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SPATIAL MOBILITY AND SETTLEMENT PATTERNS: AN APPLICATION OF A BEHAVIOURAL ENTROPY

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Preface

One of the major long run tasks of the Human Settlements and Services Research Area at IIASA is Human Settlement Systems: Development Processes and Strategies. This paper presents a generalized entropy approach to the analysis of spatial dispersion and mobility patterns. The model developed here is a synthesis of a behavioral model and an extended entropy model. Its application to one of the Dutch provinces indicates that it is useful in studying spatial interactions between production systems and settlement systems.

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8. Niles Hansen, "The Economic Development of Border Regions," RM-76-37, April, 1976.

9. Piotr Korcelli, "The Human Settlement Systems Study: Suggested Research Directions," RM-76-38, April, 1976.

10. Niles Hansen, "Alsace-Baden-Basel: Economic Integration in a Border Region," RM-76-51, June, 1976.

Spatial Mobility and Settlement Patterns: An Application of a Behavioural Entropy

Abstract

This paper is concerned with the intricate interrelation between urban and rural development. Particular attention is paid to suburbanization processes and mobility patterns, with an emphasis on the links between spatial structures and the development of settlement patterns.

In addition, a formal analysis is presented which investigates the determinants of mobility behaviour and consequences of the latter for a settlement system.

Next, a variety of entropy hypotheses is introduced to estimate the most probable spatial flows in a settlement system.

Finally, an integrated behavioural entropy approach is proposed as a reasonable method to fill in the "black box" of a settlement pattern and its associated spatial flows.

The analysis is illustrated by means of an empirical study of one of the Dutch provinces.

1. INTRODUCTION

The pattern of human settlements in Western countries has undergone significant changes during the last decades. In the past the urban agglomerations have exerted strong *attraction forces*, so that the rise of big metropolitan concentrations was stimulated. These positive attraction forces included both residential and employment conditions.

As the urban concentrations grew faster, the residential climate was affected increasingly. This has induced the *suburbanization process*, a widespread scattering of population around the big centres. Several factors behind the suburbanization process can be distinguished (see also Nijkamp [1975b]):

- qualitative and quantitative shortages of dwellings in many urban areas, caused by losses from the war and by the rise in population;
- congestion and pollution in densely populated or highly industrialized areas in big centres;

- the attractiveness of high-quality environmental conditions in regions around central cities;
- improvement of physical infrastructure from central cities to surrounding regions;
- rise in welfare, so that richer people in particular are able to overcome the rise in transportation costs from more remote residential areas to employment centres.

The factors mentioned above indicate that frequently urban residential and living conditions are far from favourable. "Urbanism as a way of life" (cf. Wirth [1938]) is being rejected increasingly as an acceptable attitude. On the contrary, anti-urbanism (cf. Glass [1955]) is becoming more and more apparent; the flight to the suburbs and to remote areas is its logical consequence.

During the last decade this suburbanization process has continued at such a scale that an urbanization of the suburbs has emerged (cf. Masotti and Hadden [1972]). This spatial shuffling process was not only due to the decline in the supply of environmental goods, but also to the rise in the demand for environmental goods. Analyses of shifts in priorities for environmental goods and of the repercussions of shifts in the preference structure upon spatial diffusion of population are contained in, for example, Cebula and Vedder [1973] and Nijkamp [1976c].

It should be noted that the agglomeration advantages of urban centres (like shopping facilities, highly qualified service institutions, governmental institutions, and the availability of a big and varied labour market) have prevented a random diffusion of settlement patterns throughout a country. Instead, a diffuse clustering of settlement patterns around big cities has taken place.

This large-scale diffuse concentration of residential areas around the traditional centres has exerted a twofold effect.

First, the traditional role of urban centres as places of creative entrepreneurship and as growth poles for surrounding (particularly lagging) areas is in serious danger due to the disintegration of the urban structure (congestion, segregation) and the weak financial base of large cities. It is not surprising that at the moment the idea of cities as significant growth centres in a spatial development process is open to doubt (cf. also Hansen [1975]). The stagnation of many big cities raises the question of whether urbanization and labour-extensive technologies form a guarantee for a balanced socioeconomic growth of a nation or region. The spatial spread of innovations and growth from big centres onwards to surrounding areas is being affected by the stagnation of big cities (cf. Lasuén [1973]).

In addition to these serious threats to an integrated development process, environmental problems also have to be mentioned. The spatial dispersion of population to areas around big cities has led to a considerable decline in their environmental quality: a decline in the quantity of natural areas due to road-building and house-building, congestion in recreation areas, congestion and pollution due to increased traffic, etc.

It is important to ask whether the spatial diffusion of settlement patterns should continue to follow the trend just mentioned. One may expect that uncontrolled development will not only destroy the urban climate, but also the rural and suburban climate. Therefore, many planning agencies (for example, in the Netherlands) are confronted with the issue of how to develop an integrated and balanced physical planning?

One of the insights recently gained is that effective spatial planning requires a significant improvement of urban residential and living conditions to reduce the uncontrolled flight from the cities. Urban renovation policies are a matter of major concern for urban planners. This renovation does not imply a complete destruction and replacement of older urban areas. The aim is to maintain the positive elements of the socioeconomic and sociopsychological structure of these (older) areas. The emphasis of such an urban policy is on improvement of dwellings, creation of small-scale recreation areas, variety of residential areas, accessibility of residential areas, maintenance of characteristic urban elements, public facilities etc. By means of extensive subsidy programs an attempt can be made to make this re-urbanization successful.

It should be mentioned that the urban policy mentioned above is of crucial importance, since it attempts to restore the position of the cities as the heart of a spatial structure. Balanced physical and urban planning is the only way to avoid the "city of the dead" (Mumford [1961]).

Apart from the question of whether the financial carrying capacity of a city is sufficient to develop balanced urban development and to provide positive external effects to surrounding regions, one should be aware of the fact that a re-urbanization policy cannot be the only means for balanced regional-urban development. A moderate dispersion of population is desirable in view of the growth of population, the decline in occupation rates for dwellings, and the flexibility of physical planning.

Obviously, there is a need for effective planning. However, a basic problem is the lack of information about mobility patterns and mobility motives of people. Clearly, such insights into the structure of settlement patterns are a necessary condition for successful physical planning.

In the following sections particular attention will be paid to mobility patterns (section 2) and spatial flows in a settlement system (section 3). Next, it will be shown that the use of multi-criteria profiles is meaningful to study the relationships between the elements of a spatial and urban structure and the development of settlement patterns (section 4). Then a behavioural model based on the theoretical ideas of section 4 will be developed (section 5); also a variety of entropy-hypotheses will be suggested to obtain insights into the most probable spatial flows in a settlement system (section 6). In section 7 a synthesis of a behavioural approach and an entropy approach will be presented. Finally, section 8 describes an empirical application.

2. Mobility Patterns in a Spatial Structure

A spatial system is not a homogeneous entity. On the contrary, there is a great variety and differentiation of phenomena through space. Heterogeneous areas, unequal spatial-temporal diffusion patterns, socioeconomic and residential discrepancies, and bottlenecks in traffic networks are phenomena which demonstrate that uniformity in space is an illusion.

The great variety in the elements of a spatial structure and the great variety in preference patterns and opportunities of people lead to a situation where differences in spatial behaviour are dominant. Consequently, mobility patterns and settlement patterns are closely linked together for various groups of people.

Mobility patterns can be assumed to arise from significant discrepancies among the elements of a spatial structure. These elements are, among others, employment, housing, transport, natural and recreation areas, pollution, and public amenities. Depending on the degree of deviation of the local supply of these elements with respect to the individual and collective priorities attached to them, a spatial dispersion from big urban centres to the surrounding areas will take place. In a rather simple way the tension between residential and living conditions on the one hand, and employment and agglomeration conditions on the other hand can be illustrated by means of Figure 1.

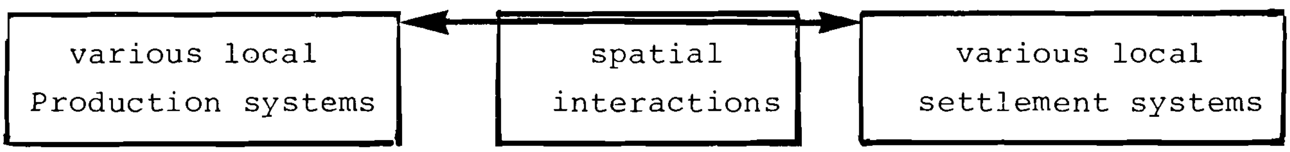

Figure 1: Spatial interaction flows.

As the qualities of the production systems at the various localities diverge more from the residential preferences of people, the degree of spatial interaction in the form of commuting flows, migration flows and recreation flows will increase.

A more extensive picture of spatial interactions associated with discrepancies in spatial structure (socioeconomic, environmental, residential and urban) is shown in Figure 2.

The relationships presented in Figure 2 may be causal, interdependent or limiting. In principle, they can be described by means of a formal mathematical model, a so-called spatial impact model (see Nijkamp [1976d]). However, there is frequently a considerable lack of information, particularly as far as the spatial interactions are concerned. On the other hand, the presentation of a spatial interaction structure in a coherent impact system provides more insight into the complexity and general features of such a structure. Such an integrated view of production systems, settlement systems, and mobility patterns is also a necessary condition to make public policy more successful, particularly in the fields of infrastructure planning, environmental management and development programming.

