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REGIONAL DEVELOPMENT AND LAND-USE MODELS: AN OVERVIEW OF OPTIMIZATION METHODOLOGY

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Preface

This paper is the first in a series entitled 'Regional Development and Land-Use Models'.

The present paper is intended to draw a coherent picture of the major issues in land-use modelling research. It represents an effort within the Human Settlements and Services Research area to come to grips with some major issues in integrated regional development. This series will be devoted to the investigation of regional land-use models as tools for understanding and planning development. The paper is concerned only with optimization models but later papers will cover both optimization and behavioural models. Further, there will be an emphasis both on theoretical and applied planning models. In all cases, the emphasis will be on the spatial implications of regional development and economic growth.

Abstract

Progress in the development of optimizing land-use design models is evaluated in this paper. Eight methodological issues are raised concerning the theoretical foundations of such models and the transition from a theoretical to an applied planning tool. Five specific land-use models are evaluated in relation to these issues. A series of extensions to these models are proposed to help meet the methodological issues raised. The main conclusion reached is that the short-term propects for an improved design model, suitable for applied planning, are not good without more research into the areas noted.

Regional Development and Land-Use Models: An Overview of Optimization Methodology*

John R. Miron

1. INTRODUCTION

One general question underlies this paper. If such a thing can be defined, what is the 'optimal' spatial arrangement of land-use activities within a region? The answer to this question is of substantial interest to governments involved in regional development. It is also a matter of considerable interest to regional economists who see implications here for economic theory in general and for the debate over the relative efficiency of alternative institutional arrangements for resource allocation. This question is approached here mainly from the viewpoint of the regional economic theorist. This involves a commitment to a certain degree of abstraction. Some application-oriented readers may be initially discouraged by the approach chosen. They should not be. The approach taken leads immediately to a discussion of methodological issues which will also necessarily underly any more-applied efforts to answer this question[1]. The methodological issues raised are of great importance. It is felt that several areas of research must be explored before realistic applications of optimization procedures are feasible.

The approach of this paper is to review some simple mathematical models which have been developed to find optimum solutions to certain kinds of land-use arrangement problems. Some of these models have even been used empirically in contemporary planning situations. However, they are

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*This paper has benefitted from the critical comments of N. Hansen and P. Korcelli on an earlier draft.

[1]To avoid later confusion, it is here noted that 'methodology' is used in the sense of a logic or rationale for a particular method.

relatively naive and their main value is still as a theoretical, rather than an applied, tool.

1.1 A Mathematical Approach

Let us begin by defining a typical mathematical land-use design model. Such a model usually has outputs of a form useable by regional planners in their Land-Use Plan. A typical Plan consists partly of a map on which is outlined zones within the the region. To clarify matters, let us assume that this region is approximately 500,000 hectares in size. The Plan indicates the amount of land within each zone which any particular land-use can occupy. Note that the Plan is a static picture. It represents what planners believe to be an optimal 'mature state' arrangement of land uses in the region at some point in the future when development has 'filled in' the region according to the Plan.

A normative mathematical design model usually consists of two parts. The first is a welfare function which translates the design choice into a unique measure or ranking of the design's value. Typically, the welfare function is cast in terms of instrument variables representing aspects of the design choice. These might well include the size and location of either public facilities or other land uses. The second part of this model consists of a set of constraints which restricts the ranges of the instrument variables. There may be restrictions on the total supply of land available for, or on the amount of land demanded by, any land-use. Alternatively, there may be design restrictions on land uses which, for instance, prohibit the contiguous location of incompatible land uses or require the proximity of complementary ones.

A generalized programming model might be constructed. Suppose that the development region is sub-divided into Z zones and that there are A land use activities to be located in the region. Let x_{az}, the instrument variables, be the amounts of land allocated to each land use 'a' in each zone 'z'. Further, suppose that there are 'q' constraints in total on the land use assignment. A generalized model is the following. Here π is the value of the welfare function and the inequalities $g_i \leq c_i$ each represent constraints[2].

$$\text{Maximize} \quad \pi = f(x_{11}, x_{12}, \ldots, x_{AZ})$$

$$\begin{aligned} \text{Subject to} \quad & g_1(x_{11}, x_{12}, \ldots, x_{AZ}) \leq c_1 \\ & g_2(x_{11}, x_{12}, \ldots, x_{AZ}) \leq c_2 \\ & \vdots \\ & g_q(x_{11}, x_{12}, \ldots, x_{AZ}) \leq c_q \end{aligned}$$

$$x_{az} \geq 0 \begin{cases} a = 1,2,\ldots,A \\ q = 1,2,\ldots,Z \end{cases} \qquad (1)$$

For a general class of mathematical programming problems, there exists a complementary dual problem to each original or primal problem[3]. This dual, which can be solved to find the shadow price on each constraint, provides valuable information to the central planner. In addition, it provides theoretical insights into the nature of the design problem.

A problem arises when one attempts to interpret the term 'zone' in a model such as (1). Some researchers have defined a zone as an area of homogeneous soil type of from 100 to about 400 hectares in size. Such a zone would almost invariably include multiple land-using activities. Others define a zone as a very small areal unit of from 0.1 to about 10 hectares which would usually include only one land-use.

Different scales can be used to answer different kinds of questions. At the 100 hectare (gross) scale, a design model can allocate land uses in a 'broad brush stroke' manner as a direct input for a regional Master Plan. At the 0.1 hectare

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[2]Note that π need not be an ordinal measurement here. It may simply be a ranking or index number. It is noted, of course, that (1) is a general form which can also represent minimization problems or those with $\geq$ constraints.

[3]It is necessary that f be convex and that $g_i = c$ be each a concave function. Refer to Balinski and Baumol (1968).

(fine) scale, a design model can be used for detailed planning at the smaller-area Site Plan scale. Since it seems technically infeasible to construct a model at the latter scale for a region of 500,000 hectares or so, a hierarchical design process using models at each scale is necessary.

To cover the complete range of design models would be too difficult in the space of this paper. Fine-scale models have therefore been omitted. The interested reader is referred to some initial sources. Lynch (1971) presents an excellent non-mathematical overview of the many issues in small-scale planning. Scott (1971) discusses sub-problems in transportation network design and public facility location which could be integrated into a general design model. Francis and White (1974) discuss layout planning and facility location sub-models to minimize flow costs or discordances between adjacent facilities.

It should be noted that gross-scale design models are ill-suited to answer some detailed design questions. Water and sewer servicing costs, road requirements and congestion levels, and common externality benefits or costs, for example, may very significantly with the layout of land uses within a 100 hectare site. Only rough approximations on these problems are possible with a gross-scale model.

1.2 The Purpose of this Paper

The central tenet of this paper is that current, gross-scale, design models such as (1) fail to grasp even the theoretical complexity of optimal planning. They do not adequately handle several important methodological issues and this is purely aside from any complaints which regional planners may have about their implementability. Further, although some of the methodological problems may be resolved by further research, it is an open question whether an adequate resolution of all these issues is forthcoming in the near future.

The purpose of this paper is to develop this argument. An identification of the major methodological issues is undertaken in Section 2. A review of some current models in English-language research follows this in Section 3. Finally, some suggestions are made about the feasibility and directions of new research in this area in the near future. This, and a conclusion about the feasibility of better applied optimization models, are presented in Section 4.

2. THE MAJOR METHODOLOGICAL ISSUES

There are at least eight major methodological issues posed when one attempts to convert the general form (1) into a specific design model. Each issue is discussed in this Section and the way in which the models of Section 3 handle it are summarized.

2.1 The Welfare Function

The welfare function in (1) presumably reflects those societal objectives which are relevant to the spatial arrangement decision. Given that the spatial pattern of human settlements and activity impinges on most aspects of human existence, it is no wonder that most societies have many diverse, and possibly conflicting, design objectives[4]. Some economic objectives, for instance, might be to minimize regional development costs, to maximize the region's contribution to national output, or to maximize the real income levels of the region's residents. Social objectives might include preserving a certain mix of income and social groups within the region, guaranteeing a certain level of access to public facilities, and ensuring a diversity of housing types. Aesthetic objectives might include those related to the spacing of buildings, to their height and bulk, and to the conformity of neighbouring land-uses. There may even be political or military objectives which dictate the spatial patterns of development.

This multiplicity of objectives generates at least two kinds of choice problems. First, how should the planner weigh different goals and choose among plans which each emphasize alternate goals? Further, within any society there are several groups of people each with some participatory power in the plan

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[4] Refer to Merlin (1973b, pp. 242-246), for example, for an international comparison of siting objectives for New Towns. Refer to Laidlaw (1972), pp. 103-137) for other examples of such objectives.

selection process. Given groups with conflicting sets of values, the second problem is to reconcile different plans which are each optimal only with respect to certain groups.

Real progress has been made only on the first of these two problems. Baecher _et al_ (1975, pp. 47-75) discuss the methods which have been used to evaluate multi-objective plans[5]. Most of these use a welfare function which is a simplification of that in (1). Suppose that there are W objectives and that $y_W = y_W(x_{11}, x_{12}, \ldots, x_{AZ})$ measures the degree to which objective 'w' is attained by a land-use allocation. The welfare function in (1) can then be re-expressed as a function of these objectives.

$$\pi = h(y_1, y_2, \ldots, y_W) \qquad (2.a)$$

This function can be simplified if it is assumed that π is a linear function of separable objectives.

$$\pi = a_1 y_1 + a_2 y_2 + \ldots + a_W y_W \qquad (2.b)$$

Such a simplification is quite restrictive in that it presumes (i) that objectives are independent of each other and (ii) that the marginal contribution of measure y_i to the value of the welfare function is independent of y_i itself. Even this simplified welfare function requires estimates of the a_i terms in (2.b) and this poses some estimation problems[6].

