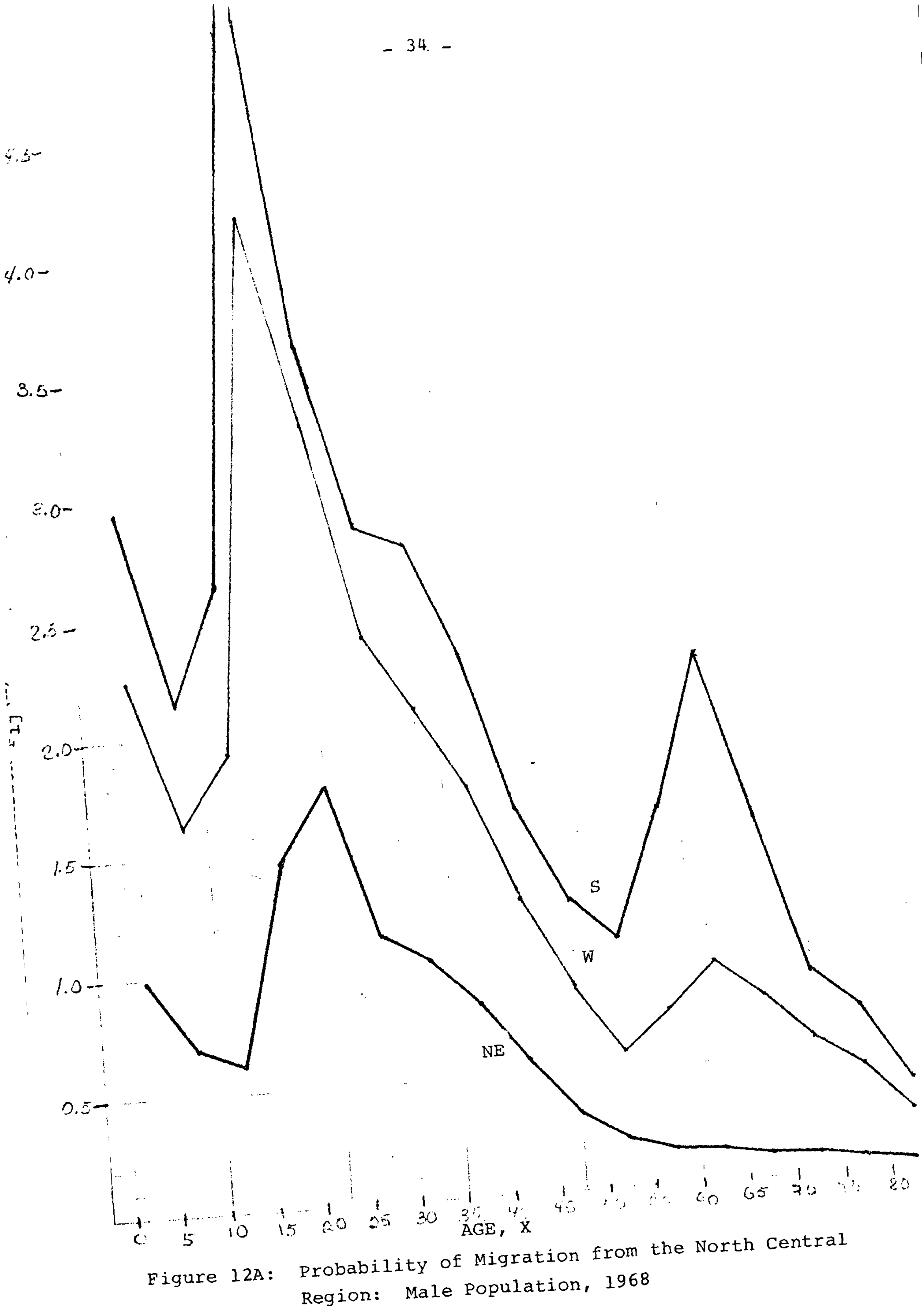

Figure 12A: Probability of Migration from the North Central Region: Male Population, 1968

FIGURE 12B: Probability of Migration from the North Central Region: Female Population, 1968

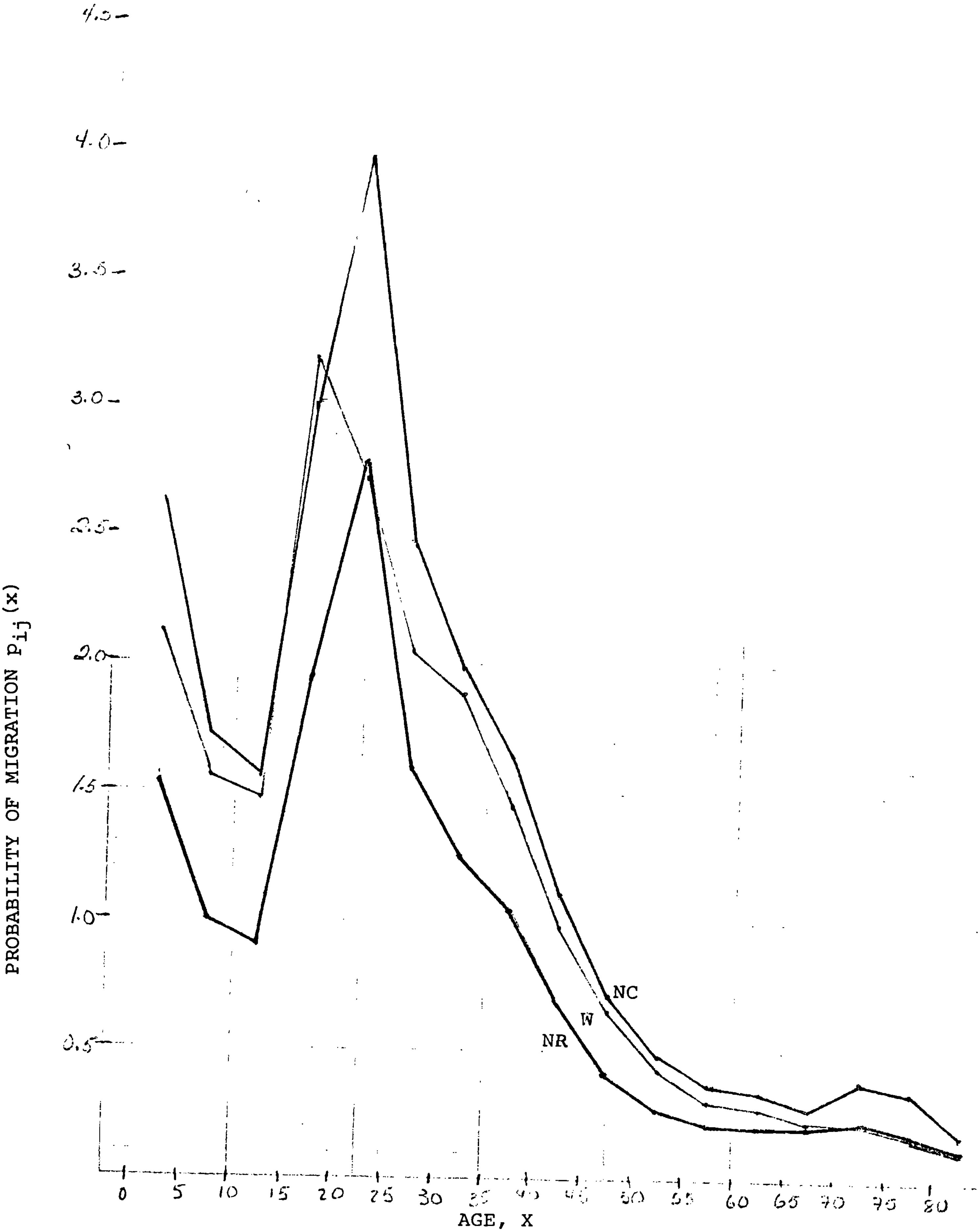

Figure 12C: Probability of Migration from the South Region: Male Population, 1968

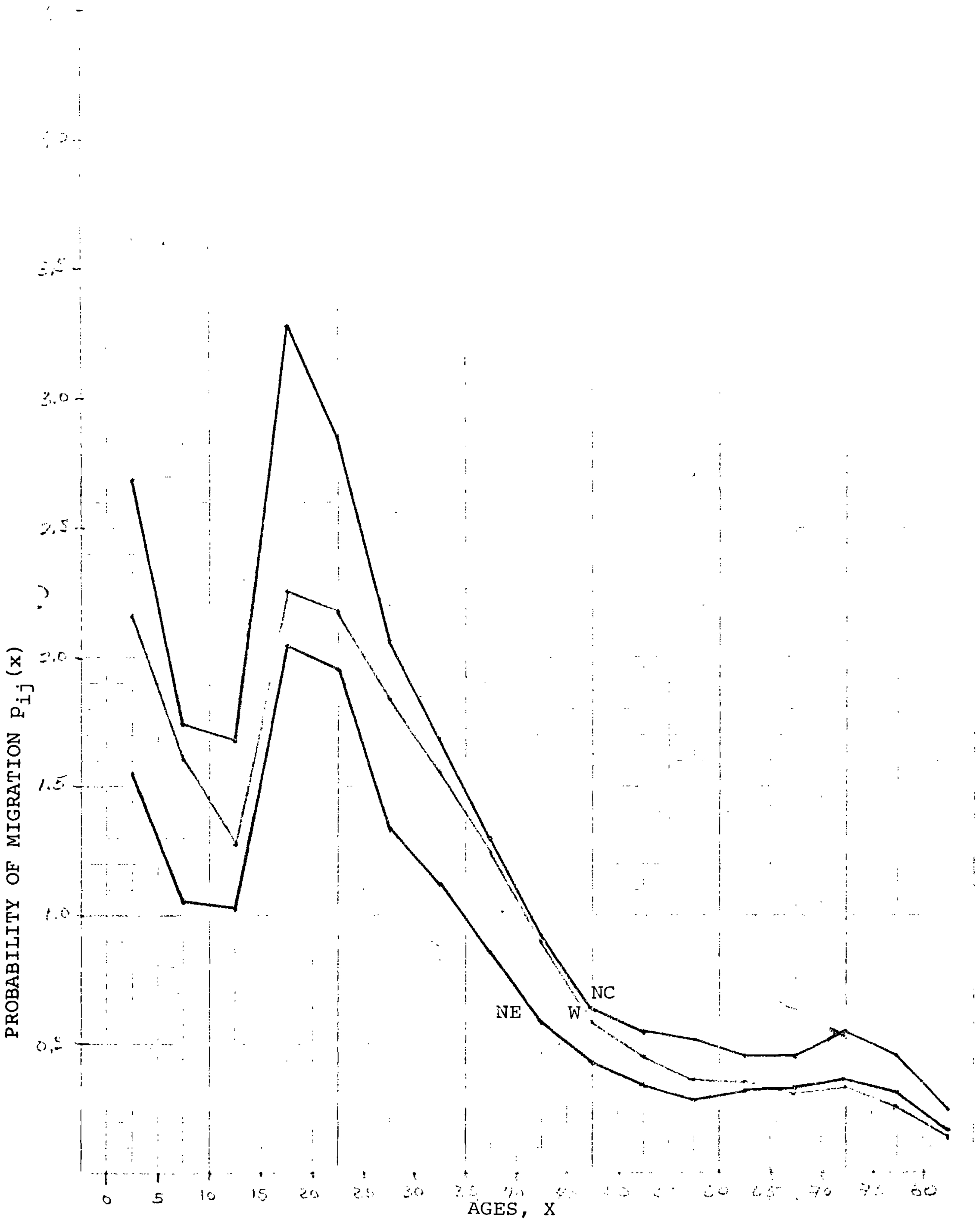

Figure 12D: Probability of Migration from the South Region: Female Population, 1968

The rest of the distribution, however, remains essentially unchanged, with peaks occurring in the 0-4, the 20-24, and the 60-64 year old age groups. Note that in some instances, the consolidation into broader age groups produces a younger than normal high peak.

Figure 11 indicates that the relative ordering of migration levels between Census regions in the U.S. did not change over the decade between 1958 and 1968. Migration out of the North Central region was highest to the South and lowest to the North East at both times (though in 1958 the flows to the West were virtually at the same level as those to the South). Migration out of the South was highest to the North Central region and lowest to the North East region both in 1958 and in 1968. The same finding also was observed for migration out of the other two regions: the North East and the West (not illustrated).

The age profiles set out in Figure 11 tend to vary more than the relative levels. Nevertheless one can readily identify a temporally unchanging fundamental difference between the retirement profiles of migration flows to the South and West and the labor force profiles of migration out of the South and to the North East. The two sets of fundamental profiles are distinguishable by the presence of a high retirement peak in the former and its virtual absence in the latter.

A well known migration differential, affirmed in numerous demographic studies, is that males migrate more than females. Figure 12 adds further support to this contention, but suggests that the difference is no longer as great as it once was and indicates that important age-specific variations do exist. In general, the high peak for males is considerably higher than that for females and occasionally comes at an older age[2]. A significant reversal in migration levels takes place in the senior age groups (i.e., those beyond age 50) at which point women tend to migrate at a higher rate than men.

Two other idiosyncracies exhibited by the age profiles of Figures 11 and 12 should be noted. These relate to the behaviors of the initial peak, $p_{ij}(0)$, and of the low point. The former tends to be higher in 1968 than in 1958 and seems to move in the same direction as the level of migration, subject to variations occasioned by the changing behavior of the peak (and, of course, to sudden changes in fertility levels). The low point varies between the 5-9 and 10-14 age groups among males, but always occurs at the latter age group among females. When disaggregated by sex, the low point appears to vary in a predictable way with respect to the high peak: the female high peak tends to immediately follow the low point, whereas the male high peak generally occurs ten years after the low point.

Some of the regularities identified above are illustrated in Figures 13, 14, and 15, respectively. We focus only on the total population but consider data for all four Census regions and for both points in time. Figure 13 shows that a strong and positive association exists between the height of the initial peak, $p_{ij}(0)$, and the level of migration as measured by, for example, ${}_i\theta_j$, the fraction of the expected lifetime of an individual born in region i that is expected to be lived in region j.

---

[2] In age-specific migration schedules disaggregated by single years of age, the high peak for women migrants almost always lies to the left of the corresponding peak for male migrants because, on the average, women tend to marry men who are several years older. However, a consolidation into five-year age groups often masks this fundamental regularity.

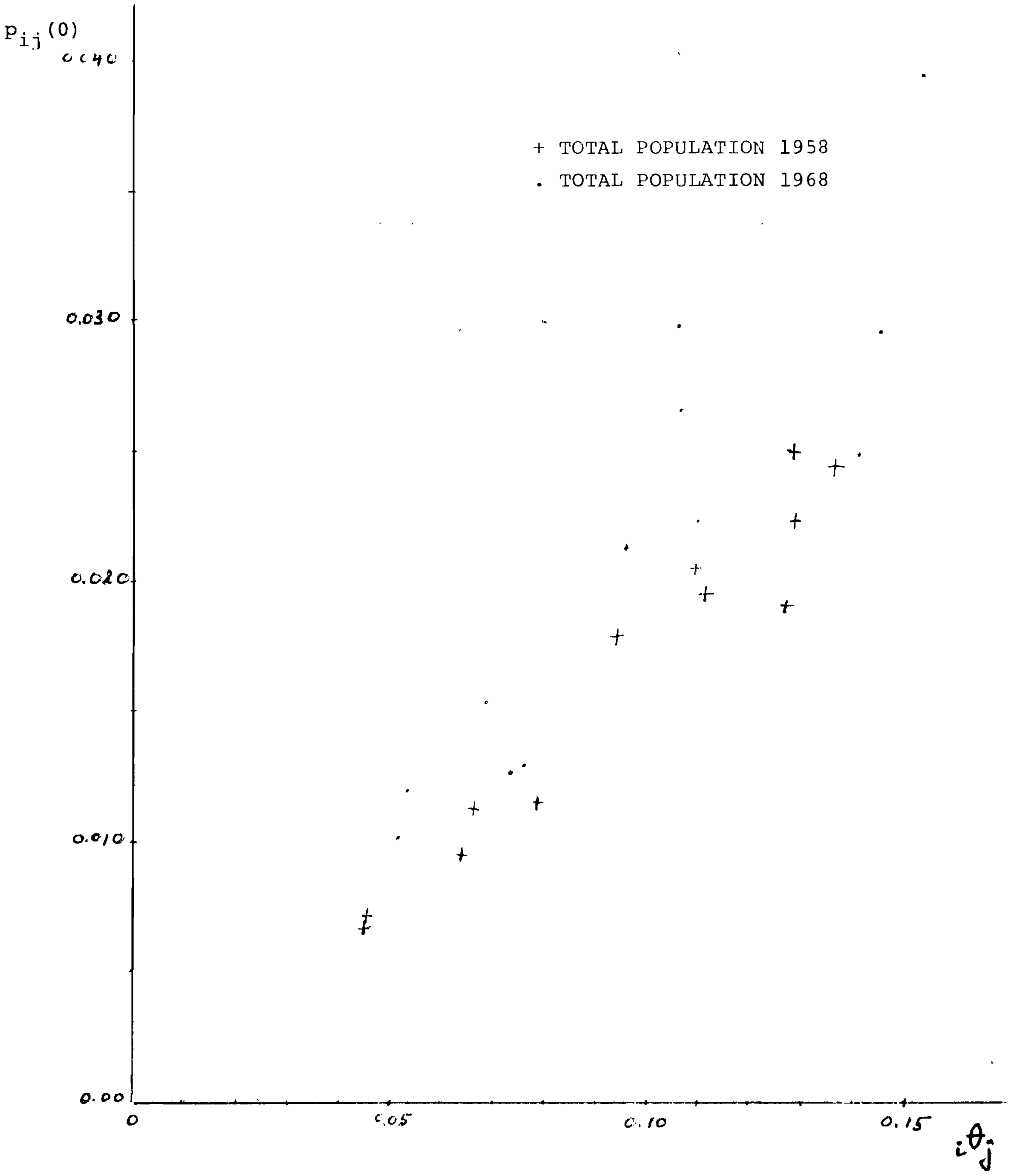

Figure 13: Relation Between Initial Peaks and Migration Levels in in Two Observed Migration Schedules

