International Institute for Applied Systems Analysis
Schlossplatz 1 • A-2361 Laxenburg • Austria
Telephone: (+43 2236) 807 342 • Fax: (+43 2236) 71313
E-mail: publications@iiasa.ac.at • Internet: www.iiasa.ac.at

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Robustness to Stochastic Shocks of Alternative Old-Age Pension Arrangements: Macroeconomic Stability

Tatiana Ermolieva (ermol@iiasa.ac.at)
Landis MacKellar (mckellar@iiasa.ac.at)
Anders Westlund (STAW@HHS.SE)

Approved by

Gordon J. MacDonald (macdon@iiasa.ac.at)
Director

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Abstract

The robustness analysis presented in this paper is a part of the work done to develop and verify the properties of a stochastic macroeconomic-demographic growth model focused on social security (MacKellar and Ermolieva 1999). The first set of experiments on robustness has been reported by Westlund et al. (1999).

Here we study the sensitivity of model solutions with respect to structural changes in exogenous variables and time varying structural interactions. The parameters varied are the productivity of capital and labor force participation rates. The sensitivity analysis focuses on three macroeconomic indices of the stability of the old-age support system: current balance and assets of the private, defined-contribution, fully-funded pension system and balance of the public, defined-benefit, Pay As You Go (PAYG) pension system. Robustness at the microeconomic level, construed as the stability of pensioners' and workers' after-tax income, is the subject of another paper in this series.

About the Authors

Landis MacKellar is Leader of the Social Security Reform (SSR) Project, and Tatiana Ermolieva is Research Assistant. Anders Westlund was visiting professor of statistics at the University of Economics, Vienna, on leave from the Stockholm School of Economics, where he is chair of the Statistics Department. His participation in the SSR Project was made possible by the financial support of the Swedish Council for Planning and Coordination of Research (FRN).

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1. Introduction

Pension regimes consist of the entire spectrum of social and economic institutions that deal with the problem of how to provide income for elderly persons who no longer engage in remunerated work. The pension regime in OECD countries is evolving, *de facto,* into a two-part system. The first component is the public, defined-benefit, Pay As You Go (PAYG) pension system which has been the mainstay of old age support for over fifty years. The second component is the private, defined-contribution, fully-funded pension system which is growing in importance as demographic pressures are felt by the PAYG system. The first component is a transfer-based system in which claims on transfers are enforced by an intergenerational contract. The second is an accumulation-based system in which individuals own assets either explicitly (in the case of private pension plans and retirement saving accounts) or implicitly (in the case of fully funded government schemes).

Pension regimes which fall in between (systems in which the public pension system is partially funded, systems in which private pension schemes contain a substantial PAYG component, etc.), can generally be expressed by some combination of these two components. For example, the PAYG component of private defined-benefit pension schemes is effectively guaranteed by the state, and can for analytical purposes be lumped together with the public PAYG system. The funded component of public defined-benefit schemes represents a net asset belonging to system participants (assuming that the accumulation of pension system reserves has not simply relaxed fiscal constraints elsewhere, which is debatable) and can, again from an analytical point of view, be combined with the private fully-funded component identified above.

Pension systems protect against risk. It is therefore important that the pension system be robust at both the macroeconomic and microeconomic levels to economic shocks. In this paper, which concentrates on the macro-level, we look at how the two system components respond to stochastic shocks to productivity and labor force participation rates. The state variables whose response is studied are

- "Current" balance (i.e., contributions minus benefits) of the private, fully funded, defined-contribution pension system. Since the work of Schieber and Shoven (1994), concern has been expressed that the retirement of the Baby Boom generation will cause net flows into the private pension system to switch from positive to negative, with far-reaching economic consequences. The popular economic press coined the term "asset meltdown" to describe this pessimistic scenario.

- Assets of the private, fully-funded, defined contribution pension system. While disbursements from private pension plans are typically thought of by beneficiaries as an income flows, in the system of national accounts, they represent a capital-account transaction, that is, the sale of a capital asset. Thus, the "current" balance as defined above is a misnomer, and can give a misleading impression if there is substantial imbalance between capital returns on assets and the paying out of benefits. More simply, while a diminishing surplus of contributions over disbursements can serve as a yellow caution light, it is ultimately the asset stock that matters.

- Balance (i.e., contributions minus benefits) of the public, defined-benefit PAYG pension system. In a classic PAYG system, there is no surplus or deficit because the payroll tax is calculated to ensure balance. In the real world, raising payroll taxes is politically difficult, as it is allowing pensioners' benefits to erode. As a result, many pension systems worldwide run deficits, which must be covered out of general government revenue. The specter of widening pension-system deficits has been one of the principal motivations for recent applied macroeconomic policy research in the area (Roseveare *et al*., 1996).

In particular, we study the sensitivity of these three variables with respect to nonstationary uncertainties, modeled by ARCH-M processes with intervention functions; explore the magnitude of responses, and identify the main contributors to variation (i.e., when slightly different parameter specifications lead to significant changes in the outcomes).

The organization of the paper is as follows. Section 2 contains a cursory description of the model. Section 3 presents the robustness analysis. Subsection 3.1 contains some theoretical issues on robustness. Subsection 3.2 defines the uncertainties incorporated in these studies. In subsection 3.3 we describe numerical results of the robustness studies. Conclusions are in section 4.

