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A Systems Approach to Modeling Catastrophic Risk and Insurability

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Abstract. This paper describes a spatial-dynamic, stochastic optimization model that takes account of the complexities and dependencies of catastrophic risks. Following a description of the general model, the paper briefly discusses a case study of earthquake risk in the Irkutsk region of Russia. For this purpose the risk management model is customized to explicitly incorporate the geological characteristics of the region, as well as the seismic hazards and the vulnerability of the built environment. In its general form, the model can analyze the interplay between investment in mitigation and risk-sharing measures. In the application described in this paper, the model generates insurance strategies that are less vulnerable to insolvency.

Key words: catastrophic risk management, earthquake risk management, natural risk insurability, stochastic optimization, risk mapping.

1. Introduction

The management of catastrophic risks involves the assessment of the hazard, vulnerability analysis, and the allocation of resources for hazard and loss mitigation, preparedness and response. Catastrophic risk management also requires strategies for risk burden sharing, for which private and public insurance plays an important role. For this reason, insurance strategies should be better integrated into the overall measures of a country in coping with catastrophic risks (UNDRO, 1991). Insurers, however, are reluctant to enter markets that expose them to a risk of bankruptcy. In the U.S., for example, many insurers pulled out of catastrophic risk markets in response to their large losses from natural catastrophes in the last decade (Cummins and Doherty, 1996; Insurance Service Office, 1994; Kunreuther, 1996).

To reduce their risk of insolvency, insurers' strategies might be based on modeling tools that account for the complexity implied by the manifold dependencies in the stochastic process of catastrophic events, decisions and losses (see discussions in: MacDonald, 1992; Swiss Re, 1997). Some of the most important dependencies include:

- the clustering of events in time in a particular region;
- the space-time correlation among climatic events in different regions (such as hurricanes, floods, droughts, extreme weather);
- the time sequence of previous events and losses, as well as the resulting policy measures (e.g. the status of preparedness and response, the dependency of property values on their degradation or restoration status);
- the correlation among different events and losses taking into account possibly cascading effects (such as earthquake → landslide → dam failure → flood → technological accident → diseases);
- the correlation among claims for losses covered by different policies (such as life, estate, car, employment, business interruption etc.), and at different locations.

Particularly the time correlations put into question the use of the Poisson distribution, and the geographical correlations emphasize the importance of insurance strategies with proper spatial diversification of the risks.

To study the problem in its complexity a spatial-dynamic, stochastic optimization model has been developed at IIASA (Ermoliev *et al.*, 1997; Ermolieva, 1997; Ermoliev *et al.*, 1998). The model is based on Monte Carlo simulations of catastrophic events in the selected regions. The key feature of the model is the stochastic search technique enabling adaptive adjustments of decision variables towards desirable outcomes on the basis of sequential simulations.

The model can be generalized to account for the interplay between *ex ante* investment in prevention/mitigation measures (on the part of the public authorities, the citizens and the insurance industry) and policies for sharing the financial costs *ex post* to the disaster. Insurance and other financial instruments can be viewed as reducing catastrophic losses *to a community* by spreading these losses over a wider region, and therefore as decreasing individual catastrophic exposure. Such instruments come into play when the costs for further prevention/mitigation are prohibitive. The model is therefore useful not only to the insurance industry but also to national authorities in informing decisions on overall catastrophic risk management.

In this paper, the model is applied to analyze the insurability of risks to the Irkutsk region in Russia, which is exposed to earthquakes risk. For this purpose the risk management model has been customized to explicitly incorporate the geological characteristics of the region, as well as the seismic hazards and the vulnerability of buildings. The purpose of the model is to generate insurance strategies that are robust with respect to dependencies and uncertainties, thus reducing the risk of bankruptcy to the insurers.

Many recent authors have stressed the need for better models to improve established insurance practices (see, for instance, Walker, 1997). Such models can be even more useful for guidance and setting regulations in countries that are moving towards market economies. In Russia, new legislative instruments and government resolutions (for industrial activities in 1997) are creating a framework

for risk management similar to that existing in the OECD countries.* However, in Russia and other transition countries** the emergence of a viable insurance industry is slow and subject to insolvency risks due to problems of the national economies, the lack of consolidated experience and practicable guidance, and the lack of sufficient surplus of the existing companies. The problems these countries face with respect to insurance regulation are now recognized (as an example of lack of guidance, when in seismic regions insurance is available, premiums are not based on probability of occurrence of earthquakes; neither they differentiate among geological situations and construction type), as well as the usefulness of research that can guide appropriate insurance policies. The model application described in the paper is a pilot exercise, which however can create the basis for cooperation with researchers, insurers and financial regulatory bodies in transition countries.