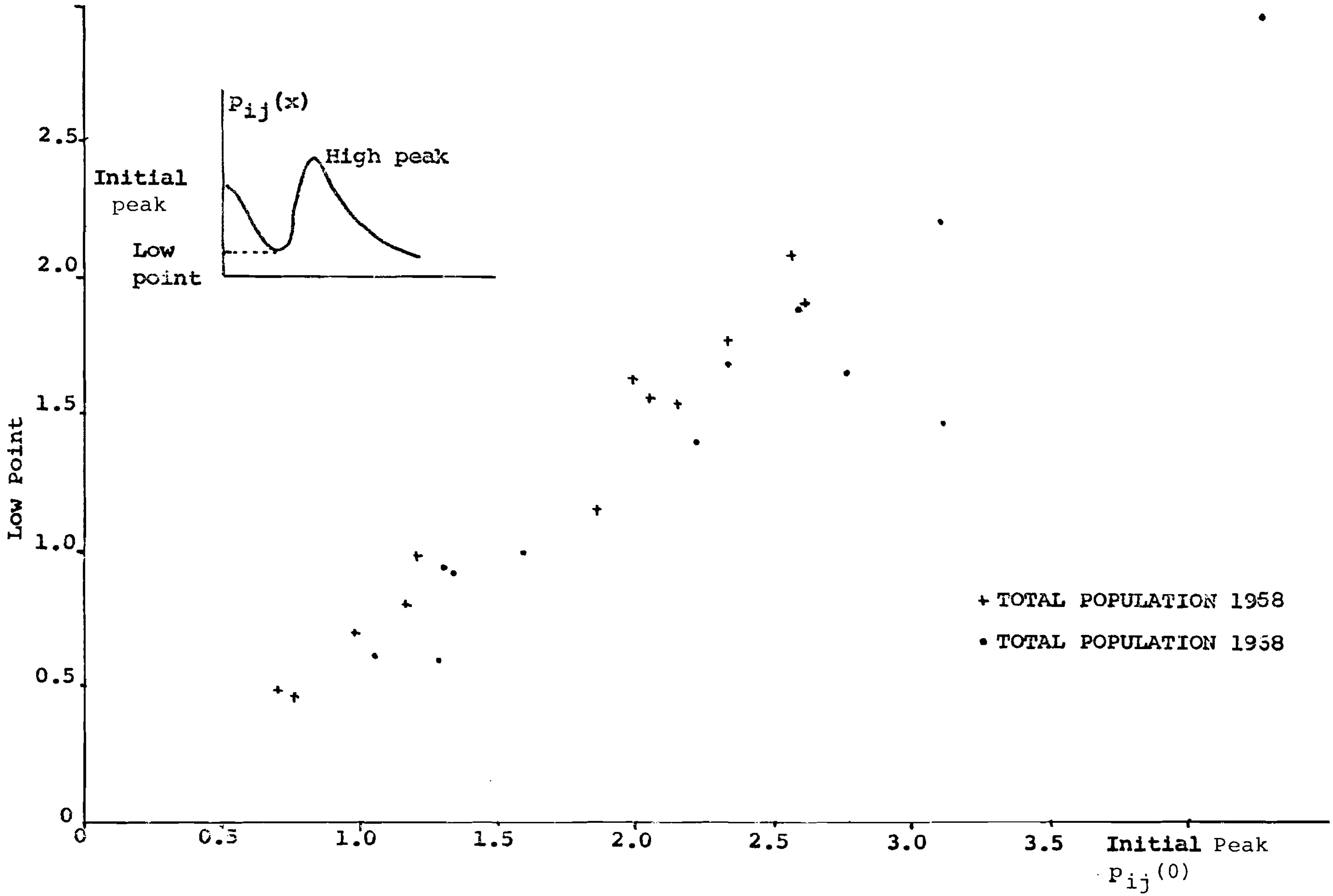

Figure 14: **Relation Between Peaks in Migration Schedules**

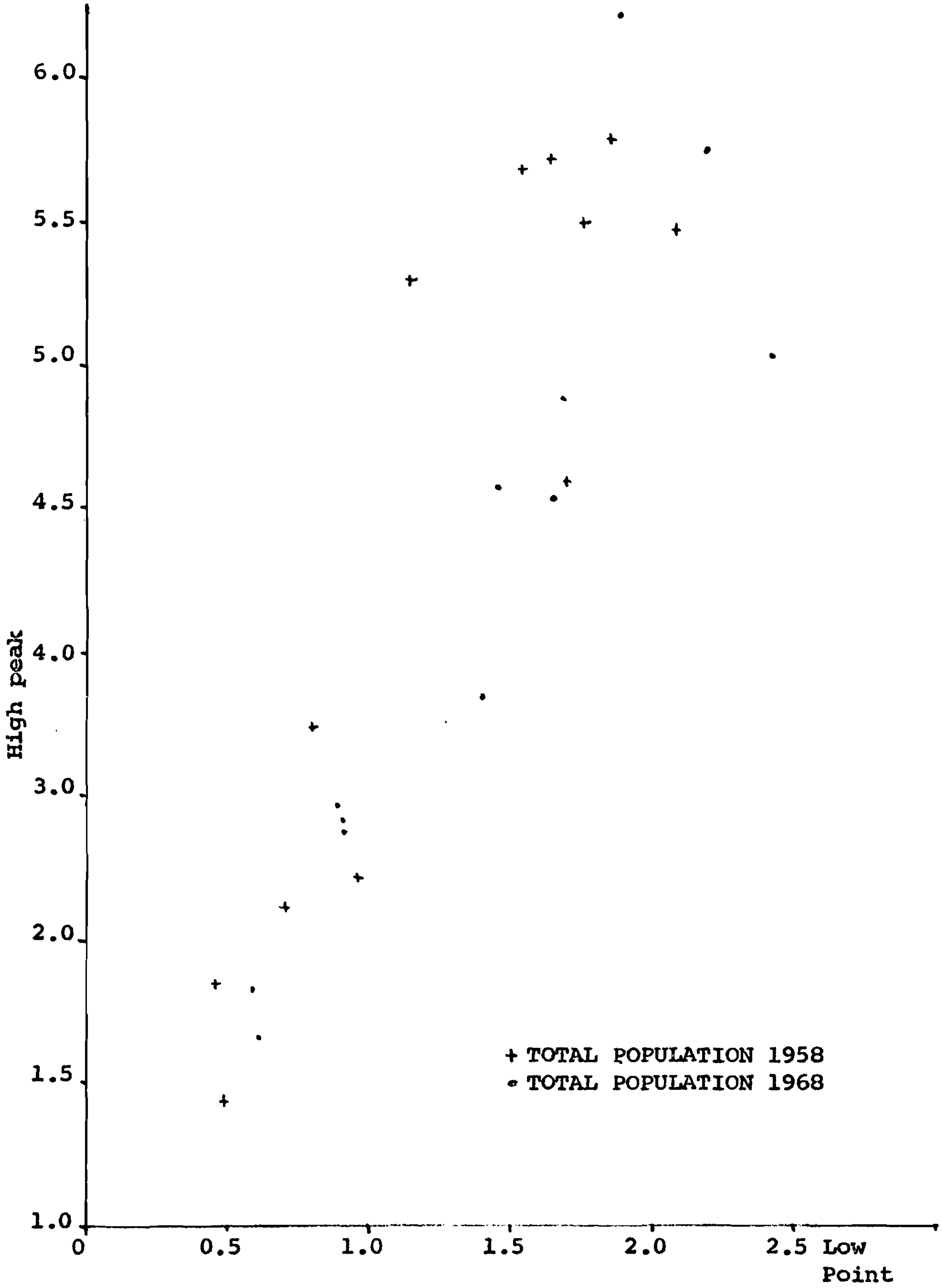

Figure 15: Relation Between High Peaks and Low Points in Migration Schedules

Figure 14 indicates that a similarly strong and positive relationship exists between the height of the low peak and the height of the initial peak. Finally, Figure 15 describes the positive association between the heights of the high peak and the low point. Thus a direct line of correlation appears to connect the general migration level between two regions to the values assumed by the corresponding age-specific probabilities of migrating. This suggests that a simple linear regression equation may be used to associate a set of probabilities of migrating at each age x, $p_{ij}(x)$, with a single indicator of migration level, say ${}_i\theta_j$. We explore this possibility in the next section.

3.3 Summarizing the Regularities

The migration risks experienced by different age and sex groups of a given population are strongly interrelated, and higher (or lower) than average migration rates among one segment of a particular population normally imply higher (or lower) than average migration rates for other segments of the same population. This association stems in part from the fact that if socioeconomic conditions at a location are good or poor for one group in the population, they are also likely to be good or poor for other groups in the same population. Since migration is widely held to be a response to spatial variations in socioeconomic conditions, these high intercorrelations between age-specific migration risks are not surprising.

Figures 13, 14, and 15 support the above conjecture and, moreover, suggest a way of summarizing the observed regularities in migration probabilities. They indicate that a relatively accurate accounting of the variation of the initial peak (and through it in the rest of the migration schedule) may be obtained by means of a straight line fitted to the scatter of points in Figure

13\. Thus a linear regression of the form

$$p_{ij}(0) = \alpha + \beta \; {}_i\theta_j$$

would seem to be appropriate[3]. But $p_{ij}(0)$ cannot take on negative values; a convenient way of ensuring that this possibility never arises is to force the line through the origin by adopting the zero-intercept simple linear regression model

$$p_{ij}(0) = \beta \; {}_i\theta_j \quad . \tag{5}$$

The least-squares fit of such an equation to the data illustrated in Figure 13 gives

$$p_{ij}(0) = 0.17392 \; {}_i\theta_j$$

for the 1958 observations, and

$$p_{ij}(0) = 0.22002 \; {}_i\theta_j$$

for the 1968 data points. The fit in each instance is quite satisfactory, yielding coefficients of determination (r^2) of 0.94 and 0.84, respectively.

Given estimates of β and ${}_i\theta_j$ we can obtain an estimate of $p_{ij}(0)$. Figures 14 and 15 suggest that with the value of $p_{ij}(0)$ fixed, we can find the corresponding value of the low point and use that, in turn, to estimate the value of the high point. Generalizing this argument to all age

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[3] Since changes in fertility also affect the height of the initial peak, a possible further refinement of the model would be to include a variable describing the level of fertility, for example, the reproduction rate.

groups beyond the first, we may adopt the simple model

$$p_{ij}(x + 5) = \alpha(x)\ p_{ij}(x) \quad , \qquad (6)$$

where $p_{ij}(0)$ is estimated by Equation 5. Thus

$$p_{ij}(5) = \alpha(0)\ p_{ij}(0) = \alpha(0)\beta_i\theta_j = \beta(5)_i\theta_j \quad , \text{ say,}$$

$$p_{ij}(10) = \alpha(5)\ p_{ij}(5) = \alpha(5)\beta(5)_i\theta_j = \beta(10)_i\theta_j \quad ,$$

and, in general,

$$p_{ij}(x) = \beta(x)\ {}_i\theta_j \quad , \qquad (7)$$

in which the β in (5) now is designated by $\beta(0)$. Note that as a consequence of our definitions

$$\alpha(x) = \frac{\beta(x + 5)}{\beta(x)} \quad ,$$

and

$$p_{ij}(x + 5) = \alpha(x)\ \frac{\beta(x)}{\beta(0)}\ p_{ij}(0) = \frac{\beta(x + 5)}{\beta(0)}\ p_{ij}(0) \qquad (8)$$

from which we conclude that the probability of migration at age x, $p_{ij}(x)$, is directly proportional to the corresponding regression coefficient $\beta(x)$.

Equation 7 may be treated as a simple (zero-intercept) linear regression equation, and its coefficient $\beta(x)$ may be estimated using the conventional least-squares procedure. Table 3 presents two sets of such coefficients for the U.S. total population. The first set was obtained using 1958

TABLE 3

Regression Coefficients for Obtaining Model Probabilities of Migration

AGE	Total (1958)		Total (1968)	
	β	r^2	β	r^2
0	0.17392	0.94	0.22002	0.84
5	0.13460	0.95	0.15553	0.89
10	0.15736	0.86	0.15040	0.94
15	0.30757	0.93	0.29195	0.85
20	0.32271	0.72	0.26370	0.72
25	0.23251	0.96	0.20037	0.90
30	0.17897	0.95	0.17907	0.94
35	0.12912	0.95	0.14392	0.96
40	0.09790	0.93	0.10397	0.95
45	0.07522	0.86	0.07378	0.91
50	0.06838	0.73	0.06352	0.76
55	0.07347	0.63	0.07362	0.54
60	0.08254	0.47	0.08320	0.43
65	0.06086	0.50	0.06425	0.47
70	0.04488	0.58	0.04919	0.64
75	0.03019	0.67	0.03951	0.64
80	0.01342	0.18	0.02058	0.63

data, the second set was estimated on the basis of 1968 data. In both instances the observed migration flows were those between the four U.S. Census regions.

The regression coefficients in Table 3 may be used in the following way. First, starting with a complete set of multiregional migration levels ${}_i\theta_j$ one calculates the matrix of migration probabilities $\underset{\sim}{P}(x)$ for every age, using Equation 7 and one of the two sets of regression coefficients in Table 3. (Figure 16 illustrates a range of such probabilities by way of example.) With $\underset{\sim}{P}(x)$ established, one then may compute the usual life table statistics such as the survivorship proportions defined in Equation 4 and the various region-specific expectations of life at each age. The collective results of these computations constitute a model multiregional life table.

Migration, like fertility, is a potentially repetitive event, and its level therefore can be expressed in terms of an expected number of events per person. However, like mortality, migration also can be measured in terms of an expected duration time, for example, the fraction of a lifetime that is expected to be lived at a particular location. The latter led to the development of a regression approach similar to one used by Coale and Demeny (1966) to summarize regularities in mortality schedules; the former suggests an alternative procedure--one which is analogous to that used by Coale and Demeny (1966, p.30) to summarize fertility schedules.

Consider, once again, the two migration schedules M(x) set out earlier in Figure 6A. Observe that the higher of the two schedules (the one illustrating the 1958 rates) describes a higher level of migration since its migration rates are greater at most ages. A convenient summary measure of migration level, then, is the total area under the curve, i.e., the sum of all age-specific rates. Working by direct analogy with a similar measure

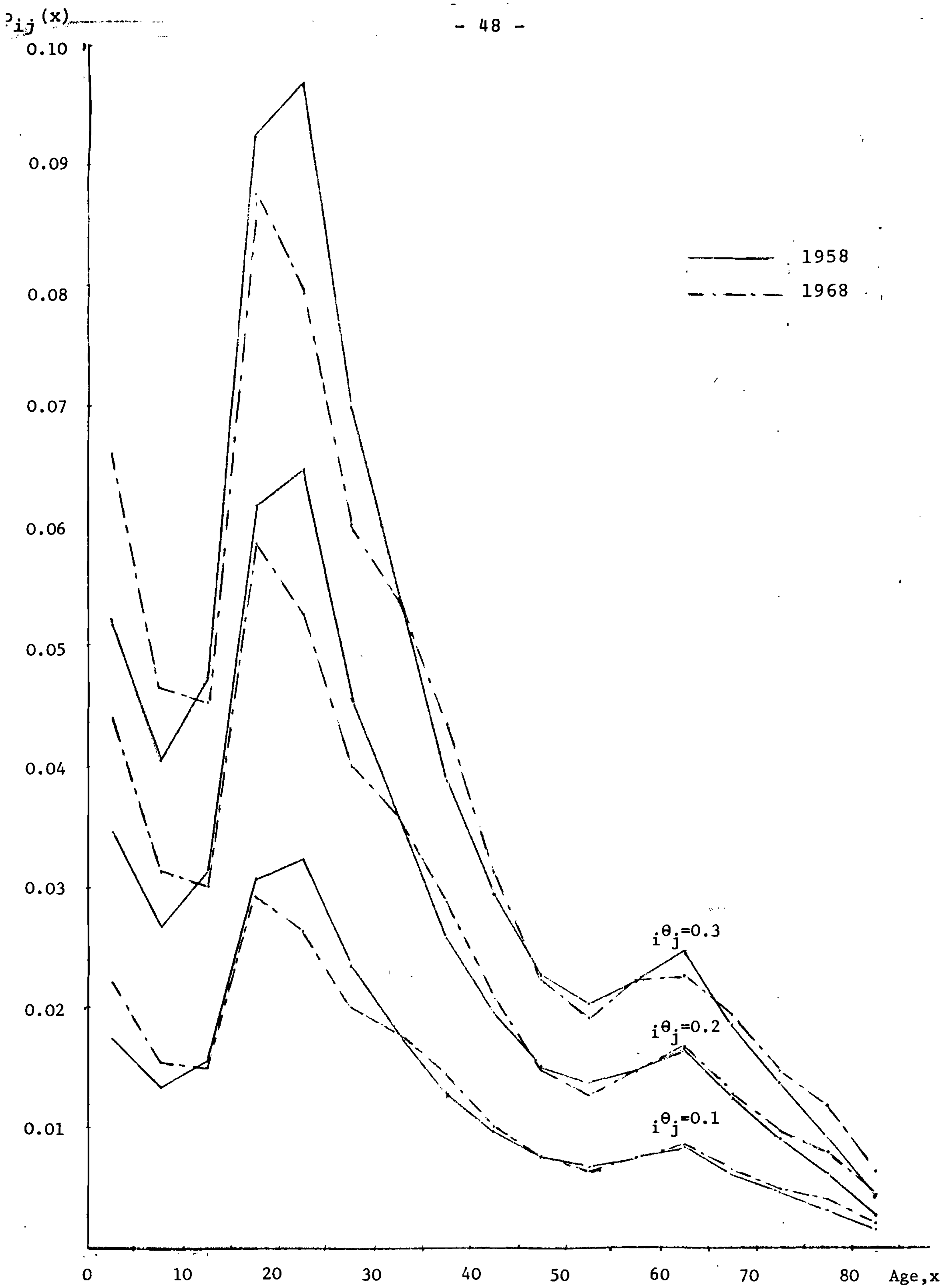

Figure 16: Age-Specific Model Probabilities of Migration at Various Levels of Migration

used in fertility analysis, we multiply this sum by five, to transform its point of reference from an annual to a five-year interval, and call it the gross migraproduction rate, GMR. Thus, recalling that

$$\mathrm{GRR} = 5 \sum_{x=0}^{z} F(x)$$

is the conventional formula for the gross reproduction rate of fertility analysis, we define

$$\mathrm{GMR} = 5 \sum_{x=0}^{z} M(x)$$

to be the corresponding migraproduction rate of migration analysis. By way of illustration, the GMR of the 1958 migration schedule in Figure 6A is 0.6488; the GMR of the corresponding 1968 schedule is 0.6546.

The GMR of a migration schedule is a summary measure of migration level. But we have seen that such schedules also vary in age profile. Thus we need to develop an aditional indicator with which to differentiate the age profiles of various migration schedules. Once again resorting to the analogy with fertility analysis, we define

$$\bar{n} = \sum_{x=0}^{z} (x + 2.5)\, M(x) \Big/ \sum_{x=0}^{z} M(x)$$

to be the mean age of the migration schedule M(x). The mean ages of the 1958 and 1968 migration schedules in Figure 6A, for example, are 29.23 and 29.73 years, respectively.

