

Interim Report

IR-00-040

**Evolutionary Branching and Sympatric Speciation
Caused by Different Types of Ecological Interactions**

Michael Doebeli (doebeli@zoology.ubc.ca)

Ulf Dieckmann (dieckman@iiasa.ac.at)

Approved by

Gordon J. MacDonald (macdon@iiasa.ac.at)

Director, IIASA

July 2000



The Adaptive Dynamics Network at IIASA fosters the development of new mathematical and conceptual techniques for understanding the evolution of complex adaptive systems.

Focusing on these long-term implications of adaptive processes in systems of limited growth, the Adaptive Dynamics Network brings together scientists and institutions from around the world with IIASA acting as the central node.

Scientific progress within the network is reported in the IIASA Studies in Adaptive Dynamics series.

THE ADAPTIVE DYNAMICS NETWORK

The pivotal role of evolutionary theory in life sciences derives from its capability to provide causal explanations for phenomena that are highly improbable in the physico-chemical sense. Yet, until recently, many facts in biology could not be accounted for in the light of evolution. Just as physicists for a long time ignored the presence of chaos, these phenomena were basically not perceived by biologists.

Two examples illustrate this assertion. Although Darwin's publication of "The Origin of Species" sparked off the whole evolutionary revolution, oddly enough, the population genetic framework underlying the modern synthesis holds no clues to speciation events. A second illustration is the more recently appreciated issue of jump increases in biological complexity that result from the aggregation of individuals into mutualistic wholes.

These and many more problems possess a common source: the interactions of individuals are bound to change the environments these individuals live in. By closing the feedback loop in the evolutionary explanation, a new mathematical theory of the evolution of complex adaptive systems arises. It is this general theoretical option that lies at the core of the emerging field of adaptive dynamics. In consequence a major promise of adaptive dynamics studies is to elucidate the long-term effects of the interactions between ecological and evolutionary processes.

A commitment to interfacing the theory with empirical applications is necessary both for validation and for management problems. For example, empirical evidence indicates that to control pests and diseases or to achieve sustainable harvesting of renewable resources evolutionary deliberation is already crucial on the time scale of two decades.

The Adaptive Dynamics Network has as its primary objective the development of mathematical tools for the analysis of adaptive systems inside and outside the biological realm.

IIASA STUDIES IN ADAPTIVE DYNAMICS

- No. 1 Metz JAJ, Geritz SAH, Meszéna G, Jacobs FJA, van Heerwaarden JS:
Adaptive Dynamics: A Geometrical Study of the Consequences of Nearly Faithful Reproduction.
IIASA Working Paper WP-95-099.
In: van Strien SJ, Verduyn Lunel SM (eds.): *Stochastic and Spatial Structures of Dynamical Systems*, Proceedings of the Royal Dutch Academy of Science (KNAW Verhandelingen), North Holland, Amsterdam, pp. 183–231 (1996).
- No. 2 Dieckmann U, Law R:
The Dynamical Theory of Coevolution: A Derivation from Stochastic Ecological Processes.
IIASA Working Paper WP-96-001.
Journal of Mathematical Biology (1996) 34, 579–612.
- No. 3 Dieckmann U, Marrow P, Law R:
Evolutionary Cycling of Predator-Prey Interactions: Population Dynamics and the Red Queen.
Journal of Theoretical Biology (1995) 176, 91–102.
- No. 4 Marrow P, Dieckmann U, Law R:
Evolutionary Dynamics of Predator-Prey Systems: An Ecological Perspective.
IIASA Working Paper WP-96-002.
Journal of Mathematical Biology (1996) 34, 556–578.
- No. 5 Law R, Marrow P, Dieckmann U:
On Evolution under Asymmetric Competition.
IIASA Working Paper WP-96-003.
Evolutionary Ecology (1997) 11, 485–501.
- No. 6 Metz JAJ, Mylius SD, Dieckmann O:
When Does Evolution Optimise? On the Relation between Types of Density Dependence and Evolutionarily Stable Life History Parameters.
IIASA Working Paper WP-96-004.
- No. 7 Ferrière R, Gatto M:
Lyapunov Exponents and the Mathematics of Invasion in Oscillatory or Chaotic Populations.
Theoretical Population Biology (1995) 48, 126–171.
- No. 8 Ferrière R, Fox GA:
Chaos and Evolution.
Trends in Ecology and Evolution (1995) 10, 480–485.

- No. 9 Ferrière R, Michod RE:
The Evolution of Cooperation in Spatially Heterogeneous Populations.
IIASA Working Paper WP-96-029.
American Naturalist (1996) 147, 692–717.
- No. 10 Van Dooren TJM, Metz JAJ:
Delayed Maturation in Temporally Structured Populations with Non-Equilibrium Dynamics.
IIASA Working Paper WP-96-070.
Journal of Evolutionary Biology (1998) 11, 41–62.
- No. 11 Geritz SAH, Metz JAJ, Kisdi É, Meszéna G:
The Dynamics of Adaptation and Evolutionary Branching.
IIASA Working Paper WP-96-077.
Physical Review Letters (1997) 78, 2024–2027.
- No. 12 Geritz SAH, Kisdi É, Meszéna G, Metz JAJ:
Evolutionarily Singular Strategies and the Adaptive Growth and Branching of the Evolutionary Tree.
IIASA Working Paper WP-96-114.
Evolutionary Ecology (1998) 12, 35–57.
- No. 13 Heino M, Metz JAJ, Kaitala V:
Evolution of Mixed Maturation Strategies in Semelparous Life-Histories: the Crucial Role of Dimensionality of Feedback Environment.
IIASA Working Paper WP-96-126.
Philosophical Transactions of the Royal Society of London Series B (1997) 352, 1647–1655.
- No. 14 Dieckmann U:
Can Adaptive Dynamics Invade?
IIASA Working Paper WP-96-152.
Trends in Ecology and Evolution (1997) 12, 128–131.
- No. 15 Meszéna G, Czibula I, Geritz SAH:
Adaptive Dynamics in a Two-Patch Environment: a Simple Model for Allopatric and Parapatric Speciation.
IIASA Interim Report IR-97-001.
Journal of Biological Systems (1997) 5, 265–284.
- No. 16 Heino M, Metz JAJ, Kaitala V:
The Enigma of Frequency-Dependent Selection.
IIASA Interim Report IR-97-061.
Trends in Ecology and Evolution (1998) 13, 367–370.

- No. 17 Heino M:
Management of Evolving Fish Stocks.
IIASA Interim Report IR-97-062.
Canadian Journal of Fisheries and Aquatic Sciences (1998) 55, 1971–1982.
- No. 18 Heino M:
Evolution of Mixed Reproductive Strategies in Simple Life-History Models.
IIASA Interim Report IR-97-063.
- No. 19 Geritz SAH, van der Meijden E, Metz JAJ:
Evolutionary Dynamics of Seed Size and Seedling Competitive Ability.
IIASA Interim Report IR-97-071.
Theoretical Population Biology (1999) 55, 324–343.
- No. 20 Galis F, Metz JAJ:
Why are there so many Cichlid Species? On the Interplay of Speciation and Adaptive Radiation.
IIASA Interim Report IR-97-072.
Trends in Ecology and Evolution (1998) 13, 1–2.
- No. 21 Boerlijst MC, Nowak MA, Sigmund K:
Equal Pay for all Prisoners. / The Logic of Contrition.
IIASA Interim Report IR-97-073.
AMS Monthly (1997) 104, 303–307.
Journal of Theoretical Biology (1997) 185, 281–294.
- No. 22 Law R, Dieckmann U:
Symbiosis without Mutualism and the Merger of Lineages in Evolution.
IIASA Interim Report IR-97-074.
Proceedings of the Royal Society of London Series B (1998) 265, 1245–1253.
- No. 23 Klinkhamer PGL, de Jong TJ, Metz JAJ:
Sex and Size in Cosexual Plants.
IIASA Interim Report IR-97-078.
Trends in Ecology and Evolution (1997) 12, 260–265.
- No. 24 Fontana W, Schuster P:
Shaping Space: The Possible and the Attainable in RNA Genotype-Phenotype Mapping.
IIASA Interim Report IR-98-004.
Journal of Theoretical Biology (1998) 194, 491–515.
- No. 25 Kisdi É, Geritz SAH:
Adaptive Dynamics in Allele Space: Evolution of Genetic Polymorphism by Small Mutations in a Heterogeneous Environment.
IIASA Interim Report IR-98-038.
Evolution (1999) 53, 993–1008.

- No. 26 Fontana W, Schuster P:
Continuity in Evolution: On the Nature of Transitions.
IIASA Interim Report IR-98-039.
Science (1998) 280, 1451–1455.
- No. 27 Nowak MA, Sigmund K:
Evolution of Indirect Reciprocity by Image Scoring. / The Dynamics of Indirect Reciprocity.
IIASA Interim Report IR-98-040.
Nature (1998) 393, 573–577.
Journal of Theoretical Biology (1998) 194, 561-574.
- No. 28 Kisdi É:
Evolutionary Branching Under Asymmetric Competition.
IIASA Interim Report IR-98-045.
Journal of Theoretical Biology (1999) 197, 149-162.
- No. 29 Berger U:
Best Response Adaptation for Role Games.
IIASA Interim Report IR-98-086.
- No. 30 Van Dooren TJM:
The Evolutionary Ecology of Dominance-Recessivity
IIASA Interim Report IR-98-096.
Journal of Theoretical Biology (1999) 198, 519-532.
- No. 31 Dieckmann U, O’Hara B, Weisser W:
The Evolutionary Ecology of Dispersal.
IIASA Interim Report IR-98-108.
Trends in Ecology and Evolution (1999) 14, 88–90.
- No. 32 Sigmund K:
Complex Adaptive Systems and the Evolution of Reciprocation.
IIASA Interim Report IR-98-100.
Ecosystems (1998) 1, 444-448.
- No. 33 Posch M, Pichler A, Sigmund K:
The Efficiency of Adapting Aspiration Levels.
IIASA Interim Report IR-98-103.
Proceedings of the Royal Society of London Series B (1999) 266, 1427-1435.
- No. 34 Mathias A, Kisdi É:
Evolutionary Branching and Coexistence of Germination Strategies.
IIASA Interim Report IR-99-014.

- No. 35 Dieckmann U, Doebeli M:
On the Origin of Species by Sympatric Speciation.
IIASA Interim Report IR-99-013.
Nature (1999) 400, 354–357.
- No. 36 Metz JAJ, Gyllenberg M:
How Should We Define Fitness in Structured Metapopulation Models? Including an Application to the Calculation of Evolutionarily Stable Dispersal Strategies.
IIASA Interim Report IR-99-019.
Research Report A39 (1999), University of Turku, Institute of Applied Mathematics, Turku, Finland.
- No. 37 Gyllenberg M, Metz JAJ:
On Fitness in Structured Metapopulations.
IIASA Interim Report IR-99-037.
Research Report A38 (1999), University of Turku, Institute of Applied Mathematics, Turku, Finland.
- No. 38 Meszéna G, Metz JAJ:
Species Diversity and Population Regulation: The Importance of Environmental Feedback Dimensionality.
IIASA Interim Report IR-99-045.
- No. 39 Kisdi É, Geritz SAH:
Evolutionary Branching and Sympatric Speciation in Diploid Populations.
IIASA Interim Report IR-99-048.
- No. 40 Ylikarjula J, Heino M, Dieckmann U:
Ecology and Adaptation of Stunted Growth in Fish.
IIASA Interim Report IR-99-050.
- No. 41 Nowak MA, Sigmund K:
Games on Grids.
IIASA Interim Report IR-99-038.
In: Dieckmann U, Law R, Metz JAJ (eds.): *The Geometry of Ecological Interactions: Simplifying Spatial Complexity*, Cambridge University Press, Cambridge, UK, pp. 135–150 (2000).
- No. 42 Ferrière R, Michod RE:
Wave Patterns in Spatial Games and the Evolution of Cooperation.
IIASA Interim Report IR-99-041.
In: Dieckmann U, Law R, Metz JAJ (eds.): *The Geometry of Ecological Interactions: Simplifying Spatial Complexity*, Cambridge University Press, Cambridge, UK, pp. 318–332 (2000).
- No. 43 Kisdi É, Jacobs FJA, Geritz SAH:
Red Queen Evolution by Cycles of Evolutionary Branching and Extinction.
IIASA Interim Report IR-00-030.

- No. 44 Meszéna G, Kisdi É, Dieckmann U, Geritz SAH, Metz JAJ:
Evolutionary Optimisation Models and Matrix Games in the Unified Perspective of Adaptive Dynamics.
IIASA Interim Report IR-00-039.
- No. 45 Parvinen K, Dieckmann U, Gyllenberg M, Metz JAJ:
Evolution of Dispersal in Metapopulations with Local Density Dependence and Demographic Stochasticity.
IIASA Interim Report IR-00-035.
- No. 46 Doebeli M, Dieckmann, U:
Evolutionary Branching and Sympatric Speciation Caused by Different Types of Ecological Interactions.
IIASA Interim Report IR-00-040.

Issues of the IIASA Studies in Adaptive Dynamics series can be obtained free of charge.
Please contact:

Adaptive Dynamics Network
International Institute for Applied Systems Analysis
Schlossplatz 1
A-2361 Laxenburg
Austria

Telephone +43 2236 807, Telefax +43 2236 71313, E-Mail adn@iiasa.ac.at,
Internet <http://www.iiasa.ac.at/Research/ADN>

Contents

1	Introduction	1
2	Evolutionary branching in models for symmetric and asymmetric competition	4
3	Evolutionary branching in coevolutionary models for mutualisms	7
4	Evolutionary branching in coevolutionary predator-prey models	12
5	Evolutionary branching in sexual populations	14
6	Discussion	20

Abstract

Evolutionary branching occurs when frequency-dependent selection splits a phenotypically monomorphic population into two distinct phenotypic clusters. A prerequisite for evolutionary branching is that directional selection drives the population towards a fitness minimum in phenotype space. This paper demonstrates that selection regimes leading to evolutionary branching readily arise from a wide variety of different ecological interactions within and between species. We use classical ecological models for symmetric and asymmetric competition, for mutualism, and for predator-prey interactions to describe evolving populations with continuously varying characters. For these models, we investigate the ecological and evolutionary conditions that allow for evolutionary branching and establish that branching is a generic and robust phenomenon. Evolutionary branching becomes a model for sympatric speciation when population genetics and mating mechanisms are incorporated into ecological models. In sexual populations with random mating, the continual production of intermediate phenotypes from two incipient branches prevents evolutionary branching. In contrast, when mating is assortative for the ecological characters under study, evolutionary branching is possible in sexual populations and can lead to speciation. Therefore, we also study the evolution of assortative mating as a quantitative character. We show that evolution under branching conditions selects for assortativeness and thus allows sexual populations to escape from fitness minima. We conclude that evolutionary branching offers a general basis for understanding adaptive speciation and radiation under a wide range of different ecological conditions.

About the Authors

Michael Doebeli
Departments of Zoology and Mathematics
University of British Columbia
6270 University Boulevard
Vancouver, B.C., Canada V6T 1Z4

Ulf Dieckmann
Adaptive Dynamics Network
International Institute for Applied Systems Analysis
A-2361 Laxenburg, Austria

Acknowledgments

We thank P. Abrams, M. Ackermann, S. Geritz, M. Heino, É. Kisdi, R. Luethy, and D. Schluter for very helpful comments. R. Luethy greatly helped us to improve the analysis of the model for coevolution in mutualisms.

