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Population, Natural Resources and Food Security Lessons from Comparing Full and Reduced Form Models

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Abstract

This paper discusses one of the most difficult issues in modeling complex population-environment interactions: The advantages and disadvantages of highly disaggregated empirical models versus highly reduced theoretical models. The analysis is carried out on the basis of the PEDA model, recently developed to capture interactions between population change, education, agricultural production, food security and natural resource degradation in Africa. An important feature of the approach is a non-linear food distribution function. Ranging in its use from direct science-policy communication with a number of African governments, to highly advanced dynamic mathematical analysis, PEDA turns out to be appropriate for the kind of sensitivity analysis and comparison of different levels of complexity attempted in this paper. In short, the results show that highly reduced models can never replace full empirical models, but that they show important additional features that complement the full models. For the researcher, the best understanding will be gained if one does not limit one's analysis to one level of complexity, but compares results from different levels.

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Introduction

Any analysis of the complex interactions between human population dynamics and the natural environment requires some sort of a model, either in one's mind or on paper or on the computer. If the objective is to come up with some quantitative estimates of future trends and interactions, explicit mathematical (and mostly computer based) models are the appropriate tool to this end. Since there are myriads of possible models determined by the choice of variables and parameters, the level of aggregation and the complexity of the model, the scientist entering this field will have to make a daunting number of choices for which he/she has often little scientific basis. While the choice of variables and the level of regional aggregation tend to be strongly influenced by the specific research question, the issue of model complexity is more ambiguous. Are complex models more or less appropriate than simple ones? This general question has been addressed in different contexts (see, e.g., in the field of population forecasting Rogers 1995b; Lee 1995; Sanderson 1999). Here we want to study it in the context of dynamic population-environment modeling by comparing a recently developed large empirical model to its reduced form derivative.

This paper is built around an interactive simulation model, the PEDA (population, environment, development, agriculture) model, which focuses on the interactions between changes in the population size and distribution, natural resource degradation, agricultural production and food security (Lutz and Scherbov 2000). This recently developed model has been inspired by the "vicious circle" reasoning (Dasgupta 1993; Nerlove 1991) which assumes a dynamic relationship between resource degradation, poverty (food insecurity) and high fertility. PEDA explicitly includes these variables which are necessary to model the "vicious circle," but it is not limited to its specific assumptions. PEDA is a much more general tool, which can calculate the longer term implications of a great range of alternative assumptions. It is also used for science-policy communication. The major message to be conveyed to policy makers is that population, education, rural development, land degradation, water, food production and food distribution are not independent issues (which are usually addressed independently by different ministries), but that these trends are interconnected in the real world and need to be addressed in a way that takes account of this nexus. PEDA is being coordinated by the UN Economic Commission for Africa and has been applied to an increasing number of countries (by mid 2000 Burkina Faso, Mali, Madagascar, Uganda, Cameroon and Zambia).

The PEDA model also inspired theoretically-oriented scholars interested in the long term dynamics of non-linear models to develop a more general reduced form of the model that included only a hand full of variables and equations but maintains the non-linear food distribution function (Lorenz curve). The explicit consideration of a concentration curve in this context seems to be an innovative feature with interesting consequences for the dynamic behavior of the model. This reduced model can complement the full empirical model in terms of sensitivity analysis and in gaining a better understanding of the nature of the underlying dynamics (Dworak *et al*. 2000). Hence, a specific feature of this paper will be the comparison between the full PEDA model with an empirical application to Mali and the reduced form PEDA, in order to learn some more general lessons about the advantages of different degrees of complexity and detail in dynamic population-environment modeling.

After this introduction the paper will consist of three parts. First, the general approach, theory and design of the full empirical PEDA will be discussed and illustrated with selected scenarios for Mali. Next, we will introduce the reduced form model, develop comparable scenarios and compare the model performance and sensitivity analysis of the full empirical and the reduced form PEDA. In a final part we draw conclusions and discuss how the two ways of capturing the dynamics complement each other by serving different purposes.

The Full PEDA Model

Logic and structure of the model

A theoretical construction, often labeled the "vicious circle model," has recently become a very influential paradigm in the discussion around population, poverty, food security and sustainable development. It essentially assumes that high fertility, poverty, low education and status of women are bound up in a web of interactions with environmental degradation and declining food production, in such a way that stress from one of the sources can trap certain rural societies, especially those living in marginal areas, into a vicious circle of increasingly destructive responses. One possible illustration of this assumed mechanism is the parable of the firewood (Nerlove 1991): In many countries the collection of firewood takes a lot of time, and more children can help to collect more firewood. But this leads to less firewood near the villages, increasing degradation of the natural resource and the desire for more children to go still further, also depriving the children of educational opportunities. Dasgupta (1993) presents this argument in a more generalized form. The condition of poverty and illiteracy of the households concerned prevents substitution of alternative fuel sources or alternative livelihoods. A gender dimension is being added through the fact that the low status of women and girls also devalues the increasing amount of time and effort that they must devote to daily fuelwood gathering (Agarwal 1994; Sen 1994). The education of girls is blocked because girls are kept at home to help their mothers. The result is faster population growth, further degradation of the renewable resource base, increasing food insecurity, stagnating education levels, and yet a further erosion of women's status.

From a theoretical point of view this vicious circle model is a useful contribution toward a more general framework in causally linking fertility, poverty, low female status and environmental degradation. It is also attractive because it explicitly addresses equity concerns. Its multi-dimensional structure helps to view different possible interventions in, e.g., reproductive health, education, environmental conservation and agricultural efficiency in a unifying context rather than in isolation from each other. Each of the interventions may, under certain conditions, contribute to breaking up the vicious circle, but – as the following applications will show – a comprehensive strategy viewing all these aspects together and recognizing their interdependencies is likely to be more successful (O'Neill *et al*. 2000).

In terms of its empirical relevance, the vicious circle assumption is more controversial. Because the economic reasoning of this model largely operates at the household level, empirical studies on the issue have been mostly confined to that level and reached mixed results. At the macro level of different population segments this model could potentially be very relevant, especially in the African context, although some of the assumptions of the stricter version of this model are empirically unconfirmed and controversial. Especially the assumption that environmental degradation may actually lead to increases in fertility is difficult to be defended at a time when fertility rates are rapidly falling all over Africa with simultaneously degrading environmental resources. This does not necessarily imply that the assumed effects are entirely absent, but it seems to imply that if they operate, they are overlaid by the powerful and dominating process of demographic transition. Hence, it may be reasonable to alternatively assume that food insecurity is associated with a slower decline in fertility, although under certain conditions and in the short run famines may well induce declining fertility. Whatever the position on this issue, the PEDA model as outlined below is general enough to represent alternative assumptions through alternative parameter choices and scenarios.

Figure 1 gives the basic structure of PEDA. The model has a rather open structure. But it can, among others, capture the basic assumed mechanism of the vicious circle, i.e., that rapid population growth due to high fertility of the illiterate, food-insecure population in marginal rural areas contributes to further degradation of the land, thus lowering agricultural production and further increasing the number of food insecure. If not broken this vicious circle would lead to ever increasing land degradation and increases in the food-insecure population. This model does not assume increasing fertility as a response. Rather the food secure and food insecure are assumed to have different fertility levels (or exogeneously-defined trends) and hence the aggregate fertility level only responds to changing food insecurity through the changing weights of the groups. The vicious circle can be broken, however, through several possible interventions in the field of food production, food distribution, education, environmental protection and population dynamics.

Such a quantitative model can help policy makers and other users to (a) view these interconnected aspects, and (b) think in terms of alternative outcomes of alternative policy scenarios.

PEDA is different from most macroeconomic models in that it uses a population-based approach. The population-based approach views human beings and their characteristics (such as age, sex, education, health, food security status, place of residence, etc.) as agents of social, economic, cultural and environmental change. But the population is also at risk of suffering from repercussions of these changes and of benefiting from positive implications. In this sense the human population is seen as a driving force of these changes and is affected by the outcomes and consequences of these changes. Economics, if it comes into the picture, e.g., through the importance of markets in distributing goods, plays only an intermediate role and is not seen as an end in itself or the primary object of modeling. In this, the population-based approach chosen here differs from much of the development-economics literature.

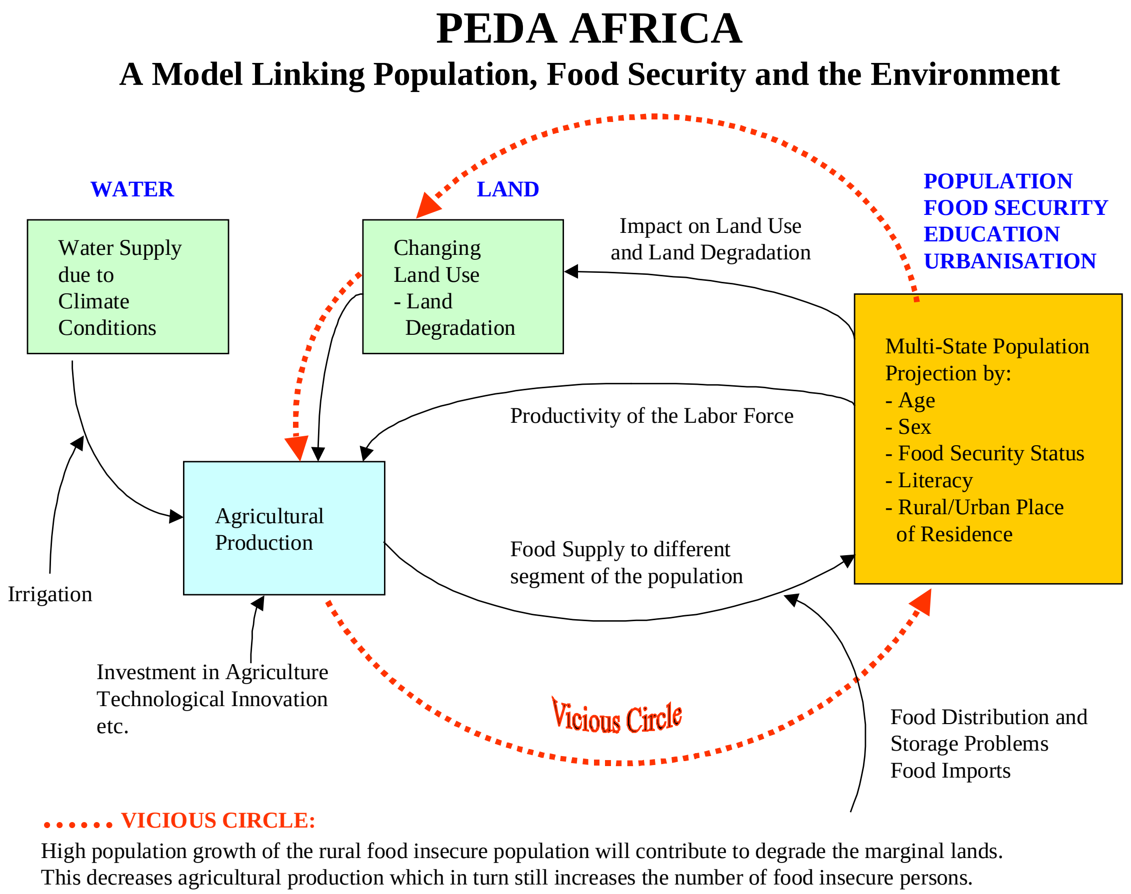

Figure 1. Basic structure of the PEDA model.