An alternative to the multi-objective welfare function is to presume that only one objective is to be included. In land use design problems, the single objective is usually development cost minimization. Other objectives, of which there may be several, are then treated as constraints with y_i being forced to a certain minimum (or maximum). One representation of such a design model, where y_1 is the only objective included in the welfare function is the following extension of (1).

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[5]These exclude cost-benefit analysis, cost-effectiveness, the planning balance sheet of Litchfield, Hill's Goals achievement Matrix, and preference theory approaches.

[6]Refer to Heal (1973, pp. 9-16).

$$\begin{aligned}
\text{Maximize} \quad & \pi = y_1(x_{11}, x_{12}, \ldots, x_{AZ}) \\
\text{Subject to} \quad & y_2(x_{11}, x_{12}, \ldots, x_{AZ}) \geq b_2 \\
& y_3(x_{11}, x_{12}, \ldots, x_{AZ}) \geq b_3 \\
& \vdots \\
& y_W(x\ \ , x\ \ , \ldots, x_{AZ}) \geq b_W \\
& g_1(x_{11}, x_{12}, \ldots, x_{AZ}) \leq c_1 \\
& g_2(x_{11}, x_{12}, \ldots, x_{AZ}) \leq c_2 \\
& \vdots \\
& g_q(x_{11}, x_{12}, \ldots, x_{AZ}) \leq c_q \\
& x_{az} \geq 0 \begin{cases} a = 1,2,\ldots,A \\ z = 1,2,\ldots,Z \end{cases}
\end{aligned} \tag{3}$$

The constant b_i represents a minimum attainment level for objective y_i not included in the welfare function[7]. These might also be termed planning standards.

In practice, design models similar to (3) might be used iteratively by the planner. He would make subjective trade-offs and raise or lower the b_i's in each iteration depending on the acceptability of the current value of π. Such a procedure, while still forcing him to implicitly consider the general welfare function in (2.a), is an improvement because it helps the planner to come to grips interactively with important trade-offs and their consequences.

Some critics assert that the search for a welfare function is a futile exercise. They point out that, in a pluralistic society, different interest groups may have different values and that this usually implies that a well-behaved welfare function does not exist[8]. Further, they argue that the political decision-making process itself, if provided with adequate information, serves as its own kind of optimizing routine.

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[7]For completeness, the possibility of upper limit constraints (e.g., $y_i \leq b_i$) should not be excluded.

[8]As discussed in Heal (1973, pp. 25-59), this is the familiar Arrow paradox.

There are, however, several reasons why a design model is useful in principle in spite of these criticisms. First, it is still necessary to reduce the large array of possible land-use plans to a manageable set wherein hopefully each plan represents a distinct alternative emphasing different sets of values. For this, a design model is useful. Secondly, these models necessitate the systematic collection of data which is important for any informed discussion of alternatives. Finally, such models force an explicit consideration of objectives and trade-offs[9].

In a preview of Section 3, the welfare functions currently in use may be described as follows. Nearly all models are of the type typified by (3). There are virtually no direct applications of (2.b) in optimizing land-use models where the included y_i's represent dis-similar objectives. Model evaluation according to (2.b) does take place in conventional land-use planning where trial-and-error schemes are used for plan generation. However, applications using a direct optimization procedure are scarce. An extension of current models to a welfare function of the (2.a) or (2.b) variety is one area of future research.

2.2 Uncertainty

The model (1) is deterministic. To use it, one must be able to specify exactly the shape of the welfare function and each constraint. This, however, is very difficult to do. Often, one does not have enough information to be able to specify (1) either at the present moment or in the future at the mature state point.

This uncertainty stems from at least two sources. First, the planner usually operates in an environment which he, at best, only partly understands and controls. To some extent, the planner is trying to optimize a spatial arrangement of activities without knowing all the interconnections that might exist among these activities. This issue is approached again in Section 2.7. The second source of uncertainty is due to changes between now and the mature state time frame in such aspects as technology and

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[9] Schlager makes similar points in Highway Research Board (1968, pp. 193-196).

societal preferences.

The problem of optimization under uncertainty about future conditions is one which planners have long recognized. The source of the problem is of course the durability of buildings and the difficulty of altering the use for which a building is originally designed. In other words, a decision made now about the land-uses allocated to a zone cannot be easily changed for many years, perhaps decades, once it is developed. Thus, if an optimal solution to (1) is very sensitive to unpredictable future conditions, what is optimal? Different strategies can be proposed based on different notions of rationality under uncertainty but the concept of a best solution may have to be drastically altered.

None of the models reviewed in this paper consider the problem of uncertainty. They all presume that whatever parameter values are required can be supplied with precision. In fact, as is to be seen, newer models tend to make greater demands for precise parameter values so that the issue of the treatment of uncertainty is becoming ever more important.

Indeed, it could be argued that the use of mathematical design models such as (1) is unwarranted because of this uncertainty. The precision of the solution to such a model is viewed as irrelevant. What one seeks, instead of a deterministic optimum, is a spatial arrangement of land-uses which is robust while being near-optimal. Robustness would here refer to the near-optimality of a solution over an expected range of parameter values. It is possible to consider robustness, in a design model such as (1), in a crude way using sensitivity analysis. Whether a mathematical design model is really the best way to find a near-optimal robust solution is, however, still an open question.

2.3 Dynamic Optimization

All references to optimization in (1) are to a 'mature state' at some future point in time. However well an optimal end state can be defined, several questions can be raised concerning the dynamics of how a society gets there. Is there an optimal path that the spatial pattern of land-use should follow over time between now and the mature state? Is there a significant difference between incremental optimization at each future time period and mature state optimization? Should consideration be given to the differences between a developing and a mature region and what might these be? None of the models reviewed in this paper consider the first and third questions. One model is used to explicitly evaluate the second queston although others could potentially be used as well.

On the third question, there are at least two phenomena which might be considered. In a developing region, there are problems in the temporal sequencing of interdependent projects. Which land-use activities should be developed first and where should they be located relative to each other? Some work has been carried out on models of the phasing of industrial complexes[10]. However, this work is essentially non-spatial and none of the land-use models to be considered incorporates such notions[11]. The second dynamic phenomenon has to do with the changing demographic structure of the population. Usually, a large part of population growth in a developing region is due to net in-migration. In-migrants, however, tend to be younger than the national average[12]. As the region evolves toward a mature state, its population age distribution can be expected to move closer to the national average. Since housing and other consumer needs of a population vary with the age distribution, the optimal land-use pattern may also vary through the development phase. Again, however, none of the design models explicitly considers this problem.

2.4 Layout and Level

There is a significant distinction inthe following review between two classes of models. One class assumes that the

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[10]Refer, for example, to Reiter and Sherman (1962) and Reiter (1963).

[11]A non-mathematical discussion of some sequencing problems in urban development is found in Friend and Jessop (1969, pp. 165-213).

[12]An exception occurring in the case of retirement areas.

aggregate amount (level) of each land-using activity in the region in the mature state is exogenously given. In these models, the only endogenous variables are the allocations (layout) of this aggregate among the different zones. A second class of models, assumes that both the aggregate amount of development and its location (level and layout) are endogenously determined. In this class of models, it is believed that the efficiency of its land-use arrangement affects the aggregate level of development of the region. Proponents of the first kind of model would argue that the level of a land-use activity is usually relatively insensitive to the efficiency of its layout. They would thus argue that the second class of models is un-necessarily complicated. Proponents of the second class would argue that level is more sensitive to layout. Further, they tend to see in this class of models explicit connections with an economic theory of regional development that is missing in the first class.

Should both level and layout be made endogenous to the model? Viewing the design model as a theoretical tool, the answer is yes. The second class of models is more general in that it clearly encompasses the solution possibilities of the first as well as others. At an operational level, the answer is not so clear. The second class of models is considerably more complex and may be infeasible to operationalize. Also, the applied planner may be faced with a given bill of activity levels which he is not free to vary. In these cases, the first class of models may be easier or more appropriate to use.

2.5 The Transportation Sector

Virtually all land use planning models treat the transportation sector very sketchily. There are at least two main reasons why a more thorough treatment of this sector is justified. The first involves a relationship between network design and land use. The second involves a relationship between the spatial pattern of urbanization and the amount of land required for transportation. In almost any kind of society, the provision of most transportation facilities is in the public sector.

Thus, the planner has usually both a responsibility to plan for the transportation sector and a capability to use this sector as a tool in encouraging an optimal land-use pattern.

The design of a transportation network gives the planner several instruments to control the general pattern of land use. These instruments include the location and geometry of the network, its flow capacities, and its pricing (including congestion costs) structure. Since a transportation network generally consists of several modal networks, further policy instruments might include the mix of modes, the congruence of junction points, and the relative pricing of each[13].

Nearly all land-use design models take a very naive view of the transportation network. If they treat it at all it is usually assumed that the cost of any movement between a pair of zones is fixed and exogenously given. In other words, the transportation network is given prior to determination of the optimal land-use pattern.

The second aspect, the demand for land by transportation, has received more attention in design models. Here emphasis has been put entirely on road transport. Besides the public role in transport in general, the planner has another interest in road provision. Since roads are rarely priced efficiently (at marginal social cost), they are subject to over-use in a socially optimal sense. The rational provision of land for roads is thus a major planning problem for many kinds of societies.

To complicate this there is a fundamental trade-off in the provision of land for transportation. Given the economic benefits of spatially concentrated production, there is an advantage in allocating as little land as possible to transportation within an urban area. On the other hand, congestion costs rise

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[13]As one simple example, consider a network with limited entry and exit points, such as a subway or a limited-access freeway, in contrast to a continuous access network such as an urban grid street system. Land-use development in the limited-access network might be expected to be clustered around access points in contrast to the spatially-homogenous pattern which might be expected with the latter network.

quickly with the ratio of traffic flow to route capacity. Given a relationship between route capacity and the amount of land used, the optimal land allocation is seen as a well-defined economic concept.