Figure 17 illustrates several basic model migration schedules with a mean age of 29 years. It is the "fertility

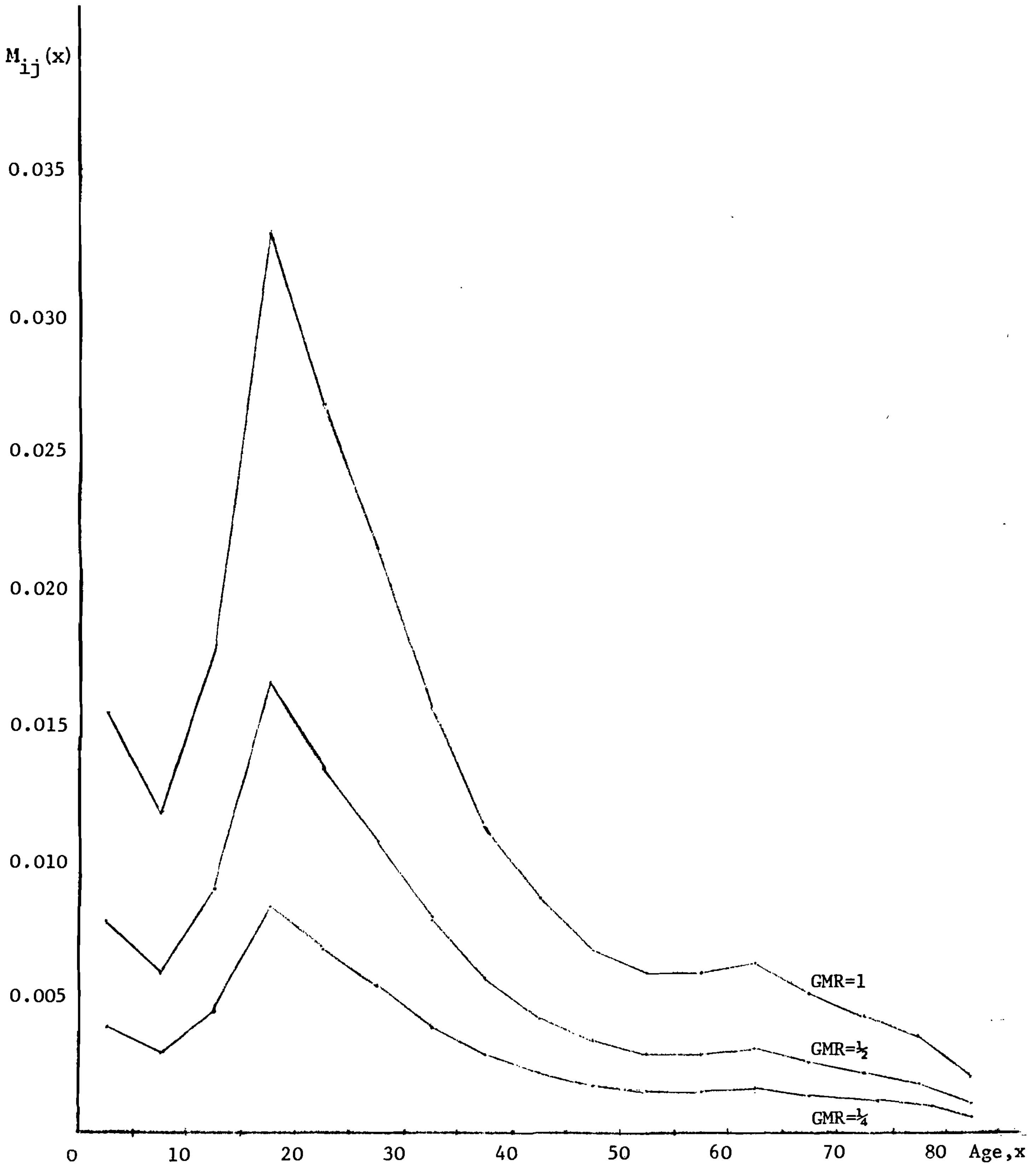

Figure 17: Basic Age-Specific Model Migration Schedules with a Mean Age of 29 Years by Various Gross Migraproduction Rates

approach" counterpart to Figure 16, which showed several basic model migration schedules obtained using the "mortality approach". The latter schedules focused on $\underset{\sim}{P}(x)$, whereas the former are expressed in terms of $\underset{\sim}{M}(x)$. This, however, is simply a matter of convention and convenience inasmuch as either set of model schedules may be expressed as a linear transformation of the other by means of Equation 3.

Figures 18 and 19 plot the gross migraproduction rate against the mean age for the migration schedules of our four-region U.S. population system. (The detailed data are included in Appendix C.) Figure 18 treats the total population in 1958 and 1968; Figure 19 considers only the 1968 data but disaggregates it by sex. In both figures we find evidence of a division of the schedules into four groups:

1. high GMR - high n;
2. high GMR - low n;
3. low GMR - high n;
4. low GMR - low n.

Migration flows from the North Central region to the South, for example, exhibit an "old" profile and a mean age of about 32.5 years. The reverse migration flows, on the other hand, takes on the shape of a "young" profile and shows a mean age that is about five years younger. This suggests that it may be useful to develop a family of basic model migration schedules in order to more accurately capture and summarize the various age profiles that are exhibited by empirical migration schedules.

3.4 A Family of Model Migration Schedules

In this section we consider the effects on the migration age profile of various disaggregations of our data on the U.S. population system. Specifically, we examine how the regression coefficients set out earlier in Table 3, and now illustrated in Figure 20 below, respond to various disaggregations of the empirical population on the basis of

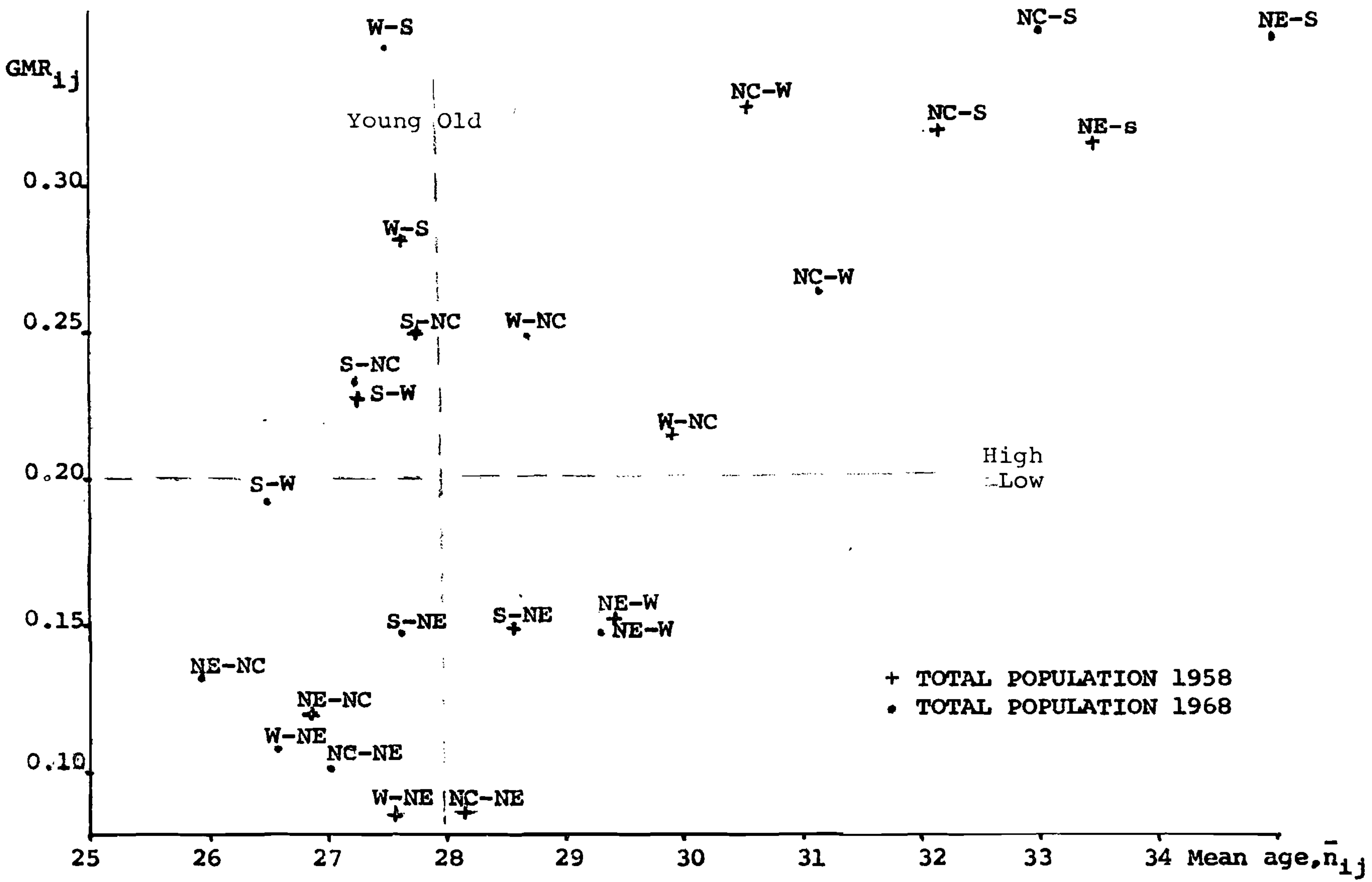

Figure 18: Relation Between Observed GMR and Mean Age of Migration Schedule: Total Populations, 1958 and 1968

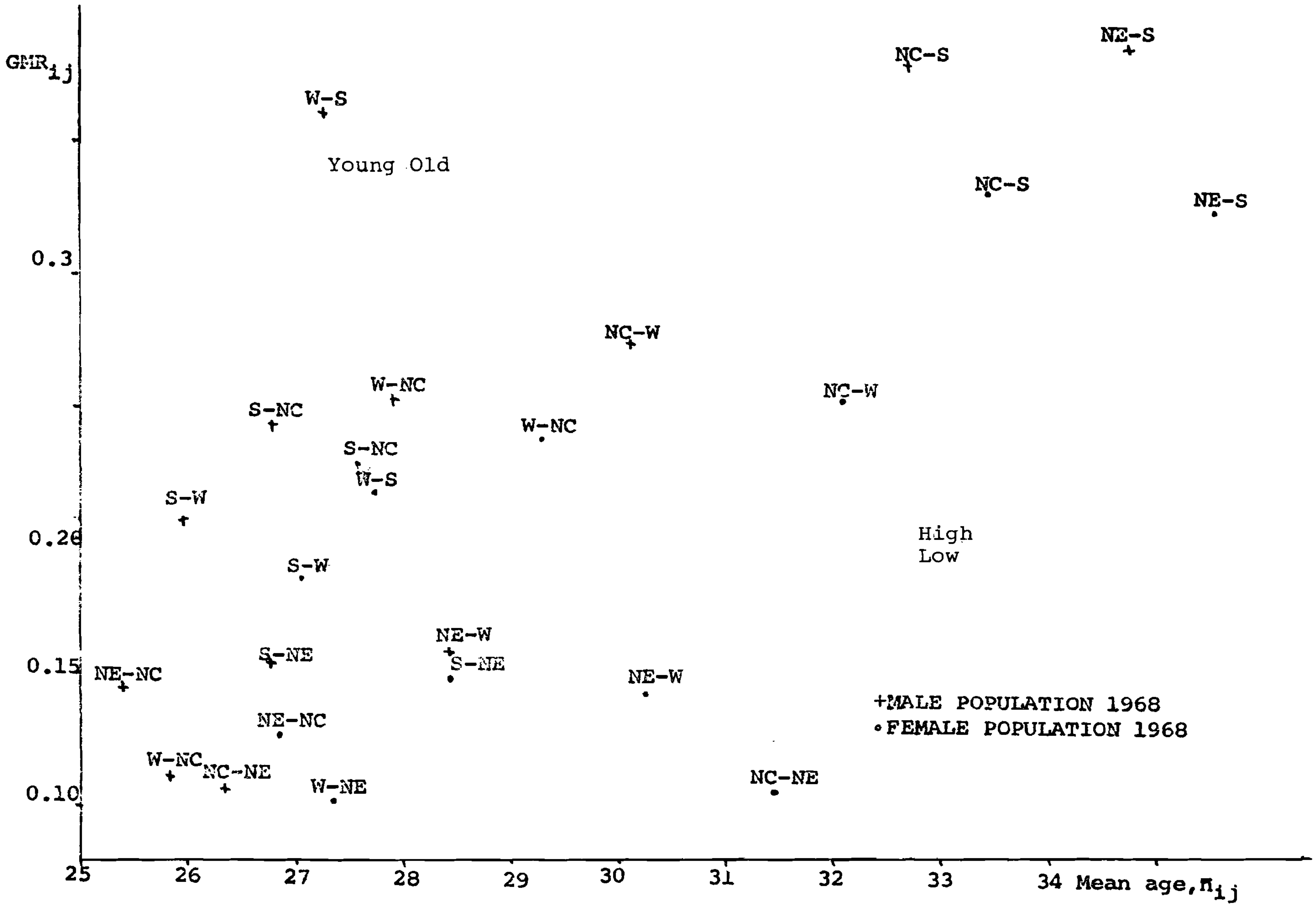

Figure 19: Relation Between Observed GMR and Mean Age of Migration Schedule: Male and Female Populations, 1968

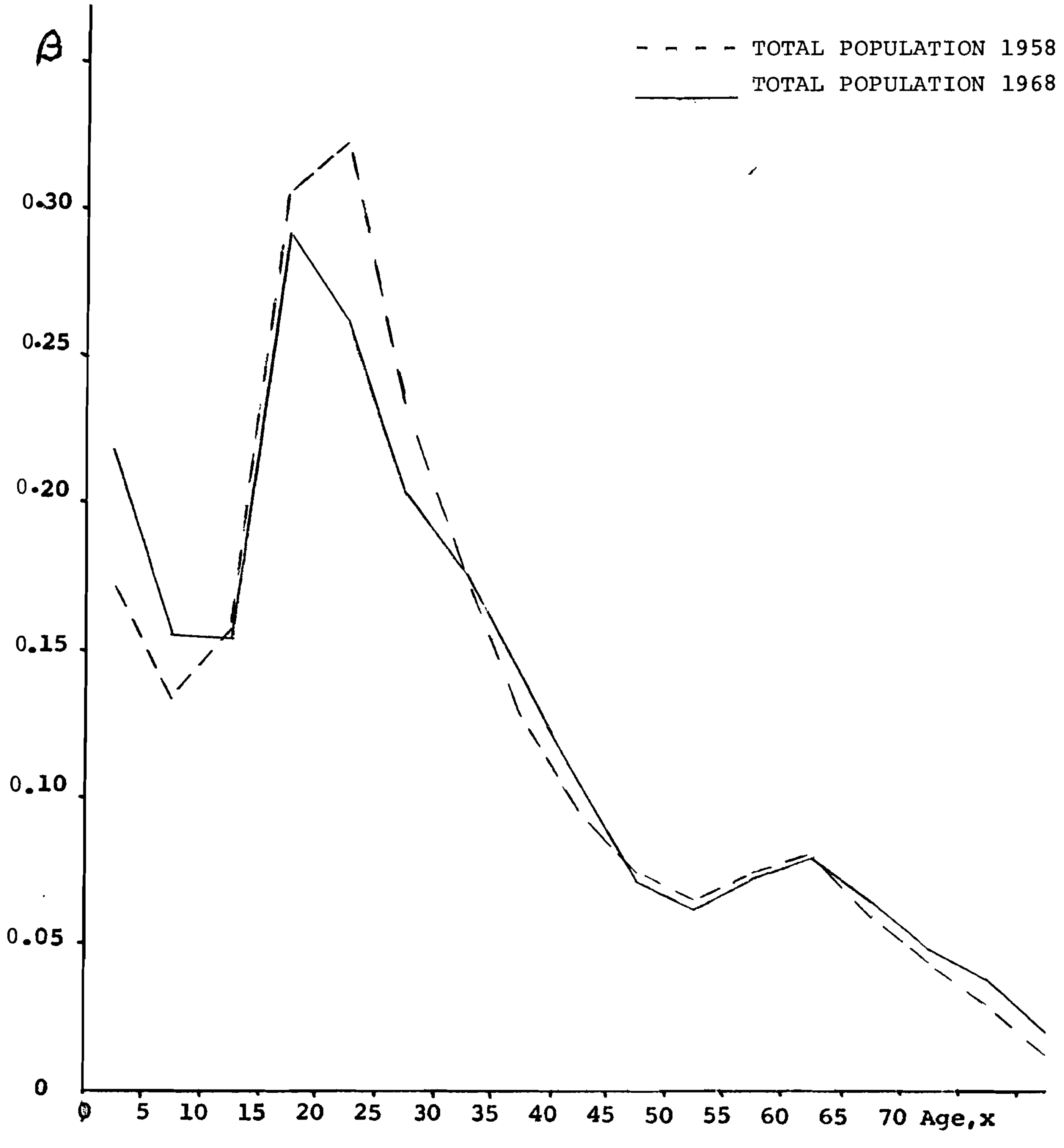

Figure 20: Regression Coefficients For Model Migration
Schedules: Total Populations, 1958 and 1968

which they were estimated. First, we disaggregate the total population by sex. Next, we introduce a further disaggregation according to mean age. Then we consider a spatial disaggregation of the four Census Regions into their constituent nine Census Divisions. Finally, we explore the impact of an even finer deconsolidation by mean age.

The two regression coefficient profiles in Figure 20 mirror the fundamental age profile of migrants that has been analyzed earlier in this paper. The principal differences between the two coefficient profiles are the higher and older high peak in the 1958 migration schedule and the higher and older low point of the corresponding 1968 schedule. Beyond the mid-thirties the two profiles are quite similar, with both showing a retirement peak in the 60-64 year old age group.

Profile Differences by Sex. A disaggregation of the 1968 regression coefficient profile introduces important variations by sex, according to Figure 21. The male coefficients are higher from the very early teens to the mid-forties and are lower at all other ages. The locations of the high peak and the retirement peak are the same in both profiles, but the low point among males comes at a younger age than in females. Also, the retirement peak among females is broader and starts at an earlier age.