The population-based approach does not assume that population growth or other demographic changes are necessarily the most important factors in shaping our future. It must not be misunderstood in the sense of a narrow view in which only demography matters. Instead the phenomena that we want to model are studied in terms of different characteristics that can be directly attached and (at least theoretically) measured with individual members of the population. Characteristics such as age, sex, literacy, place of residence and even nutritional status can be assessed at the individual level. The sum over these individual characteristics makes up the distribution in the total population. These individual characteristics are different from other frequently-used indicators such as the GNP per capita which, although it is suggestive of the average amount of money that an individual has in his/her pocket, cannot be directly measured with individuals. It results from a certain way of aggregate level national accounting with various conceptual and measurement problems. Although many of the powerful quantitative economic tools (such as general equilibrium models) cannot be applied due to this choice of approach, other very powerful but less well known tools of demographic analysis and projection can be applied. The tools of multi-state population analysis allow for the projection of the population by several characteristics (such as age, sex, education and place of residence) at the same time. Multi-state projection groups all individuals of a given population into different sub-populations which are then projected into the future, while in each time interval, people can also move from one sub-population to another (e.g., from rural to urban or illiterate to literate for each sex and age group).

As shown in Figure 2, in PEDA the population of a country under consideration is broken down into eight sub-groups according to urban/rural place of residence, education and food security status. Place of residence and food security status are two dimensions which are core elements of the vicious circle reasoning as specified in this setting. Education, or more precisely literacy status, has been introduced into the model as one of the assumed key sources of population heterogeneity, which is related to both agricultural production and land degradation. Significant educational fertility differentials give the explicit consideration of education in the model a strong rationale. There is abundant literature on the significance of literacy in population-development-environment interactions (see, e.g., Lutz 1994). The potential of explicitly including education as a demographic dimension in multi-state population projection models has recently been evaluated (see Lutz *et al.* 1999) and is strongly recommended in the case of educational fertility (or other behavioral) differentials.

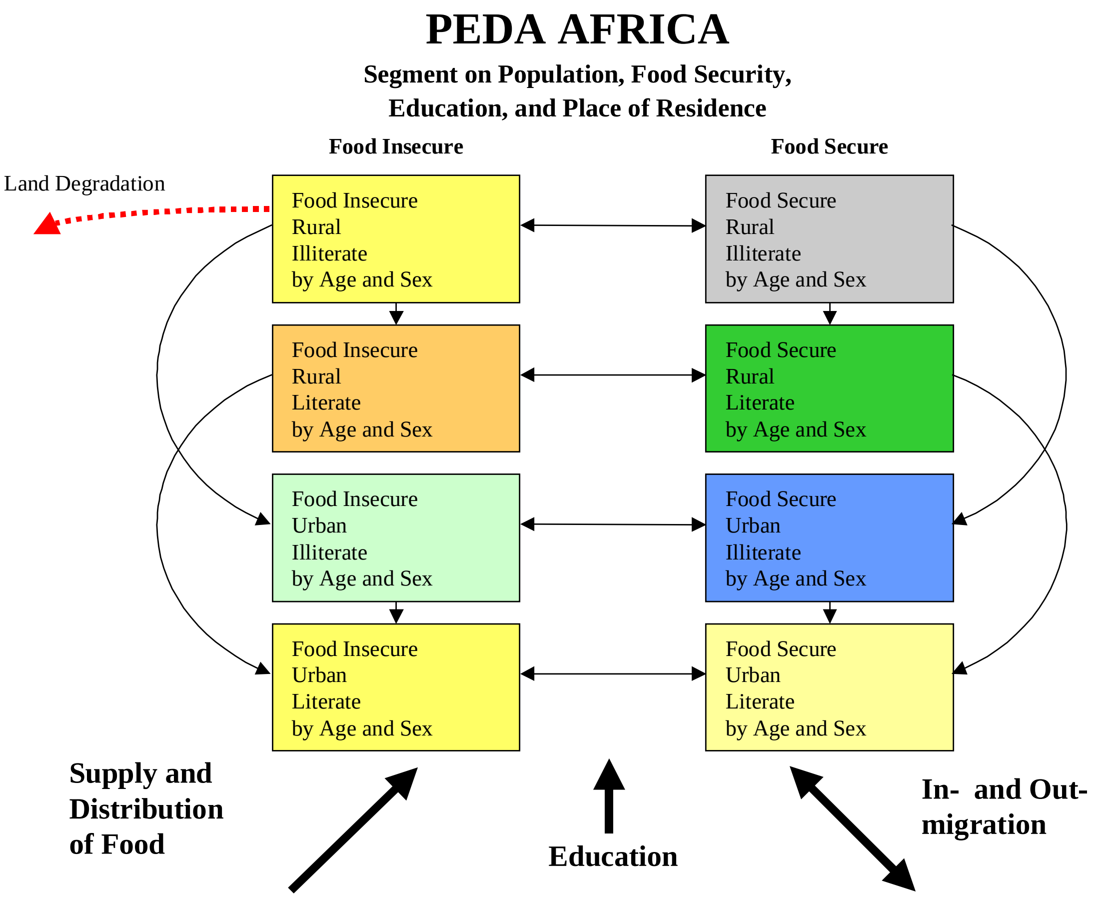

Figure 2. PEDA population segment: A multi-state model by place of residence, education and food security status.

Each of these eight sub-groups further subdivides the population by age and sex, i.e., every one of the eight groups has its own age pyramid. During each one-year simulation step, a person will move up the age pyramid by one year within the same sub-group, or move to another sub-group while aging by one year. The movements between groups that are possible within each step are shown by arrows in Figure 2. For education and rural/urban migration, the model is hierarchical, i.e., people can only move in one direction, from lower to higher education and from rural areas to urban areas. Movement between food security states can happen in both directions, depending on the food conditions in the relevant year and the food distribution function.

This PEDA population module is in itself a useful piece of software (written in Excel) for multi-state population projections. For each of the eight states the user can set age and sex-specific fertility, mortality, migration and transition rates. If such detailed empirical data is not available, standard age schedules can be applied and relational logit transformations are applied for mortality. More generally, each country first needs to be initialized in terms of all empirical data for the starting conditions. This is a significant task to be done outside the normal software, which requires more demographic skills than the use of the initialized model, which is very user friendly in terms of scenario setting and presentation of results.

As indicated in Figure 1, the population module, i.e., the population by age and sex in the eight defined categories and for each year in time, affects the total agricultural production in two different ways. The productivity of the rural labor force as measured by the proportion literate of the rural population of productive age will directly enter the agricultural production function as discussed below. The other chain of causation is a direct reflection of the vicious circle reasoning: the factor land is degraded as a function of the increase in the number of people in the rural, food-insecure and illiterate category. In the current version of PEDA, this impact is operationalized in the following manner. The total amount of high quality agricultural land enters the production function in an index form, which is assumed to combine quantity and quality aspects. The higher the increase (as compared to the starting conditions) in this critical group of food-insecure, rural, illiterate population, the more this land factor will decline (cf. Eq. (9) in Appendix A). The user can set a scenario variable, the land degradation impact factor, that determines to which degree a certain percentage increase in this critical population impacts on the land index.

Agricultural output is calculated by a Cobb-Douglas type agricultural production function as estimated by Hayami and Ruttan (1971) for a number of developing countries. In addition to the rural labor force, literacy and land, which are at least partly endogenous to PEDA, production also depends on fertilizer use, machinery and technical education, which are treated as exogenous scenario variables. PEDA also includes a water component, which directly impacts on production and which is also influenced by land degradation among other factors. The result of this process is the total amount of calories produced in the country in a year.[1]

Unfortunately, in reality, not all the production will be consumed by individuals to satisfy their food needs. Some calories will be lost during the treatment of the food; others will be lost during transport; and some will be lost due to inadequate storage. Of

[1] The details of this production part of the model are described extensively in Lutz and Scherbov (2000).

the food that will actually reach people for consumption, a certain fraction will go to urban areas and another to rural areas. All these factors can be set in PEDA as scenario variables specific for a country and can be changed over time, or alternative starting values can be assumed. More specifically, in the model there are three different scenario variables that the user can set: loss in transport and storage, food import/export, and an urban bias factor. The latter determines to what degree the total available food should be disproportionally distributed between urban and rural areas. Within these areas, however, not everybody will receive an equal amount of food. Reality shows that there are gross inequalities in access to food and, therefore, PEDA has an explicit food distribution module.