2. The Model Description

For sake of clarity the model is described by using only fundamental equations and variables of insurance risk management. The treatment of cycles of an insurance business (see Pentikainen *et al.*, 1989), inflation, interest rates, credits would require a larger number of state and decision variables (Ermoliev *et al.*, 1999), in particular, to describe the time variability of premiums, property values and transaction costs.

The study region is subdivided into compartments, or cells that can be defined on the basis of spatial data sets organized by rectangular grids. Depending on their scale, the cells might correspond to a set of households or a location with similar seismic characteristics, or a watershed, etc. It is important that the subdivision is structured such that there can be a meaningful representation of the simulated patterns of events in space and time. For each rectangular cell (i, j) there exists an estimation $W(i, j)$ of the property value or "wealth", including the value of residential property, farms, and commercial property.

Since the purpose of this region-wide integrated catastrophe risk model is to render insurers more robust to the risk of insolvency, the model can be specified for one insurer, a pool of insurers or a regulatory agency aimed at improving the stability of the insurance industry. Usually it is important to understand the conditions for insurability and to take account of the possibly competitive nature of the market. In a classical paper, Borch (1974) emphasized the cooperative efforts of insurers through risk pooling. In the case of Russia's emerging insurance industry,

* This and the following information on insurance status in Russia has been derived from a recent conference organized by the Regional Inspection of Insurance Supervision, the Russian Ministry of Finance and the Ural State University at Ekaterinburg (Oct., 1998).

** Among the countries from the former Soviet Union, the most developed system for insurance against natural risks has been implemented in Kazakhstan. This country requires mandatory property insurance for organizations and individuals in regions with maximum seismic activity larger/equal an intensity of 7 MKS.

cooperation among insurers will undoubtedly play an important role in stabilizing the insurance market.

The model assumes the existence of N ($N \geqslant 1$) insurance companies, and in this application does not consider competition among them. These N insurance companies may write contracts or policies to partially – or fully – cover catastrophic losses in any or all cells at specified transaction costs that depend on the location of the cell (i, j). Excessive transaction costs may impose restrictions on contracts in some cells. In the following only linear coverage, which is proportional to losses, is discussed. In general, it is possible to include other structures of the contract, for instance, deductibles and/or reinsurance contracts on specified layers of losses.

Each insurer k has an initial risk reserve, R_k^0. The insurer receives an annual premium $\pi_k(i, j)$ per unit of cover at (i, j) according to the specification of the contract. The risk reserve R_k for company k after a catastrophe at random time τ has the following form:

$$R_k = R_k^0 + \tau \sum_{(i,j)} \pi_k(i, j) q_k(i, j) -$$

$$- \sum_{(i,j)} D(i, j) q_k(i, j) - \sum_{(i,j)} C_k(i, j) q_k(i, j),$$

where $q_k(i, j)$ is the coverage of company k at (i, j) with $\sum_{k=1}^{N} q_k(i, j) \leqslant 1$, $C_k(i, j)$ is the corresponding transaction cost per unit of cover, and $D(i, j)$ is the random damage caused by the simulated catastrophe. This value depends on the pattern of the random catastrophic events, their intensity and duration, as well as on the mitigation measures taken. It should be especially noted that the damage or losses $D(i, j)$ may be due both to the direct impacts of the catastrophe at (i, j) and to the impacts of cascade failures in the other cells (this is discussed in more detail in the Appendix). In general, a catastrophe is modeled by an event affecting a random subset $O(\omega)$ of cells; and its magnitude in each cell (i, j) within $O(\omega)$, where ω is an element of the probability space. Therefore, the damage $D(i, j)$ is a function of $O(\omega)$, $W(i, j)$ in $O(\omega)$, the magnitude of the event and the vulnerability of the properties in each cell. The modeling of catastrophic events has become a subject of intensive research and development. A major problem for using catastrophe models to aid decisions on the extent of catastrophe coverage, mitigation measures, and so forth, has been poor historical data or their inadequacy to describe a changed environment (e.g., for flood risk) (see Insurance Service Office, 1994 and Walker, 1997). Modern computer techniques, however, allow the simulation of catastrophes close to how they happen in reality, therefore extending the databases. The broad purpose of the model presented in this paper is to combine catastrophe modeling with tools for optimal choices of the various decision variables for the purpose of lessening the financial impact of catastrophes.

The specific goals of the optimization are to improve the position of the insurers with respect to their profits, to decrease their risk of insolvency to some acceptable level, and to decrease the expected losses of the insured. The goal functions or performance indicators are, thus, the insurers' profit functions, their risk of insolvency, and the loss functions of the insured.

These indicators implicitly depend on policy variables, such as coverage at different locations, mitigation measures, transaction costs, and premiums. Although not included in this application, these performance indicators also depend on deductibles and reinsured losses (layers). At this stage the model implicitly assumes a stable regime of prices and interest rates. Consideration of their time variability would only increase the dynamic complexity of the model since another stochastic process would be superimposed onto the catastrophic events. For a theoretical treatment of this type of dynamics in the proposed model, as well as for consideration of other financial instruments (e.g., catastrophe bonds), see Ermoliev *et al.* (1999).