2. The model

The detailed description of the algebraic structure of the IIASA model is presented in MacKellar and Ermolieva (1999). It is a neoclassical two-factor multiregional economic-demographic model with a particular focus on social security. The macro-economic core employs a Cobb-Douglas production function. Gross domestic product (*GDP*) is

$$GDP(t) = \alpha(t)(1+g(t))^t K(t)^{\beta(t)} L(t)^{(1-\beta(t))} \qquad (1)$$

$g(t)$ is the average rate of total factor productivity growth between 0 and *t*. In traditional growth theory, this is a constant; however, in the general case, it is time-dependent and may be determined endogenously. Similarly, the scaling factor $\alpha(t)$ is traditionally constant; in some applications, however, it can be regarded as a stochastic risk factor $\alpha(t) = \alpha(t,\omega)$, where ω is defined on the space of all stochastic shocks, Ω.

We assume, based on factor income shares observed in developed countries, that $\beta(t) = 0.33$, $g(t) = 0.01$. The gross profit rate $R(t)$, including depreciation and indirect taxes net of subsidies, is endogenously calculated as follows:

$$R(t) = \beta(t)\left[\frac{GDP(t)}{K(t)}\right], \qquad (2)$$

The average (over age groups) wage rate, including social insurance contributions (workers´ and employers´ contributions to public and private pension schemes) is

$$\overline{Wage}(t) = [1 - \beta(t)]\left[\frac{GDP(t)}{L(t)}\right]. \qquad (3)$$

Age-specific wages are calculated based on years of experience in the labor force and scaled so that the weighted average wage rate calculated over age groups is equal to the marginal product of labor implied by the production function.

The model incorporates exogenous population projections and age-specific saving rates, age-specific labor force participation rates, and tax rates. The emphasis of the model is on tracking income and outlay of households by single-year age groups, as well as the intergenerational transfer of resources via bequests. Households accumulate assets during working years and then "dissave" in retirement, in addition to which, intergenerational transfers between the working and retired populations are mediated through the PAYG public pension system. A special feature of the model is that it is multiregional, with allowance made for capital flows between regions or countries characterized by different demographic conditions and pension arrangements. For the robustness analysis presented here, however, we have collapsed the model to its single-region form, and no attempt will be made to describe its multiregional structure.

3. Robustness: numerical experiments

3.1. Some robustness issues

A model (or methodology) is robust if the conclusions which follow from its application are not significantly changed when the assumptions it is based on are marginally different. Thus, robustness is generally a positive characteristic. It does not refer to situations where underlying assumptions are drastically changed, a case when the analysis is expected to, and also generally should provide a set of new conclusions. Robustness can be considered, not only in relation to assumptions regarding exogenous variables, but the structural parameters of the model as well (Hackl and Westlund 1991). So long as the model being employed is the correct one, model robustness can also give rise to inferences about the robustness or stability of the system being modeled. If, in the current analysis, the components of the pension system are found to be stable under reasonable changes in parameters and assumptions, then we can say that they are robust; if, on the other hand, marginal changes lead to wide divergences, they are not robust.

Robustness analysis should proceed in stages. First, the robustness towards randomizing initial parameters assumptions should be verified. Second, the assumptions of parameter time invariance should be considered, i.e. robustness towards imposing of new parametric regimes, or shocks to the parameter structure should be analyzed. In the next section, robustness properties of the model will be illustrated by introducing randomization of initial parameter values using ARCH-M processes, as well as by introducing new regimes of parameters incorporating intervention processes. The temporal properties (the duration) of intervention will be taken into account as well as its qualitative characteristics, such as gradual or impulse intervention. Attention will be given to verifying autoregressive characteristics of parameter models through introduction of the ARCH-M processes. In analyzing numerical results, attention will be focused on temporal reconvergence properties of the involved processes.

In the numerical experiments reported on here, we focus on the randomization of $\beta(t)$ and *LabForcePartRate(t)* by means of ARCH-M models. Furthermore, we introduce "intervention" processes, i.e. shocks (which may be temporary or sustained) to the level of parameters. The state variables whose responses are analyzed are the balances of the private defined-contribution (fully-funded) and public defined-benefit (PAYG) pension systems (*BalPvtDCPenSys* and *BalPubDBPenSys*, respectively) and assets of the first of these (*AssetsPvtDCPensSys*). All of these variables are normalized by GDP.

3.2. Generation of uncertainties

Many economic variables display nonstationarity (Enders, 1995), meaning that they do not have constant means and exhibit periods of different volatilities (heteroskedasticity). Their structural interactions are not time invariant. In these circumstances, ARCH processes, in particular ARCH-M, are an appropriate choice to describe structural parameters and exogenous variables.

The ARCH-M process allows the mean of the variable to depend on its own conditional variance. In the case of $\beta(t)$, for example, the process is modeled by

$$\beta(t) = \mu(t) + \varepsilon(t)\,;\ \mathrm{E}\beta_{t-1}(t) = \mu(t)\,;\ \mu(t) = \beta^* + \delta h(t)\,,\ \delta > 0\,;$$

$$h(t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2 (t-i)\,. \tag{4}$$

where β^* is the baseline parameter (equal to 0.33 in our case).

Two basic ARCH-M models were studied with $h(t)$ being ARCH(1) and ARCH(2) processes, respectively. Thus, autoregressivity stems from $h(t)$ through the error term $\varepsilon(t)$. Note that autoregressivity when combined with intervention (see below) produces persistent post-shock effects.