A useful method to obtain more adequate insight into the development of a spatial system is a scenario-analysis, which presents a series of alternative development patterns. The spatial interactions associated with each individual development pattern can be gauged by means of Figure 2. By carrying out a critical path analysis one may inspect, for example, the degree to which the existing infrastructure is satisfactory with respect to the corresponding production and settlement scenarios.

Clearly, the search for an optimal policy and for optimal decision-making has to be based on a multiplicity of criteria. In this respect, the use of recently-developed multi-criteria analyses (for example, a concordance analysis; Nijkamp [1975c]) may be extremely useful. The major advantage of these kinds of analysis is that, in addition to traditional efficiency criteria, intangible effects also can be taken into consideration. Consequently, the use of a multi-criteria analysis offers more opportunities for a balanced and integrated spatial policy.

Obviously, such a multi-criteria policy requires detailed information about spatial flows between local production systems and local settlement systems, since mobility patterns and infrastructural provisions may be one of the decisive factors in spatial and physical planning. In general, origin-destination tables of these flows are not available. This disaggregation problem will be discussed in the next section.

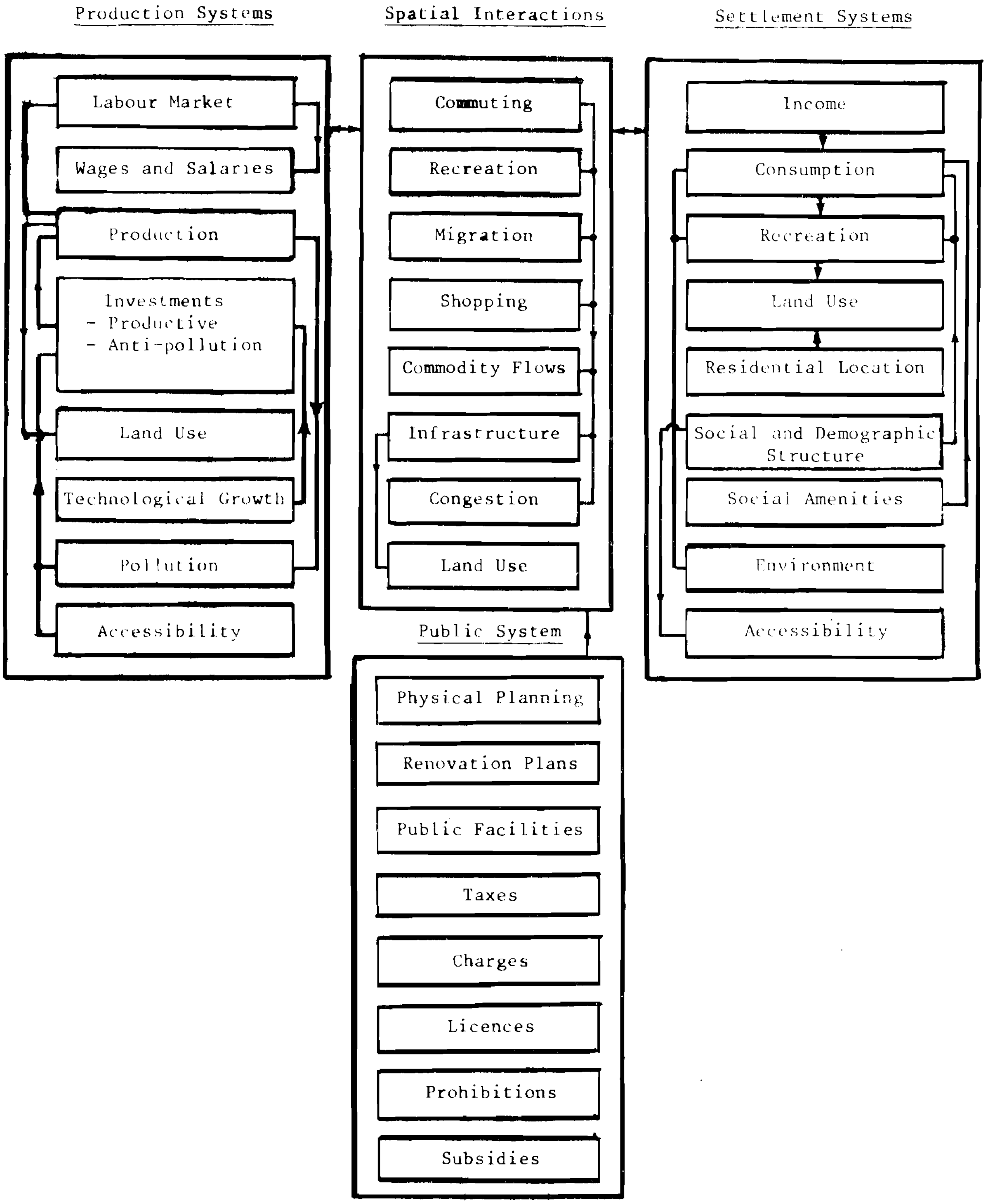

Figure 2: Spatial Interaction Structure

3. Spatial Interactions Between Production Systems and Settlement Systems

A detailed analysis of spatial interactions requires a large data set. In general, such a data set is not available or only available in an aggregated form. In consequence, the spatial classification used in many interaction analyses is frequently too rough to guarantee reliable and detailed outcomes. For example, an analysis of journeys-from-home-to-work on the basis of regional data instead of local data has only a limited meaning. Generally speaking, the degree of spatial interaction is higher and more varied as the spatial classification is more refined. For example, commuting flows in a spatial production-settlement system can be analyzed more adequately as the spatial scale is more detailed.

Adequate insight into the spatial development of an area and into effective policies requires detailed information on mobility patterns between local production systems and local settlement systems. This information is a prerequisite for predicting and evaluating spatial patterns (in a scenario-analysis, for example). Therefore, a spatial interaction analysis also has to include elements useful for prediction and planning (cf. Nijkamp [1976a]), including the specific local characteristics of the successive production and settlement systems.

In view of the foregoing remarks, a spatial interaction analysis based on an origin-destination table should be carried out at the lowest possible spatial scale. This condition requires disaggregation from regional data to interlocal flows. The remaining part of the paper will focus on the way in which global information on production and settlement systems can be disaggregated to the level of interlocal interactions (cf. Magoulas et al. [1975] and Morrison [1973]).

The decision to commute from place i to j rests on a variety of factors. Given the fact that spatial interactions and mobility patterns result from polar tensions between local production and settlement conditions (see Figure 2), the following three main factors for a decision to commute from i to j can be distinguished:

- the employment conditions of place j correspond more closely to the individual employment and income preferences than the employment conditions of place i (in other words, the employment attractiveness of place j (denoted by w_j) is higher than the employment attractiveness of place i (denoted by w_i)).
- the environmental and residential conditions of place i correspond more closely to the individual settlement preferences than the environmental and residential conditions of place j (in other words, the environmental-residential attractiveness of place i (denoted by m_i) is higher than the environmental-residential attractiveness of place j (denoted by m_j)).

- the distance from place i to j (measured in time, money or miles) is such that the net (employment and environmental-residential) attractiveness compensates for the disadvantages arising from the distance friction.

These notations can be integrated in a probability matrix for spatial mobility patterns (see Figure 3 and Figure 4).

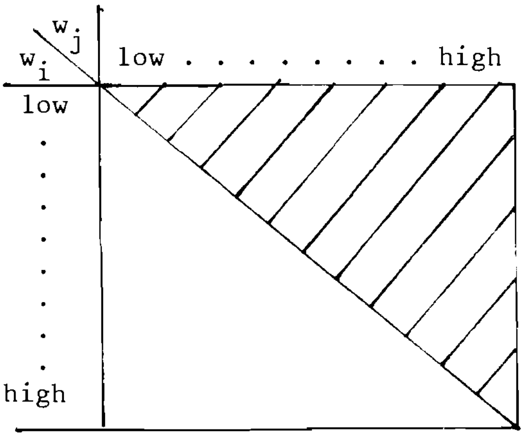

Figure 3: A probability matrix for mobility patterns (commuting) between place i and j on the basis of employment attractiveness.

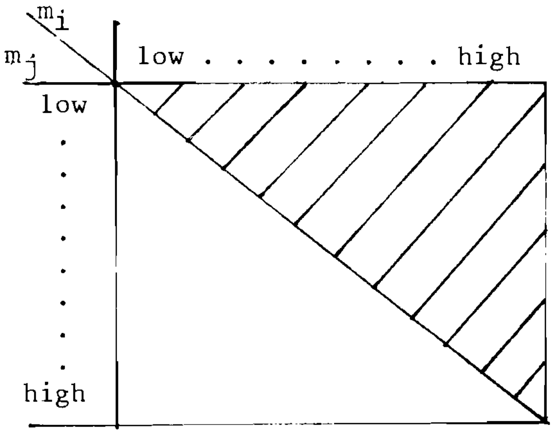

Figure 4: A probability matrix for mobility patterns (commuting) between place i and j on the basis of environmental-residential attractiveness.