Profile Differences by Mean Age. Our earlier division of migration schedules into "young" and "old" categories in Figures 18 and 19 suggests that such a classification might be a useful way of disaggregating the regression coefficients illustrated in Figures 20 and 21. Figure 22 indicates that this is indeed the case. It shows two basic age profiles which are distinguishable by the presence of a high retirement peak in one profile and its virtual absence in the other. We have earlier designated the former profile as a retirement profile and the

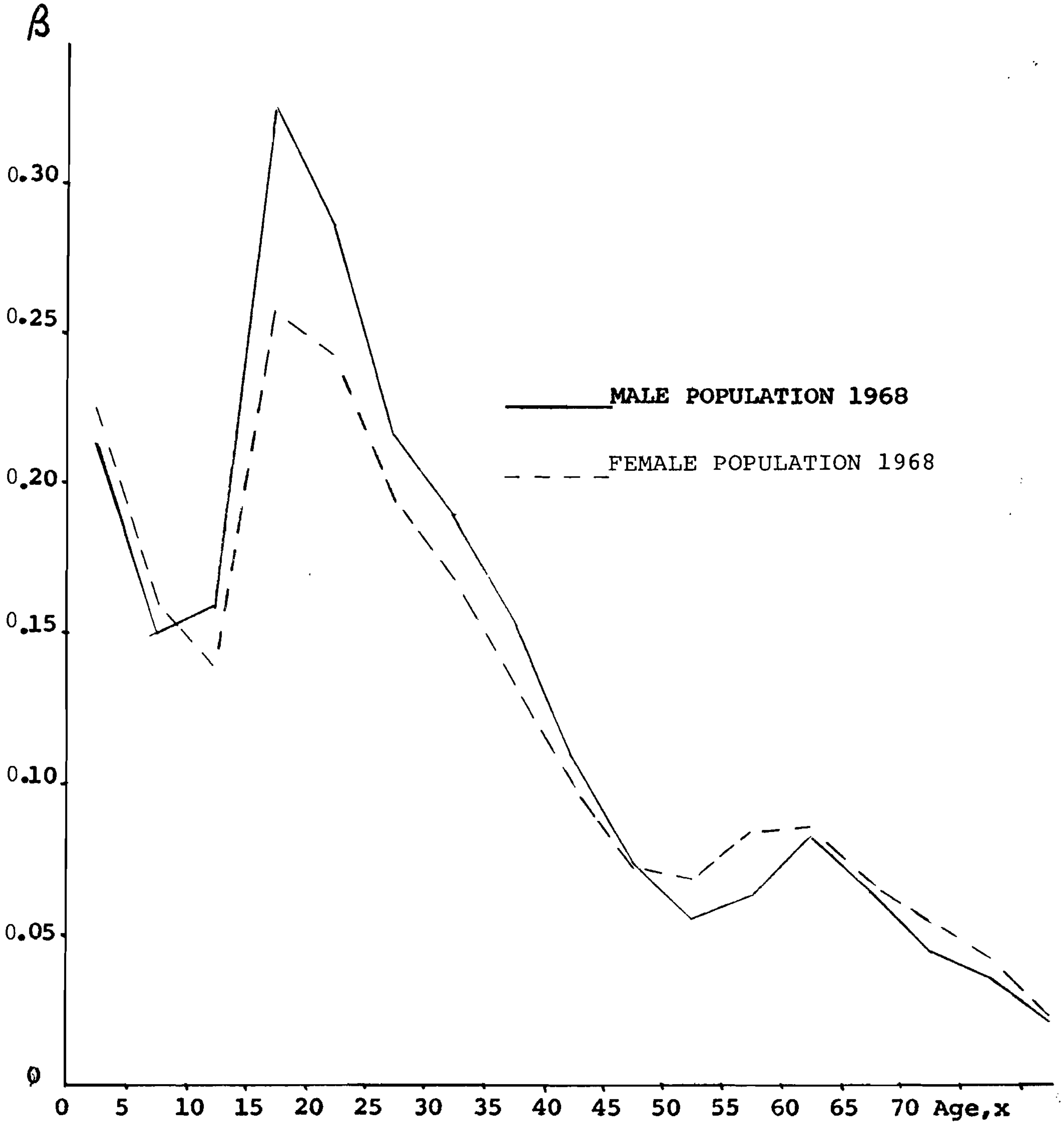

Figure 21: Regression Coefficients For Model Migration Schedules: Male and Female Populations, 1968

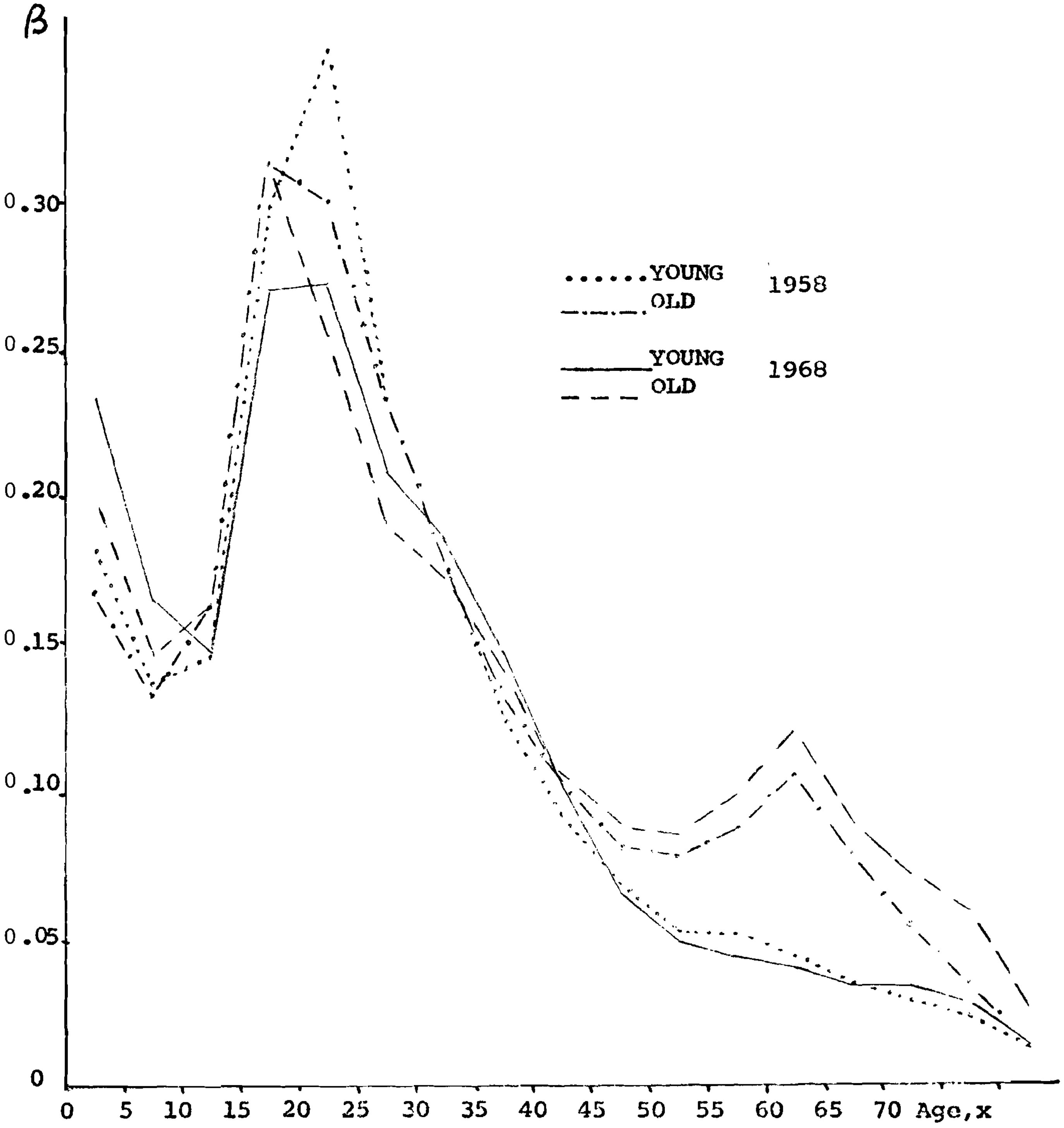

Figure 22: Regression Coefficients by "Young" and "Old" Classification ($\bar{n}_{ij} \leq 28$ and $\bar{n}_{ij} > 28$): Total Populations, 1958 and 1968

latter as a labor force profile. An alternative designation is "old" and "young" profile, respectively.

A disaggregation of these two basic profiles by sex reveals an important further difference (Figure 23). Whereas a clear division into young and old categories may be made for males, in the case of females the two basic profiles are remarkably alike and, moreover, both show a retirement peak. Also, the retirement peak of the "younger" profile is for some reason higher than that of the "old" profile. However, in light of the very small sample sizes used to estimate the regression coefficients defining the "young" and "old" profiles, little significance can be attributed to this particular feature[4].

Profile Differences by Size of Areal Unit. Because migration normally is defined as a crossing of a regional boundary, it is clear that reducing the size of a spatial unit should increase the level of outmigration from that unit, since some of the moves that previously did not cross over the old borders now will be recorded as migrations over the new borders. But what of the age profile in each case? Should not this feature of the observed migration flows remain essentially unchanged, at least for the relatively large areal units? Figure 24 (like Figure 5B before it) gives some evidence that this conjecture is valid. The two regression coefficient profiles that it illustrates were estimated on the basis of the same data set, using first a nine- and then a four-region spatial delineation of the total 1958 U.S. population. The fact that the former is always higher than the latter is perhaps a

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[4] According to Table C.4 in the Appendix, the mean age of the female migration schedule from the South to the Northeast was 28.33 years, and therefore is an "old" schedule. Yet the corresponding male and total schedules are "young" schedules. To maintain consistency we therefore treated the female schedule as a "young" schedule. An analogous argument led to the inclusion of the male schedule of migration from the West to the North Central Region in the class of "old" schedules.

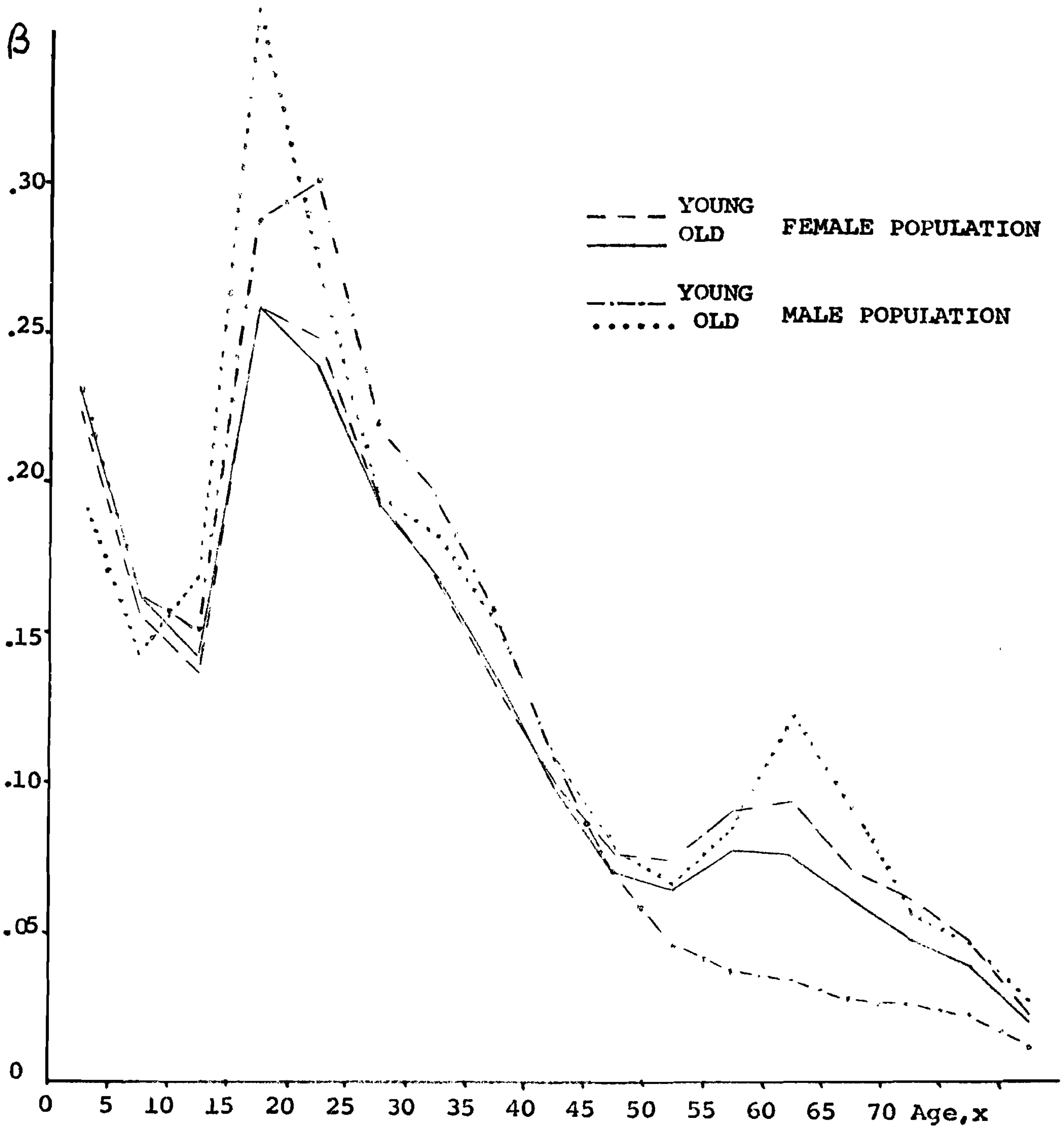


Figure 23: Regression Coefficients by "Young" and "Old" Classification ($\bar{n}_{ij} \leq 28$ and $\bar{n}_{ij} > 28$): Male and Female Populations, 1968

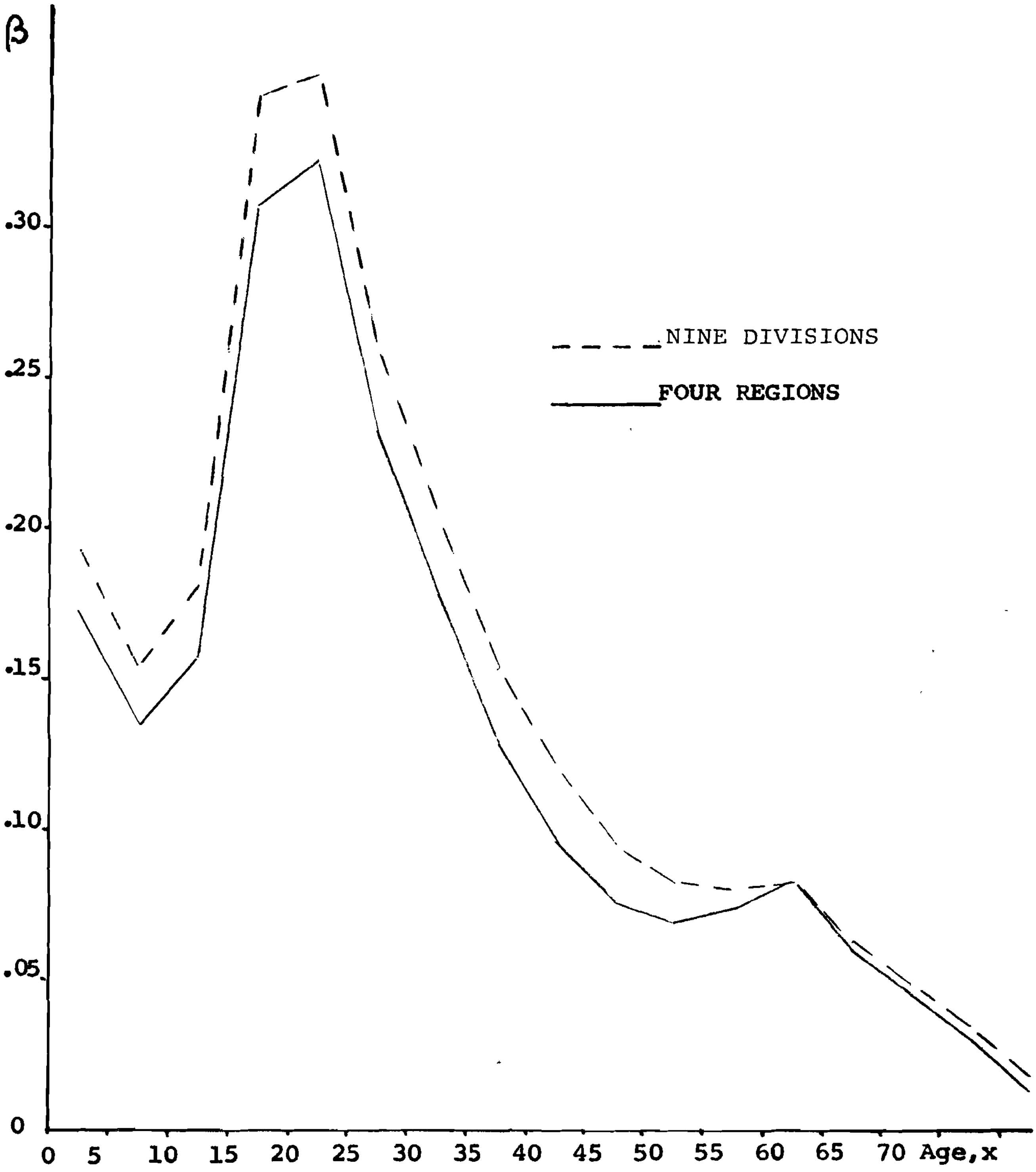


Figure 24: Regression Coefficients For Model Migration Schedules: Total Population, 1958, by Region and Divisions

consequence of some confounding between profile and level introduced by aggregation bias.

Profile Differences by Several Mean Age Groupings. The spatial disaggregation of our data from four to nine areal units increases the number of observations from 12 to 72 and thereby affords us an opportunity to examine the impact of a finer classification by mean age. Specifically, we now consider the disaggregation of the 1958 regression coefficient profile into four instead of two mean age categories: "very young" ($\bar{n}_{ij} \leq 26$); "young" ($26 < \bar{n}_{ij} \leq 28$); "old" ($28 < \bar{n}_{ij} \leq 30$); and "very old" ($\bar{n}_{ij} > 30$).

Except for variations with respect to the retirement peak, the principal impact of the finer disaggregation by mean age appears not so much in the age profile as in the relative height of that profile for a given value of the migration level ${}_i\theta_j$. Thus, for example, the age curve of the "very old" profile in Figure 25 is almost everywhere higher than the corresponding curve of the "very young" profile, for the same level of migration. The reason for this is not immediately apparent and merits further study. A possible explanation may lie in the fact that ${}_i\theta_j$ is an index which combines an age-specific migration pattern with a specific (life-table) age composition. This particular confounding of schedule and composition could perhaps generate the variations in profile heights that appear in Figure 25, although the underlying dynamics of this are by no means self-evident. Consequently, it may well be the case that the "fertility approach" with its focus on the GMR as an index of migration level has a built-in advantage over the "mortality approach" that we have been following in this section. This possibility is considered further in the conclusion of this paper.

The regression coefficients set out in Tables D.1 through D.5 of Appendix D, and illustrated above in Figures

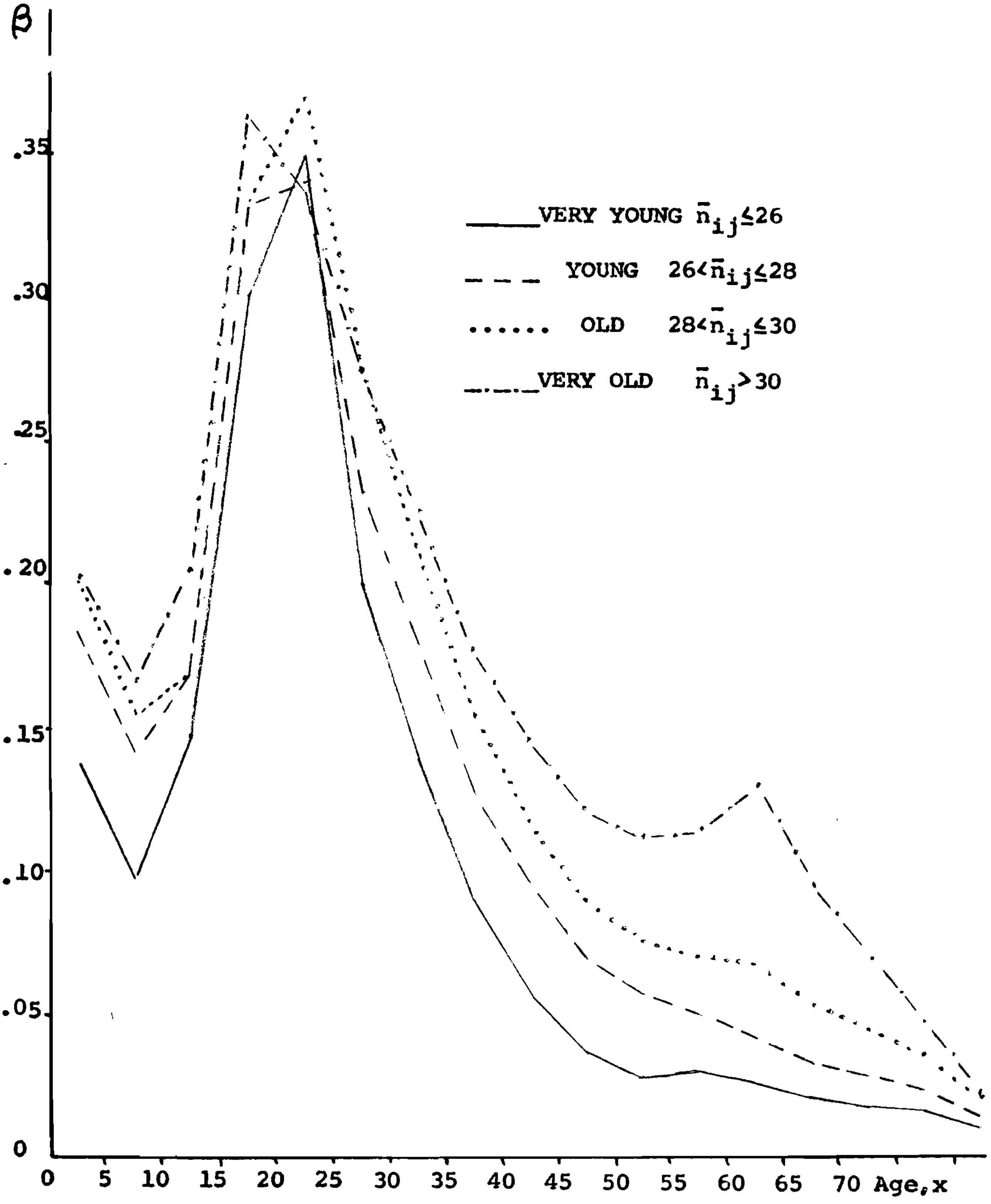


Figure 25: Regression Coefficients For Model Migration Schedules: Total Population, 1958, by Several Mean Age Classes

20 through 25, may be said to form a family of model migration probabilities or schedules. Those associated with different categories of mean age give "young" and "old" profiles; those that do not consider mean age as an index give "average" profiles. We next illustrate an application of the female "average" profile by constructing a specimen model multiregional life table and then comparing some of its characteristics with those of the corresponding empirical life table.

