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SECOND BEST ENERGY POLICIES

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Abstract

The paper considers the problem of resource allocation when factor groups attempt to obtain a share of real income which is greater than what would be imputed by classical economies. A formulation stressing the Divvy nature of the problem is given both in theoretical terms and with a framework which is susceptible to empirical estimation. Policy questions resulting from the formation of OPEC are discussed and a framework for policy analysis is given.

1. Introduction

Classical economic theory argues that for an economy to achieve an efficient level of production and distribution a necessary condition is that factor inputs should be remunerated based on their marginal product. The behavioral model underlying this so-called competitive solution is a continuous supply of the factor input at different factor prices. The supply is determined for a given price with the equilibrium resulting from the mutual determination of supply and demand. Even though deviation from the competitive model will effect the overall efficiency of the economic system, there has been recently increasing efforts by input factor groups to improve their total welfare at the expense of other groups. For example, labor unions controlling a particular factor input offer the total supply at a particular wage structure. If agreement is not reached the factor is withdrawn (strike). The willingness of the employer group to accept the factor demands depends on the eventual damage that a withdrawal would cause. A prominent example of a factor group controlling a raw material input is OPEC. Again there is an attempt to increase the share of real output flows that the group can control to the detriement of the consumer groups of industrial nations.

What those examples would suggest is the need of integrating political and economic considerations in the problems of resource allocation and welfare distribution. Divvy Economy [6] would be an example of how a political bargaining process constrains the solution to the economic problem. The Divvy approach provides a very valuable conceptual framework for analysis that we think is important to pursue.

The purpose of this paper is to present a framework of analysis for problems arising from factor rewards which are influenced by non-economic elements and which differ from the reward structure assumed in a competitive economic model. Our goal is the development of a model of an economic system incorporating these phenomena and allowing for the determination of economic policy which can adapt the system to these new realities. The form of the model we present will allow for its elaboration so that it could eventually be amenable to statistical study. For this reason we develop a fairly general, though static, model of an economic system incorporating a consumer and producer sector. In the first part of the paper we discuss the input-output framework and its relationship to the general equilibrium model used in economics. Starting with a general equilibrium description incorporating a new constraint relating factor inputs to outputs, an input-output model results from the choice of a special functional form to represent the production technology. On the other hand more complex specifications of econometric production function lead to models which can be perceived as generalizations of traditional input-output analysis. The form and derivations of the extension of input-output models is described in section 2.

In section 3 we analyze the effect of one factor group's demand on the production sector of an economic system. We want to present the framework for economic policies which reduce the distortion effect of the political demands. The particular form of this factor demand is represented as an extra constraint. A new second best optimization problem is formed and the necessary optimality conditions are determined.

While a considerable amount of literature has been accumulated which concentrates on theoretical questions surrounding the theory of the Second Best, relatively little has been attempted in the way of a framework for developing concrete economic policies. By introducing a notation that allows for a disaggregated model and by relating the input-output framework to general equilibrium models we allow for the eventual formulation of specific second best policies.

The final part of the paper discusses the questions concering the practical application of the results.

2. General Equilibrium Modelling

Figure 1 shows a schematic representation of the interrelated subparts that constitute a General Equilibrium Model, GEM. The model is static in the sense that no explicit reference is made about the behavior of the economic units. Rather, they are assumed to interact in the "black boxes" representing the input resources and final goods markets. The outcomes of the behavior of the markets are a set of prices of inputs λ and outputs p for which supply and demand for inputs (r_s and r_d respectively) are equal, and supply and demand for outputs (Y_S and Y_D) are also equal. Exogenous variables are those out of control of the system and those under control of the policy maker. A is a $n \times n$ matrix of interindustry flows per unit of sector output (for sector 1 to n), and R is a $m \times n$ matrix of primary input flows. The column $[a_{\cdot j}:r_{\cdot j}]' = b_{\cdot j}$ is a vector representing the technological coefficients of sector j. Input-output models with substitution provide for various $b_{\cdot j}$ in each industry. For given values of Y_D, equation $x = (I - A)^{-1}Y_D$ gives the vector of activity levels for sectors $i = 1,\ldots,n$. x can be used to evaluate the primary inputs requirements from the equation $r = Rx$. Detail on input-output modeling and computation can be found in [10].

Traditional applications of input-output models have taken the technological coefficients as fixed, independent of the relative prices of competitive inputs. This assumption has been used in practice in the way to compute those coefficients. The coefficient a_{ij} would be the ratio between input to sector j from sector i and the total inputs of j, for some observed data values.

The purpose of this section is to show the relationships between the coefficients of A and R with the formulation of the resource allocation problem by a profit maximization model, and to evaluate a general equilibrium model that has used this result.