In an empirical analyses parameters δ, α_0, α_i have to be estimated separately; in our analyses we allowed for an exogenous specification and initialized them as $\delta = 1$, $\alpha_0 = 0$, $\alpha_1 = 0.65$, $\alpha_2 = 0.65$.

In introducing intervention processes we distinguished between two regimes:

- The first implies a gradual change of a parameter from the base assumption to a new structural level, which then remains in place for the rest of the simulation period. Schematically the regime is defined as $1 \to 2 \to 2$.
- In the second, $1 \to 2 \to 1$, the gradual change as described in the first case is, after a number of periods, followed by a return to the base assumption.

The intervention function is characterized by the magnitude and duration of the shock applied. Moderate interventions incorporate a 5% gradual change of the variable from its base scenario value, extreme interventions correspond to a 15% change. Short-term interventions are defined as those characterized by 4 time periods of gradual increase in the case $1 \to 2 \to 2$ and 4 periods of increase followed symmetrically by four periods of decay in the case $1 \to 2 \to 1$. Medium-term interventions are characterized by 10 time periods of increase in the case $1 \to 2 \to 2$ and ten periods of increase followed by ten periods of decay in the case $1 \to 2 \to 1$.

We illustrate the scheme of the numerical experiments with the parameter $\beta(t)$, an ARCH(1) process, and an extreme-magnitude, moderate-duration $1 \to 2 \to 1$ intervention:

1.1. At $t = 1$, year 1995, the baseline parameter value $\beta^* = 0.33$ is shocked by a random variable $\varepsilon(t) \sim N(0, 0.05\beta^*)$.

1.2. For $1 < t < 30$ (until 2025) the parameter is simulated according to the formula

$\beta(t) = \mu(t) + \varepsilon(t)$, $\mu(t) = 0.33 + h(t)$, $h(t) = 0.65\varepsilon_{t-1}^2$, where $\varepsilon(t)$ is as described above.

1.3. The intervention process is introduced with respect to the mean value $\beta^* = 0.33$. For $30 \le t < 40$ (starting 2025 and till 2035) there is a gradual increase of the mean, each year by 1.5%. Thus, in 10 years the mean of the ARCH-M model is

$$\mu(t) = (1 + 0.15)(0.33) + h(t)$$

1.4. For $40 \le t < 50$ the model employs a gradual decline from the value $\mu(t) = (1 + 0.15)(0.33) + h(t)$ back to $\mu(t) = 0.33 + h(t)$

1.5. For $t \ge 50$ (or after year 2045) $\beta(t)$ develops as for $1 < t < 30$.

The different types of intervention are summarized schematically Table 1.

	Regime 1 2 2				Regime 1 2 1			
Change	Moderate 5		Extreme 10		Moderate 5		Extreme 10	
Time	Short 4	Medium 10	Short 4	Medium 10	Short 8 (4+4)	Medium 20 (10+10)	Short 8 (4+4)	Medium 20 (10+10)

Table 1. Scheme of parameter randomization (in each case above, ARCH(q), with q=1, or 2 is used).

The results of simulations are presented in graphical form providing some important percentiles of samples. Different cases are identified by $C(i, j, k, l, m, n)$, where $i = 1,2$ indexes the type of the parameter model (ARCH(1) or ARCH(2)); $j = 1,2$ indexes the intervention regime ($1 \to 2 \to 1$ or $1 \to 2 \to 2$); $k = 1,2$ identifies the duration of the (short or medium; $l = 1,2$ identified the magnitude of the intervention (moderate or extreme); $m = 1,2$ identified the parameter perturbed parameter ($\beta(t)$ or *ASLFPR*); and $n = 1,2$; for the response variable (*BalPvtDCPenSys* and *BalPubDBPenSys*). For example, $C(1,1,1,1,1,1)$ represents the case of ARCH(1), regime $1 \to 2 \to 1$, short-term intervention of moderate magnitude in parameter change of $\beta(t)$. In this case we analyze responses of *BalPvtDCPenSys*.

To save space we here present only some of the results and group them by the response variable.

3.3 Numerical results

The baseline scenario from 1995-2100 was produced using generic assumptions suitable to an industrial economy. The robustness results reported here are, of course, subject to the baseline scenario employed.

3.3.1. *BalPvtDCPenSys(t) / GDP(t), uncertainties in parameter* $\beta(t)$*, cases* $C(:,:,:,:,1,1)$.

The variable *BalPvtDCPenSys(t)* is defined by the identity

$$AssetsPvtDCPenSys(t) = AssetsPvtDCPenSys(t-1)(1 + r(t-1)) + BalPvtDCPenSys(t) \quad (5)$$

In other words, assets of the private defined contribution pension system are equal to assets in the last period plus investment income plus current receipts minus disbursements, where the latter two taken together are defined as the balance of the system. Receipts are contributions out of wages and the profits of household-operated unincorporated enterprises, whereas disbursements are pension payments and the "paying out" of pension assets upon death of the beneficiary.[1]

We normalize *BalPvtDCPenSys(t)* by *GDP(t)*. Analytically speaking, when $\beta(t)$ is increased, *GDP(t)* increases while the wage share $1-\beta(t)$ declines. The proportional increase in the former is larger than the proportional decline in the latter, as a result of which the nominal wage bill increases, resulting in a roughly equiproportional increase in contributions into the private pension system.[2] Outflow from the pension system is unaffected in the near term; only in the long-term, when higher wages are translated via higher contributions into higher pension entitlements is the impact on pension system disbursements felt. Therefore, the impact of an increase in $\beta(t)$ is a near-term increase in *BalPvtDCPenSys(t) / GDP(t)* followed by gradual decline as outflows increase with a long time lag.

