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RR-03-04
March 2003

Reprinted from *International Journal of Technology Management*, **23**(7/8): 691–717 (2002).

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Optimal feedbacks in techno-economic dynamics

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Abstract: The objective of this work is twofold: to design control strategies which optimise production, technology and their rates in a nonlinear model of economic growth; and to demonstrate the significance of this modelling approach by means of an empirical analysis. We formulate a problem of optimal R&D investment for a dynamic model, which binds production to technology. A discounted utility function, which correlates the amount of sales with the diversity in production, gives a criterion of optimality. We use the Pontryagin maximum principle for the design of an optimal nonlinear dynamics. On the basis of the theoretical analysis, we carry out an empirical analysis, which attempts to demonstrate the practical significance of the approach. For Japan's major manufacturing sectors, we compare optimal and actual levels of R&D intensities and identify sources of 'pseudo innovation' in high-tech industries.

Keywords: Economic growth; dynamic optimality principles; optimal investment feedback; evaluation of R&D intensity.

Reference to this paper should be made as follows: Tarasyev, A.M., Watanabe, C. and Zhu, B. (2002) 'Optimal feedbacks in techno-economic dynamics', *Int. J. Technology Management*, Vol. 23, Nos. 7/8, pp.691-717.

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1 Introduction

The problem of optimal regulation of R&D investments arises naturally due to the presence of the 'growth' and 'decline' trends in the interaction between production and technology. The technology intensity modelled as a positive exponential term in the economy's dynamics is responsible for sustainable growth. Investments in R&D stimulate new sales on the one hand; on the other hand, they lead to a redistribution of resources between production and technology and thus introduce a risk factor in innovation. A utility function is an instrument to correlate the amount of sales with the diversity in production. The amount of sales is determined by growth in production and the diversity in production depends on the accumulated and current R&D investments. Qualitatively, a utility function expresses preferences of the investors in the process of the simultaneous growth of production, technology and technology rates. The problem of optimal R&D investment consists of finding an optimal innovation policy, which maximises the utility function. Due to the structure of the utility function, an optimal innovation policy optimises production, technology and their rates. An important task in the analysis of optimal dynamics is to specify the rates of growth of the productivity capacities and the knowledge stocks.

The present research is connected with classical problems of economic growth and optimal allocation of resources [1–4] and refers to endogenous growth theory [2,3,5]. Unlike [2,3,5], which treat the dynamics of the knowledge stock as a function of the prices for the technology outputs, we deal with a dynamic model, which correlates growth with sales to R&D investments. The model is constructed through the differentiation of the production function with respect to time. The model has been adjusted to a real econometric time series [6]. We define the utility as a discounted integral of a logarithmic consumption index assuming no variations in the elasticity of substitution of the invented products [3,7]. We apply the Pontryagin maximum principle [8] in order to find an optimal R&D investment strategy. When studying the components of the optimal solution (the value function, an optimal feedback and its approximations), we use the optimality principles of the theory of Hamilton-Jacobi equations [9,10], theory of guaranteed control [11] and theory of robust control [4]. Also, our analysis addresses the qualitative methods and methods of approximations to the value functions and optimal feedbacks in control problems and differential games with discounted pay-off integrals [12,13]. Finally, we refer to research on planning dynamic investments under uncertainties [14].

Optimality principles are expressed in the nonlinear system of differential equations of the fourth order. We find the first integral for this system, reduce it to the system of the third order and decompose to the second order. Equilibrium of this Hamiltonian system is generated by optimal solutions. The existence and uniqueness results are valid for the saddle type equilibrium and optimal trajectories converge to it.

The optimal feedback which generates optimal trajectories is given implicitly and we provide explicit approximations of the rational type – suboptimal feedbacks. The obtained suboptimal feedbacks have reasonable interpretations in terms of econometric characteristics. We indicate growth and decline properties of optimal trajectories and feedbacks generated by R&D intensities. The statistic analysis demonstrates that the synthetic optimal scenarios turn out to be robust qualitatively. They reflect the actual qualitative trends in production, technology, technology productivity and R&D intensities of the real econometric time series correctly within a wide domain of the model's parameters: discount coefficients and elasticities in consumption index, growth rates, delay and obsolescence parameters in technology dynamics.

On the basis of the theoretical analyses, empirical analyses are attempted to demonstrate the practical significance of this approach. By utilising the developed approach, evaluations of R&D intensity in major Japanese manufacturing sectors are conducted by comparing optimal and actual levels identifying 'pseudo innovation' in high technology and its sources.

Section 2 constructs an optimal control model for the decision of R&D intensity in techno-economic dynamics. Section 3 analyses the trajectories of this optimal control problem. Section 4 empirically conducts the evaluation of R&D intensity in Japan's major manufacturing industries. Section 5 briefly summarises the implication of the optimal feedback in techno-economic dynamics.

2 The optimal control model

2.1 The system model

To construct the nonlinear growth model which describes dynamics of aggregated production $y = y(t)$ and technology $T = T(t)$, let us introduce the following notations. Aggregated production factors: labour L, capital K, materials M and energy E are decomposed into two parts: one with index y goes to production and another with index T goes to technology

$$L = L_y + L_T, \qquad K = K_y + K_T, \qquad M = M_y + M_T, \qquad E = E_y + E_T.$$

Assuming that production function depends on production factors L_y, K_y, M_y, E_y and technology T.

$$y = F(t, L_y, K_y, M_y, E_y, T)\ [15]$$

and differentiating it by time t we obtain the following time series

$$\frac{\dot{y}}{y} = \frac{\partial F}{\partial t}\frac{1}{y} + \frac{\partial F}{\partial L}\frac{L}{y}\frac{\dot{L}}{L} + \frac{\partial F}{\partial K}\frac{K}{y}\frac{\dot{K}}{K} + \frac{\partial F}{\partial M}\frac{M}{y}\frac{\dot{M}}{M} + \frac{\partial F}{\partial E}\frac{E}{y}\frac{\dot{E}}{E}$$
$$-\left(\frac{\partial F}{\partial L}\frac{\partial L_T}{\partial T} + \frac{\partial F}{\partial K}\frac{\partial K_T}{\partial T} + \frac{\partial F}{\partial M}\frac{\partial M_T}{\partial T} + \frac{\partial F}{\partial E}\frac{\partial E_T}{\partial T}\right) + \frac{\partial F}{\partial T}\frac{\dot{T}}{y}$$