Suppose we have for sector j,

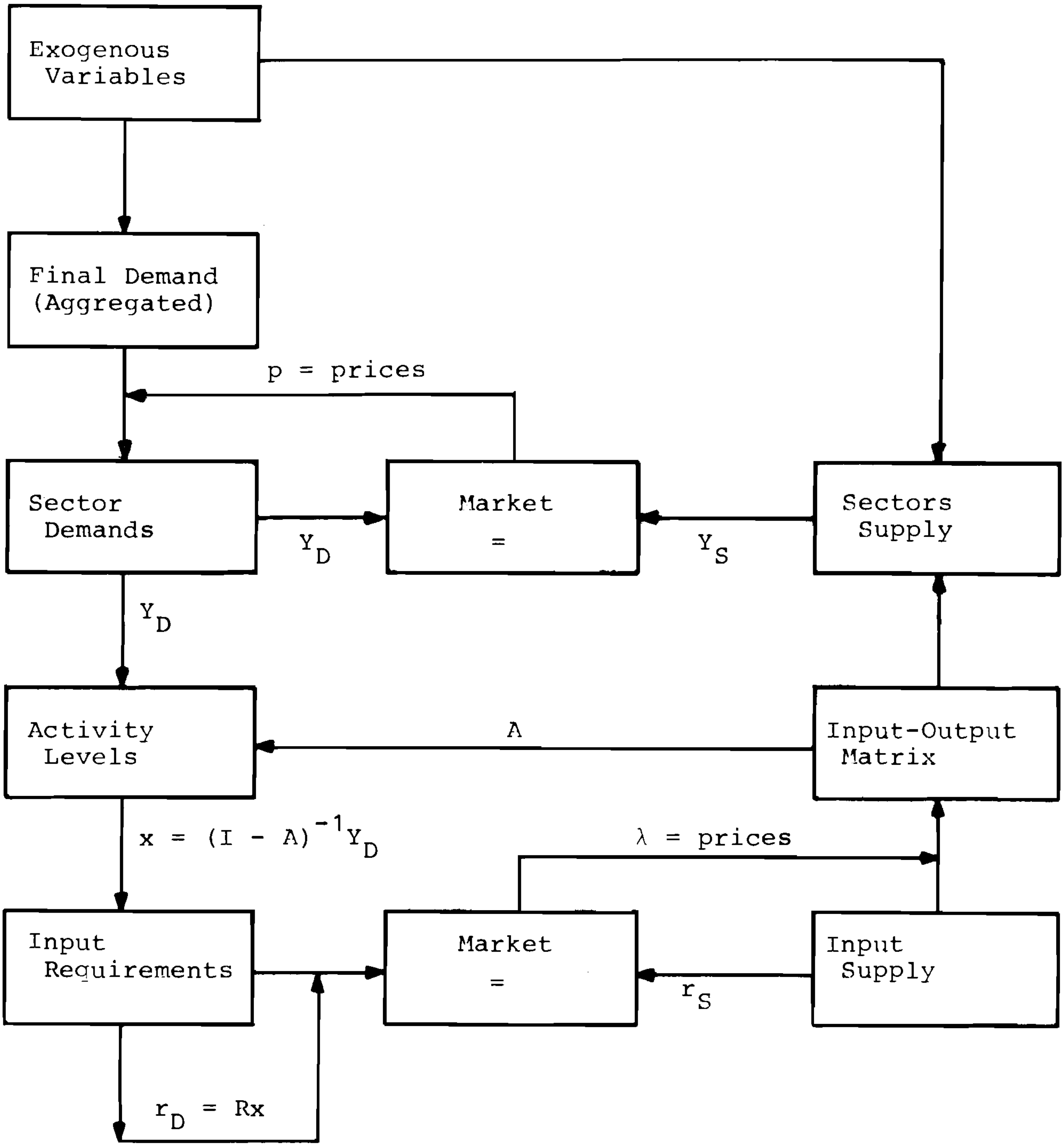

Figure 1. Aggregated General Equilibrium Model

$$\max\ p_j x_j - \sum_{i=1}^{n+m} \lambda_i x_{ij}$$

$$x_j \geq 0 \quad , \qquad x_{ij} \geq 0 \tag{2.1}$$

$$\text{subject to } x_j = F_j(x \cdot j)$$

where F_j is the production function for sector j and represents the technological possibilities. (2.2) gives the necessary conditions where F'_{ji} is the derivative of F_j with respect to input i.

$$p_j x_j - w x_j = 0$$

$$-\lambda_i x_{ij} + w F'_{ji} x_{ij} = 0 \tag{2.2}$$

$$F_j(x_{ij}) = x_j \quad .$$

Under the assumption of concavity on F_j, (2.2) are sufficient conditions. Homogenity of F_j on x_{ij} would give $\sum_i F'_{ji} x_{ij} = F_j$ and

$$w = p = \frac{\sum_i \lambda_i x_{ij}}{x_j} \quad .$$

Substituting this value for w in (2.2) we have

$$\frac{\lambda_i x_{ij}}{p_j x_j} = \frac{F'_{ji} x_{ij}}{x_j} \tag{2.3}$$

or in real terms,

$$\frac{x_{ij}}{x_j} = \frac{F'_{ji} x_{ij}}{x_j} \cdot \frac{p_j}{\lambda_i} \quad . \tag{2.4}$$

Equations (2.3) and (2.4) give the elements of A in monetary and real terms respectively. As we see, they depend on the form of the production technology. For example, if F_j is of the Cobb-Douglas form $x_j = \prod_i x_{ij}^{\alpha_{ij}}$ then $\frac{\lambda_i x_{ij}}{p_j x_j} = \alpha_{ij}$, which is a parameter. However, for more general forms of F_j like the Transcendental Logarithmic [4]

$$Lnx_j = \alpha_0 + \sum_i \alpha_{ij} Lnx_{ij} + \frac{1}{2} \sum_\ell \sum_i \beta_{i\ell j} Lnx_{ij} Lnx_{\ell j} \qquad (2.5)$$

then

$$\frac{\lambda_i x_{ij}}{p_j x_j} = \frac{\partial Lnx_j}{\partial Lnx_{ij}} = \alpha_{ij} + \sum_\ell \beta_{i\ell j} Lnx_{\ell j}$$

which is clearly dependent on the values of x_{ij}.