One of the models to be discussed in Section 3 does incorporate a specific land-use role for transportation. A very simple model of transportation is used; a ground-level road network. Alternative modes in which the ratio of capital to land is higher, such as subways or elevated monorails, have not been discussed.

A broad criticism may be made of the approach of such land-use design models towards land for road transportation. These models invariably assume two engineering relationships. The first is a link between congestion costs and the ratio of traffic to route capacity. The second is a relationship between capacity and land input. Both of these relationships are subject to some variation in reality depending on the very detailed attributes of the route link in question[14]. That gross-scale design models rely too heavily on an over-simplified model of road traffic flow is a serious criticism. It indicates a limitation on the usefulness of this kind of model.

2.6 Handling of Locational Interdependencies

The desirability of a zone to a certain land-use activity may depend on the kinds of activities which locate in nearby zones. Such distance-related locational interdependencies arise from at least three sources. First, one land-use activity may purchase the outputs of another activity. If they vary systematically with distance, the transportation costs associated with such transactions are one source of locational interdependency. Included in such costs would be commuting and consumer trip expenses as well as intermediate good flow expenses among producers or distributors. A second kind of interdependency arises because of communications and information flow costs which vary systematically with distance. There, the time costs created by distance and the necessity of face-to-face communication are another source of locational interdependence. Thirdly,

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[14]Refer to Wohl and Martin (1967, pp. 322-373) for example.

locational interdependencies may be created by the externality effects flowing from one land-use to nearby ones. A typical negative example is air pollution while a positive externality might be a view of an adjacent scenic park.

Gross-scale design models are generally of two types. In one type of model, all locational interdependencies are ignored. In the second type, only transportation costs are considered. There are no gross-scale models which incorporate either of the other two sources of locational interdependence endogenously[15].

2.7 The Private Sector

To this point, the treatment of the design problem has been quite abstract. The optimal layout, and sometimes even the level, of all land-uses has been sought with no concern for the ability of a government operating within a particular institutional setting to effect that spatial pattern. Even the concept of a 'planner' as used until now has been in terms of the academic notion of a complete dictator operating his own economy. This has been consistent with our emphasis on regional economic theory.

The methodological issues raised by the existence of a private sector in society become important when one begins to move from a theoretical to an applied model. Of course, the particular legal and institutional frameworks within which it operates makes the private sector in each society unique to some extent. However, in most societies a role is accorded to the private sector which makes the public sector only partly able to effect an optimal plan. Two questions are raised. How does the behaviour of the private sector in a particular society affect the level and layout of land-use activities? What tools are available to the regional planner to effect an optimal plan? The first question is considered below while the second is discussed in Section 2.8.

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[15]Some fine-scale design models have been developed in which it is possible to represent any kind of interdependency. Refer, for example, to the models described in Francis and White (1974), Chapter 3). However, such models usually have only a vague kind of weighting scheme for such interdependencies. In addition, such models tend not to have analytical or even numerical solution algorithms.

A rather large literature has developed on models of private sector behaviour. Most of this literature can be subdivided into models of the aggregate level of regional development and models of the role of private sector behaviour in shaping land-use or layout patterns. Richardson (1973) presents a recent summary on models of aggregative development applicable to Britain and North America. Behavioural models of layout are reviewed in Senior (1973 and 1974), King (1972), Batty (1972), and Lowry (1968).

In spite of this large literature, there has been no attempt to integrate behavioural and design models. Some of the behavioural land-use models, such as that of Lowry, allow for specific planning tools such as zoning and density restrictions. However, they do not contain an explicit optimization procedure to qualify them as design models.

In the models reviewed in this paper, there is no general treatment of private sector behaviour. The only behavioural aspects contained in any of these models concern transportation flows. In these cases, the models estimate the total transportation flows between land-use activities in each zone or pair of zones. The costs associated with these flows are then included as part of the aggregate regional development cost to be minimized. That design models have not been extended to consider other behavioural aspects is surprising in view of the number of societies where the private sector plays a significant role.

The methodological issues raised by a private sector go beyond just problems of application. There is also a substantive theoretical question raised. In the abstract world of perfectly planned and perfectly competitive societies, does a design model indicate anything about the ability of a decentralized private sector or market economy to achieve the same maximum efficiency of the centralized economy?

The dual solution to a design programming model is helpful in establishing the (non)equivalence of centralized versus decentralized decision-making. In market economies, the land market is partly relied upon to distribute land among potential users. The Ricardian rent pattern which might be established in a perfectly competitive market economy may be compared with the shadow price for land in a programming model. Equivalent prices imply that land is not mis-allocated in the market solution.

2.8 Policy Tools and Their Efficiency

As argued earlier, no design model explicitly considers the tools to be used in effecting a land use plan. The tools available vary widely from one society to the next and it is difficult to discuss them more specifically without choosing particular societies as examples. In most societies, however, the planner at least has some say in the creation of public infrastructure such as transportation facilities, utility lines, and parkland. This can sometimes include such aspects as location, quantity, and pricing. Further, in most societies, planners usually also have the power to implement permissive zoning[16]. Other powers are less widely available.

When one considers societies which allocate more authority to planners, there are at least three directions in which such controls might go. The first is towards more restrictive zoning in which the range of uses permitted under a given zoning is reduced. Another is toward development control. Unlike zoning which permits a new land-use anywhere that the zoning requirement is met, development control tools usually allow the planner to dictate a unique location where a new land-use is to be permitted. The final direction is toward complete centralized planning in which the planner can dictate when and where land-uses are to be developed.

Thus, to encourage an optimal development plan, the planners in different societies may have different tools. In all but the central planning case, however, the planner cannot explicitly dictate the location, timing, and mix of development. He must

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[16] Under permissive zoning, all possible land uses are ranked on some basis from least obnoxious (usually parkland) to most obnoxious (usually heavy industry) and each zone of a region is designated by the most obnoxious use permitted there. Thus a zoning of 'regional commercial use' would also permit less obnoxious uses such as parkland, churches, residential uses at several densities, and local shopping for example.

rely on his zoning, development controls, and infrastructure planning tools to help convince industries and households to locate appropriately. Further, he may find that he has several different combinations of tools which might be used to attack a particular problem. How can he decide which combination is most efficient in an economic or political sense? Such a question is beyond the scope of the design models considered in this paper. Currently-available design models are most appropriate in centrally-planned economies because of their abstract formulation. The whole issue of reconciling tools and plans in a non-centrally controlled economy is a relatively untouched area of analytical research.

3. OPTIMAL LAND-USE MODELS

A systematic review of some basic design models is undertaken in this section. The models are arranged in order of complexity to show how they have developed over time. In each case, the primal and, generally, the dual model are presented. The structure of the design model is discussed and related to the major issues raised in Section 1.

3.1 The Schlager Land-Use Model

One of the earliest gross-scale design models is that of Schlager (1965). Let us begin consideration of this model by noting his definition of a land-use allocation, x_{az}. Subsequently, his constraints and objective function are introduced. Finally, the dual to his problem is presented and interpreted.

The allocation of an amount of land, x_{az}, to use 'a' in zone 'z' of a region is a gross concept. It included allocations to complementary uses. For example, the allocation for residential land includes the land required for streets, neighbourhood shopping, schools, and local parks. Thus land-use activities are broadly-defined classes in this model[17]. Further, the allocation of land to transportation is not determined endogenously in the model.

[17] In fact, Schlager describes only eight land-use activities in his Waukesha (Wisc.) study of which five represent residential uses at different densities.

In each zone 'z' there is an amount of developable land, S_z. This amount is net of the parts of the zone which are unsuitable for development because of slope conditions, soil type, and drainage patterns. Thus the total amount of land allocated to all uses must be less than or equal to this amount.

$$\sum_{a=1}^{A} x_{az} \leq S_z \qquad z = 1,2,\ldots,Z \quad . \tag{4.a}$$

A second set of constraints are posed by planning design standards. Schlager argues that planning standards may be represented as minimum or maximum constraints on the ratio of any pair of land uses in the same or different zones. Thus, we might have

$$x_{az} - \ell_{ab}^{zy} x_{by} \geq 0 \begin{cases} y,z \in (1,2,\ldots,Z) \\ \\ a,b \in (1,2,\ldots,A) \end{cases} \tag{4.b}$$

where

$$\ell_{ab}^{zy} \, \ell_{ba}^{yz} \leq 1$$

is assumed for consistency.
In the above, ℓ represents a minimum constraint on the ratio of x_{az} to x_{by}. Such constraints need (or may) not be defined for all combinations of zone and land use pairings. In these cases, ℓ might be thought to take on a zero value.

The third and final set of constraints relate to the demand for land by each activity. Schlager assumes that the aggregate level or demand for land, D_a, by each activity 'a' is known[18].

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[18] Schlager discusses the use of a regional economic simulation model to forecast these values.

There is thus a constraint on the minimum amount of land which can be allocated to any use 'a' throughout the region[19].

$$\sum_{z=1}^{Z} x_{az} \geq D_a \qquad a = 1,2,\ldots,A \quad . \tag{4.c}$$

Finally, Schlager assumes a cost-minimizing welfare function. He defines c_{az} to be the cost of establishing one area unit of land-use 'a' in zone 'z'. This includes the cost of raw land, the cost of servicing the land with public infrastructure, and the cost of building construction to house the activity. In a market economy, this cost thus includes private and public development expenses.

The unit cost, c_{az}, is assumed to be fixed and invariant with respect to the land use pattern itself. Three observations may be made on this. First, in a fine-scale design model, c_{az} would not usually be fixed. As noted earlier, the cost of physical infrastructure such as water, sewer, and power systems tends to be sensitive to the particular spatial arrangement of activities. Secondly, c_{az} does not measure the whole social cost of development since it ignores the costs imposed by externalities. These are also sensitive to the particular land-use pattern selected. Thirdly, any effect on the aggregate level of demand, D_a, by use 'a' arising from the costs of putting it in different zones is ignored. However, land-uses of type 'a' may be very reluctant to enter the development region at all if they are restricted to zones in which the development cost appears to be unreasonably high.