Even under these simplified assumptions, optimal choices of coverages with respect to the above mentioned performance indicators are analytically intractable because they are implicitly defined by the simulated patterns of catastrophes. Therefore, the goal functions can only be optimized using methods of adaptive Monte Carlo optimization (Ermoliev and Wets (eds), 1988; Ermoliev *et al.*, 1999). Formally, the goals of this model can be expressed with the following functions:

$$\begin{aligned} I_k(q,m) &= E\sum_{i,j}[\tau\pi_k(i,j) - C_k(i,j) - D(i,j)]q_k(i,j) + \\ &\quad + \sum_{(i,j)} w_k(i,j)E\min\{0, R_k\}, \end{aligned}$$

$$\begin{aligned} L_{i,j}(q,m) &= E\left[D(i,j)\sum_k q_k(i,j) - \tau\sum_k \pi_k(i,j)q_k(i,j)\right] + \\ &\quad + \nu(i,j)E\min\left\{0, D(i,j)\sum_k q_k(i,j) - \tau\sum_k \pi_k(i,j)q_k(i,j)\right\}. \end{aligned}$$

The expectation E is over a random time span, over patterns of catastrophes, their magnitudes, and the random variables characterizing vulnerability of properties. The variable m in $I_k(q,m)$, $L_{i,j}(q,m)$ denotes a vector of decision variables characterizing feasible mitigation measures at different locations (i, j). This notation makes explicit the dependency of I and L on m: in reality both π and D depend on m. The function I_k expresses a trade-off between the expected profits of insurers (first term) and their risk of insolvency (second term). Similarly, $L_{i,j}(q,m)$ defines a trade-off between the expected compensation of damages for insured properties (first term) and the risk of overestimating losses (second term), i.e., the risk of paying too high premiums.

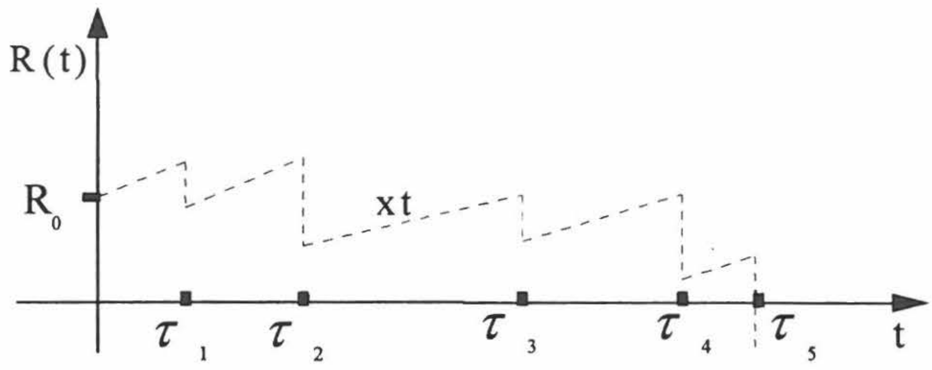

Figure 1. A sample trajectory.

The weights $w_k(i, j) \geqslant 0$ and $\nu(i, j) \geqslant 0$ can be adjusted to take account of various additional constraints, for example, fairness to the policy holders or the stability of the insurers, etc. In particular, it can be proven that if the weights w_k become large enough the corresponding term of the first risk function is equivalent to the constraints requiring that the probability of insolvency for each insurer does not exceed a given "level of survival" (see Ermoliev *et al.* (1998) for details). In the computational process these coefficients can be adjusted to guarantee a desirable level of insolvency simply by using histograms of risk reserves (as will be shown in Section 4). The solution technique is based on stochastic optimization methods (see, for example, Ermoliev and Wets (eds) (1998) for a general overview). These methods attempt to find a strategy that is "optimal" (robust) with respect to all possible catastrophic events. In contrast, the well-known, one-by-one scenario analysis provides a set of optimal strategies for each possible catastrophic event. The number of such "if-then" strategies rapidly increases to infinity without providing insights with respect to the choice of a desirable strategy.