The results above show that the evaluation of a_{ij} and r_{kj} by computing $\frac{\lambda_i x_{ij}}{p_j x_j}$ would be consistent with the assumption of a Cobb-Douglas form of the production function in which case the input shares are independent of the prices. For general forms of technology the result would not be correct.

The generalization of the input-output coefficients is a simple but rather important result. Diewert [8] suggested a generalized Leontief production function but used the dual cost function associated with it (more precisely its approximation) to compute the $\frac{x_{ij}}{x_j}$ coefficients which in his case were given as a function of the prices of the inputs. Hudson and Jorgenson [9] use the dual Transcendental Cost function to compute the coefficients as a function of the prices; the coefficients were then evaluated simultaneously with the prices. The practical application of their model in creating scenarios that would provide a basis for energy policy formulation uses the computational advantage of an input-output table together with more general assumptions on the form of the production technology for each sector.

The consumer side of the GEM is formulated in terms of the utility maximization behavior of the consumption units subject to their budget constraint, for a given set of prices that satisfy the market equilibrium conditions between supply and demand. In [9], a transcendental logarithmic form of utility function is used.

GEM as presented above makes the important assumption that the economic units (consumers and producers) interact in competitive markets and the resulting allocation of resources is strictly determined by economic and technological forces. However, the presence of political factors in this process and their influencing of the results is not given the attention that the empirical evidence of its importance would suggest. Divvy Economy [5] would be a preliminary attempt in the direction of including the political constraints into the system, but as is shown in [12] the statement of those constraints in monetary terms and the flexibility on the sizes of the consumer groups to freely vary implies that they do not affect the real results. Political or institutional constraints should be stated in real terms and the resource allocation should be done under the assurance that they are satisfied. If the constraint can be relaxed it should be done so to begin with. But if the constraint can not be eliminated, the results obtained without assuring it is satisfied may not be meaningful. Moreover, if our interest is policy formulation we want to use models to evaluate alternatives whose consequences we know satisfy the full set of constraints.

The economic theory of the Second Best can provide a framework for modeling resource allocation problems under the presence of constraints that do not appear in competitive markets. In [2] optimal pricing under the presence of the Government's budget constraint is discussed. In [6] the problem of monopolistic behavior is discussed. But until the present, there has not been any empirical study of quantitative policy evaluation under situations where not all the decision units follow the Paretian rules.

In the coming section we develop some GEM models in the field of energy policy and advance some preliminary results that would allow quantitative evaluation of policies under situations when only second best results are possible.

3. A Second Best Model on Distribution

The GEM presented in section 2 formalizes the resource allocation problem under the assumption of competitive market conditions. Under them, the decision units, producers and consumers, take prices as given and choose the quantities of inputs and outputs by solving their respective behavioral models. An important property of the competitive market is the achievement of a Pareto optimal allocation of resources [8] in a completely decentralized manner; that is by each decision unit using an optimal decision rule which involves only its decision variables, (independent of the decisions of the others). The market mechanism together with technological constraints also determine the optimal distribution of welfare among social groups. This distribution is directly related to what each group receives as a remuneration from its participation in the production process in terms of its marginal contribution to it.

The unrealistic nature of the assumptions characterizing a competitive economy have been extensively recognized. In the previous section we have outlined the new directions that the modeling of the allocation process may take. Numerous examples exist on how the bargaining at the political level for shares of welfare among social groups modifies the strictly economic solution. This politization has been aided by the growth in the relative importance of the government which not only provides services but administers taxes and subsidies creating considerable transfers of income among groups. The understanding of the ways by which each group imposes its demands is still limited, but it seems to rely upon the damaging effects on the economic system that would result from the withdrawal of the resource controlled by the demanding group. Union strikes and walk outs would be examples of such withdrawals.

A strong motivation for modelling the resource allocation by economic and political bargaining has been the behavior of the Organization of Petroleum Exporting Countries (OPEC) in controlling the price of oil to assure themselves a share of the welfare of the world. In this section we formalize the OPEC demands and show how they would be translated into an additional constraint that would distort the allocation of resource in the productive sector. Although the model is built around a particular example, it could be easily generalizable to similar cases where the demands are stated in terms of shares. The framework used is the Economic Theory of the Second Best which also provides a methodology for systematic definition of policies.

Consider the welfare problem

$$\max U(z_{ij})$$

$$z,x,y \geq 0$$

$$\text{Subject to } \sum_{j=1}^{r} z_{ij} \leq y_i \qquad \forall i \qquad (3.1.1)$$

$$F(y,x) = 0 \qquad (3.1.2) \quad (3.1)$$

$$x_\ell \leq h_\ell \qquad \forall \ell \;. \qquad (3.1.3)$$

The objective function represents an aggregate utility index for the countries of the world, $j = 1,\ldots,r$. The aggregation factor is the reciprocal of the marginal utility of income for each country[1], which converts the value of the utility function from utils to monetary units. z_{ij} is the consumption of good $i, i = 1,\ldots,n_j$ in country j. Constraint (3.1.1) is an availability constraint in final good i, $i = 1,\ldots,n$. (3.1.2) gives the aggregate production of the vector of outputs y as a function of the vector of inputs x. $F(y,x)$ is assumed to be a continuous, concave, separable and homogeneous function of y and x, i.e., $F(y,x) = F_1(y) - F_2(x)$ with $\sum_{i=1}^{n} F'_{1i} y_i = F_1(y)$, $\sum_{\ell=1}^{m} F'_{2\ell} x_\ell = F_2(x)$. (3.1.3) is the resource availability constraint for input ℓ, $\ell = 1,\ldots,m$ and h_ℓ is a parameter.