However, given these fixed unit development costs, the Schlager model welfare function can now be expressed as a

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[19]Schlager uses an equality constraint for (4.c). However, provided that D_a/D_b is greater than ℓ_{ab}^{zy} for any 'y' and 'z', the optimal solutions in either case will be equivalent. The present version has the advantage of yielding a more easily interpreted dual.

linear function of the land use allocations.

$$\pi = \sum_{a=1}^{A} \sum_{z=1}^{Z} c_{az} \, x_{az} \qquad (4.d)$$

A linear programming problem is thus formed in which (4.d) is minimized subject to (4.a), (4.b), (4.c) and a non-negativity constraint.

$$x_{az} \geq 0 \qquad \forall a,z \qquad (4.e)$$

This model leaves several things unsaid about the issues raised in Section 2. There is, for instance, no explicit treatment of the transportation network although some account may be made of spatial separation. Given the location of major public utility facilities such as water, sewage, and power plants, there may be substantial spatial patterns implicit in the development cost parameters, c_{az}. Other issues ignored in the model are uncertainty and dynamic optimization. The model's solution is only optimal in the time frame of a known long-run mature state. Finally, the model ignores private sector behaviour and policy tools.

The dual solution is derived most transparently by considering first a simplified Schlager model without any planning standards. Let r_z be the shadow price of land in zone 'z' and let v_a be the shadow price for activity 'a'. Then, the dual to the problem of minimizing (4.d) subject to (4.a), (4.c), and (4.e) is the following.

$$\text{minimize:} \quad \sum_{z=1}^{Z} S_z r_z - \sum_{a=1}^{A} D_a v_a \qquad (5.a)$$

Subject to: $r_z \geq v_a - c_{az}$ $\quad \forall a,z$ (5.b)

$r_z \geq 0$ $\quad z = 1,2,\ldots,Z$ (5.c)

$v_a \geq 0$ $\quad a = 1,2,\ldots,A$ (5.d)

An interpretation of the dual to this simplified model is quite straightforward. The term v_a is the marginal cost of accommodating the last areal unit of land use 'a' in the region. Thus $v_a - c_{az}$ is the marginal reduction in development cost if the last areal unit of use 'a' had been allocated to zone 'z' instead. Now, it is noted that, from (5.c), r_z cannot be negative while, from (5.a), it must be as small as possible. Therefore, it is now seen from (5.b) that the shadow price of land in zone 'z', r_z, is zero unless $v_z > c_{az}$ for some 'a'. Of course, $v_z \leq c_{az}$ for any zone with unused developable land. The expression $v_a - c_{az}$ is positive only where the constraint (4.a) is optimally binding. In that case, r_z is the maximum that total development costs could be reduced by having one additional areal unit of land in zone 'z'.

The dual to the full Schlager model, including (4.b), can now be seen. Each constraint of the form (4.b) has an associated, non-negative shadow price w_{ab}^{zy} whose indices correspond to the constraint involving ℓ_{ab}^{zy}. By the Complementary Slackness Theorem, w_{ab}^{zy} is zero unless the associated constraint is binding. The dual to the full problem is identical to that for the simplified problem except that (5.b) is replaced by the following[20].

$$r_z \geq V_a - c_{az} + \sum_{b=1}^{A} \sum_{y=1}^{Z} w_{ab}^{zy} - \sum_{b=1}^{A} \sum_{y=1}^{Z} \ell_{ba}^{yz} w_{ba}^{yz} \qquad (5.b)^1$$

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[20] Formally, a non-negative condition should also be added.

$$w_{ab}^{zy} \geq 0 \qquad (5.e)$$

where $w_{aa}^{zz} = 0$

by definition. The first double summation term is the cost reduction from placing the marginal unit of 'a' in zone 'z' that arises from a better land-use allocation to meet the constraint (4.b). In other words, by being able to place that marginal unit of 'a' at 'z', some other unit of 'a' previously put at 'z' merely to satisfy (4.b) may be 'freed up' to relocate more efficiently elsewhere. The second double summation term in $(5.b)^1$ represents the incremental costs of relocating other land-uses necessitated via (4.b) by the placing of that marginal unit of 'a' at 'z'. Thus, the entire right-hand expression in $(5.b)^1$ is the net cost reduction in locating a marginal areal unit of 'a' at 'z' taking into account the costs imposed by the necessarily altered location of other uses.

The dual to the simple Schlager problem suggests the equivalence there of centralized and market decision-making. Suppose that, instead of a Schlager-like central planner, we have a perfectly competitive land market with many small land users of each of the A types. Suppose further that each land-user makes a bid for every site in which he is interested and that land in each zone is allocated to the highest bidder. Suppose further that, as a result of this market process, D_a units of land in the region are allocated to land-users of type 'a'. The bid rent by use 'a' for land in zone 'z' is the opportunity cost of locating a marginal unit of 'a' anywhere in the region less the cost of locating it in 'z'. This opportunity cost is v_a and the condition that land be allocated to the highest bidder is merely (5.b) Thus, the shadow price on land in the simple Schlager planning model is merely the Recardian rent in a competitive model.

The equivalence of centralized and market decision-making in the full Schlager model is somewhat more difficult to establish. The problem arises because the design standards impose externalities through locational interdependencies among land-users in the competitive market analogy. One way in which these externalities can be internalized in the market is through a system of transfer payments between land-users. In the simplest case, it is usually assumed that there are no bargaining or transactions costs in effecting this system of payments. Finally, assume that the legal onus is on each user to ensure that all design standards are met[21].

The marginal user of type 'a' seeking to locate in 'z' must now take account of two new elements in deciding his bid offer. First, he must induce whatever other new land-uses are required by (4.b) to locate appropriately. The opportunity cost to these other users is $\sum_{b=1}^{A} \sum_{y=1}^{Z} \ell_{ba}^{yz} w_{ba}^{yz}$, which would have to be borne by him. Secondly, his locational decision may benefit other users by helping them to satisfy their design standards requirements. If so, he can, in principle, extract from them a transfer payment equal to the opportunity cost of satisfying their requirements which is $\sum_{b=1}^{A} \sum_{y=1}^{Z} w_{ab}^{zy}$. Thus, the bid rent of a marginal user of type a at z is adjusted by the difference between these

$$V_a - c_{az} + \sum_{b=1}^{A} \sum_{y=1}^{Z} w_{ab}^{zy} - \sum_{b=1}^{A} \sum_{y=1}^{Z} w_{ba}^{yz} \ell_{ba}^{yz} \qquad (6)$$

Now, it can be seen that (5.b)[1] is merely the generalization of (6) when the market rent for land is equal to the highest bid by any use. Thus, a competitive market solution generates no

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[21]Note the well-known hypothesis of Coase (1958) on the irrelevance of liability assignment for market resource allocation.

market mis-allocation but only when a perfect system of transfer payments exists[22].

3.2 The Transportation Model

A major criticism of the Schlager model is that it ignores the behavioural activity patterns created by a given land-use plan. Several models have been developed to consider, in particular, the transportation flows and costs generated by a land-use pattern. In fact, the Transportation Linear Program considered in this Section emphasizes only the transportation cost aspect of development. Subsequent models attempt to integrate transportation and other development costs.

The Transportation Model can be viewed at two different levels[23]. At one level, it can be viewed as a problem in centralized planning. Suppose that conditions on the spatial arrays of production and demand points for a commodity, the supply capacity or demand requirement at each point, the unit production cost at each supply point, and the cost of shipment from each supply to each demand point are given. The model's solution indicates the minimum cost of producing and shipping the commodity under these conditions. In a centralized economy, the model can thus be used to direct the production level and output assignment of each factory or production point. In a

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[22] It is interesting to speculate on the efficiency of design standards. A common reason for imposing these standards is to reflect the external costs or benefits created by land-use at a given site. Suppose that e_{ab} is the measurable external cost on use b of the adjacent location of a unit of use 'a'. This could be compared with each $w_{ba}^{yz} \cdot \ell_{ba}^{yz}$. Suppose this shadow price product is greater than e_{ab} for some adjacent zones 'y' and 'z'. It indicates that the development cost savings by marginally slackening the constraint for that pair of zones is greater than the increase in externality costs. Thus, information on the actual externality costs can be used iteratively in conjunction with the design standard shadow prices to modify the nature of the standards themselves.

[23] This is the common Linear Programming, often attributed to Koopmans, as described, for instance, in Scott (1971, pp. 60-62).

less centralized economy the model can be viewed at another level as a prediction of the outputs and transportation flows which would occur in a competitive economy. In this latter view, the Transportation Model simulates one behavioural response of land-uses to a given locational pattern; the resulting transportation flows.

Let us now define the Transportation Model more specifically. For simplicity, assume that only one commodity is produced in the region. The model is easily extended to multiple commodities. Let x_{ij} be the flow of this commodity from source 'i' to demand point 'j'.[24] The source point may, for example, be a factory, a residential area, a warehouse, or a railway. The demand point may be another factory, a store, a warehouse, or a shipping facility. Suppose, further, that at the point 'i', this commodity has a unit f.o.b. cost of c_i which is fixed. Here, c_i consists of production and overhead costs including site, structure, and infrastructure charges. In addition, there are fixed unit costs of shipping from 'i' to 'j' of t_{ij}. If there are M demand points and N supply points, the total cost of producing and shipping the commodity within the region is the following

$$\pi = \sum_{i=1}^{M} \sum_{j=1}^{N} (c_i + t_{ij})x_{ij} \tag{7.a}$$

Note that the supply and demand points must be given even though this is what the planner seeks to find.