Evolutionary Branching and Sympatric Speciation Caused by Different Types of Ecological Interactions

Michael Doebeli

Ulf Dieckmann

1 Introduction

Understanding the origin of new species remains to be one of the core problems in evolutionary biology. Whether or not one believes that there are many (Hutchinson 1959) or actually only a few species (Felsenstein 1981) compared to what one would expect from general niche space considerations, the actual mechanisms by which a phenotypic cluster of individuals splits into two distinct descendent clusters with restricted gene flow between them are only poorly understood. Traditionally, there are two basic approaches to understanding this evolutionary process. In the first, subpopulations of a given species are thought to become geographically isolated, after which they follow separate evolutionary paths, eventually leading to different species that are reproductively isolated even after secondary contact (Mayr 1963). Since the particular cause for geographic isolation is not part of the description, this is a kind of 'black box' mechanism. Given such isolation, it is rather easy to imagine that separate evolution in different habitats during many generations would lead to phenotypic differences and to reproductive incompatibility. Indeed, such allopatric speciation is quite well understood experimentally (Rice and Hostert 1993). Theoretically, the basic ingredients for allopatric speciation are isolated habitats in which selection pressures are different and in which different realizations of the stochastic process underlying genetic drift can occur. Habitat differences have also been the starting point for the second traditional approach, which studies the conditions under which speciation occurs in sympatry, i.e. when gene flow is possible between two incipient species. In this approach it is also often assumed that there are different habitats favoring different genotypes, but the different genotypes occur sympatrically and are pooled for mating. One then studies the conditions necessary for reproductive isolation to evolve between the genotypes that are favored in the different habitats (e.g. Maynard Smith 1966, Felsenstein 1981, Kawecki 1996). Depending on whether one thinks that such conditions are easy or hard to meet, one leans towards sympatric or allopatric speciation as the dominant mode for the origin of new species.

The difficulties in the theory of sympatric speciation are two-fold (Kondrashov and Mina 1986). On the one hand, ecological conditions must induce disruptive selection in such a way that the population does not become monomorphic for one of the favored phenotypes. On the other hand, given such ecological conditions, the mating system must evolve such that reproductive isolation ensues between the phenotypes that are favored by disruptive selection. In the history of the theory of sympatric speciation the focus has shifted between these two difficulties (e.g. Maynard Smith 1966, Felsenstein 1981). In this paper, we present an integrative framework for studying sympatric speciation that

simultaneously addresses both issues.

In genetic models for sympatric speciation the maintenance of a polymorphism in the presence of disruptive selection is often a delicate affair (Maynard Smith 1966, Udovic 1980, Felsenstein 1981, Kawecki 1996), thus narrowing the basis for the evolution of reproductive isolation. In our theory, such problems of ecological fine tuning do not arise, because disruptive selection is not an externally imposed assumption. Instead, the system automatically converges to a state in which it experiences disruptive selection, and in which the conditions for the evolution of polymorphisms are satisfied. We will show that such a course of events is a general phenomenon and can occur under a wide range of ecological interactions.

Our approach is based on the phenomenon of *evolutionary branching* and on the theory of adaptive dynamics (Metz et al. 1992, Dieckmann 1994, Metz et al. 1996, Dieckmann and Law 1996, Geritz et al. 1997, Dieckmann 1997, Geritz et al. 1998, Meszéna et al. 2000). In this approach ecological interactions are the evolutionary driving force, and the feedback between evolutionary change and the ecological conditions experienced by individuals is considered. Evolutionary dynamics are studied using the concept of invasion fitness (Metz et al. 1992). This quantity measures the long-term per capita growth rate of a rare mutant in an environment that is determined by externally fixed parameters on the one hand, and by the population density and the phenotype of the resident population(s) on the other hand. In particular, the invasion fitness of a mutant explicitly depends on the resident phenotype, reflecting the consequences of frequency-dependent ecological interactions. For deriving analytical results, one makes a number of simplifying assumptions, e.g. that mutations are sufficiently rare so that mutants encounter monomorphic resident populations which are at their ecological equilibrium. This corresponds to assuming a separation of ecological and evolutionary time scales, with the ecological dynamics occurring faster than the evolutionary dynamics. Under the further assumption that mutants whose invasion fitness is larger than zero can not only invade (with some probability) but can also replace the former resident and thus become the new resident, it is possible to study the evolutionary dynamics by analyzing a function $f(y, x)$ describing the invasion fitness of a mutant y in a resident population x . Here x may be a multi-dimensional vector, either because the trait under study has more than one component, or because there are more than one species involved. Evolutionary dynamics then follows selection gradients determined by derivatives of the invasion fitness function $f(y, x)$, as will be explained in the next sections.

Phenotypes of special interest are those where the selection gradient is zero, and the first question is whether these points actually are evolutionary attractors. In classical optimization models of evolution, reaching such attractors implies that evolution comes to a halt, because evolutionary attractors only occur at fitness maxima. However, in the wider framework of adaptive dynamics this need not be the case (Geritz et al. 1998, Meszéna et al. 2000). When frequency-dependent ecological interactions drive the evolutionary process, it is possible that an evolutionary attractor represents a fitness minimum at which the population experiences disruptive selection. In fact, evolution towards fitness minima due to frequency-dependent selection has been found in a number of previous studies (e.g. Ludwig and Levin 1991, Christiansen 1991, Abrams et al. 1993). Adaptive dynamics takes these analyses one step further by asking what happens after the fitness minimum has been reached. The ensuing evolutionary dynamics can indeed be very interesting, because after attaining the fitness minimum the population may split into two distinct and diverging phenotypic clusters. Thus, in adaptive dynamics, evolutionary convergence towards a fitness minimum can lead to evolutionary branching. It is important to note

that such a sequence of events is entirely explained by frequency-dependent ecological interactions, and does not require any extrinsically imposed disruptive selection pressures.

Evolutionary branching has been found in a number of models including models for the evolution of dispersal rates (Doebeli and Ruxton 1997, Parvinen 1999) and for the evolution of seed size (Geritz et al. 1999), in host-parasite models (Koella and Doebeli 1999, Boots and Haraguchi 1999), in models for habitat specialization (Geritz et al. 1998), and in models for the evolution of quantitative characters determining competitive interactions (Metz et al. 1996, Dieckmann and Doebeli 1999, Kisdi 1999). These results already hint at the possibility that evolutionary branching might be a ubiquitous phenomenon. To support and substantiate this claim in a more systematic way, our goal in this paper is to demonstrate the phenomenon of evolutionary branching in a number of classical models covering a wide range of ecological interactions. In Section 2, we review evolutionary branching in single-species models for symmetric and asymmetric resource competition. This section serves to recall basic concepts of the theory of adaptive dynamics and paves the way for the more complicated two-species models analyzed in Sections 3 and 4, where we extend our considerations to coevolutionary scenarios and show that evolutionary branching is a generic and robust phenomenon in ecological models for two mutualistic species and for predator-prey interactions. Our analytical theory is always supplemented by computer simulations of stochastic individual-based models, which overall confirm the analytical results and thereby show that the assumptions used to derive the analytical theory are sensible.

The framework of adaptive dynamics has so far mainly been developed as an asexual theory that lacks population genetic considerations. In particular, most previous models of evolutionary branching have assumed clonal organisms (for recent exceptions see Van Dooren 1999, Dieckmann and Doebeli 1999, Kisdi and Geritz 1999). Therefore, the theory of adaptive dynamics may appear not to be relevant for evolution in sexual populations. Quite to the contrary, we suggest that adaptive dynamics, and in particular processes of evolutionary branching, can have interesting implications for understanding general evolutionary principles. In particular, we think that evolutionary branching is a unifying concept that helps us to understand speciation under a wide range of ecological conditions. However, for evolutionary branching to become a model for sympatric speciation, population genetics and mating mechanisms must be incorporated into the underlying ecological models. In this paper, we do this by assuming that the quantitative characters influencing ecological interactions are determined by many additive diploid loci (for an alternative approach see Kisdi and Geritz 1999). In fact, there is a valid caveat against considering evolutionary branching in asexual models as a basis for understanding aspects of speciation: in sexual populations, branching could be prevented by the continual production of intermediate offspring phenotypes through recombination between incipient branches. If mating is random, this indeed is the case. However, if mating is assortative with respect to the ecological characters under study, evolutionary branching is possible in sexual populations and can lead to speciation. This will be demonstrated in Section 5, where we will also show that assortative mating is a mechanism allowing the escape from fitness minima. In initially randomly mating populations under branching conditions, assortativeness can therefore be favored by natural selection. Our conclusion is that evolutionary branching can serve as a general paradigm for sympatric speciation once multi-locus genetics and the evolution of assortative mating are included in the underlying ecological models.

2 Evolutionary branching in models for symmetric and asymmetric competition

In order to introduce some basic concepts and notation, we begin with single-species competition models that are based on the classical Lotka-Volterra population dynamics. More extensive discussions of related material can be found in Metz et al. (1996), Dieckmann and Doebeli (1999) and Kisdi (1999). We assume that individuals are characterized by a quantitative trait x , e.g. body size, which affects intraspecific competition between individuals. Let $N(x, t)$ be the population density of individuals with character value x at time t . Then their ecological dynamics are given by

$$\frac{dN(x, t)}{dt} = r \cdot N(x, t) \cdot \left(1 - \frac{N_{eff}(x, t)}{K(x)}\right). \quad (1)$$

Here $K(x)$ is the carrying capacity of populations that are monomorphic for trait x . For simplicity, it is assumed that $K(x)$ varies with the trait x , and that the intrinsic growth rate r is independent of x . The quantity $N_{eff}(x, t)$ is the effective population density that an individual with character value x experiences at time t . The effective density is determined by the distribution of phenotypes in the population, and by a function $\alpha(x - y)$ which measures the strength of competition exerted by an individual with phenotype y on an individual with phenotype x . Here we take the function $\alpha(x - y)$ to be

$$\alpha(x - y) = \exp(\sigma_\alpha^2 \beta^2 / 2) \cdot \exp\left(\frac{-(x - y + \sigma_\alpha^2 \beta)^2}{2\sigma_\alpha^2}\right), \quad (2)$$

which has been previously used in the study of character displacement (Rummel and Roughgarden 1985, Taper and Case 1992). For $\beta = 0$, this function describes symmetric competition, that is, $\alpha(x - y)$ is a symmetric function of the difference $x - y$ with a maximum at 0, see Fig. 1A. This implies that individuals with similar phenotypes compete more strongly with each other than individuals with dissimilar phenotypes, as, e.g., when beak size in birds determines the type of seeds eaten. If $\beta > 0$, $\alpha(x - y)$ describes asymmetric competition, with $\alpha(x - y)$ being maximal for some negative difference in character values, see Fig. 1A. This implies that larger individuals tend to have a competitive advantage over smaller individuals. In contrast to the asymmetric competition models in Law et al. (1997) and in Kisdi (1999), in which the competitive advantage or disadvantage increased monotonically with phenotypic distance, the function $\alpha(x - y)$ used here implies that competition between very different phenotypes is always weak. Such asymmetric competition would for example occur when overlaps in resource utilization between different phenotypes are asymmetric but vanish with increasing phenotypic distance.

For both symmetric and asymmetric competition, the effective density $N_{eff}(x, t)$ is obtained as a weighted sum over all densities $N(x, t)$:

$$N_{eff}(x, t) = \int \alpha(x - x') \cdot N(x', t) \cdot dx'. \quad (3)$$

To determine the invasion fitness $f(y, x)$ of a rare mutant y in a resident population that is monomorphic for the character value x , we assume that mutants invade sufficiently rarely, so that residents are always at (or very close to) their ecological equilibrium $K(x)$ when new mutants appear. Since the mutant is initially rare, its own density is negligible compared to that of the resident, and hence the effective density that the mutant experiences is simply the resident density $K(x)$ weighted by the strength of competition $\alpha(y - x)$ between the

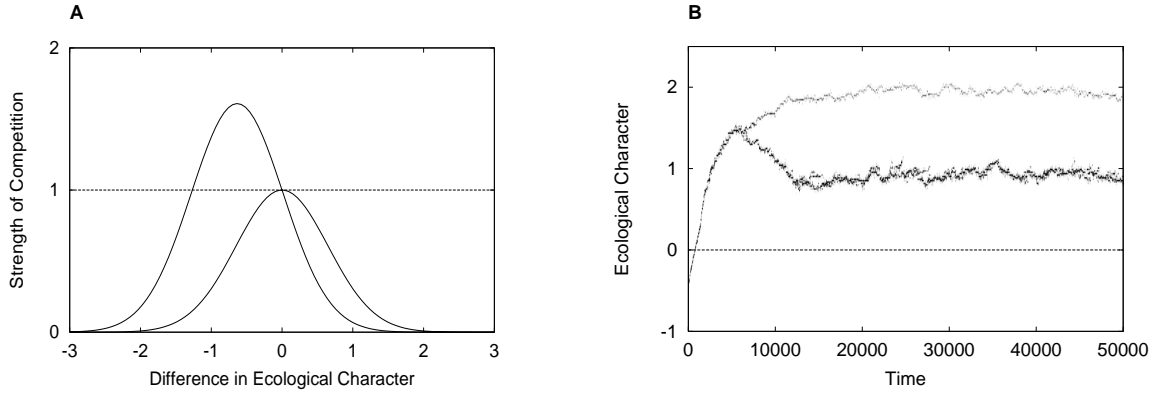


Figure 1: **Asexual resource competition and evolutionary branching.**

A. Strength of competition as a function of phenotypic difference between competitors. Symmetric competition is described by the function $\alpha(x - y)$ (see text) with $\beta = 0$. An example of asymmetric competition is shown for $\beta = 1.5$. σ_α is set to 0.65 in both cases. B. Evolutionary dynamics with the asymmetric competition function shown in 1A. The distribution of character values (shown by scales of gray; black: highest frequency, white: absence) first converges toward the ecological character's evolutionary branching point. Since higher values of the ecological character confer an advantage under asymmetric competition, branching occurs at a phenotype that is larger than the one with maximal resources, $x_0 = 0$. At the branching point directional selection turns into disruptive selection which splits the character distribution into two phenotypic clusters. The two resulting branches differ in their population size: the upper branch, which is further away from the carrying capacity's maximum, consists of fewer individuals, although its individuals possess a competitive advantage over those in the lower branch. Parameters: $r = 1$, $K_0 = 1000$, $\sigma_K = 1$, $\sigma_\alpha = 0.65$, $\beta = 1.5$, $x_0 = 0$.

mutant and the resident. Thus, in the initial phase of the invasion when the mutant is rare, the population dynamics of the mutant are given by

$$\frac{dN(y, t)}{dt} = r \cdot N(y, t) \cdot \left(1 - \frac{\alpha(y - x) \cdot K(x)}{K(y)}\right). \quad (4)$$

The invasion fitness of the mutant is its long-term per capita growth rate when rare (Metz et al. 1992, Dieckmann 1994, Rand et al. 1994, Metz et al. 1996, Dieckmann and Law 1996), hence the invasion fitness of the mutant y in the resident x is

$$f(y, x) = r \cdot \left(1 - \frac{\alpha(y - x) \cdot K(x)}{K(y)}\right). \quad (5)$$

To determine the evolutionary dynamics one calculates the derivative of $f(y, x)$ with respect to y and evaluates it at the resident value x . Thus, the crucial quantity is the selection gradient

$$g(x) = \left. \frac{\partial f(y, x)}{\partial y} \right|_{y=x}. \quad (6)$$

If $g(x) > 0$, then invasion fitness increases for mutants with higher trait values than the resident, while invasion fitness decreases for mutant with lower trait values. Since $f(x, x) = 0$ by necessity (i.e., the resident neither grows nor declines in its own equilibrium population), this means that mutants with higher trait values can invade, i.e. are favored

by natural selection, while mutants with lower trait values are selected against. Analogous statements in the opposite direction hold for $g(x) < 0$. Thus, as long as $g(x) \neq 0$, selection is directional. For the evolutionary dynamics those values x^* are important for which $g(x^*) = 0$. These trait values are called evolutionarily singular (Metz et al. 1996, Geritz et al. 1998). A singular value x^* is an attractor for the evolutionary dynamics if and only if

$$\frac{dg(x)}{dx}\Big|_{x=x^*} < 0, \quad (7)$$

for in that case $g(x) < 0$ for $x > x^*$ and $g(x) > 0$ for $x < x^*$, hence lower trait values are favored when the resident is larger than x^* and higher trait values are favored when the resident is smaller than the singular value x^* . Singular points that are not attractors are of little practical interest, since even starting with resident populations that are very close to such a point, evolution will drive the trait away from the singular point. A critical issue to realize, however, is that there are two different types of evolutionary attractors.