Unless specified by the derivative sign, superscript $'$ will denote derivative and subindex i, ℓ, j will be combined to represent the variable with respect to which we take the derivative.

The necessary conditions for (3.1) are, after multiplying by the respective variables,

$$U'_{ij}z_{ij} = p_i z_{ij} \qquad \forall_{ij} \qquad (3.2.1)$$

$$p_i y_i = wF'_i y_i \qquad \forall_i \qquad (3.2.2) \quad (3.2)$$

$$wF'_\ell x_\ell = \lambda_\ell x_\ell \qquad \forall_\ell \qquad (3.2.3)$$

where p, w, λ are the vectors of dual variables of (3.1.1), (3.1.2) and (3.1.3) respectively.

If x_k represents input factor oil, $\lambda_k x_k$ will be the total revenue assigned to that input factor. We will assume that this is the total revenue of the OPEC countries. This does not seem an unreasonable assumption given the present level of specialization of those countries.

In our utility function subindex j = k will identify the OPEC group which consumes goods i = 1,...,n_k. From (3.2) we can obtain the relation between the level of income of group k and its share of total welfare

$$\sum_{i=1}^{n_k} U'_{ik}z_{ik} = \sum_{i=1}^{n_k} p_i z_{ik} \leq \lambda_k x_k$$

or in relative terms

$$\frac{\sum_{i=1}^{n_k} U'_{ik}z_{ik}}{\sum_j \sum_{i=1}^{n_j} U'_{ij}z_{ij}} = \frac{\sum_{i=1}^{n_k} p_i z_{ik}}{\sum_j \sum_{i=1}^{n_j} U'_{p_j} z_{ij}} \leq \frac{\lambda_k x_k}{\sum_{i=1}^{n_k} p_i y_i} \qquad (3.3)$$

where $\sum_j \sum_{i=1}^{n_j} p_i z_{ij} = \sum_{i=1}^{n} p_i y_i = \sum_{\ell=1}^{m} \lambda_\ell x_\ell$ is the aggregate income. Equation (3.3) shows that the share of welfare for group k is proportional to its share of income over the total world consumption. This intuitive result can be used to understand the behavior of a group like OPEC where consumption depends on the imports of final goods from which it derives a level of welfare. The increase in oil prices should not be seen as merely increasing their monetary income, but as a way of maintaining a stable or increasing share of welfare with respect to the rest of the world. Similar conclusions can be made if we look at the indirect utility function[2] for the k^{th} group, $V_k(p_1,\ldots,p_{n_k},I_k)$. To maintain a certain value for V_k implies the maintenance of a certain relationship between the income of the group and the level of prices of the goods it consumes. Thus claims of raw material exporting countries for an indexing of raw material prices to prices of industrial goods can be seen as an attempt to fix a level of indirect utility.

The value of the share of real income that the OPEC group is able to fix is the result of a political process in which OPEC's bargaining power is based in the damaging effect that an oil embargo (withdrawal of the resource) would be for the world economies.[3] The rest of the world acknowledges this power and has shown willingness to negotiate an indexing of the price of oil with the prices of selected commodities (presumably those entering the utility function of the OPEC group).

Although the outcome of the bargaining is not determined in this paper, it is likely to arrive at a political arrangement different from the one determined by the market mechanisms. That is, if U_j is the share of welfare under market conditions for group j, and $\bar{U}_j$ is then negotiated, $\bar{U}_j = KU_j$ where $K \neq 1$, and most likely > 1. Our interest now is to show how this would effect the production sector. From (3.2.2) and (3.2.3) we have

$$\frac{\lambda_k x_k}{\sum\limits_{i=1}^{n_k} p_i y_i} = \frac{F'_k x_k}{\sum\limits_{i=1}^{n_k} F'_i y_i} \quad . \tag{3.4}$$

The left hand side of (3.4) will be affected by $K = \bar{U}_j/U_j$, so we have the deviant form of (3.4) to be written as

$$\frac{\lambda_k x_k}{\sum\limits_{i=1}^{n_k} p_i y_i} = K \frac{F'_k x_k}{\sum\limits_{i=1}^{n_k} F'_i y_i} \tag{3.5}$$

Although the result of (3.5) has been justified in terms of allocation of welfare in a world basis, we can impose a constraint in the form of (3.5) for each country. The results given below would apply then to a particular economy.

To look at the effects of (3.5) on the production sector we can solve the following problem for given values of λ and p (dual variables of (3.1.1) and (3.1.3)).

$$\max_{y,x\geq 0} \sum_{i=1}^{n} p_i y_i - \sum_{\ell=1}^{m} \lambda_\ell x_\ell$$

$$\text{subject to } F(y,x) = 0 \tag{3.6.1) (3.6}$$

$$\frac{\lambda_k x_k}{\sum\limits_{i=1}^{n_k} p_i y_i} - K \frac{F'_k x_k}{\sum\limits_{i=1}^{n_k} F'_i y_i} = 0 \quad . \tag{3.6.2}$$