Figure 1 illustrates case $C(1,1,2,2,1,1)$, that is, an ARCH(1) process with randomization applied in the form of a medium-term, extreme-magnitude intervention. The thick black line represents the baseline deterministic scenario. Its negative slope corresponds to the gradually declining rate of net financial inflow to the pension system which is expected as the population ages.[3] The jagged lines around the baseline give frequency bounds which result when $\beta(t)$ is shocked upward. In 5% of all cases, the resulting value of the variable *BalPvtDCPenSys(t) / GDP(t)* lies below the lower bound; in 95% of all cases, it lies below the upper bound. Therefore, the area in between has the interpretation of a 90 percent confidence interval. The mean stochastic case is also given, as well as the 50th percentile line representing the median.

The trajectories start around the same level, roughly 0.04. As time goes by, the distance between the 5-th and the 95-th percentile increases as a result of the cumulative nature of the ARCH(1) process. The impact of the increase in $\beta(t)$ between 2025 and 2035 is clearly visible, as is the decay in the period 2035-45. Following the intervention, a pronounced asymmetry develops, with many values of *BalPvtDCPenSys(t) / GDP(t)* lying below the baseline while few lie above it. Following 2045, in over 5% of all scenarios, the balance is persistently negative and even in the median case, the balance skirts zero over much of the simulation period. This is explicable as follows. The impact of higher $\beta(t)$ on GDP (and on contributions)

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[1] The latter term can become rather complicated, as the treatment of pension bequests varies. We make the simplifying assumption that, upon death of a pension beneficiary, his or her remaining pension assets are paid out in a lump sum to survivors, who convert some of the bequest into consumption and save the remainder. Saving out of bequests takes place outside the pension system, that is, no provision is made for "rolling over" inherited pension assets into another pension plan.

[2] In fact, the proportional increase in contributions is slightly greater than the increase in the nominal wage bill, since it reflects growth in contributions out of profits of unincorporated enterprises as well. The proportional increase in profits is (ignoring second-order effects) equal to the proportional increase in GDP plus the proportional increase in $\beta(t)$; this is clearly greater than the proportional increase in the wage bill. It can be shown that the more capital-intensive the economy, i.e. the higher the capital-labor ratio, the more sensitive are GDP, wages, and profits to $\beta(t)$.

[3] Nothing in the baseline scenario gives credence, however, to the extreme "asset meltdown" scenario alluded to above.

diminishes as the parameter returns to its pre-intervention level. However, each dollar of added contributions produced in the immediate wake of the intervention must eventually be paid out, plus accumulated capital returns.

What about a sustained, as opposed to transitory, shock? Figure 2 represents case $C(1,2,2,2,1,1)$, which differs from the case illustrated in Figure 1 in the type of intervention, which is here permanent ($1 \rightarrow 2 \rightarrow 2$) as opposed to transient ($1 \rightarrow 2 \rightarrow 1$). In years 2025 to 2035, the time of gradual elevation of $\beta(t)$, the percentile curves replicate those in Figure 1. Since $\beta(t)$ stays on the higher position, the curves do not decline rapidly as in Figure 1. However, even in this case of a permanent increase in the productivity of capital, the 5% "worst case" probability band corresponds, in the outer years of the simulation, to a negative balance on current flows.

However, as we have discussed above the current balance is a biased index since it includes the disbursement of benefits, which is from an accounting point of view a capital transaction, but does not include capital returns, in the form of income on assets. It is arguably the stock of private pension system assets which is important for the real economy. Figures 3 and 4 illustrate the path of total assets of the pension system *AssetsPvtDCPenSys(t) / GDP(t)*. The flow variable *GDP(t)* is more sensitive to changes in $\beta(t)$ than is the stock variable *AssetsPvtDCPenSys(t)*; thus, the positive shift of $\beta(t)$ reduces the ratio of pension system assets to GDP. The mean and median shocked scenarios track the deterministic baseline fairly closely, returning to baseline from above and below in cases ($1 \rightarrow 2 \rightarrow 1$) and ($1 \rightarrow 2 \rightarrow 2$), respectively. Even at its widest, the 95% certainty band is only about 1.5 percent of GDP and none of the "worst scenario" cases come close to indicating financial distress for the pension system. As we will see later, however, the sensitivities in Figures 3 and 4 are much greater than those of the defined benefit, PAYG pension system.

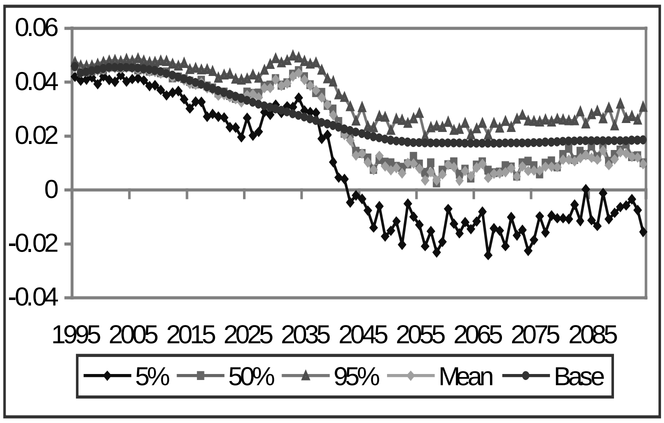

Figure 1. *BalPvtDCPenSys(t) / GDP(t)*, case $C(1,1,2,2,1,1)$.

