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APPLICATION OF THE KALMAN FILTER TO CYCLONE FORECASTING:

1. METHODOLOGY

2. TYPHOON FORECASTING

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PREFACE

The Water Resources Group of the IIASA has been engaged in various research tasks, including short-term forecasting and control problems in the water resources field. Cyclone motion forecasting was the initial part of this task to further the effort of similar technical applications to climatological, hydrological and water quality forecast and control.

This is the first of a two-part report. Methodology and a few experiments on typhoon forecasting are presented.

ABSTRACT

The Kalman filtering technique was applied to the problem of cyclone forecasting. This paper presents the methodology and the preliminary results of typhoon experiments. Further typhoon experiments and hurricane experiments will be reported in a forthcoming paper.

The purpose of the study is to establish a methodology which will better utilize existing models. In this paper, the SFC.700 mb model and SNT models, developed respectively by Dr. Arakawa and the Japan Meteorological Agency, were selected as examples of existing models. The case study was conducted using the typhoon data observed in August 1974. The results, the improvement of the performance of original models, were demonstrated in terms of the percent reduction in prediction errors which appeared to be 30% to 50% on an average. Improvement of the 24 hour forecast is recognized more than that of the 12 hour forecast. The Kalman filter application is concluded to be promising in tropical cyclone forecast probelms in the sense that it improves the performance of any models whose residual errors are correlated.

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I. INTRODUCTION

Cyclone motion forecasting has long been one of the major subjects in meteorology. Insights, experiences, aerodynamical analyses and their combinations have been explored and applied. The cyclone studied in this paper is a tropical cyclone, including tropical depressions and storms[1], although the basic techniques employed apply to the forecast of motions of any cyclones and other similar meteorological phenomena.

Among a number of cyclone motion forecast models, the statistical model plays a very important role amending the shortcomings of both empirical and purely theoretical approaches. It is well acknowledged that the numerical solution to the aerodynamical equations has made a significant improvement in recent years in forecasting the general flows in the atmosphere but does not and will not give satisfactory forecasts for cyclone motions, at least in the near future. A further knowledge of dynamics of the atmosphere and a denser and wider observation network to obtain meteorological data are necessary. In the meantime, improvement of the statistical models is of great importance.

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[1] According to the Japan Meteorological Agency, tropical cyclones are classified in the following four categories depending on the maximum velocity of winds in the cyclone area:

1. Tropical depression	17.2 m/sec or less (~34kt)
2. Tropical storm	17.2~24.5 m/sec (34~48kt)
3. Severe tropical storm	24.5~32.7 m/sec (48~64kt)
4. Typhoon	32.7 m/sec or more (64kt~)

Objective

The objective of this paper is to demonstrate a method to better utilize the 'existing' statistical cyclone motion forecast models. The cyclone motion in this context means the advancing direction and speed of the eye of a tropical cyclone after its formation. The forecasting time spans are twelve and twenty-four hours. It is not intended to create any new statistical models but rather intended to devise a better use of already existing models. The statistical models to be used should have a simple linear regression form.

Methodology

Most, if not all, statistical models developed thus far for tropical cyclones have a simple linear regression form which can be represented by

$$\Delta = f\ (\text{persistence data, climatological data, synoptic data})$$

where Δ is a predictant, the location of the cyclone eye at the forecast time or the displacement of the cyclone eye per unit time interval such as 12 hours, 24 hours, ..., 72 hours and f is a linear function. Persistence data refer to the location and advancing direction of a cyclone at current and preceeding times. The simplest model which is not yet a statistical model uses only these data, assuming that a storm will continue to move in the direction and speed that it has been moving at. The climatological data usually stand for the cyclone motion data accumulated in the past. These data are used to determine the motion of a cyclone, assuming that it moves in the same direction and at the same speed as other cyclones moved which occurred during the same month and in the same location. These two methods and their combinations, often denoted as $\frac{1}{2}$ (C + P), sometimes give as accurate a forecast as other more sophisticated methods especially for cyclones in the lower latitude area.

The synoptic data refer to the meteorological data at and around a cyclone, such as surface pressure distribution, geopotential height distribution and the distribution of their time rate of changes. The prognostic synoptic data are also used in practice, by obtaining the prognostic map through numerical simulation of dynamic equations.

Although quite a variety of statistical models exist and their performances depend upon the application areas, seasons, and other geotopographical and meteorological factors, the choice of models to be worked on in this paper is insignificant as long as the model has a linear regression form. In fact, the performance of any model, good or bad, will be improved by a magnitude depending upon the characteristics of prediction errors of the original model.

The technique used in this paper is the Kalman filter, which is a filtering technique used to obtain optimal estimates of state variables that can only be observed indirectly. The Kalman filter is nothing but a least square estimation technique, but possesses very important capabilities; namely, a non-stationary system can be dealt with in an adaptive way and the computational burden required is negligible. To illustrate what the Kalman filter does, consider a simple linear model having the form:

$$y_t = a_1 x_{1t} + a_2 x_{2t} + \cdots + a_n x_{nt}$$

where y_t is a predictant at time t, x_{it} is a predictor to be used at time t, as a_i is a corresponding coefficient. A regression equation of this type is usually used with fixed coefficients, thus the system is stationary. In reality, however, the governing nature in meteorology is highly nonstationary and accordingly, the system seldom performs well if its structure is set constant. Even in a case where the real system can be approximated stationary, its structure may be imprecisely known and the choice of predictors may be irrelevant. The nonlinearity part that cannot be incorporated with the form of a model described above and various uncertainties and errors are also involved both in the system structure and in the measurements. All of these factors lead to an irrelevant performance of the model.

One of the simplest ways to mitigate such difficulties is to relax the constraint on the constancy of the coefficients, namely, to allow them to vary while leaving the structure of a model fixed.

In order to proceed with this line of thought, one can treat the coefficients as unknown state variables. Then the Kalman filtering technique can be used to obtain the best estimates of the unknown variables in an adaptive way responding to the difference observed between a predicted and an observed value.

The motivation which initiated this analysis lies in the fact that the performance of a forecast model differs from cyclone to cyclone. A model works satisfactorily for a cyclone or a portion of the cyclone and for another, it does not. This phenomena seems quite natural because each cyclone at any point in time has specific inner and surrounding physical characteristics. The constant coefficient regression model cannot respond to time variant peculiarities. Consequently, the forecast errors often appear in the same direction or the errors show sequential correlations. Once some coefficients in the model are slightly changed in accordance with the forecast errors observed, such simple errors must disappear.

II. SELECTION OF MODELS AND DATA REQUIREMENT

Since the aim of this study is the development of a method through which existing models can be put to better use, the object models should be selected. Such models should have a form of simple linear regression, but also preferably be in current practice in competent meteorological agencies. Considering these conditions, the following three models have been selected:

1. SFC.700 mb model (Arakawa, 1963)
2. SNT model (Nomoto, Takenaga & Hara, 1973)
3. NHC 72 model (NHC, NOAA, 1972)

The SFC.700 mb model belongs to the earliest group of statistical forecast models. Nevertheless, its performance is not inferior to other later and more sophisticated models.

The SNT (Statistical and Numerical forecasting Technique) model is presently in use as a part of the comprehensive cyclone forecast management in the Japan Meteorological Agency. The major difference between the SNT model and others is the use of prognostic synoptic field data as predictors. The prognostic data are obtained through the numerical simulation of a dynamical equation, independently of the statistical model. This approach is new, having been developed in the past few years, but it is considered to be one of the most promising techniques as the progress of a more accurate solution of dynamical equations is expected to be firm and constant. The NHC 73 model, the latest forecast model in the US National Hurricane Center, also takes this approach.

The NHC 72 model (Neumann, Hope and Miller, 1972) is the latest model among those using only observation analysis data. The original version was developed as the NHC 67 model (Miller, Hill and Chase, 1968) and revised to the current form as part of the NHC 72 model, which consists of the revised NHC 67 and the so-called CLIPER model. The reformed NHC 67 model is referred to simply as the NHC 72 model in this paper.

A complete description of a 12 hour forecast SFC.700 model and 24 hour forecast SNT model, the predictors and their coefficients, is given in Tables 1 and 2. The necessary data required to use these models are obvious from the tables. The data were supplied by the Japan Meteorological Agency and the US National Hurricane Center. Table 3 lists the data used in the analyses. The large amount of data was sent from these agencies on magnetic tapes. Only data 1 were read from the typhoon truck map.

All the models use the moving coordinate system fixed to the eye of a cyclone, which moves as the cyclone moves (see Figures 1 and 2). Therefore, in case the data are given on grid points fixed on the earth, the values corresponding to the grid points on the moving coordinate are determined through recalculation. This process necessitated an interpolation. A simple weighted average was used for this purpose with weights being inverse to the distance. Figure 3 depicts the procedure.

III. APPLICATION OF THE KALMAN FILTER

Brief Review of the Theory

The term 'filtering' stands for a process computing an estimate of a state variable x_k at time t_k using the sequence of observations up to time t_k, i.e.

$$z^{(k)} = \{z_0, z_1, \ldots, z_k\} \quad .$$

Independent variable t assumes the discrete times $t_0 < t_1 < t_2 < \ldots < t_{k-1} < t_k < t_{k+1} < \ldots$. An estimate of x_k using $z^{(j)}$ is denoted by $\hat{x}_{k|j}$. Therefore, a filtered estimate is $\hat{x}_{k|k}$, while a predicted estimate is $\hat{x}_{k|j}$, $j < k$ and a smoothed estimate is $\hat{x}_{k|j}$, $j \geq k$.

The discrete time Kalman filtering technique used to obtain the optimal estimate $\hat{x}_{k|k}$ assumes the system governed by the following linear, vector difference equation

$$x_k = F(k,k-1) \cdot x_{k-1} + G(k,k-1) \cdot v_{k-1} \quad k = 1,2,3\ldots \qquad (1)$$

The state of the system at time t_k is given by the (n x 1) vector x_k. v_k is a (n x p) stochastic random vector, the input to the system at time t_k. F(k,k-1) is a (n x n) state transition matrix which relates the state at time t_{k-1} to the state at time t_k. G(k,k-1) is a (n x p) matrix that relates the stochastic inputs at time t_{k-1} to the state at time t_k. The states of the system are assumed measurable through output z_k, an (r x 1) observation vector at time t_k which is linearly related to the state, corrupted by an additive noise w_k, namely

$$z_k = H(k) \cdot x_k + w_k \quad . \qquad (2)$$

H(k) is an (r x n) observation matrix. The (r x 1) vector w_k is called an observation or measurement disturbance. The number of observations r may be smaller than the number of state variables n. This assumption is quite realistic and convenient for application since only a few measurements are often available for the system involving many states. The statistical properties of disturbance vectors v_k and w_k are assumed to be known as

$$\begin{aligned} E(v_k) &= 0 \\ E(v_k v_j^T) &= Q(k)\delta_{kj} \\ E(w_k) &= 0 \\ E(w_k w_j^T) &= R(k)\delta_{kj} \\ E(v_k w_j^T) &= 0 \qquad \text{for all } k \text{ and } j \end{aligned} \tag{3}$$

where δ_{kj} is the Kronecker's delta. Since the statistical properties of v_k and w_k are assumed to be fully represented by the means and covariences, their distributions are necessarily Gaussian. If this is not the case, as one might expect, the estimates obtained through the Kalman filter do not possess the various convenient properties such as maximum likelihood estimates. They will merely be the least squared estimates.