Even if the total amount of food reaching the (urban and rural total) population would be theoretically sufficient to provide the necessary minimum diet for everybody, in practice, the distribution of food is unequal because some persons have more purchasing power than others, or have privileged access to food by other means. This will result in the fact that some people remain food insecure even when the average total amount of food reaching the population is above the minimum.

There is abundant empirical evidence backed up by theoretical considerations, clearly showing that the distribution of food is at least as important as the total production of food in explaining food insecurity. Especially the work of Amartya Sen (1994) demonstrated that some of the worst famines occurred under conditions in which theoretically there would have been enough food for everybody, if the distribution had been appropriate. For this reason it is evident that a model focusing on food security without paying attention to the distributional aspects would be incomplete, if not misleading. The main problem with considering such distributions, however, lies in the fact that hardly any empirical data exist on distributive mechanisms in the countries of Africa today, and that theoretical distributions are hardly appropriate because conditions tend to vary significantly from one country to another. As a solution to this problem, in PEDA we chose to approximate the food distribution function through an income distribution function, which exists for a number of African countries based on household income surveys. This allocation of food to urban and rural populations and the food distribution within these populations then determines the new sizes of the food-secure and food-insecure sub-populations in the following year.

Figure 3 shows such a food distribution function that is applied after allocating the total available food to urban and rural populations (according to an exogenously-defined "urban bias" variable). This figure shows a Lorenz curve with the cumulated proportion of the population on one axis and the cumulated calories available for distribution on the other. The available food is then distributed from right to left along the black curve. The given curve indicates that in this case, the first (most privileged) 10 percent of the population use 30 percent of the available food. Going further down the curve, about 23 percent of the population use half of the food, and half of the population uses 75 percent of the food. The borderline between the food-secure and the food-insecure population is then established by applying an externally-defined minimum calorie requirement per person. At the point where the remaining food supply falls below the minimum requirement times the remaining population, the border line for the population considered to be food insecure is established. Over time the proportions food insecure may change as a consequence of changes in the calories available for distribution or possible changes in the assumed food distribution function.

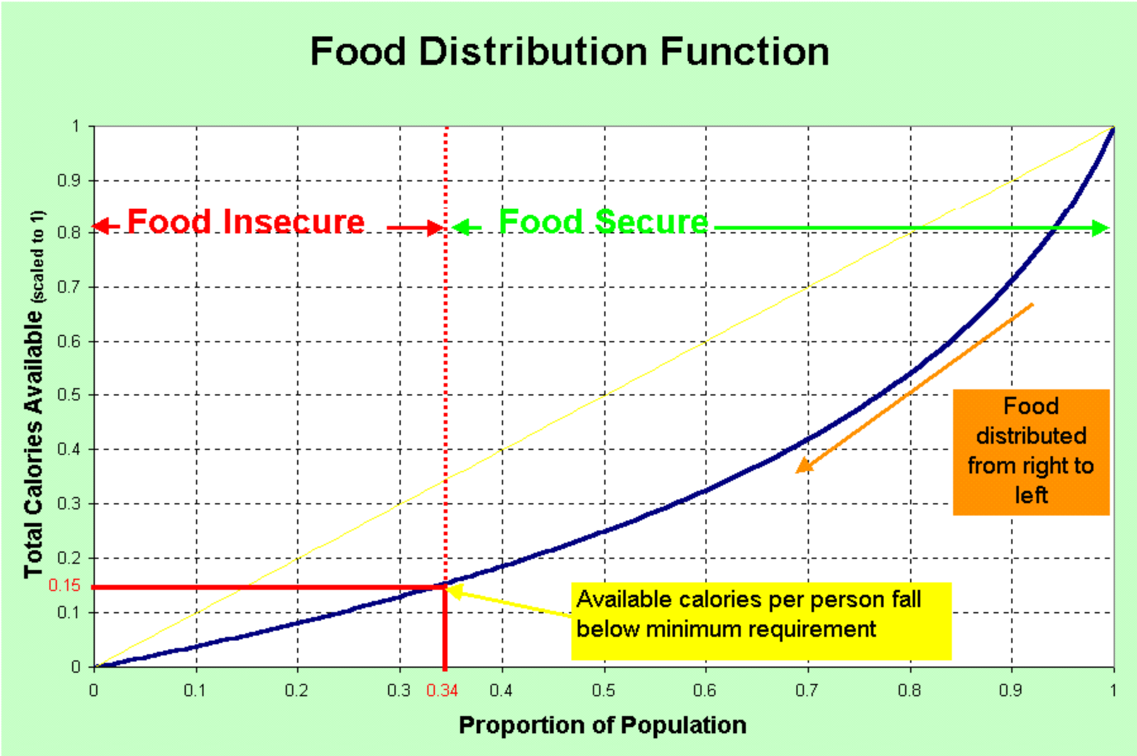

Figure 3. Food distribution function.

Alternative scenarios for Mali

Due to space limitation, we can only give a very concise example of an actual application of PEDA to one country. We chose the example of Mali, a country with serious land degradation problems, a total fertility rate of still 6.6 and an estimated proportion of food insecure of around 30 percent. To compare the consequences of alternative possible future trends for the country, we defined six different scenarios for Mali from the starting year 1995 (latest data available) up to the year 2030.

Scenario 1: All rates remain constant at their 1995 level.

Scenario 2: Fertility in all groups declines to half of its 1995 level (i.e., on average to 3.3) by 2030; all other rates are constant.

Scenario 3: In addition to the fertility decline as in scenario 2, primary school enrollment rates increase to 80 percent immediately, i.e., 80 percent of young cohorts become literate.

Scenario 4: On top of the assumptions of scenario 3, some civil unrest (or other problems) results in a loss of 30 percent of the harvest for the years 2005-2009.

Scenario 5: Fertility even declines to 2.2 (on average) by 2020; 80 percent of young cohorts become literate; otherwise like scenario 1.

Scenario 6: Like scenario 5, but fertilizer use also increases by 1 percent per year.

These six scenarios result in very extensive output tables. Detailed results for single year age groups could be studied for each of the eight sub-populations or any combination of them, e.g., food-insecure illiterate rural women between age 60 and 65. In terms of human welfare the most interesting aggregate level result is probably the evolution of the proportion food insecure.

Figure 4 depicts the trend in the total proportion food insecure under the six alternative scenarios as described above. Since the graph is on a relative scale, one needs to know that under scenario 1(constant rates) the total population of Mali would increase from 9.7 million in 1995 to 31.7 million in 2030. Because of the great momentum of population growth, even under the fertility decline scenario (scenario 2), total population would increase to 23.1 million, and under extremely rapid fertility decline (scenario 5) total population would still almost double to 17.6 million by 2030. For the first two scenarios the food security deteriorates significantly over time, even though an assumed fertility decline (scenario 2) would ameliorate the situation. Somewhat additional strong educational efforts (scenarios 3 and 4) clearly make a significant difference, and the proportion food insecure would only increase to somewhat above 40 percent. This is because education has three partly independent beneficial effects on food security: it increases productivity, it slows down land degradation, and it reduces population growth through a higher proportion in the more educated sub-populations that have lower fertility. Scenario 5, which assumes a temporary problem in terms of loss of food, has a clearly visible negative effect on the graph, but has only insignificant lasting consequences. A sustainable stabilization in food security, however, only results from scenario 5 that assumes an even more rapid fertility decline to 2.2 by 2020. If improvements in fertilizer use (scenario 6) are added to that, then the situation would even improve somewhat in the longer run.

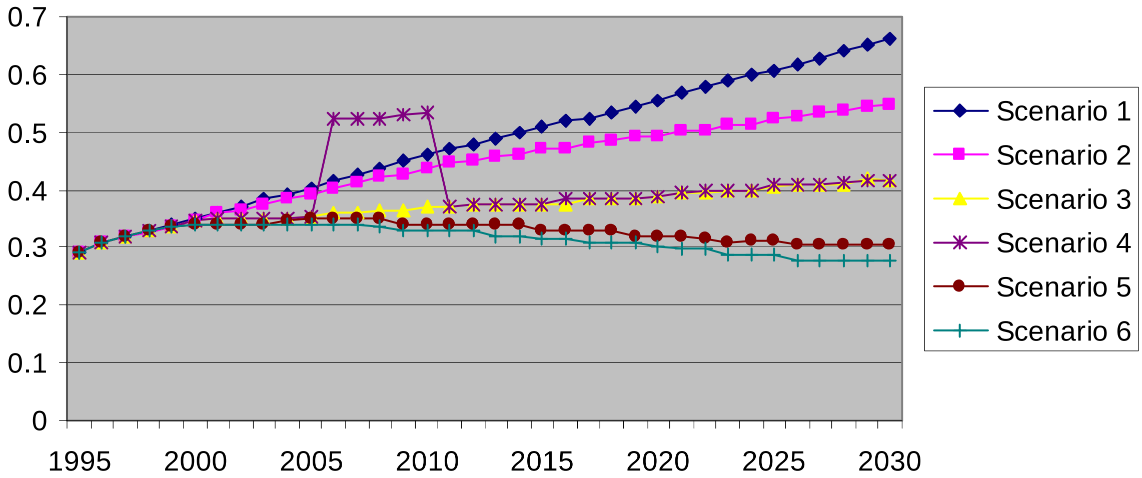

Figure 4. Trends in the total proportion of the population that will be food insecure in Mali under different scenarios.

Figure 5 shows the same 6 scenarios for the absolute number of food-insecure children below age 5 in the whole country. The picture has the same qualitative structure as the one in Figure 4. Under the (unrealistic) constant rates scenario the absolute number of food-insecure children would increase by an incredible factor of more than 8. Here the difference between scenarios 1 and 2 (fertility goes to half the level by 2030) is already very significant and would result in only less than half of the number of food-insecure children. Education also makes a difference, but less strongly than in the case of total proportions food insecure. Finally, increased fertilizer use makes little difference with respect to the number of food-insecure children. In conclusion the comparison between Figures 4 and 5 shows that for the absolute number of children that will be food insecure, the assumed level of future fertility is of overriding importance, whereas for the total proportions food insecure, education and some of the technical inputs such as fertilizer use also have considerable impact.