This can be seen by considering the second derivative of the invasion fitness at the evolutionary attractor x^* . If $\frac{\partial^2 f(y, x^*)}{\partial y^2}\Big|_{y=x^*} < 0$, then the point x^* is a fitness maximum with respect to the mutant trait value y (recall that $\frac{\partial f(y, x^*)}{\partial y}\Big|_{y=x^*} = g(x^*) = 0$, since we assume that x^* is a singular point). The evolutionary attractor x^* is therefore stable against invasion of neighboring phenotypes, i.e., it is an evolutionarily stable strategy, or ESS, as it is traditionally called.

If x^* is an ESS, selection first drives the population towards x^* and then comes to a halt, i.e., x^* is an evolutionary equilibrium, also called a continuously stable strategy or CSS (Eshel 1983). A very different scenario occurs when

$$\frac{\partial^2 f(y, x^*)}{\partial y^2}\Big|_{y=x^*} > 0. \quad (8)$$

In this case evolution still drives the population towards x^* , since we assumed the singular point x^* to be an evolutionary attractor. However, once at x^* , the population is actually located at a fitness minimum and therefore experiences disruptive selection. As a consequence, evolutionary branching can occur, that is, the population can split into two different and diverging phenotypic clusters (Metz et al. 1996, Geritz et al. 1997, Geritz et al. 1998).

Metz et al. (1996), Dieckmann and Doebeli (1999) and Kisdi (1999) have shown that evolution towards a fitness minimum, and hence evolutionary branching, is a generic phenomenon in models for resource competition similar to the ones described above. To complement this theory, we use eq. (2) above for the function $\alpha(x - y)$ describing the frequency dependence in the competitive interactions, and we take the resource distribution to be of Gaussian form with a maximum at some intermediate phenotype x_0 :

$$K(x) = K_0 \cdot \exp\left(\frac{-(x - x_0)^2}{2\sigma_K^2}\right). \quad (9)$$

We then calculate $g(x)$, eq. (6), from eqs. (4) and (5) as

$$\begin{aligned} g(x) &= -r(\alpha'(0) - \frac{K'(x)}{K(x)}) \\ &= -r \cdot \left(\frac{x - x_0}{\sigma_K^2} - \beta\right). \end{aligned} \quad (10)$$

It follows that $g(x^*) = 0$ for $x^* = x_0 + \beta\sigma_K^2$. Note that the singular point is always larger than the trait value maximizing the carrying capacity. The derivative of $g(x)$ at the

singular point x^* is

$$\left. \frac{dg(x)}{dx} \right|_{x=x^*} = -\frac{r}{\sigma_K^2} < 0, \quad (11)$$

hence x^* always is an evolutionary attractor. In addition, straightforward calculations reveal that condition (8) becomes

$$\left. \frac{\partial^2 f(y, x^*)}{\partial y^2} \right|_{y=x^*} = r \cdot \left(\frac{1}{\sigma_\alpha^2} - \frac{1}{\sigma_K^2} \right) > 0. \quad (12)$$

This condition is satisfied, and hence x^* is a fitness minimum, if

$$\sigma_\alpha < \sigma_K. \quad (13)$$

Thus, for a given width σ_K of the resource distribution, the singular point x^* is a branching point, i.e. an evolutionarily attracting fitness minimum, if the parameter σ_α is small enough. Since σ_α measures the strength of the frequency dependence in the competitive interactions, this implies that, in the model considered, asymmetric competition leads to evolutionary branching whenever the frequency dependence is strong enough. (Note that this result is also true if competition is symmetric, i.e. if $\beta = 0$, see Dieckmann and Doebeli 1999). An example of the corresponding evolutionary dynamics is shown in Figure 1B. (This figure is based on an individual-based model, the details of which are described in Appendix A.) Starting with small phenotypic values, the evolutionary dynamics show a steady increase in the trait value until the system reaches the branching point. Because larger phenotypes have an intrinsic advantage, branching occurs at a phenotypic value that is larger than the value that maximizes the carrying capacity. After branching, the phenotype in one branch continues to increase, while in the other branch it decreases again. Note that the population size is different in the two branches, with the branch that consists of larger individuals and is located farther away from the optimal carrying capacity having fewer individuals. For a more thorough treatment and additional examples of evolutionary branching under symmetric and asymmetric competition see Dieckmann and Doebeli 1999; Kisdi 1999.

3 Evolutionary branching in coevolutionary models for mutualisms

The classical models for studying processes of diversification are based on competition (MacArthur and Levins 1967), and other types of ecological interactions have received less attention. In the next two sections we therefore examine evolutionary branching in models for mutualism and for predator-prey interactions. In the models considered, coevolutionary dynamics of quantitative characters in two separate species are driven by interspecific ecological interactions. The mathematics of adaptive dynamics in two-species models are more involved than in their one-species counterparts (Abrams et al. 1993, Dieckmann and Law 1996, Matessi and Di Pasquale 1996, Metz et al. 1996, Marrow et al. 1996, Leimar 2000). However, conditions for evolutionary branching are analogous to those for the one-species models: frequency-dependent selection results in convergence to an evolutionary attractor on which either one or both species find themselves at fitness minima. Accordingly, evolutionary branching can occur either in only one species, or in both species simultaneously, or in both species sequentially (see below).

In the present section we extend standard Lotka-Volterra models for mutualisms to coevolutionary models in which quantitative characters affect the strength of mutualistic

interactions. Simple models of mutualistic dynamics (see, for example, Vandermeer and Boucher 1978) arise in a very similar way from one-species models as do two-species competition models, namely by adding a second density-dependent term to per capita growth rates in the basic Lotka-Volterra equation for one species:

$$\begin{aligned}\frac{dN_1(t)}{dt} &= r_1 \cdot N_1(t) \cdot \left(1 - \frac{N_1(t)}{K_1} + a_{12} \cdot N_2(t)\right) \\ \frac{dN_2(t)}{dt} &= r_2 \cdot N_2(t) \cdot \left(1 - \frac{N_2(t)}{K_2} + a_{21} \cdot N_1(t)\right).\end{aligned}\tag{14}$$

Here a_{12} and a_{21} are positive real numbers describing the strength of mutualistic support that the two species provide to each other. Vandermeer and Boucher (1975) have studied the ecological dynamics of system (14) (for an extension see Bever (1999)), but we are not aware of extensions of this system to situations where the interaction coefficients a_{ij} are determined by evolving quantitative characters in the two species. Such an extension can be formulated based on the theory of mutualism developed in Kiestler et al. (1984), who analyzed coevolution in mutualistic systems using the quantitative genetics framework of Lande (1982). They considered a pair of mutualistic species, e.g. a flowering plant and its insect pollinator, in which quantitative characters, e.g. related to flower morphology and to feeding apparatus, determine the level of mutualistic support. Specifically, let x_1 and x_2 denote the characters in the two species, and let $p_1(x_1)$ and $p_2(x_2)$ be the phenotype distributions in the two species. In other words, the frequency of individuals with phenotypes in the interval $(x_1, x_1 + dx_1)$ in species 1 is $p_1(x_1)dx_1$, and similarly for $p_2(x_2)$. Following Kiestler et al. (1984), we assume that the support $\alpha_{12}(x_1 - x_2)$ that a species-1 individual with phenotype x_1 receives from a species-2 individual with phenotype x_2 is given by

$$a_{12} \cdot \frac{\alpha(x_1 - x_2)}{\int \alpha(x'_1 - x_2) \cdot p_1(x'_1) \cdot dx'_1},\tag{15}$$

where $\alpha(x_1 - x_2) = \exp\left(\frac{-(x_1 - x_2)^2}{2\sigma_\alpha^2}\right)$, and where a_{12} is a positive constant. This expression takes into account that the total amount of support provided to species-1 individuals by any one species-2 individual is limited and equal to a_{12} , and that this total amount is distributed among species-1 individuals according to the weights $\alpha(x_1 - x_2)$. Thus, individuals of species 2 do not vary in their total amount of support given to species-1 individuals. Instead, they vary in how the support is distributed among species-1 individuals according to differences in character values. The support $\alpha_{21}(x_2 - x_1)$ provided by species-1 individuals with phenotype x_1 to species-2 individuals with phenotype x_2 is obtained in an analogous way as

$$a_{21} \cdot \frac{\alpha(x_2 - x_1)}{\int \alpha(x'_2 - x_1) \cdot p_2(x'_2) \cdot dx'_2},\tag{16}$$

where a_{21} is the total amount of support provided by species-1 individuals. The rationale behind expressions (15) and (16) is applicable to many mutualistic species pairs, as is explained in detail in Kiestler et al. (1984). Here we combine their approach with the classical models of Vandermeer and Boucher (1978), eq. (14) above.

For populations that are monomorphic for character values x_1 and x_2 , respectively, the ecological dynamics are given by

$$\begin{aligned}\frac{dN_1(x_1, t)}{dt} &= r_1 \cdot N_1(x_1, t) \cdot \left(1 - \frac{N_1(x_1, t)}{K_1(x_1)} + a_{12} \cdot N_2(x_2, t)\right) \\ \frac{dN_2(x_2, t)}{dt} &= r_2 \cdot N_2(x_2, t) \cdot \left(1 - \frac{N_2(x_2, t)}{K_2(x_2)} + a_{21} \cdot N_1(x_1, t)\right).\end{aligned}\tag{17}$$

Here $N_1(x_1, t)$ and $N_2(x_2, t)$ are the population sizes of the monomorphic populations of species 1 and 2 at time t , and we again assume that individuals do not vary within species in their intrinsic growth rates, but that, owing to variation in resource availability, there is within-species variation in the carrying capacities $K_1(x_1)$ and $K_2(x_2)$. These functions are assumed to be of the form $K_1(x) = K_{11} + K_{12}N(x_{10}, \sigma_{K_1}, x)$ for species 1 and $K_2(x) = K_{21} + K_{22}N(x_{20}, \sigma_{K_2}, x)$ for species 2, where the $N(x_{i0}, \sigma_{K_i}, x)$ are Gaussian functions of the variable x with maximum at $x = x_{i0}$ and variance equal to σ_{K_i} . The parameters K_{i1} represent a uniform background level of resources. Note that the maxima x_{10} and x_{20} of the two resource distributions will generally occur at different character values in the two species.

As in the previous section, we assume a separation of ecological and evolutionary time scales in order to analyze the coevolutionary dynamics of the two traits x_1 and x_2 . That is, we start from monomorphic resident populations at their ecological equilibrium and then determine the fate of rare mutants as they arise in both species. For certain choices of parameters a_{12} and a_{21} , the ecological system (14) does not converge towards an equilibrium (Vandermeer and Boucher 1978), and instead population sizes increase without bounds. Here we restrict our attention to those biologically feasible cases in which the monomorphic residents reach a stable equilibrium.

The equilibrium population sizes of such resident populations are functions $N_1^*(x_1, x_2)$ and $N_2^*(x_1, x_2)$ of the resident character values x_1, x_2 in both species and can be calculated from (17) by setting right-hand sides equal to zero. Given these values, the dynamics of a rare mutant y_1 in species 1 are given by

$$\frac{dN_1(y_1, t)}{dt} = r_1 \cdot N_1(y_1, t) \cdot \left(1 - \frac{N_1^*(x_1, x_2)}{K_1(y_1)} + a_{12} \frac{\alpha(y_1 - x_2)}{\alpha(x_1 - x_2)} \cdot N_2^*(x_1, x_2)\right). \quad (18)$$

Here the support given to species-1 individuals with character value y_1 by species-2 individuals with character value x_2 is calculated from eq. (15) using the fact that the mutant y_1 is very rare initially, so that the phenotype distribution $p_1(x'_1)$ in species 1 is concentrated at the resident phenotype x_1 , $\int \alpha(x'_1 - x_2) p_1(x'_1) dx'_1 = \alpha(x_1 - x_2)$. Thus, the initial per capita growth rate of a rare species-1 mutant y_1 in the resident (x_1, x_2) -population, i.e. the invasion fitness $f_1(y_1, x_1, x_2)$, is given by

$$f_1(y_1, x_1, x_2) = r_1 \cdot \left(1 - \frac{N_1^*(x_1, x_2)}{K_1(y_1)} + a_{12} \frac{\alpha(y_1 - x_2)}{\alpha(x_1 - x_2)} \cdot N_2^*(x_1, x_2)\right). \quad (19)$$

Analogously, the invasion fitness of a rare species-2 mutant y_2 in the resident (x_1, x_2) -population is

$$f_2(y_2, x_1, x_2) = r_2 \cdot \left(1 - \frac{N_2^*(x_1, x_2)}{K_2(y_2)} + a_{21} \frac{\alpha(y_2 - x_1)}{\alpha(x_2 - x_1)} \cdot N_1^*(x_1, x_2)\right). \quad (20)$$

The evolutionary dynamics of this system are determined by evaluating the derivatives of the invasion fitness functions with respect to the mutant trait value at the respective resident values, i.e. by

$$g_1(x_1, x_2) = \left. \frac{\partial f_1(y_1, x_1, x_2)}{\partial y_1} \right|_{y_1=x_1} \quad (21)$$

and by

$$g_2(x_1, x_2) = \left. \frac{\partial f_2(y_2, x_1, x_2)}{\partial y_2} \right|_{y_2=x_2}. \quad (22)$$

In analogy with the 1-dimensional system considered in the last section, the singular points of the 2-dimensional coevolutionary system studied here are resident values x_1^*, x_2^*

such that $g_1(x_1^*, x_2^*) = g_2(x_1^*, x_2^*) = 0$ (Dieckmann and Law 1996, Metz et al. 1996, Geritz et al. 1998, Leimar 2000). Whereas in 1-dimensional systems singular points are either attractors or repellers for the evolutionary dynamics, 2-dimensional systems offer more possibilities, including saddle points (Dieckmann and Law 1996, Leimar 2000) and cyclic evolutionary dynamics (Dieckmann et al. 1995, Marrow et al. 1996). In Appendix B we indicate how to determine the evolutionary stability properties of a singular point (x_1^*, x_2^*) . We are interested in singular points that are evolutionary attractors, and we wish to determine the conditions under which those evolutionary attractors are also fitness minima for either one or both of the trait values x_1^* and x_2^* . That is, we are looking for evolutionary attractors (x_1^*, x_2^*) for which the invasion fitness functions have a minimum with respect to the mutant trait values,

$$\frac{\partial^2 f_1(y_1, x_1^*, x_2^*)}{\partial y_1^2} \Big|_{y_1=x_1^*} > 0 \quad (23)$$

and

$$\frac{\partial^2 f_2(y_2, x_1^*, x_2^*)}{\partial y_2^2} \Big|_{y_2=x_2^*} > 0. \quad (24)$$

If either one or both of these conditions are satisfied at an evolutionary attractor, then the evolutionary dynamics will first converge to the singular point (x_1^*, x_2^*) , after which evolutionary branching will occur in one or both species (see Appendix B). It is not our aim here to give an exhaustive classification of all the possibilities of coevolutionary dynamics in our model for mutualism (although this would seem to be a worthwhile and interesting endeavor). Instead, we confine ourselves to pointing out that, again, evolutionary branching is a generic feature of this model and proceed by describing the general conditions under which branching occurs.

We first consider a symmetric case, in which the mutual support is equal for both species (i.e., $a_{12} = a_{21}$), and in which the two species only differ in the location of the maxima of their resource distributions. Then, if these maxima are far enough apart, evolutionary branching is likely to occur. This can be understood as follows. If the resource distributions have different maxima, say $x_{10} < x_{20}$, then there is an evolutionarily singular point (x_1^*, x_2^*) which lies symmetrically between these maxima, that is, such that $x_{10} < x_1^* < x_2^* < x_{20}$ and such that the distance to their respective resource maxima is the same in both species (Appendix B). Such a configuration results from a tradeoff between being close to the own resource maximum and being close to the phenotype of the other species in order to benefit from its mutualistic support. With increasing distance between the two resource maxima, the distance between the singular values x_1^* and x_2^* constituting a symmetric singular point increases as well. Intuitively speaking, an increasing distance between the resource distributions increases the strain on the evolutionary compromise between the two species. If this strain is large enough, and if the symmetric singular point is an evolutionary attractor (for technical details see Appendix B), then simultaneous evolutionary branching in both species occurs, an example of which is shown in Figure 2A.