3. The Stochastic Optimization Procedure

The proposed approach can be illustrated by using a standard model of an insurance business (see Pentikainen *et al.* (1989), see also the discussion in Ermolieva *et al.* (1997)). Assume that an insurer operates in a market with no reinsurance and no transaction costs. The long-term stability of his or her firm is defined at time periods $t = 1, 2, \dots$ by the risk reserve R^t,

$$R^t = R^0 + P^t - S^t, \quad t > 0,$$

where P^t is the aggregated premium on $(0, t]$, S^t is the aggregated claim (which may depend on the mitigation measures), and R^0 is the initial risk reserve. A possible trajectory of R^t is shown in Figure 1. For catastrophes occurring at random times τ_i, $i = 1, 2, \dots$, claims push the reserve down, whereas premiums, $P^t = xt$, $x > 0$, push it up. In this sample trajectory, claims exceed aggregate premiums

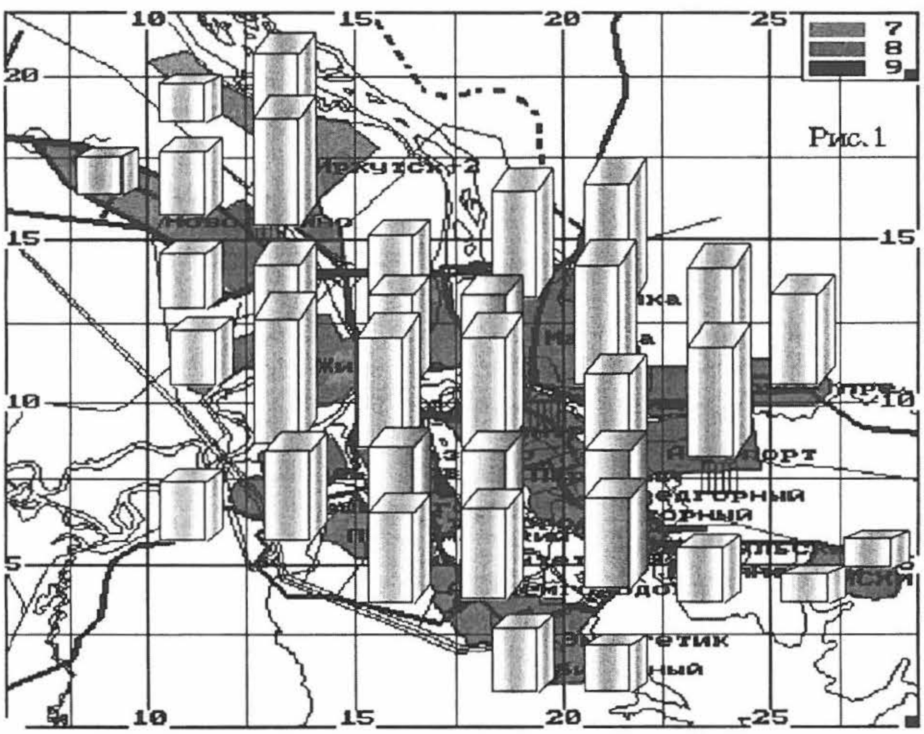

Figure 2. Landscape of property values:* Irkutsk, Russia.

plus initial reserves at time τ_5. In general cases, the value x incorporates levels of premiums $\pi_k(i, j)$ and coverages $q_k(i, j)$. In this simple model, the premium rate, x, is the decision variable. In this way, alternative pricing or premiums strategies can be examined.

The long-term stability of R^t can be characterized by the probability of ruin (insolvency)

$$\Psi(R^0, x) = \Pr\{R^t \leqslant 0 \text{ for some } 0 < t \leqslant T\},$$

where T is a time horizon. Let the desirable strategy be to choose a minimal premium x, which ensure a given risk of insolvency $\Psi(R^0, x) \leqslant Y$, where Y is the highest acceptable risk of insolvency.

A straightforward application of the Monte Carlo method for each combination of policy variables x would be impossible since the number of such combinations might approach infinity. Since ruin is a rare event, this may require a large number of simulations for a consistent estimate of Ψ. As a methodological alternative, we have used fast Monte Carlo estimation procedures with importance sampling and special search techniques for the desirable decision variables, in this case x. To

* In the map the scale 7, 8, 9 indicates maximum possible intensities of expected earthquakes in MSK scale taking into account the nature of the soil.