Using $\dot{T} \approx r$, rewrite the above equation in the form of:

$$\frac{\dot{y}}{y} = f - p\frac{r}{y} + q\frac{r}{y}$$

where function $f = f(t)$

$$f = f(t) = \frac{\partial F}{\partial t}\frac{1}{y} + \frac{\partial F}{\partial L}\frac{L}{y}\frac{\dot{L}}{L} + \frac{\partial F}{\partial K}\frac{K}{y}\frac{\dot{K}}{K} + \frac{\partial F}{\partial M}\frac{M}{y}\frac{\dot{M}}{M} + \frac{\partial F}{\partial E}\frac{E}{y}\frac{\dot{E}}{E}$$

decrease in manufacturing due to R&D spending L_T, K_T, M_T, E_T is collected into function p

$$p = p(t) = \frac{\partial F}{\partial L}\frac{\partial L_T}{\partial T} + \frac{\partial F}{\partial K}\frac{\partial K_T}{\partial T} + \frac{\partial F}{\partial M}\frac{\partial M_T}{\partial T} + \frac{\partial F}{\partial E}\frac{\partial E_T}{\partial T}$$

increase of R&D knowledge stock is described by function q which coincides with the marginal productivity of technology

$$q = q(t) = \frac{\partial F}{\partial T}$$

In the general case function f depends on the accumulated R&D investment T. Let us assume that this dependence is given by the formula

$$f = f_1 + f_2\left(\frac{T}{y}\right)^{\gamma}, \qquad f_1 = f_1(t),\ f_2 = f_2(t)$$

Now we can get a nonlinear growth model which describes dynamics of aggregated production y and technology T depending on cumulative R&D investment $r = r(t)$

$$\frac{\dot{y}}{y} = f_1 + f_2\left(\frac{T}{y}\right)^{\gamma} - g\frac{r}{y} \qquad (1)$$
$$\dot{T} = r$$

where $g = p - g$.

One can treat dynamic process (1) as the balanced equations of spending resources between the productivity rate $\dot{y}/y$ and R&D intensity r/y. Function $f = f(t)$ presents the non-R&D contribution to the productivity growth rate r/y. The term $(T/y)^Y$ with the coefficient $f_2 = f_2(t)$ in the first equation in (1) shows the growth effect of the technology intensity T/y on production rate $\dot{y}/y$. The negative sign $-g$, $g = g(t) = p(t) - q(t) > 0$, of the net contribution by R&D means that in the short run, spending $p = p(t)$ on R&D prevails on the rate of return $q = q(t)$ to R&D and provides the decline and risk factor of R&D investment.

Change in technology T due to time lag m and obsolescence effect σ in technology, is not precisely equal to the current R&D investment r_t and is connected mainly with the R&D investment in initial stage r_{t-m}

$$\dot{T} = r = \frac{1}{(1-\sigma)}(-\sigma T + r_{t-m}), \quad 0 \le \sigma < 1 \tag{2}$$

Relation (2) means that a part of contribution r_{t-m} to R&D at time $t - m$ is spent on compensation of obsolescence σT of technology T and the rest $r_{t-m} - \sigma T \ge 0$ affects the current change of technology $\dot{T} = r$ with the time lag m (see [6]).

The production y and the accumulated R&D investment T stand for the phase parameters in dynamics (1). The current change r in technology T is the control parameter. The control parameter $r = r(t)$ is not fixed and can be chosen for obtaining 'good' properties of trajectories of dynamics (1). If the trajectories $T = T(t)$ and their rates $r = r(t)$ with 'good' properties are constructed, then the real investment r_{t-m} is expressed from equation (2) by relation

$$r_{t-m} = (1-\sigma)r + \sigma T$$

2.2 *Utility of the system trajectories*

We formulate now the utility principle for evaluating the quality of economic trajectories $(y(\cdot), T(\cdot), r(\cdot))$. For this purpose we introduce the discounted integral which measures utility in the long-run term (see [1,3])

$$U_t = \int_t^{+\infty} e^{-\eta(s-t)} \ln D(s)ds \tag{3}$$

Here the logarithm of the consumption index $D(s)$ represents the instantaneous utility of products (technologies) at time s, η is the discount rate, s is the running time, t is the fixed initial time.

For the consumption index D we choose a specification that imposes a constant and equal elasticity of substitution between every pair of products (see [3])

$$D = D(s) = \left(\int_0^n x^{\alpha}(j)dj \right)^{1/\alpha}, \quad n = n(s) \tag{4}$$

Here j is the current index of invented products, $x(j)$ is the quantity of production of the brand with index j, n is the quantity of available (invented) products, α is the parameter of elasticity and ε is the elasticity of substitution between any two products

$$\varepsilon = \frac{1}{(1-\alpha)}$$

Similarly to [3] we assume that quantities $x(j)$ are equal for each index j

$$x(j) = \frac{y}{n}, \quad y = y(s), \quad n = n(s) \tag{5}$$

let us suppose that quantity of invented products n depends on the accumulated R&D investment T and the technology rate r according to the exponential rule (see [6])

$$n = n(s) = be^{\kappa s} T^{\beta_1} r^{\beta_2}, \quad T = T(s), \quad r = r(s) \tag{6}$$

Formulas (5) and (6) mean that innovation n depends upon the forefront R&D activities demonstrated by the technology rate r. At the same time it owed accumulation of past R&D activity given by technology stock T. In addition, innovation n has a general tendency to a decaying nature which can be expressed by term e^{KS}. All three effects lead to a decrease in the respective brand production x and imply diversification.