3.5 A Specimen Model Multiregional Life Table

Table B.4 in Appendix B gives the four regional expectations of life at birth and the dozen migration levels that together characterize the patterns of regional mortality and interregional mobility of U.S. females in 1968. Interpolating in the "West" family of model life tables developed by Coale and Demeny (1966), we first obtain the appropriate set of model probabilities of dying at each age for each of our four Census Regions. Inserting, in turn, each of the dozen values of ${}_i\theta_j$ into Equation 7, with $\beta(x)$ taking on the column of "average" values set out for females in Table D.2 of Appendix D, we next derive initial approximations for $p_{ij}(x)$. These probabilities of migration then may be used in conjunction with the associated interpolated model probabilities of dying to obtain the matrix of survivorship proportions defined in Equation 4. By appropriately manipulating Equation 3, we also can find the associated model migration rates. And then, following the normal computational procedures of multiregional life table construction (Rogers, 1975, Ch. 3), we may derive, for example, the corresponding matrix of expectations of life at birth, appropriately disaggregated by region of birth and region of residence. Unfortunately this latter matrix usually will not yield the same migration levels that were used to generate the $\underset{\sim}{P}(x)$ matrix. Such inconsistencies occasionally occur in model life table construction and appear, for example, in the model life tables of Coale and Demeny (1966). To eliminate them one must resort to iteration[5]. Only in this way can one obtain a model multiregional life table whose statistics and parameters are internally consistent.

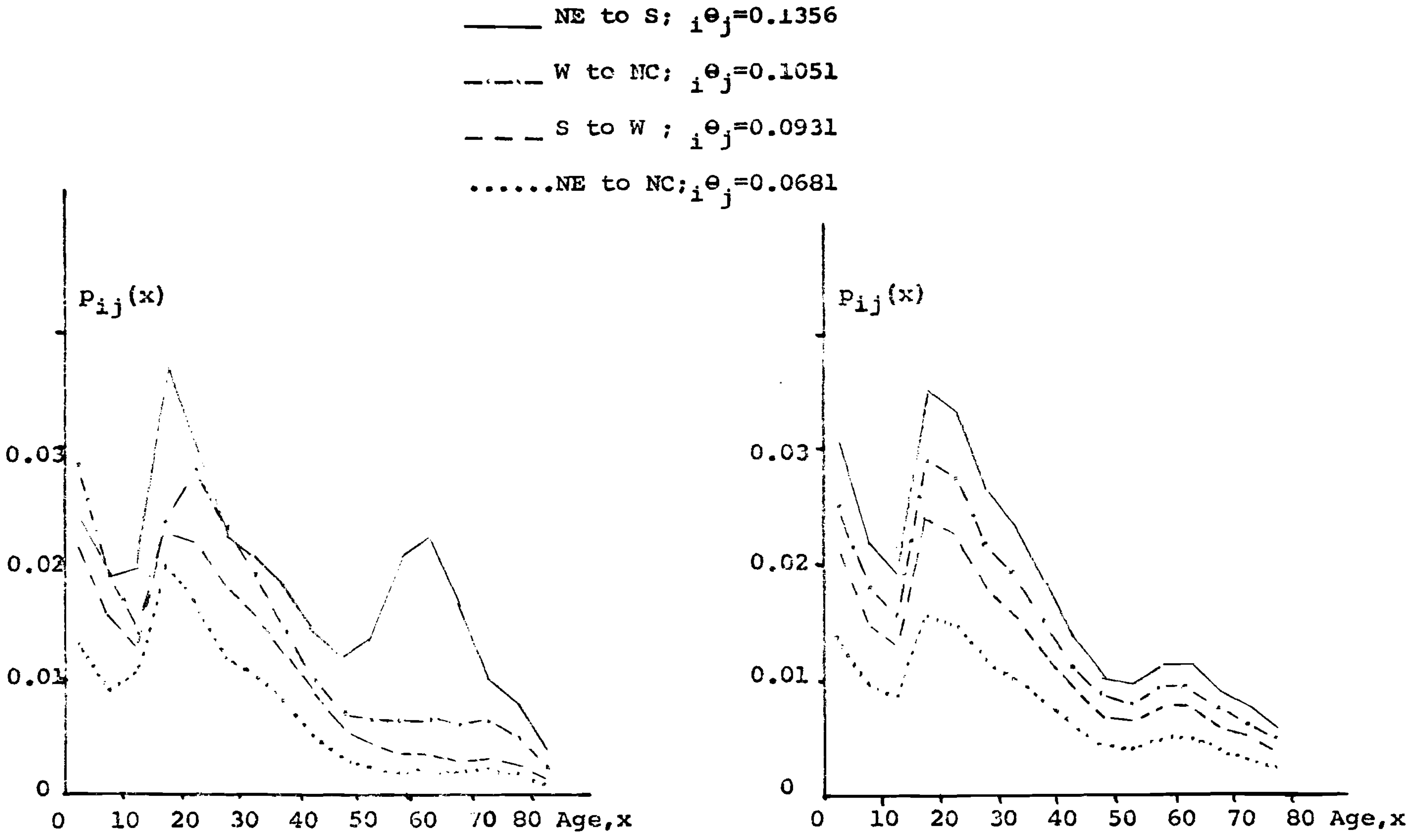
Figures 26, 27, and 28 illustrate several of the model probabilities, proportions, and rates that were generated in the course of constructing our specimen model multiregional life table for U.S. females. Adjoining each of the model schedules is the corresponding observed empirical schedule. A comparison of the two sets of schedules suggests that, although the degree of correspondence is fairly close, further improvement would be highly desirable.

Because migration, like mortality, affects all age groups, it is likely that (as with mortality) minor shifts in migration patterns will have a negligible impact on population projection[6]. This will be explored further in the next part of this paper, where we examine population projections carried out to stability using model schedules of fertility, mortality, and migration.

---

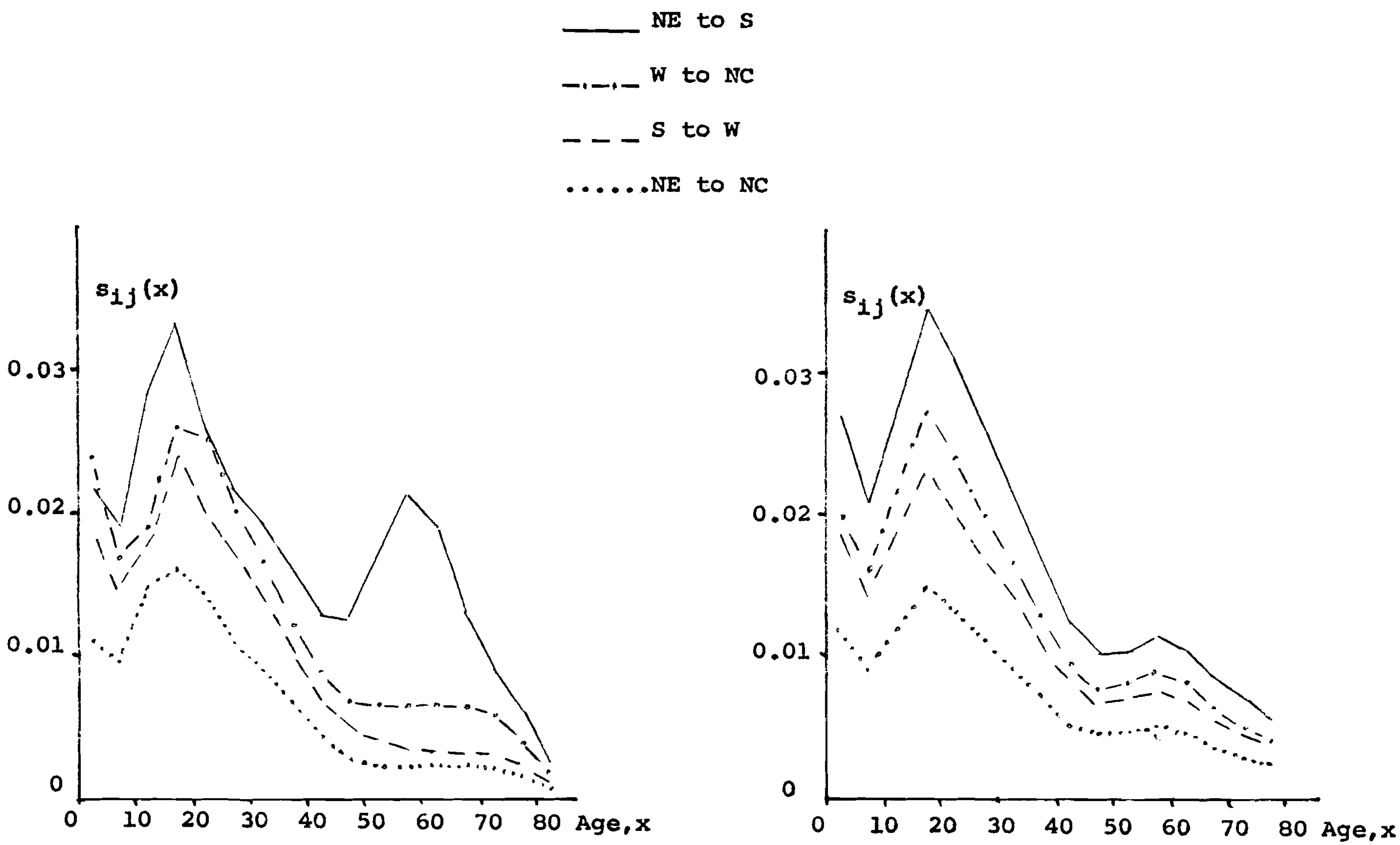
[5] The particular iteration problem that is involved in the multiregional case is a subtle and difficult one because variations in the regional levels of mortality combine in a perverse way with the mathematical model's basic assumption that migrants immediately assume the characteristics of the growth regime operating at their region of destination. The net result is that the convergence of the iteration procedure is not assured. However, such purely technical problems are beyond the scope of this particular paper and are therefore not examined here.

[6] In contrast, small changes in fertility patterns, because they immediately affect the first age group, can produce a significant and immediate shift in the projected age structure.



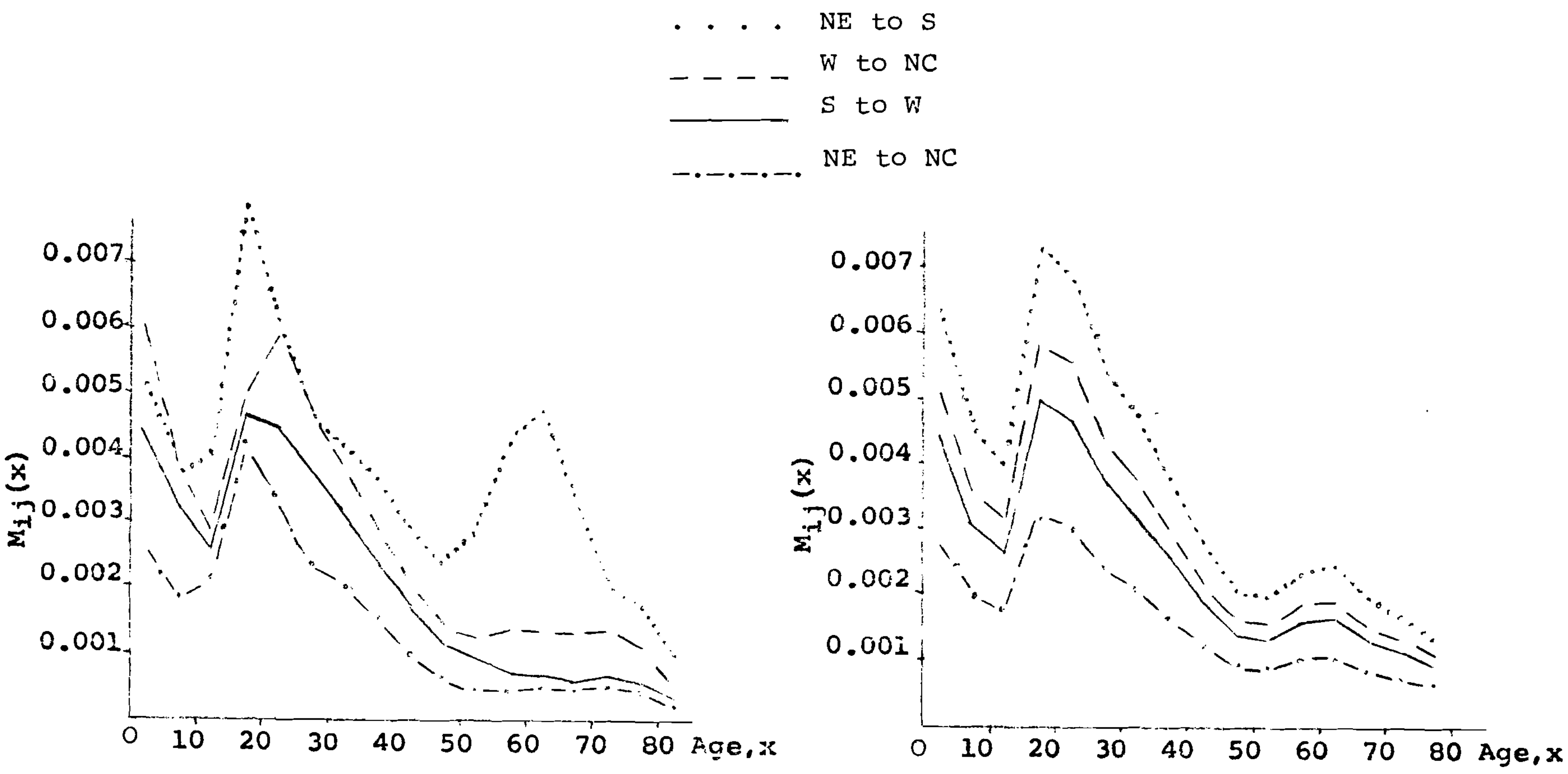
A. Observed Probabilities of Migration B. Model Probabilities of Migration

Figure 26: Observed and Model Female Probabilities of Migration



A. Observed Migration Proportions B. Model Migration Proportions

Figure 27: Observed and Model Female Migration Proportions



A. Observed Migration Schedules

B. Model Migration Schedules

Figure 28: Observed and Model Female Migration Schedules

4. Model Multiregional Stable Populations

A particularly useful way of understanding the evolution of the regional age compositions and regional shares of a closed multiregional population is to imagine them as describing a population that has been subjected to fertility, mortality, and migration schedules which have remained unchanged for a relatively long period of time. Such a population may be said to have been subjected to a fixed regime of growth and is called a multiregional stable population. Its principal characteristics are: unchanging regional age compositions and regional shares; constant regional annual rates of birth, death, and migration; and a fixed multiregional annual rate of growth that is everywhere the same (Rogers, 1975).

In this section of our paper we examine the multiregional stable populations that evolve out of particular histories of fertility, mortality, and internal migration. Such a tracing through of the ultimate consequences of alternative fixed regimes of growth, gives one a fuller understanding of the spatial dynamics of the hypothetical populations that they describe.

4.1 Alternative Representations of Model Multiregional Stable Populations

The most common mathematical representation of a (single-sex) multiregional population growth process focuses on a population disaggregated into 18 five-year age groups, starting with the 0-4 year age group and extending through the open-ended terminal age interval of 85 years and over. If only the ages 10 through 50 are assumed to be capable of childbearing, then such a representation involves 8 age-specific birth rates, 18 age-specific death rates, and 18(m-1) age- and destination-specific migration rates for each of the m regions comprising the multiregional system. However, because such rates exhibit persistent regularities, a remarkably accurate description of spatial population dynamics can be realized by means of models that adopt "model" schedules of growth which have been generated on the basis of a relatively small number of indices of variation.

The study of population dynamics by means of model schedules of growth and model stable populations has been pioneered by Ansley Coale. In a series of articles and books published during the past decade, he and his collaborators have established a paradigm that has become the standard approach of most mathematical demographers. This paradigm is developed in an early study in which Coale and Demeny (1966) present two sets of model (single-region) stable populations that evolve after a long and continued exposure to particular combinations of unchanging schedules of growth. Each population is identified by two nonredundant indices of variation relating to fertility and mortality, respectively, and evolves out of a particular combination of a model life table and an intrinsic rate of growth or gross reproduction rate. The former are referred to as the "growth rate" stable populations; the latter are called the "GRR" stable populations and rely on a model fertility schedule with a given mean age of childbearing $\bar{m}$, which is assumed to be 29 years. Symbolically, the two sets of model stable populations may be expressed as:

1. Growth Rate Stable Populations: $f(e(0),r)$
2. GRR Stable Populations : $g(e(0), GRR)$,

where $e(0)$ is the expectation of life at birth, r is the intrinsic annual rate of growth, and GRR is the gross reproduction rate.

The paradigm introduced by Coale and Demeny may be extended to multiregional populations. In such an extension, a particular model <u>multiregional</u> life table is linked with an intrinsic rate of growth or set of gross reproduction rates. In the former case one must also specify a set of additional indices that relate to spatial distribution, for example, the spatial distribution of births or of people (Rogers, 1975, and Rogers and Willekens, 1975). Symbolically, the two sets of model multiregional stable populations may be expressed as:

1. Growth Rate Multiregional Stable Populations : $f(\underset{\sim}{EXP},r,\underset{\sim}{SRR},\underset{\sim}{\theta})$ or $h(\underset{\sim}{EXP},r,\underset{\sim}{SHA},\underset{\sim}{\theta})$;
2. GRR Multiregional Stable Populations : $g(\underset{\sim}{EXP},\underset{\sim}{GRR},\underset{\sim}{\theta})$,

where $\underset{\sim}{EXP}$ is a diagonal matrix of regional expectations of life at birth ${}_ie(0)$, $\underset{\sim}{SRR}$ is a matrix of stable radix ratios SRR_{ji}; $\underset{\sim}{SHA}$ is a diagonal matrix of stable regional shares SHA_i; $\underset{\sim}{\theta}$ is a matrix of migration levels ${}_j\theta_i$; and GRR is a diagonal matrix of regional gross reproduction rates GRR_i. (Alternatively, we could instead have adopted gross migraproduction rates GMR_{ji} in place of the migration levels ${}_j\theta_i$. In this event the matrix $\underset{\sim}{\theta}$ would be replaced by the matrix $\underset{\sim}{GMR}$.)