The planner minimizes π, a single-objective welfare function, subject to certain constraints. One set of constraints asserts that the supply capacity, S_i, of site 'i' not be exceeded. Another set asserts that the exogenously-given demand requirements, D_j, of site 'j' be met. Finally, the flows of the commodity are assumed to be non-negative

[24] A ubiquitous single product is presumed for simplicity. The analysis is easily extended to include multiple commodity flows.

$$\sum_{j=1}^{N} x_{ij} \leq S_i \qquad i = 1,2,\ldots,M \tag{7.b}$$

$$\sum_{i=1}^{M} x_{ij} \geq D_j \qquad j = 1,2,\ldots,N \tag{7.c}$$

$$x_{ij} \geq 0 \qquad \forall i,j \tag{7.d}$$

In delineating this model, several issues have remained untouched. The levels of each activity, D_j, are assumed to be given and are not generated within the model. In addition, no aspect of uncertainty is considered. There is no treatment of the transportation network other than its representation as a fixed cost, unlimited-flow system. Also, there is little treatment of the private sector and none of policy tools. The generation of commodity flows in this model can be viewed at one level as a behavioural aspect but this is the only step taken in this direction.

The dual to this problem has an interesting interpretation. Let r_i be the shadow price on the supply constriants and v_j be the shadow price on the demand constraints. The dual takes the following well-known form.

$$\text{Minimize:} \quad \sum_{i=1}^{M} S_i r_i - \sum_{j=1}^{N} D_j v_j \tag{8.a}$$

$$\text{Subject to:} \quad r_i \geq v_j - (c_i + t_{ij}) \qquad \forall i,j \tag{8.b}$$

$$r_i \geq 0 \qquad i = 1,2,\ldots,M \tag{8.c}$$

$$v_j \geq 0 \qquad j = 1,2,\ldots,N \tag{8.d}$$

This dual suggests that the shadow price on site 'j' is the larger of zero or the largest difference between the opportunity cost of supplying any site 'j' with a marginal unit of the commodity less the cost of production and shipping from 'i'. Note that r_i is the rent on the amount of capacity needed to produce one unit of output per unit of time. This can be translated into a rent per unit land area per unit time (comparable to the rents in the Schlager dual) if there is a well-defined relationship between output capacity and land input.

As in the case of the simple Schlager Model, the Ricardian land rents emerging in a competitive land market correspond to the shadow price on a production point in the Transportation Model. In a competitive market, a potential profit of $v_j - c_i - t_{ij}$ is realized in suppling the marginal unit of the commodity to 'j' from 'i'. Through competition for land at site 'j', the rent at that site will rise to absorb this potential profit. If, for instance, the land requirement for production is in a fixed ratio of 'ℓ' to the per-period production level, the bid rent per unit land at 'i' will rise to $(v_j - c_i - t_{ij})/\ell$. The market rent established at site 'i' will either be zero or the maximum $v_j - c_i - t_{ij}$ offered by suppliers to any point 'j'. Thus, there is no misallocation of land in a competitive market model equivalent to the Transportation Model.

There are two ways in which the Transportation Model might be used in land-use planning. The first is in the case where we begin with an effectively empty region to be developed. In this case, the problem is to systematically vary the D_i and S_j terms as well as x_{ij} to minimize π. A usual consequence of such an extension is that constraints must be placed on the amount of development at any site 'i' or 'j'. A model based on these considerations is the Koopmans-Beckman model which is discussed below.

The second case is to assume that the study region has some substantial amount of development already and that the planner's task is to allow for increments to this pattern. In this case, the model (7), its dual (8), and sensitivity analysis can be used to provide direct answers. The shadow prices on demand requirements and supply capacities, when translated into rents per unit land provide direct indicators as to the optimal location of new supply or demand facilities. Harris (1973) describes the application of this approach to the projection of regional economic activities in U.S. counties. However, this approach is equally feasible for the smaller zonal areas usually conceived of in a gross-scale design model.

The emphasis in this kind of model is on sequential optimization. Development can be thought to occur in a set of phases during each of which the optimized increments to total land-use are small relative to the land-uses already in place. There exists, of course, the possibility that this sequential, incrementally optimized solution will be inferior to a mature state optimizing solution. Nevertheless, this model can be used in a crude way to develop a dynamic sequence of static solutions each of which are optimal in a myopic sense. This is one step closer to the dynamic optimization solution than the earlier Schlager model.

3.3 The Koopmans-Beckmann Model

Koopmans and Beckmann (1957) have considered a design model in which the profitability of a development plan is maximized. The instrument variable in their model, x_{ki}, is now the assignmentof a plant of type 'k' to site 'i'. This variable takes on a value of one if an assignment is made and zero if it is not. For each possible assignment there is a "semi net revenue", c_{ki}, which is the annual gross revenues earned at site 'i' earned by 'k' less the annual cost of primary inputs (utilities, structures, equipment, and labour for example). In addition each factory 'k' ships a fixed weight, x_{ks}^{ij}, of its output to factory 's' at 'j'[25]. Further, for simplicity, it is assumed that all of a factory's output is sold to other factories. The unit weight cost of shipment from site 'i' to site 'j' is t_{ij}. Thus, the aggregate profitability of an assignment of factories to sites is the aggregated semi-net revenue less the aggregated transport flows.

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[25] It is assumed that there are Z sites and plants.

$$\pi = \sum_{k=1}^{Z} \sum_{i=1}^{Z} c_{ki}x_{ki} - \sum_{k=1}^{Z} \sum_{s=1}^{Z} \sum_{i=1}^{Z} \sum_{j=1}^{Z} t_{ij}\, x_{ks}^{ij} \qquad (9.a)$$

In the Koopmans-Beckmann Model, this aggregate profitability is the only objective in the welfare function. It is maximized subject to certain constraints of which the first is that for each site 'i' and any pair of factories 'k' and 's', a physical trade balance must hold. If n_{ks} is the required weight flow of output from plant 'k' to plant 's', this requires

$$n_{ks}\, x_{ki} + \sum_{j=1}^{Z} x_{ks}^{ji} = n_{ks}X_{si} + \sum_{j=1}^{Z} x_{ks}^{ij} \qquad \forall\ k,s,i \qquad (9.b)$$

This condition asserts that the amount of 'k' consumed by 's' at 'i' ($n_{ks}x_{si}$) must be equal to the amount of 'k' produced at 'i' and destined for 's' ($n_{ks}x_{ki}$) plus the difference between the inflow of 'k' destined for 's' ($\sum_{j=1}^{Z} x_{ks}^{ji}$) and the outflow ($\sum_{j=1}^{Z} x_{ks}^{ij}$). The other conditions assert that one factory of each type is placed within the region, that one plant be allocated to a site, that x_{ki} be a non-negative integer, and that there be no intra-site flows.

$$\sum_{i=1}^{Z} x_{ki} = 1 \qquad k = 1,2,\ldots,Z \qquad (9.c)$$

$$\sum_{k=1}^{Z} x_{ki} = 1 \qquad i = 1,2,\ldots,Z \qquad (9.d)$$

$$x_{kk}^{ij} = 0 \qquad \forall\ k,i,j \qquad (9.e)$$

$$x_{ki} \in I \qquad I = (0,1) \qquad (9.f)$$

Several comments are in order on this model. Koopmans and Bechmann did not create this model for design purposes. Rather, they sought to assert the principle that a market economy could not sustain an optimal allocation of factories to sites because of the externality effects created by intermediate good flows between plants. This issue is considered below. The model they use is therefore quite abstract and unwieldy. Residential land-use, for instance, is very difficult to handle in this model unless one treats it as an activity to be allocated to one site only. As in the earlier models, any notion of policy tools is ignored. Further, like the earlier Transportation Model, the present one ignores all aspects of the transportation network except the shipment costs. Also, it ignores all kinds of locational interdependencies other than commodity flows even to the extent of completely ignoring residential location and job commuting. Finally, as in the Schlager Model, the present one optimizes only for a known mature state and ignores both dynamic issues and uncertainty.

In principle, this model looks promising as a starting point for new design models. Its emphasis on profitability maximization instead of cost minimization suggests that one can introduce the effect of design on the level as well as the layout of land-use within the region. To undertake this, it is necessary to release the constraint (9.c) that all plants be located in the region. Further, it is necessary to specify import points and costs so that each plant can have a choice between purchasing from inside or outside the region[26]. With these amendments, it should be possible to construct a design model in which the level and layout of each land-use is endogenously determined.

It is possible to generate a dual program to this problem. If we delete the integer constraint (9.f.) and treat the remaining constraints as suitably-defined inequalities, the dual is [27]

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[26]Nijkamp (1972), especially Chapter 3, discusses some non-spatial models of industrial complexes which emphasize the inclusion or exclusion of plants. These emphasize the price difference between intermediate goods produced outside and inside the complex.

[27]Heffley (1972, pp. 1158-1161).

Minimize: $\sum_{i=1}^{Z} v_i + \sum_{j=1}^{Z} r_j$ (10.a)

Subject to: $U_{ks}^{j} - U_{ks}^{i} \leq t_{ij}$ $\forall k,s,i,j$ (10.b)

$r_i \geq c_{ki} - v_k + \sum_{s=1}^{Z} N_{ks}U_{ks}^{i} - \sum_{s=1}^{Z} N_{sk}U_{sk}^{i}$ $\forall k,i$ (10.c)

$r_i \geq 0$ $i = 1,2,\ldots,Z$ (10.d)

$v_k \geq 0$ $k = 1,2,\ldots,Z$ (10.e)

$U_{ks}^{i} \geq 0$ $\forall k,s,i$ (10.f)

where r_i is the shadow price on site 'i', v_k is the marginal profitability of plant 'k', and U_{ks}^{i} is the opportunity cost of supplying plant k's output to plant 's' at site 'i'. Condition (10.b) asserts that the difference in this latter shadow price between two locations can not exceed the cost of shipment. The value of site 'i' to plant 'k' is the semi net revenue (c_{ki}) at site 'i' less its best alternative marginal profitability (v_k) plus the marginal opportunity cost savings in supplying other plants from 'i' ($\sum_{s=1}^{A} N_{ks}U_{ks}^{i}$) less the marginal opportunity cost increment in supplying plant 'k' at 'i' ($\sum_{s=1}^{A} N_{sk}U_{sk}^{i}$). Thus, the dual indicates that the shadow price on site 'i' is merely the highest value placed on it by any plant or zero (if all the plant valuations are non-positive).