Combining equations (3)-(6) we obtain the following expression for the utility function

$$U_t = \int_t^{+\infty} e^{-\eta(s-t)} (\ln y(s) + a_1 \ln T(s) + a_2 \ln r(s)) ds + A \int_t^{+\infty} e^{-\eta(s-t)} (\kappa s + \ln b) ds \tag{7}$$

$$a_1 = A\beta_1, \quad a_2 = A\beta_2, \quad A = \frac{(1-\alpha)}{\alpha}$$

The structure of the utility function U_t (7) means that investors (governments, financial groups) are interested in growth of production y as well as in growth of the accumulated R&D investment T and the current change of technology r (new goods, technologies, etc.).

3 Trajectories of the optimal R&D control system

3.1 Optimality principle

The problem of optimal R&D investment is to find such level of the technology rate $r^0 = r^0(t)$ – optimal investment, the corresponding optimal production $y^0 = y^0(t)$ and the optimal accumulated R&D investment $T^0 = T^0(t)$ subject to dynamics (1) which maximise the utility function (7).

Let us note that problem (1), (7) is a classical problem of the optimal control theory. For its solution one can use the maximum principle of Pontryagin [8]. Applications of this optimality principle to problems of economic growth were developed by pioneer economists (e.g. [1,7]).

Let us define the value function of the optimal control problem (1), (7)

$$V(y,T) = \sup_{r(\cdot)} U_t(y(\cdot), T(\cdot), r(\cdot)), \quad y(t) = y, \quad T(t) = T \tag{8}$$

Results of the pioneer works by Dolcetta [12] and [13] show that the value function $(y,T)\to V(y,T)$ has finite values and is continuous. Taking this fact into account and based on the compactness property of the set of admissible control $r(\cdot)$ one can prove that the supremum in the value function V (8) is realised and hence the optimal control problem (1), (7) has the solution

$$V(y,T)=\max_{r(\cdot)} U_t(y(\cdot),T(\cdot),r(\cdot))=U_t(y^0(\cdot),T^0(\cdot),r^0(\cdot))$$

$$y(t)=y^0(t)=y\,,\ \ T(t)=T^0(t)=T$$

Let us compose the Hamiltonian of the problem (1), (7)

$$H(y,T,r,\psi_1,\psi_2)=\ln y+a_1\ln T+a_2\ln r+\psi_1(f_1 y+f_2 T^{\gamma}y^{(1-\gamma)}-gr)+\psi_2 r \qquad (9)$$

The Hamiltonian H (9) describes the current flow of utility from all sources. The current control $r = r(t)$ is chosen to maximise this flow. Note that boundaries r_l, r_u in restrictions on control parameter r

$$r_l\le r\le r_u$$

are given not precisely and can scarcely be identified from the real econometric data. Therefore, we will be interested in such regimes of optimal control $r^0 = r^0(t)$ which are realised at points of global maximum of the Hamiltonian H (9) for technology rates $r > 0$. It happens that these optimal values are restricted and the corresponding optimal trajectory converges to the equilibrium point with finite, positive values. Taking this fact into account we calculate maximum of the Hamiltonian H (9) over parameter r as follows

$$\frac{\partial H}{\partial r}=a_2\frac{1}{r}-g\psi_1+\psi_2=0$$

So the maximum value is attained at the optimal technology rate r^0

$$r^0=a_2\frac{1}{(g\psi_1-\psi_2)} \qquad (10)$$

The value function $V(y,T)$ at points of differentiability should satisfy the Hamilton-Jacobi equation

$$-\eta V(y,T)+\ln y+a_1\ln T+a_2(\ln a_2-1)+\frac{\partial V}{\partial y}(f_1 y+f_2 T^{\gamma}y^{(1-\gamma)})-a_2\ln(g\frac{\partial V}{\partial y}-\frac{\partial V}{\partial T})=0 \quad (11)$$

Let us introduce constructions of the Pontryagin maximum principle which plays the role of the method of characteristics for the Hamilton-Jacobi equation (11). For this purpose we consider dynamics of the conjugate (adjoint) variables ψ_1, ψ_2 which can be interpreted as 'prices' of production y and technology T

$$\begin{aligned}\dot\psi_1&=\eta\psi_1-\frac{\partial H}{\partial y}=\eta\psi_1-\frac{1}{y}-(1-\gamma)\psi_1 f_2 T^{\gamma}\frac{1}{y^{\gamma}}-\psi_1 f_1\\ \dot\psi_2&=\eta\psi_2-\frac{\partial H}{\partial T}=\eta\psi_2-\frac{a_1}{T}-\gamma\psi_1 f_2\left(\frac{y}{T}\right)^{(1-\gamma)}\end{aligned} \qquad (12)$$

Prices ψ_1, ψ_2 measure the marginal contribution of variables y, T to the utility function.

Differential equations (12) for prices ψ_1, ψ_2 can be interpreted as an equilibrium condition: the increment in flow plus the change in price should be zero.

Combining dynamics of real (1) and adjoint variables (12) with the maximum condition for the Hamiltonian (10) we obtain the following closed system of differential equations

$$
\begin{aligned}
&\frac{\dot{y}}{y} = f_1 + f_2\left(\frac{T}{y}\right)^{\gamma} - \frac{g a_2}{(g\psi_1 - \psi_2)y} \\
&\dot{T} = \frac{a_2}{(g\psi_1 - \psi_2)} \\
&\frac{\dot{\psi}_1}{\psi_1} = \eta - \frac{1}{\psi_1 y} - (1-\gamma) f_2\left(\frac{T}{y}\right)^{\gamma} - f_1 \\
&\frac{\dot{\psi}_2}{\psi_2} = \eta - \frac{a_1}{\psi_2 T} - \gamma f_2 \frac{\psi_1}{\psi_2}\left(\frac{y}{T}\right)^{(1-\gamma)}
\end{aligned}
\tag{13}
$$