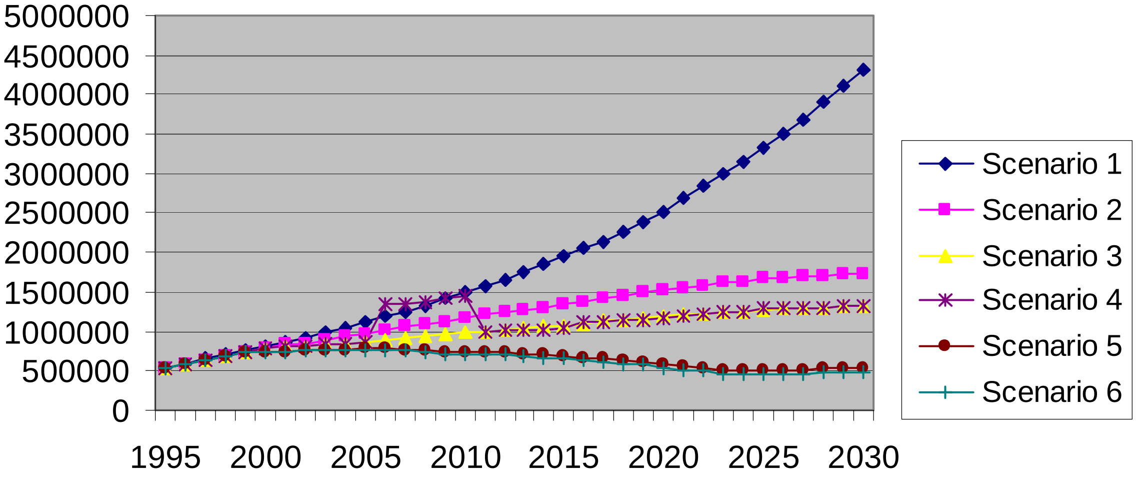

Figure 5. Trends in the absolute numbers of children aged 0-4 that will be food insecure in Mali under different scenarios.

The applications of the full empirical PEDA model to different African countries under large sets of alternative assumptions have been and will be extensively described elsewhere (Lutz and Scherbov 2000). In the context of this chapter it was only important to outline the basic structure and philosophy of the model and give a taste of its potential for applications. This will now allow us to compare it to a highly reduced form of PEDA with much lower empirical content and complexity, but higher flexibility and transparency.

Set Up and Assessment of a Reduced Form PEDA Model

When deciding on a reduced form model we have to keep in mind that it will depend on the specific context that determines the degree of aggregation. As the food distribution function is a key feature that governs the dynamics in the PEDA model, we aim to build up a reduced form model that focuses on gaining a better understanding of how the degree in inequality in food distribution affects population and resource dynamics and their interactions, and reduces complexity in the other parts of the model.

In the reduced model we, therefore, disregard the urban population (which in PEDA is not involved in food production) and consider only rural food-secure P_S and rural food-insecure population P_I . We also aggregate over education, age and sex. This simplification comes at the cost of losing age-structural impacts such as momentum, as well as the possibility of looking at age and sex differentials of the results. But since we want to focus primarily on the food distribution mechanism, the significant reduction in cells from 1600 (8 states x 2 sexes x 100 age groups) to just two (food-secure and food-insecure population) seems to be worth the costs. In addition to these two population stocks, the reduced form model, similar to the full PEDA model, also includes the resource stock R as the third dynamic variable.

The basic structure of the reduced form PEDA model is sketched in Figure 6 (for a detailed summary of the equations of motion of the reduced form model, see Appendix A). Each time period, the stock of rural food-secure and rural food-insecure populations, together with the amount of available resources, determines the total food production. The stock of resources will in turn depend on indigenous resource growth and previous degradation as caused by the rural food-insecure population and the prevailing population density. As food is not equally distributed among the population, and some minimum level of calories is necessary to be food secure, the prevailing food distribution will determine the proportion of the population that becomes food insecure or food secure for each time period. Together with the prevailing natural growth rates of the population, the movement between food security and food insecurity status will determine the next period's stocks of the rural food-secure and rural food-insecure populations.

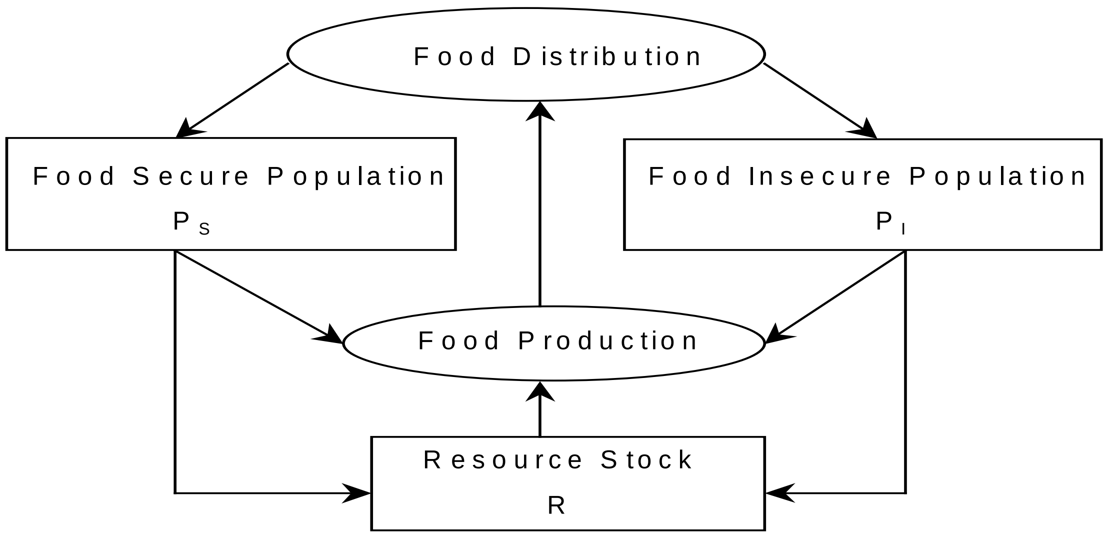

Figure 6. Reduced form PEDA model.

To calibrate the reduced form model such that it matches the full empirical model, we first run the full empirical model, using the same functional form of the Lorenz curve, resource dynamics and production function as summarized in Appendix A. Additionally, all transitions between rural/urban and literate/illiterate population states have been set equal to zero and all production elasticities except those for resources and labor were set equal to zero in the production function. From the resulting time series of rural population states by age and sex, we construct the time series of the total stock of rural food-secure and rural food-insecure populations. The corresponding time series of births and deaths for rural food-secure and rural food-insecure populations can then be used to construct the natural population growth rates of each sub-population. Next, we run the reduced form model with the exogenous time series of the population growth rates as obtained by the empirical model. The other parameters, as well as the initial conditions for the rural food-secure and rural food-insecure populations and the resource stock, have been set accordingly to the empirical model. Obviously, after this calibration, the reduced form model will produce very similar results to the empirical model.

In Figures 7a-7c we plot the time series of the food-secure and food-insecure rural populations, and the resource stock for the full empirical and the reduced form model for various degrees of inequality in the food distribution as measured by the parameter α.

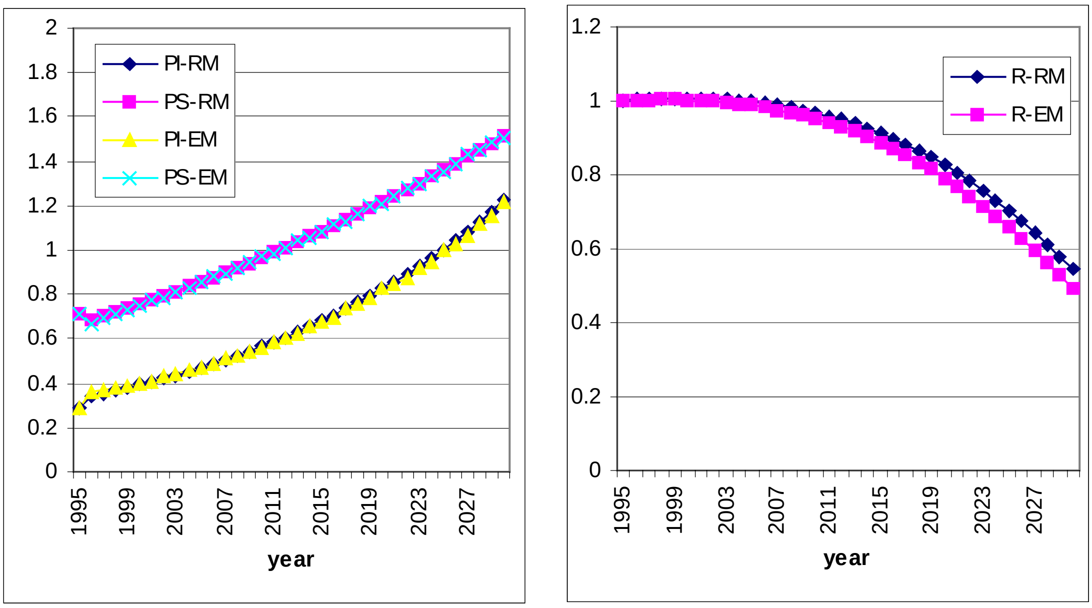

Figure 7a. Simulations of food-secure and food-insecure rural populations and resources for the reduced form model (RM) and the full empirical model (EM) with the degree of inequality in food distribution set at α=*2.667086.*

Remarks: The population stock of the food-insecure and food-secure rural populations has been scaled to the initial stock of the whole rural population, i.e., $P_I(t)/P(0)$, $P_S(t)/P(0)$. Resources are scaled to start at the value of R(0)=1. Parameters have been set at $a=0.02$, $\overline{R}=1.5$, $\gamma=0.02$, $\eta=1$, $T=2.912$, $\beta_1=0.088$, $\beta_2=0.534$, $y^*=1.2$.