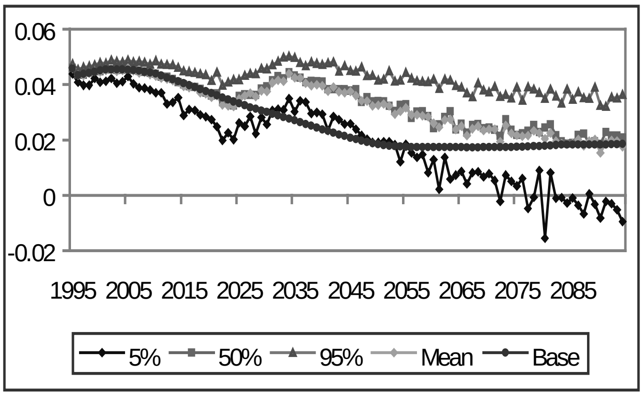

Figure 2. *BalPvtDCPenSys(t) / GDP(t)*, case $C(1,2,2,2,1,1)$.

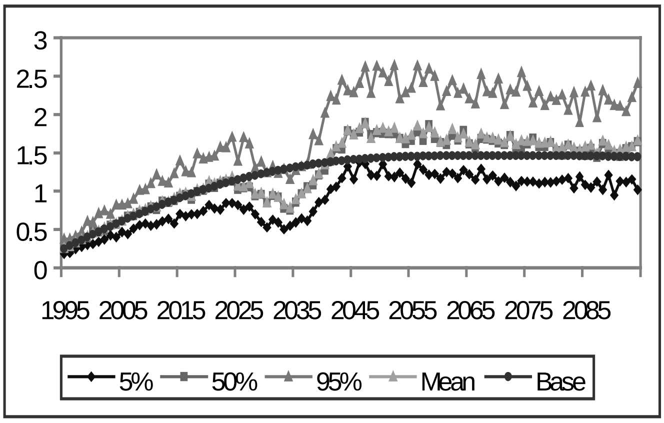

Figure 3. *AssetsPvtDCPenSys(t) / GDP(t)*, case $C(1,1,2,2,1,1)$.

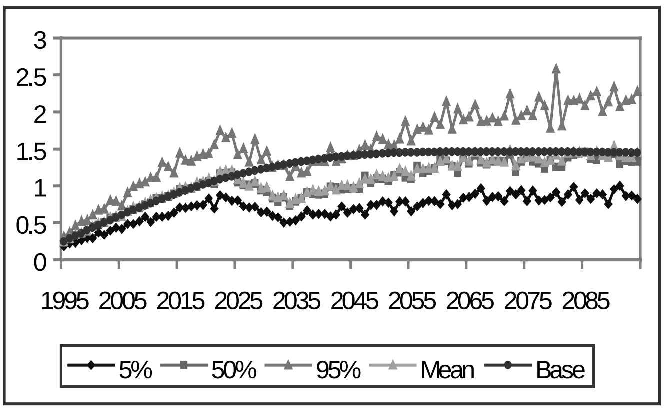

Figure 4. *AssetsPvtDCPenSys(t) / GDP(t)*, case $C(1,2,2,2,1,1)$.

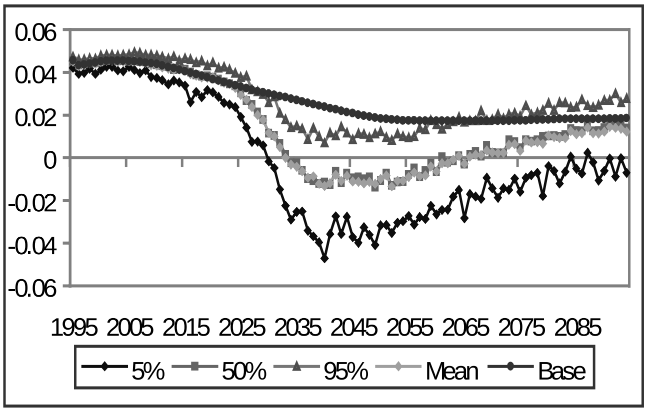

Figure 5. *BalPvtDCPenSys(t) / GDP(t)*, case $C(2,2,2,2,1,1)$.

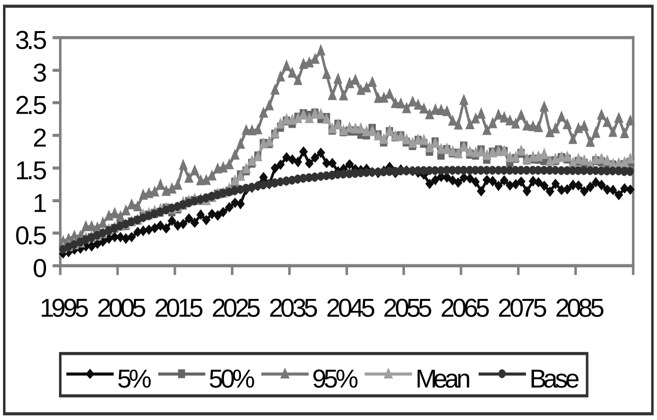

Figure 6. *AssetsPvtDCPenSys(t) / GDP(t)*, case $C(2,2,2,2,1,1)$.