The governing system parameters (F,G,H), should be a priori known at each time, but not necessarily be constant. This latter property is one of the most important advantages of the Kalman filter in contrast to the Wiener filter which applies only to stationary processes.

The following is a brief sketch of the solution to the problem. Only the general concept is described in an intuitive fashion. Suppose that one has the observation $z^{(k-1)}$ at the beginning of time t_k and that one gets the optimal filtered estimate of x_{k-1} as $\hat{x}_{k-1|k-1}$. The first task is to obtain the best estimate of $\hat{z}_k$, the predicted observation. For this purpose, equation (2) suggests that the estimate of $\hat{x}_k$ be found. This can be obtained through equation (1), by substituting x_{k-1} with $\hat{x}_{k-1|k-1}$ and taking the expectation of v_k. The estimate is a predicted estimate $\hat{x}_{k|k-1}$, i.e.

$$\hat{x}_{k|k-1} = F(k,k-1) \cdot \hat{x}_{k-1|k-1} \quad . \tag{4}$$

This, in turn, gives the observation prediction

$$\hat{z}_{k|k-1} = H(k) \cdot \hat{x}_{k|k-1} \quad . \tag{5}$$

With this forecast at hand, one gets a new observation z_k at the end of time t_k. Naturally, the difference between the predicted and the observed is recognized. This difference is important information by which the state estimate can be updated from $\hat{x}_{k|k-1}$ to $\hat{x}_{k|k}$, which is the second task. In order to obtain the best filtered estimate $\hat{x}_{k|k}$, the Kalman filter assumes the relation

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K(k) \cdot (z_k - \hat{z}_{k|k-1}) \tag{6}$$

where K(k) is an (n x r) matrix called Kalman gain. $(z_k - \hat{z}_{k|k-1})$ is a prediction error, whose sequence is often referred to as an innovation sequence. The Kalman gain is determined under a criterion to minimize the squared state estimation error, namely

$$\min_{K(k)} \mathcal{E} \left\{ (x_k - \hat{x}_{k|k})^T \cdot (x_k - \hat{x}_{k|k}) \right\} \quad . \tag{7}$$

In general, this criterion is equivalent to the weighted mean square error criterion such that

$$\min_{K(k)} \mathcal{E} \left\{ (x_k - \hat{x}_{k|k})^T \cdot W_k \cdot (x_k - \hat{x}_{k|k}) \right\}$$

where W_k is an arbitrary (n x n) positive definite symetric matrix. It is obvious, therefore, that the Kalman filter is nothing but a least squared estimate. This criterion is equivalent to the orthogonality between $(x_k - \hat{x}_{k|k})$ and $z^{(k)}$. This simply implies that the estimate $\hat{x}_{k|k}$ is optimal if and only if, its error $(x_k - \hat{x}_{k|k})$ does not include any part that can be further explained by the information obtained up to time t_k. The complete solution to the problem is as follows

$$\hat{x}_{k|k-1} = F(k,k-1) \cdot \hat{x}_{k-k|k-1} \qquad (8)$$

$$\hat{z}_{k|k-1} = H(k) \cdot \hat{x}_{k|k-1} \qquad (9)$$

$$P_{k|k-1} = F(k,k-1) \cdot P_{k|k-1} \cdot F^T(k,k-1) + G(k,k-1) \cdot Q(k) \cdot G^T(k,k-1) \qquad (10)$$

$$K(k) = P_{k|k-1} \cdot H^T(k)[H(k) \cdot P_{k|k-1} \cdot H^T(k) + R(k)]^{-1} \qquad (11)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K(k)[z_k - \hat{z}_{k|k-1}] \qquad (12)$$

$$P_{k|k} = [I - K(k) \cdot H(k)] \cdot P_{k|k-1} \qquad (13)$$

$P_{i|j}$ is a covariance matrix of state estimation error $(x_i - \hat{x}_{i|j})$, i.e.

$$P_{i|j} = \mathcal{E}(x_i - \hat{x}_{i|j}) \cdot (x_i - \hat{x}_{i|j})^T \qquad (14)$$

Besides the system parameters $F(k,k-1)$, $G(k,k-1)$ and $H(k)$, and the covariance structure of random vectors $Q(k)$ and $R(k)$, the initial conditions $\hat{x}_{0|0}$ and $P_{0|0}$ are also assumed to be given.

Application to Cyclone Forecast Models

The Kalman filtering technique is applied to cyclone forecast models. The procedure is illustrated by using the 12 hour latitudinal prediction equation of the Arakawa's SFC.700 mb model as an example. It reads from Table 1

$$\varphi_{12} = -33.0 + 1.7710\varphi_0 - 0.7770\varphi_{-12} - 0.0076H_{147} + 0.0568X_{26} \qquad (15)$$

where φ_{12}, φ_0, φ_{-12} are the latitude (0.1 degree unit) of the cyclone eye at 12 hours ahead, at current time, and at 12 hours past, respectively.

H_{147} is 700 mb gph (in meters) at grid point 147 at current time, and

X_{26} is surface pressure (in mb) at grid point 26 at current time.

The location of grid point relative to the cyclone eye is defined in Figure 1. In the Kalman filter application, the coefficients of the regression model are treated time variant. To make this model compatible with the theory described in the previous section, the equation is rewritten in the following way:

$$\varphi_{12} = [1 \quad \varphi_0 \quad \varphi_{-12} \quad H_{147} \quad X_{26}] \begin{bmatrix} -33.0 \\ 1.7710 \\ -0.7770 \\ -0.0076 \\ 0.0568 \end{bmatrix} + w \qquad (16)$$

where φ_{12} is now the real observation and w is the observation error. Equation (16) is considered the measurement equation

$$z_k = H(k) \cdot x_k + w_k \qquad (17)$$

where $z_k = \varphi_{12}$ is the cyclone eye position at time t_k,
$H(k) = [1 \quad \varphi_0 \quad \varphi_{-12} \quad H_{147} \quad X_{26}]$ is the observation matrix at time t_k composed of the observations obtained at time t_{k-1}.

The (5 x 1) vector x_k is the coefficient vector, now treated as a time variant state vector, whose change is governed by the system transition equation

$$x_{k+1} = F(k+1,k) \cdot x_k + G(k+1,k) \cdot v_k \qquad (18)$$

The initial estimate of the state $\hat{x}_{0|0}$ is regarded as the given coefficient vector,

$$\hat{x}_{0|0} = \begin{bmatrix} -33.0 \\ 1.7710 \\ -0.7770 \\ -0.0076 \\ 0.0568 \end{bmatrix} \tag{19}$$

The transition matrices F and G can be considered identity matrices since the true transition structure is unknown and the changes are supposedly gradual. If the process is considered first order Markovian, F could be calculated from correlation matrices such that

$$F(k+1,k) = \mathcal{E}(x_{k+1} \cdot x_k^T) \cdot (x_k \cdot x_k^T)^{-1} \quad . \tag{20}$$

This approach is, however, unfeasible here for the determination of F since the sample states are not available.

The question left is the statistical structure of disturbance sequences v_k and w_k. In theory, they should be zero mean white sequences. The assumption of zero mean may be reasonable since the original model (15) is already in good approximation. On the other hand, the disturbance sequences are quite likely to be sequentially correlated. This is obvious from the fact that the forecasts of the original model are often biased to the same direction for consecutive periods, as mentioned in the section on Methodology. This difficulty along with the problem of unknown covariances Q and R are considered in the next section.

Shaping Filter and Adaptive Filter

A linear dynamical system whose output u_k has zero mean and covariance matrix D_{jk}, when the input μ_k is a zero mean white noise, is called a 'shaping filter'. This filter can be used as a converter from a colored sequence to a white sequence. Consider linear differential dynamical systems

$$v_k = W_1(k,k-1) \cdot v_{k-1} + W_2(k,k-1) \cdot \theta_{k-1} \tag{21}$$

$$w_k = W_3(k,k-1) \cdot w_{k-1} + W_4(k,k-1) \cdot \omega_{k-1} \tag{22}$$

where θ_k and ω_k are zero mean white noise processes having the covariance $Q_{\theta(k)}$ and $R_{\omega(k)}$. The dimensions of vectors θ_k and ω_k are the same as those of v_k and w_k, respectively. The transition matrices $W_{i(k,k-1)}$'s have to be determined so that the covariance structure of v_k and w_k are preserved. Similarly to the transition matrices of F and G, however, W's cannot be analytically determined in the cyclone problem. They are all assumed identity matrices here, since the sequential correlations of v_k and w_k are likely to be very high and their changes gradual.

Now the shaping filter developed and the original formulation (17) and (18) should be combined. The three state transition systems (18), (21) and (22)

$$x_k = F(k,k-1) \cdot x_{k-1} + G(k,k-1) \cdot v_{k-1}$$

$$v_k = W_1(k,k-1) \cdot v_{k-1} + W_2(k,k-1) \cdot \theta_{k-1}$$

$$w_k = W_3(k,k-1) \cdot w_{k-1} + W_4(k,k-1) \cdot \omega_{k-1}$$

can be rewritten as

$$\begin{bmatrix} x_k \\ v_k \\ w_k \end{bmatrix} = \begin{bmatrix} F(k,k-1) & G(k,k-1) & 0 \\ 0 & W_1(k,k-1) & 0 \\ 0 & 0 & W_3(k,k-1) \end{bmatrix} \begin{bmatrix} x_{k-1} \\ v_{k-1} \\ w_{k-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ W_2(k,k-1) & 0 \\ 0 & W_4(k,k-1) \end{bmatrix} \begin{bmatrix} \theta_{k-1} \\ \omega_{k-1} \end{bmatrix} \tag{23}$$

The observation equation (17) becomes

$$z_k = \begin{bmatrix} H(k) & 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ v_k \\ w_k \end{bmatrix} \tag{24}$$

Redefining the vectors and matrices as

$$\mathbf{x}_k \equiv \begin{bmatrix} x_k \\ v_k \\ w_k \end{bmatrix} \quad : \quad \text{dimension } (2n + 1) \times 1 \tag{25}$$

$$\mathbf{F}(k,k-1) \equiv \begin{bmatrix} F(k,k-1) & G(k,k-1) & 0 \\ 0 & W_1(k,k-1) & 0 \\ 0 & 0 & W_3(k,k-1) \end{bmatrix} \tag{26}$$

$: (2n + 1) \times (2n + 1)$

$$\mathbf{G}(k,k-1) \equiv \begin{bmatrix} 0 & 0 \\ W_2(k,k-1) & 0 \\ 0 & W_4(k,k-1) \end{bmatrix} : (2n + 1) \times (n + 1) \tag{27}$$

$$\mathbf{v}_k \equiv \begin{bmatrix} \theta_k \\ \omega_k \end{bmatrix} \quad : (n + 1) \times 1 \tag{28}$$

$$\mathbf{H}(k) \equiv [H(k) \quad 0 \quad I] \quad : \quad r \times (2n + 1) \quad , \tag{29}$$

the equation (23) and (24) returns to a simple form

$$x_k = F(k,k-1) \cdot x_{k-1} + G(k,k-1) \cdot v_{k-1} \tag{30}$$

$$z_k = H(k) \cdot x_k \quad . \tag{31}$$

The dimensions of 0 and I are omitted but should be obvious. It is worth noting that the observation equation does not have any explicit noise terms. They are now imbedded in the state equations. The covariance of the zero mean white noise sequence v_k is, as already defined

$$Q(k) = \begin{bmatrix} Q_\theta(k) & 0 \\ 0 & R_\omega(k) \end{bmatrix} \quad : \quad (n+1) \times (n+1) \quad . \tag{32}$$

The corresponding Kalman filter solution is exactly the same as those given in the previous section with R(k) being a zero matrix[2)].