Let us pass to the transversality conditions for the system (13). On the finite time horizon $[t, \vartheta]$, $t \leq \vartheta < +\infty$ the transversality conditions have the following form

$$\psi_i(\vartheta) = 0, \quad i = 1,\ 2 \tag{14}$$

The transversality conditions (14) and the nonpositive rates $\dot{\psi}_i(s) \leq 0$ in (12) provide the shift of prices ψ_i, $i = 1,2$ to the positive domain in the inverse time

$$\psi_i(s) \geq 0, \quad s \in [t, \vartheta], \quad i = 1,\ 2 \tag{15}$$

Let us introduce notations for costs of production y and technology T

$$z(s) = z_1(s) + z_2(s), \quad z_i(s) = \psi_i(s) y(s), \quad s \geq t, \quad i = 1,\ 2 \tag{16}$$

Since variables y, T are strictly positive, we can derive the equivalent transversality conditions in terms of costs

$$z(\vartheta) = 0, \quad z_i(\vartheta) = 0, \quad i = 1,\ 2 \tag{17}$$

The cost $z = z(s)$ satisfies the following differential equation

$$\dot{z}(s) = \eta z(s) - \eta p^0, \quad p^0 = \frac{(a_1 + a_2 + 1)}{\eta} \tag{18}$$

Its solution which meets transversality conditions (17) can be presented by the Cauchy formula

$$z(s) = p^0(1 - e^{-\eta(\vartheta - s)}) \tag{19}$$

Solution $z = z(s)$ (19) and its components $z_i = z_i(s)$, $i = 1,2$ are bounded and, hence, adjoint variables $\psi_i = \psi_i(s)$, $i = 1,2$ are also bounded

$$0 \le z(s) \le p^0, \quad 0 \le z_i(s) \le p^0, \quad 0 \le \psi_i(s) \le p^0, \quad i = 1,\ 2 \tag{20}$$

It means that for times $\vartheta \to +\infty$ there exists a sequence of components of optimal solutions $y^k(\cdot)$, $T^k(\cdot)$, $z^k(\cdot)$, $z_i^k(\cdot)$, $\psi_i^k(\cdot)$, $I = 1{,}2$ for the problems with finite horizons ϑ_k which converges to the optimal solution of the problem (1), (7) with the infinite horizon.

The uniform estimate is valid for the sequence $z^k(\cdot)$

$$\sup_{s \ge t} | e^{-\eta s} z_k(s) - e^{-\eta s} p^0 | = e^{-\eta \vartheta_k} p^0 \tag{21}$$

For the terminal times growing to infinity $\vartheta_k \to +\infty$ the sequence of costs $\{e^{-\eta s} z_k(s)\}$ converges uniformly to the optimal cost $e^{-\eta s} p^0$ and therefore the constant

$$z = p^0 \tag{22}$$

is the limit function for costs $z^k(\cdot)$. The constant function (first integral) $z = p^0$ is the unique solution of differential equation (18) which meets the well known transversality condition (see [1,3])

$$\lim_{s \to +\infty} e^{-\eta s} z(s) = 0 \tag{23}$$

Transversality condition (23) means that the total cost $z = z(s)$ should not grow faster than exponent $e^{\eta s}$.

We introduce new variables

$$x_1 = \frac{y}{T}, \quad x_2 = \psi_1 y, \quad x_3 = \frac{1}{T}, \quad x_4 = \psi_2 T \tag{24}$$

Here x_1 – technology productivity, x_2 – the cost of production, x_3 – the inverse of technology, x_4 – the cost of technology.

The system of new variables (24) transforms system (13) to the system with the separable structure

$$\begin{aligned}
\dot{x}_1 &= f_1 x_1 + f_2 x_1^{(1-\gamma)} - \frac{a_2 (x_1 + g) x_1}{(g x_2 - x_1 x_4)} \\
\dot{x}_2 &= \eta x_2 + \gamma f_2 x_2 \frac{1}{x_1^{\gamma}} - 1 - \frac{a_2 g x_2}{(g x_2 - x_1 x_4)} \\
\dot{x}_3 &= -\frac{a_2 x_1 x_3}{(g x_2 - x_1 x_4)} \\
\dot{x}_4 &= \eta x_4 - \gamma f_2 x_2 \frac{1}{x_1^{\gamma}} - a_1 + \frac{a_2 x_1 x_4}{(g x_2 - x_1 x_4)}
\end{aligned} \tag{25}$$

Analysis of system (25) shows that under the certain conditions (for details see [16]) it has the unique stationary point $x^0 = (x_1^0, x_2^0, x_3^0, x_4^0)$ of the saddle type and the optimal trajectory is the only trajectory which converges to it (see [17]).

Taking into account that system (25) has the first integral

$$z = x_1 + x_4 = p^0 \tag{26}$$

and using its block structure one can reduce this system with four variables to the two dimensional system

$$\dot{x}_1 = f_1 x_1 + f_2 x_1^{(1-\gamma)} - \frac{a_2(x_1+g)x_1}{((x_1+g)x_2 - p^0 x_1)}$$
$$\dot{x}_2 = \eta x_2 + \gamma f_2 x_2 \frac{1}{x_1^{\gamma}} - 1 - \frac{a_2 g x_2}{((x_1+g)x_2 - p^0 x_1)} \qquad (27)$$

In the general case optimal control r^0 which provides convergence of the system (25) to equilibrium x^0 has a very complicated structure. Analysis of system (27) at the equilibrium point x^0: evaluation of the Jacobi matrix of the right hand sides of the system (27), estimation of its eigenvalues and eigenvectors, allows the substitution of optimal control r^0 by a series of suboptimal feedbacks r^* with the rational structure

$$r^* = r^*(y,T) = \frac{a_2 y}{(d + (k_1\omega + k_2)(y/T - x_1^0) + \omega(y/T - x_1^0)^2)} \qquad (28)$$

$$d = (x_1^0 + g)x_2^0 - p^0 x_1^0, \qquad k_1 = x_1^0 + g, \qquad k_2 = -(p^0 - x_2^0)$$

The suboptimal feedback (28) is derived from the first equation of the nonlinear system (27) and the structure of optimal control r^0 (10) under the condition of the linearisation of the second coordinate

$$x_2 = x_2^0 + \omega(x_1 - x_1^0), \qquad \omega \geq 0$$

The convergence result to the equilibrium point x^0 is valid for trajectories of the controlled process (1), generated by feedbacks r^* with slopes ω_0 corresponding to the eigenvector with the negative eigenvalue of the Jacobi matrix. The rational feedback $r^* = r^*(\omega_0)$ with the slope ω_0 can be interpreted as the linear approximation of the optimal control r^0.