The shaded parts of Figure 3 and Figure 4 indicate that spatial interaction (commuting) flows are probable if $w_j > w_i$ and $m_j < m_i$. This probability is higher as w_j diverges more from w_i, and m_i more from m_j. In other words, the commuting probability between a local production system and a local settlement system is at its maximum in the upper right corner of Figure 3 and Figure 4.

It should be noted that the third factor (distance friction or accessibility) may play a prohibitive role with respect to the occurrence of spatial commuting flows. Consequently, the accessibility from i to j (denoted by b_{ij}) is of crucial importance. This can be illustrated by means of the following probability block for spatial interactions. The shaded area of Figure 5 reflects the fact that a spatial link between places i and j will be more probable as w_j, m_i and b_{ij} are higher. This probability is equal to 1 at the extreme point P in Figure 5.

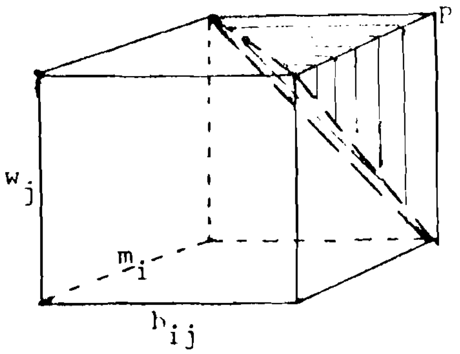

Figure 5: An integrated probability block for mobility patterns (commuting) between place i and j.

The foregoing notions about spatial interactions can be generalized by taking into account spatial spillover effects. An example of the latter situation is the case where the environmental attractiveness of place i is codetermined by the environmental and recreational conditions of an adjacent place k. The existence of such spatial spillover effects may influence to a considerable degree the development of a settlement pattern or, in general, of a multiregional socioeconomic system.

It is clear, that the preceding notions of spatial interactions and spatial attractiveness phenomena have to be elaborated in an operational sense. Therefore, in the next section a method will be presented by means of which the attractiveness of local systems can be quantified and integrated into a spatial interaction framework.

4. A Multi-Criteria Profile Analysis of Spatial Interaction

In the foregoing paragraph attention was paid to the qualities of local settlement and production systems. Concepts like environmental-residential attractiveness and employment attractiveness can be described by means of a recently developed multi-criteria profile analysis (see Nijkamp [1975a, 1976b] and Paelinck and Nijkamp [1975]).

A multi-criteria profile can be conceived of as a vector representation of a series of elements that characterize a certain phenomenon. For example, the environmental residential attractiveness of a place i can be represented by means of the following profile $\underline{p}_i$, in which each element p_{ik} (h = 1,...,K) is a quantitative characteristic of place i, which can be considered as one of the determinants of settlement behaviour:

$$\underline{p}_i = \begin{bmatrix} p_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_{ik} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_{iK} \end{bmatrix} \begin{bmatrix} \text{size of recreation areas} \\ \text{cultural facilities} \\ \text{stock of dwellings} \\ \text{population dispersion} \\ \text{medical provisions} \\ \text{educational provisions} \\ \text{availability of natural areas} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad (4.1)$$

In principle, each element of the foregoing profile can be quantified for each place i in a regional system. By means of generalized distance measures (or similarity measures) the relative discrepancy Δ_{ij} of the environmental-residential profile between place i and j can be determined, after standardisation and normalisation, as follows (see also Paelinck and Nijkamp [1975]):

$$\Delta_{ij} = \sqrt{(\underline{p}_i - \underline{p}_j)'(\underline{p}_i - \underline{p}_j)} \qquad (4.2)$$

In this way Δ_{ij} can be used as one of the explanatory variables for spatial interactions from place i to j. This formal presentation corresponds to the ideas presented in Figure 4. If necessary the environmental-residential attractiveness indicators can be split up into residential factors in a narrow sense, environmental and recreational factors in a broader sense, and social-cultural factors. In the last case various discrepancy indices instead of one discrepancy index would be obtained.

If for theoretical or practical reasons a discrepancy-index between places is less desirable or useful, one can also calculate an environmental-residential attractiveness indicator for each place i separately on the basis of the profile vector $\underline{p}_i$ described above. Such a uni-dimensional indicator can be calculated from a multi-dimensional profile vector in various alternative ways:

- transform the elements of $\underline{p}_i$ into an interval scale. This implies that each local characteristic k of the profile is transformed into an index on the basis of its relative magnitude with respect to other places.

- use a multi-variate technique (factor-analysis, e.g.) to transform the original multi-dimensional data matrix (over all places) into a limited number of mutually independent components.
- use an interdependence analysis to reduce the multi-dimensional data matrix to a limited number of original variables which reflect the original multi-dimensional profile to a maximum degree (see also Boyce, Fahri and Weischedel [1974]).

In the empirical part of our study the first method has been used.

In addition to the environmental-residential attractiveness indicator of a place i, one can determine in a similar way the average first-order contiguity attractiveness of adjacent places (corrected for distance frictions; see Nijkamp [1976b]). In this way, the spatial spillover effects can be taken into account in a mobility analysis of settlement systems.

So far, only environmental-residential attractiveness has been discussed. Clearly, in an analogous way employment attractiveness can be determined. The employment attractiveness indicator can be constructed on the basis of local employment profiles which may include the number of job opportunities, the average wage rate, the degree of social stability, the quality of labour, etc. By means of the employment profile the corresponding attractiveness indicator w_j for a place j can be calculated (see also Figure 3). Here again spatial spillover effects due to contiguous attractiveness can be distinguished.

The foregoing multi-criteria profile analysis can be seen as an operational method to describe in an integrated way the various elements of production and settlement systems as represented in Figure 2. Given the institutional framework, given a certain spatial governmental policy, and given the development of exogenous variables (population growth, e.g.), the multi-criteria framework may be extremely useful in describing and predicting mobility patterns associated with changes in the production and settlement profiles (by means of a scenario-analysis, e.g.). In a next section an attempt will be made to construct a formal model for dealing with this problem.

5. A Behavioural Model for Mobility Patterns in Production-Settlement Systems

In this section a formal model will be developed to describe the mobility patterns arising from the forces between the various attraction profiles in a production-settlement system. Particular attention will be paid to commuting flows, although the analysis can be generalised directly to all other types of mobility phenomena. Given the theoretical notions presented in section 4 the following formal model will be assumed:

$$v_{ij} = f(d_{ij}, w_j, m_i^o, m_i^r, m_i^s, m_i^k) \quad , \qquad (5.1)$$

where:

v_{ij} = volume of interaction (commuting) from settlement place i to production place j ,

d_{ij} = distance from i to j (measured in time, money or miles) ,

w_j = employment attractiveness indicator of place j ,

m_i^o = attractiveness indicator for educational facilities in place i ,

m_i^r = attractiveness indicator for recreational facilities in place i ,

m_i^s = attractiveness indicator for sports accomodations in place i ,

m_i^k = attractiveness indicator for cultural facilities in place i.

In addition to the foregoing local characteristics of places i and j, the spatial spillover effects arising from attraction effects of contiguous places can be included as well. Normally, the foregoing model might be estimated by means of least-squares procedures, provided an extensive data set on spatial flows between settlement places and production places is available.

However, such an interlocal data matrix is only rarely available. Sometimes, data on commuting flows between a limited number of specific places are available, but information on the entire spatial pattern is generally missing. Frequently, however, at a more aggregated level, particularly at a regional level, a spatial data matrix on commuting flows is available, because many (un)employment data are gathered at a regional level.

If an interregional mobility matrix for journeys-from-home-to-work is available, the basic problem is to transform this matrix into a disaggregated, local form. This implies, however, that a traditional least-squares method for estimating (5.1) is not useful. Therefore, an adaptation has to be carried out to gauge interlocal flows on the basis of interregional flows.

Clearly, the abundant amount of degrees of freedom with respect to the spatial allocation of mobility flows cannot be solved without additional information.

Three possibilities exist to fill this gap. A first possibility to arrive at a meaningful disaggregation is to take into account the specific local characteristics (the local attractiveness indicators) of all places in the multiregional system, given the additivity conditions with respect to the known interregional flows (section 5). The second way is to use some adapted entropy hypotheses to calibrate the most probable spatial mobility pattern at a local level (section 6). The final way is to integrate both methods, so that both behavioural and entropy elements are included in the analysis (section 7).

Now the first method will be considered. This method is directly based on relationship (5.1). As set out above, information on v_{ij} is not available. However, the variables v_{ij} have to satisfy the interregional additivity conditions. These conditions can be formalized for regions r and s as follows:

$$\sum_{i_r \in r} \sum_{j_s \in s} v_{i_r j_s} = v_{rs} \quad , \qquad (5.2)$$

where $v_{i_r j_s}$ represents the (unknown) commuting flow from place i in region r to place j in region s, and v_{rs} the (known) commuting flow from region r to region s. The elements $v_{i_r j_s}$ can be included in a vector $\underline{v}$ of order I(I - 1) x 1, where I indicates the total number of places in the spatial system concerned. In an analogous way, the elements v_{rs} can be included in a vector $\underline{\bar{v}}$ of order R(R - 1) x 1, where R represents the number of regions in the spatial system. Condition (5.2) can be written in matrix notation as:

$$H \underline{v} = \underline{\bar{v}} \qquad (5.3)$$

where H is a regional summation matrix of order R(R - 1) x I (I - 1), which adds up the interlocal flows between two regions to an interregional flow:

$$H = \left[\begin{array}{c|c} 1 \cdots\cdots 1 & 0 \cdots\cdots\cdots\cdots\cdots 0 \\ & \ddots \\ 0 \cdots\cdots\cdots\cdots\cdots 0 & 1 \cdots\cdots 1 \end{array}\right] \qquad (5.4$$

If relationship (5.1) is assumed to be a linear function of the attractiveness indicators, the following specification may be adopted:

$$\underline{v} = X \, \underline{\beta} \quad , \qquad (5.5)$$

where X is the matrix of K known local attractiveness indicators (of order I(I - 1) x K), and $\underline{\beta}$ is a vector of unknown parameters (of order K x 1). Since $\underline{v}$ is unknown, a procedure has to be developed to calibrate $\underline{v}$ and $\underline{\beta}$ simultaneously. This procedure, which rests mainly on (5.3), attempts to calibrate $\underline{\beta}$ such that a maximum correspondence is achieved between the estimated flows and the known flows according to (5.3).