Choosing Figure 7a as the baseline scenario, Figures 7b and 7c demonstrate the effect of reducing or increasing the degree of inequality in the food distribution, respectively.

The results clearly evidence that environmental degradation and population dynamics are closely interlinked with institutional settings as represented by the food distribution function. When food is more equally distributed among the population, the share of the food-insecure population will be lower, and the pressure on the resource stock will be less, as evidenced in Figure 7b. A very unequal food distribution (Figure 7c) may initiate the vicious circle between population growth and environmental degradation.

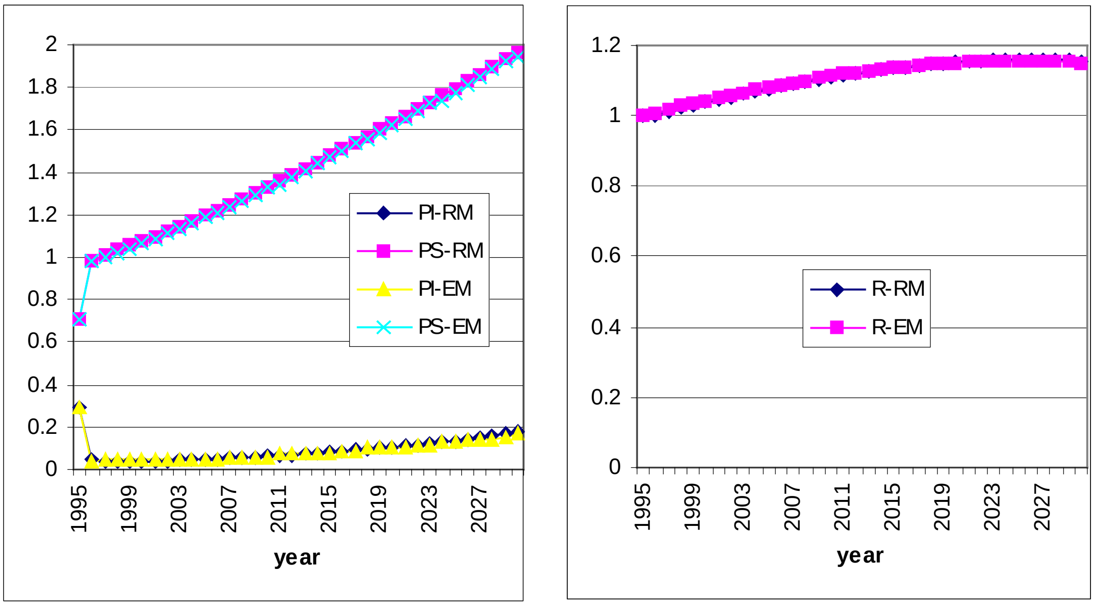

Figure 7b. Simulations of food-secure and food-insecure rural populations and resources for the reduced form model (RM) and the full empirical model (EM) with the degree of inequality in food distribution set at α=1.334172.

Remarks: The population stock of the food-insecure and food-secure rural populations has been scaled to the initial stock of the whole rural population, i.e., $P_I(t)/P(0)$, $P_S(t)/P(0)$. Resources are scaled to start at the value of $R(0)=1$. Parameters are set as in Figure 7a.

The correspondence between the full empirical and the reduced form PEDA model is very persuasive, even more so since we did not re-calibrate our reduced form model when we changed the form of the food distribution function in Figures 7b and 7c. More specifically, we ran the reduced form model with the same exogenous natural population growth rates as in Figure 7a, and only varied the parameter of the food distribution function in Figures 7b and 7c. In the full empirical model, any change in the form of the food distribution function will feed back on the natural population growth rates of each sub-population. Even though we keep fertility and mortality constant in the full empirical model as well, the transition between population states will affect the age composition and henceforth the number of births and deaths. But as our reduced form model shows, these compositional changes in the age structure may not be very strong, at least not for the range of parameter values we consider.

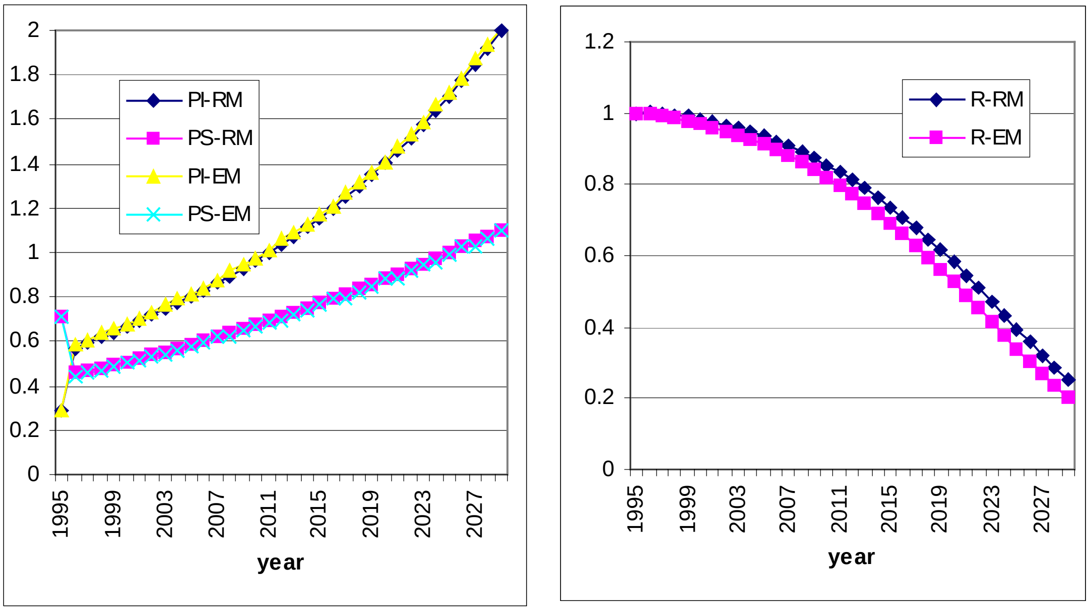

Figure 7c. Simulations of food-secure and food-insecure rural populations and resources for the reduced form model (RM) and the full empirical model (EM) with the degree of inequality in food distribution set at α=5.334172.

Remarks: The population stock of the food-insecure and food-secure rural populations has been scaled to the initial stock of the whole rural population, i.e., $P_I(t)/P(0)$, $P_S(t)/P(0)$. Resources are scaled to start at the value of $R(0)=1$. Parameters are set as in Figure 7a.

The simulations in Figures 7a-7c demonstrate various important roles of reduced form models. We can use such aggregate models to better understand in which way changes of, e.g., the food distribution will affect the dynamics of population structures and natural resources. That there will be effects is obvious, but it is equally important to understand whether such effects will mainly work through the transition between population states or whether they might feed back on population processes as the natural population growth rates. Although we do not re-calibrate the population growth rates, we nevertheless get quantitatively similar results for the reduced form and the full empirical model in Figures 7b and 7c. This indicates that changes in the food distribution function mainly affect the composition of population states as opposed to the age composition of various population states. Hence, the sensitivity analysis as conducted in Figures 7a-7c acts as a test, whether the reduced form model is adequate to investigate the role of food distribution, and thereby contributes to a better understanding of the underlying mechanisms.

Once we have set up a reduced form model that yields qualitatively the same dynamics as the full empirical model, and have investigated whether this qualitative equivalence persists under specific sensitivity analysis, we can use the reduced form model to investigate the spectrum of possible dynamics. Not only can we perform simulations much faster by running a reduced form model, but we may also be able to explicitly derive analytical expressions. The latter suggestion is most relevant when we consider the long-run behavior of our model. While the transient behavior may be very sensitive to parameter changes, the long-run values of the population and resource stock may be very stable across various parameter changes.

Recalling the equations of motion of the rural food-secure and rural food-insecure populations as stated in Appendix A, it can easily be verified that for positive values of the natural population growth rates, the system does not yield any sustainable long run equilibrium. More specifically, since population grows exponentially and resources only logistically, resources will tend towards zero in the long run, while the whole population becomes food insecure. To facilitate a non-trivial long run equilibrium we have altered the total fertility rate of the food-secure and food-insecure populations to 1.5 and 3.0, respectively, while keeping all other parameters as in Figure 7a. Simulations of the full empirical model and aggregation over age, sex and education as outlined above implies that for this alternative scenario the population growth rates of the food-secure and food-insecure rural populations decline to n_S=-0.015 and n_I=0.006 by 2030.

We then use these values of the population growth rates and iterate the reduced form model until all state variables remain stationary, i.e., until the system converges to an equilibrium with constant stocks of food-secure and food-insecure rural populations and resources. For this parameter set the reduced model converges to a state where the whole population becomes extinct and resources equilibrate at their maximum carrying capacity $\overline{R}$ (compare Figure 9a). But as analytical calculations (Appendix B) show, there may exist a non-trivial equilibrium with strictly positive values of food-secure and food-insecure rural populations and a positive level of resources below the maximum carrying capacity. Obviously this non-trivial equilibrium must be either unstable or be separated by an unstable equilibrium, since we would have otherwise converged towards this equilibrium by applying forward iteration of our reduced form model. Analytical calculations confirm the existence of a non-trivial equilibrium that is unstable (we have indicated the values of the state variables for this equilibrium by drawing a vertical line in Figures 8a and 8b at point A).

Similar to the concept of comparative static in economics we can investigate the dependence of the level of the unstable equilibrium on changes in, e.g., the degree of inequality of food distribution as given by the parameter α. By applying a more sophisticated continuation method of the equilibrium (Khibnik *et al*. 1993) we can trace the stability of the equilibrium in addition to its level as we change the parameter α. The plot of the equilibrium value of population stocks (Figure 8b) and resources (Figure 8a) versus the parameter α is termed “bifurcation diagram.” The term “bifurcation” refers to the fact that the diagram may not be restricted to a plot of only a single equilibrium but multiple equilibria can emerge or disappear through various kinds of bifurcations. As evidenced in Figures 8a and 8b, the reduced form model is only characterized by a unique equilibrium, at least for the parameter range we consider. Moreover, this equilibrium is unstable over the range of values of α, which we consider.