The evolutionary outcome resulting from branching is interesting: branching leads to two species pairs, in each of which one of the species is close to its resource optimum and provides a large amount of mutualistic support to the other species in the pair, which in turn is far away from its resource optimum and therefore can give only little support. Thus, in each of the two original species, branching leads to one mutualistic branch, which is close to its resource optimum, and one “exploitative branch”, which is far from its resource optimum, and each branch interacts mainly with its respective opposite in the

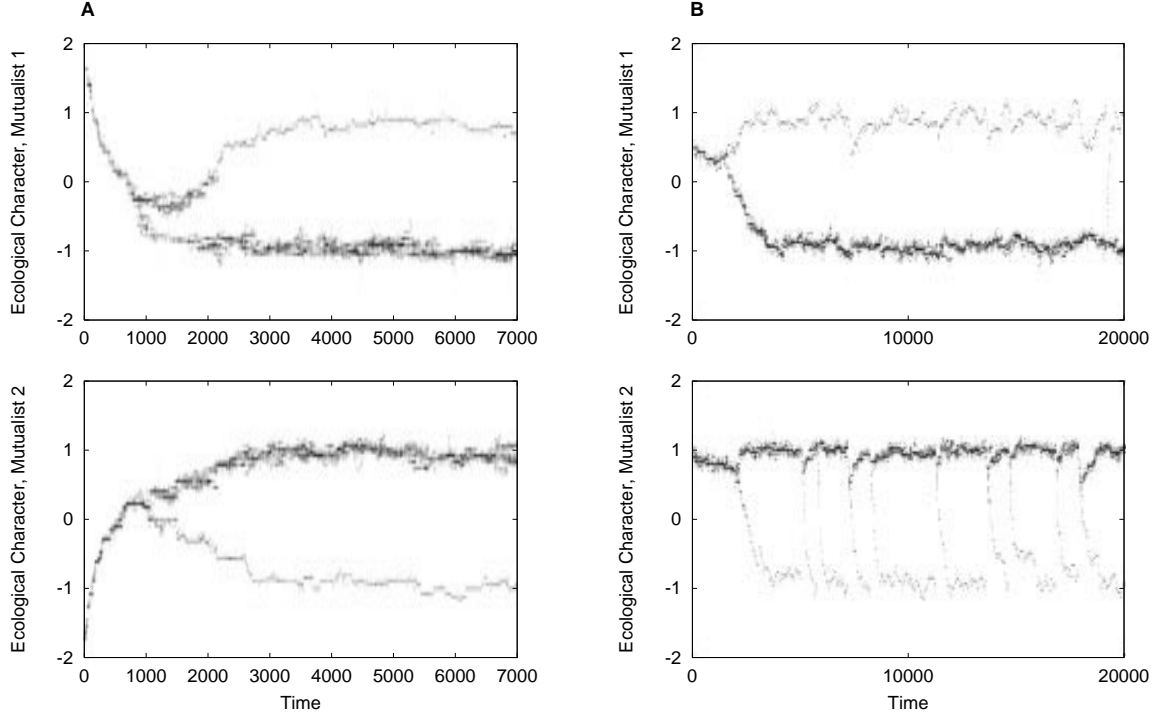


Figure 2: **Asexual coevolutionary dynamics of mutualistic interactions.**

A. Simultaneous evolutionary branching in two symmetric mutualistic species leads to two asymmetric pairs of mutually specialized populations. In each pair, one branch is close to its resource optimum (high abundance), while the other (low abundance) mainly exploits its mutualistic partner. Parameters: $r_1 = r_2 = 1$, $K_1(x_1) = K_{11} + K_{12} \exp(-(x_1 - x_{10})^2/2\sigma_{K_1})$, $K_2(x_2) = K_{21} + K_{22} \exp(-(x_2 - x_{20})^2/2\sigma_{K_2})$, $K_{11} = K_{21} = 300$, $K_{12} = K_{22} = 400$, $x_{10} = -1$, $x_{20} = 1$, $\sigma_{K_1} = \sigma_{K_2} = 1.0$, $a_{12} = a_{21} = 0.00016$, $\alpha(x_1 - x_2) = \exp(-(x_1 - x_2)^2/2\sigma_\alpha^2)$, $\sigma_\alpha = 0.4$, $\mu_1 = \mu_2 = 0.01$, $\sigma_{M_1} = \sigma_{M_2} = 0.05$.

B. Primary evolutionary branching in one of two asymmetric mutualistic species results in repeated secondary evolutionary branching and extinction in the partner species. After the two branches of the first mutualist are established, subsequent branching occurs in the second mutualist. This causes one of the resulting secondary branches to evolve to exploit the highly abundant branch of its mutualistic partner. This adaptation requires a large departure from the second species' resource optimum, upon which the newly established branch goes extinct again, triggering a continual cyclic sequence of repeated branching and extinction in the second mutualist. Parameters are the same as in (a), except for $K_{11} = K_{21} = 50$, $K_{12} = K_{22} = 200$, $\sigma_{K_1} = 0.8$, $\sigma_{K_2} = 0.55$, $a_{12} = a_{21} = 0.001$, $\sigma_\alpha = 0.3$.

other species. In this way, an initially symmetric configuration, with each species being at equal distance from its resource optimum and giving equal amounts of support to its partner, evolves into two asymmetric species pairs, in each of which one species essentially only survives due to the mutualistic support from the other species.

These results are robust in the sense that introducing asymmetries in the system will not qualitatively change them. With large asymmetries, however, new phenomena such as repetitive evolutionary branching can be observed. An example is shown in Figure 2B. Here the evolutionarily attracting singular point (x_1^*, x_2^*) is a fitness minimum for only one of the two species and, therefore, branching first occurs only in that species. Subsequently, the evolving dimorphism also induces evolutionary branching in the second species, because emergence of the two branches in the first species leads to disruptive selection for mutualistic support in the second species. The two branches in the second species, however, do not persist indefinitely, and, instead, the branch that is far away from its resource optimum eventually goes extinct. The remaining branch then again undergoes secondary branching, which leads to a repeating, cyclic pattern of speciation and extinction in this lineage. That frequency-dependent ecological interactions can lead to adaptation-driven extinctions has been observed before (see, e.g. Dieckmann et al. 1995).

4 Evolutionary branching in coevolutionary predator-prey models

To complete our survey of evolutionary branching under different fundamental types of ecological interactions, we study coevolution in predator-prey systems. For this purpose, we use an extension of classical Lotka-Volterra predator-prey models, similar to the ones used in Dieckmann et al. (1995) and in Brown and Vincent (1992); see also Doebeli (1997). Brown and Vincent (1992) already concluded that frequency-dependent predation could lead to coexistence of multiple predator and prey species at evolutionary stable states. However, their approach differs in at least two ways from the predator-prey models described below: first, they did not study the actual mechanisms by which multiple species would arise (i.e. speciation), and second they included frequency-dependent competition among the prey as an additional diversifying agent.

Here we assume that the interaction parameter describing predation efficiency depends on two quantitative characters, one in the prey and the other in the predator. These characters are scaled such that the interactions are the stronger the more similar prey and predator characters are. If x_1 denotes the character in the prey and x_2 that in the predator, the predation efficiency of a predator with phenotype x_2 on prey individuals with phenotype x_1 is

$$\alpha(x_1 - x_2) = \alpha_0 \cdot \exp\left(\frac{-(x_1 - x_2)^2}{2\sigma_\alpha^2}\right). \quad (25)$$

As a consequence, the ecological dynamics of monomorphic prey and predator populations with trait values x_1 and x_2 are given by

$$\begin{aligned} \frac{dN_1(x_1, t)}{dt} &= r \cdot N_1(x_1, t) \cdot \left(1 - \frac{N_1(x_1, t)}{K(x_1)} - \alpha(x_1 - x_2) \cdot N_2(x_2, t)\right) \\ \frac{dN_2(x_2, t)}{dt} &= N_2(x_2, t) \cdot (-d + c \cdot \alpha(x_1 - x_2) \cdot N_1(x_1, t)). \end{aligned} \quad (26)$$

Here $N_1(x_1, t)$ is the size at time t of the prey population monomorphic for x_1 , $N_2(x_2, t)$ is the size of the predator population monomorphic for x_2 , and r , d , and c are positive rate

constants describing, respectively, the intrinsic growth rate of the prey, the death rate of the predator, and the conversion efficiency of captured prey into predator offspring. For simplicity, we assume that these parameters are not influenced by the quantitative characters x_1 and x_2 . However, we again assume that resource availability for the prey varies with the quantitative character x_1 such that the resource distribution function $K(x_1)$ is of Gaussian form $N(x_0, \sigma_K)$ with a maximum at x_0 . Note, however, that we do not assume frequency dependence in the competitive interactions among the prey.

Yet, the existence of the predator system imposes frequency-dependent selection on the prey, because common prey phenotypes have the disadvantage that the predator phenotype that preys upon them most efficiently is thriving. We show that this frequency dependence can easily lead to evolutionary branching in the prey. To see this, we again calculate the growth rate of rare mutants in both prey and predator while assuming that the resident phenotypes are at their ecological equilibria, $N_1^*(x_1, x_2)$ and $N_2^*(x_1, x_2)$, which can easily be calculated by setting the right hand sides of (26) to zero. The population dynamics of a rare prey mutant y_1 in the resident population is given by

$$\frac{dN_1(y_1, t)}{dt} = r \cdot N_1(y_1, t) \cdot \left(1 - \frac{N_1^*(x_1, x_2)}{K(y_1)} - \alpha(y_1 - x_2) \cdot N_2^*(x_1, x_2)\right), \quad (27)$$

so that the invasion fitness becomes

$$f_1(y_1, x_1, x_2) = r \cdot \left(1 - \frac{N_1^*(x_1, x_2)}{K(y_1)} - \alpha(y_1 - x_2) \cdot N_2^*(x_1, x_2)\right). \quad (28)$$

Similarly, the dynamics of a rare predator mutant y_2 is given by

$$\frac{dN_2(y_2, t)}{dt} = N_2(y_2, t) \cdot (-d + c \cdot \alpha(x_1 - y_2) \cdot N_1^*(x_1, x_2)), \quad (29)$$

so that the invasion fitness for the predator becomes

$$f_2(y_2, x_1, x_2) = -d + c \cdot \alpha(x_1 - y_2) \cdot N_1^*(x_1, x_2). \quad (30)$$

As in the previous models, the quantities that determine the monomorphic evolutionary dynamics of the system are the selection gradients, i.e. the derivatives of the invasion fitness functions with respect to mutant phenotypes, evaluated at the resident values,

$$g_1(x_1, x_2) = \left. \frac{\partial f_1(y_1, x_1, x_2)}{\partial y_1} \right|_{y_1=x_1} = r \frac{N_1^*(x_1, x_2) \cdot K'(x_1)}{K^2(x_1)} - r \cdot \alpha'(x_1 - x_2) \cdot N_2^*(x_1, x_2) \quad (31)$$

and

$$g_2(x_1, x_2) = \left. \frac{\partial f_2(y_2, x_1, x_2)}{\partial y_2} \right|_{y_2=x_2} = c \cdot \alpha'(x_1 - x_2) \cdot N_1^*(x_1, x_2). \quad (32)$$

The quantities $g_1(x_1, x_2)$ and $g_2(x_1, x_2)$, together with the rate constants for the mutation processes in the two species, determine the coevolutionary dynamics (Appendix B). Singular points of this system are points (x_1^*, x_2^*) with $g_1(x_1^*, x_2^*) = g_2(x_1^*, x_2^*) = 0$. Their stability is determined as described in Appendix B. It is intuitively clear, and easy to see analytically from eqs. (31) and (32), that the only singular point in this system is $x_1^* = x_2^* = x_0$. If this point is an evolutionary attractor, the prey character will evolve to the optimum of the resource distribution, and the predator will evolve to maximize its predation efficiency. It is also intuitively clear that for a given prey character, it is best for the predator to have the same character value as the prey to maximize its predation efficiency. This implies that the singular point (x_0, x_0) will always be a fitness maximum for the predator, and hence can never be a branching point for the predator character.

However, the singular point will be a branching point for the prey character as soon as the frequency dependence given by the function $\alpha(x_1 - x_2)$ is strong enough (Appendix B). In other words, for fixed values of the other parameters in the system, evolutionary branching in the prey character occurs when the predation efficiency decreases sufficiently fast with increasing distance between prey and predator characters. An example of evolutionary branching in the prey is shown in Figure 3A. In the example shown, the predator does not undergo evolutionary branching. As a consequence of branching in the prey, the predator therefore becomes a generalist with a lower predation efficiency on each of the two emerging prey species than it had on the single prey species that existed prior to branching.

Evolutionary branching in the prey can sometimes induce secondary branching in the predator. When the prey character splits into two clusters, one might, at first glance, expect that it would always be best for the predator to undergo such a split as well. However, whether this happens depends on the details of the system: predator individuals having character values deviating from the intermediate between the two prey branches have the advantage of being closer to one of the two branches, but at the same time they have the disadvantage of being farther away from the other one. As is shown in Appendix B, it is the distance of the two prey branches from the intermediate singular point x_0 that determines whether or not branching in the prey induces secondary branching in the predator. This is exemplified in Figure 3B, for which parameters are the same as in Figure 3A, except for increased widths of the resource distribution and the predation efficiency. This allows the two phenotypic branches in the prey to diverge farther and hence to induce secondary branching in the predator. In this case, the outcome of the evolutionary process are two prey species, each being exploited by a specialist predator, as opposed to the generalist predator emerging with a narrower resource distribution for the prey (Figure 3A).

To end this section, we give an example of how a change in the relative evolutionary speed, resulting from different mutation rates in the two species, can critically affect the adaptive dynamics. In the system shown in Figure 3C everything is the same as in Figure 3B, except that now the mutation rate in the predator is five times lower than the mutation rate in the prey. This destabilizes the singular point (x_0, x_0) , as described in Appendix B: instead of convergence to the singular point, the coevolutionary adaptive dynamics now show sustained evolutionary cycling. Thus, the evolution in the predator-prey system gives rise to a cyclic type of arms race. This scenario has been extensively studied by Dieckmann et al. (1995); see also Abrams and Matsuda (1996) and Doebeli (1997).