The procedure used here attempts to select values of $\underline{\beta}$ which minimize the difference between estimated and known interregional flows by means of an approximation of this difference via a power function (of an arbitrary degree).

This procedure stems from a direct search method for extreme points of functions, developed by Hooke and Jeeves [1961]. It is an iterative method which attempts to obtain a minimum discrepancy between calibrated and known values of $\underline{\bar{v}}$ by means of a successive adaptation of the unknown parameter vector $\underline{\beta}$. The procedure starts off from an initial value of $\underline{\beta}$, while next $\underline{\bar{v}}$ is calculated on the basis of (5.3) and (5.4). Then the Hooke and Jeeves procedure searches in all K directions (and in the intermediate directions) to determine whether a shift of $\underline{\beta}$ in one of these directions will lead to a lower discrepancy between the calibrated and known values of $\underline{\bar{v}}$. If so, a new starting point of $\underline{\beta}$ is calculated and the procedure is repeated. By means of a limited number of analogous steps an extremum can be found, so that the interlocal flow matrix can be calculated directly by means of (5.5). Given the underlying behavioural assumptions, this matrix can be conceived of as the best calibration of the mobility pattern associated with a given production and settlement system. A more detailed exposition of the method of Hooke and Jeeves is contained in Appendix A.

A serious problem of the aforementioned method is the fact that a statistical test of the interlocal results is hardly possible, because these results cannot be compared with observed data. There is only one test possible, viz. the degree to which the observed and the calibrated <u>interregional</u> data differ. In the previous case, however, these differences can be made arbitrarily small (up to any ε - limit).

In view of these test problems an alternative specification of the problem may be considered, viz. by writing (5.5) as a regression equation:

$$\underline{v} = X\,\underline{\beta} + \underline{\varepsilon} \tag{5.6}$$

where $\underline{\varepsilon}$ is a vector of disturbance terms of order I(I - 1) x 1. By premultiplying (5.6) with the summation matrix H, the following result is obtained:

$$H\underline{v} = H\,X\,\underline{\beta} + H\underline{\varepsilon} \tag{5.7}$$

or:

$$\underline{\bar{v}} = P\ \underline{\beta} + \underline{u} \quad ,$$

where $\bar{v}$ and P are known. With the aid of the latter regression equation the parameter vector $\underline{\beta}$ can also be estimated. The obvious advantage of the regression procedure is the fact that now a possibility for testing the results does exist, although it should be noted that this test is only relevant for inter-regional flows and not for interlocal flows. In this respect, the regression procedure offers no significant improvement.

The methods described above can be used to gauge $\underline{\beta}$ and also the interlocal mobility pattern. Given the structure of such a model, the results may also be used for predicting the mobility pattern in a scenario-analysis for the future. In this way, the aforementioned method can be used to gauge the interlocal mobility pattern associated with alternative spatial policies concerning production and settlement systems, assuming at least that the behavioural structure will remain more or less constant in the future. Otherwise, a sensitivity analysis with respect to shifts in behavioural patterns may be carried out.

Changes in the spatial structure (for example, increased accessibility arising from an improved infrastructure) can also be analyzed by means of the foregoing model. In conclusion, in spite of lack of information the foregoing behavioural model may be useful in analyzing mobility patterns associated with local production and settlement systems.

In the next section an alternative analysis will be presented and elaborated.

6. Entropy Hypotheses for Spatial Mobility Patterns[1]

In the field of spatial interaction models and of mobility analyses entropy models are becoming increasingly popular (cf. Wilson [1970]). Examples include migration studies, shopping models, and traffic studies.

Entropy hypotheses can be conceived of as the foundation stones for the use of gravity models in various spatial analyses. By means of the hypothesis of a maximum entropy the most probable configuration of a spatial system can be derived. An exposition of the background of entropy is found in Nijkamp and Paelinck [1974], and an interpretation in terms of behavioural assumptions (particularly, the minimization of generalized costs) is contained in Nijkamp [1975d].

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[1] This part rests on Nijkamp and Wiersma [1976].

The traditional entropy approach rests on the idea that, given the marginal totals of a flow matrix and given a total distance budget (in terms of time, money or miles), the most probable guess concerning the total flow matrix can be obtained by maximizing the corresponding entropy function. The entropy model associated with the mobility pattern between local production and settlement systems (see section 5) can be formalized as:

$$\begin{aligned}
&\max \quad \omega = -\sum_{i_r} \sum_{j_s} (v_{i_r j_s} \ln v_{i_r j_s} - v_{i_r j_s}) \\
&\sum_{i_r} v_{i_r j_s} = D_{j_s} \quad , \forall\, j_s \\
&\sum_{j_s} v_{i_r j_s} = O_{i_r} \quad , \forall\, i_r \qquad\qquad (6.1)\\
&\sum_{i_r} \sum_{j_s} v_{i_r j_s} d_{i_r j_s} = T \quad ,
\end{aligned}$$

where D_{j_s} is the (known) volume of flows which have place j in region s as destination, O_{i_r} the (known) volume of flows which have place i in region r as origin, and T the (known) total distance budget.

The solution of this traditional entropy model is:

$$v_{i_r j_s} = A_{i_r} B_{j_s} O_{i_r} D_{j_s} \exp (- \beta d_{i_r j_s}) \quad , \qquad (6.2)$$

where:

$$A_{i_r} = \{ \sum_{j_s} B_{j_s} D_{j_s} \exp (- \beta d_{i_r j_s}) \}^{-1} \qquad (6.3)$$

and:

$$B_{j_s} = \{ \sum_{i_r} A_{i_r} O_{i_r} \exp (- \beta d_{i_r j_s}) \}^{-1} \qquad (6.4)$$

Clearly, the foregoing entropy model is only useful when the marginal totals D_{j_s} and O_{i_r} are known. It was already indicated in the foregoing paragraph that frequently not all these local marginal totals are known. Sometimes only the <u>interregional</u> flows are known. This again raises the question of how to disaggregate the interregional flows to interlocal flows. The solution requires an adaptation of the original entropy formulation.

The entropy formulation for the disaggregation problem is:

$$\max \quad \omega = -\sum_{i_r} \sum_{j_s} (v_{i_r j_s} \ln v_{i_r j_s} - v_{i_r j_s})$$

$$\sum_{i_r \in r} \sum_{j_s \in s} v_{i_r j_s} = v_{rs} \quad , \forall\ r,s \tag{6.5}$$

$$\sum_{i_r} \sum_{j_s} v_{i_r j_s} \, d_{i_r j_s} = T$$

where the total distance budget can be calculated on the basis of the known interregional flow matrix.

The solution of this particular entropy model is (see also Appendix B):

$$v_{i_r j_s} = C_{rs} \, v_{rs} \exp(-\beta d_{i_r j_s}) \quad , \tag{6.6}$$

where:

$$c_{rs} = \{ \sum_{i_r \in r} \sum_{j_s \in s} \exp(-\beta d_{i_r j_s}) \}^{-1} \quad . \tag{6.7}$$

It should be noted that this last entropy model contains less information than (6.2), since information about local flows is included. Therefore, one may expect that (6.2) will provide more reliable results than (6.5), provided the additional information is available. Furthermore, one should take account of the fact that the backgrounds of both entropy models are different: model (6.2) aims at gauging the interlocal flow matrix by means of local marginal totals, whereas model (6.6) aims at gauging the interlocal flow matrix by means of interregional flows.

Finally, if information on both the local marginal totals and the interregional flows is available, a meaningful approach would be to integrate both models. Then the entropy of the mobility pattern between local production and settlement systems has to be maximized subject to the marginal totals and the interregional flows. This entropy model can be specified as follows:

$$\max \omega = - \sum_{i_r} \sum_{j_s} (v_{i_r j_s} \ln v_{i_r j_s} - v_{i_r j_s})$$

$$\sum_{i_r} v_{i_r j_s} = D_{j_s} \quad , \quad \forall\ j_s$$

$$\sum_{j_s} v_{i_r j_s} = O_{i_r} \quad , \quad \forall\ i_r \tag{6.8}$$

$$\sum_{i_r \in r} \sum_{j_s \in s} v_{i_r j_s} = v_{rs} \quad , \quad \forall\ r,s$$

$$\sum_{i_r} \sum_{j_s} v_{i_r j_s} d_{i_r j_s} = T$$

The solution of the latter model is (see Appendix B):

$$v_{i_r j_s} = C_{rs}\ A_{i_r}\ B_{j_s}\ v_{rs}\ O_{i_r}\ D_{j_s}\ \exp(- \beta d_{i_r j_s}) \tag{6.9}$$

This generalized model rests on two sources of information. If these sources are available, the latter model can be considered as the most adequate representation of an entropy model for mobility patterns. The solution technique of this generalized entropy model is analogous to that of the traditional entropy model, in the sense that the iterative procedure includes now a switch among four parameters, viz. C_{r_s}, A_{i_r}, B_{j_s} and β. Therefore, the computing time for arriving at a converging solution will be somewhat longer (cf. Bouchard and Pijers [1964]).