One can show that that there exist intervals for parameter ω around the 'optimal' slope ω_0 which give different combinations of growing and declining properties of R&D intensities r/y, r_{t-m}/y. Trajectories $(y^*(\cdot), T^*(\cdot), r^*(\cdot))$, $(y^0(\cdot), T^0(\cdot), r^0(\cdot))$, generated by rational r^* and optimal r^0 feedbacks respectively has the analogous growth properties: production $y(\cdot)$, technology $T(\cdot)$ and investment $r(\cdot)$, are growing to infinity with the equal exponential growth rates.

3.2 Analytic solution

Let us note that the nonlinear system (13) of the optimal process is rather complicated and at the first glance does not have the analytic solution expressed in the explicit functions. In order to obtain explicit solutions we consider now the reduced version – the test optimal control problem, as the first approximation of the nonlinear system (13). To obtain the simplified dynamics assume that

$y = 0$ in (13) and, hence, function f does not depend on the technology parameter T

$$f = f_1 + f_2$$

So we deal with the following dynamics

$$\frac{\dot{y}}{y} = f - g\frac{r}{y} \tag{29}$$

Let us stress that we examine here a nonstationary model with the time dependent functions

$$f = f(s)\,,\; g = g(s)\,.$$

Equation (6) can be developed as follow,

$$n = n(s) \approx bT^{\beta_1} r^{\beta_2} \approx b\left(\frac{r}{\theta+\sigma}\right)^{\beta_1} r^{\beta_2} \qquad (30)[18]$$

$$\ln n = (\ln b - \beta_1 \ln(\theta+\sigma)) + (\beta_1+\beta_2)\ln r \tag{31}$$

Combining (3), (4), (5) and (31), we obtain the following expression for the utility function

$$U_t = \int_t^\infty e^{-\eta(s-t)}(\ln y + \frac{1-\alpha}{\alpha}((\ln b - \beta_1\ln(\theta+\sigma)) + (\beta_1+\beta_2)\ln r))ds \tag{32}$$

The Hamiltonian has the form

$$H(y,r,\psi) = \ln y + \frac{(1-\alpha)}{\alpha}((\ln b - \beta_1\ln(\theta+\sigma)) + (\beta_1+\beta_2)\ln r) + \psi(fy - gr) \tag{33}$$

Its maximum by parameter r is determined by the formula:

$$\frac{\partial H}{\partial r} = \frac{(1-\alpha)}{\alpha}(\beta_1+\beta_2)\frac{1}{r} - g\psi = 0 \tag{34}$$

So its maximum value is attained at the optimal R&D investment r^0

$$r^0 = \frac{(1-\alpha)}{\alpha}\frac{(\beta_1+\beta_2)}{g\psi} \tag{35}$$

Here ψ is the marginal price of production y. It can be expressed as the gradient $\psi = \partial W/\partial y$ of the value function W.

The Hamilton-Jacobi equation has the following form

$$\frac{\partial W}{\partial t} + H(y,r,\frac{\partial W}{\partial y}) = \frac{\partial W}{\partial t} + H(y,r,\psi) = 0 \tag{36}$$

It implies the adjoint equation for the marginal price ψ

$$\frac{\partial}{\partial y}\left(\frac{\partial W}{\partial t}\right) + \frac{\partial H}{\partial y} = \frac{\partial}{\partial t}\left(\frac{\partial W}{\partial y}\right) + \frac{\partial H}{\partial y} = \frac{\partial \psi}{\partial t} + \frac{\partial H}{\partial y} = 0 \tag{37}$$

Utility function (32) requires the following Hamiltonian in addition to the Hamiltonian (33):

$$H^{*}(s-t,y,r,\psi^{*})=e^{-\eta(s-t)}(\ln y+\frac{1-\alpha}{\alpha}((\ln b-\beta_1\ln(\theta+\sigma))+(\beta_1+\beta_2)\ln r))+\psi^{*}(fy-gr) \tag{38}$$

Under the optimality condition we have

$$\frac{\partial H}{\partial r}=\frac{\partial H^{*}}{\partial r}=0$$

The following relations bind two Hamiltonians (33) and (38)

$$\frac{\partial H}{\partial y}=\frac{1}{y}+f\psi \tag{39}$$

$$\frac{\partial H^{*}}{\partial y}=e^{-\eta(s-t)}\frac{1}{y}+f\psi^{*} \tag{40}$$

$$\psi^{*}=e^{-\eta(s-t)}\psi \tag{41}$$

$$\frac{\partial H^{*}}{\partial y}=e^{-\eta(s-t)}\left(\frac{1}{y}+f\psi\right)=e^{-\eta(s-t)}\frac{\partial H}{\partial y} \tag{42}$$

The marginal price ψ' satisfies the following adjoint equation

$$\frac{\partial H^{*}}{\partial y}=-\frac{\partial \psi^{*}}{\partial t}=-\left(-\eta e^{-\eta(s-t)}\psi+e^{-\eta(s-t)}\dot{\psi}\right) \tag{43}$$

From equations (42) and (43) it follows

$$\frac{\partial H}{\partial y}=\eta\psi-\dot{\psi} \tag{44}$$

Therefore, for dynamics of the conjugate variable ψ one can compose the adjoint equation:

$$\dot{\psi}=\eta\psi-\frac{\partial H}{\partial y}=\eta\psi-\frac{1}{y}-f\psi \tag{45}$$