The response to an extreme downward shock, which might correspond to some sort of technological crisis, is symmetrical. Figures 5 and 6 illustrate a 15% negative sustained intervention in $\beta(t)$, case $C(2,2,2,2,1,1)$. The pension system balance immediately begins a rapid decline, entering negative territory, on average, 10 years after administration of the shock. The 5-th percentile remains decisively below zero throughout the simulation period. However, the relative stability of the pension system considered as a stock of assets is clear in Figure 6.

3.3.2. BalPvtDCPenSys(t)/GDP(t), uncertainties in parameter LabForcePartRate(t), cases $C(:,:,:,:,2,1)$ *.*

There have been large structural changes in the OECD labor market. For example, the labor force participation rate of elderly men in some OECD countries has fallen by some 20 percentage points in the post-War period while the labor force participation rate of women has risen. Lower labor supply of the elderly is often cited as a cause of the weakness of public PAYG pension systems, because when an elderly worker withdraws from the labor force, the number of persons drawing funds out of the pension system rises while the number of persons paying in falls. The case of a private defined contribution fully funded pension scheme is different, because more such schemes require participants to begin drawing down benefits at some fixed age (here, age 60) whether they continue to work or not. Thus, the impact of reduced labor supply (anywhere over the age-span) is to reduce system receipts, but only in the long run as the asset stock is affected is there any impact on system disbursements. Two additional stabilizers apply equally to PAYG and fully-funded schemes. First, reduced labor supply drives up the wage rate, as a result of which, those persons who are still working are, for given saving rates, paying higher contributions. Second, and of interest only in the context of the index chosen for study here, while lower labor supply reduces the current balance of the pension system, so too does it reduce GDP, so in a relative sense, the impact is muted.

The relative insensitivity of an accumulation-based pension systems to the labor force participation rate is illustrated in Figures 7-10. In the baseline scenario, the average (over age groups) labor force participation rate for persons aged 20-65 was approximately 80 percent, and for persons aged over 65 was approximately 10 percent. Stochastic shocks, administered in 1995, reflect standard deviations equal to 5 percent of baseline age-specific labor force participation rates (i.e., 4 percentage points in the case of a baseline rate of 80 percent, and so on). Interventions took the form of downward adjustment of labor force participation. Figure 7 represents an extreme, medium-term intervention of the regime $1 \rightarrow 2 \rightarrow 1$, that is, a 15 percent reduction in the labor force participation rate over ten years followed by a return to baseline value. The system balance is reduced relative to GDP, but only slightly. The shocked trajectory converges to a level marginally higher than the base line, which is explained by less asset accumulation during the intervention period, and hence lower benefit disbursements in the longer run. Figure 8 illustrates a persistent intervention (regime $1 \rightarrow 2 \rightarrow 2$).[4] Even in the persistent case, the balance of the system is quite robust.

[4] The stochastic process illustrated in Figures 8 and 10 is ARCH(2). Introduction of the ARCH(2) characteristics does not seem to radically change the shape of the curves in Figures 8 and 10 in comparison to Figures 7 and 9 (for the time period of gradual increase). However, not many experiments have been performed on the verification of coefficients a_0 and a_1.

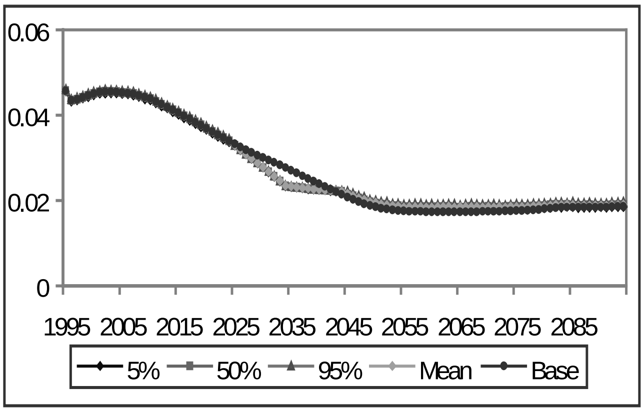

Figure 7. *BalPvtDCPenSys(t) / GDP(t)*, case $C(1,1,2,2,2,1)$.

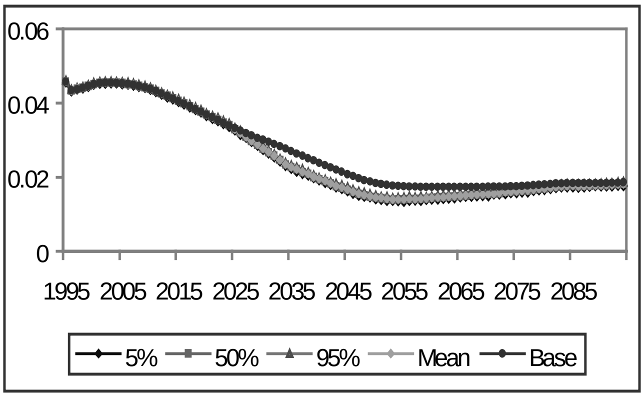

Figure 8. *BalPvtDCPenSys(t) / GDP(t)*, case $C(2,2,2,2,2,1)$.