The only question remaining now is the determination of unknown noise covariance Q(k), which is huristically treated in this analysis based on the following facts. The Kalman filter is a linear filter applicable to nonstationary processes. Nonstationary process has, by definition, time variant statistical properties. In fact, the covariance of disturbance sequence v_k is defined as Q(k) which is a function of time. This matrix, however, cannot be determined a priori unless the physical process involved is statistically known or some prediction procedure for error covariances available. In the cyclone problem, neither is the case. Instead of prediction, however, the updating procedure is available, which is called an 'adaptive filter'. An adaptive filter updates the unknown parameters of the system during the operation of a model, using

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2) The condition $R(k) = 0$ implies $H(k) \cdot P_{k|k} \cdot H^T(k) = 0$ which could lead to some computational difficulty, and can be avoided by the method described in (Jazwinski, 1970. p.213). This difficulty was, however, not encountered in this cyclone analysis.

the predicted measurement errors as new information.

The various techniques of adaptive filters are available (Mehr, 1972). Among them is one that has practical computational feasability and is based on the so-called 'innovation sequence' defined as $\{\nu_k\}$ such that

$$\nu_k = z_k - H(k) \cdot \hat{x}_{k|k-1} \quad . \tag{33}$$

It can be shown that if the optimal filter is used for the estimation of $\hat{x}_{k|k}$ with respect to the least squared error criterion, the innovation sequence should be a zero mean white noise sequence (Kailath, 1968). If in reality the innovation sequence is found to be correlated, the filter used must be suboptimal rather than optimal. This fact, in turn, can be used to improve the filter and the estimates of error covariance matrices Q and R.

Two difficulties are involved in this line of development. One is that a number of ν_k observations are necessary to obtain a reasonable correlation covariance of ν_k sequence, while in a cyclone case more than ten ν_k's are seldom available.

The other difficulty is more profound, that is, most of the theories currently available deal with the time variant system. The system can be parameterized as (F,G,H) where in this paper the observation matrix H varies over time. An example of a theory dealing with a time variant case is in Jazwinski, (1970, p.311), whose performance is doubtful when the number of stages of forecast is limited as in the cyclone problem.

Considering such circumstances, the following simple procedure is used in this analysis where the covariance structure is considered time invariant. First, arbitrary but intuitively reasonable covariance matrices are assumed, that is

$$Q_\theta(k) = \alpha^2 \begin{bmatrix} (\hat{x}^1_{0|0})^2 & & & & 0 \\ & (\hat{x}^2_{0|0})^2 & & & \\ & & (\hat{x}^3_{0|0})^2 & & \\ & & & (\hat{x}^4_{0|0})^2 & \\ 0 & & & & (\hat{x}^5_{0|0})^2 \end{bmatrix} \tag{34}$$

$$Q_\omega(k) = \beta^2 \left[\operatorname*{mean}_k \, (z_k - \hat{z}_{k|k-1})\right]^2 \tag{35}$$

where $\hat{x}^i_{0|0}$ is the i-th component of the initial estimate of vector x_0, given equation (19), α and β are the scalar parameters to be optimized. The mean value of $(z_k - \hat{z}_{k|k-1})$ is an unknown a priori but the prediction accuracy of the original model is used as a rough estimate. The choice of these particular forms is based on the assumption that the variance of a variable should roughly be proportional to the magnitude of the value of the variable; namely, the variance of a variable having a large absolute value is quite likely larger than that of having a small absolute value. This assumption does not hold in general, but must be reasonable under various practical situations. Note that the covariance of ω_k is set not proportional to z_k, but to its estimate error since the error magnitude is roughly known while the estimate error of x_k is completely unknown before the calculation.

The next task is to optimize the parameter α and β, which is performed under the criterion

$$\min_{\alpha,\beta} \frac{1}{N} \sum_{k=1}^{N} \left| z_k - \hat{z}_{k|k-1} \right| \tag{36}$$

where N is the number of prediction cases. Once an optimal set of α and β are obtained by considering several cyclones, they are set constant for other cyclones.

The last problem is to determine the initial estimate of $P_{0|0}$, the error covariance of state variables. Since the Kalman filter solution (8) and (13) includes the updating of $P_{k|k}$, the accuracy of its initial value is less important than Q (and R). A similar assumption as applied to equations (33) and (34) is used for this matrix, too; that is

$$P_{0|0} = \begin{bmatrix} P_1 & & 0 \\ & P_2 & \\ 0 & & P_3 \end{bmatrix} \tag{37}$$

where

$$P_1 = \gamma_1^2 \cdot \begin{bmatrix} (\hat{x}^1_{0|0})^2 & & & 0 \\ & (\hat{x}^2_{0|0})^2 & & \\ & & \ddots & \\ 0 & & & (\hat{x}^5_{0|0})^2 \end{bmatrix}$$

$$P_2 = \gamma_2^2 \cdot \begin{bmatrix} (\hat{x}^1_{0|0})^2 & & 0 \\ & \ddots & \\ 0 & & (\hat{x}^5_{0|0})^2 \end{bmatrix}$$

$$P_3 = \gamma_3^2 \cdot \left[\underset{k}{\text{mean}} \ (z_k - \hat{z}_{k|k-1})\right]$$

About the magnitude of parameters α, β, γ_1, γ_2 and γ_3, the equations (17), (18), (21) and (25) suggest that the following relation may quite likely hold

$$\begin{cases} \gamma_1 > \alpha \simeq \gamma_2 \\ \gamma_3 > \beta \end{cases} \quad . \tag{38}$$

It is simply because γ_1 and γ_3 are the coefficients of the error covarience of x_k and z_k, while the others are the coefficients of the error covariances of noises attached to x_k and z_k. Finally, it should be noted that all the error covariances are assumed diagonal matrices, since the cross correlations of state variables and their noises are absolutely unknown and quite likely smaller in magnitude than those of variances.

Returning to the original shaping filter formulation (21) and (22), one can conceive of simpler cases where the observation disturbances are not correlated while the system disturbances are.

This situation may occur in the cyclone forecast when the highly correlated prediction errors of the original model are mostly due to the fixed coefficients and not due to the correlated observation errors. In this case, the shaping filter to the observation disturbance (22) can be omitted, resulting in the corresponding system and measurement equations:

$$\underset{(2n \times 1)}{\begin{bmatrix} x_k \\ v_k \end{bmatrix}} = \underset{(2n \times 2n)}{\begin{bmatrix} F(k,k-1) & 0 \\ 0 & W_1(k,k-1) \end{bmatrix}} \underset{(2n \times 1)}{\begin{bmatrix} x_{k-1} \\ v_{k-1} \end{bmatrix}} + \underset{(2n \times n)}{\begin{bmatrix} 0 \\ W_2(k,k-1) \end{bmatrix}} \cdot \underset{(n \times 1)}{\theta_k} \qquad (39)$$

$$\underset{(1 \times 1)}{z_k} = \underset{(1 \times 2n)}{[H(k) \quad 0]} \underset{(2n \times 1)}{\begin{bmatrix} x_k \\ v_k \end{bmatrix}} + \underset{(1 \times 1)}{w_k} \quad . \qquad (40)$$

Redefining the vectors and matrices as

$$x_k \equiv \begin{bmatrix} x_k \\ v_k \end{bmatrix}$$

$$F(k,k-1) \equiv \begin{bmatrix} F(k,k-1) & 0 \\ 0 & W_1(k,k-1) \end{bmatrix}$$

$$G(k,k-1) \equiv \begin{bmatrix} 0 \\ W_2(k,k-1) \end{bmatrix}$$

$$v_k \equiv \theta_k$$

$$H(k) \equiv [H(k) \quad 0]$$

(39) and (40) become

$$x_k = F(k,k-1) \cdot x_{k-1} + G(k,k-1) \cdot v_{k-1} \qquad (41)$$

$$z_k = H(k) \cdot x_k + w_k \tag{42}$$

which are identical to the original forms (1) and (2). The error covariances are now assumed

$$Q(k) = Q_\theta(k) \tag{43}$$

$$R(k) = R_\nu(k) \tag{44}$$

$$P_{0|0} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \tag{45}$$

where Q_θ, R_ν, P_1 and P_2 are given in (34), (35) and (37). In these covariance matrices the number of unknown parameters are four, i.e. α, β, γ_1 and γ_2. A corresponding foreseeable relation (38) becomes

$$\gamma_1 > \alpha \simeq \gamma_2 \quad . \tag{46}$$

The model represented by (39) and (40) will be referred to as the filtered model A, while the model represented by (23) and (24) is referred to as the filtered model B.

IV. PRELIMINARY COMPUTATIONAL RESULTS [3)]

Computational Procedure

The purpose of the computational experiments is to demonstrate the feasibility of filtered models and their performances and not necessarily to establish an operational program ready for implementation. The number of cases used for experimentation are, therefore, limited to the least satisfactory amounts. The optimization of parameters is also undertaken only to a crude extent.

The typhoon data examined are only those which occurred in August 1974. During this month, five typhoons occurred. Typhoon No.14 (TYPH7414) lasted 15 days and others only two to

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3) Based only on the typhoons which occurred in August 1974. Results based on more data will be reported in the forthcoming paper.

five days. The amount of data are thus very unsatisfactory, but nevertheless they are enough to withdraw the basic information necessary to make a preliminary evaluation of the models developed in this paper.