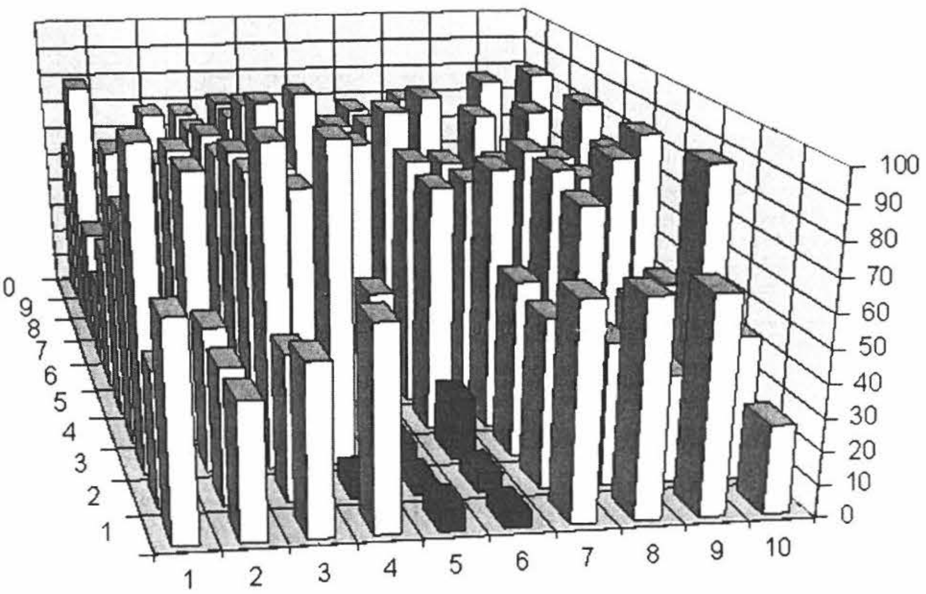

Figure 3. Landscape of damaged property values: Scenario 1.

continue, let the overall risk portfolio of an insurer be constituted by a "normal" part, M_t, associated with ordinary, independent claims, and a "catastrophic" part, L, associated with catastrophic claims. The reason for this is that an insurer's reserves accumulate not just from his or her catastrophic risk policies, but also from other types of policies. Assume that time is represented by a number of discrete time intervals, $t = 0, 1, \ldots, T$. The random variable M_t constitutes the full risk reserve, including catastrophic risk. The probability $\underline{p} \leqslant p \leqslant \bar{p}$ of a catastrophic event at time t is unknown and characterized by an a priori probability distribution. The probability of ruin from the first catastrophe is defined as the expectation

$$\Psi(R^0, x) = E \sum_{t=1}^{T} p(1-p)^{t-1} \Pr[M_t + xt - L_t < 0], \tag{3.1}$$

where L_t is the catastrophic claim generated at time t, and it is assumed that ruin can only occur due to a catastrophe. For the sake of simplicity, we assume that only one catastrophe leads to ruin or insolvency, and the simulation proceeds until this catastrophic event occurs. The term under the symbol of the mathematical expectation E is an estimator of Ψ. If the probability distribution $V_t(z) = \Pr[M_t < z]$ can be evaluated analytically, the variance of this estimator is reduced by taking the conditional expectation.

$$\Psi(R^0, x) = E \sum_{t=1}^{T} p(1-p)^{t-1} V_t(L_t - xt). \tag{3.2}$$

This simple formula permits a faster estimation of $\Psi(R^0, x)$ than formula (3.1).

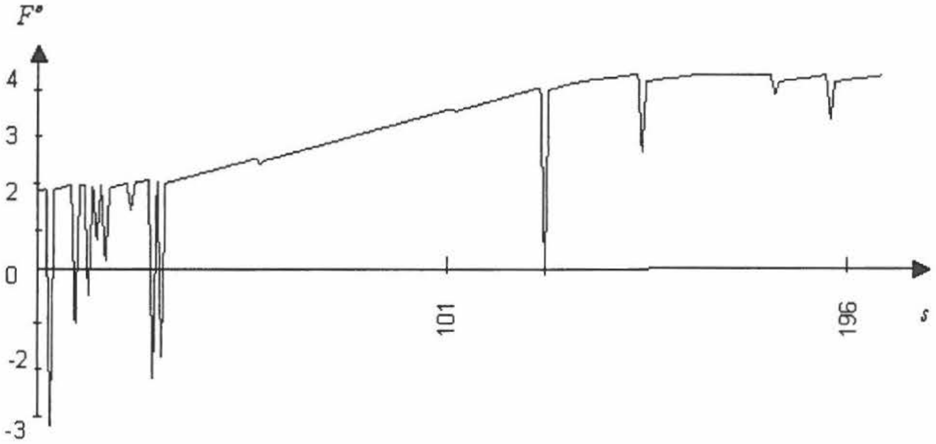

Figure 4. Pattern of performance indicator.

The stochastic search procedure starts with a given initial combination of policy variables. In this case it is only the value of the premium x^0. Let us denote by x^k the value of x after k simulations. Step $k+1$: choose t_k with probability $1/T$ from the set $\{1, 2, \dots, T\}$, generate $p_k \in [\underline{p}, \bar{p}]$ (from the given a priori distribution of p) and the claim $L^k_{t_k}$. Adjust the current value x^k according to the feedback:

$$x^{k+1} = \max\left\{0, x^k + \frac{\rho}{k+1}[Tp_k(1-p_k)^{t_k-1}V_{t_k}(L^k_{t_k} - x^k t^k) - \gamma]\right\},$$

where ρ is a positive constant. The value x^k converges with probability 1 to the desired value of the premium such that $\Psi(R^0, x) = \gamma$. This follows from the fact that the term $Tp(1-p)^{t_k-1}V_{t_k}(L^k_{t_k} - x^k t^k)$ is an estimate of $\Psi(R^0, x)$ at $x = x^k$. This type of approach, which can be viewed as an adaptive Monte Carlo simulation, is also used for the general problems outlined in Section 2.