5 Evolutionary branching in sexual populations

So far we have considered clonally reproducing populations. That is, the quantitative characters of offspring were identical to those of their parents, except for changes resulting from mutations. While such a theory is applicable to simple organisms, it clearly requires incorporation of population genetics in order to serve as a basis for understanding evolution in sexual populations. In this section we describe ways of including multi-locus genetics into models of adaptive dynamics, and we identify some of the conditions under which branching can occur in sexual populations. In particular, we show that evolutionary branching can easily arise in sexual species if mating is assortative with respect to traits determining the ecological interactions. The evolution of assortative mating therefore is a crucial issue, which we address by assuming that assortativeness is also a quantitative trait. We show that under branching conditions selection favors genotypes that mate

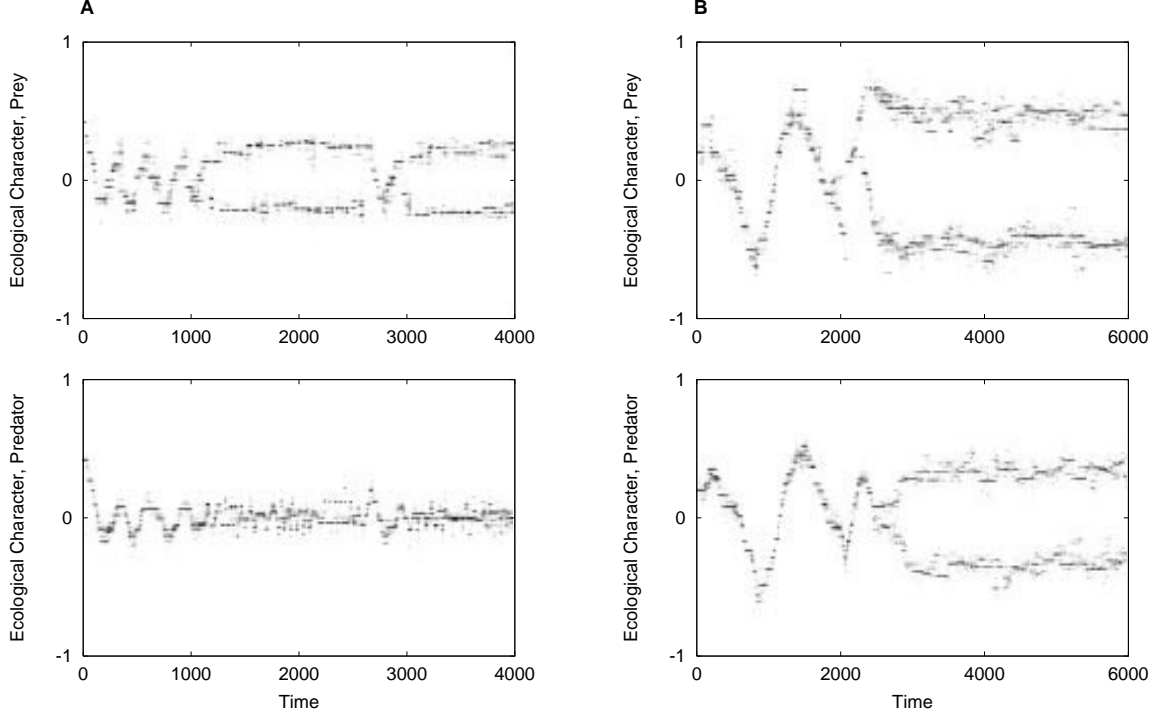


Figure 3: Asexual coevolutionary dynamics of predator-prey interactions.

A. Evolutionary branching only in the prey. While the ecological character in the prey converges (with oscillations) to a fitness minimum, leading to branching in the prey character, the resulting branches do not move far enough apart to induce secondary branching in the predator. Parameters: $r = d = 1$, $c = 2$, $\alpha_0 = 0.001$, $K(x_1) = K_0 \cdot \exp(-(x_1 - x_0)^2/2\sigma_K)$, $K_0 = 2000$, $x_0 = 0$, $\sigma_K = 0.27$, $\alpha(x_1 - x_2) = \exp(-(x_1 - x_2)^2/2\sigma_\alpha^2)$, $\sigma_\alpha = 0.23$, $\mu_1 = \mu_2 = 0.01$, $\sigma_{M_1} = \sigma_{M_2} = 0.05$.

B. Secondary evolutionary branching in the predator. In this case, the prey branches diverge sufficiently for the predator to experience a fitness minimum, resulting in secondary branching in the predator character. Parameters are the same as in (A), except for $\sigma_K = 0.7$, $\sigma_\alpha = 0.4$.

C. Evolutionary cycling of predator-prey interactions. Under identical ecological conditions, changes in mutation rates can dramatically effect outcomes of asexual coevolution. In the case shown here, the prey evolves so fast relative to the predator that it always evades the predator's adaptation, instead of being caught at a fitness minimum and undergoing evolutionary branching. Parameters are the same as in (B), except for $\mu_2 = 0.002$.

assortatively, which leads to evolutionary branching in initially randomly mating sexual populations.

To introduce population genetics into our ecological models, we assume that the quantitative characters determining ecological interactions in these models are genetically determined by many equivalent, additive, diploid and diallelic loci with free recombination. This assumption is easily implemented into individual-based models, as described in Appendix A. Examining parameter values that ensure evolutionary branching in the corresponding asexual models, simulations of the individual-based sexual models readily reveal that evolutionary branching cannot occur in sexual populations if mating is random (Figure 4A). This is intuitively clear, for random mating produces intermediate offspring phenotypes from extreme parent phenotypes, and therefore results in the continual production of intermediate phenotypes from two incipient branches. This prevents the branches from developing into distinct phenotypic clusters.

However, the situation drastically changes if mating is assortative rather than random. To model this case, we assume that the probability of mating between individuals with similar ecological characters is relatively higher than the probability of mating between individuals with dissimilar ecological characters. Such a situation would for example arise if mating is assortative with respect to body size, and if body size at the same time has a strong effect on the ecological interactions, conditions that appear to be satisfied in many species (Schluter and Nagel 1995). Another example is given by quantitative characters that control diet preferences: often individuals mate where they feed so that mating probabilities increase with similarity of feeding preferences, as is the case e.g. for the apple maggot fly (Feder et al. 1988). To be specific, we assume that the probability of mating between two individuals with ecological character values x and y is proportional to

$$\exp\left(\frac{-(x-y)^2}{2\sigma_{\text{mate}}^2}\right), \quad (33)$$

where the parameter σ_{mate} determines the degree of assortativeness: if σ_{mate} is small, mating is strongly assortative, and if $\sigma_{\text{mate}} = \infty$, mating is random. In a randomly mating population, the probability of mating between phenotypes x and y is $p(x) \cdot p(y)$, where $p(x)$ and $p(y)$ are the frequencies of the two phenotypes. In a population with assortative mating, however, the probability of mating between phenotypes x and y is proportional to $\exp\left(\frac{-(x-y)^2}{2\sigma_{\text{mate}}^2}\right) \cdot p(x) \cdot p(y)$, with a proportionality factor arising from a normalization ensuring that, as with random mating, the total probability of mating equals 1 for all phenotypes. (This may not be a realistic assumption for all systems: when assortative mating has a large cost extreme phenotypes may have a disadvantage. However, we expect our results to be robust against introducing small to moderate costs of assortative mating.)

With this setup, evolutionary branching occurs in sexual populations if mating is sufficiently assortative. An example is shown in Figure 4B, for which we have implemented the genetic assumptions in the model for the evolution of mutualism described in Section 4. The ecological parameters for this figure are the same as those used for Figure 2A, which shows simultaneous branching in two asexual mutualistic species. In Figure 4A, mating is random, and branching does not occur in either species. In Figure 4B, however, mating is assortative with respect to the quantitative characters determining the mutualistic interaction, and evolutionary branching again occurs simultaneously in both species. In other words, each species splits into two distinct phenotypic and genotypic clusters. Note that because mating is assortative with respect to the phenotypes forming the clusters, these clusters eventually are almost completely reproductively isolated, so that there is very little gene flow between them. Similar observations hold for our models for resource

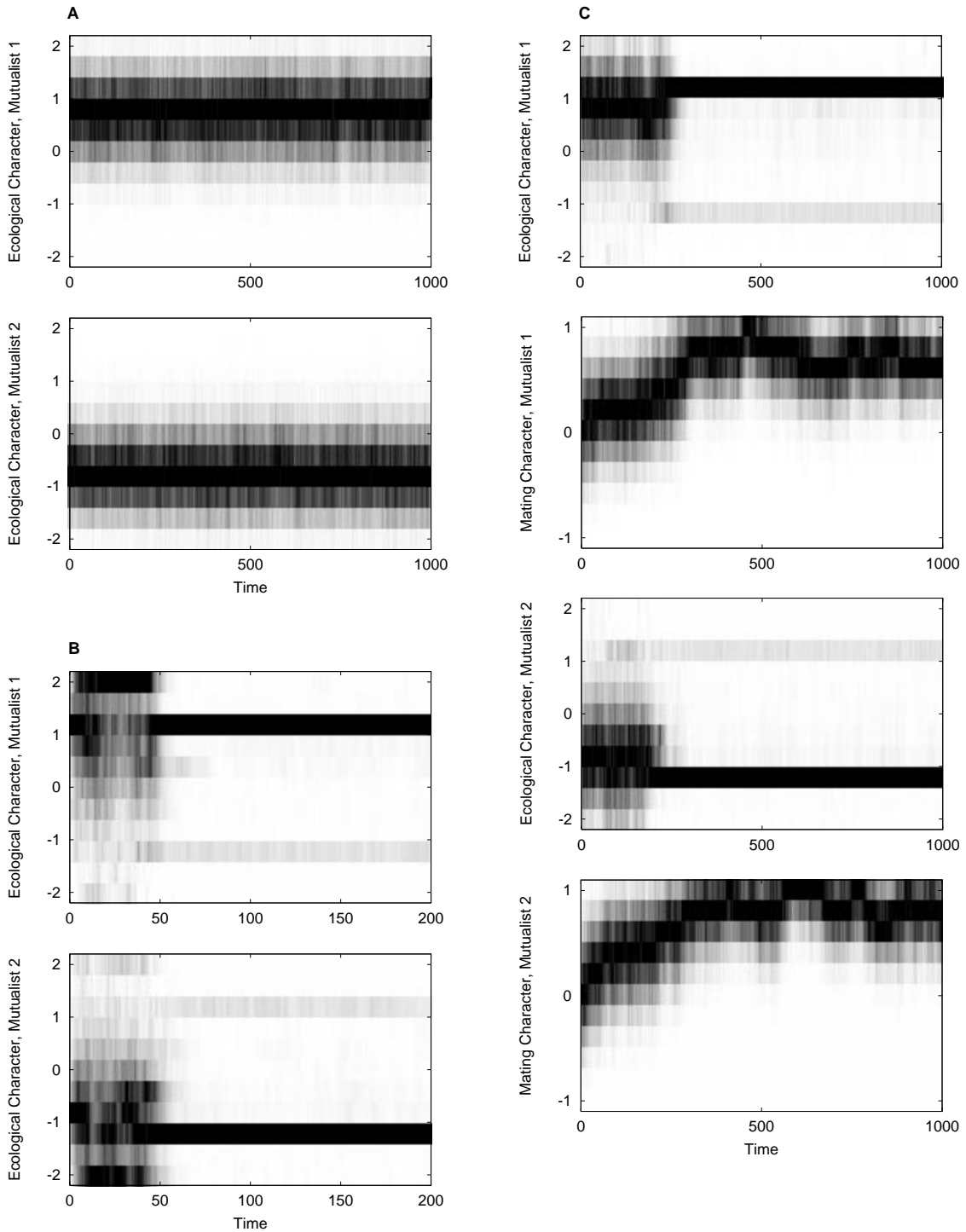


Figure 4: **Sexual coevolutionary dynamics of mutualistic interactions.**

A. Coevolution under random mating. Without assortative mating, the two sexually reproducing mutualistic species cannot undergo evolutionary branching: the continual generation of intermediate offspring characters prevents any bimodality in character values to persist.

B. Coevolution under fixed assortative mating. Given a sufficiently high, yet fixed, degree of assortativeness ($\sigma_{\text{mate}} = \frac{1}{20}$), each of the two sexually reproducing mutualistic species rapidly splits into two reproductively isolated branches.

C. Coevolution under evolving degrees of assortative/disassortative mating. When allowing for adaptation in the degree of assortativeness (second and fourth panel), disruptive selection at the fitness minima favors the evolution of assortative mating, thus enabling evolutionary branching in the two sexually reproducing mutualistic species (first and third panel).

Ecological characters and mating characters are determined by 5 diploid loci; other parameters are the same as in Figure 2A.

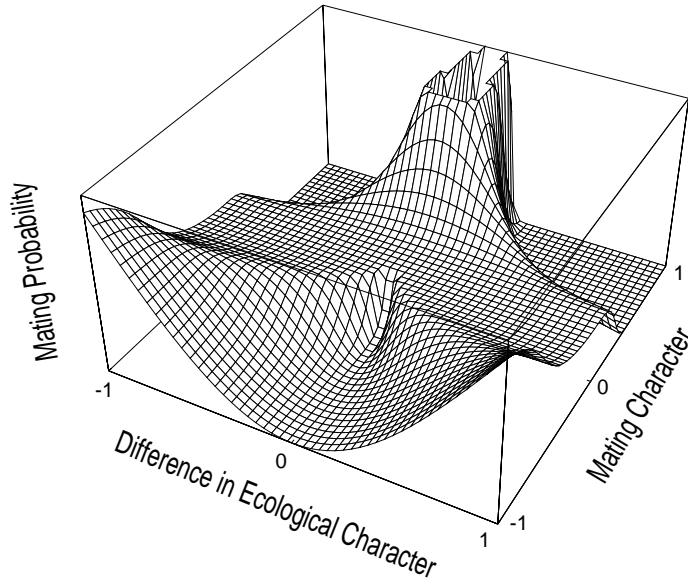


Figure 5: **Character-dependent degrees of assortative/disassortative mating.** Mating probabilities as determined by mating character and difference in ecological characters between mates. If the mating character in a focal individual is close to +1 (only + alleles at mating loci), it has a high probability of mating only with similar individuals. If its mating character is close to -1 (only - alleles), it is more likely to mate with dissimilar individuals. Intermediate mating characters (close to 0) correspond to random mating.

competition and for predator-prey interactions: under branching conditions in the clonal versions, evolutionary branching occurs in the sexual multi-locus versions of these models if assortative mating with respect to the characters determining the ecological interactions is strong enough.

The remaining question is why mating should be assortative in the first place. To investigate this, we regard the degree of assortativeness itself as a quantitative character that is determined by many loci. This allows us to study the evolution of assortative mating in initially randomly mating populations. To exert no bias on the evolution of mating preference we also allow for the possibility of disassortative mating in our individual-based models, i.e., for the possibility that mating probabilities increase with the distance between ecological characters of partners. Thus, individuals having an intermediate mating character mate randomly, while individuals having character values towards one extreme of the mating character's range mate disassortatively, and individuals with values towards the other extreme mate assortatively, i.e. preferentially with ecologically similar partners (Figure 5).

The basic result emerging from our individual-based simulations is that under branching conditions positive assortative mating is selectively favored and typically evolves to a degree that allows for evolutionary branching. That is, given parameter values that ensure branching in the corresponding asexual model, mating evolves from being random to a degree of assortativeness that enables branching in sexual populations. For the case of symmetric competition this is described in more detail in Dieckmann and Doebeli (1999). Here we give examples for the evolution of assortative mating and subsequent evolutionary branching in the sexual mutualistic model (Figure 4C), and in the sexual predator-prey model (Figure 6).

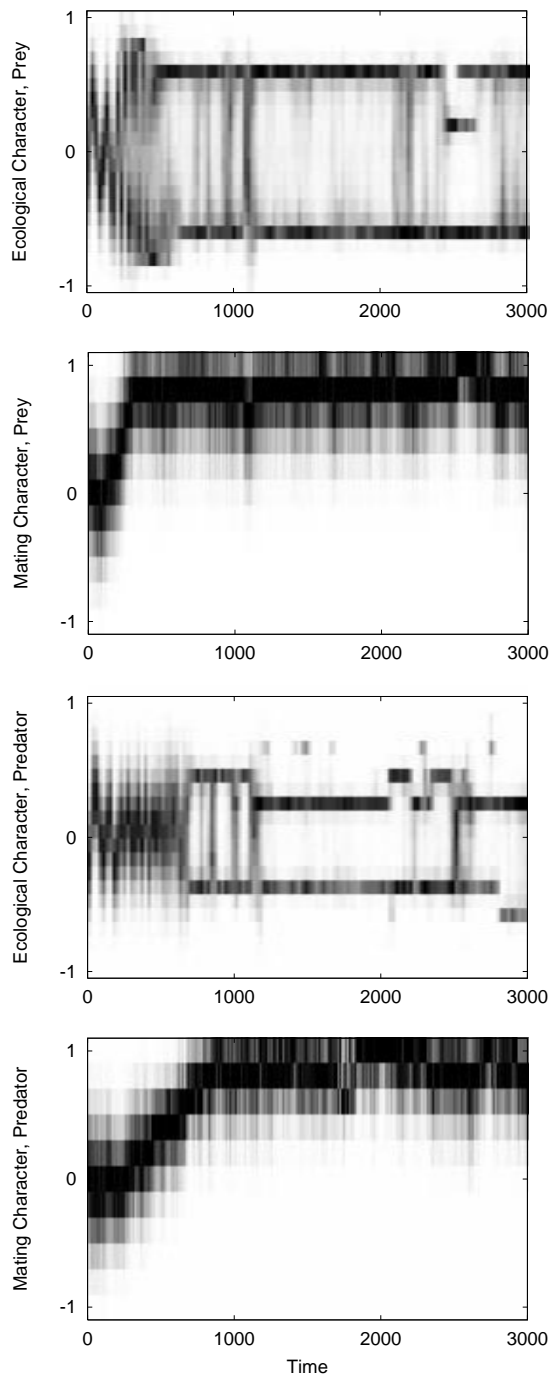


Figure 6: Sexual coevolutionary dynamics of predator-prey interactions.