Regarding the optimization of error covariance parameters, $(\alpha, \beta, \gamma_1, \gamma_2)$ for filtered model A and $(\alpha, \beta, \gamma_1, \gamma_2, \gamma_3)$ for filtered model B, the following strategy was taken:

1. To an estimation of mean $(z_k - z_{k|k-1})$, 0.8 degree is used regardless of the forecast models. This value is the 12 hour prediction error associated with the simple persistence method reported in the Typhoon Forecast Manual (Japan Weather Association, 1974, p.76, Table 4.1.2). But it is used for all models since this value is simply a normalizing factor to value β and its choice does not have any substantial effect on the model performance as long as β is chosen properly.

2. The parameter optimization was undertaken through random sampling with intuitive judgement. The foreseeable relationships among parameters, (46) for filtered model A and (38) for filtered model B were not necessarily strictly considered, since the examination of the validity of these relations were also in question.

3. Although the criterion is formally expressed as

$$\min_{\alpha,\beta,\gamma} \frac{1}{N} \sum_{k=1}^{N} \left| z_k - \hat{z}_{k|k-1} \right| ,$$

this is not operational since all possible combinations of α, β, γ can by no means be examined.

Instead, a crude stopping rule was used, that is, the optimization procedure terminates when the first satisfactory results are encountered. The satisfactory level is, of course, very subjective, but nevertheless, this approach is considered reasonable in the light of the purpose of this paper. For practical use, a further detailed search of better parameters may be necessary.

4. The parameter optimization was conducted only for the data of TYPH7414. The other data are used as independent cases. The TYPH7414 was selected because it had the longest

duration of all the typhoons and its behaviour was complex enough (as seen in Figure 4) to be a good representative of other typhoons.

Results and Evaluation

The first series of results are those pertaining to the parameter optimization, or the model calibration. Table 4 lists the 'optimal' parameters and corresponding forecasts. This optimality is by no means the real optimality due to the limited number of trials. One immediately notices that the forecast improvement of 12 hours SFC.700 model for φ (latitude) is minus, meaning worse than the original model. For this particular model, many combinations of parameters were examined, yet better parameters were not found except for a set of parameters which leads to the filtered model identical to the original model, that is, $\alpha = 0$ (and $\beta = 0$ in model B), or $Q = 0$, implying no system error. The case of $Q = 0$ was not considered as optimal since it did not use Kalman filter algorism at all, although in practice the condition $Q = 0$ may be more preferable than the parameters listed in Table 4.

A careful examination of the performance of the filtered model reveals the fact that when the original model has large prediction errors, the filtered model works very well. For instance, 12 hours SFC.700 model for λ (longitude) has original mean absolute error 1.68 degree while that for φ has 0.7 degree and the improvement in λ is remarkable. The same applies to 24 hours SNT model. The φ model has the error 3.27 degree while the λ model has 1.82 degree, and the improvement in φ is much larger than in λ. The question is then why. In Table 5, a comparison of observations and predictions by the SFC.700 and the filtered model A, gives a clue. A surprising difference can be seen in the predictions of the original models for λ and φ, that is, all the λ errors have minus signs or the constant bias to the east side, while the φ errors have both signs. This implies that the λ errors are highly correlated while the errors are not. It may also be correct to say that the φ errors are closer to the white sequences than the φ errors. When the residual errors are white, it is difficult to improve the model by any updating procedure based on past histories. The Kalman filter is by no means an exception. In fact, the residual errors of the filtered model show quasi-whiteness.

Even if a small decline (a minus improvement) is obtained in the predictions whose original errors are small, however, the overall prediction in terms of vector error will not be heavily affected. Because the improvement in the predictions whose original errors are large is much larger than just to compensate its partner's decline. The vector errors are reduced by 30% and 45% in 12 hours and 24 hours respectively. Such reduction is certainly substantial.

This overall improvement is more obvious in Figures 4 and 5 which show only the model A results. The filtered model is considered practical since in a wandering part of the cyclone track, from the 20th to 23rd August, the forecast errors by filtered model are considerably smaller than those by the original model. This part is the most difficult part in practical forecasting.

With respect to the difference between filtered models A and B, no significant differences can be observed. The small difference may be attributed to the difference in the extent of parameter optimization. It is, however, too soon to conclude that the measurement disturbances are independent. The filtered model B is simply incapable of taking away the various errors involved in the original model. Such errors include the non-linearity of the cyclone behaviour and the time variant nature of error covariances and the model structure. Because model A is superior to B in perthimony (fewer number of parameters), only model A will be used from now on.

The sensitivity of covariance parameters is found to be quite high. To illustrate parameter effects on the forecast, only the examples from the 12 hours SFC.700 through filtered model A are listed in Table 6. From this table, one notices that the relation $\gamma_1 > \alpha$ (and $\gamma_3 > \beta$ in model B) holds, but $\alpha \simeq \gamma_2$ is not necessarily true. This may be attributed to the different sensitivities of various elements in covariance matrices, of which one has higher influence to the forecast than others. In this case the parameter attached to this particular element becomes more important than to others, and the relation $\alpha \simeq \gamma_2$ may not hold since the most important elements in two matrices do not necessarily coincide.

The optimized parameters have quite interesting matches in different models, that is, α is always 0.001 and β is 3.0. Although these values are not quite optimal and accordingly the credibility of this match is not so high, still this fact implies that the error covariances of systems noises and measurement noises are similar in magnitudes in the four models. Only the goodness of initial estimates of the states, $P_{0|0}$ is different. It may not be true, however, for the prediction models of even longer periods, such as 48 hours or 72 hours.

The second series of results are those for the independent cases which are not used for parameter determination. Unfortunately, there is not enough data yet to be examined. Only four other short period typhoons are examined. They are TYPH 13, 15 16 and 17. As a total, 34 12 hour forecasts and 10 24 hour forecasts are made. Tables 7 and 8 list the summary of the results.

In the 12 hour forecasts by SFC.700 model, λ error is originally 1.5 degrees and φ error is a 0.7 degree. Therefore, the worse model λ was improved by 29% whereas there was an improvement of only 1% in the better model φ. The case of TYPH15 shows a large decline in the prediction but it should be considered as a sample variation. Only the total mean of this small set of 35 forecasts has significance if any at all. In this sense, the improvement in vector error is more realistic measure, which shows a remarkable improvement of 31%. In real distance this stands for the vector error reduction from 220 km to 150 km on an average. TYPH16 and 17 are plotted in Figures 6 and 7. From these figures, it can be concluded that the improvement is significant.

In the 24 hour SNT forecast, the forecast of λ is more difficult than φ. The original mean absolute error was 1.72

degree versus 3.74 degree. In fact, the errror in prediction λ was not improved at all but even became worse by 2% as a total mean. The TYPH17 had the worst case, about 20% decline in errors. The prediction of φ is remarkable which leads to a 50% vector error reduction as a total, implying the distance error reduction from 450 km to 220 km on an average. The significance of this improvement is more clearly seen in Figures 8 and 9.

Conclusions

Despite that the number of typhoon cases used for parameter optimization and for predictions is extremely limited, the following can be concluded:

1. The Kalman filtering approach to get better performance in existing models is promising.
2. The magnitude of improvement depends upon the performance of the original model. In the cases when the residual errors of the original model are small and little correlated, the improvement by the Kalman filter is little, but when the original model's predictions are inaccurate and errors are highly correlated, the improvement is substantial.
3. Since the cases in which both λ and φ models predict accurately are rare, the vector errors are more often improved than each component error. In the 12 hour forecast based on the SFC.700 model, the vector error reduction was roughly 30% and in the 24 hour forecast, based on the SNT model, it was roughly 50%.
4. Since the original model predictions are inaccurate when a typhoon movement is stagnated in a small area, the filtered model can improve the predictions substantially. This fact is important and beneficial to the practical application of the filtered model.
5. The shaping filter applied to the measurement noise does not give significant improvement in results.

Table 1

Description of 12 hours SFC.700 Model

φ (Latitude)		λ (Longitude)	
Predictor	Coefficient	Predictor	Coefficient
Const.	-33.0	Const.	-32.4
φ_0	1.7710	λ_0	1.7792
φ_{-12}	-0.7770	λ_{-12}	-0.7922
H_{147}	-0.0076	H_{124}	-0.0076
X_{26}	0.0568	X_{89}	0.0576

Note φ, λ : 0.1 degree unit.

H : 700 mb geopotential height in meters.

X : surface pressure in mb.

Table 2

Description of 24 hours SNT model

φ (Latitude)		λ (Longitude)	
Predictor	Coefficient	Predictor	Coefficient
Const.	-16.515	Const.	-1.894
φ_0	2.089	λ_0	2.639
φ_{-12}	-1.142	λ_{-12}	-1.705
Z_{26}	-0.248	Z_{25}	-0.102
Z_{16}	0.121	Z_{56}	0.188
Z_{44}	0.103	Z_{26}	-0.126
Z_{13}	0.049	$\Delta\lambda(T_0 - T_{-24})$	-0.314
Z_{56}	0.189	Z_{42}	0.159
Z_{58}	-0.156	Z_{31}	-0.095

Note φ : 0.1 degree unit

λ : (longitude -100) in 0.1 degree (ex. 135.8E → 358).

Z : (500mbgph - 5000) in meter (ex. 5765gpm → 765).

Table 3

Description of Data Supply

Data (period)	Arrangements
From the Japan Meteorological Agency	
1. Observed Typhoon eye location (whole 1974)	every 12 hours (at 0 and 12 GMT).
2. Observed surface pressure and 700 mb gph distribution (June ~ September 1974)	every 12 hours (at 0 and 12 GMT) at 381 km square grid points over the Northern Hemisphere.
3. Prognostic 250 mb and 500 mb gph distribution (June ~ September 1974)	12 hour prognosis at 0 GMT and 24 hour prognosis at 12 GMT at 509.6 km square grid points over the Northern Hemisphere.
From the US National Hurricane Center	
4. Observed Hurricane eye location (1945 ~ 1974)	every 12 hours (at 0 and 12 GMT).
5. Observed 500, 700 and 1000 mb gph distribution (1945 ~ 1974)	every 12 hours (at 0 and 12 GMT) at 5^{o} square grid points on moving coordinates.