Does this dual suggest anything about the equivalence of centralized and market land allocations? This has become a confused point in the literature because there are really two questions. First, does the planning model dual bear a similarity to market decision behaviour? Secondly, does the dual in (10) in fact correspond to the primal problem (9)?

At first glance, the answer to the first question would seem to be affirmative. The shadow price of a site in (10.c) is very similar to conditions found earlier in the Schlager and Transportation models. The difficulty here in carrying through the analogy to the equivalence of market behaviour lies in the definition of the problem. In the present case, there is only one plant of each type. Unless there is a very large number of plants, there is no reason to expect competitive behaviour in the land market. Further, the maximum bid for a site need never be r_i as defined in (10.c). The plant occupying the site need bid only marginally more than the next highest offer by another plant. Thus, although the rent bid for site 'i' will not exceed r_i, it may be somewhat less. Thus, because of the incongruence between the definition of the Koopmans-Beckmann problem and the notion of perfect competition, it may not be possible to think of a competitive market analogy.

Koompans and Beckmann avoid this first question by re-phrasing it to ask if the planning model solution is price sustainable. They ask if there exists a set of land prices, one price per site, which would induce each plant to remain at its optimal location. This avoids the question of whether a market allocation process could find that set of prices. By rephrasing the first question in this way however, the answer to it becomes less meaningful for our purposes.

The second question is the one which has dominated the literature on this model. There has been no satisfactory answer to this question yet. Koopmans and Beckmann used a very roundabout and somewhat weak argument to conclude not only that the dual (10) does not correspond to the primal (9), when this includes integer constraint (9.f), but also that this implies that optimal location sustainable land prices do not exist. They begin by assuming that the semi net revenues for plant 'k' at all sites are equivalent

$$c_{ki} = c_k \quad \forall_{i,k} \tag{11}$$

They argue that the only solution to the Linear Programming Problem (9), omitting the integer constraint (9.f) and including (11), is always a fractional assignment of factories to sites. The dual solution (10), with its analogy to location-sustaining rents, does not hold for an integer solution to the primal. Since the dual is invalid, they assert that the integer primal has no dual constraints with optimal location-sustaining land prices[28]. The basis for this last assertion remains unclear.

Two more recent researchers have attempted to qualify these conclusions. Heffley (1972) shows that a necessary condition for an integer solution to (9) omitting (9.f) is that the semi-net revenues be spatially variant. In other words, assumption (11) necessarily leads to fractional assignments and this can not be generalized to the case when (11) is dropped. Hartwick (1974) presents some counter-examples to the argument of Koopmans and Beckmann. He presents some examples where, even given (11), an integer solution may have location-sustaining land prices although the linear programming dual is not applicable. Both of these researchers fail to set out the general conditions under which an optimal integer solution with location-sustaining land prices exists. However, their examples suggest the seriousness of the flaws in the Koopmans-Beckmann argument.

3.4 The TOPAZ Model

Another design model which considers the transportation costs associated with a land use pattern is the TOPAZ Model[29]. This model has been used for both gross and fine scale planning problems. An extension of the Schlager Model it includes a behavioural model of transportation flows. The simple Gravity Model from transportation planning is used to estimate these flows.

The model has a welfare function with a single objective; to minimize the sum of development costs and transportation costs.

Minimize: $$\pi = \sum_{a=1}^{A} \sum_{z=1}^{Z} c_{az} x_{az} + \sum_{a=1}^{A} \sum_{b=1}^{A} \sum_{y=1}^{Z} \sum_{z=1}^{Z} t_{ab}^{zy} (x_{az} + e_{az}) (x_{by} + e_{by}) \quad (12)$$

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[28] ...any solution of the quadratic assignment problem...being by definition an integral assignment is not a solution of the linear problem...hence does not have associated with it a price system meeting the conditions (of the dual (10)). Koopmans and Beckmann (1957, page 69).

[29] TOPAZ is an acronym for "Technique for the Optimal Placement of Activities in Zones".

Here, c_{az} is defined as in the Schlager Model. The constant e_{az} is the existing amount of land-use of type 'a' in zone 'z'. It contrasts with the amount to be added (x_{az}). The term t_{ab}^{zy} is a composite constant reflecting the effect of the intensities of activity 'a' at 'z' and 'b' at 'y' on the transportation cost of the resulting flow between them. A derivation of t_{ab}^{zy} is presented in Appendix A. The constraints in the TOPAZ Model are the familiar ones from the simple Schlager model.

$$\sum_{a=1}^{A} x_{az} \leq S_z \qquad z = 1,2,\ldots,Z \tag{13.a}$$

$$\sum_{z=1}^{Z} x_{az} \geq D_a \qquad a = 1,2,\ldots,A \tag{13.b}$$

$$x_{az} \geq 0 \qquad \forall\ a,z \tag{13.c}$$

Several additional comments can be made with regard to the issues raised in this paper. The model does not make the level of a land-use activity endogenous to the model. Further, uncertainty is not considered at all. Also like earlier models, it does not treat the transportation sector either endogenously or in much detail. Further it ignores locational interdependencies other than those created by transportation flows. Perhaps more obviously than with the Koopmans-Beckmann formulation, the present model determines transportation flows using a behavioural hypothesis. It thus does represent a first attempt to introduce some of the behavioural consequences of a land-use pattern into the design problem.

This model shares a feature with the Transportation Model in that it may be used to assign land-uses incrementally over time. Since the model allows, in e_{az}, for an existing set of land-uses, D_a and S_z represent the new land-uses to be allocated and the existing undeveloped or redevelopable land respectively. Thus, the model can be used recursively to allocate a temporal sequence of developments to then-available sites. As in the Transportation Models, these increments must be specified exogenously. This model shares another feature in that its optimal assignments are myopic: it optimizes for each time period ignoring subsequent development.

This is also the first nonlinear programming model considered. To solve it, one can not rely on the Simplex Algorithm with its optimal solution properties. Dickey and Najafi (1973) present an empirical application of this model together with an algorithm to solve it. Their algorithm will find a local optimal point but does not necessarily find the best solution. In general, such algorithms require a good initial guess as to the solution.

A dual problem to the TOPAZ Model can also be found. However, since TOPAZ is nonlinear, the dual is somewhat more difficult to work with. Following the approach of Balinski and Baumol (1968), the dual to (12) subject to (13) is the following.

Minimize:

$$- \sum_{a=1}^{A} v_a D_a + \sum_{z=1}^{Z} r_z S_z + \sum_{a=1}^{A} \sum_{b=1}^{A} \sum_{y=1}^{Z} \sum_{z=1}^{Z} (t_{ba}^{yz} x_{az} - t_{ab}^{zy} e_{az})(x_{by} + e_{by}) \qquad (14.a)$$

Subject to:

$$r_z \geq v_a - c_{az} - \sum_{b=1}^{A} \sum_{y=1}^{Z} (t_{ab} + t_{ba})(x_{by} + e_{by}) \qquad (14.b)$$

$$v_a \geq 0 \qquad a = 1,2,\ldots,A \qquad (14.c)$$

$$r_z \geq 0 \qquad z = 1,2,\ldots,Z \qquad (14.d)$$

A difficulty with this dual is that it requires pre-knowledge of the solution to the primal. Condition (14.b) nevertheless can be interpreted. The final double summation term is the increase in transportation costs on flows both into and out of zone 'z' with a marginal change there in the amount of land use 'a'. This term plus c_{az} is thus the total marginal cost of so allocating a unit of 'a'. Thus, (14.b) asserts that the shadow price on land is at least as large as the difference between the opportunity cost of location for activity 'a' and its location in zone 'z'.

Is there a direct analogy with the market allocation of land? As in the case of the full Schlager model with its design standards constraints, the ability of a competitive system to replicate the planning solution's shadow price for land depends on whether an extensive but efficient system of payments can be set up so that one land use can induce another to take its transportation cost interests into account. In the absence of such a system, there is no possibility that an equivalent market price for land could be established.

3.5 The Ripper-Varaiya Model

There has, in recent years, emerged a group of design models of a degree more complex than any of those discussed to this point. These have the models of Mills (1972), Ripper and Varaiya (1974), and Hartwick and Hartwick (1974 and 1975). Since these all are very similar, only the Ripper-Varaiya model is considered here.

It is not intended here to reproduce the full Ripper-Varaiya model because of its intricate structure. Rather, a sketch is made which indicates the new contributions of the model. It is first necessary to define the geography of the region. A uniform undeveloped plane is assumed on which is overlaid a square grid of zones of an arbitrary size. One zone, labelled 'O' is preselected as a central export zone for the city and might include, for instance, a rail depot. All other zones are identified according to their manhatten distance from O. As shown below, all zones 'u' units distance away from O are labelled 'u' and there are 4u of these otherwise-homogeneous zones.

				4				
			4	3	4			
		4	3	2	3	4		
	4	3	2	1	2	3	4	
4	3	2	1	O	1	2	3	4
	4	3	2	1	2	3	4	
		4	3	2	3	4		
			4	3	4			
				4				

For any zone, $x_{rs}(u)$ is the planner's instrument variable. It represents the output level of industry 'r' at distance 'u' using activity technology 's'. There are c + d + 1 industries; 'c' centralized industries which can locate only at 'O', 'd' decentralized industries which can locate anywhere else, and the housing service industry which can also locate anywhere where $u \geq 1$. The technology 's' can be thought of as the number of stories in the plant or housing thus allowing the planners to specify production technologies in a three-dimensional sense.