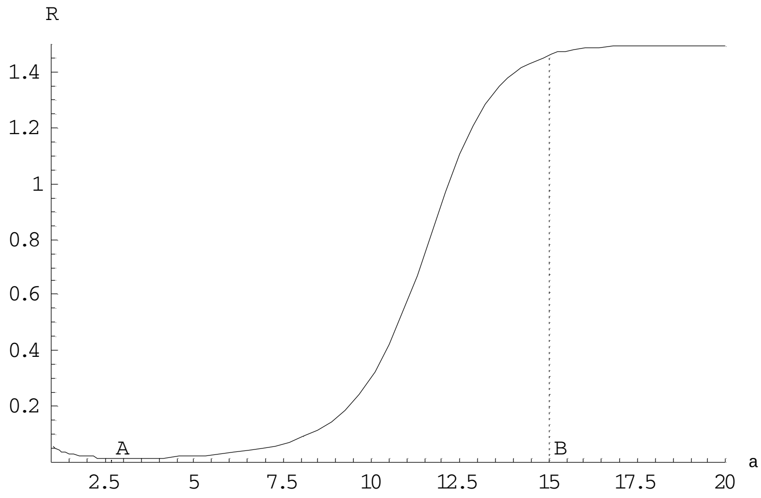

Figure 8a. Bifurcation diagram of the resource stock with respect to the parameter α which measures the degree of inequality in food distribution.

Remarks: Resources are scaled to start at the value of R(0)=1. Parameters are set at $a=0.02$, $\overline{R}=1.5$, $\gamma=0.02$, $\eta=1$, $T=2.912$, $\beta_1=0.088$, $\beta_2=0.534$, $y^*=1.2$, $n_I=0.005528$, $n_S=-0.0153$.

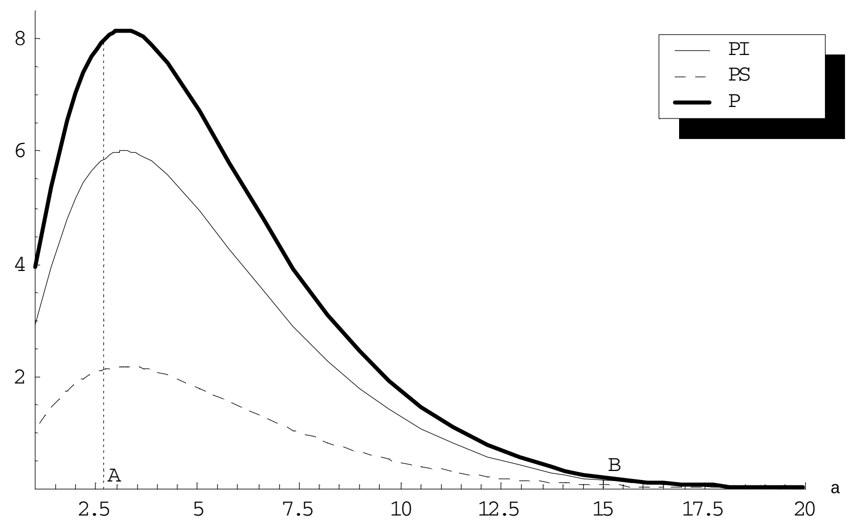

Figure 8b. Bifurcation diagram of the rural food-insecure population (solid line), the rural food-secure population (dashed line), and total rural population (bold line) with respect to the parameter α which measures the degree of inequality in food distribution.

Remarks: The population stock of the food-insecure and food-secure rural populations has been scaled to the initial stock of the whole rural population, i.e., $P_I(t)/P(0)$, $P_S(t)/P(0)$ and $P(t)/P(0)$. Parameters are set as in Figure 8a.

As we increase the parameter α, i.e., as food distribution becomes more unequal, the equilibrium values of population first increase and then start to decrease (Figure 8b). At the same time resources continuously increase, reaching the upper level of carrying capacity for high values of α (Figure 8a).

How can we interpret these results? As the unique equilibrium that exists is unstable, it is more straightforward to first consider the possible long-run behavior of the model for various values of α. We choose the values of α that correspond to points A and B in Figures 8a and 8b, and iterate the model forward 400 years in time. Initial conditions are set as in Figure 7a, implying that we start the simulations above (below) the value of the unstable equilibrium in the case of α equal to point A (point B). For α equal to point A, the system converges towards the trivial state where the whole population becomes extinct and the resources equilibrate at their carrying capacity (Figure 9a). Figure 9b illustrates an alternative long-run behavior of the model of α equal to point B. In this case resources will vanish in the long run and all populations end up being food insecure.

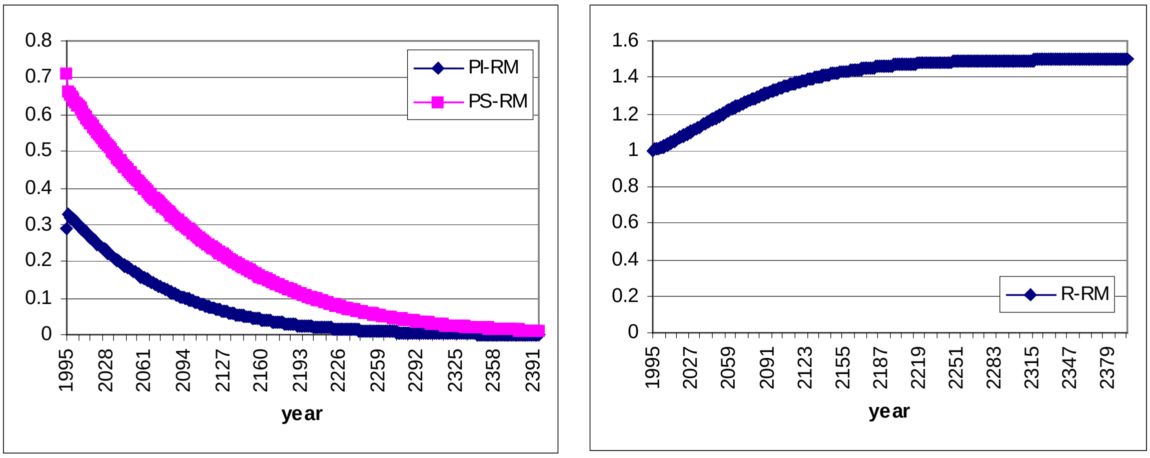

Figure 9a. Long-run behavior of rural food-secure and rural food-insecure populations and resources for the reduced model, and setting the degree of inequality in food distribution at α= 2.667086.

Remarks: The population stock of the rural food-insecure, rural food-secure populations and rural total population has been scaled to the initial stock of the whole population, i.e., $P_I(t)/P(0)$, $P_S(t)/P(0)$ and $P(t)/P(0)$. Parameters are set as in Figure 8a.

Recalling that we assume a negative rate of population growth for the rural food-secure population and a positive rate of population growth for the rural food-insecure population, the following results. In the case of low values of α and moderate levels of initial resources, part of the rural food-insecure population will become food secure in each time period. But as the rural food-secure population declines over time, food availability for the rural food-insecure population even increases over time and reinforces the transition towards food security status. Hence, in the long run, the whole population will become food secure; this is the population that ultimately vanishes as its natural population growth rate is negative. On the other hand, if inequality in food distribution is high, only a small amount of people will move from the food-insecurity status towards the food-security status. This amount may not be sufficient to counteract the indigenous decline of the rural food-secure population, nor may it relieve the population growth of the rural food-insecure population over time. Hence, a vicious circle of increasing food insecurity and environmental degradation may set in (Figure 9b) with the rural food-secure population vanishing in the long run.

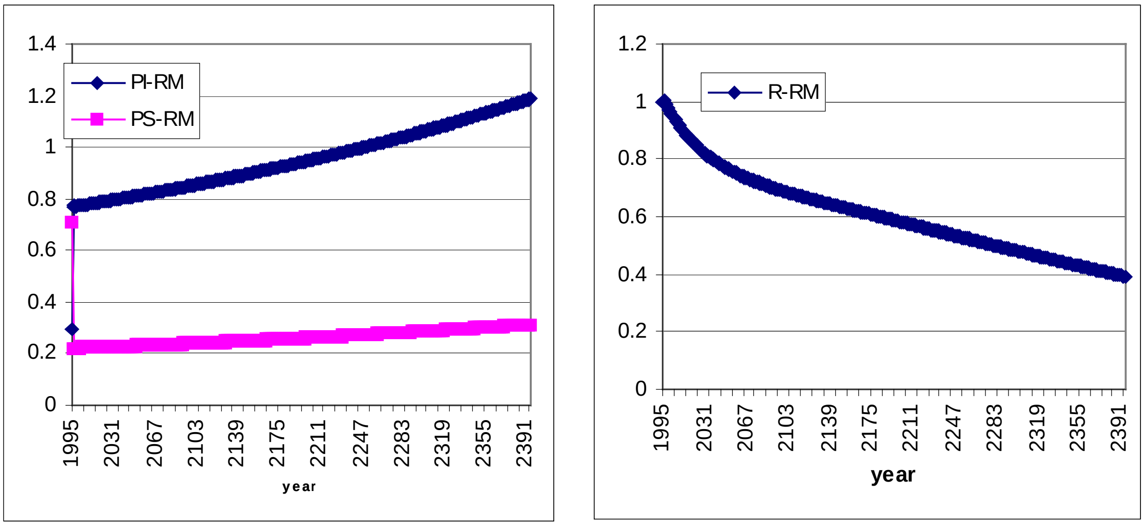

Figure 9b. Long-run behavior of rural food-secure and rural food-insecure populations and resources for the reduced model, and setting the degree of inequality in food distribution at α= 15.

Remarks: The population stock of the rural food-insecure, rural food-secure populations and rural total population have been scaled to the initial stock of the whole population, i.e., $P_I(t)/P(0)$, $P_S(t)/P(0)$ and $P(t)/P(0)$. Parameters are set as in Figure 8a.