Note: 500 mb prognostic gph was calculated through linear approximation:

$$500 \text{ mb gph} = \frac{1}{6} \cdot 250 \text{ mb gph} + \frac{5}{6} \cdot 550 \text{ mb gph}$$

Table 4

Calibrated Parameters and Corresponding Forecast Improvements

Model	α	β	γ_1	γ_2	γ_3	Improvement in degree (filtered/original)	Improvement in distance (km) (filtered/original)
SFC.700(A)							
λ	0.001	3.	1.0	0.1		62%(0.65/1.68)	40%(115/192)
φ	0.001	3.	0.01	0.001		-3%(0.72/0.7)	
SFC.700(B)							
λ	0.001	2.0	1.0	0.1	3.	57%(0.74/1.68)	38%(119/192)
φ	0.001	2.0	0.01	0.001	3.	-4%(0.73/0.7)	
SNT(A)							
λ	0.001	2.	0.01	0.001		23%(1.41/1.82)	40%(223/414)
φ	0.001	2.	0.1	0.01		60%(1.32/3.27)	
SNT(B)							
λ	0.001	2.	1.	0.1	3.	17%(1.51/1.82)	44%(232/414)
φ	0.001	1.	0.01	0.001	2.	57%(1.41/3.27)	

Table 5

Comparison of Observations and Prediction errors by SFC.700 model

	M	D	H	LONGITUDE OBSERVED	LONGITUDE SFC.700	(error)	LONGITUDE FILTERED	(error)	LONGITUDE IMPROVE	LATITUDE OBSERVED	LATITUDE SFC.700	(error)	LATITUDE FILTERED	(error)	LATITUDE IMPROVE	VECTOR ERROR SFC.700	VECTOR ERROR FILTERED
0	8	12	0	153.00	151.53	-1.47	151.53	-1.47	0.00	17.80	18.45	0.65	18.45	0.65	0.00	171.31	171.31
1	8	12	12	151.00	149.55	-1.45	151.02	0.02	1.43	19.10	18.07	-1.03	18.06	-1.04	-0.01	190.60	115.67
2	8	13	0	149.20	147.56	-1.64	148.98	-0.22	1.42	20.40	20.07	-0.33	20.08	-0.32	0.01	174.98	42.00
3	8	13	12	147.40	145.95	-1.45	147.64	0.24	1.21	21.90	21.35	-0.55	21.37	-0.53	0.02	161.61	63.94
4	8	14	0	145.20	144.18	-1.02	145.71	0.51	0.51	23.60	23.00	-0.60	23.05	-0.55	0.04	123.57	80.76
5	8	14	12	143.40	141.74	-1.66	142.88	-0.52	1.14	25.10	24.85	-0.25	24.93	-0.17	0.08	169.31	55.90
6	8	15	0	142.10	140.29	-1.81	141.69	-0.41	1.39	26.50	26.18	-0.32	26.28	-0.22	0.10	183.41	47.81
7	8	15	12	140.30	139.44	-0.86	141.07	0.77	0.09	26.80	27.50	0.70	27.64	0.84	-0.13	115.28	120.00
8	8	16	0	138.30	137.25	-1.05	138.51	0.21	0.83	26.90	26.95	0.05	27.00	0.10	-0.05	104.11	24.09
9	8	16	12	136.40	135.11	-1.29	136.23	-0.17	1.12	27.30	26.89	-0.41	26.92	-0.38	0.03	135.72	45.41
10	8	17	0	133.90	133.30	-0.60	134.44	0.54	0.07	28.10	27.53	-0.57	27.62	-0.48	0.09	86.71	74.91
11	8	17	12	132.00	130.35	-1.65	131.23	-0.77	0.88	28.60	28.62	0.02	28.80	0.20	-0.18	160.80	78.52
12	8	18	0	130.20	128.95	-1.25	130.05	-0.15	1.10	28.50	28.88	0.38	29.03	0.53	-0.16	129.01	60.96
13	8	18	12	127.30	127.26	-0.04	128.39	1.09	-1.05	28.60	28.30	-0.30	28.36	-0.24	0.06	33.71	109.88
14	8	19	0	124.50	123.49	-1.01	124.30	-0.20	0.81	28.70	28.54	-0.16	28.65	-0.05	0.11	99.78	20.46
15	8	19	12	122.30	120.80	-1.50	121.60	-0.70	0.80	28.80	28.64	-0.16	28.76	-0.04	0.12	147.32	68.33
16	8	20	0	120.70	119.12	-1.58	120.01	-0.69	0.89	29.00	28.74	-0.26	28.87	-0.13	0.13	156.46	68.26
17	8	20	12	120.00	118.04	-1.96	119.08	-0.92	1.04	28.70	29.01	0.31	29.18	0.48	-0.17	194.13	104.20
18	8	21	0	119.60	118.08	-1.52	119.51	-0.09	1.43	28.00	28.31	0.31	28.38	0.38	-0.07	153.42	43.62
19	8	21	12	120.50	117.94	-2.56	119.49	-1.01	1.55	27.60	27.31	-0.29	27.30	-0.30	-0.01	254.40	104.80
20	8	22	0	121.40	119.86	-1.54	122.09	0.69	0.85	28.60	27.16	-1.44	27.20	-1.40	0.04	220.28	169.66
21	8	22	12	122.50	120.78	-1.72	122.77	0.27	1.45	29.10	29.25	0.15	29.58	0.48	-0.33	167.48	59.25
22	8	23	0	124.50	122.02	-2.48	123.99	-0.51	1.97	28.40	29.36	0.96	29.62	1.22	-0.25	263.88	143.94

errors (longitude) ↑ ↑ errors (latitude) ↑ ↑

Table 5 -- continued

						errors ↓		↓				errors ↓		↓			
23	8	23	12	125.80	124.71	-1.09	126.99	1.19	-0.10	27.50	27.74	0.24	27.74	0.24	0.00	110.24	120.22
24	8	24	0	127.40	125.42	-1.98	127.38	-0.02	1.95	26.80	26.70	-0.10	26.64	-0.16	-0.06	196.80	18.61
25	8	24	12	130.20	127.26	-2.94	129.27	-0.93	2.01	26.10	26.18	0.08	26.14	0.04	0.04	293.34	93.04
26	8	25	0	132.90	130.96	-1.94	133.28	0.38	1.56	26.80	25.52	-1.28	25.46	-1.34	-0.06	239.95	153.67
27	8	25	12	135.70	133.59	-2.11	135.93	0.23	1.88	29.80	27.33	-2.47	27.51	-2.29	0.19	343.71	255.17
28	8	26	0	137.50	136.43	-1.07	138.80	1.30	-0.22	34.40	32.13	-2.27	32.85	-1.55	0.72	271.50	209.96
29	8	26	12	144.00	137.46	-6.54	139.50	-4.50	2.04	40.90	37.92	-2.98	39.25	-1.65	1.33	651.86	424.71
30	8	27	0	148.80	147.61	-1.19	148.61	-0.19	1.00	44.00	45.88	1.88	48.16	4.16	-2.28	229.34	463.13
						1.68		0.65	1.04			0.70		0.72	-0.02	192.09	114.70
																	(40.29% RED.)

Table 6

Parameter Sensitivity to Forecasts:

12 hours SFC.700(A)

λ

α	β	γ_1	γ_2	Degree of improvement (original error: 1.68°)	
0.001	2.	1.	0.1	1.00°	
0.0001	2.	1.	0.1	0.96	
0.01	2.	1.	0.1	0.89	
0.001	1.	1.	0.1	0.94	
0.001	3.	1.	0.1	1.04	: selected
0.001	4.	1.	0.1	0.92	
0.001	2.	0.1	0.01	0.91	
0.001	2.	2.	0.2	0.95	

φ

				(original error : 0.7°)	
0.001	2.	0.01	0.001	-0.02°	
0.0001	2.	0.01	0.001	-0.08	
0.01	2.	0.01	0.001	-0.08	
0.001	1.	0.01	0.001	-0.03	
0.001	3.	0.01	0.001	-0.02	: selected
0.001	4.	0.01	0.001	-0.11	
0.001	2.	0.001	0.0001	-0.03	
0.001	2.	0.1	0.01	-0.05	

Table 7

12 hours SFC.700 forecast errors of independent typhoons

TYPH	number of forecasts made	λ error (degree original	λ error (degree filtered	φ error (degree) original	φ error (degree) filtered
13	8	1.17	0.44	0.62	0.58
15	4	0.76	1.32	1.55	1.46
16	11	1.52	0.87	0.58	0.62
17	11	2.17	1.74	0.61	0.61
mean	34 (Total)	1.56	1.10	0.71	0.71
(error reduction)		(29.2%)		(0.9%)	

TYPH	Vector error (km) original	Vector error (km) filtered
13	147	82.8
15	198	228
16	176	121
17	236	199
mean	217	150
	(31%)	

Table 8

24 hours SNT forecast errors for independent tyhpoons

TYPH	number of forecasts made	λ error (degree) original	λ error (degree) filtered	φ error (degree) original	φ error (degree) filtered
13	2	2.37	2.10	3.69	1.26
16	4	1.39	1.27	3.20	0.69
17	4	1.73	2.05	4.05	1.16
mean	10 (Total)	1.72	1.76	3.74	1.00
(error reduction)		(-2.0%)		(72.7%)	

TYPH	Vector error(km) original	Vector error(km) filtered
13	474	256
16	384	159
17	498	269
mean	447	222
	(50.4%)	

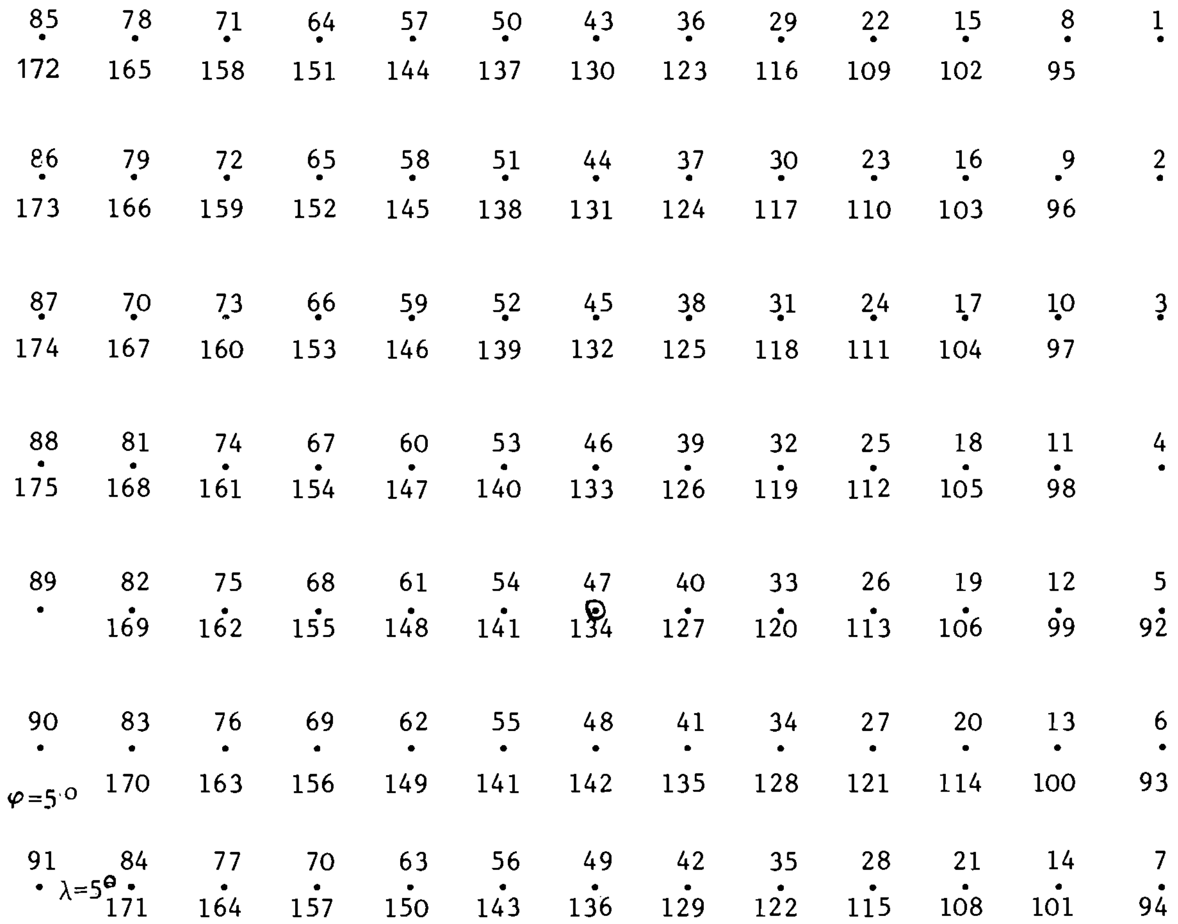


Figure 1 The grid system is used to express the distribution of the surface pressures and teh 700 mb gph for the SFC.700 mb model.