7. A Synthesis of Behavioural and Entropy Hypotheses

The use of entropy models has been criticized from several sides owing to the rather mechanical nature of the entropy hypotheses, particularly as far as the physical backgrounds are concerned. This criticism is valid, in as far as behavioural assumptions are not introduced explicitly in the derivation of entropy models. However, it should be noted that an entropy hypothesis in a spatial interaction model can be interpreted by means of a dual formulation in terms of behavioural hypotheses about a collective mobility behaviour (see Nijkamp [1976d]). This behavioural hypothesis states that all people together will choose a mobility pattern which minimizes a certain generalized travel cost function, based on the differential attractiveness of places of origin and destination.

In view of the latter behavioural interpretation one may wonder whether it would not be meaningful to introduce behavioural assumptions a priori and explicitly in entropy models. So far this possibility has received only little attention. In this paragraph an attempt will be made to analyze this question.

The introduction of behavioural assumptions in an entropy model has two advantages: (a) the mechanical nature of entropy models will be abandoned; (b) the probability background of entropy models will be maintained.

Now the question arises as to how behavioural elements can be introduced into entropy models. In principle, there are three possibilities of including behavioural hypotheses in an entropy model: (a) in the entropy function itself; (b) in the traditional entropy constraints; (c) in the form of new entropy constraints (for example, by means of (5.1)).

The last possibility is very hard to realize, since a behavioural model like (5.1) is deterministic, so that the use of this behavioural model would assign the flows in a unique way without taking into account the entropy assumptions.

Possibility (b) might be applied in principle, but includes many arbitrary elements. For example, one might assume that the original distance budget might be transformed into a sociopsychologic distance budget by means of the attractiveness of the origins and destinations. However, it appears to be extremely difficult to find a reasonable specification which is not completely arbitrary.

Possibility (a) appears to be the most meaningful one. This possibility, which is based on some ideas from information theory and Bayesian statistics, will be discussed here more thouroughly.

According to Theil [1967], entropy can be conceived of as the expected information content of a message. In a more general sense, one may state that entropy is linked up with the degree of uncertainty prevailing in a choice situation with many different possibilities. A reduction in the entropy of a system reflects a decline in the degree of uncertainty. Therefore, additional information will lead to a decline in entropy and to a higher degree of certainty.

These ideas correspond to a certain extent to a number of elements from Baysesian statistics, in which the influence of prior information in the ultimate (posterior) results is analyzed (cf. Raiffa and Schaifer [1961]). A basic idea is the distinction between prior probabilities and posterior probabilities. Prior probabilities reflect the chance that a certain event will occur on the basis of prior information introduced from outside the analysis itself (subjective ideas, external information, alternative analyses, etc.). Posterior probabilities reflect the ultimate chance that an event will occur taking into account the prior information.

Now the behavioural model specified in (5.1) can be used to deduce prior probabilities for a mobility pattern between local production and settlement systems. The assumption made here is that the behavioural model provides only a best first guess for the spatial commuting flows, since due to lack of information this model cannot be tested in an entirely satisfactory manner. Therefore, the results of this model can be conceived of as a reasonable approximation, although several disturbances may exist. This first approximation can now be considered as a prior information for the entropy model. Therefore, we define $p_{i_r j_s}$ as the prior probability that a certain commuting flow $v_{i_r j_s}$ from place i in region r to place j in region s will occur according to model (5.1). In other words:

$$p^o_{i_r j_s} = \frac{v^o_{i_r j_s}}{v} \quad , \tag{7.1}$$

where:

$$v = \sum_{i_r} \sum_{j_s} v^o_{i_r j_s} \quad , \tag{7.2}$$

so that the additivity conditions for probabilities are satisfied. The foregoing approach implies that the behavioural model (5.1) provides the prior information by means of (7.1). This prior information can now be introduced into the entropy model to calculate the posterior probabilities of the spatial flow matrix (see also Hobson and Cheng [1973] and Kullback [1959]). These posterior probabilities will be denoted by $p_{i_r j_s}$.

Next, one may define a <u>conditional</u> probability as the chance that an event will occur, given the prior probability. This conditional probability gives rise to the notion of an <u>average conditional entropy</u> (see Nijkamp and Paelinck [1974] and Theil [1967]. This average conditional entropy can be written as:

$$\omega = - \sum_{i_r} \sum_{j_s} p_{i_r j_s} \ln(p_{i_r j_s} / p^o_{i_r j_s}) \tag{7.3}$$

This implies that model (6.9) can be written ultimately as:

$$v_{i_r j_s} = v^o_{i_r j_s} C_{rs} A_{i_r} B_{j_s} v_{rs} O_{i_r} D_{j_s} \exp(-\beta d_{i_r j_s}) \qquad (7.4)$$

The latter model contains both behavioural elements (owing to the prior information $v^o_{i_r j_s}$ and probability elements (from the generalized entropy model). The calibration of this model can be carried out in a way analogous to the traditional entropy model. The conclusion may be that the foregoing conditional entropy theory links the advantages of a behavioural model to those of a gravity model[1].

8. Empirical Application

The analysis described in the preceding paragraphs was applied to one of the Dutch provinces, viz. North-Holland[2]. The southern part of this province in particular is confronted with serious problems with respect to its spatial structure: the declining function of the city of Amsterdam, urban renovations in various cities, congestion and environmental quality decline in many areas, wide-spread suburbanization in many directions, noise nuisance from the airport of Amsterdam, distance friction caused by the North-sea canal, the development of the harbour of Amsterdam and of the neighbouring steel industry, etc.

In addition to information on production, incomes, investments, congestion, and pollution, a prerequisite for effective spatial planning is insight into the interlocal spatial patterns of this area. The developments of settlement patterns and production patterns are here closely linked to infrastructural patterns and mobility patterns. It is obvious that a spatial divergence between a settlement system and a production system will induce significant spatial interaction problems. The question as to whether the present modal split of traffic flows needs changing is also important in this respect; the North Sea canal in particular forms a spatial barrier which restricts

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[1] A reverse procedure may be to use the result of the extended entropy model (6.9) as 'fictitious' observations on the v_{ij} -variables from the behavioural model (5.1), so that then a regression analysis can be used to gauge the parameter vector $\underline{\beta}$.

[2] Data about this province are contained, for example, in De Donnea [1971], Gedeputeerde Staten van Noord-Holland [1973, 1975], De Langen et al. [1974], Netherlands Economic Institute [1972, 1975], Projectgroep Noordzeekanaalgebied [1971], Ronteltap [1968], Schreuder [1974], and Verstedelijkingsnota [1976].

possibilities to influence mobility patterns by means of physical planning. On the other hand, any change in the settlement profile or in the production profile, generated by (endogenous or exogenous) economic developments or by a governmental policy, will affect the spatial flow pattern.

The analysis was applied for the year 1971 to eight labour regions including, in total, fifty places (see Appendix C and Map 1). During the first stage of the analysis only an interregional flow matrix (of order 8 x 8) was available. For each of the fifty places, the employment attractiveness indicators and the environmental-residential attractiveness indicators were determined (see section 4).

Next, the behavioural model, specified in (5.1), was estimated according to the lines set out in section 5.

The next step was to calculate the interlocal flow matrix by means of entropy model (6.5), based on interregional flows only. These results were less satisfactory, because the flow matrix contained many zero's; they can be explained by the small amount of information used in this approach. The large number of zero's arises here from the implicit distance minimization which forms the dual background of the entropy concept. Hence, this procedure seeks, within each interregional flow, two places which have a minimum distance. Then the flow total is assigned to these two places, whereas the flows between remaining places are set equal to zero. The small amount of information used in this approach appears to be insufficient to provide reasonable results. The conclusion may be that entropy analyses are less useful to disaggregate interregional flows to interlocal flows.

A more satisfactory result can be achieved by applying entropy model (6.1), although the application of this model requires more information, viz. the marginal totals of an interlocal flow matrix. These marginal totals were estimated for 1971 on the basis of a number of known data and on the basis of data from 1966. These results showed a much higher degree of dispersion of the interlocal flows through the matrix, indicating that indeed more adequate results are achieved as more information is introduced.

In applying model (6.1), however, the information about interregional flows was not used. Therefore, the generalized entropy model (6.8) was calibrated to include both the data on marginal totals and on interregional effects. These results appeared to be already rather satisfactory.

Next, the final step was undertaken, in which the prior information from model (5.1) was included in the extended entropy model (7.4). This model satisfied the following three conditions: (a) an explanatory model for spatial behaviour as prior information; (b) interregional additivity conditions for spatial flows; (c) additivity conditions for local marginal totals. The results of this model are represented in Table 1.