4. Numerical Experiments

After some preliminary numerical experiments (Ermolieva *et al.*, 1997; Ermolieva, 1997), this spatial-dynamic, stochastic optimization model has been applied empirically to the region Irkutsk, Russia (Gitis *et al.*, 1996; Pavlov *et al.*, 1995). The property values in a modeled region can be viewed as a histogram where the heights of each cell (i, j) equal $W(i, j)$. Figure 2 shows this for Irkutsk. The evaluation of $W(i, j)$ for each (i, j) requires data on the types of buildings and other constructions , their density in each cell (i, j), and the values of different buildings and constructions. The evaluation of damages $D(i, j)$ requires data on the vulnerability of the buildings, etc. as a function of earthquake magnitude, type of soil, etc., as it is described in Pavlov *et al.*, (1995). Figure 3 illustrates damages caused by a randomly generated catastrophe, which may cause insolvency of some insurers if their coverage is not properly spread among different locations.

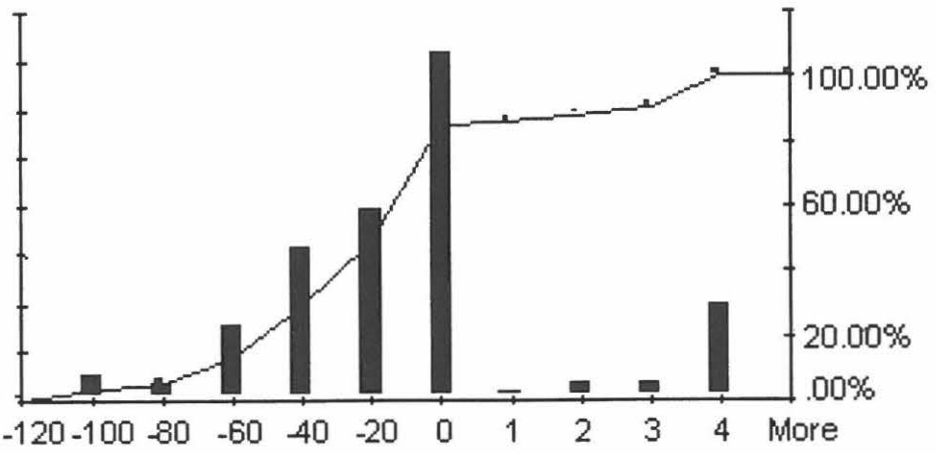

Figure 5. Histogram* of initial risk reserve: Insurer 1.

After each simulation run the stochastic optimization procedure (similar to that outlined in Section 3) adjusts the desirable coverage of the insurers operating in the region and mitigation measures to improve insurer stability, insurer profits, and the reduction of regional losses. The dynamics of improvements is controlled by a specially designed goal function F (performance indicator), reflecting the trade-off between profits, losses and the stability of insurers.

The pattern in Figure 4 versus the number of optimization steps shows the value of this function often jumping suddenly downwards either due to the insolvency of some insurers, excessive damages of individuals or lack of insurance coverage (see Figure 4). Step by step, the general situation is improved, and the performance indicators are increasingly robust to catastrophes. The corresponding values of the decision variables then define risk management decisions. Improvements in the insurability of catastrophic risks can also be shown by the dynamics of the histograms of the risk reserves as illustrated for one insurer in Figure 5 and Figure 6. Figure 5 shows that the insolvency of this insurer for initial coverage occurs in more than 60% of simulated catastrophes. In this figure, the numbers -20, -40, $\ldots -120$ indicate simulated levels of risk reserve shortage. Figure 6 shows a significant improvement of the insurer's risk reserve as a result of better diversification of coverage and implementation of appropriate mitigation measures.

5. Conclusions

This paper has demonstrated a systems approach to decision making on catastrophic risk management that can be applied to both the public and the private sectors. This novel approach permits the use of hazard and vulnerability inform-

* In the Figures 5 and 6 the left axis expresses the frequency of risk reserves; the dotted line and the right vertical axis express the cumulative distribution of risk reserve.