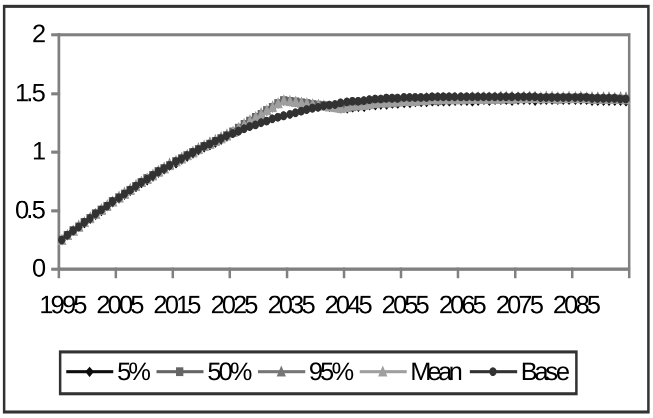

Figure 9. *AssetsPvtDCPenSys(t) / GDP(t)*, case $C(1,1,2,2,2,1)$.

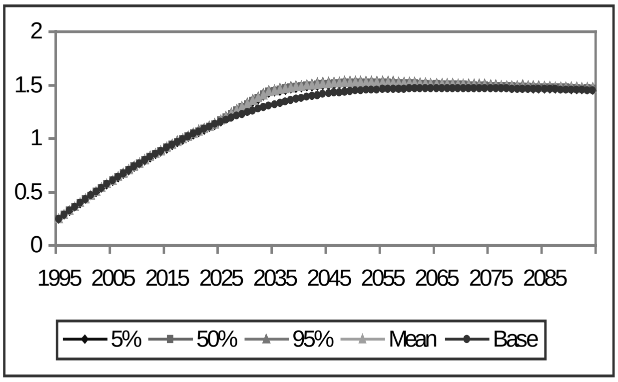

Figure 10. *AssetsPvtDCPenSys(t)/GDP(t)* case $C(2,2,2,2,2,1)$.

Figures 9 and 10 illustrate pension system asset stocks, which barely respond to the intervention. In addition to the stabilizers mentioned above, a third built-in stabilizer cushions the impact of reduced labor supply on pension system assets. When a system participant withdraws from the labor force, the capital returns to his/her assets continue to flow into the pension system until the retirement age and, after retirement, while these assets are gradually drawn down.

3.3.3. BalPubDBPenSys(t) / GDP(t), uncertainties in parameter $\beta(t)$*, cases* $C(:,:,:,:,1,2)$

The balance of the defined-benefit PAYG pension system is defined as total contributions by the system's members minus total pension payments. The variable is normalized by *GDP*. The baseline case illustrated below corresponds to the rapid deterioration of public pension systems under "business as usual" conditions which has featured in a number of other analyses (Roseveare et al 1996). In order to illustrate the response of the variable we present results only for cases $C(1,1-2,2,2,1,2)$

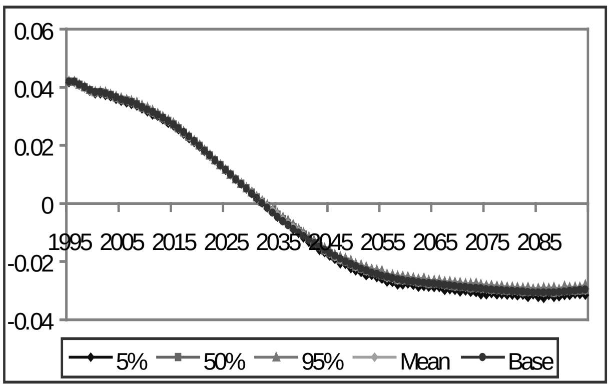

Figure 11. *BalPubDBPenSys(t) / GDP(t)*, case $C(1,1,2,2,1,2)$.

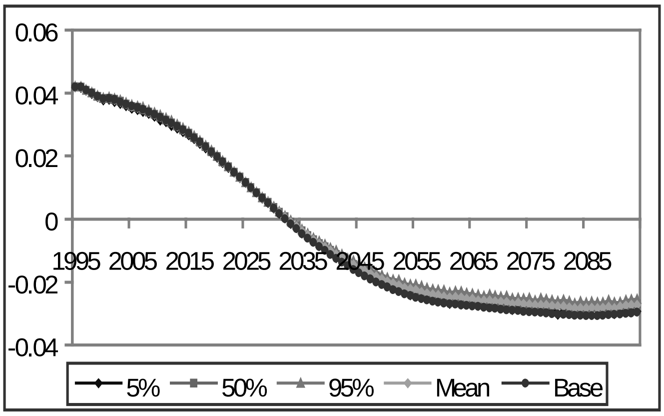

Figure 12. *BalPubDBPenSys(t)/GDP(t),* case $C(1,2,2,2,1,2)$.

The conclusion gained from Figures 11 - 12 is that the PAYG pension system balance is extremely robust with respect to uncertainties in $\beta(t)$. An increase of $\beta(t)$ leads to the increase of wages, as a result of which contributions to the system increase. However, the increase in wages leads to an immediate increase in pension entitlements for new retirees, shifting pension system payments up as new retirees take their place in the pool of system beneficiaries. In this way, the increase in system balance is smaller in absolute terms than in the case of the fully-funded pension system considered above.

3.3.4. BalPubDBPenSys(t) / GDP(t), uncertainties in parameter LabForcePartRate(t), cases $C(:,:,:,:,2,2)$

On the assumption that system obligations per retiree and the payroll tax rate are both fixed, lower labor force participation, especially among the elderly, shifts the PAYG social security system balance towards deficit by increasing the system dependency ratio. In the short-term, the effect is straightforward and pronounced. As opposed to the case of most private defined contribution fully funded pension schemes, in most public pension schemes, elderly persons start to receive benefits only when they actually retire. Thus, whereas reduced elderly labor force participation affects only the contribution side of most private schemes, it affects both the contribution and benefit sides of most public schemes. In the longer term, the impact of lower labor force participation is more complex because two opposing forces (which also affect private pension schemes) must be taken into account. Lower labor force participation among the young eventually raises the ratio of elderly who never worked to elderly who worked; all else being equal, this reduces system obligations. Because of lower labor supply, however, those who worked earned higher wages and, thus, have a higher pension entitlement; all else being equal, this increases system obligations.