These results appear to be very meaningful. There is a considerable dispersion of flows through the table, the employment attractiveness places (like Amsterdam) are significant, and typical commuting places are also significant.

The final conclusion is that the generalized entropy approach, developed here as a synthesis of a behavioural model and an extended entropy model, provides useful results. Its use in analyzing spatial dispersion and mobility patterns in the field of production and settlement systems is worthwhile.

Appendix A

The Method of Hooke and Jeeves

The direct search method of Hooke and Jeeves [1961] is a recursive solution technique for finding the extremum of a nonlinear function without boundary conditions (see also Kowalik and Osborne [1968]). Each stage of the method includes two kinds of movements: an exploration and a pattern. The explorative movement serves to detect the local behaviour of the function, whereas on the basis of this exploration a pattern is developed to find in an efficient way the extremum of the function.

Assume the following function:

$$\omega = f\ (\underline{x}) \tag{A.1}$$

as well as the following argument vector:

$$\underline{x}' = (x_1, \ldots, x_I) \tag{A.2}$$

The exploration to find the minimum of f $(\underline{x})$ includes the following steps:

- take an initial value $\underline{x}_1$ and calculate $\omega_1 = f\ (\underline{x}_1)$.
- let $\underline{x}_1$ shift in one of the directions $\underline{e}'_i = (0, \ldots, 0, 1, 0, \ldots, 0)$ with an amount Δx_i $(i = 1, \ldots, I)$, and calculate the change in the function value denoted as $\Delta\omega_1^i$.
- if $\Delta\omega_1^i < 0$, use $\underline{x}_1^{*\prime} = (x_1, \ldots, x_i + \Delta x_i, \ldots, x_I)$ as a new starting point, and carry out the same step for a change Δx_{i+1} in the $(i+1)^{th}$ element of $\underline{x}_1^*$.
- if $\Delta\omega_1^i > 0$, use $\underline{x}_1^{**\prime} = (x_1, \ldots, x_i - 2\Delta x_i, \ldots, x_I)$ as a new starting point. If then the change in the function value is positive, the i^{th} element is abandoned, and the $(i+1)^{th}$ element is increased with an amount Δx_{i+1}. Otherwise, $\underline{x}_1^{**}$ will be used as the new starting point, and the $(i+1)^{th}$ element of $\underline{x}_1^{**}$ will be increased with an amount Δx_{i+1}.

When in all I directions, as well as in intermediate directions, these explorative movements are carried out, a new initial value $\underline{x}_2$ is obtained. This new basic point $\underline{x}_2$ is now used together with the first point $\underline{x}_1$ to determine a pattern. The pattern movement is determined as follows:

$$\overset{o}{\underline{x}} = \underline{x}_2 + \Delta\underline{x}_{12} \qquad (A.3)$$

where Δx_{12} is defined as:

$$\Delta\underline{x}_{12} = \underline{x}_2 - \underline{x}_1 \qquad (A.4)$$

On the basis of the new pattern point $\overset{o}{\underline{x}}$ the foregoing steps will be repeated. If then a lower value of ω is found, the corresponding value of $\underline{x}$ will be considered as a new initial point for the explorative movement. In the reverse case, the second point $\underline{x}_2$ will be used as the starting point for a further exploration.

If from a certain point onwards the explorative movements do not result in a lower value of $f(\underline{x})$, the step lengths will be reduced and the procedure will be repeated. A convergence will occur, if these step lengths are ultimately smaller than a certain ε-limit.

The method of Hooke and Jeeves is rather easily applicable for convex functions. In the case of non-convexity, problems of local optima do arise. The advantage of this method is that, owing to the possibility of varying the step lengths, insight may be obtained into the existence of local optima. These problems, however, did not occur in the present case of determining the spatial interaction pattern.

Finally, it should be noted that the method of Hooke and Jeeves can be considered as a particular case of the solution technique of Rosenbrock [1960] (see also Walsh [1966]). The latter procedure attempts to find a minimum function value in I mutually independent directions from an initial point by means of a Gram-Schmidt orthogonalization method.

Appendix B

The Lagrange function L associated with (6.5) is:

$$L = \omega + \sum_r \sum_s \lambda_{rs} (v_{rs} - \sum_{i_r \in r} \sum_{j_s \in s} v_{i_r j_s}) + \beta (T - \sum_{i_r} \sum_{j_s} v_{i_r j_s} d_{i_r j_s}) \quad \text{(B.1)}$$

The first-order conditions for a maximum are:

$$\frac{\partial L}{\partial v_{i_r j_s}} = - \ln v_{i_r j_s} - \lambda_{rs} - \beta d_{i_r j_s} = 0 \quad \text{(B.2)}$$

or:

$$v_{i_r j_s} = \exp(- \lambda_{rs} - \beta d_{i_r j_s}) \quad \text{(B.3)}$$

or:

$$v_{i_r j_s} = C_{rs} \, v_{rs} \exp(- \beta d_{i_r j_s}) \quad , \quad \text{(B.4}$$

where C_{rs} is defined as:

$$C_{rs} = \exp \, (- \lambda_{rs}) \, v_{rs}^{-1} \quad \text{(B.5}$$

It is easily seen that this derivation shows the validity of (6.6) and (6.7).

The Lagrange function associated with (6.8) is:

$$L = \omega + \sum_{j_s} \nu_{j_s} (D_{j_s} - \sum_{i_r} v_{i_r j_s}) + \sum_{i_r} \Pi_{i_r} (O_{i_r} - \sum_{j_s} v_{i_r j_s}) + \quad \text{(B.6)}$$

$$+ \sum_r \sum_s \lambda_{rs} (v_{rs} - \sum_{i_r \in r} \sum_{j_s \in s} v_{i_r j_s}) + \beta (T - \sum_{i_r} \sum_{j_s} v_{i_r j_s} d_{i_r j_s})$$

The first-order conditions for a maximum entropy are:

$$\frac{\partial L}{\partial v_{i_r j_s}} = - \ln v_{i_r j_s} - \nu_{j_s} - \Pi_{i_r} - \lambda_{rs} - \beta d_{i_r j_s} \qquad (B.7)$$

or:

$$v_{i_r j_s} = \exp(- \nu_{j_s} - \Pi_{i_r} - \lambda_{rs} - \beta d_{i_r j_s}) \qquad (B.8)$$

or: $$v_{i_r j_s} = C_{rs} A_{i_r} B_{j_s} v_{rs} O_{i_r} D_{j_s} \exp(- \beta d_{i_r j_s}) \quad , \qquad (B.9)$$

where:

$$C_{rs} = \exp(- \lambda_{rs}) v_{rs}^{-1} \qquad (B.10)$$

$$A_{i_r} = \exp(- \Pi_{i_r}) O_{i_r}^{-1} \qquad (B.11)$$

$$B_{j_s} = \exp(- \nu_{j_s}) D_{j_s}^{-1} \qquad (B.12)$$

By making use of the three types of additivity conditions it can easily be derived that:

$$C_{rs} = \{\sum_{i_r \in r} \sum_{j_s \in s} A_{i_r} B_{j_s} O_{i_r} D_{j_s} \exp(- \beta d_{i_r j_s})\}^{-1} \qquad (B.13)$$

$$A_{i_r} = [C_{rs} v_{rs} \{\sum_{j_s} B_{j_s} D_{j_s} \exp(- \beta d_{i_r j_s})\}]^{-1} \qquad (B.14)$$

$$B_{j_s} = [C_{rs} v_{rs} \{\sum_{i_r} A_{i_r} O_{i_r} \exp(- \beta d_{i_r j_s})\}]^{-1} \qquad (B.15$$

Appendix C

Labour Regions and Places in the Southern Part of North-Holland

Labour region IJmond

1. Beverwijk
2. Castricum
3. Heemskerk
4. Uitgeest
5. Velsen

Labour region Haarlem

6. Bennebroek
7. Bloemendaal
8. Haarlem
9. Haarlemmerliede en Spaarnwoude
10. Heemstede
11. Zandvoort

Labour region Zaandam

12. Assendelft
13. Jisp
14. Koog a/d Zaan
15. Krommenie
16. Oostzaan
17. Westzaan
18. Wormer
19. Wormerveer
20. Zaandam
21. Zaandijk

Labour region Hoofddorp

22. Aalsmeer
23. Haarlemmermeer
24. Uithoorn

Labour region Amsterdam

25. Amstelveen
26. Amsterdam
27. Diemen
28. Landsmeer
29. Ouder-Amstel

Labour region Purmerend

30. Beemster
31. Broek in Waterland
32. Edam
33. Graft de Rijp
34. Ilpendam
35. Katwoude
36. Marken
37. Monnickendam
38. Purmerend
39. Wijde Wormer
40. Zeevang