(The center of a typhoon is located at point 47. Grid spacing is 5° both in latitude and longitude).

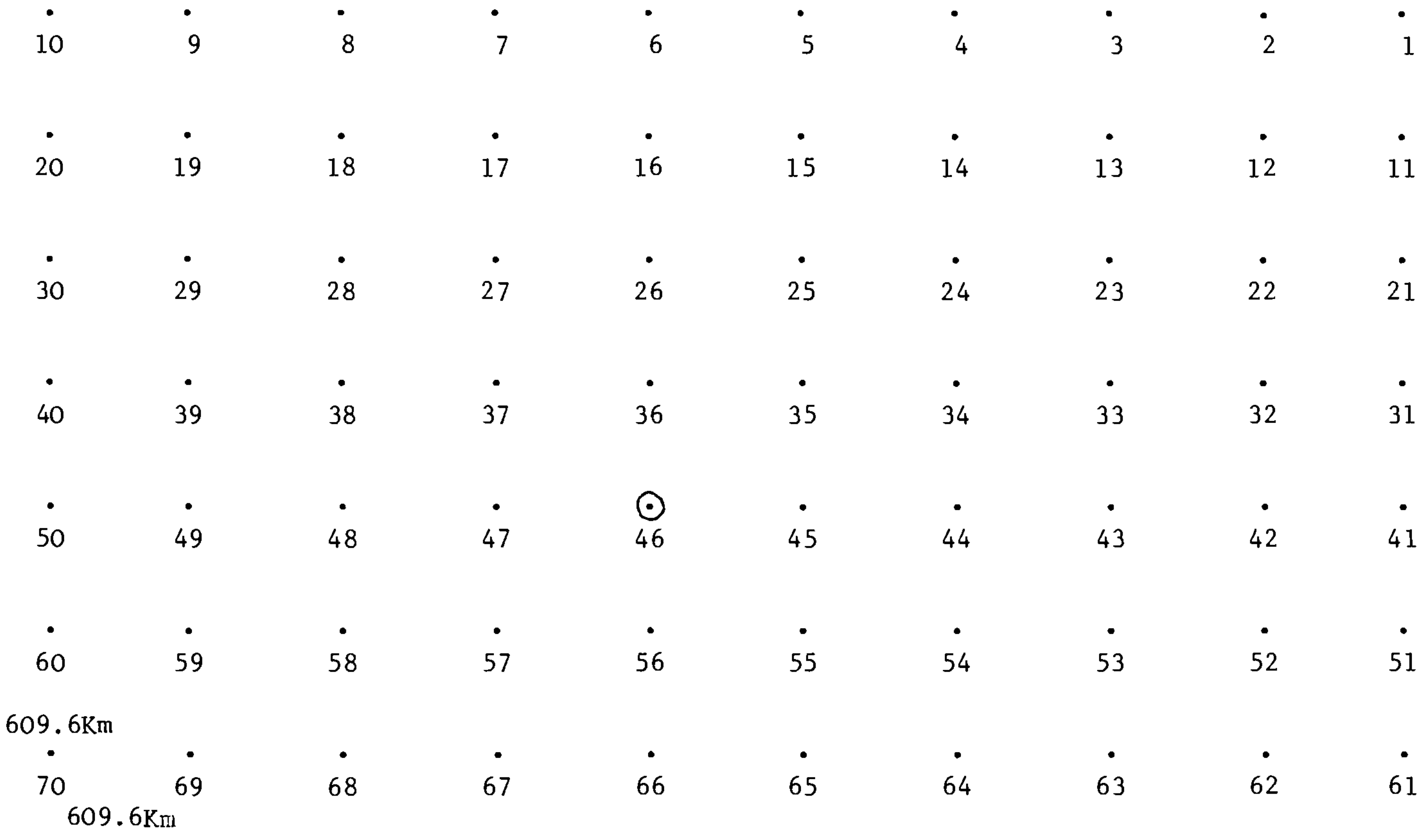
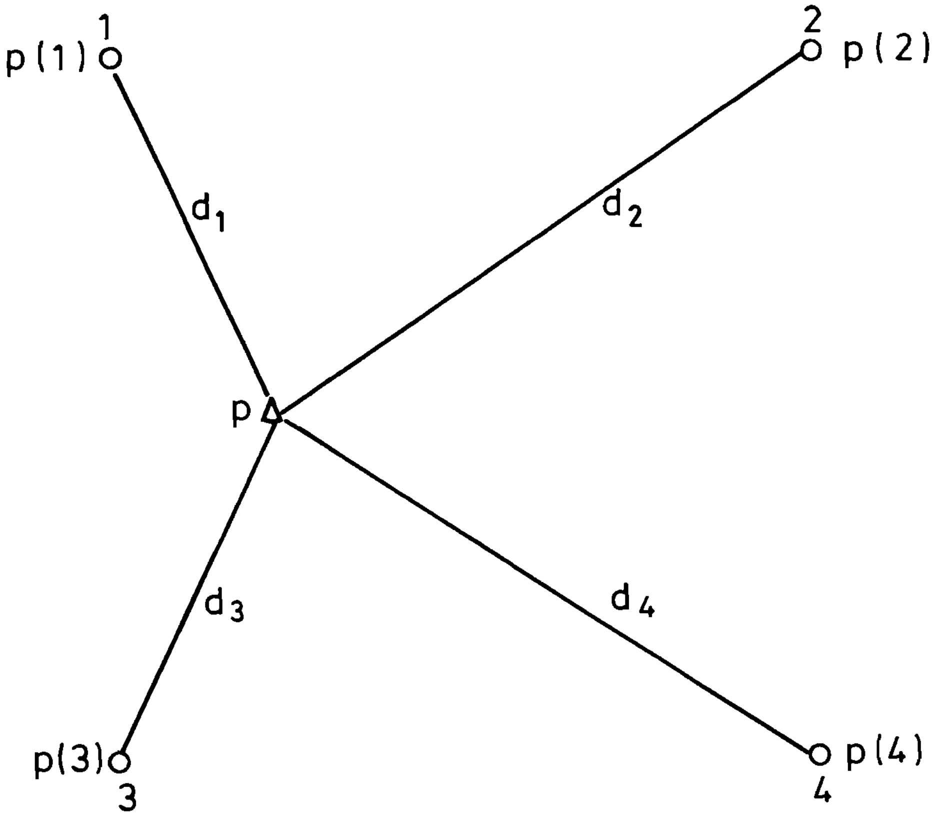


Figure 2 The grid system is used to express the distribution of 500 mb gph for the SNT model.

(The center of a typhoon is located at point 46. Grid spacing is 609.6Km).

Figure 3 Interpolation.



p(i) indicates the value attached to the grid point i fixed to the earth, one of the nearest four points surrounding the grid point Δ in the moving coordinate. The value p at the point Δ is determined by the formula

$$p = \frac{\sum_{i=1}^{4} \frac{1}{d_i} \cdot p(i)}{\sum_{i=1}^{4} \frac{1}{d_i}}$$

FIGURE 4

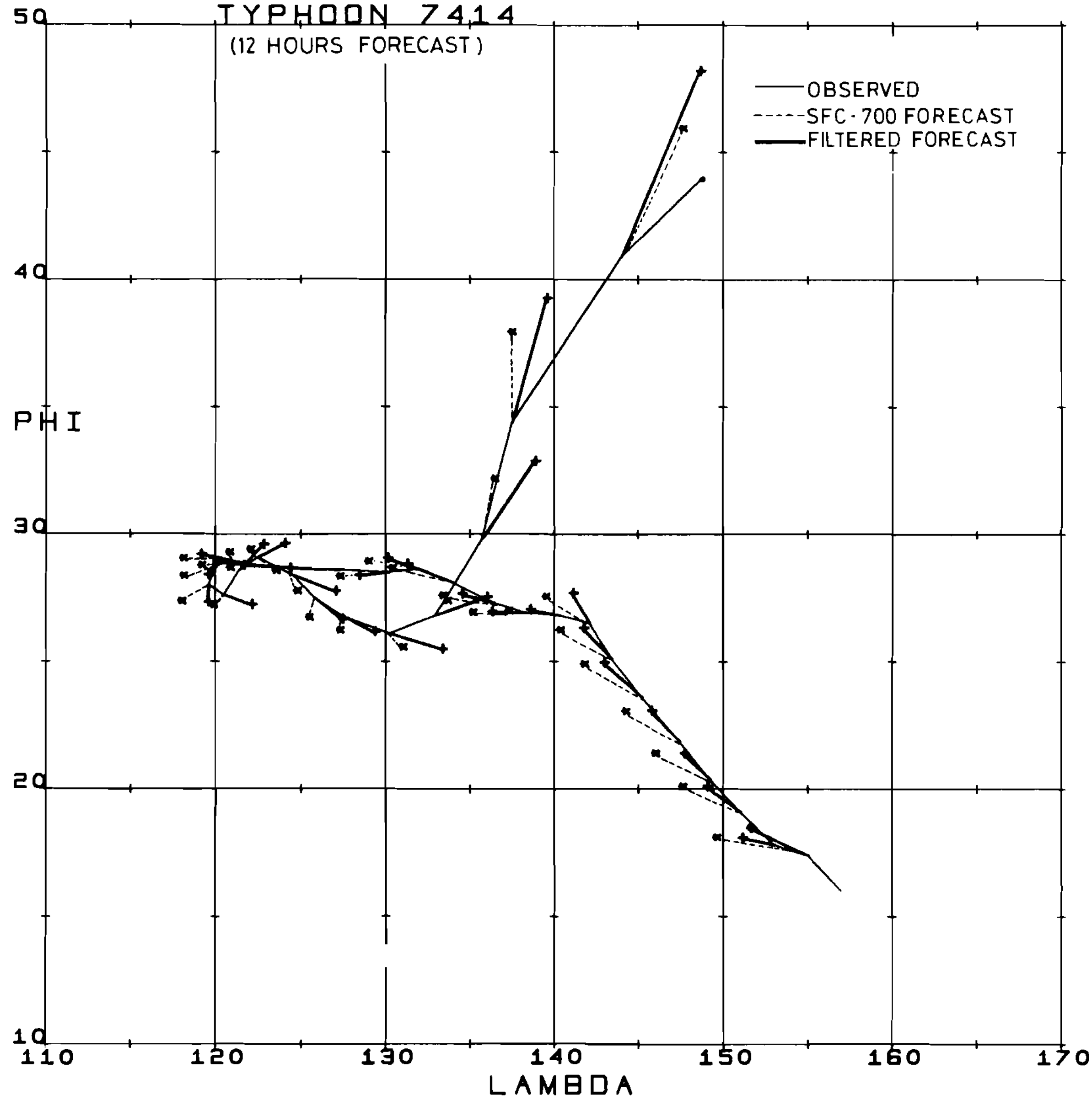

TYPHOON 7414
(12 HOURS FORECAST)
OBSERVED
SFC-700 FORECAST
FILTERED FORECAST
50
40
30
20
10
PHI
110
120
130
140
150
160
170
LAMBDA