The planner can specify exogenously two important sets of limits. First, he can specify $\bar{s}(r)$ which is the maximum number of stories permitted (or economically feasible) for industry 'r'.[30] Secondly, he can specify $\bar{u}$ which is the maximum outer radius permitted for the city. The planner may, of course, set these arbitrarily high to permit unconstrained solutions.

As in earlier models, the present one assumes that each industry in the region faces an exogenous minimum condition. Here, however, it is not the land requirement of each industry which is fixed as in earlier models, but the industry's regional export level, D_r. The industry's required output can be satisfied by two means. The first is through regional imports, M_r, whose level is determined endogenously in the model. The second is through production within the region itself. Such production processes are assumed to use four kinds of inputs; intermediate goods purchased from the c + d centralized and decentralized industries, labour (ℓ), land (t), and capital (m).[31] Let a_{vrs} be the input requirement from sector v required for the production of one unit of output in sector 'r' within an s-storey production process. The export requirement thus states that regional production by industry 'r' plus imports less intermediate demands by other industries must be at least as large as the export requirement.

[30] Ripper and Varaiya consider $\bar{s}$ to be fixed for all 'r'. However, there is no apparent reason why a more general formulation could not be used.

[31] Each unit of labour has an inelastic unit demand for housing. Therefore, the supply of housing services and the labour supply can be equated.

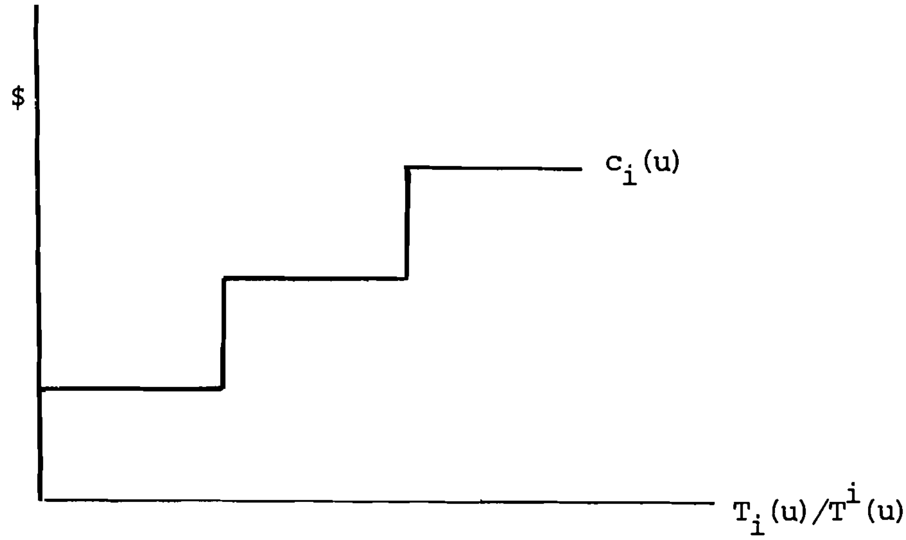
$$\sum_{s=1}^{\bar{s}(r)} \sum_{u=1}^{\bar{u}} 4ux_{rs}(u) + \sum_{s=1}^{\bar{s}(r)} x_{rs}(O) + M_r$$

$$- \sum_{v=1}^{i} \sum_{s=1}^{\bar{s}(r)} \sum_{u=1}^{\bar{u}} 4ua_{vrs}x_{vs} - \sum_{v=1}^{i} \sum_{s=1}^{\bar{s}(r)} a_{vrs}x_s(O) \geq D_r \qquad \text{or} = 1,2,\ldots,i \tag{15.a}$$

where $i = c + d + 1$

One important implication of this constraint is that no production need necessarily take place within the region. A region's export requirements could be met strictly by imports. In fact production will take place only if the social cost of production in the region is lower than the price of import replacements. This model is thus an important extension because it makes the level of activity as well as its layout endogenously-determined.

The notion of traffic also plays an important role. Traffic arises from two sources. First, all exported goods are assumed to be shipped either from the O zone or from the periphery $\bar{u}$. Thus, production destined for export must be transported to one of these two places. Secondly, intermediate goods flows exist between producers or between industries and households. These two kinds of flows, aggregated over industries by flow characteristics such as weight, generate traffic levels in two directions for any zone. Let us denote the traffic at 'u' directed towards 'O' and the periphery as $T_1(u)$ and $T_2(u)$ respectively. Further, imagine a transportation sector which produces traffic capacities of $T^1(u)$ and $T^2(u)$ in the same directions using a Leontief production function with land and capital. Finally, the marginal cost of transportation, $c_i(u)$, to reflect congestion, is assumed to be an increasing step function of the ratio of traffic to capacity in direction 'i' as the following example displays. The model seeks a balance between the increasing costs of congestion and the cost of land and capital in deciding how much land should be allocated to transportation.



To complete the model, it is assumed that all inputs and imports are available at fixed prices. Imports of any of the c + d + 1 industries are permitted and these may occur via either the 'O' zone or the periphery (at assumedly two sets of different exogenously-given prices). Labour, land, and capital are the three other inputs also available at given exogenous prices.

The objective function is to minimize the daily cost of the regional development scheme. This includes the daily interest charges on all capital used by industries, by the housing sector, and by the transportation sector. It also includes the land rentals and wage charges incurred by all three sets of activities. Finally, it includes all congestion and import costs. This is minimized subject to (15.a) as well as to constraints on the amount of land available, L(u), in each zone[32].

$$\sum_{r=1}^{c+d+1} \sum_{s=1}^{\bar{s}(r)} a_{trs} x_{rs}(u) + b_t [T^1(u) + T^2(u)] \leq L(u) \qquad u = 1,2,\ldots,\bar{u} \qquad (15.b)$$

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[32] b_t is the land input required per unit of road capacity.

Ripper and Varaiya also discuss two extensions to their basic model. One is to permit multiple classes of workers with different wage levels. The other is a dynamic version of the model in which existing stocks of development are permitted. Again, a myopic optimization procedure is used in which the implications of future developments are ignored.

How does this model perform in terms of the issues raised in this paper? In terms of objective functions and constraints, it is the most ambitious one examined. By making both the level and layout of activity endogenous, it rephrases the design problem wholly in terms of the rationale for the existence of urban areas. Further, although the single objective is stated as cost minimization, the model really maximizes the efficiency or profitability of regional development. A simple way to see this is to imagine a solution in which the region imports all of its export requirements. In this case, no industry is located in the region and no labour is necessary: the region is merely an intermediate shipping point in an interregional economic system. If some industry is to be located in the region, it must be because outputs can be produced there at lower cost than imports. The lower the regional production costs, the greater is the amount of development in an optimal solution. The amount of industry in the region thus corresponds to its efficiency vis-a-vis the price of imports.

Another of the strengths of this model is that it attempts to treat certain behavioural aspects of a land use plan. The earlier models which included interdependencies among land uses displayed these mathematically as relationships between quantities of land use. The present model envisages a Leontief economy in which the connections among industries (including the household sector) are relationships between output levels. Further, unlike most Leontief models, this one allows for some amount of factor substitution because each industry can vary, to some extent, the 's' parameter in its production function.

In terms of some of the other issues raised in this paper, the model does not fare as well. Treatment of the transportation sector, for instance, is very sketchy. There is no concern for network design and only one mode of transportation is considered. Further, only an aggregate amount of land in each zone is allocated to transportation without regard to its spatial layout. This latter point would be similar for all other models considered so far except that Ripper and Varaiya try to draw out an inference regarding congestion costs. Using such temporally and spatially aggregated variables as total daily zonal traffic and capacity to get congestion costs is a very tenuous exercise. Much of this relationship depends on the network configuration and on the time pattern of the traffic flows.

The treatment of locational interdependencies in this model is similar to that in most of the other models considered. The transportation flow element is considered in detail but there are no other interdependencies considered. In addition, the treatment of dynamic optimization is of the same myopic type found in earlier models[33]. Also, uncertainty is not considered at all.

Since this is a linear programming model, it has a well-defined dual. For each constraint of the type (15.b), there exists a shadow price on land in that zone. There also exists an opportunity cost of supplying one extra unit of export for any commodity. Without specifying the dual exactly, can we speculate on whether a competitive market analogy results in an equivalent pricing of land? The presence of congestion costs alone might be sufficient to drive a wedge between the optimal and market prices. Where the government fails to properly price traffic, individual industries may not make pricing and location decisions which are socially optimal. In the absence (or efficient pricing) of congestion, there may exist a competitive market equivalent but only if there exists an extensive system of compensating transfers so that each industry takes into account the full effect of its locational choice on those other industries whose transportation costs are thus affected.

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[33] Rupper and Varaiya (1974, pp. 160-163) describe a numerical experiment showing a large divergence between an optimal static solution and incremental myopic dynamic solutions.

4. FUTURE AREAS OF RESEARCH

To conclude, five models of increasing complexity have been reviewed. The aim has been to show how these models have evolved and how they still remain frail. The class of models typified by the Ripper-Varaiya model is among the most advanced available today. Its structure is both rich in theory and realistic in its extensive detail. To operationalize this model for plan design work would be no small feat in terms of its input data requirements. Nonetheless, the simplicity of even this model might well make a verteran planner blush.

What are the short-term prospects for the development of better design models along the lines of those described above? On several methodological issues, the research out look for the near future is dim. On other issues, there has been a parallel development of models in other research areas which might usefully be incorporated into a better design model. Let us now consider each issue in turn and speculate on potential research gains in that area. The interests of both the theoretician and the applied researcher are considered.