Labour region het Gooi

41. Blaricum
42. Bussum
43. 's-Graveland
44. Hilversum
45. Huizen
46. Laren
47. Naarden

Labour region Weesp

48. Muiden
49. Nederhorst den Berg
50. Weesp

Map 1. The Southern part of North-Holland

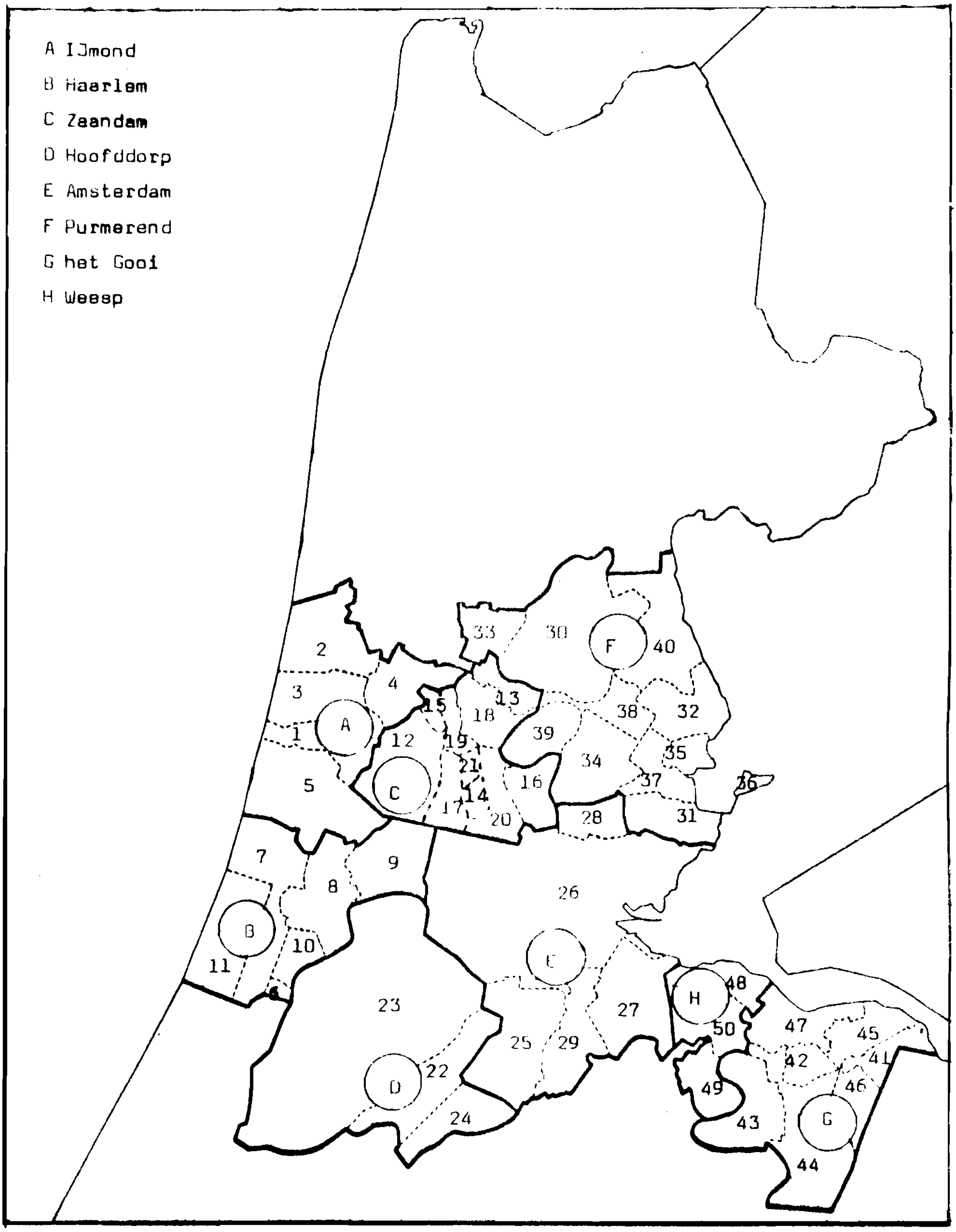

Table 1A: Commuting Pattern According to Behavioural Entropy Model

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	0	68	102	8	259	8	123	533	6	72	94	18	2	9	36	5	8	16	27	128	9	1	3	0	13
2	2	54	0	72	3	146	2	64	189	1	29	50	4	0	2	13	1	1	3	6	48	2	0	1	0	6
3	3	82	76	0	3	216	3	99	282	1	43	76	5	0	2	16	1	2	3	6	68	2	1	2	0	9
4	4	17	19	24	0	45	0	21	59	0	9	16	1	0	0	4	0	0	1	1	17	0	0	0	0	2
5	5	184	86	127	26	0	28	154	1255	19	147	122	56	5	29	99	17	26	54	89	297	30	2	8	1	20
6	6	5	3	5	0	13	0	3	9	0	2	3	0	0	0	0	0	0	0	0	0	0	0	1	0	2
7	7	24	14	21	1	62	1	0	49	0	7	10	0	0	0	0	0	0	0	0	1	0	1	2	0	5
8	8	267	89	145	48	407	28	77	0	15	128	58	6	1	3	9	2	3	5	9	36	3	13	58	8	72
9	9	3	2	3	0	8	0	1	5	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
10	10	25	15	22	1	64	1	14	65	0	0	14	0	0	0	0	0	0	0	0	2	0	1	4	0	9
11	11	19	12	18	1	54	1	11	35	0	7	0	0	0	0	0	0	0	0	0	1	0	1	2	0	5
12	12	19	15	21	1	48	0	3	8	0	1	2	0	0	1	3	0	0	1	1	17	0	0	1	0	3
13	13	2	2	2	0	4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
14	14	10	8	11	0	24	0	1	5	0	1	1	1	0	0	2	0	0	1	1	16	1	0	0	0	2
15	15	34	29	40	2	81	0	4	14	0	2	3	3	0	2	0	1	1	3	5	36	2	0	1	0	5
16	16	6	5	7	0	14	0	1	3	0	1	1	0	0	0	1	0	0	0	0	10	0	0	0	0	2
17	17	9	7	10	0	22	0	1	4	0	1	1	0	0	0	2	0	0	0	0	10	0	0	0	0	2
18	18	18	17	22	1	45	0	2	8	0	1	2	1	0	1	5	1	1	0	2	24	1	0	1	0	4
19	19	30	25	34	2	69	0	4	14	0	2	3	4	0	2	10	2	2	5	0	42	3	0	1	0	6
20	20	105	58	86	17	189	2	12	93	2	12	9	34	4	28	63	22	20	45	66	0	28	3	9	2	34
21	21	10	9	12	0	26	0	2	5	0	1	1	1	0	0	2	0	0	1	1	15	0	0	0	0	2
22	22	8	5	7	0	19	7	68	314	3	65	62	1	0	0	1	0	0	1	1	10	0	0	24	4	74
23	23	36	17	26	4	71	73	254	2116	33	379	234	7	1	4	11	3	4	7	11	60	4	46	0	22	219
24	24	6	4	6	0	15	4	52	200	2	43	47	0	0	0	1	0	0	0	0	7	0	10	15	0	78
25	25	19	12	17	1	43	3	30	153	2	28	25	2	0	2	5	2	1	3	4	33	2	40	80	17	0
26	26	365	68	127	125	281	152	101	4601	157	549	72	201	29	166	329	161	120	284	396	1897	170	626	2160	700	566
27	27	2	2	3	0	6	0	4	13	0	2	3	0	0	0	0	0	0	0	0	4	0	3	5	1	5
28	28	3	2	3	0	7	0	3	11	0	2	2	0	0	0	0	0	0	0	0	4	0	2	3	0	3
29	29	2	1	2	0	5	0	3	12	0	2	3	0	0	0	0	0	0	0	0	3	0	4	6	1	6
30	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3	0	0	0	0	1
31	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
32	32	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1	1	6	0	0	0	0	2
33	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
34	34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
35	35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	1
38	38	0	0	0	0	1	0	0	1	0	0	0	1	0	0	2	0	0	1	1	10	0	0	0	0	2
39	39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
41	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
42	42	0	0	0	0	1	0	1	5	0	1	1	0	0	0	0	0	0	0	0	2	0	0	0	0	9
43	43	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
44	44	1	1	1	0	2	1	2	18	0	3	1	1	0	1	1	1	0	1	2	9	1	1	2	1	29
45	45	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	4
46	46	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3
47	47	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	4
48	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	4
49	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	4
50	50	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	1	7	0	1	1	0	15