The necessary conditions on (3.6) are, for the multipliers w and v,

$$i \varepsilon n_k \quad p_i y_i - w F'_i y_i - v \left(- \frac{\lambda_k x_k p_i y_i}{\left(\sum\limits_{i=1}^{n_k} p_i y_i\right)^2} + K \frac{F'_k x_k \left(\sum\limits_{j \varepsilon n_k} F''_{ji} y_i y_j + F'_i y_i \right)}{\left(\sum\limits_{i=1}^{n_k} F'_i y_i\right)^2} \right) = 0$$

$$i \notin n_k \quad p_i y_i - wF'_i y_i - vK \frac{F'_k x_k \left(\sum\limits_{j \varepsilon n_k} F''_{ji} y_i y_j \right)}{\left(\sum\limits_{i=1}^{n_k} F'_i y_i \right)^2} = 0 \tag{3.7}$$

$$-\lambda_k x_k + wF'_k x_k - v \left(\frac{\lambda_k x_k}{\sum\limits_{i=1}^{n_k} p_i y_i} - K \frac{F''_{kk} x_k^2 + F'_k x_k}{\sum\limits_{i=1}^{n_k} F'_i y_i} \right) = 0$$

$$\ell \neq k \qquad -\lambda_\ell x_\ell + wF'_\ell x_\ell + vK \frac{F''_{k\ell} x_k x_\ell}{\sum\limits_{i=1}^{n_k} F'_i y_i} = 0$$

together with the constants (3.6.1) and (3.6.2).

We recall that given a function G(x), $e_i \equiv \frac{G'_i x_i}{G}$ is the elasticity of G(x) with respect to x_i, and $\sigma_{ij} = \frac{G'_i G'_j}{GG''_{ij}}$ is the Allen Elasticity of Substitution of i by j.[4] We can substitute for the new parameters in the necessary conditions above, so we would have, after simplification

$$i \varepsilon n_k \quad p_i y_i - wFe_i - v \left(\frac{Ke_k}{\left(\sum\limits_{i=1}^{n_k} e_i \right)} \left(- \frac{p_i y_i}{\sum\limits_{i=1}^{n_k} p_i y_i} + \frac{e_i}{\sum\limits_{i=1}^{n_k} e_i} \right) + K \frac{e_k \left(\sum\limits_{j \varepsilon n_k} \sigma^1_{ji} e_i e_i \right)}{\left(\sum\limits_{i=1}^{n_k} e_i \right)^2} \right) = 0 \tag{3.8.1}$$

(3.8)

$$i \notin n_k \quad p_i y_i - wFe_i - vK \frac{e_k \sum\limits_{j \varepsilon n_k} \sigma^1_{ji} e_j e_i}{(\sum\limits_i e_i)^2} = 0 \tag{3.8.2}$$

$$-\lambda_k x_k + wFe_k + vK\frac{\sigma^1_{kk} e_k^2}{\sum\limits_{i=1}^{n_k} e_i} = 0 \qquad (3.8.3)$$

$$\ell \neq k \qquad -\lambda_\ell x_\ell + wFe_\ell + vK\frac{\sigma^1_{k\ell} e_k e_\ell}{\sum\limits_{i=1}^{n_k} e_i} = 0 \qquad (3.8.4)$$

where F is the index value of total output: $F = F_1(y) = F_2(x)$ under the assumption that (3.6.1) is of the form $F_1(y) - F_2(x) = 0$ (separability of inputs and outputs). σ^1 is the reciprocal of σ defined above.

Substituting (3.8.1) and (3.8.3) in (3.6.2) we can solve for v:

$$\frac{we_k + \frac{vK}{F}\,\frac{\sigma^1_{kk} e_k^2}{\sum\limits_{i=1}^{n_k} e_i}}{w\left(\sum\limits_{i=1}^{n_k} e_i\right) + \frac{vK}{F}\,\frac{e_k}{\left(\sum\limits_i e_i\right)^2}\left(\sum\limits_{j\varepsilon n_k}\ \sum\limits_{i=1}^{n_k} \sigma^1_{ji} e_j e_i\right)} = \frac{Ke_k}{\sum\limits_{i=1}^{n_k} e_i} \qquad (3.9)$$

and

$$\frac{vK}{F} = \frac{w(K-1)\left(\sum\limits_{i=1}^{n_k} e_i\right)}{\sigma^1_{kk} e_k - K\frac{e_k}{\left(\sum\limits_{i=1}^{n_k} e_i\right)^2}\left(\sum\limits_{j\varepsilon n_k}\ \sum\limits_{i=1}^{n_k} \sigma^1_{ji} e_j e_i\right)} \ . \qquad (3.10)$$

Using (3.10) in (3.8) we can obtain the second best relative shares

$$\frac{\lambda_\ell x_\ell}{\lambda_k x_k} = \frac{e_\ell(1 + u\sigma^1_{\ell k})}{e_k(1 + u\sigma^1_{kk})} \tag{3.11}$$

$$s \notin n_k \qquad \frac{p_s y_s}{\sum_{i=1}^{n_k} p_i y_i} = \frac{e_s\left(1 + u\frac{\sum_{j\varepsilon n_k} \sigma^1_{js} e_i}{\sum_{i=1}^{n_k} e_i}\right)}{\sum_{i=1}^{n_k} e_i + u \sum_{i=1}^{n_k} \sum_{j\varepsilon n_k} \sigma^1_{ji} e_j} \tag{3.12}$$

where

$$u = \frac{(K - 1)}{\sigma^1_{kk} - \frac{K}{\left(\sum_{i=1}^{n_k} e_i\right)^2}\left(\sum_{i=1}^{n_k} \sum_{j\varepsilon n_k} \sigma^1_{ji} e_j e_i\right)} \ . \tag{3.13}$$

For $K = 1$, $u = 0$ (3.11), (3.12) would just be the first best results. However, as we justified before, it is expected $K > 1$ from the bargaining process so the first best results would no longer hold.