Coale and Demeny point out that growth rate stable populations are more useful for analyzing the consequences of various observed intercensal rates of growth, whereas GRR stable populations are more suitable for studies of the impacts of different fertility and mortality levels. An analogous observation may be made with respect to multiregional populations. Growth rate multiregional stable populations are more useful for examining the implications of various observed intercensal rates of growth and regional allocations of total births or people, whereas GRR multiregional stable populations are more convenient for assessing the impacts of different combinations of regional levels of fertility, mortality, and migration.

Tables 4 and 5 set out several specimen model multiregional stable populations which were generated by means of specific combinations of model schedules of fertility, mortality, and migration. The model fertility schedules were obtained by applying Coale and Demeny's (1966) basic age profile, for a mean age of childbearing of 29 years, to different values of GRR_j model mortality schedules were taken from their "WEST" family; and the model migration

TABLE 4

Model Growth Rate Multiregional (Two-Region) Female Stable Populations with Equal Mortality Levels: ${}_1e(0) = {}_2e(0) = 70$ years

		Intrinsic Rate of Growth (r)											
		r = 0.00			r = 0.01			r = 0.02			r = 0.03		
		Region			Region			Region			Region		
Growth Rate Set*		1 + 2	1	2	1 + 2	1	2	1 + 2	1	2	1 + 2	1	2
A.	SHA	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000
${}_1\theta_2 = {}_2\theta_1 = 0.3$	b	0.0143	0.0143	0.0143	0.0203	0.0203	0.0203	0.0276	0.0276	0.0276	0.0358	0.0358	0.0358
$SRR_{12} = SRR_{21} = 1$	Δ	0.0143	0.0143	0.0143	0.0103	0.0103	0.0103	0.0076	0.0076	0.0076	0.0058	0.0058	0.0058
	a	37.92	37.92	37.92	32.82	32.82	32.82	28.16	28.16	28.16	24.11	24.11	24.11
B.	SHA	1.0000	0.5999	0.4001	1.0000	0.5919	0.4081	1.0000	0.5839	0.4162	1.0000	0.5762	0.4238
${}_1\theta_2 = 0.2;\ {}_2\theta_1 = 0.4$	b	0.0143	0.0119	0.0179	0.0203	0.0172	0.0249	0.0276	0.0236	0.0331	0.0358	0.0311	0.0422
$SRR_{12} = SRR_{21} = 1$	Δ	0.0143	0.0119	0.0179	0.0103	0.0072	0.0149	0.0076	0.0036	0.0131	0.0058	0.0011	0.0122
	a	37.92	39.24	35.94	32.82	34.20	30.82	28.16	29.52	26.26	24.11	25.38	22.37

*Parameters under stability: regional share, SHA; birth rate, b; absence rate, Δ; average age, a; stable radix ratio, SRR

TABLE 5

Model GRR Multiregional (Two-Region) Female Stable Populations with Equal Mortality Levels: $_1e(0) = {}_2e(0) = 70$ years

		Gross Reproduction Rate (GRR)								
		$GRR_1 = 1$, $GRR_2 = 1$			$GRR_1 = 2$, $GRR_2 = 1$			$GRR_1 = 3$, $GRR_2 = 1$		
		Region			Region			Region		
GRR Set*		1 + 2	1	2	1 + 2	1	2	1 + 2	1	2
A.	SHA	1.0000	0.5000	0.5000	1.0000	0.6168	0.3832	1.0000	0.6801	0.3199
$_1\theta_2 = {}_2\theta_1 = 0.3$	b	0.0131	0.0131	0.0131	0.0232	0.0282	0.0152	0.0331	0.0409	0.0165
	Δ	0.0153	0.0153	0.0153	0.0091	0.0140	0.0010	0.0063	0.0141	-0.0103
	r	-0.0022	----	----	0.0142	----	----	0.0268	----	----
	a	39.08	39.08	39.08	30.80	28.84	33.96	25.34	23.06	30.17
	SRR_{21}	1.000	----	----	0.335	----	----	0.189	----	----
B.	SHA	1.0000	0.6667	0.3333	1.0000	0.7556	0.2444	1.0000	0.7976	0.2024
$_1\theta_2 = 0.2$; $_2\theta_1 = 0.4$	b	0.0131	0.0131	0.0131	0.0254	0.0286	0.0156	0.0363	0.0413	0.0167
	Δ	0.0153	0.0153	0.0153	0.0082	0.0114	-0.0016	0.0057	0.0107	-0.0139
	r	-0.0022	----	----	0.0172	----	----	0.0306	----	----
	a	39.08	39.08	39.08	29.42	28.25	33.04	23.88	22.56	29.09
	SRR_{21}	0.500	----	----	0.176	----	----	0.103	----	----
C.		$GRR_1 = 1$, $GRR_2 = 1$			$GRR_1 = 1$, $GRR_2 = 2$			$GRR_1 = 1$, $GRR_2 = 3$		
$_1\theta_2 = 0.2$; $_2\theta_1 = 0.4$	SHA	1.0000	0.6667	0.3333	1.0000	0.5391	0.4609	1.0000	0.4550	0.5450
	b	0.0131	0.0131	0.0131	0.0208	0.0148	0.0277	0.0293	0.0161	0.0404
	Δ	0.0153	0.0153	0.0153	0.0101	0.0042	0.0171	0.0071	-0.0061	0.0182
	r	-0.0022	----	----	0.0106	----	----	0.0222	----	----
	a	39.08	39.08	39.08	32.52	35.08	29.52	27.22	31.52	23.63
	SRR_{21}	0.500	----	----	1.603	----	----	3.010	----	----

*Parameters under stability: Regional share, SHA; birth rate, b; Absence rate, Δ; average age, a; stable radix ratio, SRR schedules were calculated using our own "AVERAGE" regression equations set out in Appendix Table D.2. Each of the populations in the two tables may be expressed symbolically by any one of the three forms listed earlier. For example, the first multiregional stable population in Table 5 may be expressed as a function of

$$\underset{\sim}{EXP} = \begin{bmatrix} 70 & 0 \\ 0 & 70 \end{bmatrix} \quad r = -0.0022 \quad \underset{\sim}{SRR} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \underset{\sim}{\theta} = \begin{bmatrix} 7/10 & 3/10 \\ 3/10 & 7/10 \end{bmatrix} ,$$

in which $\underset{\sim}{SRR}$ could be replaced by

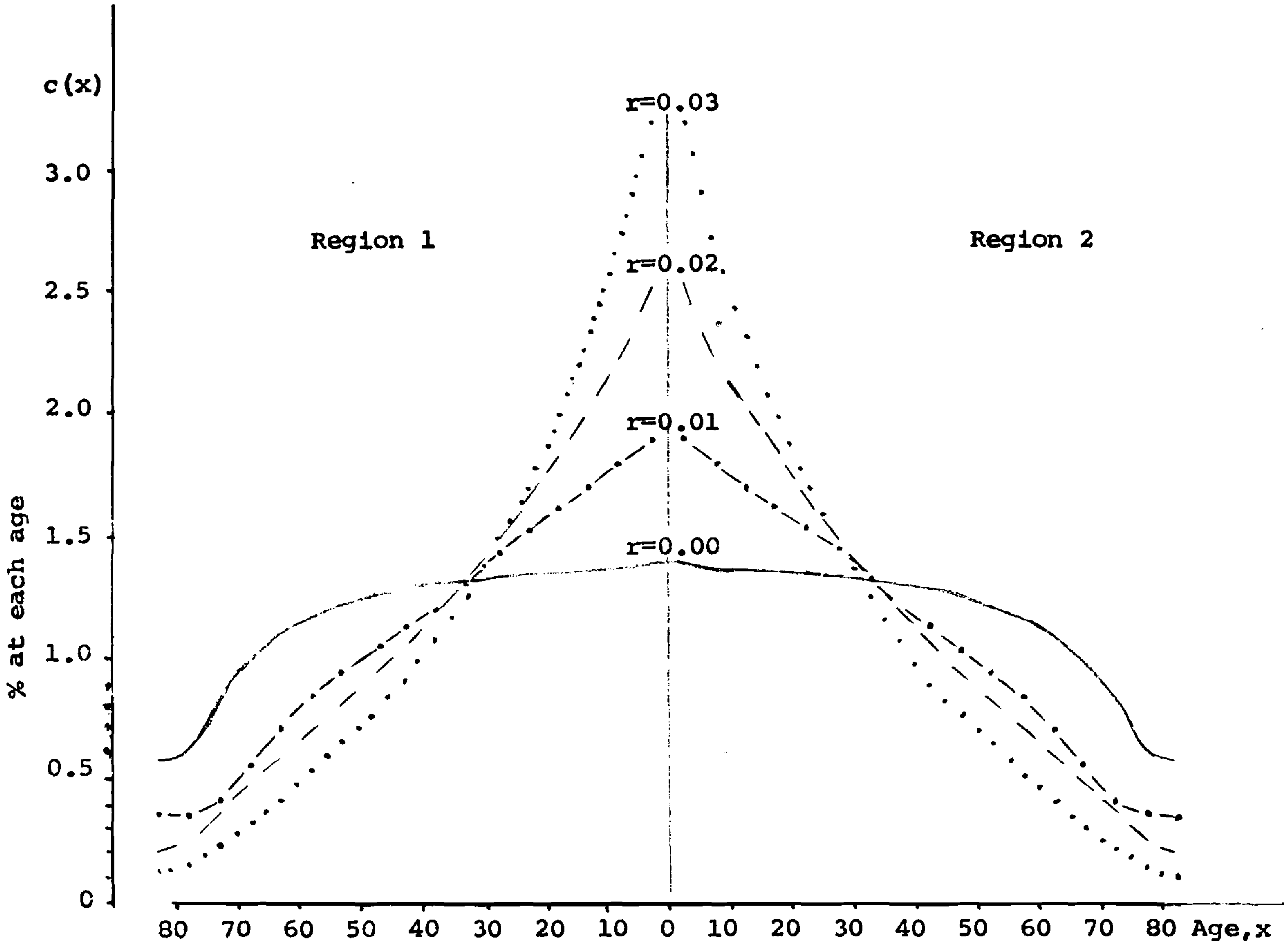
$$\underset{\sim}{SHA} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} .$$

Alternatively, the same population also may be described as a function of the same $\underset{\sim}{EXP}$ and $\underset{\sim}{\theta}$ matrices but with r and $\underset{\sim}{SRR}$ (or $\underset{\sim}{SHA}$) replaced by

$$\underset{\sim}{GRR} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

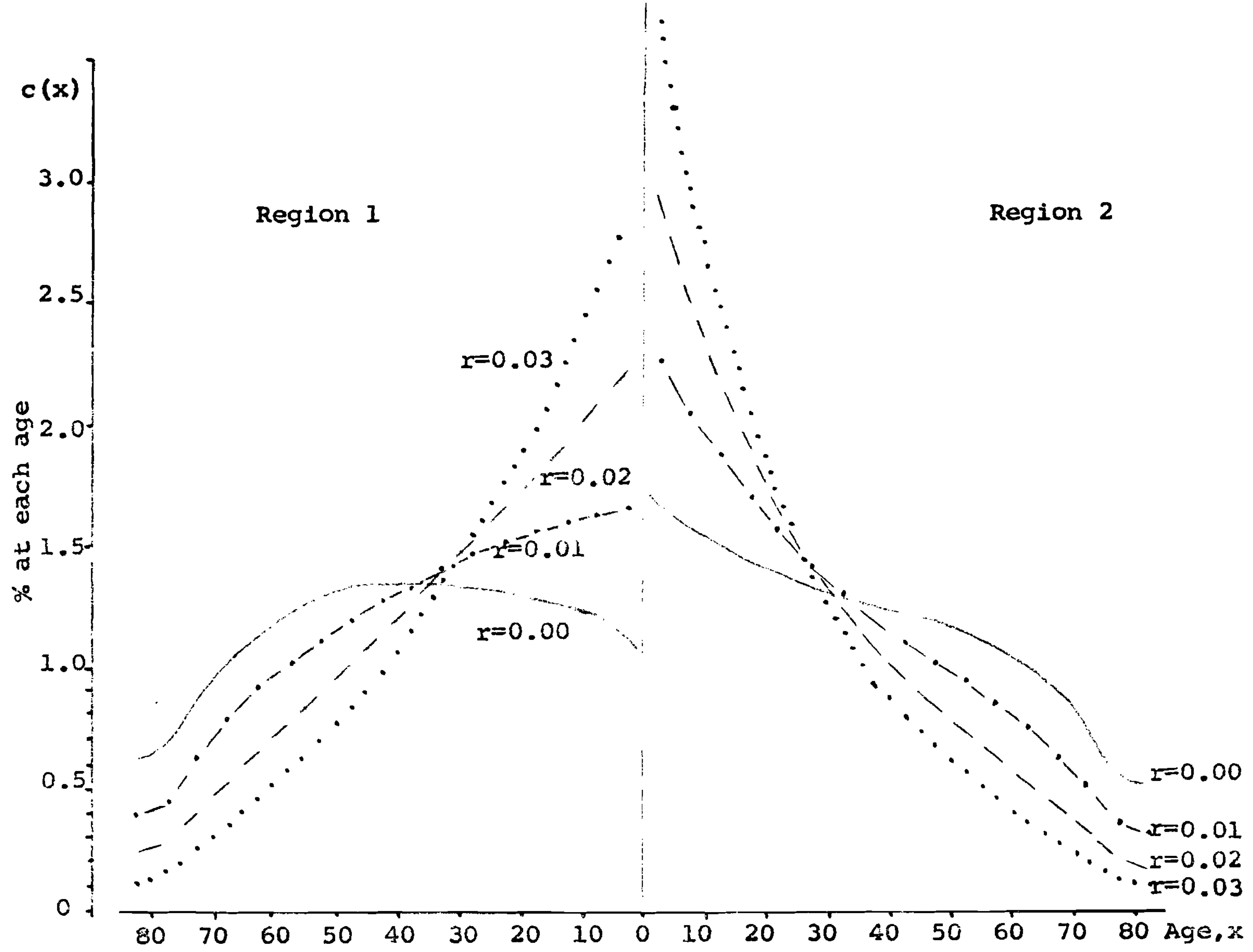
4.2 Dynamics of Model Multiregional Stable Populations

Model multiregional stable populations readily reveal the long-run consequences of particular changes in fertility, mortality, and migration levels. For example, consider several of the more interesting aspects of population dynamics that are manifested in the stable populations presented in Tables 4 and 5 and illustrated in Figures 29 and 30 below. First, identical schedules of regional fertility and mortality produce identical stable regional age compositions. The stable regional shares of such populations, however, will vary inversely with the ratio of their respective migration levels. Second, higher values of the



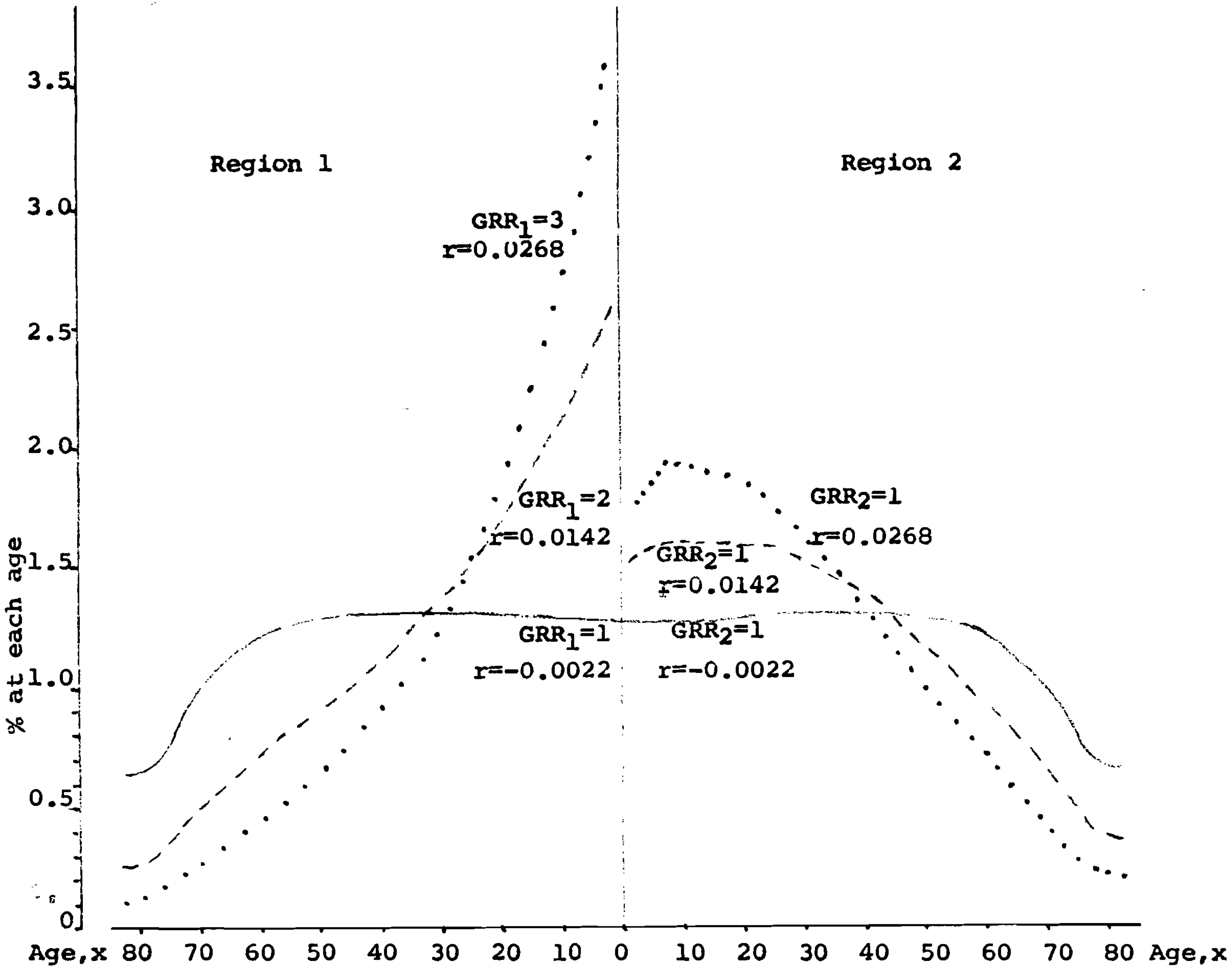
A. Identical regional mortality levels: ${}_1e(0) = {}_2e(0) = 70$; identical regional migration levels: ${}_1\theta_2 = {}_2\theta_1$; identical stable radix ratios: $SRR_{12} = SRR_{21} = 1$.