Combining equations (29) and (34) and transforming (45), we obtain the following closed system of differential equations

$$\frac{\dot{y}}{y}=f-\frac{(1-\alpha)}{\alpha}(\beta_1+\beta_2)\frac{1}{y\psi} \tag{46}$$

$$\frac{\dot{\psi}}{\psi}=\eta-\frac{1}{y\psi}-f \tag{47}$$

Introducing notation $z=y\psi$ for the production cost and summarising equations (46) and (47) the following differential equation is obtained:

$$\dot{z} = \eta z - \left[\frac{(1-\alpha)}{\alpha} (\beta_1 + \beta_2) + 1 \right] \tag{48}$$

The general solution of equation (48) has the following form

$$z(t) = Ce^{\eta t} + \frac{1}{\eta} \cdot \left[\frac{(1-\alpha)}{\alpha} (\beta_1 + \beta_2) + 1 \right] \tag{49}$$

The unique solution which meets the transversality condition of the maximum principle is the steady state solution:

$$\lim_{t \to \infty} e^{-\eta t} z(t) = 0 \tag{50}$$

Since coefficient C in equation (49) should be 0 to satisfy the steady solution condition, the following formula is obtained:

$$z = z(t) = \frac{(B+1)}{\eta}, \quad B = \frac{(1-\alpha)}{\alpha} (\beta_1 + \beta_2) \tag{51}$$

Substituting solution (51) into equations (46), (47) we obtain dynamics of the optimal process

$$\frac{\dot{y}}{y} = f - \frac{B}{(B+1)} \eta$$

$$\frac{\dot{\psi}}{\psi} = \frac{B}{(B+1)} \eta - f$$

Assuming that $f = f(t)$ is a nondecreasing function with positive rate $(f - \eta B/(B+1)) > 0$ and introducing notations

$$Q(t) = \int_{t_0}^{t} (f(\tau) - \frac{B}{(B+1)} \eta) d\tau > (f(t_0) - \frac{B}{(B+1)} \eta)(t - t_0)$$

we obtain the optimal model with the exponentially growing production y

$$y = y(t) = y_0 e^{Q(t)}, \quad y(t_0) = y_0$$

and the exponentially decreasing price ψ

$$\psi = \psi(t) = \psi_0 e^{-Q(t)}, \qquad \psi(t_0) = \psi_0$$

Substituting solution (51) into optimal control (35), we obtain the relation between the optimal investment r and the optimal production y

$$r = \frac{\eta y}{g} \cdot \frac{(1-\alpha)}{((1-\alpha)(\beta_1 + \beta_2) + \alpha)} = \frac{1}{(\varepsilon - 1 + (\beta_1 + \beta_2))} \cdot \frac{\eta}{g} y \tag{52}$$

Here $\varepsilon = 1/(1-\alpha)$ is the elasticity of substitution.

In the case where the number of available varieties *n(s)* in equation (6) satisfies the condition of the constant returns to scale with respect to *r* and *T*, we have $\beta_1 + \beta_2 = 1$ (see [19]), and the optimal investment *r* is defined by the formula

$$r = \frac{\eta}{\varepsilon g} y \tag{53}$$

The optimal R&D intensity is presented by the following equation

$$\frac{r}{y} = \frac{1}{(\varepsilon - 1 + (\beta_1 + \beta_2))} \cdot \frac{\eta}{g} \tag{54}$$

Under the condition that $\beta_1 + \beta_2 = 1$, we have

$$\frac{r}{y} = \frac{\eta}{\varepsilon g} = \frac{(1-\alpha)\eta}{(p-q)} \tag{55}$$

Let us note that formula (53) for optimal investment *r* can be treated as the first approximation of optimal feedback *r** (28) for zero slope $\omega = 0$ of R&D intensity and maximum price of production $x_2^0 = p^0$. For R&D intensity r/y, equation (55) describes dependence of optimal R&D intensity on the substitution parameter α, the subjective discount rate η and the discounted marginal productivity of technology $-g = q - p$. When the cost p for sustaining the accumulated R&D investment T is high, then the R&D intensity r/y is low. Vice versa, increase of the rate of return to R&D q leads to growth of the research intensity r/y. Assuming that the positive function $g = g(t)$ is non-increasing over time *t* we get the growth property of the R&D intensity r/y.

Taking into account relation (53) for optimal R&D investment *r* we can derive the growth process for technology

$$T = T_0 + \frac{B}{(B+1)} \eta y_0 \int_{t_0}^{t} \frac{e^{Q(\tau)}}{g(\tau)} d\tau, \quad T(t_0) = T_0 \tag{56}$$

For technology intensity $P = T/y$ one can obtain the following differential equation

$$\dot{P} = \frac{(\dot{T}y - \dot{y}T)}{y^2} = -\frac{\dot{y}}{y} P + \frac{r}{y} = -(f(t) - \frac{B}{(B+1)}\eta)P + \frac{B}{(B+1)} \frac{\eta}{g(t)}$$

Its solution is presented according to the Cauchy formula

$$P(t) = P_0 e^{-Q(t)} + \frac{B}{(B+1)} \eta \int_{t_0}^{t} \frac{e^{(-Q(t)+Q(\tau))}}{g(\tau)} d\tau, \qquad P(t_0) = P_0$$

Technology intensity *P* has the zero velocity on the curve

$$P^0(t) = \frac{B\eta}{(B+1)g(t)(f(t) - \eta B/(B+1))}$$

If the initial position (t_0, P_0) is located below the curve P^0, $P_0 < P^0(t_0)$, then technology intensity $P(t)$ is growing. If the initial position (t_0, P_0) is located above the curve P^0, $P_0 > P^0(t_0)$, then technology intensity $P(t)$ is declining over time t.