Equation (3.11) and (3.12) give the conditions for producer equilibrium under the second best. As we see, the first best results for the inputs $\frac{e_\ell}{e_k}$ are "corrected" by the value $\frac{1 + u\sigma^1_{\ell k}}{1 + u\sigma^1_{kk}}$ which depends on the output estasticities and the Allen elasticities of substitution. That is, for any two factors q and t, the deviation from the first best will be larger, with respect to k, for the factor with larger $\sigma^1_{\ell k}$. Or, since $\sigma^1_{\ell k} = \frac{1}{\sigma_{\ell k}}$, where $\sigma_{\ell k}$ is the Allen elasticity of substitution between ℓ and k, the deviation will be greater for the factor with lower elasticity of substitution with oil.

Similarly, from (3.1.2), the ratios of shares of final goods are appropriately modified, depending on the sum of elasticities of transformation of the final good with the commodities inside the index. Note also that if for factors q and t, $t \neq q \neq k$, $\sigma_{tk} = \sigma_{qk}$ the relative shares between t and q will still be the same as in the first best.

The Allen elasticity of substitution is a parameter which will depend on the form chosen to represent the technology. For example, for a Cobb-Douglas function $F_1(y) = \prod_{i=1}^{n} y_i^{\beta_i}$, $F_2(x) = \prod_{\ell=1}^{n} x_\ell^{\alpha_\ell}$, the elasticities of substitution between i and j and ℓ and k are constant and equal to one, for $i \neq j$ and $\ell \neq k$ while σ_{ii} and $\sigma_{\ell\ell}$ are $\frac{\beta_i}{1-\beta_i}$ and $\frac{\alpha_\ell}{1-\alpha_\ell}$ respectively. For the Transcendental Logarithmic function, however, the elasticity of substitution is a variable that depends on the share value of the factor (or output). For example, for the function given in (2.5), the elasticity of substitution between i and k would be $\sigma_{ik} = M_i M_k/(\beta_{jik} + M_i M_k)$ where M_i is the share of i.

The selection of the form of the production technology becomes then a major issue. From the computational point of view, the Cobb-Douglas form is the most attractive. An extension of the Cobb-Douglas is the Constant Elasticity of Substitution production function, CES [2] which still has constant elasticity of substitution between the variables, but is not restricted to the value of one. The Transcendental Logarithmic Frontier [5], the Generalized Leontief function [9] or the function given in [12] would all allow for variable elasticities of substitution and consequently will be more general, but would also have more computational difficulty.

At this point it may be important to mention the use of the concept of duality [12] in general equilibrium modeling. In [10], Hudson and Jorgenson use transcendental logarithmic functions to represent the profit frontier as a second order approximation of an arbitrary profit function. Presumably this would be the dual form of a production function in terms of inputs and outputs. This representation allows them to write the share values for the inputs as functions of the prices and consequently allows share values to vary according to substitution effects when there are changes in those prices.

The prices are originally obtained from the solutions of a set of general equilibrium equations in the dual space. The general equilibrium in the primal space (physical inputs and outputs) is represented in a Leontief Input-Output table with variable coefficients (depending on prices). No check is made however, after the computation of the activity levels whether the dual prices associated with the solution coincide with the prices in the dual problem originally solved. In general, it does not seem likely for them to coincide at first; however it would be expected for them to converge through an iterative procedure similar to the one used to prove the existence of a general equilibrium solution using the fixed point theorem. Finally a remark on the empirical evidence [4] of observed differences on the estimated values of cross-elasticities of substitution when they are estimated from the profit frontier and when they are estimated from the production frontier. The sensitivity of the parameter to this choice when it should be indifferent should be clarified before further use of the Transcendental Logarithmic function is made.

In our model we gave the results in terms of the primal problem represented by the transformation technology. The application of the model would require the selection of a particular form for $F(y,x) = 0$. The choice of a function with variable elasticities of substitution would make equations (3.11) and (3.12) nonlinear since u and σ would be functions of x and y, which would create a computational problem. A certain compromise could be achieved by using a CES function

$$\left[\sum_{i=1}^{n} \gamma_i y_i^{-\rho}\right]^{-1/\rho} = \left[\sum_{\ell=1}^{n} \delta_\ell x_\ell^{-\mu}\right]^{-1/\mu}$$

where γ_i and δ_ℓ are parameters ≥ 0 and $\sum_{\ell=1}^{n} \gamma_i = \sum_{\ell=1}^{n} \delta_\ell = 1$.

The elasticity of substitution is related to the parameter μ, $\sigma_{\ell q} = \frac{1}{\mu + 1}$ for all ℓ, q, $\ell \neq q$, and the elasticity of transformation to ρ : $\sigma_{ij} = \frac{1}{1 + \rho}$ for all i, j, $i \neq j$.

4. The Use of the Model in Policy Evaluation

The main motivation of this work has been to try to relate political and economic considerations in a GEM. From the Theory of the Second Best we have introduced a constraint in the economic model justified by political consideration. The new conditions for optimality can be estimated under certain restrictions in the technology used to represent the production sector. The conditions take explicitly into account the existence of the behavioral rule of OPEC trying to maintain certain levels of real surpluses transferred to them. The use of the model in a complete GEM framework (as the one in Figure 1) for policy evaluation will be now much more meaningful since the results shown by the different alternatives will be feasible results, that is, it will already incorporate the satisfaction of the OPEC demands.

Apart from the adequate feasible set to evaluate policies, the Second Best provides a methodology to decide how to select a policy and where it should be directed. Below we will use theoretical results from the Second Best literature in the context of our practical problem of energy policy. We will represent our additional constraint (3.6.2) by $m(y,x)$ where $y = F(x)$ since it involves relationships between inputs and outputs.