Coevolution under evolving degrees of assortative/disassortative mating in sexual predator and prey populations leads to primary and secondary evolutionary branching. Initially, the prey character converges (with oscillations) toward the fitness minimum at its resource optimum (first panel). There, the degree of assortativeness is evolutionarily adjusted to high values (second panel). Under these conditions, primary evolutionary branching occurs in the prey (first panel). This induces a fitness minimum for the predator, leading to increased levels of assortativeness in this species (fourth panel), and subsequently allowing for secondary evolutionary branching in the predator (third panel). Ecological characters are determined by 10 diploid loci, mating characters by 5 diploid loci; other parameters are the same as in Figure 3B.

Our extensive numerical simulations indicate that for the models considered in this paper, parameter requirements for the evolution of assortative mating, and hence for evolutionary branching in sexual populations, are not significantly more restrictive than the conditions for branching in the corresponding clonal models. The reason why assortative mating evolves under branching conditions is rather straightforward. At the branching point, the population is trapped at a fitness minimum if mating is random (Figure 4A). Assortative mating is a mechanism which allows for evolutionary branching and hence for a departure from the fitness minimum, because it prevents the generation of intermediate offspring phenotypes from extreme parent phenotypes. Because their offspring are less likely to have a phenotype corresponding to the fitness minimum, individuals that mate assortatively are favored by natural selection. Once assortative mating has evolved, evolutionary branching can occur in sexual populations. In conclusion, when multi-locus genetics and the evolution of assortative mating are incorporated into ecological models, adaptive dynamics leading to evolutionary branching offers a unifying framework for studying and understanding sympatric speciation.

6 Discussion

Evolutionary branching is a type of adaptive dynamics that naturally occurs in phenotypic evolutionary models involving frequency-dependent selection (Metz et al. 1996, Geritz et al. 1998). It consists of two phases: first, there is convergence in phenotype space to an evolutionarily attracting fitness minimum, and then the population splits into two diverging phenotypic clusters. In the present paper, we have shown that this evolutionary phenomenon readily occurs in a range of basic models in which evolutionary dynamics are driven by different types of ecological interactions. Symmetric and asymmetric resource competition, mutualistic interactions, and predator-prey interactions can all lead to evolutionary branching, which is therefore a general and robust feature of adaptive dynamics.

Evolutionary branching is a mechanism that can explain speciation in asexual populations. By including population genetics into the underlying ecological models it becomes an integrating paradigm for studying sympatric speciation in sexual populations. To illustrate this finding we have assumed that the quantitative characters that determine ecological interactions are controlled by many loci, and we have investigated the evolution of assortative mating in our models. The results show that, due to the evolution of positive assortative mating with respect to the characters determining ecological interactions, evolutionary branching also generically arises in models for sexual populations, in which it can lead to sympatric speciation.

Previous models for sympatric speciation

One of the main differences between our approach and previous work is that in the framework presented here disruptive selection is not an externally imposed assumption, as is the case in the majority of earlier models, where some form of habitat difference is usually needed to generate disruptive selection (e.g. Maynard Smith 1966, Udovic 1980, Felsenstein 1981, Johnson et al. 1996, Kawecki 1996). In such models the maintenance of a polymorphism for the phenotypes favored in the different habitats (a prerequisite for the evolution of reproductive isolation) is problematic. By contrast, in the framework of adaptive dynamics, the emergence of disruptive selection is an intrinsic process: disruptive selection dynamically arises from the underlying ecological interactions in such a way that the conditions for the evolution and maintenance of a polymorphism, i.e. for evolutionary

branching, are automatically satisfied. The basic insight is that this is a generic phenomenon. (It should be noted that evolutionary branching in traits determining habitat specialization also readily occurs in multiple-habitat models, see Geritz et al. 1998).

Two approaches similar to our theory were previously put forward by Seger (1985) and by Kondrashov (Kondrashov 1986, Kondrashov and Kondrashov 1999). In Seger (1985) frequency-dependent resource competition could generate a mismatch between the phenotype distribution and the resource distribution, the consequences of which were explored by means of a simple genetic model leading to the conclusion that over-dispersed resources are needed for sympatric speciation. Seger (1985) modeled assortative mating as a discrete rather than as a continuous trait, which may have contributed to the fact that unimodal (binomial) resource distributions did not favor speciation in his model. Nevertheless, if his model is put into the framework of adaptive dynamics, his results can be reformulated in terms of conditions for evolutionary branching points. From the perspective of population genetics, Kondrashov's studies of sympatric speciation (Kondrashov 1986, Kondrashov and Mina 1986, Kondrashov and Kondrashov 1999) are more similar to ours, although these authors did not use an ecological embedding that would generate the selection regimes. Instead, an unspecified mechanism is assumed to favor marginal phenotypes at all times. Moreover, these authors did not model the degree of assortative mating as a quantitative trait. Our results can be viewed as an extension and generalization of these previous results, suggesting that, in theory, sympatric speciation is a common evolutionary process.

The model that comes closest to ours – because it is a precursor – is Doebeli (1996). This paper showed that frequency-dependent competition for unimodally distributed resources can lead to sympatric speciation if mating is assortative, and that assortative mating should evolve continuously towards a degree inducing speciation. It used a deterministic multi-locus genetic model for the ecological traits, but did not consider genetics for assortative mating. It also did not put results into the framework of adaptive dynamics and evolutionary branching, and therefore did not highlight the obtained results as a special case of a general phenomenon.

Felsenstein (1981), who used a model of the type involving habitat differences to generate disruptiveness, classified models for sympatric speciation into two groups: the 'one-allele' models, in which the same alleles need to be substituted in the subpopulations forming the two incipient branches in order to achieve divergence and reproductive isolation, and the 'two-allele' models, in which different alleles must be substituted in the two incipient species. Felsenstein (1981) noted that sympatric speciation occurs relatively easily in 'one-allele' models, but is much harder to obtain in 'two-allele' models. The models presented here would all be classified as 'one-allele' models, because in our models evolutionary branching occurs in sexual populations if assortative mating evolves in both incipient branches, that is, if the alleles coding for stronger assortativeness are substituted in both branches. To make the genetic theory more general, one therefore wants to consider models in which assortative mating is not based on the characters that determine ecological interactions, but instead on selectively neutral marker traits. In this case, evolution of assortative mating is not sufficient anymore to induce speciation under branching conditions. In addition, a linkage disequilibrium between loci coding for the marker trait on the one hand and for the ecological character on the other is required. This implies that different marker alleles must be substituted in the two incipient branches: this scenario thus corresponds to the 'two-allele' models of Felsenstein (1981). The reason why speciation is more difficult when mating is based on a marker trait is that recombination between the marker loci and the loci determining the ecological character prevents the

buildup of a linkage disequilibrium between the marker trait and the ecological trait. As Felsenstein (1981) pointed out, deterministic models predict that sympatric speciation is unlikely under these conditions (see also Seger 1985).

However, a rather different picture emerges when stochastic, individual-based models are employed. In Dieckmann and Doebeli (1999) we have shown that, once demographic stochasticity and resulting genetic drift are included in the description, evolutionary branching is a robust phenomenon in sexual populations even when assortative mating is based on a neutral marker trait. The reason is that genetic drift due to demographic stochasticity leads to small and genetically localized linkage disequilibria. These local and temporary disequilibria select for assortative mating, which in turn magnifies the local disequilibria into a global and stable linkage disequilibrium between marker trait and ecological character, allowing for evolutionary branching. Thus, symmetry breaking due to genetic drift can trigger evolutionary branching in sexual populations, despite the opposing force of recombination. We have described this phenomenon in detail in Dieckmann and Doebeli (1999) for the case of symmetric resource competition, and similar observations are expected to hold for the coevolutionary models described in the present paper. This establishes a strong link between our theory and models for sympatric speciation based on sexual selection (Lande 1982). That assortative mating based on marker traits can lead to evolutionary branching and sympatric speciation in our ecological models means that the evolution of sexually selected traits might strongly promote phenotypic cluster formation and divergence if sexually selected characters become correlated with characters affecting ecological interactions. Recent results of Kondrashov and Shpak (1998) indicate that sympatric speciation is unlikely without such a correlation, i.e., with assortative mating based on a selectively neutral trait alone. Notice that a wide range of potential marker traits coexist in typical organisms. For branching to occur in sexual populations, only one of these needs to latch on to the ecologically relevant character. The likelihood for such an event is further enhanced by the existence of spatial phenotypic heterogeneity and by the occurrence of cryptic speciation (Metz et al. 1996): fleeting patterns of cryptic variation may become "frozen" at a branching point.

Empirical evidence for sympatric speciation

Evidence for the sympatric origin of many species groups is accumulating rapidly in recent years. Strong support comes from phylogenetic reconstruction studies based on genetic analyses, e.g. in cichlids (Meyer et al. 1993, Schlieuwen et al. 1994), and it is increasingly recognized that extent populations show large degrees of population subdivision, which may often be the raw material for processes of speciation. Relevant studies (e.g. Boursot et al. 1996, De Leon et al. 1997) may reveal cases of incipient speciation which can be investigated experimentally, for example in light of the theory presented here. This seems to be promising because there already are a few cases in which our theory agrees particularly well with empirical insights about the mechanisms of speciation. For example, a recent study of incipient speciation in a pair of cichlid morphs in a crater lake in Cameroon by Schlieuwen et al. (in preparation) argues that competition for resources and size-assortative mating are responsible for reproductive isolation between two monophyletic morphs that only differ in size. More generally, sympatric speciation driven by niche separation due to resource competition is an attractive explanation for the radiation of the monophyletic cichlids in large lakes in Africa. Recent studies by Seehausen et al. (1997) show that sexually selected traits are used for maintaining species boundaries, which seems to correspond well with the extension of our theory to the evolution of assortative mating based on marker traits that are correlated with ecological characteristics (Dieckmann and Doebeli 1999).

Another example where our theory would seem to be applicable are sticklebacks in lakes of British Columbia (Schluter and McPhail 1992, Schluter 1994, Taylor and McPhail 1999). Although it is not clear at present whether the two forms that exist in some lakes, a large benthic one and a small limnetic one, arose sympatrically, there is evidence from experimental research (Schluter 1994, Nagel and Schluter 1998, Rundle and Schluter 1998) as well as from phylogenetic studies (Taylor and McPhail 1999) that supports this hypothesis. In these species, body size is an ecologically important trait, and studies of assortative mating (Nagel and Schluter 1998, Rundle and Schluter 1998) match with the theoretical prediction that size assortative mating could have led to a sympatric split into a large and a small form due to competition for resources.

Further empirical evidence for sympatric speciation via evolutionary branching comes from studies on organisms such as intertidal snails (Johannesson et al. 1995), *Anolis* lizards (Losos et al. 1998) and senecio trees (Knox and Palmer 1995), in which resource competition is likely to have been a major driving force of speciation. Evolutionary branching due to resource competition may also help explain the much discussed speciation events in Darwin's finches, a group that, because of the biogeography of the Galápagos islands, is regarded as a stronghold for allopatric speciation theory. In fact, it is known that size-assortative mating is common in these birds, and that body size, and in particular beak size, strongly influences their diet (Grant et al. 1985). While these birds are a classical study system for ecological character displacement (e.g. Schluter et al. 1985, Schluter 1988), the possibility of sympatric speciation has not been a major focus of interest, and it may be worthwhile to reconsider even this apparently long-resolved case of allopatric speciation in the light of evolutionary branching.

There are also cases of radiation and divergence in mutualistic and predator-prey systems for which our models may help unravel the underlying mechanisms. For example, Kiestler et al. (1984) argue that the exceptional diversity of orchids may be due to coevolution with orchid bees, and our model for facultative mutualism could provide a theoretical basis for understanding speciation in this group of species. Another example comes from carnations in Middle Europe, whose diversity matches the diversity of their pollinators. It is believed that coevolutionary interactions dominate these mutualistic systems (A. Erhardt, pers. comm.), and it is likely that many more examples of adaptive speciation due to mutualistic interactions are revealed once mutualisms are investigated more rigorously from the perspective of ecologically driven radiations (see Pellmyr and Leebens-Mack, this volume).

When viewed from the predator's perspective, predator-prey interactions lead to familiar resource competition, but for the prey a different type of selection pressure emerges, termed apparent competition by Holt (1977), under which prey compete for predator-free phenotype space. Recently, it has been emphasized that apparent competition may be a strong diversifying force (Brown and Vincent 1992, Abrams, this volume), and our theory shows that apparent competition can lead to sympatric speciation through evolutionary branching in the prey. Empirical support for this comes for instance from mollusks, where it has been argued that the great diversity of ornamentation in some groups is due to evolutionary responses to predation (e.g. Stone 1998, Van Damme and Pickford 1995, Leighton 1999). Another example comes from sticklebacks, where it has been proposed that predation is important for the evolution of divergent body shapes (Walker 1997). Finally, Chown and Smith (1993) suggested that size-selective predation by mice, in combination with size-assortative mating, is causing sympatric speciation in sub-Antarctic weevils, a scenario that corresponds well with the models presented in Section 5.

In sum, there appear to exist many case studies supporting a theory of sympatric speci-

ation and radiation driven by ecological interactions, as presented in this paper. However, evidence for sympatric speciation from field research and from experimental studies involving relatively large organisms with long generation times such as fish will almost always be circumstantial to some extent (the example of Schliewen et al. (unpublished work) mentioned above provides a convincing exception). A promising alternative for empirically testing the theory of evolutionary branching may be found in experimental evolution of microorganisms. Recent work by Rainey and Travisano (1998, and this volume) shows that sympatric divergence can be induced in bacteria by placing them in a novel, heterogeneous environment. It would appear that such approaches could be developed into direct tests of evolutionary branching by experimentally generating the ecological conditions and interactions predicted by the theory to induce adaptive speciation. Such experiments could greatly advance our knowledge about one of the central problems in biology: understanding the processes that lead to the origin of new species.

References

- Abrams, P.A. 1991. The effects of interacting species on predator-prey coevolution. *Theoretical Population Biology* 39: 241–262.
- Abrams, P.A., and Matsuda, H. 1996. Fitness minimization and dynamic instability as a consequence of predator-prey coevolution. *Evolutionary Ecology* 10: 167–186.
- Bever, J.D. 1999. Dynamics within mutualism and the maintenance of diversity: inference from a model of interguild frequency dependence. *Ecology Letters* 2: 52–61.
- Boots, M. and Haraguchi, Y. 1999. The evolution of costly resistance in host-parasite systems. *American Naturalist* 153: 359–370.
- Boursot, P., Din, W., Anand, R., Darviche, D., Dod, B., Von Deimling, F., Talwar, G.P., and Bonhomme, F. 1996. Origin and radiation of the house mouse: Mitochondrial DNA phylogeny. *Journal of Evolutionary Biology* 9: 391–415.
- Brown, J.S. and Vincent, T.L. 1992. Organization of predator-prey communities as an evolutionary game. *Evolution* 46: 1269–1283.
- Chown, S.L., and Smith, V.R. 1993. Climate change and the short-term impact of feral house mice at the sub-Antarctic Prince Edwards Islands. *Oecologia* 96: 508–516.
- Christiansen, F.B. 1991. On conditions for evolutionary stability for a continuously varying character. *American Naturalist* 138:37–50.
- De Leon, F.J.G., Chikhi, L., and Bonhomme, F. 1997. Microsatellite polymorphism and population subdivision in natural populations of European sea bass *Dicentrarchus labrax* (Linnaeus, 1758). *Molecular Ecology* 6: 51–62.
- Dieckmann, U. 1994. Coevolutionary dynamics of stochastic replicator systems. *Reports of the Research Center Juelich* 3018
- Dieckmann, U., and Doebeli, M. 1999. On the origin of species by sympatric speciation. *Nature* 400: 354–357.
- Dieckmann, U., and Law, R. 1996. The dynamical theory of coevolution: a derivation from stochastic ecological processes. *Journal of Mathematical Biology* 34: 579–612.
- Dieckmann, U., Marrow, P., and Law, R. 1995. Evolutionary cycling in predator-prey interactions: population dynamics and the Red Queen. *Journal of Theoretical Biology* 176: 91–102.