Finally we consider the value function $\varphi(t, y)$ which assigns the optimal result φ of the utility function (32) along the optimal process (y^0, r^0) to an initial position (t, y). The value function φ is the solution of the Hamilton-Jacobi equation for the reduced control problem

$$\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial y} fy + e^{-\eta t} \ln y + \max_r \{-\frac{\partial \varphi}{\partial y} gr + e^{-\eta t} B \ln r\} = 0 \qquad (57)$$

Let us find the value function φ in the class of the following structure

$$\varphi(t, y) = e^{-\eta t}(\mu(y) + \nu(t))$$

Substituting the optimal control r^0 (35) into the Hamilton-Jacobi equation (57) and considering price ψ as the gradient of the function $\mu(y)$

$$\psi = \frac{\partial \mu}{\partial y}$$

we derive equations for components $\mu(y)$, $\nu(t)$

$$-\eta(\mu + \nu) + \dot{\nu} + \frac{\partial \mu}{\partial y} fy + \ln y - B + B(\ln B - \ln g - \ln \frac{\partial \mu}{\partial y}) = 0$$

Using indeterminate coefficients in the expression for function μ

$$\mu(y) = C \ln y$$

we obtain the explicit expression for parameter C

$$C = \frac{(B+1)}{\eta}$$

and the linear differential equation for function ν

$$\dot{\nu}(t) = \eta \nu(t) + h(t), \qquad h(t) = B \ln g - \frac{(B+1)}{\eta} f - B(\ln \frac{B}{(B+1)} - \ln \eta - 1)$$

The general solution of this equation has the following form

$$\nu(t) = De^{\eta t} + F(t), \qquad F(t) = \int_0^t e^{-\eta(s-t)} h(s) ds$$

The transversality condition for component ν

$$\lim_{t \to +\infty} e^{-\eta t} \nu(t) = 0$$

provides the explicit expression for parameter C

$$D = -\int_{0}^{+\infty} e^{-\eta s} h(s) ds$$

Finally, we obtain the following explicit expressions for functions μ and v

$$\mu(y) = \frac{(B+1)}{\eta} \ln y, \qquad v(t) = -\int_{t}^{+\infty} e^{-\eta(s-t)} h(s) ds$$

In particular, if h is a constant, then v is also a constant determined by the formula

$$v = -\frac{h}{\eta}$$

According to the explicit expressions for the value function φ we can conclude that in the considered model the optimal result has the decomposition property. The first term μ depends only on the discount parameterη, parameter of the elasticity of substitution α and in the logarithmic way (not very intensively) on the initial production y and does not depend on the specific features of the dynamic system – functions f and g. On the contrary, the second term v is determined mainly by dynamics aggregated in function h and does not depend on initial production y.

4 Empirical analyses and evaluation of the results

4.1 Optimal R&D intensity

As developed in section 3.2, the optimal R&D intensity can be obtained by solving the optimal R&D control model:

$$\frac{r}{y} = \frac{\eta}{\varepsilon g} \qquad (55)[19]$$

Equation (46) suggests that the optimal R&D intensity depends on the elasticity of substitution ε, the discount rate η and the discounted marginal productivity of technology g and its level increases as ε and g decrease and η increases. After the model is constructed and the analytic optimal solution is obtained, the key process moves to the measurement of core factors in the model and to the empirical analysis of actual industrial activities by using the model.

4.2 Measurement of core factors

4.2.1 Measurement of elasticity of substitution

Under the condition of the equilibrium between demand and supply, the elasticity of substitution measured by demand-side factors (e.g. substitution between innovative goods) could be interpreted by the elasticity of substitution measured by supply-side factors (e.g. substitution among production factors). Given that production is represented by GDP (V, value added), the substitution elasticity should be between labour, capital and technology. By using a technology incorporation model to treat technology *(T)* embodied in labour *(L)* and capital *(K)*, the substitution elasticity could be treated as a bilateral substitution issue only between labour and capital $\varepsilon(K(T), L(T))$.

On the basis of technology incorporated production function [20], the elasticity of substitution between capital $K(T)$ and labour $L(T)$ can be formulated as follows:

$$\varepsilon = \frac{d\ln \frac{K(K',T)}{L(L',T)}}{d\ln \frac{P_l}{P_k}}$$

$$= \left[\left(\frac{\partial \ln K(K',T)}{\partial \ln T}\right)\left(\frac{d\ln T}{d\ln \frac{P_l}{P_k}}\right) - \left(\frac{\partial \ln K(K',T)}{\partial \ln K'}\right)\left(\frac{d\ln K'}{d\ln P_k}\right)\right] - \left[\left(\frac{\partial \ln L(L',T)}{\partial \ln T}\right)\left(\frac{d\ln T}{d\ln \frac{P_l}{P_k}}\right) + \left(\frac{\partial \ln L(L',T)}{\partial \ln L'}\right)\left(\frac{d\ln L'}{d\ln P_l}\right)\right] \quad (58)$$

where

L':	labour without technology incorporation [21]
K':	capital without technology incorporation
P_l:	price of labour and
P_k:	price of capital.

The following regression equations are used to measure the different terms in equation (58):

$$ln(T/L) = a_{l0} + b_{l1}\, lnT + b_{l2}\, ln(P_l/P_{t0}) \quad (59)$$

$$ln(T/K) = a_{k0} + b_{k1}\, lnT + b_{k2}\, ln(P_k/P_{t0}) \quad (60)$$

$$ln(GLC/GTC) = b'_{l3} + b'_{l4}\, lnT \quad (61)$$

$$ln(GCC/GTC) = b'_{k3} + b'_{k4}\, lnT \quad (62)$$

$$ln(P_l(T)/P_k(T)) = h + i\, lnT \quad (63)$$

where

P_{t0}:	price of technology
GLC:	gross labour cost
GCC:	gross capital cost and
GTC:	gross technology cost.

4.2.2 Measurement of discounted marginal productivity of technology

In the case using value-added (*V*) as production output, the discounted marginal productivity of technology *(g)* can be formulated as follows:

$$g \equiv p - q \quad (64)$$

$$p = \sum_{L,K} \frac{\partial V}{\partial X'} \cdot \frac{\partial X_T}{\partial T} \tag{65}$$

$$q = \frac{\partial V}{\partial T} \text{ (marginal productivity of technology)} \tag{66}$$

where

X': L' and K'; and

X_T: L_T (labour for technology) and K_T (capital stock for technology).