Having discussed the alternative long-run behavior of the model, we can now interpret the specific values for the unstable equilibrium as given in Figures 8a and 8b. The existence of a non-trivial equilibrium requires a moderate but continuous positive flow of population that becomes food secure in each time period (otherwise the population stocks will not equilibrate at strictly positive values). More specifically, the flow needs to be large enough to countervail the negative population growth rate of the rural food-secure population and to relieve the positive population growth rate of the rural food-insecure population. Obviously the degree of inequality in the food distribution will determine the transition between food insecurity and food security; it is the resource stock that will ultimately determine the extent of this transition. For low values of α, the equilibrium resource stock will be low (Figure 8a), otherwise, the transition from food insecurity to food security is too great and we end up in a long-run state as indicated in Figure 9a. On the other hand, for high values of α, the resource stock must be high in equilibrium (Figure 8a), otherwise, the transition between food insecurity and food security is too low, and all populations will be food insecure in the long run, with resources tending towards zero as illustrated in Figure 9b.

What is the message conveyed by the long-run dynamics of the reduced model as illustrated in Figures 8 and 9? One of the most important lessons is that focus on only transient sustainable paths of population and resource dynamics is dangerous. Though the statement by Keynes, “In the long run we are all dead,” is persuasive for today’s society, the irreversibility of resource degradation cannot be neglected, and the responsibility of today’s society, therefore, has to exceed the lifespan of an individual. Hence, consideration of long-run dynamics is especially justified in research on population-environment interrelationships.

By offering alternative scenarios for long-term developments of resources and population, we can draw attention towards the sensitivity of the model assumptions as it regards specific parameter values and/or functional forms. For instance, for the parameter values chosen in Figure 8a, a sustainable future is only guaranteed, if we exactly start our simulations in the unstable equilibrium. Any deviation from the equilibrium values will imply convergence towards one of the trivial states as illustrated in Figures 9a and 9b. The specific long-run level of population and resources will furthermore depend on the degree of inequality in food distribution. Though the interpretation of the long-run dynamics as outlined above seems almost obvious and not really surprising, a mere sensitivity analysis of the full empirical model would be much more difficult and less intuitive.

Conclusions

A computer model is nothing but a tool to extend the capabilities of our human brain. Similar to using a pencil (or a keyboard for text processing) for writing down notes on thoughts that we will not be able to remember, a computer can help us to consider more things simultaneously than our human brain is usually able to do. It is true that a chess master can still be as good as a computer, but this may only be true for very few people after very long training and for a very limited set of problems. A second advantage of a computer model over the human brain is that the assumptions and steps that lead to a result are completely transparent and can be repeated by anybody interested in doing so. Similar again to the pencil (or a printer), thoughts and relationships are being nailed down in a lasting and open manner. This is something the human brain, even aided by speech, cannot do.

In the field of population and environment analysis – as in many other fields – it is therefore the wrong dichotomy to ask whether one should either use computer models or other more qualitative or anthropological approaches. It is simply a matter of the choice of the right tools for a given research question. If the objective is to anticipate future trends or give warning of pending dangers in a quantitative way that is explicit, so that it can be repeated by others, then the choice of tool will most likely be a computer model. But this is not the end of the decision process because there is an infinite number of different possible specifications of computer models, even for rather specific future-related questions.

While the questions of choice of variables, regional focus and level of regional aggregation can usually be resolved in a straightforward way, depending on the context of the question asked, in this paper we address one of the more difficult issues, namely that of the appropriate level of complexity and empirical content.

In judging the usefulness of reduced form models versus full empirical models, it is useful to recall the recent debate in the population forecasting literature on “simple vs.

complex models" (Rogers 1995a). "The most fundamental rationale for disaggregation is the demographer's eternal hope that, when the right disaggregation is discovered, temporal changes that initially appear disorderly will be resolved into the effects of a distorted underlying structural composition applied to constant, or at least smoothly changing, refined rates" (Lee 1995:218). Multistate population models, such as the population component of the PEDA model, are partly based on this assumption. When studying the process of changing food security disaggregated by age, sex and education (assumed to be important sources of heterogeneity), it is hoped to produce a more accurate estimate of the future food-secure population than would result from the simple projection of a population which is assumed to be completely homogeneous. For total population size, Lutz *et al.* (1999) have recently shown that explicit consideration of educational groups in addition to age and sex tends to make significant differences (as opposed to considering only age and sex) for the projected total population size, especially for populations with strong educational fertility and mortality differentials and recent improvements in school enrollment. This discussion, however, only concerns different sources of heterogeneity, assuming that one is only interested in the aggregate outcome. If there is interest in the relative status of women, of specific age categories or of certain educational groups, then there is no alternative to the explicit consideration of these dimensions. Increasing female education can only be considered as a strategy and studied in terms of its consequences in a model that includes female education.

But Lee (1995:220) also notes: "More complex disaggregations may simply conceal dynamic regularities that would otherwise be apparent." This aspect best captures the rationale of the reduced form PEDA model. It refers to the danger of not seeing the wood for the trees. On the other hand, there is no wood without trees, and the aggregate picture in the end results from the sum of individual behavior. Obviously, both the wood and the trees matter.

What does this tell us about the relationship between reduced and full models? If we can verify that the qualitative dynamics persist under the processes of aggregation and reduction, then the reduced form model helps to elucidate the sensitivity of the dynamics as depending on parameter changes. Moreover, the reduced form model may help to derive analytical expressions for long-run values of the state variables and thereby facilitate comparative static analysis.

A strongly reduced model can never replace a full empirical model; it should rather serve as a complimentary tool to (a) get more insight into specific dynamics, and (b) to aid numerical sensitivity analysis. Hence, we should neither become a victim of the fallacy of "simple is beautiful" nor of the belief that adding on complexity always improves understanding. For the researcher in population and environment analysis, this implies that judgment should be based on analysis of models of different complicity. And here, human intuition, experience, and plausibility consideration also qualify as models.

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Appendix A: The Reduced Form PEDA Model

The model consists of three difference equations that describe the dynamics of the rural food-insecure population P_I, the rural food-secure population P_S, and the natural resources R:

$$
\begin{aligned}
&P_I(t+1) = [1+n_I(t)]P_I(t) + M(P_I(t), P_S(t), R(t)) \\
&P_S(t+1) = [1+n_S(t)]P_S(t) - M(P_I(t), P_S(t), R(t)) \\
&R(t+1) = R(t) + g(R(t)) - D(P_I(t), P_S(t), R(t))
\end{aligned}
\qquad (1)
$$

where n_I(t) and n_S(t) denote the exogenous given trajectories of natural growth rates for the rural food-insecure and the rural food-secure populations. In addition to the natural population growth rates, there will be a movement (transition) between the two sub-populations as described by the function $M(P_I(t),P_S(t),R(t))$. The functions *g(.)* and *D(.)* stand for indigenous growth and degradation of the natural resource stock.

Per capita food production, together with the degree of inequality of food distribution, will determine the share of the food-insecure population and hence, the value of the function *M(.,.,.)* for each period. Whenever the percentage of the food-insecure population remains below the percentage of the population that falls short of some exogenously-fixed level of minimum calories, *M(.,.,.)* will be positive. This implies that part of the food-secure population will become food insecure in the next period. On the other hand, part of the food-insecure population might become food secure in the next period, if *M(.,.,.)* is negative.

Total food production each period *Y(t)* depends on total population $P(t) = P_I(t) + P_S(t)$, resources *R(t)* and an exogenous technology parameter *T*:

$$
Y(t) = T R(t)^{\beta_1} (P_I(t) + P_S(t))^{\beta_2} \qquad (2)
$$

where β_1, β_2 denote the production elasticities of resources and population.

As a measure of the prevailing inequality we chose the Lorenz curve *L(F(z))* (see Figure A1) which “plots cumulative shares of income *L(F(z))* as a function of cumulative population shares *F(z)* when individuals are ranked in increasing order of income *z*” (Lam 1988:143ff).

To determine the percentage of the food-insecure population, we exploit the fact that the slope of the Lorenz curve at any point is inversely proportional to the mean income y(t) = Y(t)/P(t) (Atkinson 1970; Lam 1988) and given by

$$
dL(F(z))/dF(z) = l(F(z)) = z/y\,. \qquad (3)
$$

Note that we have skipped the time argument in Eq. (3) and retain this simplification of notation for visibility for the following explanations that refer to the Lorenz curve.

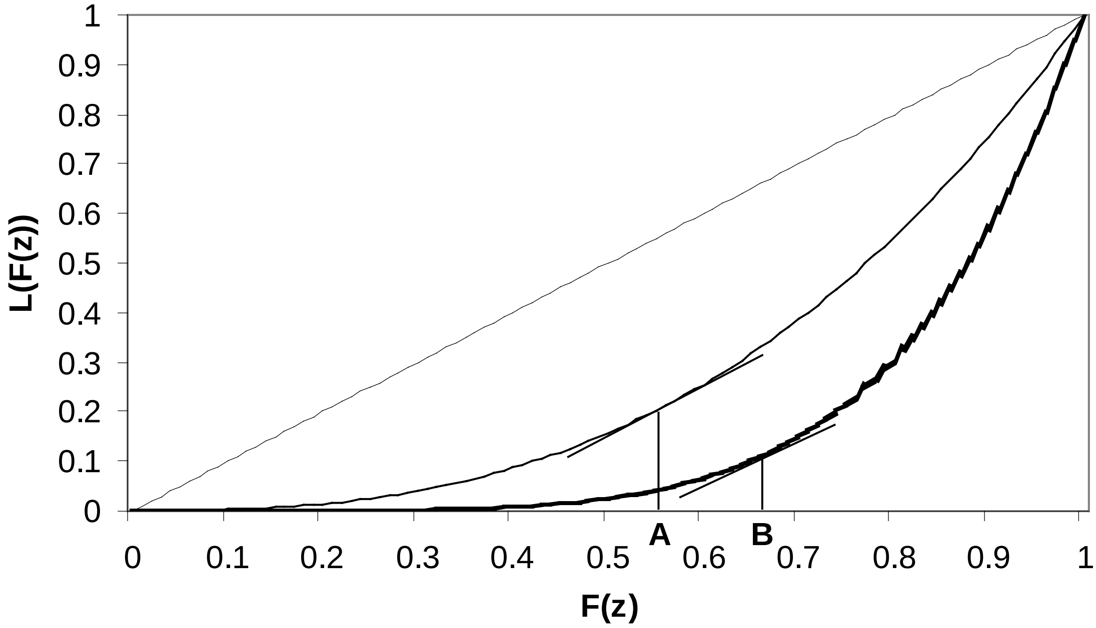

Figure A1. Lorenz curve.