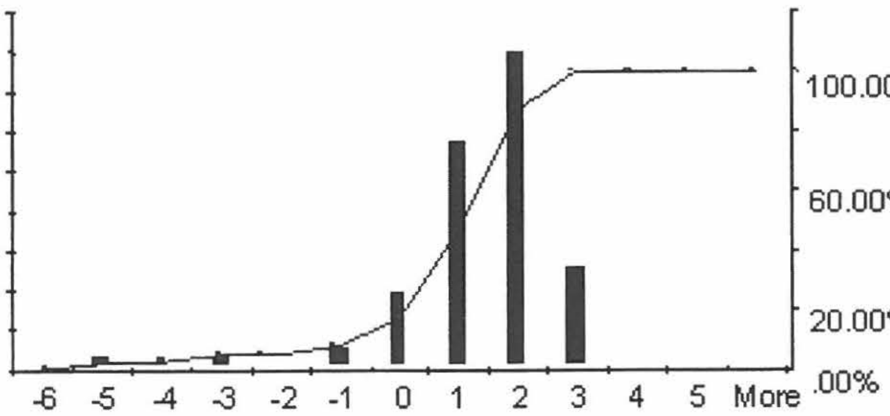

Figure 6. Histogram of improved risk reserve: Insurer 1.

ation systems as illustrated by the Irkutsk case study, in order to find optimal strategies for mitigation measures and loss spreading through insurance contracts.

The approach is computationally possible with the use of fast Monte Carlo and stochastic optimization techniques. It is general and flexible enough to allow decision makers to shape the performance indicators or goal functions according to different objectives of their policy strategies (e.g., fairness and equity in risk burden sharing, resource allocation in prevention and mitigation, optimal insurance strategies, etc.). The analyst can also incorporate various sub-models of catastrophes. Moreover, this system approach can be used for decisions involving a single catastrophe (e.g., earthquake, flooding etc.) as well as for an all-hazards perspective. By showing the range of policy options and the complex nature of the tradeoffs involved, this systems approach can provide insights to public and private policy makers, particularly in the transitional countries with nascent insurance industries.

Appendix: Direct and Indirect Losses: Catastrophe Chains

The main concern of the catastrophe modeling (see Insurance Service Office (1994)) is the estimation of damages $D(i, j)$ and risk reserves of insurers for any given combination of decision variables, such as premium and coverage. Mitigation measures (such as land use strategies, building codes, spatial diversification of units to decrease vulnerability of industrial systems, etc.) affect the distribution of risk reserve by reducing or increasing damages $D(i, j)$. As it was already mentioned in Section 1, the development of an appropriate model reflecting dependencies affecting $D(i, j)$ requires special attention. A catastrophe may produce a chain of indirect damages. For example, a seismic event may cause landslides and formation of damps, lakes; overfilling and breakdowns of dams may further cause floods and destruction of buildings, communication networks, and transportation systems. Fires may affect computer networks and destroy important information, etc. The

indirect catastrophe losses can even significantly exceed direct impacts. Therefore it is important to develop a model capable of analyzing the propagation of catastrophic events through the region and their total direct and indirect impacts. In the following a simple model is described, which is related to notions such as random fields and Bayesian nets (Spiegelhalter *et al.*, 1993, and references therein). A static version of this model was proposed previously (Gitis *et al.*, 1994). The model distinguishes N elements (buildings or their elements, locations, etc.) $l = 1, \ldots, N$ of a system (region). Possible damage at each l is characterized by random variable ζ_l assuming M levels: for sake of simplicity $1, 2, \ldots, M$. Hence damages of the system (region) are described by the random vector $\zeta = (\zeta_1, \ldots, \zeta_N)$. A fixed value of this vector is denoted by z and the set of all possible damages by Z. Let us denote by p_k^{lt} the probability that the damage at l is equal k at time t, $\sum_{k-1}^{M} p_k^{lt} = 1$, $p_k^{lt} \geqslant 0$. Dependencies between locations are represented as a graph, where elements $i = 1, \ldots, N$ are nodes of the graph and links between locations are represented by arrows between nodes. The dependency graph $G = (V, U)$ is characterized then by the set of nodes $V = \{1, 2, \ldots, N\}$ and the set of arrows (directed arcs) U. If nodes l, s belong to V, $l, s \in V$, and there is an arrow from l to s, then l is an adjacent to s node. Define as V_s the set of all adjacent to s nodes and z_{V_l} is sub-vector of the vector of damages indexed by V_l. For example, $z_{V_l} = (z_2, z_5)$ for $V_l = (2, 5)$.