A. The possibility of restoring the first best result

We have already argued that the existence of the constraint is a result of the bargaining power that the OPEC possesses. If OPEC attempted to satisfy its demands in the form of a one time boost in the price of oil, there would be an immediate policy dicision that would restore the first best result: adjust all the prices monetarily so that their relative position with respect to the new oil price is maintained as before [1]. The adjustment would propogate through the economic sectors modifiying also the prices of the final goods, but after the adjustment process no effect on the real sector would actually occur. The policy makers of the consuming countries have been somehow reluctant to expand the money supply in the amount required to make the compensation not only under the fear of inflation (which would be just apparent but not real) but under the fear that a new rise in the price of oil would follow from OPEC's realization that their relative advantage had been eliminated. To some extent current economic policies reflect this behavior. Pressures from inside countries to stimulate their economy and reduce unemployment that originally had increased as the result of anti-inflationary policies, forced the governments to make expansive decisions (including moderate increases in the money supply). In recent years there has been a high rate of price increase in most of the oil consuming nations that has made more expensive the imports by the OPEC group. The cartel has responded by a new price increase of oil. To avoid the spiral that those counter policies would stimulate the OPEC and the consuming countries have agreed to discuss the idea of indexing the price of oil in terms of the price of final goods.

Our conclusion is that no restoration of the first best is possible and the discussion reinforces the need for the constraint to be explicitly taken into account.

B. The identification of sectors not affected by the constraint

The first order conditions of problem (3.6) involve the derivative of m(y,x). If the derivative is equal to zero for a certain variable, m(y,x) will not affect directly the optimality conditions. This would be the case for an industrial sector not included in the index as a final good and not using any oil or related products in tis process. Mathematically this is true for y_s such that

$$\frac{\partial m}{\partial y_s} \cdot \frac{\partial y_s}{\partial x_k} = 0 \quad . \tag{4.1}$$

Also for the input such that it is not used in any final good, included in the index and does not substitute with oil; for x_ℓ such that

$$\frac{\partial m}{\partial y_i} \cdot \frac{\partial y_i}{\partial x_\ell} = 0 \quad \text{or} \quad \frac{\partial^2 m}{\partial x_k \partial x_\ell} = 0 \quad , \quad i \in n_k \tag{4.2}$$

Note that $\frac{\partial^2 m}{\partial x_k \partial x_\ell} = 0$ implies $F''_{k\ell} = 0$ and $\sigma_{k\ell} = -\infty$ which gives $\sigma^1_{k\ell} = \frac{1}{\sigma_{k\ell}} = 0$.

The set of variables satisfying those conditions above would be practically empty given the presence of oil as an input in most of the production process in a direct of indirect way.

Following the arguments in [1], it can be said that the relative optimality conditions between variables i and j will still be first best if their rate of distortion is the same. In our case this would happen under the conditions of constant and equal elasticities of substitution and constant and equal elasticities of transformation (recall equations (3.11) and (3.12). This is a sufficient condition.

As we mentioned before the elasticities of substitution and transformation are dependent on our assumption on the technology so in the verification of this condition we may introduce a certain bias.

The identification of variables satisfying the first best results is important since in general it is not necessary to take policy actions to modify their decision rules. Note that the set of variables will have to have an equivalence class relation, i.e., the relative distortion will be the same among all the elements of the class.

C. Policy recommendations

Based on the theory of the Second Best specific recommendations to the policy maker on how to influence the behavioral rules of the economic units so that the second best solution is obtained in a decentralized manner can be made. The point is that if we let the market behave by itself, with the economic units allocating the resources under the guidance provided by observed prices as in the situations with perfect competition, not only the first best is not attainable, but neither the second best since the constraint is completely ignored. Our interest is to avoid the alternative of a completely centralized resource allocation problem that would solve problem (3.6) and then would order the economic units to do exactly as specified in the second best conditions, but rather we want to preserve as much as possible of the decentralized nature of the resource allocation through a market mechanism. We thus have to focus on the design of what has been called "piecemeal" policy [8]. That is, a policy by the coordinating unit that affects the decision variables of the economic units as if the constraint would have been explicitly taken into account. Further we want the new decision rule to still be only a function of the decision variables under the control of the single unit.

The possibility of piecemeal policy is challenged in [1] where it is argued that it is precisely the impossibility of eliminating the decision variables of the deviant unit from the decision function of the others which imposes a second best solution and in the cases where this elimination is possible a first best result is obtained.

More precisely, for the first best result for factor ℓ we have

$$\lambda_\ell x_\ell = e_\ell = w\frac{F'_\ell x_\ell}{F}$$

or

$$\lambda_\ell = \frac{wF'_\ell}{F}$$

while in our case we have

$$\lambda_\ell = \frac{wF'_\ell}{F}\left(1 + u\sigma^1_{k\ell}\right) \qquad (4.3)$$

where u is a function of x_k and so will be $\sigma^1_{k\ell}$ in the case of a variable elasticities of substitution production function. The price decision variable which is second best optimal depends then on x_k, the decision variable of the deviant factor.

If there is no piecemeal policy, what may be the role of the model in policy formulation for the sectors operating under second best? First of all it has already shown to be useful in identifying the sectors under first best. Second, it provides guidelines for policy and action as we will try to show.