FIGURE 5

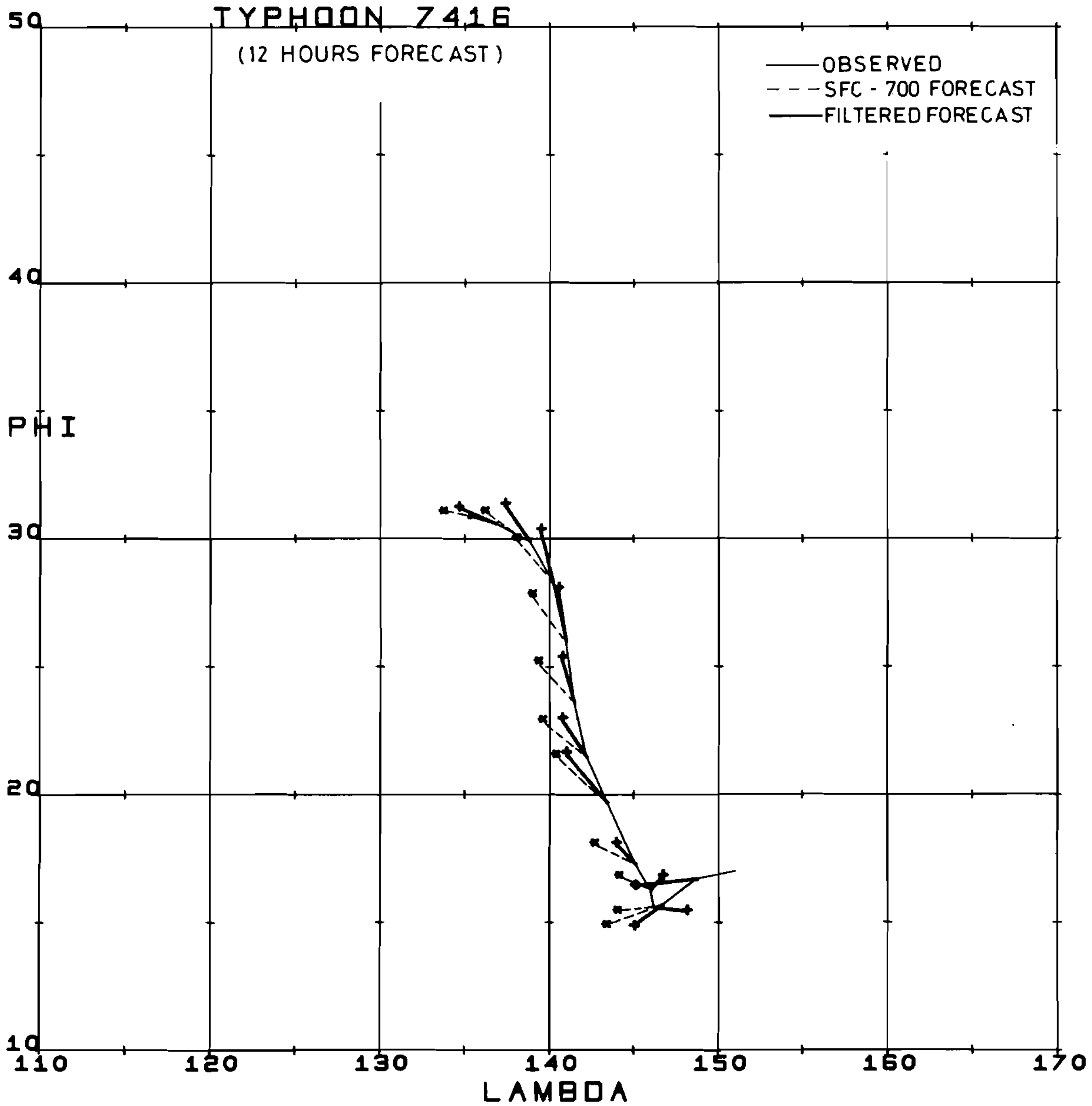

TYPHOON 7416
(12 HOURS FORECAST)
OBSERVED
SFC - 700 FORECAST
FILTERED FORECAST
50
40
30
20
10
PHI
110
120
130
140
150
160
170
LAMBDA

FIGURE 6

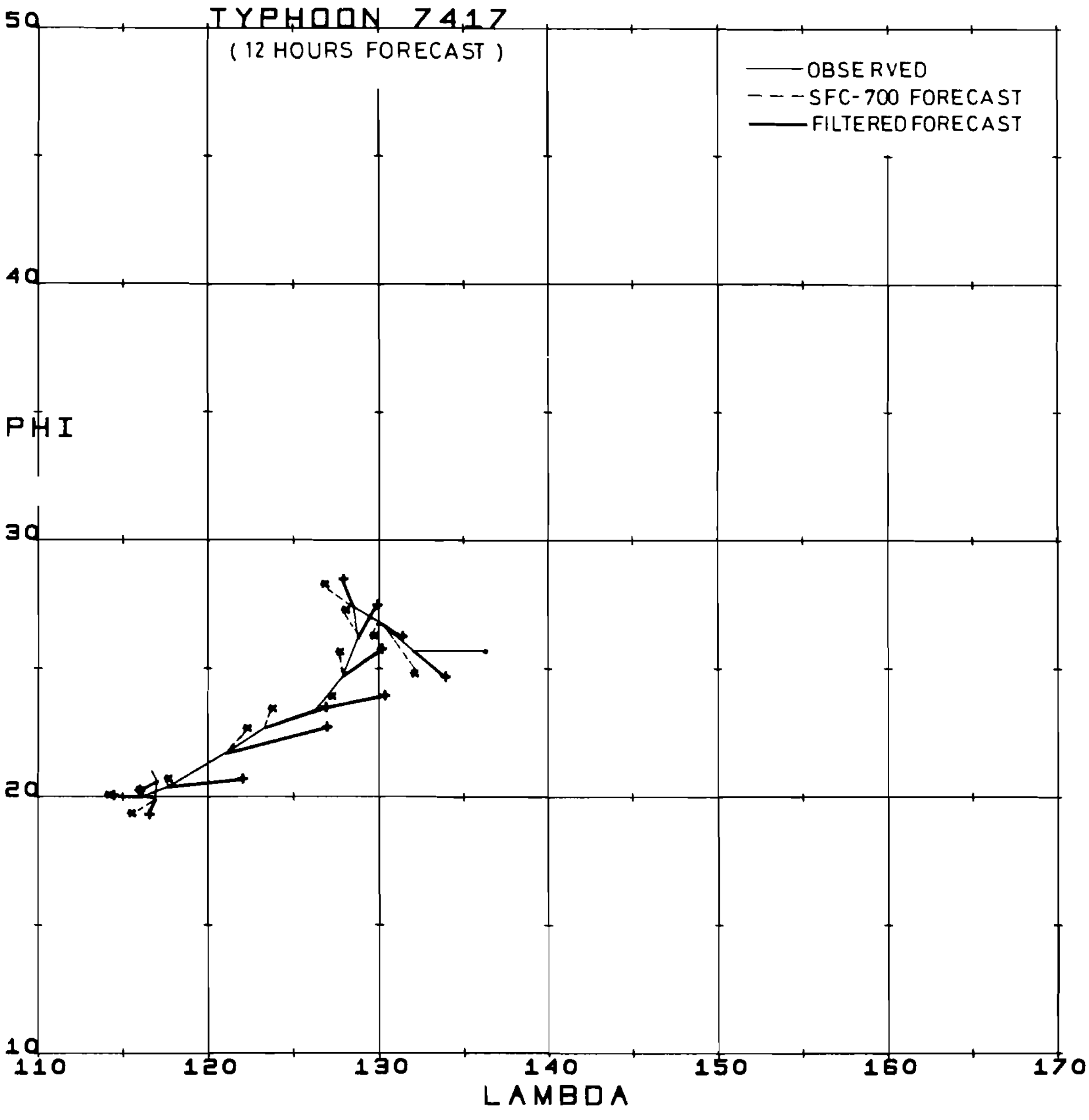

TYPHOON 7417
(12 HOURS FORECAST)
OBSERVED
SFC-700 FORECAST
FILTERED FORECAST
PHI
LAMBDA
50
40
30
20
10
110
120
130
140
150
160
170