Damages z_l are described by a conditional probability $H^l(z_1|z_{V_l}, m)$, i.e., damages at l depend on current values of damages at l and adjacent nodes as a function of available mitigation measures m. Let this function is known for each l. Functions H^l define the propagation of indirect catastrophic damages through the system according to the following relation:

$$p_k^{l,t+1} = \sum_{z_{V_l} \in Z} H^1(\zeta_l^t = k|\zeta_{V_l}^{t-1} = z_{V_l}, m) P(\zeta_{V_l}^{t-1} = z_{V_l}),$$

where $p_k^{l,t} = P(\zeta_l^t = k)$. To define completely the propagation of catastrophic damages it is necessary to fix an initial distribution of ζ_l^t for $t = 0$, i.e., at the moment when the catastrophic event occurred. This equation together with initial distribution allow the exact calculation (under certain assumptions on the structure of graph G) or the estimation of $p_k^{l,t}$ for any $t \geqslant 0$. Of course, for complex graphs it is practically impossible to derive analytical formulas for $p_k^{l,t}$ as functions of decision variables m and the most important approaches are based on the use of fast Monte Carlo simulation. Hence the damages $D(i, j)$ in the model in Section 2 may have rather complex implicit dependencies on decision vector m. The values $p_k^{l,t}$ reflect the dynamic of propagation of initial (direct) catastrophic impacts through the system after the occurrence of a catastrophe. The distribution of $D(i, j)$ can be approximated by using values p_k^{lt} at any $t \geqslant 0$. For example, $t = 0$ corresponds to the distribution of direct catastrophic damages.

References

Borch, K.: 1974, *The Mathematical Theory of Insurance*, Lexington Books, Lexington.

Cummins, J. D. and Doherty, N.: 1996, *Can Insurers Pay for the "Big One"? Measuring Capacity of an Insurance Market to Respond to Catastrophic Losses*, Wharton Risk Management and Decision Processes Center, University of Pennsylvania, W.P.

Gitis, V., Vainstock, A., Koff, G., and Karagodina, M.: 1996, *Geo-Informational Technology for Evaluation of Seismic Risk and Damages from Earthquakes*, Joint Report of the Institute for Information Transmission Problems and the Institute for Lithosphere (in Russian).

Gitis, V., Petrova, E., and Pirogov, S.: 1994, *Catastrophic Chains, Natural Hazards* **10**, 117–191.

Ermoliev, Y., Ermolieva, T., MacDonald, G., Norkin, V., and Amendola, A.: 1999, A system approach to management of catastrophic risks, Accepted for publication in the *Europ. J. Operational Res.*

Ermoliev, Y. and Wets, R.. (eds): 1988, *Numerical Techniques for Stochastic Optimization*, Springer series in Computational Mathematics, Springer-Verlag.

Ermoliev, Y., Ermolieva, T. Y., MacDonald, G., and Norkin, V.: 1998, On the design of catastrophic risk portfolios, IIASA, Interim Report IR-98-056.

Ermolieva, T. Y.: 1997, The design of optimal insurance decisions in the presence of catastrophic risks, IIASA, Interim Report IR-97-068.

Ermoliev, Y., Ermolieva, T. Y., and Norkin, V.: 1997, Spatial stochastic model for optimization capacity of insurance networks under dependent catastrophic risks: numerical experiments, IIASA, Interim Report, IR-97-028.

Ermolieva, T. Y., Ermoliev, Y., and Norkin, V.: 1997, On the role of advanced modeling in managing catastrophic risks, In: B.-M. Drottz-Sjöberg (ed.), *Proc. New Risk Frontiers: Conference for the 10th Anniversary of the Society for Risk Analysis – Europe,* The Center for Risk Research, Stockholm, pp. 68–74.

Insurance Service Office: 1994, The impact of catastrophes on property insurance, New York, In: H. Kunreuther (ed.), *Mitigating Losses and Providing Protection Against Catastrophic Risks. The Role of Insurance and Other Policy Instruments*, University of Pennsylvania, W.P. (1996).

MacDonald, G.: 1992, *Persistence in Climate*, JSR-91-340, The MITRE Cooperation, McLean, Virginia, 22102-3481

Pavlov O., Koff, G., and Frolova, N.: 1995, Seismic hazard and seismic risk mapping for Irkutsk City, *Proc. 5th Int. Conf. on Seismic Zonation*, Nice, France, pp. 1999–2019.

Pentikainen, T., Bonsdorf, H., Pesonen, M., Rantala, J., and Ruohonen, M.: 1989, *Insurance Solvency and Financial Strength*, Finnish Insurance Training and Publishing Company Ltd., Helsinki.

Spiegelhalter, D. J., Dawid, A. P., and Lauritzen, S. L.: 1993, Bayesian analysis in expert systems, *Statistical Science*.

Swiss, Re: 1997, Natural catastrophes and major losses in 1996: High losses from man-made disasters, but no extremely costly losses from natural catastrophes, *Sigma 3, a journal of the Swiss Reinsurance Company*.

UNDRO: 1991, *Mitigating Natural Disaster: Phenomena, Effects and Options. A Manual for Policy Makers and Planners*, United Nations, New York.

Walker, G. R: 1997, Current developments In: catastrophe modelling, In: N. R. Britten and J. Oliver (eds), *Financial Risk Management for Natural Catastrophes*, Proceedings of a Conference sponsored by the Aon Group Australia Limited.

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