We have emphasized the possibility of integrating the producer vector model into a general equilibrium model which would explicitly take into account the existence of the constraint. The model could then be used to make predictions on what the values of the input and output variables would be under certain assumptions on growth and consumption. The piecemeal policy could then be built around this preliminary result since we will have approximate values for the variables in equation (4.3), as well as for the equivalent decision rules in the final goods sector. In the case where σ would be variable the process would have to be iterative, starting with the values of the variables with no constraint and adjusting progressively the correction value in the shares until some equilibrium solution could be obtained. That would allow us to compute the value $\frac{u}{x_\ell}\sigma^{1}_{k\ell}$. This value would be administered in the form of taxes and subsidies to be added to the prices of the input factors, and/or final goods.

The use of a large scale integrated model of the economy to predict values for these variables which then would be used to compute policy values (taxes and subsidies) to administer a second best solution, may be an important tool to overcome the problem of information in implementing second best solutions. With the use of econometric models to represent and estimate utility and production functions the policy administrator can gain information on the behavior of the subunits, provided the model is correct in specifying the behavior. The process would be similar to an abstract solution of (3.6) by a centralized unit, but the implementation would still be left to the economic units who would make the final choices under more realistic information.

Some other suggestions can also be made about general policy. The need for direct regulation and the distortion in the economy will be minimized by increasing the size of the set of industries operating under first best rules (not affected by the constraint as discussed in B). The policies directed towards the encouragement of research and development of new energy sources that would substitute for oil as an input factor would contribute to reducing the distortion by increasing the number of industries for which (4.2) would hold. In the cases when this substitution could not be completed, the effort should be directed towards increasing the elasticity of substitution between oil and other inputs. The later is true since we have seen that the distortion is inversely proportional to this elasticity. As a corollary, the economy would become more oil independent and would lower the bargaining power of the resource controlling group forcing the value of the parameter progressively closer to 1 and the effect of the constraint would completely vanish.

5. Concluding Remarks

In the previous sections we presented a Second Best model on factor share distribution together with a methodological framework for combining modeling with policy analysis so that second best solutions are attainable.

The results are still theoretical, although some indication is given on how the model could be used to quantitatively evaluate policies. It is important to emphasize that the methodological procedure to evaluate policies would not be much different from what is done in other studies ([9] for example), but our framework should provide a basis for identification of economic sectors whose first best rules are not affected by the additional constraint so no direct policy action would have to be taken for them. For the industries not included in this class, the difficulties of "piecemeal policy" are partially overcome by using the model to predict values of the decision variables of the deviant economic unit. This approximated value can then be used to find the second best optimal decisions of the other economic units (Equation (4.3)): Since the economic units can not be expected to take into account the overall constraint the role of the policy maker is to modify their behavioral rules to make them consistent with the new optimality condition. To do this, the policy maker would administer taxes and subsidies by the equivalent of the correction factor given by equations of the form (4.3).

Finally, we point out the importance of technological specification in GEM, and the trade offs between generality and computability that the analyst will have to make in actual implementation. Another important practical consideration is the level of aggregation used to build the model, that is, the number of economic sectors and primary inputs selected as variables.

All these remarks should make clearer the imaginative role that the analyst would still have to take in implementing the model and the important questions still left to his discretion, but we believe we have offered him a more realistic and theoretically sound starting point.

Footnotes

1. For example we could write U(z) of the form

$$U(z_{ij}) = \prod_{j=1}^{r}\left(\prod_{i=1}^{n_j} z_{ij}^{\alpha_i}\right)^{\beta_j}$$

where β_j would be the aggregation factor equal to the reciprocal of the marginal utility of income for country j.

2. The indirect utility function is derived by solving the problem,

$$\max\ U_k(y_{ik})$$

$$\text{subject to } \sum_{i=1}^{n_k} p_i y_{ik} \leq I_k \qquad (1)$$

where I_k is the income level for the OPEC. The solution to (1) gives $V_k\left(\frac{p_1}{I_k},\ldots,\frac{p_{nk}}{I_k}\right)$, which is the representation of the utility in terms of prices of the final goods and income level allocated to consumption. Evaluation of V_k over time gives an index measure of the evolution of the welfare level for the group. The OPEC demands can thus be interpreted as stabilizing V_k's growth over time.

3. It may be of interest to contrast the behavioral rule introduced in the paper with the situation where we have a monopsony which is supplying the resource. The monopsony will have a perceived demand equation for its services, and will determine the quantity to supply by equating marginal revenue to marginal cost. For this quantity the price will be determined from the demand equation. The behavioral rule of the monpsomist is not Paretian since it does not result in a quantity that maximizes the consumer surplus (this would be the quantity for which marginal cost equals demand). The appropriation of part of the consumer surplus by the monopsonist is an indirect result of the structure of the market

(no competition on the side of the supply), but is consistent with the behavioral rule of profit maximization which prevails in the overall economy.

In our case, the resource controlling group specifically bargains for a share of the world surplus, as it affects its level of utility. The indexing of the price of the resource is directly aimed at this appropriation, and in the bargaining process the group makes use of its knowledge of the damaging effect that the withdrawal of the resource would cause to the consumer groups forcing them to accept its conditions.

4. The elasticity of substitution (σ) is a number that measures the rate at which the substitution between inputs or between outputs takes place, and it is defined as the proportionate rate of change of the input ratio by the proportionate rate of change of the rate of technical substitution:

$$\sigma = \frac{d \log(x_2/x_1)}{d \log(G_1'/G_2')} = \frac{G_1'/G_2'}{x_2/x_1} \cdot \frac{d(x_2/x_1)}{d(G_1'/G_2')}$$

in obtaining σ we also make use of the property that $F_1(y)$ and $F_2(x)$ in the main text are homogeneous [J.M. Henderson, R.E. Quandt, Microeconomic Theory, p. 62].

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