FIGURE 7

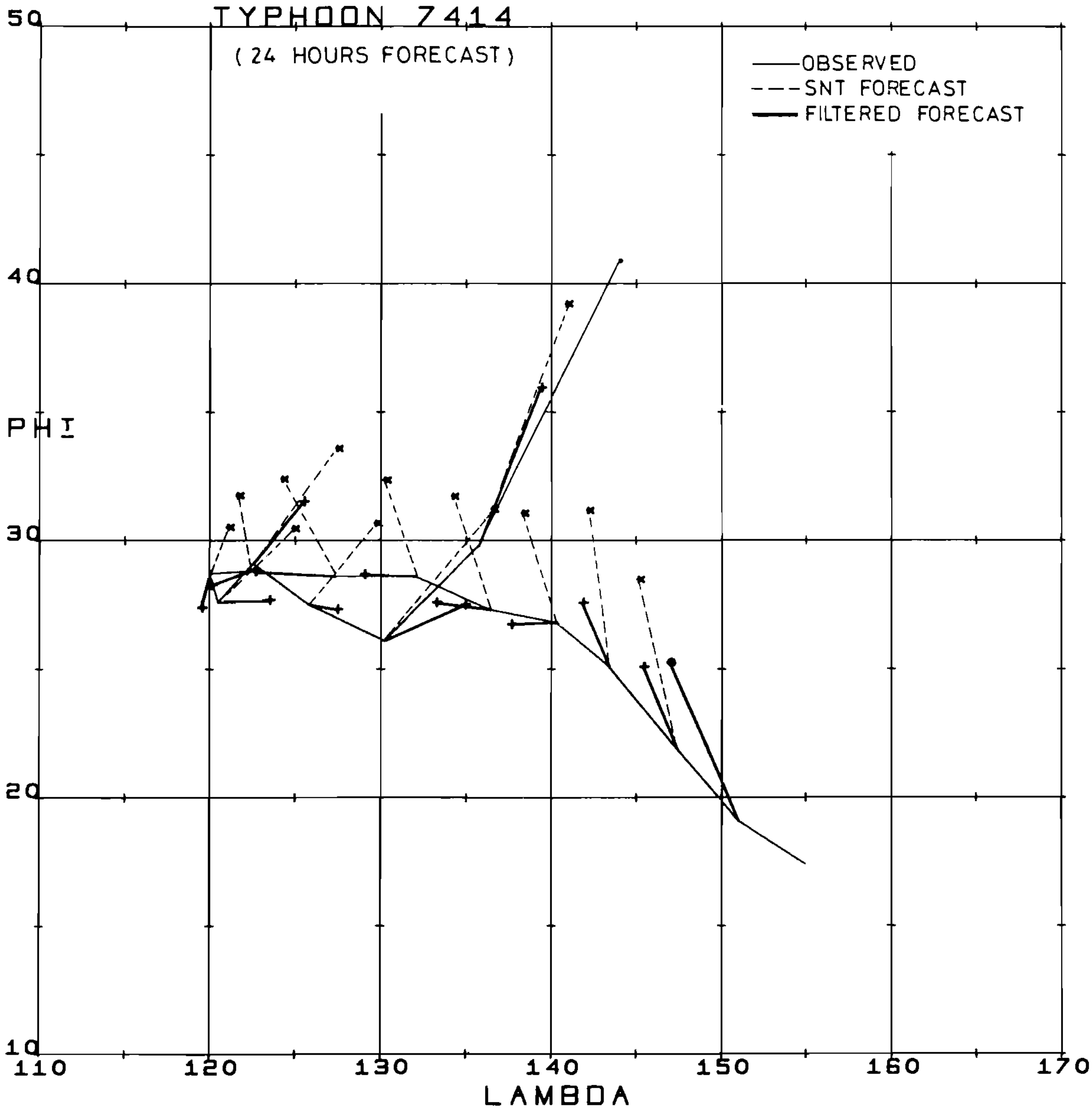

TYPHOON 7414
(24 HOURS FORECAST)
OBSERVED
SNT FORECAST
FILTERED FORECAST
PHI
50
40
30
20
10
110
120
130
140
150
160
170
LAMBDA

FIGURE 8

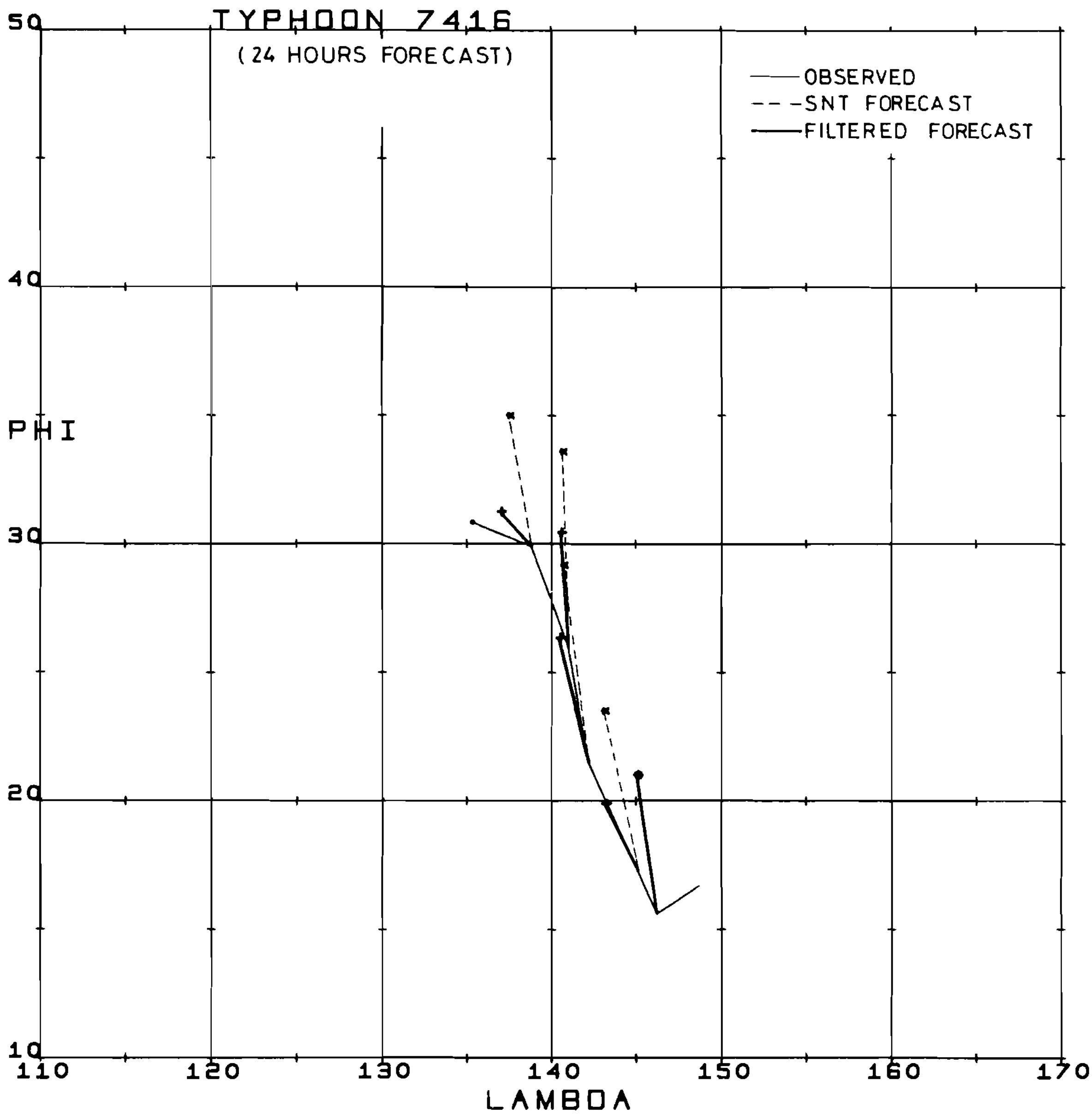
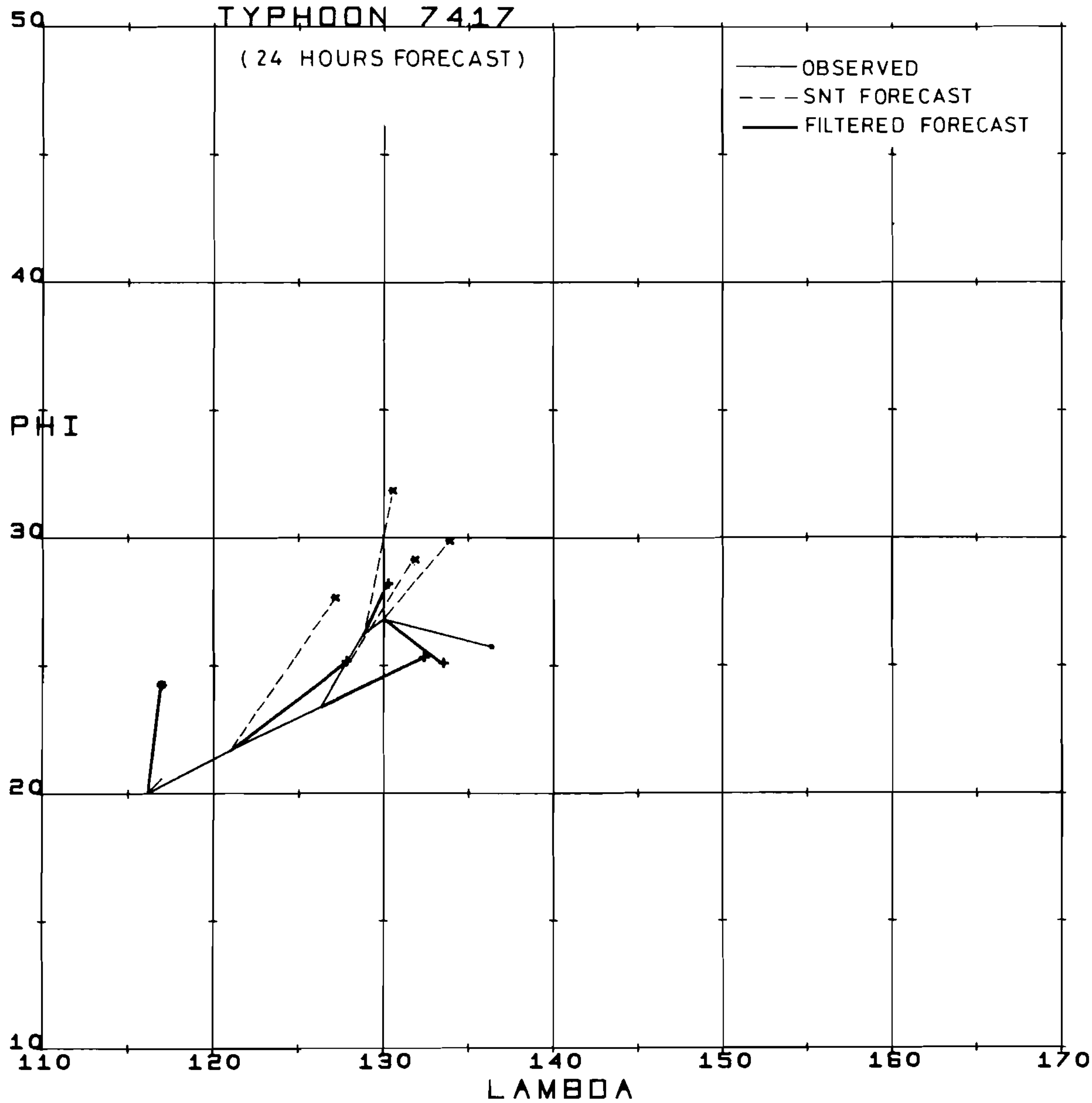

TYPHOON 7416
(24 HOURS FORECAST)
OBSERVED
SNT FORECAST
FILTERED FORECAST
PHI
50
40
30
20
10
110
120
130
140
150
160
170
LAMBDA

FIGURE 9


TYPHOON 7417
(24 HOURS FORECAST)
OBSERVED
SNT FORECAST
FILTERED FORECAST
PHI
LAMBDA
50
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