- Doebeli, M. 1996. A quantitative genetic competition model for sympatric speciation. *Journal of Evolutionary Biology* 9: 893–909.
- Doebeli, M. 1997. Genetic variation and the persistence of predator-prey interactions in the Nicholson-Bailey model. *Journal of Theoretical Biology* 188: 109–120.
- Doebeli, M. and Ruxton, G.D. 1997. Evolution of dispersal rates in metapopulation models: branching and cyclic dynamics in phenotype space. *Evolution* 51: 1730–1741.
- Eshel, I. 1983. Evolutionary and continuous stability. *Journal of Theoretical Biology* 103: 99–111.
- Feder, J.L., Chilcote, C.A., and Bush, G.L. 1988. Genetic differentiation between sympatric host races of the apple maggot fly *Rhagoletis Pomonella*. *Nature* 336: 61–64.
- Felsenstein, J. 1981. Skepticism towards Santa Rosalia, or why are there so few kinds of animals? *Evolution* 35: 124–138.
- Geritz, S.A.H., Metz, J.A.J., Kisdi, E., Meszéna, G. 1997. The dynamics of adaptation and evolutionary branching. *Physical Review Letters* 78: 2024–2027.
- Geritz, S.A.H., Kisdi, E., Meszéna, G., and Metz, J.A.J. 1998. Evolutionarily singular strategies and the adaptive growth and branching of the evolutionary tree. *Evolutionary Ecology* 12: 35–57.
- Geritz, S.A.H., Van der Meijden, E. and Metz, J.A.J. 1999. Evolutionary dynamics of seed size and seedling competitive ability. *Theoretical Population Biology* 55: 324–343.
- Grant, P.R., Abbott, I., Schluter, D. Curry, R.L., and Abbott, L.K. 1985. Variation in the size and shape of Darwin’s Finches. *Biological Journal of the Linnean Society* 25: 1–39.
- Holt, R.D. 1977. Predation, apparent competition and the structure of prey communities. *Theoretical Population Biology* 12: 197–229.
- Hutchinson, G.E. 1959. Homage to Santa Rosalia *or* why are there so many kinds of animals? *American Naturalist* 93: 145–159.
- Johannesson, K., Rolan-Alvarez, E., and Ekendahl, A. 1995. Incipient reproductive isolation between two sympatric morphs of the intertidal snail *Littorina saxatilis*. *Evolution* 49: 1180–1190.
- Johnson, P.A., Hoppenstaedt, F.C., Smith, J.J., and Bush, G.L. 1996. Conditions for sympatric speciation: a diploid model incorporating habitat fidelity and non-habitat assortative mating. *Evolutionary Ecology* 10: 187–205. Kawecki, T. 1996.
- Kawecki, T. 1996. Sympatric speciation driven by beneficial mutations. *Proceedings of the Royal Society of London B*, 263: 1515–1520.
- Kisdi, E. 1999. Evolutionary branching under asymmetric competition. *Journal of Theoretical Biology* 197: 149–162.
- Kisdi, E. and Geritz, S.A.H. 1999. Adaptive dynamics in allele space: evolution of genetic polymorphism by small mutations in a heterogeneous environment. *Evolution* 53: 993–1008.
- Knox, E.B., and Palmer, J.D. 1995. Chloroplast DNA variation and the recent radiation of the Giant Senecios (*Asteraceae*) on the tall mountains of Eastern Africa. *Proceedings of the National Academy of Sciences USA* 92: 10349–10353.

- Koella, J.C. and Doebeli, M. 1999. Population dynamics and the evolution of virulence in epidemiological models with discrete host generations. *Journal of Theoretical Biology* 198: 461-475.
- Kondrashov, A.S. 1986. Multilocus model of sympatric speciation III. Computer simulations. *Theoretical Population Biology* 29: 1-15.
- Kondrashov, A.S., and Mina, M.V. 1986. Sympatric speciation: when is it possible? *Biological Journal of the Linnean Society* 27: 201-223.
- Kondrashov, A.S., and Shpak, M. 1998. *Proceedings of the Royal Society of London B* 265: 2273-2278.
- Lande, R. 1982. A quantitative genetic theory of life history evolution. *Ecology* 63: 607-615.
- Law, R., Marrow, P. and Dieckmann, U. 1997. On evolution under asymmetric competition. *Evolutionary Ecology* 11: 485-501.
- Leighton, L.R. 1999. Possible latitudinal predation gradient in middle Paleozoic oceans. *Geology* 27: 47-50.
- Leimar, O. 2000. Multidimensional convergence stability and the canonical adaptive dynamics. in *Elements of Adaptive Dynamics* (U. Dieckmann and J.A.J. Metz, eds.) Cambridge University Press, Cambridge, UK (to appear)
- Losos, J.B., Jackman, T.R., Larson, A., de Queiroz, K. and Rodríguez-Schettino, L. 1998. Contingency and determinism in replicated adaptive radiations of island lizards. *Science* 279: 2115-2118.
- Ludwig, D. and Levin, S.A. 1991. Evolutionary stability of plant communities and the maintenance of multiple dispersal types. *Theoretical Population Biology* 40: 285-307.
- MacArthur, R.H., and Levins, R. 1967. The limiting similarity, convergence and divergence of coexisting species. *American Naturalist* 101: 377-385.
- Matessi, C. and Di Pasquale, C. 1996. Long-term evolution of multi-locus traits. *Journal of Mathematical Biology* 34: 613-653.
- Maynard Smith, J. 1966. Sympatric speciation. *American Naturalist* 100: 637-650.
- Meszéna, G., Kisdi, E., Dieckmann, U., Geritz, S.A.H. and Metz, J.A.J. 2000. Evolutionary optimization models and matrix games in the unified perspective of adaptive dynamics. *Selection* (in press).
- Metz, J.A.J., Nisbet, R.M. and Geritz, S.A.H. 1992. How should we define fitness for general ecological scenarios? *Trends in Ecology & Evolution* 7: 198-202.
- Metz, J.A.J., Geritz, S.A.H., Meszéna, G., Jacobs, F.J.A., and van Heerwaarden, J.S. 1996. pp. 183-231 in *Stochastic and Spatial Structures of Dynamical Systems*. (S.J. van Strien and S.M. Verduyn Lunel, eds.) North Holland, Amsterdam.
- Meyer, A., Kocher, T.D., Basasibwaki, P., and Wilson, A.C. 1990. Monophyletic origin of Lake Victoria cichlid fishes suggested by mitochondrial DNA sequences. *Nature* 347: 550-553.
- Nagel, L. and Schluter, D. 1998. Body size, natural selection, and speciation in sticklebacks. *Evolution* 52: 209-218.
- Parvinen, K. 1999. Evolution of migration in a metapopulation. *Bulletin of Mathematical Biology* 61: 531-550.

- Petren, K., Grant, B.R., and Grant, P.R. 1999. A phylogeny of Darwin's finches based on microsatellite DNA length variation. *Proceedings of the Royal Society of London B* 266: 321–329.
- Rainey, P.B., and Travisano, M. 1998. Adaptive radiation in a heterogeneous environment. *Nature* 394: 69–72.
- Rand, D.A., Wilson, H.B. and McGlade, J.M. 1994. Dynamics and evolution: evolutionarily stable attractors, invasion exponents and phenotype dynamics. *Philosophical Transactions of the Royal Society London B* 345: 261–283.
- Rundle, H.D., and Schluter, D. 1998. Reinforcement of stickleback mate preferences: Sympatry breeds contempt. *Evolution* 52: 200–208.
- Schliewen, U.K., Tautz, D., and Pääbo, S. 1994. Sympatric speciation suggested by monophyly of crater lake cichlids. *Nature* 368: 629–623.
- Schluter, D. 1988. Character displacement and the adaptive divergence of finches on islands and continents. *American Naturalist* 131: 799–824.
- Schluter, D. 1994. Experimental evidence that competition promotes divergence in adaptive radiation. *Science* 266: 798–801.
- Schluter, D. and McPhail, J.D. 1992. Ecological character displacement and speciation in sticklebacks. *American Naturalist* 140, 85–108.
- Schluter, D. and Nagel, L.M. 1995. Parallel speciation by natural selection. *American Naturalist* 146: 292–301.
- Schluter, D., Price, T.D., and Grant, P.R. 1985. Ecological character displacement in Darwin's Finches. *Science* 227: 1056–1059.
- Seehausen, O., van Alphen, J.J.M., and Witte, F. 1997. *Science* 277: 1808–1811.
- Seger, J. 1985. Intraspecific resource competition as a cause of sympatric speciation. in *Evolution. Essays in honour of John Maynard Smith* (Greenwood, P.J., Harvey, P.H. and Slatkin, M. eds.), Cambridge University Press, Cambridge, UK.
- Stone, H.M.I. 1998. On predator deterrence by pronounced shell ornament in epifaunal bivalves. *Palaeontology* 41: 1051–1068.
- Taper, M.L., and Case, T.J. 1992. Coevolution among competitors. Pages 63–111 in *Oxford Surveys in Evolutionary Biology, Volume 8* (D. Futuyma and J. Antonovics, eds.) Oxford University Press, Oxford.
- Taylor, E.B., and McPhail, J.D. 1999. *Biological Journal of the Linnean Society* 66: 271–291.
- Udovic, D. 1980. Frequency-dependent selection, disruptive selection, and the evolution of reproductive isolation. *American Naturalist* 116: 621–641.
- Van Damme, D., and Pickford, M. 1995. The late Cenozoic ampullariidae (mollusca, gastropoda) of the Albertine Rift Valley (Uganda-Zaire). *Hydrobiologia* 316: 1–32.
- Vandermeer, J.H., and Boucher, D.H. 1978. Varieties of mutualistic interaction in population models. *Journal of Theoretical Biology* 74: 549–558.
- Van Dooren, T.J.M. 1999. The evolutionary ecology of dominance-recessivity. *Journal of Theoretical Biology* 198: 519–532.
- Walker, J.A. 1997. Ecological morphology of lacustrine threespine stickleback *Gasterosteus aculeatus* L. (*Gasterosteidae*) body shape. *Biological Journal of the Linnean Society* 61: 3–50.

Appendix A: Description of the individual-based models

In this appendix we summarize the algorithmic procedures underlying the simulation results presented in this paper.

For the individual-based asexual simulations, we follow the polymorphic stochastic model developed by Dieckmann (1994) and employed in Dieckmann et al. (1995) and Dieckmann and Doebeli (1999). In this model, individuals in an ecological community can belong to different species and can possess distinct phenotypes. At any given time, each individual can reproduce or die according to stochastic rates that can depend on the abundance and phenotypic composition of all species in the community. When an individual gives birth, its offspring will either inherit the parent's phenotype or undergo a mutation that results in a random displacement between offspring and parent phenotype.

For the evolutionary models of symmetric and asymmetric intraspecific competition (Section 2), the community comprises a single species. The distribution of phenotypes x at time t is described by $n(x, t)$. From this, the phenotypic frequency distribution $p(x, t)$, used in eqs. (15) and (16), is obtained as $p(x, t) = n(x, t) / \int n(x', t) dx'$. The stochastic per capita rates for birth and death of an individual with phenotype x at time t are then given by

$$\begin{aligned} b(x, n) &= r \\ d(x, n) &= r \cdot \frac{1}{K(x)} \int \alpha(x - x') \cdot n(x', t) \cdot dx' = r \cdot \frac{N_{eff}(x, t)}{K(x)}. \end{aligned} \quad (\text{A1})$$

Mutations in offspring phenotypes occur with probability μ . For a parent phenotype x , mutated offspring phenotypes are chosen according to a Gaussian distribution $N(x, \sigma_M)$ with mean x and variance σ_M^2 .

For the coevolutionary models of interspecific mutualisms (Section 3), the stochastic per capita birth and death rates in the two mutualistic species are set to

$$\begin{aligned} b_1(x_1, n_1, n_2) &= r_1 \cdot \left(1 + a_{12} \cdot \int \frac{\alpha(x_1 - x_2) \cdot \int n_1(x'_1) \cdot dx'_1}{\int \alpha(x'_1 - x_2) \cdot n_1(x'_1) \cdot dx'_1} \cdot n_2(x_2) \cdot dx_2 \right) \\ d_1(x_1, n_1, n_2) &= r_1 \cdot \frac{1}{K_1(x_1)} \int n_1(x'_1) \cdot dx'_1 \end{aligned} \quad (\text{A2})$$

and to

$$\begin{aligned} b_2(x_2, n_1, n_2) &= r_2 \cdot \left(1 + a_{21} \cdot \int \frac{\alpha(x_2 - x_1) \cdot \int n_2(x'_2) \cdot dx'_2}{\int \alpha(x'_2 - x_1) \cdot n_2(x'_2) \cdot dx'_2} \cdot n_1(x_1) \cdot dx_1 \right) \\ d_2(x_2, n_1, n_2) &= r_2 \cdot \frac{1}{K_2(x_2)} \int n_2(x'_2) \cdot dx'_2. \end{aligned} \quad (\text{A3})$$

As for the single-species models, mutation probabilities for two-species models are determined by μ_1 and μ_2 and mutation distributions around parent phenotypes are given by $N(x_1, \sigma_{M_1})$ and $N(x_2, \sigma_{M_2})$.

For the coevolutionary predator-prey models (Section 4), per capita birth and death rates for the prey are

$$\begin{aligned} b_1(x_1, n_1, n_2) &= r \\ d_1(x_1, n_1, n_2) &= r \cdot \left(\frac{1}{K(x_1)} \int n_1(x'_1) \cdot dx'_1 + \int \alpha(x_1 - x_2) \cdot n_2(x_2) \cdot dx_2 \right) \end{aligned} \quad (\text{A4})$$

and for the predator

$$\begin{aligned} b_2(x_2, n_1, n_2) &= c \cdot \int \alpha(x_1 - x_2) \cdot n_1(x_1) \cdot dx_1 \\ d_2(x_2, n_1, n_2) &= d. \end{aligned} \tag{A5}$$

For the individual-based sexual simulations, we generalize the genetic model introduced by Dieckmann and Doebeli (1999) to coevolutionary dynamics. In this extended model, individuals in an ecological community can belong to different species and can possess distinct genotypes. The stochastic per capita rates for birth and death are affected by the abundance and phenotypic composition of all species in the community in the same way as in the asexual models.

However, in the sexual models phenotypic diversity is no longer only generated by mutations, but now also arises from the recombination of genotypes. Each individual carries two diploid sets of diallelic loci (with alleles + and -), one set determining the ecological phenotype, the other set affecting mate choice. Loci have equivalent and additive phenotypic effects and recombine freely. Under these assumptions, 5 diploid loci, for example, can code for 11 equidistant phenotypic values. The ecological character is chosen to range from -1 to +1 for the predator-prey system and from -2 to +2 for the system of mutualists. The mating character always assumes values between -1 to +1 and is given by the difference between the number of + and - alleles at the mating loci divided by the total number of mating alleles.

An individual with ecological character x and mating character m reproduces by choosing a partner with ecological character x' depending on the difference $x' - x$. The probability distribution for this choice is depicted in Fig. 5. For $m > 0$, mating is assortative with mating probabilities following a Gaussian function $N(x, \sigma_{\text{mate}})$ with $\sigma_{\text{mate}} = \frac{1}{20m^2}$. For $m = 0$, mating is random. For $m < 0$, mating is disassortative with mating probabilities following a Gaussian function $1 - N(x, \sigma_{\text{mate}})$ with $\sigma_{\text{mate}} = \frac{1}{m^2}$. In order to avoid a bias against marginal phenotypes in the population, mating probabilities are normalized, so that their sum over all potential partners is 1 for all ecological phenotypes. Upon reproduction, one offspring allele at each diploid locus is chosen randomly from the two alleles at the corresponding homologous locus of the first parent, and the other allele from those of the second parent. With a small probability ($\mu = 0.001$), a mutation occurs in the inherited alleles and reverses their value.