Figure 29: Regional Age Compositions of Model Multiregional (Two-region) Growth Rate Stable Populations



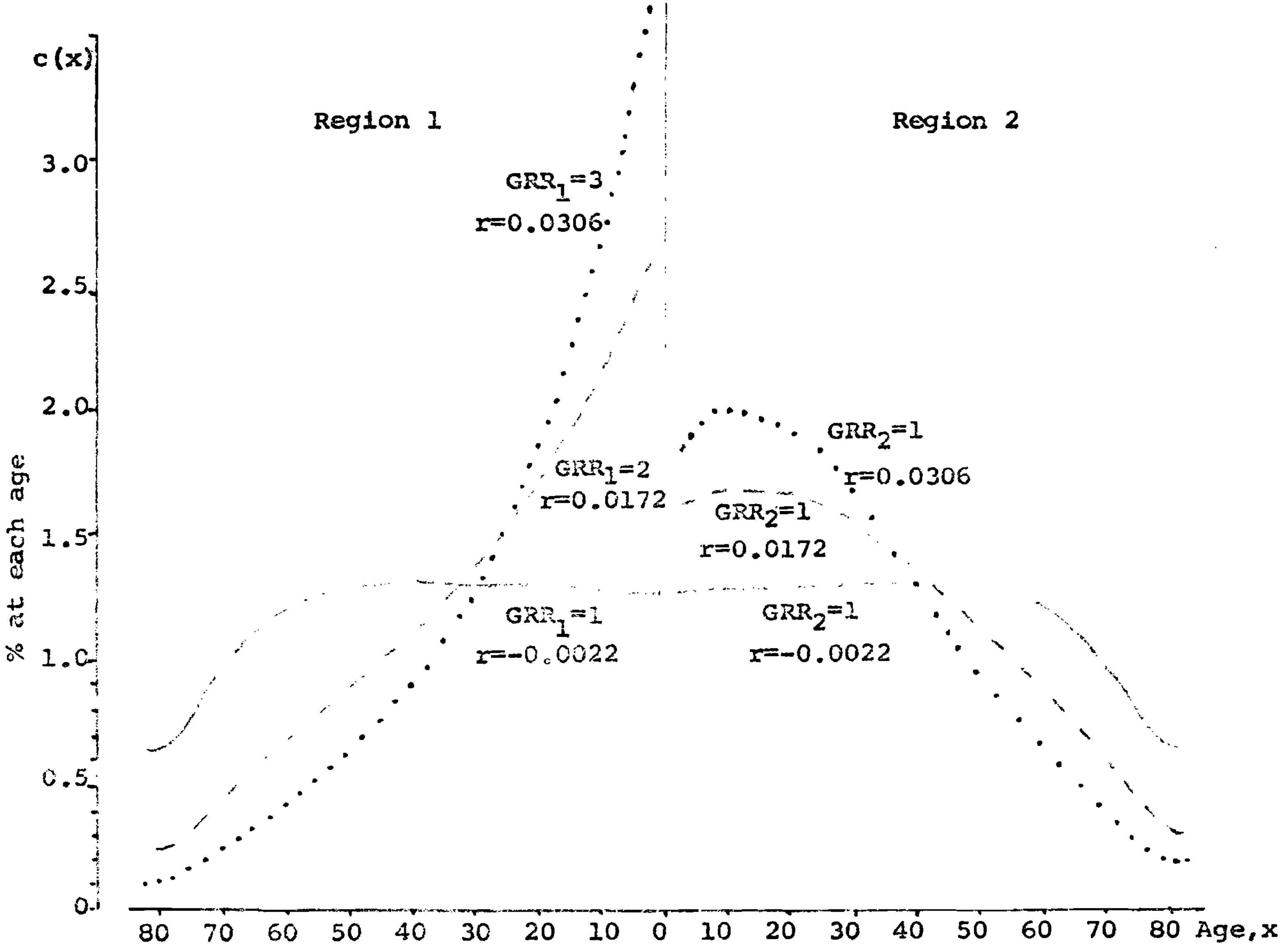
B. Identical regional mortality levels: ${}_1e(0) = {}_2e(0) = 70$; different regional migration levels: ${}_1\theta_2 = 14/70 = 0.2$, ${}_2\theta_1 = 28/70 = 0.4$; identical stable radix ratios: $SRR_{12} = SRR_{21} = 1$.

Figure 29: Regional Age Compositions of Model Multiregional (Two-region) Growth Rate Stable Populations



A. Identical regional mortality levels: ${}_1e(0) = {}_2e(0) = 70$; identical regional migration levels: ${}_1\theta_2 = {}_2\theta_1 = 0.3$; different regional fertility levels: $GRR_1 = 1,2,3,$; $GRR_2 = 1$.

Figure 30: Regional Age Compositions of Model Multiregional (Two-region) GRR Stable Populations



B. Identical regional mortality levels: ${}_{10}e(0) = {}_{20}e(0) = 70$; different regional migration levels: ${}_1\theta_2 = 14/70 = 0.2$, ${}_2\theta_1 = 28/70 = 0.4$; different regional fertility levels: $GRR_1 = 1,2,3$; $GRR_2 = 1$.

Figure 30: Regional Age Compositions of Model Multiregional (Two-region) GRR Stable Populations

intrinsic growth rate lead to stable (regional) populations that taper more rapidly with age and, in consequence, include a higher proportion of the population below every age. Third, fertility affects not only the rate of growth of a stable regional distribution. Fourth, mortality and migration schedules affect the form of the stable regional age compositions and the stable regional shares in an obvious way, and any idiosyncracies in the age patterns of such schedules will be reflected in the age patterns of the corresponding regional populations.

Somewhat surprising is the relative insensitivity of regional age compositions and birth rates to changes in migration levels. For example, consider the case of unequal migration levels with $GRR_1 = 1$, $GRR_2 = 3$ and with $GRR_1 = 3$, $GRR_2 = 1$, respectively. In the first case the region with the larger (by a factor of 2) outmigration has the higher fertility level; in the second case the situation is reversed. Yet in both instances the population of the region with the higher fertility level has an average age of approximately 23 years and exhibits a birth rate of approximately 41 per 1000. This insensitivity to migration behavior does not extend to aggregate systemwide measures, however. For the same example, the intrinsic growth rate and systemwide birth rate are considerably lower in the first case than in the second; the higher fertility region, however, assumes a stable regional share of only 54 percent in the first case but receives 80 percent in the second.

Finally, it is important to underscore the powerful influence that past patterns of fertility, mortality, and migration play in the determination of present regional age compositions and shares, inasmuch as the latter arise out of a history of regional births, deaths, and internal migration. For example, a region experiencing high levels of fertility will have a relatively younger population, but if this region also is the origin of high levels of outmigration, a large proportion of its young adults will move to other regions, producing a higher growth rate in the destination regions while lowering the average age of its own population. This suggests that inferences made about fertility, say, on the basis of a model that ignores internal migration may be seriously in error. For example, Figure 30A illustrates the significant impact on the ultimate stable age composition and regional share of region 2 that is occasioned by a doubling and tripling of fertility levels in region 1 while holding everything else constant. The mean age of the population in region 2 declines by 5.1 and 8.9 years, respectively, while its regional share decreases by 24 percent in the first instance and by 36 percent in the second.

5. Conclusion

It has been said that models are always based on assumptions which are known to be false, and that this is what differentiates them from the phenomena they purport to describe. Demographic models are no exception to this dictum and all population projections, for example, and generated on the basis of assumptions that are almost certain to be violated. Yet mere mortals cannot foresee the future, and important insights about the dynamics of human populations are revealed by relatively simple linear models which are based on rather restrictive assumptions. Such models can be used to structure data collection efforts; they often generate hypotheses for empirical confirmation; they can suggest potential policy problems and issues; and they provide indices useful for comparative studies (Keyfitz, 1971).

This study has examined regularities in empirical migration schedules and has applied model schedules in combination with demographic growth models to develop model multiregional stable populations that illuminate important aspects of spatial population dynamics. Much of the analysis has been exploratory and most of the results are tentative. Substantial further research appears to be both warranted and necessary. A particularly rewarding direction for such research lies in the development of alternative methods for summarizing the regularities exhibited by empirical migration schedules.

This study has focused on what might be called the "mortality" approach toward the construction of model migration schedules. It may well be true that the "fertility" approach, with its focus on gross migraproduction rates classified by various mean ages of migration, may be a more robust alternative.

Consider, for example, the decomposition of a typical migration profile into three broad sets of age groups: 1.) the pre-labor force migrants (0-14 years old, say), 2.) the labor force migrants (15-64 years old), and 3.) the post-labor force migrants (65 years and over). Migration by the first group may be related to levels of fertility, in addition to the usual association with the migration levels of parental age groups. Migration by the labor force age groups may be related to indices such as labor force participation rates and ages of entry and exit from the labor force. Finally, retirement migration may be expressed as a function of variables such as climate and the general quality and quantity of social services. Such a partitioning suggests an approach that in many respects is analogous to the one adopted by Coale and Trussell (1974) for the development of model fertility schedules. It will be developed further in a forthcoming paper.

ACKNOWLEDGEMENT: The authors gratefully acknowledge the computational assistance of Frans Willekens.

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APPENDIX

Contents

TABLE A.1

Projected Annual Regional Rates of Growth $[r_i(t)]$: Total United States Population

A. <u>Base Year: 1958</u>

Region i Time t	1. Northeast	2. North Central	3. South	4. West	Total
1958	0.008484	0.011421	0.016831	0.027227	0.014777
1968	0.009335	0.013217	0.017296	0.026612	0.015896
1978	0.012085	0.015817	0.018111	0.026624	0.017776
1988	0.014067	0.017446	0.019041	0.026256	0.019060
1989	0.016221	0.019284	0.020158	0.026261	0.020483
2008	0.018264	0.020653	0.021190	0.025739	0.021574
Stability	0.021810				

B. <u>Base Year: 1968</u>

Region i Time t	1. Northeast	2. North Central	3. South	4. West	Total
1968	0.003808	0.006633	0.011606	0.014698	0.008890
1978	0.005500	0.008549	0.011317	0.014101	0.009734
1988	0.004323	0.006853	0.008900	0.011126	0.007756
1998	0.004663	0.007056	0.008621	0.010408	0.007703
2008	0.005085	0.006953	0.008088	0.009466	0.007435
2018	0.004555	0.006175	0.007204	0.008380	0.006630
Stability	0.005769				

TABLE A.2

Observed and Projected Regional Shares [$SHA_i(t)$]: Total United States Population

A. Base Year: 1958

Region i Time t	1. Northeast	2. North Central	3. South	4. West	Total
1958	0.2503	0.2955	0.3061	0.1481	1.0000
1968	0.2347	0.2861	0.3122	0.1670	1.0000
1978	0.2202	0.2792	0.3157	0.1850	1.0000
1988	0.2084	0.2740	0.3164	0.2012	1.0000
1998	0.1986	0.2699	0.3161	0.2154	1.0000
2008	0.1907	0.2668	0.3150	0.2275	1.0000
Stability	0.1443	0.2525	0.3061	0.2971	1.0000

B. Base Year: 1968

Region i Time t	1. Northeast	2. North Central	3. South	4. West	Total
1968	0.2413	0.2784	0.3090	0.1713	1.0000
1978	0.2306	0.2728	0.3198	0.1768	1.0000
1988	0.2216	0.2699	0.3243	0.1841	1.0000
1998	0.2143	0.2676	0.3280	0.1901	1.0000
2008	0.2082	0.2660	0.3307	0.1950	1.0000
2018	0.2035	0.2647	0.3328	0.1989	1.0000
Stability	0.1764	0.2617	0.3425	0.2194	1.0000

TABLE A.3

Observed and Projected Regional Age Compositions [$C_i(x,t)$]: Total United States Population

A. Base Year: 1958

Region i / Age x	1. Northeast			2. North Central			3. South			4. West		
	1958	2008	Stab.	1958	2008	Stab.	1958	2008	Stab.	1958	2008	Stab.
0-4	0.1024	0.1267	0.1287	0.1142	0.1375	0.1380	0.1170	0.1348	0.1353	0.1144	0.1365	0.1345
5-9	0.0913	0.1131	0.1146	0.1016	0.1207	0.1211	0.1072	0.1184	0.1188	0.1026	0.1195	0.1184
10-14	0.0820	0.1015	0.1025	0.0878	0.1067	0.1073	0.0976	0.1062	0.1067	0.0891	0.1063	0.1064
15-19	0.0672	0.0912	0.0919	0.0704	0.0941	0.0947	0.0811	0.0952	0.0957	0.0712	0.0955	0.0957
20-24	0.0591	0.0835	0.0844	0.0613	0.0846	0.0850	0.0679	0.0843	0.0846	0.0653	0.0855	0.0844
25-29	0.0636	0.0766	0.0778	0.0635	0.0763	0.0763	0.0655	0.0745	0.0746	0.0676	0.0762	0.0742
30-34	0.0706	0.0688	0.0703	0.0675	0.0672	0.0673	0.0669	0.0663	0.0664	0.0709	0.0680	0.0661
35-39	0.0736	0.0611	0.0630	0.0688	0.0584	0.0592	0.0682	0.0585	0.0590	0.0741	0.0598	0.0589
40-44	0.0698	0.0542	0.0559	0.0641	0.0506	0.0519	0.0620	0.0511	0.0520	0.0667	0.0518	0.0523
45-49	0.0652	0.0498	0.0489	0.0598	0.0452	0.0452	0.0577	0.0454	0.0455	0.0603	0.0458	0.0460
50-54	0.0593	0.0436	0.0421	0.0540	0.0397	0.0388	0.0504	0.0393	0.0393	0.0516	0.0394	0.0399
55-59	0.0525	0.0372	0.0351	0.0485	0.0341	0.0327	0.0413	0.0346	0.0334	0.0443	0.0338	0.0339
60-64	0.0454	0.0303	0.0284	0.0418	0.0274	0.0267	0.0350	0.0289	0.0278	0.0372	0.0272	0.0281
65-69	0.0382	0.0216	0.0219	0.0362	0.0195	0.0208	0.0313	0.0213	0.0222	0.0321	0.0190	0.0224
70-74	0.0281	0.0154	0.0157	0.0274	0.0141	0.0153	0.0227	0.0154	0.0166	0.0244	0.0136	0.0167
75-79	0.0173	0.0118	0.0100	0.0178	0.0108	0.0101	0.0145	0.0153	0.0111	0.0156	0.0101	0.0111
80-84	0.0091	0.0081	0.0055	0.0096	0.0075	0.0058	0.0977	0.0078	0.0064	0.0082	0.0067	0.0065
85+	0.0052	0.0056	0.0032	0.0055	0.0056	0.0037	0.0043	0.0064	0.0046	0.0046	0.0052	0.0044
Total	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A.3 - Continued

Observed and Projected Regional Age Compositions $[C_i(x,t)]$: Total United States Population

B. Base Year: 1968

Region i / Age x	1. Northeast			2. North Central			3. South			4. West		
	1968	2018	Stab.	1968	2018	Stab.	1968	2018	Stab.	1968	2018	Stab.
0-4	0.0814	0.0854	0.0860	0.0855	0.0885	0.0883	0.0858	0.0840	0.0834	0.0844	0.0834	0.0820
5-9	0.0938	0.0834	0.0829	0.1007	0.0858	0.0847	0.0992	0.0811	0.0799	0.0986	0.0803	0.0782
10-14	0.0971	0.0807	0.0800	0.1055	0.0831	0.0817	0.1036	0.0793	0.0777	0.1022	0.0783	0.0760
15-19	0.0881	0.0755	0.0762	0.0955	0.0781	0.0779	0.0967	0.0766	0.0758	0.0942	0.0764	0.0752
20-24	0.0754	0.0713	0.0737	0.0775	0.0729	0.0744	0.0840	0.0722	0.0729	0.0857	0.0737	0.0740
25-29	0.0646	0.0704	0.0722	0.0648	0.0707	0.0720	0.0663	0.0690	0.0698	0.0712	0.0712	0.0713
30-34	0.0546	0.0712	0.0701	0.0551	0.0710	0.0692	0.0566	0.0691	0.0674	0.0597	0.0712	0.0685
35-39	0.0549	0.0705	0.0674	0.0532	0.0702	0.0662	0.0548	0.0690	0.0650	0.0565	0.0705	0.0658
40-44	0.0618	0.0653	0.0645	0.0579	0.0646	0.0629	0.0577	0.0645	0.0624	0.0590	0.0653	0.0629
45-49	0.0634	0.0571	0.0610	0.0585	0.0556	0.0594	0.0573	0.0562	0.0593	0.0602	0.0566	0.0597
50-54	0.0595	0.0508	0.0569	0.0541	0.0485	0.0553	0.0520	0.0490	0.0557	0.0534	0.0490	0.0561
55-59	0.0535	0.0558	0.0516	0.0487	0.0540	0.0502	0.0475	0.0550	0.0517	0.0464	0.0549	0.0518
60-64	0.0450	0.0526	0.0451	0.0419	0.0513	0.0441	0.0422	0.0538	0.0470	0.0386	0.0527	0.0467
65-69	0.0368	0.0421	0.0377	0.0338	0.0411	0.0372	0.0351	0.0448	0.0412	0.0307	0.0427	0.0407
70-74	0.0293	0.0298	0.0297	0.0272	0.0284	0.0297	0.0262	0.0329	0.0337	0.0238	0.0315	0.0334
75-79	0.0204	0.0188	0.0212	0.0200	0.0178	0.0216	0.0177	0.0205	0.0252	0.0169	0.0200	0.0251
80-84	0.0119	0.0103	0.0131	0.0122	0.0100	0.0137	0.0103	0.0118	0.0164	0.0105	0.0114	0.0165
85+	0.0077	0.0089	0.0108	0.0080	0.0084	0.0114	0.0069	0.0111	0.0154	0.0081	0.0108	0.0160
Total	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE B.1

Expectations of Life at Birth and Migration Levels by Region of Residence and Region of Birth: Total United States Population, 1958

A. Expectations of Life at Birth: ${}_ie_j(0)$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	50.90	4.49	8.88	5.50	69.76
2. North Central	3.18	48.45	9.10	9.60	70.32
3. South	4.58	7.52	49.21	7.67	68.98
4. West	3.18	6.60	8.95	51.22	69.94

B. Migration Levels: ${}_i\theta_j$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	0.7295	0.0643	0.1273	0.0788	1.00
2. North Central	0.0452	0.6889	0.1294	0.1365	1.00
3. South	0.0664	0.1091	0.7134	0.1111	1.00
4. West	0.0454	0.0944	0.1279	0.7322	1.00

TABLE B.2

Expectations of Life at Birth and Migration Levels by Region of Residence and Region of Birth: Total United States Population, 1968

A. Expectations of Life at Birth: ${}_ie_j(0)$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	50.61	5.06	10.00	5.15	70.83
2. North Central	3.69	49.19	10.37	7.75	70.99
3. South	4.81	7.45	51.39	6.63	70.28
4. West	3.87	7.71	11.20	48.53	71.31

B. Migration Levels: ${}_i\theta_j$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	0.7146	0.0714	0.1412	0.0738	1.00
2. North Central	0.0519	0.6929	0.1460	0.1092	1.00
3. South	0.0685	0.1060	0.7313	0.0942	1.00
4. West	0.0543	0.1081	0.1570	0.6806	1.00

TABLE B.3

Expectations of Life at Birth and Migration Levels by Region of Residence and Region of Birth: Male United States Population, 1968

A. Expectations of Life at Birth: ${}_ie_j(0)$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	47.15	5.05	9.77	5.18	67.15
2. North Central	3.55	46.19	9.99	7.54	67.28
3. South	4.60	7.14	48.02	6.54	66.30
4. West	3.70	7.25	10.57	46.18	67.70

B. Migration Levels: ${}_i\theta_j$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	0.7022	0.0752	0.1456	0.0771	1.00
2. North Central	0.0528	0.6865	0.1485	0.1121	1.00
3. South	0.0694	0.1077	0.7243	0.0986	1.00
4. West	0.0547	0.1071	0.1562	0.6821	1.00

TABLE B.4

Expectations of Life at Birth and Migration Levels by Region of Residence and Region of Birth: Female United States Population, 1968

A. Expectations of Life at Birth: ${}_ie_j(0)$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	54.13	5.08	10.11	5.25	74.56
2. North Central	3.76	52.14	10.48	8.05	74.44
3. South	5.06	7.88	54.53	6.93	74.40
4. West	3.90	7.94	11.32	52.41	75.57

B. Migration Levels: ${}_i\theta_j$

Region of Birth	Region of Residence				
	1	2	3	4	Total
1. Northeast	0.7260	0.0681	0.1356	0.0704	1.00
2. North Central	0.0506	0.7005	0.1408	0.1081	1.00
3. South	0.0680	0.1060	0.7328	0.0931	1.00
4. West	0.0516	0.1051	0.1497	0.6936	1.00

TABLE B.5

Expectations of Life at Birth and Migration Levels by Division of Residence and Division of Birth: Total United States Population, 1958.