Table 1B: Commuting Pattern According to Behavioural Entropy Model

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	382	1	1	1	0	0	1	0	0	0	0	0	1	0	0	0	3	0	21	2	2	2	0	0	1
2	217	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	12	1	1	1	0	0	0
3	331	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3	0	18	2	2	1	0	0	0
4	86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	5	1	0	0	0	0	0
5	454	2	2	2	1	0	2	1	0	0	0	0	3	0	1	1	5	1	23	3	2	2	1	1	4
6	42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
7	133	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0
8	1101	7	6	7	4	1	7	3	1	0	1	2	12	1	2	1	6	1	21	3	2	2	1	1	3
9	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
10	138	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	5	0	0	0	0	0	0
11	108	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
12	120	0	0	0	1	0	6	1	0	0	0	1	10	0	1	0	3	0	22	3	2	2	0	0	0
13	17	0	0	0	0	0	1	0	0	0	0	0	2	0	0	0	0	0	4	0	0	0	0	0	0
14	96	0	0	0	2	0	6	1	0	0	0	1	10	0	1	0	3	0	18	2	2	1	0	0	0
15	203	0	0	0	6	1	17	4	1	0	1	2	29	1	3	1	6	0	39	5	4	3	0	0	0
16	97	0	0	0	1	0	4	0	0	0	0	0	7	0	0	0	3	0	20	2	2	2	0	0	0
17	70	0	0	0	1	0	3	0	0	0	0	0	5	0	0	0	2	0	13	1	1	1	0	0	0
18	161	0	0	0	3	0	13	2	1	0	0	1	22	0	2	1	5	0	33	4	3	3	0	0	0
19	211	0	0	0	8	1	21	5	2	0	1	3	35	1	4	1	6	0	41	5	4	3	0	0	0
20	1013	3	4	3	76	16	169	44	26	2	13	47	269	13	42	5	38	4	181	26	19	19	2	2	8
21	99	0	0	0	1	0	6	1	0	0	0	0	10	0	1	0	3	0	19	2	2	2	0	0	0
22	1203	2	1	4	0	0	1	0	0	0	0	0	2	0	0	0	1	0	8	1	1	0	1	1	2
23	3341	13	8	16	3	1	7	2	1	0	1	2	11	1	2	0	3	0	16	2	1	1	3	3	12
24	1277	1	1	4	0	0	1	0	0	0	0	0	1	0	0	0	1	0	12	1	1	1	1	0	1
25	961	2	1	3	5	2	19	3	2	0	1	5	26	1	3	9	71	4	502	49	44	35	6	4	16
26	0	145	95	98	597	201	1478	324	257	25	164	560	2002	109	359	190	1237	281	1809	582	318	485	295	223	1155
27	180	0	0	0	0	0	3	0	0	0	0	0	3	0	0	2	14	0	105	11	10	8	1	0	1
28	126	0	0	0	0	0	3	0	0	0	0	0	4	0	0	1	6	0	46	5	4	4	0	0	0
29	133	0	0	0	0	0	1	0	0	0	0	0	2	0	0	1	11	0	88	8	7	6	1	0	1
30	28	0	0	0	0	0	3	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0
31	9	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
32	69	0	0	0	2	0	0	1	1	0	1	2	14	0	2	0	0	0	0	0	0	0	0	0	0
33	15	0	0	0	0	0	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
34	12	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
35	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	7	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
37	25	0	0	0	0	0	2	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
38	95	0	0	0	3	0	13	1	1	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0
39	5	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
40	16	0	0	0	0	0	2	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0
41	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	24	3	3	1	1	0	1
42	229	1	0	1	0	0	0	0	0	0	0	0	0	0	0	3	0	1	119	15	13	10	7	6	21
43	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	27	2	2	1	1	0	1
44	531	4	2	4	0	0	1	0	0	0	0	0	1	0	0	10	60	8	0	38	38	23	28	29	100
45	124	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	10	0	66	0	9	5	3	2	6
46	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	7	0	58	6	0	3	1	1	3
47	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	7	0	45	6	5	0	2	1	4
48	146	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	9	0	65	7	6	5	0	7	24
49	95	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	7	0	63	5	5	4	13	0	18
50	426	1	0	1	0	0	2	0	0	0	0	0	2	0	0	3	27	1	192	20	18	14	86	44	0

References

Bouchard, R.J., and C.E. Pyers (1964), Use of Gravity Models for Describing Urban Travel, Highway Research Record, 88, pp. 1-43.

Boyce, D.A., A. Fahri, and R. Weischedel (1974), Optimal Subset Selection, Springer-Verlag, Berlin.

Cebula, R.J., and R.K. Vedder (1973), A Note on Migration, Economic Opportunity, and the Quality of Life, Journal of Regional Science, 13, 1, pp. 105-211.

Donnea, F.X., de (1971), The Determinants of Transport Mode Choice in Dutch Cities, Rotterdam University Press, Rotterdam.

Gedeputeerde Staten van Noord-Holland (1973), Nota Overloopprolematiek, Haarlem.

Gedeputeerde Staten van Noord-Holland (1975), Nota van Uitgangspunten voor het Amsterdam-Noordzeekanaalgebied, Haarlem.

Glass, R. (1955), Anti-urbanism, Current Sociology, 4, pp. 5-19.

Hansen, N.M., (1975), An Evaluation of Growth Center Theory and Practice, Environment and Planning, 7, pp. 821-831.

Hobson, A., and B.K. Cheng (1973), A Comparison of the Shannon and Kullback Information Measures, Journal of Statistical Physics, 7, pp. 301-310.

Hooke, R., and T.A. Jeeves (1961), Direct Search Solution of Numerical and Statistical Problems, Journal Assoc. Comput. Mach., 8, pp. 212.

Kowalik, J., and M.R. Osborne (1968), Methods for Unconstrained Optimization Problems, American Elsevier Publ. Co., New York.

Kullback, S., (1959), Statistics and Information, Wiley, New York.

Langen, M. de, I.L. van Leeuwen, and A.C.P. Verster (1974), The Influence of Accesibility on the Spatial Distribution of Activities in an Urban System, Series: Foundations of Empirical Economic Research, Nr. 16, Netherlands Economic Institute, Rotterdam.

Lasuén, J.R., (1973), Urbanisation and Development: The Temporal Interaction between Geographical and Sectoral Clusters, Urban Studies, 10, pp. 163-188.

Magoulas, G., K. Peschel and M. Wadehn (1975), Determinants of Commuting and Migration, Institüt fur Regionalforschung der Universität Kiel, Kiel (mimeographed).

Masotti, L.H., and H.K. Hadden, eds. (1972), The Urbanisation of the Suburbs, Sage Publ., Beverly Hills.

Morrison, P.A., (1973), Theoretical Issues in the Design of Population Mobility Models, Environment and Planning, 5, pp. 125-134.

Mumford, L. (1961), The City and History: Its Work, Its Transformations and Its Prospects, Harcourt Brace and World, New York.

Netherlands Economic Institute (1972), Integrale Verkeers- en Vervoersstudie, Rotterdam.

Netherlands Economic Institute (1975), De invloed van de Verkeers- en Vervoers-infrastructuur op de Ruimtelijke Spreiding van Bevolking en Werkgelegenheid in de Noordvleugel van de Randstad, Rotterdam.

Nijkamp, P. (1975a), An Impact Analysis of Environmental Attraction Profiles and Spatial Mobility, Paper British Meeting Regional Science Association, London.

Nijkamp, P. (1975b), Urbane en Suburbane Leefbaarheidsperikelen, Stedelijk Perspektief (P. Nijkamp and C. Verhage, eds.), Stenfert Kroese, Leiden, pp. 1-36.

Nijkamp, P. (1975c), A Multi-criteria Analysis for Project Evaluation, Papers of the Regional Science Association, 35, 1975, pp. 87-111.

Nijkamp, P. (1975d), Reflections on Gravity and Entropy Models, Regional Science and Urban Economics, 5, 3, pp. 203-225.

Nijkamp, P. (1976a), Harmonie en Konflikt in het Ruimtelijk-Ekonomisch Denken, Stenfert Kroese, Leiden.

Nijkamp, P. (1976b), "Environmental Attraction Factors and Regional Tourist Effects", Analytical Systems of Tourism (C.E. Gearing and W.A. Swart, eds.), (forthcoming).

Nijkamp, P., (1976c), Socio-Economic and Environmental Indicators as Determinants of Interregional Migration Flows, Social Indicators Research, (forthcoming).

Nijkamp, P., (1976d), The Use of Spatial Impact Models in Physical Planning, Geoforum (forthcoming).

Nijkamp, P., and J.H.P. Paelinck (1974), A Dual Interpretation and Generalization of Entropy Maximization Models in Regional Science, Papers of the Regional Science Association, 33, pp. 13-31.

Nijkamp, P., and J.S. Wiersma (1976), Pendel-Perikelen in het Regionale Onderzoek, Research Memorandum no. 46, Dept. of Economics, Free University, Amsterdam.

Paelinck, J.H.P., and P. Nijkamp (1975), *Operational Theory and Method in Regional Economics*, Saxon House, Farnborough.

Projectgroep Noordzeekanaalgebied (1971), *Streekplan van Streek*, Planologisch en Demografisch Instituut, University of Amsterdam.

Raiffa, H., and R. Schlaifer (1961), *Applied Statistical Decision Theory*, Graduate School of Business Administration, Harvard University, Cambridge.

Ronteltap, R. (1968), *Industriele Ontvikkelingsmogelijkheden in Hollands Noorderkwartier*, Industriecommissie Hollands Noorderkwartier, Alkmaar.

Rosenbrock, H.H. (1960), An Automatic Method for Finding the Greatest or Least Value of a Function, *Computer Journal*, *3*, p. 175.

Schreuder, F.W., (1974), "Het Forensisme naar Amsterdam", Free University, Amsterdam (mimeographed).

Theil, H., (1967), *Economics and Information Theory*, North-Holland Publ. Co., Amsterdam.

Verstedelijkingsnota (1976), Staatsuitgeverij, Den Haag.

Walsh, J., ed. (1966), *Numerical Analysis*, Academic Press, New York.

Wilson, A.G., (1970), *Entropy in Urban and Regional Modelling*, Pion, London.

Wirth, L., (1938), Urbanism as a Way of Life, *American Journal of Sociology*, *44*, pp. 1-24.