Appendix B: Coevolutionary adaptive dynamics of two traits in two species

We briefly review some properties of the dynamical systems describing the coevolution of two quantitative traits in two species. Let x_1 and x_2 denote the current resident values of the two traits in species 1 and 2, respectively, and let y_1 and y_2 denote trait values of rare mutants appearing in the populations. Then the fate of these mutants is determined by the invasion fitness functions $f_1(y_1, x_1, x_2)$ and $f_2(y_2, x_1, x_2)$. The quantities that determine the direction of gradual evolutionary change are the selection gradients

$$\begin{aligned} g_1(x_1, x_2) &= \left. \frac{\partial f_1(y_1, x_1, x_2)}{\partial y_1} \right|_{y_1=x_1} \\ g_2(x_1, x_2) &= \left. \frac{\partial f_2(y_2, x_1, x_2)}{\partial y_2} \right|_{y_2=x_2}. \end{aligned} \quad (\text{B1})$$

More precisely, if mutations are sufficiently rare and sufficiently small the adaptive dynamics of the trait vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is given by

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} m_1(x_1) \cdot g_1(x_1, x_2) \\ m_2(x_2) \cdot g_2(x_1, x_2) \end{pmatrix}. \quad (\text{B2})$$

Here $m_1(x_1)$ and $m_2(x_2)$ are quantities describing how the mutational process that governs evolution in the two traits (and affects the rates and the distributions at which new mutations occur) influences the speed of evolution.

Expression (B2) has been derived in Dieckmann (1994) and in Dieckmann and Law (1996), to which we refer for a much more detailed and general discussion of this so-called canonical equation of adaptive dynamics (see also Leimar 2000). The singular points of the evolutionary dynamics given by (B2) are those points (x_1^*, x_2^*) in trait space for which both selection gradients vanish, $g_1(x_1^*, x_2^*) = g_2(x_1^*, x_2^*) = 0$. Whether a singular point is an evolutionary attractor or not can be seen from the Jacobian $J(x_1^*, x_2^*)$ of the dynamical system (B2) at the singular point (x_1^*, x_2^*) (see Marrow et al. 1996 and Leimar 2000):

$$J(x_1^*, x_2^*) = \begin{pmatrix} m_1(x_1^*) \cdot \frac{\partial g_1}{\partial x_1}(x_1^*, x_2^*) & m_1(x_1^*) \cdot \frac{\partial g_1}{\partial x_2}(x_1^*, x_2^*) \\ m_2(x_2^*) \cdot \frac{\partial g_2}{\partial x_1}(x_1^*, x_2^*) & m_2(x_2^*) \cdot \frac{\partial g_2}{\partial x_2}(x_1^*, x_2^*) \end{pmatrix}, \quad (\text{B3})$$

The singular point (x_1^*, x_2^*) is an evolutionary attractor if and only if the Jacobian $J(x_1^*, x_2^*)$ has eigenvalues with negative real parts. Note that the stability of a singular point depends on the constants $m_1(x_1^*)$ and $m_2(x_2^*)$, characterizing the mutation process: for given selection gradients $g_1(x_1, x_2)$ and $g_2(x_1, x_2)$ a singular point may be an attractor for some values of $m_1(x_1^*)$ and $m_2(x_2^*)$, but not for others (Marrow et al. 1996; see also Abrams and Matsuda 1996).

Just as in 1-dimensional (i.e., in single-trait, single-species) adaptive dynamics, a singular point that is an attractor for system (B2) may not represent the endpoint of the evolutionary process if one or both species find themselves at a fitness minimum at the singular point. The conditions for fitness minima at the singular point are simply that the invasion fitness has a minimum with respect to the mutant trait values,

$$\frac{\partial^2 f_1}{\partial y_1^2}(y_1, x_1^*, x_2^*) \Big|_{y_1=x_1^*} > 0 \quad (\text{B4})$$

and

$$\frac{\partial^2 f_2}{\partial y_2^2}(y_2, x_1^*, x_2^*)|_{y_2=x_2^*} > 0. \quad (\text{B5})$$

(Note that $g_1(x_1^*, x_2^*) = \frac{\partial f_1}{\partial y_1}(y_1, x_1^*, x_2^*)|_{y_1=x_1^*} = 0$ and $g_2(x_1^*, x_2^*) = \frac{\partial f_2}{\partial y_2}(y_2, x_1^*, x_2^*)|_{y_2=x_2^*} = 0$ for any singular point.) In 1-dimensional adaptive dynamics a fitness minimum at an attracting singular point is sufficient for evolutionary branching (Geritz et al. 1998). In higher-dimensional adaptive dynamics this need not be true anymore. For example, in adaptive dynamics of two correlated traits in a single species, convergent stable fitness minima alone need not generate evolutionary branching (U. Dieckmann, personal observation).

However, in all the cases considered in this article it can be shown that if one (or both) of the species are at a fitness minimum at an attracting singular point, then evolutionary branching occurs in one (or both) species. This simplification relative to two-dimensional adaptive dynamics in a single species occurs because in our coevolutionary models the two traits are necessarily uncorrelated since they belong to different species. A time scale separation argument then shows that, at the singular point, each species' resident phenotype can be considered as a parameter for the evolutionary dynamics of the other species, which in effect reduces the 2-dimensional problem at the singular point to the 1-dimensional case, where convergence to fitness minima is sufficient for branching.

To understand evolutionary branching in the adaptive dynamics of mutualism defined by system (17) in Section 3, we first have to find trait combinations (x_1^*, x_2^*) for which the selection gradients vanish and then check the conditions for stability from (B3) and for branching from (B4). The selection gradients are

$$\begin{aligned} g_1(x_1, x_2) &= \frac{\partial f_1(y_1, x_1, x_2)}{\partial y_1}|_{y_1=x_1} = \frac{r_1 \cdot N_1^* \cdot K_1'(x_1)}{K_1^2(x_1)} + \frac{r_1 \alpha'(x_1 - x_2)}{\alpha(x_1 - x_2)} \cdot a_{12} \cdot N_2^* \\ g_2(x_1, x_2) &= \frac{\partial f_2(y_2, x_1, x_2)}{\partial y_2}|_{y_2=x_2} = \frac{r_2 \cdot N_2^* \cdot K_2'(x_2)}{K_2^2(x_2)} + \frac{r_2 \alpha'(x_2 - x_1)}{\alpha(x_2 - x_1)} \cdot a_{21} \cdot N_1^*, \end{aligned} \quad (\text{B6})$$

where $N_1^* = N_1^*(x_1, x_2)$ and $N_2^* = N_2^*(x_1, x_2)$ are the ecological equilibrium points of system (17).

We briefly discuss the symmetric case, in which the only ecological difference between the two mutualistic species is the position of the maxima of the resource distribution. In this case there is always a singular point (x_1^*, x_2^*) that lies symmetrically around the midpoint between the two resource maxima x_{10} and x_{20} and satisfies $x_{10} < x_1^* < x_2^* < x_{20}$. This symmetric singular point may or may not be an evolutionary attractor. For example, the symmetric singular point may lose its stability through a pitchfork bifurcation, during which two new asymmetric and convergent stable singular points are borne. Numerical simulations indicate that if the symmetric singular point is the only singularity, and if the mutational functions $m_1(x_1)$ and $m_2(x_2)$ are the same, then this point is always an evolutionary attractor. To see whether such a singular point is an evolutionary branching point, we calculate the second derivatives of the invasion fitness functions, eqs. (19) and (20), at the singular point:

$$\begin{aligned} \frac{\partial^2 f_1(y_1, x_1^*, x_2^*)}{\partial y_1^2}|_{y_1=x_1^*} &= \frac{r_1 \cdot N_1^* \cdot K_1''(x_1^*)}{K_1^2(x_1^*)} - \frac{2r_1 \cdot N_1^* \cdot K_1'^2(x_1^*)}{K_1^3(x_1^*)} + \frac{r_1 \alpha''(x_1^* - x_2^*)}{\alpha(x_1^* - x_2^*)} \cdot a_{12} \cdot N_2^* \\ \frac{\partial^2 f_2(y_2, x_1^*, x_2^*)}{\partial y_2^2}|_{y_2=x_2^*} &= \frac{r_2 \cdot N_2^* \cdot K_2''(x_2^*)}{K_2^2(x_2^*)} - \frac{2r_2 \cdot N_2^* \cdot K_2'^2(x_2^*)}{K_2^3(x_2^*)} + \frac{r_2 \alpha''(x_2^* - x_1^*)}{\alpha(x_1^* - x_2^*)} \cdot a_{21} \cdot N_1^*. \end{aligned} \quad (\text{B7})$$

Two factors tend to make these expressions positive: first, x_1^* and x_2^* must lie far enough away from their respective resource optimum, i.e., beyond the inflection points of the

corresponding resource distributions, so that $K_1''(x_1^*) > 0$ and $K_2''(x_2^*) > 0$, and, secondly, x_1^* and x_2^* should be far enough apart from each other, so that the distance $x_1^* - x_2^*$ lies beyond the inflection point of the function α , so that $\alpha''(x_1^* - x_2^*) > 0$. These conditions can be met by increasing the distance between the maxima of the resource distributions, which is equivalent to decreasing the widths of the resource distributions and the width of the mutualistic interaction function α . It is easy to do this in such a way that the symmetric singular point does not lose its stability. Consequently, it is easy to allow for simultaneous evolutionary branching in this system (see Figure 2A).

Finally, we discuss the adaptive dynamics in the predator-prey system (26). It follows from the expression for the selection gradient in the predator, eq. (32), that a singular point (x_1^*, x_2^*) must satisfy $\alpha'(x_1^* - x_2^*) = 0$. We therefore must have $x_1^* - x_2^* = 0$, i.e., $x_1^* = x_2^*$. It then follows from the selection gradient in the prey that $K'(x_1^*) = 0$ for the singular prey character x_1^* , hence $x_1^* = x_0$, so that the singular point occurs at the maximum of the resource distribution. Straightforward calculations reveal that the Jacobian at the singular point is given by

$$J(x_0, x_0) = \begin{pmatrix} m_1 \cdot r \cdot (N_1^* \cdot K''(x_0)/K^2(x_0) - \alpha''(0) \cdot N_2^*) & m_1 \cdot r \cdot \alpha''(0) \cdot N_2^* \\ -m_2 \cdot c \cdot \alpha''(0) \cdot N_1^* & m_2 \cdot c \cdot \alpha''(0) \cdot N_1^* \end{pmatrix}, \quad (\text{B8})$$

where N_1^* and N_2^* are the ecological equilibrium sizes of populations monomorphic for the singular character values (x_0, x_0) , and where $m_1 = m_1(x_0)$ and $m_2 = m_2(x_0)$ are the mutational parameters at the singular point. Using the functional form for the predation efficiency α , eq. (25), and for the resource distribution K , eq. (9), and calculating the equilibrium populations sizes N_1^* and N_2^* using $x_1 = x_2 = x_0$ in eqs. (26), one obtains

$$J(x_0, x_0) = \begin{pmatrix} \frac{m_1 \cdot r}{c \cdot \alpha_0 \cdot K_0} \cdot \left(\frac{c \cdot \alpha_0 \cdot K_0 - d}{\sigma_\alpha^2} - \frac{d}{\sigma_K^2} \right) & -\frac{m_1 \cdot r \cdot (c \cdot \alpha_0 \cdot K_0 - d)}{c \cdot \alpha_0 \cdot K_0 \cdot \sigma_\alpha^2} \\ \frac{m_2 \cdot d}{\sigma_\alpha^2} & -\frac{m_2 \cdot d}{\sigma_\alpha^2} \end{pmatrix}. \quad (\text{B9})$$

Both eigenvalues of the Jacobian $J(x_0, x_0)$ have negative real parts if the determinant of $J(x_0, x_0)$ is positive and its trace is negative. The determinant of $J(x_0, x_0)$ is equal to $\frac{d^2 \cdot m_1 \cdot m_2 \cdot r}{c \cdot \alpha_0 \cdot K_0 \cdot \sigma_\alpha^2 \cdot \sigma_K^2}$ and hence is always positive. The trace is the sum of the diagonal elements and may be positive or negative. In the present situation, the diagonal elements of the Jacobian are, up to the constants m_1 and m_2 respectively, equal to the second derivatives of the invasion fitness functions with respect to the mutant traits, evaluated at the singular point:

$$\frac{\partial^2 f_1}{\partial y_1^2}(y_1, x_0, x_0)|_{y_1=x_0} = \frac{\partial g_1}{\partial x_1}(x_0, x_0) = \frac{r}{c \cdot \alpha_0 \cdot K_0} \cdot \left(\frac{c \cdot \alpha_0 \cdot K_0 - d}{\sigma_\alpha^2} - \frac{d}{\sigma_K^2} \right) \quad (\text{B10})$$

and

$$\frac{\partial^2 f_2}{\partial y_2^2}(y_2, x_0, x_0)|_{y_2=x_0} = \frac{\partial g_2}{\partial y_2}(x_0, x_0) = -\frac{m_2 \cdot d}{\sigma_\alpha^2}. \quad (\text{B11})$$

In particular, the singular point is never a branching point for the predator, because $\frac{\partial^2 f_2}{\partial y_2^2}(y_2, x_0, x_0)|_{y_2=x_0}$ is always negative. Thus, to obtain attraction to the singular point and subsequent evolutionary branching, the upper diagonal element of the Jacobian $J(x_0, x_0)$ must be positive, which would make the singular point a fitness minimum for the prey, whereas adding the negative lower diagonal element must give a negative number, ensuring that the singular point is an attractor. These conditions are easy to meet. For example, setting $m_1 = m_2$ and $r = d = 1$, it follows that the trace of the Jacobian is negative, and, provided that $c \cdot \alpha_0 \cdot K_0 > d$, the upper diagonal element will be positive as soon as σ_α is small enough.

Once the prey has branched, selection pressures for the predator change. Due to the symmetry in the system, the two prey branches are at equal distances δ on opposite sides of the singular predator character x_0 . It follows that, at time t , the growth rate of a rare mutant predator with character value y_2 is

$$f_2(y_2, \delta, x_0, t) = -d + c \cdot \alpha(x_0 + \delta - y_2) \cdot \frac{N_1(t)}{2} + c \cdot \alpha(x_0 - \delta - y_2) \cdot \frac{N_1(t)}{2}, \quad (\text{B12})$$

where x_0 is the singular predator trait value, and $N_1(t)$ is the prey population size at time t . Taking the second derivative with respect to mutant trait value y_2 and evaluating at the resident trait value x_0 reveals whether the predator is located at a fitness minimum after the prey has branched:

$$\begin{aligned} \frac{\partial^2 f_2(y_2, \delta, x_0, t)}{\partial y_2^2} \Big|_{y_2=x_0} &= c \cdot \alpha''(\delta) \cdot \frac{N_1(t)}{2} + c \cdot \alpha''(-\delta) \cdot \frac{N_1(t)}{2} \\ &= c \cdot \alpha''(\delta) \cdot N_1(t), \end{aligned} \quad (\text{B13})$$

since α is symmetric about the origin. This expression is positive for all t , and hence the singular point x_0 is a fitness minimum for the predator, if δ is large enough, i.e., if the two prey branches have moved far enough away from x_0 . Once this has happened, the branching in the prey induces secondary branching in the predator (Figure 3B).

That a difference in the mutational parameters m_1 and m_2 can greatly affect the adaptive dynamics is shown in Figure 3C. With m_2 decreased by a factor of five, the lower diagonal element of the Jacobian $J(x_0, x_0)$ given in (B8) is not negative enough any more to outweigh the positive upper diagonal element describing the fitness minimum in the prey. Therefore, the trace of the Jacobian becomes positive, and hence the branching point of Figure 3B is no longer an attractor for the evolutionary dynamics.