A. Expectations of Life at Birth: ${}_ie_j(0)$

Division of Birth	Division of Residence									
	1	2	3	4	5	6	7	8	9	Total
1. New England	4.75	6.16	3.03	1.04	6.46	0.82	1.52	1.16	5.06	70.00
2. Middle Atlantic	2.50	48.71	3.58	0.89	6.70	0.87	1.31	1.05	4.07	69.68
3. East North Central	0.89	2.56	47.14	2.61	5.16	2.05	2.08	1.85	5.82	70.17
4. West North Central	0.79	1.75	6.32	39.56	3.45	1.20	3.98	4.13	9.57	70.75
5. South Atlantic	1.58	5.16	4.82	1.28	45.39	2.57	2.31	1.23	4.46	68.81
6. East South Central	0.77	2.27	8.94	1.68	8.36	37.48	3.81	1.28	4.25	68.83
7. West South Central	0.76	1.76	3.85	3.16	3.98	2.25	41.90	3.39	8.48	69.54
8. Mountain	0.97	2.00	3.87	3.89	3.47	1.17	5.28	33.22	15.90	69.78
9. Pacific	1.03	2.10	3.35	2.55	3.72	1.08	3.56	4.19	48.65	70.21

TABLE B.5 - Continued

Expectations of Life at Birth and Migration Levels by Division of Residence and Division of Birth: Total United States Population, 1958

B. Migration Levels: ${}_{i}\theta_{j}$

Division of Birth	Division of Residence									
	1	2	3	4	5	6	7	8	9	Total
1. New England	0.6393	0.0880	0.0433	0.0149	0.0923	0.0117	0.0217	0.0166	0.0723	1.00
2. Middle Atlantic	0.0357	0.6991	0.0514	0.0128	0.0962	0.0125	0.0188	0.0151	0.0584	1.00
3. East North Central	0.0127	0.0365	0.6718	0.0372	0.0735	0.0292	0.0296	0.0264	0.0829	1.00
4. West North Central	0.0112	0.0248	0.0893	0.5592	0.0488	0.0170	0.0563	0.0584	0.1353	1.00
5. South Atlantic	0.0230	0.0750	0.0700	0.0186	0.6596	0.0373	0.0336	0.0179	0.0648	1.00
6. East South Central	0.0112	0.0330	0.1299	0.0244	0.1215	0.5445	0.0554	0.0186	0.0617	1.00
7. West South Central	0.0109	0.0253	0.0554	0.0454	0.0572	0.0324	0.6025	0.0487	0.1219	1.00
8. Mountain	0.0139	0.0287	0.0555	0.0557	0.0497	0.0168	0.0757	0.4761	0.2279	1.00
9. Pacific	0.0147	0.0299	0.0477	0.0363	0.0530	0.0154	0.0507	0.0597	0.6929	1.00

TABLE C.1

Gross Migraproduction Rate and Mean Age of Migration, by Region of Origin and Region of Destination: Total United States Population, 1958

A. Gross Migraproduction Rate: GMR_{ij}

Region of Origin	Region of Destination				
	1	2	3	4	Total
1. Northeast	-	0.1202	0.3168	0.1532	0.5902
2. North Central	0.0891	-	0.3201	0.3289	0.7381
3. South	0.1504	0.2511	-	0.2299	0.6314
4. West	0.0887	0.2167	0.2819	-	0.5873

B. Mean Age of Migration: $\bar{n}_{ij}$

Region of Origin	Region of Destination			
	1	2	3	4
1. Northeast	-	26.99	33.46	29.43
2. North Central	28.15	-	32.16	30.54
3. South	28.59	27.77	-	27.27
4. West	27.73	30.03	27.61	-

TABLE C.2

Gross Migraproduction Rate and Mean Age of Migration, by Region of Origin and Region of Destination: Total United States Population, 1968

A. Gross Migraproduction Rate: GMR_{ij}

Region of Origin	Region of Destination				
	1	2	3	4	Total
1. Northeast	-	0.1352	0.3524	0.1480	0.6356
2. North Central	0.1022	-	0.3540	0.2638	0.7200
3. South	0.1486	0.2343	-	0.1948	0.5777
4. West	0.1082	0.2504	0.3476	-	0.7062

B. Mean Age of Migration: $\bar{n}_{ij}$

Region of Origin	Region of Destination			
	1	2	3	4
1. Northeast	-	26.14	34.98	29.34
2. North Central	26.98	-	33.00	31.13
3. South	27.64	27.27	-	26.52
4. West	26.64	28.68	27.50	-

TABLE C.3

Gross Migraproduction Rate and Mean Age of Migration, by Region of Origin and Region of Destination: Male United States Population, 1968

A. Gross Migraproduction Rate: GMR_{ij}

Region of Origin	Region of Destination				
	1	2	3	4	Total
1. Northeast	-	0.1457	0.3849	0.1595	0.6901
2. North Central	0.1063	-	0.3790	0.2742	0.7595
3. South	0.1534	0.2434	-	0.2077	0.6045
4. West	0.1106	0.2515	0.3607	-	0.7228

B. Mean Age of Migration: $\bar{n}_{ij}$

Region of Origin	Region of Destination			
	1	2	3	4
1. Northeast	-	25.44	34.75	28.48
2. North Central	26.33	-	32.71	30.13
3. South	26.78	26.82	-	25.96
4. West	25.83	27.92	27.27	-

TABLE C.4

Gross Migraproduction Rate and Mean Age of Migration, by Region of Origin and Region of Destination: Female United States Population, 1968

A. Gross Migraproduction Rate: GMR_{ij}

Region of Origin	Region of Destination				
	1	2	3	4	Total
1. Northeast	-	0.1258	0.3253	0.1377	0.5888
2. North Central	0.0978	-	0.3296	0.2526	0.6800
3. South	0.1462	0.2296	-	0.1853	0.5611
4. West	0.1005	0.2374	0.3186	-	0.6565

B. Mean Age of Migration: $\bar{n}_{ij}$

Region of Origin	Region of Destination			
	1	2	3	4
1. Northeast	-	26.80	35.53	30.28
2. North Central	27.50	-	33.46	32.12
3. South	28.33	27.60	-	27.05
4. West	27.37	29.31	27.76	-

TABLE C.5

Gross Migraproduction Rate and Mean Age of Migration, by Division of Origin and Division of Destination: Total United States Population, 1958

A. Gross Migraproduction Rate: GMR_{ij}

Division of Origin	Division of Destination									
	1	2	3	4	5	6	7	8	9	Total
1. New England	-	0.2194	0.0801	0.0252	0.2528	0.0180	0.0361	0.0307	0.1520	0.8143
2. Middle Atlantic	0.0900	-	0.1050	0.0196	0.2699	0.0200	0.0298	0.0290	0.1153	0.6787
3. East North Central	0.0243	0.0760	-	0.0940	0.2060	0.0758	0.0564	0.0651	0.1818	0.7794
4. West North Central	0.0201	0.0424	0.2374	-	0.1018	0.0348	0.1430	0.1880	0.3365	1.1040
5. South Atlantic	0.0535	0.1911	0.1690	0.0347	-	0.1044	0.0699	0.0343	0.1223	0.7711
6. East South Central	0.0177	0.0548	0.3521	0.0484	0.3393	-	0.1488	0.0321	0.1031	1.0963
7. West South Central	0.0192	0.0428	0.1098	0.1182	0.1178	0.0904	-	0.1405	0.2711	0.9098
8. Mountain	0.0296	0.0574	0.1296	0.1825	0.1039	0.0357	0.2333	-	0.7560	1.5280
9. Pacific	0.0300	0.0592	0.0968	0.0885	0.1074	0.0295	0.1152	0.1864	-	0.7130

TABLE C.5 - Continued

Gross Migraproduction Rate and Mean Age of Migration, by Division of Origin and Division of Destination: Total United States Population, 1958

B. Mean Age of Migration: $\bar{n}_{ij}$

Division of Origin	Division of Destination								
	1	2	3	4	5	6	7	8	9
1. New England	-	28.91	25.73	25.57	34.05	24.26	23.51	27.06	28.42
2. Middle Atlantic	29.04	-	27.54	26.17	35.40	24.97	25.53	29.10	30.13
3. East North Central	26.42	29.39	-	29.03	36.71	28.31	27.78	31.62	30.89
4. West North Central	24.56	27.43	28.47	-	30.57	27.44	28.49	29.41	29.95
5. South Atlantic	28.38	29.94	29.21	27.19	-	28.66	26.10	27.61	27.15
6. East South Central	24.03	26.18	27.15	27.06	28.65	-	27.95	27.61	26.32
7. West South Central	24.14	26.21	26.54	28.07	26.50	28.19	-	28.15	26.98
8. Mountain	25.69	28.19	29.66	31.43	28.66	27.05	29.55	-	30.97
9. Pacific	25.74	28.89	29.27	30.75	27.01	25.51	27.64	29.86	-

TABLE D.1

Regression Coefficients for Obtaining Model Probabilities of Migration: Four Region Total Population, 1958

	Average		$\bar{n}_{ij} \leq 28$ years		$\bar{n}_{ij} > 28$ years	
Age	ß	r^2	ß	r^2	ß	r^2
0	0.17392	0.94	0.18272	0.96	0.16829	0.94
5	0.13460	0.95	0.13706	0.95	0.13303	0.95
10	0.15736	0.86	0.14784	0.95	0.16346	0.84
15	0.30757	0.93	0.29658	0.94	0.31461	0.93
20	0.32271	0.72	0.35190	0.90	0.30404	0.61
25	0.23251	0.96	0.23452	0.99	0.23122	0.95
30	0.17897	0.95	0.18026	0.95	0.17814	0.95
35	0.12912	0.95	0.12616	0.95	0.13101	0.95
40	0.09790	0.93	0.09200	0.95	0.10166	0.94
45	0.07522	0.86	0.06447	0.93	0.08211	0.91
50	0.06838	0.73	0.05240	0.91	0.07860	0.82
55	0.07347	0.63	0.05181	0.89	0.08733	0.74
60	0.08254	0.47	0.04473	0.87	0.10673	0.64
65	0.06086	0.50	0.03505	0.89	0.07737	0.69
70	0.04488	0.58	0.02899	0.86	0.05504	0.77
75	0.03019	0.67	0.02288	0.67	0.03487	0.84
80	0.01342	0.18	0.01305	0.37	0.01366	0.07

TABLE D.2.A

Regression Coefficients for Obtaining Model Probabilities of Migration: Four Region Total Population, 1968

	Average		$\bar{n}_{ij} \leq 28$ years		$\bar{n}_{ij} > 28$ years	
Age	β	r^2	β	r^2	β	r^2
0	0.22002	0.84	0.23718	0.95	0.20529	0.61
5	0.15553	0.89	0.16541	0.94	0.14705	0.78
10	0.15040	0.94	0.14760	0.95	0.15280	0.91
15	0.29195	0.85	0.27014	0.92	0.31068	0.76
20	0.26370	0.72	0.27326	0.79	0.25550	0.42
25	0.20037	0.90	0.21088	0.98	0.19135	0.66
30	0.17907	0.94	0.18563	0.96	0.17343	0.89
35	0.14392	0.96	0.14656	0.96	0.14165	0.96
40	0.10397	0.95	0.10180	0.94	0.10584	0.95
45	0.07378	0.91	0.06680	0.93	0.07977	0.94
50	0.06352	0.76	0.04949	0.92	0.07557	0.82
55	0.07362	0.54	0.04426	0.82	0.09883	0.63
60	0.08320	0.43	0.04008	0.87	0.12022	0.56
65	0.06425	0.47	0.03469	0.89	0.08963	0.59
70	0.04919	0.64	0.03429	0.81	0.06198	0.80
75	0.03951	0.64	0.02817	0.77	0.04924	0.78
80	0.02058	0.63	0.01478	0.72	0.02557	0.75

TABLE D.2.B

Regression Coefficients for Obtaining Model Probabilities of Migration: Four Region Male Population, 1968

	Average		$\bar{n}_{ij} \leq 28$ years		$\bar{n}_{ij} > 28$ years	
Age	β	r^2	β	r^2	β	r^2
0	0.21391	0.82	0.23058	0.94	0.19981	0.54
5	0.15082	0.88	0.16105	0.93	0.14216	0.76
10	0.16065	0.90	0.15183	0.92	0.16811	0.85
15	0.32595	0.79	0.28818	0.94	0.35790	0.69
20	0.28574	0.57	0.30276	0.66	0.27134	0.34
25	0.20713	0.87	0.21991	0.97	0.19633	0.54
30	0.18954	0.94	0.19711	0.96	0.18313	0.86
35	0.15380	0.95	0.15796	0.95	0.15028	0.93
40	0.10802	0.94	0.10764	0.93	0.10833	0.90
45	0.07439	0.92	0.07002	0.91	0.07809	0.94
50	0.05768	0.82	0.04774	0.89	0.06610	0.86
55	0.06393	0.54	0.03825	0.79	0.08567	0.63
60	0.08265	0.40	0.03545	0.78	0.12258	0.52
65	0.06310	0.40	0.02832	0.83	0.09253	0.52
70	0.04363	0.56	0.02724	0.81	0.05749	0.67
75	0.03643	0.56	0.02330	0.79	0.04753	0.65
80	0.02009	0.54	0.01290	0.75	0.02617	0.62

TABLE D.2.C

Regression Coefficients for Obtaining Model Probabilities of Migration: Four Region Female Population, 1968

	Average		$\bar{n}_{ij} \leq 28$ years		$\bar{n}_{ij} > 28$ years	
Age	β	r^2	β	r^2	β	r^2
0	0.22609	0.86	0.22267	0.86	0.22843	0.86
5	0.16045	0.91	0.15787	0.93	0.16221	0.90
10	0.13985	0.95	0.13620	0.97	0.14234	0.95
15	0.25814	0.85	0.25799	0.90	0.25825	0.81
20	0.24275	0.86	0.24930	0.89	0.23826	0.84
25	0.19373	0.93	0.19471	0.94	0.19306	0.93
30	0.16857	0.95	0.16835	0.98	0.16872	0.94
35	0.13404	0.97	0.13354	0.98	0.13439	0.96
40	0.10003	0.95	0.10144	0.94	0.09906	0.95
45	0.07344	0.87	0.07772	0.87	0.07051	0.89
50	0.06952	0.69	0.07537	0.77	0.06552	0.65
55	0.08356	0.53	0.09126	0.64	0.07828	0.45
60	0.08458	0.46	0.09524	0.66	0.07728	0.36
65	0.06615	0.54	0.07212	0.76	0.06207	0.43
70	0.05458	0.68	0.06186	0.84	0.04960	0.60
75	0.04258	0.68	0.04695	0.85	0.03959	0.58
80	0.02134	0.67	0.02315	0.86	0.02010	0.56

TABLE D.3

Regression Coefficients for Obtaining Model Probabilities of Migration: Four Region and Nine Division Total Population, 1968

Age	Nine Divisions Total (1958)		Four Regions Total (1958)	
	β	r^2	β	r^2
0	0.19587	0.93	0.17392	0.94
5	0.15409	0.92	0.13460	0.95
10	0.18129	0.91	0.15736	0.86
15	0.34251	0.95	0.30757	0.93
20	0.35111	0.94	0.32271	0.72
25	0.26246	0.95	0.23251	0.96
30	0.20666	0.93	0.17897	0.95
35	0.15453	0.90	0.12912	0.95
40	0.12148	0.87	0.09790	0.93
45	0.09493	0.84	0.07522	0.86
50	0.08231	0.81	0.06838	0.73
55	0.07948	0.77	0.07347	0.63
60	0.08150	0.61	0.08254	0.47
65	0.06208	0.67	0.06086	0.50
70	0.04859	0.75	0.04488	0.58
75	0.03565	0.81	0.03019	0.67
80	0.01827	0.74	0.01342	0.18

TABLE D.4

Regression Coefficients for Obtaining Model Probabilities of Migration:
Nine Division Total Population, 1958

	Average		$\bar{n}_{ij} \leq 26$ years		$26 < \bar{n}_{ij} \leq 28$ years		$28 < \bar{n}_{ij} \leq 30$ years		$\bar{n}_{ij} > 30$ years	
Age	β	r^2	β	r^2	β	r^2	β	r^2	β	r^2
0	0.19587	0.93	0.13961	0.90	0.18369	0.94	0.19995	0.88	0.20238	0.93
5	0.15409	0.92	0.09678	0.91	0.14116	0.93	0.15397	0.86	0.16578	0.94
10	0.18129	0.91	0.14779	0.92	0.16873	0.89	0.16913	0.86	0.20492	0.95
15	0.34251	0.95	0.30603	0.89	0.33225	0.92	0.33006	0.94	0.36501	0.96
20	0.35111	0.94	0.35183	0.89	0.34029	0.95	0.37131	0.92	0.33630	0.93
25	0.26246	0.95	0.19759	0.92	0.23397	0.96	0.27574	0.94	0.27048	0.95
30	0.20666	0.93	0.13817	0.91	0.17824	0.96	0.21016	0.92	0.22558	0.93
35	0.15453	0.90	0.08934	0.92	0.12782	0.96	0.15328	0.88	0.17737	0.92
40	0.12148	0.87	0.05708	0.91	0.09685	0.93	0.11756	0.84	0.14578	0.90
45	0.09493	0.84	0.03550	0.85	0.06994	0.87	0.08931	0.82	0.12114	0.91
50	0.08231	0.81	0.02669	0.85	0.05617	0.85	0.07471	0.84	0.11132	0.93
55	0.07948	0.77	0.02853	0.85	0.05046	0.91	0.06974	0.88	0.11264	0.87
60	0.08150	0.61	0.02491	0.84	0.04154	0.92	0.06661	0.85	0.12816	0.64
65	0.06208	0.67	0.02045	0.88	0.03272	0.91	0.05252	0.89	0.09484	0.73
70	0.04859	0.75	0.01740	0.89	0.02792	0.89	0.04344	0.92	0.06999	0.86
75	0.03565	0.81	0.01445	0.83	0.02225	0.85	0.03428	0.87	0.04740	0.92
80	0.01827	0.74	0.00910	0.66	0.01278	0.79	0.01917	0.73	0.02149	0.72