4.1 The Welfare Function Issue

This is not foreseen as a major stumbling block in the development of design models. Using design models in the form of (3) when multiple objectives are present is felt to be an adequate resolution of that problem. While this approach restricts the notion of optimality rather severely, it has some advantages. In the real world of planning, it would permit the planner to maintain some secrecy about his values and tradeoffs among goals. Further, the interactive nature of problem-solving using (3) allows the planner to come to grips with his own subjective preferences. Finally, the problem of public choice in a pluralistic society is better handled in (3) where the b_i's can be varied to suit the values of each interest group. Although research into multi-objective welfare functions is of interest and provides an alternative approach, the welfare function does not pose a critical methodological problem in design models.

4.2 The Uncertainty Issue

The methodological issue raised by uncertainty is much more critical. The models reviewed have been unanimous in their disregard of this important problem. One is left feeling that this whole class of models, while of theoretical interest, are completely inappropriate as applied planning models. If one seeks a robust land-use design, a completely different land-use model is required. Research into this topic should be a high priority if the gap between theoretical and applied design work is to be narrowed. However, since no models of this type exist already, one must be pessimistic about the short-term prospects of broad advances in this issue.

4.3 The Dynamic Optimization Issue

The rather slender development of the reviewed design models on this issue is somewhat misleading. At a theoretical level, the most interesting economic aspect of this issue concerns the difference between long-run optimization and incremental myopic optimization; an issue approached by Ripper and Varaiya. However, applied planners might be more interested in the sequencing and transition problems faced by a region moving towards a mature state. Here, there exists bodies of models dealing with, for instance, the dynamics of both industrial complex investment and demographic change. Although such considerations are not included in current design models, there is some hope that a design model could be constructed in the near future incorporating at least these two dynamic aspects. Undoubtedly, such a model would be quite complex and its main value would likely be at an operational rather than theoretical level.

4.4 The Layout and Level Issue

The endogenous determination of both level and layout in a land-use design model has been achieved by Ripper and Varaiya. This model is of considerable value to the economic theory of cities because it explains one basis for each city existence and growth. At the same time, applied planners might argue that its theoretical elegance is purchased at a considerable cost in terms of data requirements and detailed structural assumptions. Whether it is useful to try to further develop models along this line is an open question[34]. At a theoretical level, the Ripper-Varaiya model may be approaching the feasible limit for a pedagogical tool. At an operational level, the data requirements of the model raises questions related to the uncertainty issue.

4.5 The Transportation Network Issue

There are several sources of surveys of theory and models in network design. Some general kinds of network design problems are considered by Scott (1971). Steenbrink (1974) considers road network design problems and MacKinnon (1976) surveys models of network extension in the face of development.

In metropolitan planning, there have been many attempts to integrate land-use planning and transportation network design. Most of these models use a heuristic, iterative approach to find an 'optimal' network design given the uncontrolled response of the private sector, through land-use and traffic activity, to different network configurations. Several of these kinds of applied models are reviewed in H.J. Brown et al (1972). A recent critical review of these approaches is found in Lee (1973).

There thus appears to be some hope that the problem of integrating land-use and network design models is one which can be resolved in the short-term future. Undoubtedly, such a model would be quite complex and there is some question as to its theoretical value in that case. However, this extension is of considerable value to applied planners.

4.6 The Locational Interdependency Issue

There are several kinds of locational interdependencies which have not yet been explicitly considered in design models. One of these is air pollution. In the United States, there has been much recent effort to model the process of diffusion of air pollutants for the purpose of environmental planning. Horie (1974) indicates

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[34] For example, one extension might be to incorporate, using minimum scale thresholds for factories, a central place hierarchy within the region as in Puryear (1975).

that the Air Quality Display Model (AQDM) and the Air Quality Implementation Planning Program (IPP) are two comprehensive spatially-disaggregated models in wide use. One of these models, or perhaps some other, could be combined with a current land-use design model.

Another kind of locational interdependency is posed by communications and information flow costs. In the research in this area, much emphasis has been placed on the importance of face-to-face (otherwise referred to as 'contact') communications as a determinant of the locational choice of certain kinds of activities. Tornqvist (1970) has laid out some of the research issues in this area. However, formal models of contact processes in a spatial setting are still lacking. Much work remains to be done here before such concepts could be integrated into a design model.

A final kind of locational interdependency has to do with simple neighbourhood externalities which lead similar kinds of activities to locate near each other. Although this phenomenon is easily observable, formal models of the processes underlying it are lacking. The main research effort here has been in descriptive models of the 'Social Area' or "Ecological" variety and these do not provide very direct means of planning for optimal new development. Research is needed to develop some analytical models of neighbourhood externality effects.

Research advances in this area in the near future can thus be foreseen. Again, most of the models which are forthcoming would likely be too complex to be of theoretical value. An exception, though, might occur in the case of an integrated air pollution-design model which might be simple enough to be of much interest to theoreticians.

4.7 The Private Sector Issue

Models of household and industry locational behaviour reflect to a considerable degree the social and institutional framework within which any given society operates. In part, of course, this reflects the varying degree of central planning control exercized by a government. However, even within societies with comparable levels of central control, there may be substantial differences in the historically-accumulated institutional mechanisms for the allocation of land and other resources. For this reason, unlike the widely applicable structures of the optimal design models reviewed in this paper, behavioural models tend to be very specific to the society being examined.

What kinds of advantages are there in linking a behavioural model to a design model? The first is that most behavioural models are not suitable for generating optimal policy solutions. They merely predict locational behaviour given exogenous factors, including planning decisions. The second advantage is that, unlike the design models considered above, some behavioural models specifically include planning tools such as zoning, transportation network design (in terms of cost, speed, access to certain points, and modal mix), and public facility provisions. Thus, the problem of plan implementation, which is avoided in current design models, might be specifically considered in a combined behavioural-design approach. Thirdly, by building a behavioural model with a detailed transportation network configuration, it is possible to approach the congestion problem much more realistically than before. Thus, a combined behavioural-design model might correct many application problems with current design models.

A qualification is in order here. Many of the current behavioural models have been criticized on the grounds that they lack a theoretical structure and that they merely build on empirical regularities without questioning the basis for such regularities. Recently, Stokes (1974) heightened this attack on one kind of model (the EMPIRIC model) by showing its temporal instability in an <u>ex post</u> evaluation. Concurrently, the foundations of contenpory land-use theory, as evolved from Alonso (1964), have been attacked as being too naive. Some considerable work appears to be required in sorting out what kinds of theoretical models are most appropriate for different societies and how these might best be expressed in empirical models.

4.8 The Policy Tool Issue

A central point in this paper has been that design models have abstracted away from the issue of the tools required to achieve an optimal solution. Given the variety of societies, each with its historically-developed institutions, the range of tools and the efficiency of using different tools to achieve a plan will vary drastically.

Although the main value of putting policy tools in a design model is in the sphere of applications, some theoretical benefits would also be forthcoming. This is because, to develop the role of policy tools in shaping land-use, a set of formal theoretical models are required. Such models would trace out the response of the private sector, under different assumed conditions, to these policy tools. In a competitive market economy for instance, this would involve assessing the impact of tools on the Ricardian land rent structure. There are few of such models at present and much remains to be done in developing theoretical models of this type before more-applied design models can be created.

4.9 Concluding Assessment

The conclusion of this paper is that one would be mistaken to believe that an optimal land-use design model is just waiting to be implemented. The models reviewed here, which represent the development of English-language theory or this topic, are in much need of extension before a satisfactory theoretical or applied model can be formulated. Of all the methodological issues raised, the lack of treatment of uncertainty is potentially the most troublesome. While the other issues may be handled by an extension of existing models, resolving the uncertainty issue would seem to require an entirely new approach. Because of this, the short-term prospects for a satisfactory theoretical or applied design model are not good. Much work remains to be done.

APPENDIX A: Derivation of t_{ab}^{zy} in TOPAZ Model.

The interpretation of t_{ab}^{zy} is straightforward assuming a simple gravity model. The total number of trips generated by use 'a' at zone 'z' is hypothesized to be proportional to the amount of activity located there.

$$g_a \cdot (x_{az} + e_{az}) \tag{16.a}$$

The proportion of these trips ending at use 'b' in zone 'y' is assumed to be directly proportional to the amount of land use there and inversely related to the square of the distance between the zones, d_{zy}.

$$\frac{h_y \cdot (x_{by} + e_{by})/d_{zy}^2}{\sum_{y=1}^{Z} h_y \cdot (x_{by} + e_{by})/d_{zy}^2} \tag{16.b}$$

Thus, the total annual number of trips, T_{ab}^{zy}, from 'a' at 'z' to 'b' at 'y' is

$$T_{ab}^{zy} = \frac{g_a \cdot h_y/d_{zy}^2}{\sum_{y=1}^{Z} h_y \cdot (x_{by} + e_{by})/d_{zy}^2} (x_{az} + e_{az})(x_{by} + e_{by}) \tag{16.c}$$

where g_a and h_y are constants. Let q_{ab}^{zy} be the present value of the cost of a stream of these annual trip flows. Given that T_{ab}^{zy} is fixed over time, the present value of all trip flows is therefore

$$\sum_{a=1}^{A} \sum_{b=1}^{A} \sum_{z=1}^{Z} \sum_{y=1}^{Z} q_{ab}^{zy} \, T_{ab}^{zy} \tag{16.d}$$

$$= \sum_{a=1}^{A} \sum_{b=1}^{A} \sum_{z=1}^{Z} \sum_{y=1}^{Z} t_{ab}^{zy} (x_{az} + e_{az})(x_{by} + e_{by}) \tag{16.e}$$

where it is now seen that

$$t_{ab}^{zy} = q_{ab}^{zy} \frac{g_a \cdot h_y \ / \ d_{zy}^2}{\sum_{y=1}^{Z} h_y (x_{by} + e_{by})/d_{zy}^2} \qquad (16.f)$$

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