Figure 13 illustrates case $C(1,1,1,1,2,2)$ with positive intervention (increase of labor force participation rates). The most important conclusion is that the variable is insensitive to slight, short-term interventions. In the long term, shocked trajectories are almost not distinguished from the base line. Figure 14 displays properties of the variable with respect to a negative extreme intervention of regime $1 \to 2 \to 1$. After a rather sharp near-term deviation, the randomized trajectories converge to the base line.

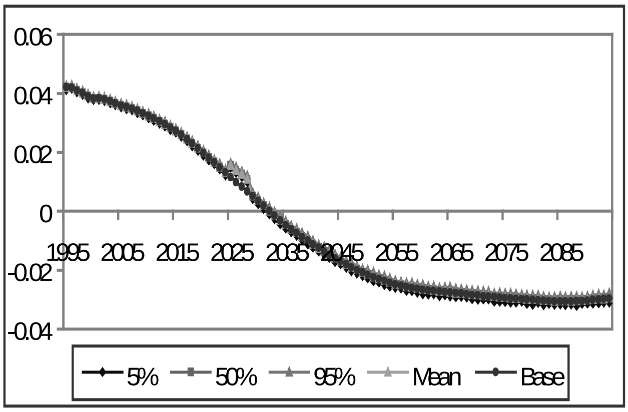

Figure 13. *BalPAYGPenSys(t) / GDP(t)*, case $C(1,1,1,1,2,2)$.

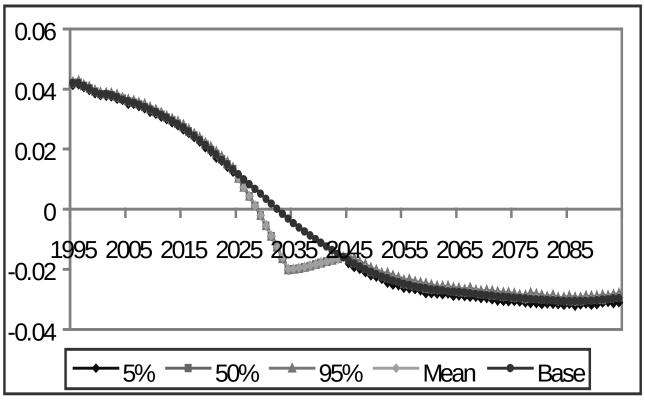

Figure 14. *BalPAYGPenSys(t) / GDP(t)*, case $C(1,1,2,2,2,2)$.

Figure 15 presents case $C(2,2,2,2,2,2)$, an extreme, persistent decline in labor force participation rates (regime $1 \to 2 \to 2$). The randomized trajectories do not converge to the base line and occupy decisively lower paths. This is in marked contrast to the case of the fully-funded pension system illustrated in Figure 8.

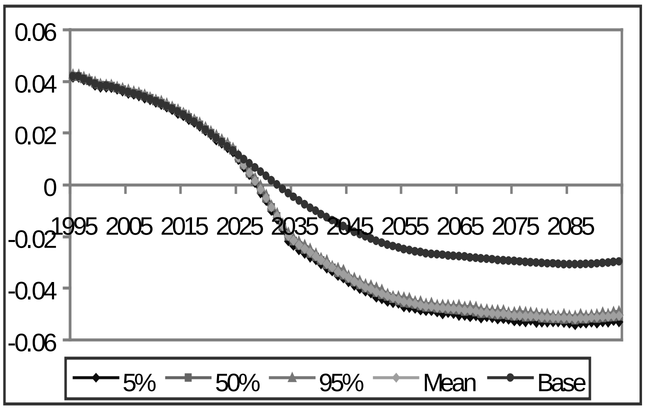

Figure 15 *BalPAYGPenSys/GDP*, case C(2,2,2,2,2,2)

4. Conclusions

We have modeled an economy in which retired persons receive income from a public defined-benefit pension system financed on a Pay As You Go (PAYG) basis and a private defined-contribution system financed on a fully funded or capital reserve basis. The question asked was how stable each of the systems was at the aggregate level when the real economy was shocked. The shocks considered involved the productivity of capital and labor supply.

We have found that the PAYG system, which has been the main source of old-age income for most of the population in the OECD since the War, was robust to shocks to productivity, but was sensitive to sustained shocks to labor force participation. The defined contribution, fully-funded system, while more sensitive to productivity shocks, could still be classified as basically robust given the observed magnitude of impacts on the asset stock relative to GDP. In contrast the PAYG system, it was found to be insensitive to labor supply shocks.

These insights are useful, but must be put in the context of two qualifications. First, we have not looked at shocks from the price side. It is widely known that PAYG pension systems can be designed so that they are relatively insensitive to price shocks. The possibly rapid depreciation of the asset value of a capital-reserve system under the shock of sustained inflation is often mentioned as a disadvantage of such an approach. Second, this paper has looked only at aggregate pension system stability measures, in this case, flows and asset stocks relative to GDP. Equally important are the stability of individual benefits and income levels, which are the subject of another paper in this series.

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