If the Lorenz curve coincides with the 45° line (cf. Figure A1), the slope of the Lorenz curve is equal to 1 at each point, and everyone in the economy will receive the mean income, i.e., "perfect equality" will prevail. Therefore, the convexity of the Lorenz curve measures the degree of inequality that prevails in the economy. For example, the faded and the bold lines in Figure A1 indicate two Lorenz curves which represent different degrees of inequality in food distribution.

Recalling Eq. (3), we can determine the share of the population that receives an income less than or equal to the mean income in the economy for each Lorenz curve by marking the points on each Lorenz curve which have slope 1. Drawing these lines in Figure A1 shows that the bold line Lorenz curve relating to point B is more convex and obviously more people will have income below the mean income as compared to point A.

Assuming that the derivative of the Lorenz curve, as denoted by l, is invertible, Eq. (3) can be written as

$$F(z) = l^{-1}(z/y)\,. \tag{4}$$

In the numerical analysis we shall postulate a stationary functional relationship for the Lorenz curve L(F(z)) instead of starting from an income distribution with density function f(x). In particular, for all further numerical calculations, we shall postulate the following flexible and simple analytical form of L(.) (which has also been postulated in Figure A1):

$$L(F(z)) = [F(z)]^{\alpha} \text{ with } \alpha > 1. \tag{5}$$

By using this special form of the Lorenz curve, Eq. (4) reduces to

$$F(z) = (z/(\alpha y))^{1/(\alpha-1)}. \tag{6}$$

Since *F(.)* represents a distribution function, at the most it can take the value 1. The corresponding income level z_{max} for which $F(z_{max})=1$ holds, indicates the maximum income level in the economy for which 100 percent of the population receives income less than z_{max}. Eq. (6) implies that the maximum income level in the economy is given by $z_{max} = \alpha\, y$ and will therefore be constrained by the mean income *y* and the degree of inequality α. Unless the economy is in a stationary state, the mean income *y* and the maximum per capita income z_{max} will vary over time. *Ceteris paribus*, the higher the prevailing income inequality and the higher the mean income, the higher the maximum income level in the economy.

Summing up, we have derived a formula to calculate the proportion of the population *F(z)* that receives income less than or equal to *z*. In order to highlight the fact that the proportion of the population *F(z)* that receives income less than or equal to *z* depends on the mean income *y*, we redefine the function F(.) as *F(z,y)*, i.e., we add the mean income as a second argument. Assuming a threshold level of calories *y** a person needs to be food secure, the proportion of the food-insecure population of the total population is determined by

$$F(y^*, y) = \begin{cases} (y^*/(\alpha y))^{1/(\alpha-1)} \, if \; y^* < \alpha y \\ 1 \, if \; y^* \geq \alpha y \end{cases}. \tag{7}$$

If $m := F(y^*,y) - P_I/P > 0$ holds, a number of $m \cdot (PI+PS)$ of former food-secure people will become food insecure. Therefore, the movement between the food-insecure and food-secure populations is given by

$$M(t) = \left[F(y(t), y^*) - \frac{P_I(t)}{P_I(t) + P_S(t)} \right] \left[P_I(t) + P_S(t) \right]. \tag{8}$$

Finally, the change of the resource stock is described by indigenous growth $g(R(t))=a\ (\overline{R} - R(t))$ and reduced by environmental degradation $D(P_I(t), P_S(t), R(t))$. The coefficient $\overline{R}$ determines the saturation level of the resource stock (i.e., $\overline{R}$ is the stationary solution of *R* if the resource is not degraded) and the parameter *a* determines the speed at which the resource regenerates.

Degradation *D(.,.,.)* of the resource stock depends on the stock of available resources and on the number of food-insecure and food-secure people. We assume that environmental degradation is increasing in the stock of resources and the amount of food-insecure population. More specific, we assume that environmental degradation increases linearly in P_I but increases at a decreasing rate with the level of the resources *R*. Moreover, we assume that the scale of environmental degradation is a function of population density as measured by the factor $P/\overline{R}$. Consequently, environmental degradation is given by

$$D(t) = \gamma \frac{P(t)}{\overline{R}} P_I(t) \frac{R(t)}{R(t) + \eta} \tag{9}$$

where γ and η are fixed parameters. If resources are completely degraded, i.e., $R=0$, environmental degradation is zero, since there is nothing to be depleted. Similarly, if the stock of food-insecure population is zero, environmental degradation will be zero. The assumption that environmental degradation will increase in the stock of resources, but at a decreasing rate, implies that there exists an upper limit of environmental degradation; this is given by $(P/\bar{R})\,\gamma\, P_I$.

Having presented the model equations we need to state some specific peculiarities on the simulations.

Since the resource stock is measured in index form, we scale population variables to one in the degradation function D(t) in order to increase numerical stability.

In the scenario, where the population growth rate of the food-secure population is negative, the movement $M(P_I(t),P_S(t),R(t))$ between the food-secure and food-insecure populations has to be changed slightly in order to guarantee the non-negativity of the stock of food-secure population. It may be the case that Eq. (8) results in a number of food-secure population that becomes food insecure, which exceeds the current number of food-secure people. This could happen, since the transition between food-insecurity and food-security status relates to the population stocks of the previous period, while the current population stock may have already declined due to a negative natural growth rate. Therefore, the movement $M(P_I(t),P_S(t),R(t))$ between the food-secure and food-insecure populations has to be defined as follows:

$$M(t) = \min\left\{\left[F(y(t), y^*) - \frac{P_I(t)}{P_I(t)+P_S(t)}\right]\left[P_I(t)+P_S(t)\right], (1+n_S)P_S(t)\right\}. \qquad (10)$$

Appendix B: Derivation of the Non-Trivial Equilibrium in the Reduced Form PEDA Model

From the evolution of the stock of the rural food-insecure and rural food-secure populations, we derive

$$\begin{aligned} P_I(t+1) - P_I(t) &= n_I P_I(t) + M(P_I(t), P_S(t), R(t)) \\ P_S(t+1) - P_S(t) &= n_S P_S(t) - M(P_I(t), P_S(t), R(t)) \end{aligned} \quad . \tag{11}$$

Setting both equations in (11) equal to zero and summing up yields

$$n_I P_I(t) + n_S P_S(t) = 0 \, . \tag{12}$$

Eq. (12) proves that there exists no sustainable long-run equilibrium, if both population growth rates are positive or both are negative. Furthermore, Eq. (12) shows that in equilibrium the stock of the rural food-insecure population is proportional to the stock of the rural food-secure population, and vice versa.

Recalling the definition of *M(t)* in case of a negative natural growth rate of the rural food-secure population (Eq. (10)), we need to distinguish between two possibilities: (1) If $M(t) = (1+n_S)P_S(t)$ holds, the equilibrium value of the number of rural food-secure population is equal to zero; similarly, the number of rural food-insecure population is equal to zero in equilibrium. In what follows, we concentrate on the second case: (2) where the transition between food-security and food-insecurity status is given by $M(t) = \left[F(y(t), y^*) - \frac{P_I(t)}{P_I(t) + P_S(t)} \right] [P_I(t) + P_S(t)]$.

Substituting the latter expression into one of the equations in (11) and taking condition (12) into account, we derive the following equilibrium condition:

$$P_I(t) = \left(\frac{\alpha T}{y^*} \frac{(1-n_I)^{\alpha-1}}{\left(1 - \frac{n_I}{n_S}\right)^{\alpha-\beta_2}} \right)^{\frac{1}{1-\beta_2}} R^{\frac{\beta_1}{1-\beta_2}} \, . \tag{13}$$

To calculate the equilibrium value for resources, we proceed in a similar way. Setting

$$R(t+1) - R(t) = a(\overline{R} - R(t)) - \frac{(P_I(t) + P_S(t))}{\overline{R}} \gamma P_I(t) \frac{R(t)}{R(t)+\eta} = 0 \, ,$$

and taking condition (13) into account yields

$$P_I(t)^2 = \frac{a\overline{R}(\overline{R} - R)(R+\eta)}{\gamma \qquad R\left(1 - \frac{n_I}{n_S}\right)} \, . \tag{14}$$

Combining Eq. (13) and Eq. (14) yields

$$\gamma\left(1-\frac{n_I}{n_S}\right)\left(\frac{\alpha T}{y^*}\frac{(1-n_I)^{\alpha-1}}{\left(1-\frac{n_I}{n_S}\right)^{\alpha-\beta_2}}\right)^{\frac{2}{1-\beta_2}} R^{\frac{2\beta_1}{1-\beta_2}+1} = a\overline{R}\left(\overline{R}-R\right)\left(R+\eta\right), \tag{15}$$

which implicitly defines the equilibrium resource stock. Eq. (15) can be solved only numerically, but we can easily determine the number of equilibria geometrically. The right-hand side defines a parabola with zeros at $R=\overline{R}$ and $R=-\eta$ and the vertex lies at $\left(\frac{\overline{R}+\eta}{2}, a\overline{R}\left(\frac{\overline{R}+\eta}{2}\right)^2\right)$. The left-hand side is an increasing function of R with a unique zero at the origin. Consequently, there exists only one intersection for positive values of R. Hence, there exists a unique non-trivial equilibrium.