In order to measure the discounted marginal productivity of technology, it is requested to simultaneously solve the following equations (67), (68) and (69) in advance (see [22]):

$$P_t = (1-gs) \cdot [(Rls \cdot Dl + Rms \cdot Dm + Res \cdot De) + Rks \cdot Dk \cdot (\bar{r} + \rho)/(1-ct)] \tag{67}$$

$$\frac{\partial V}{\partial T} = \frac{GTC \cdot (P'_t / P_t)}{GLC + GCC + GTC \cdot (P'_t / P_t)} \cdot \frac{V}{T} \tag{68}$$

$$e^{mr} = \int_0^\infty \frac{\partial V}{\partial T} e^{-(\bar{r}+\rho)} dt = \frac{\partial V}{\partial T} / (\bar{r} + \rho) \tag{69}$$

where

P_t:	service price of technology
P'_t:	capital price of technology
Rls, *Rks*, *Rms* and *Res*:	shares of R&D expenditures for labour costs, tangible fixed assets, materials and energy respectively
Dl, *Dk*, *Dm* and *De:*	wage index, investment goods deflator, wholesale price indices of materials and energy respectively
gs:	ratio of government financial support
ct:	ratio of corporate tax and
$\bar{r}$:	rate of internal return to R&D investment
m:	time-lag from R&D to commercialisation and
ρ	rate of obsolescence of technology.

Assume that factor input directing to R&D ($X_T = L_T, K_T, M_T, E_T$) which composes T and consists of labour, capital, materials and energy takes similar marginal productivity as production factors at the initial year. By introducing price indices D_V, D_X, D_T (here D_T is equal to P_t) and D_{XT} (initial year =1) corresponding to V, X, T and X_T, finally we can develop equation (54) into equation (70):

$$p \approx \sum_{L,K} \frac{D_X}{D_V} \cdot \frac{D_T}{D_{XT}} \cdot \frac{P_{y0}}{P_{V0}} \tag{70}$$

As shown in equation (66), term q is exactly the marginal productivity of technology. The discounted marginal productivity of technology (g) can be obtained from the balance of term p and q.

4.2.3 Measurement of discount rate

Normally the discount rate was treated as the average rate of bank loans. However, in recent years there has been an argument to introduce weighted average capital cost for discount rate (e.g. [23]). Stimulated by this argument an attempt to introduce composite discount rate was conducted. Composite discount rate η can be measured by the following equation:

$$\eta = r_1 \cdot w_1 + r_2 \cdot w_2 / (1 - Tax) + r_3 \cdot w_3 \tag{71}$$

where

r_1	interest rate (average rate of bank loans)
r_2	real dividend yield (= $DIVD/(CAP+CAPRV)$)
r_3	risk free rate (government bond yield)
w_1	the share of interest-bearing liabilities to gross assets (= LI/GA)
w_2	the share of capital stock and capital reserve to gross assets (=$(CAP + CAPRV)/GA$)
w_3	the share of the other reserves to gross assets (=PS/GA)
Tax	corporate tax rates
$DIVD$	dividend
CAP	capital stock
$CAPRV$	capital reserve
LI	interest-bearing liabilities
SE	shareholders' equity
PS	the other reserves (= $SE-CAP-CAPRV$)
GA	gross assets (= $LI + CAP + CAPRV + PS = LI+SE$)

Our analysis demonstrates that the composite discount rate introduced here seems to reflect the reactions of respective sectors' behaviour in the market.

4.3 Results of empirical analyses

On the bases of the measurement of core factors (elasticity of substitution, discounted marginal productivity of technology and composite discount rate), Table 1 evaluates the optimal R&D intensity of Japan's manufacturing industry (manufacturing average (MA), food (FD), chemicals (CH) and electrical machinery (EM)) over the last two decades (1975-1996). The evaluation is conducted by dividing the period of the analysis into five periods: 1975-1978 (after the first energy crisis and before the second energy crisis);

1979-1982 (after the second energy crisis and before the fall of international oil prices); 1983-1986 (after the fall of international oil prices and before the bubble economy); 1987-1990 (during the period of the bubble economy); and 1991-1996 (after the bursting of the bubble economy). In addition to R&D intensity using the 1990 fixed prices, the nominal value (current prices) of optimal R&D intensity is also calculated.

Table 1 Calculation of optimal level of R&D intensity in major sectors of the Japanese manufacturing industry (1975-1996)

				1975-78	*1979-82*	*1983-86*	*1987-90*	*1991-96*
Manufacturing		η	(%)	8.45	8.55	7.21	6.11	4.70
Average		ε		1.01	1.01	1.01	0.71	0.42
		g		1.05	1.23	1.23	1.20	1.42
	1990 prices	$(r/V)_{opt.}$	(%)	7.97	6.88	5.80	7.17	7.88
		$(r/V)_{act.}$	(%)	4.36	4.72	5.90	6.77	6.91
	Current prices	$(r/V)_{opt.}$	(%)	6.15	6.05	5.34	6.87	8.20
		$(r/V)_{act.(n)}$	(%)	3.36	4.15	5.43	6.49	7.19
Food		η	(%)	9.12	8.94	7.58	6.02	4.60
		ε		0.69	0.69	0.691	0.59	0.41
		g		2.41	2.55	2.31	1.90	2.13
	1990 prices	$(r/V)_{opt.}$	(%)	5.48	5.07	4.75	5.33	5.32
		$(r/V)_{act.}$	(%)	0.75	0.84	1.09	1.67	1.64
	Current prices	$(r/V)_{opt.}$	(%)	6.18	6.10	5.04	5.13	5.05
		$(r/V)_{act.}$	(%)	0.85	1.01	1.16	1.60	1.56
Chemicals		η	(%)	8.45	8.49	6.96	6.14	4.76
		ε		1.31	1.31	1.31	1.12	0.63
		g		0.34	0.51	0.63	0.67	0.87
	1990 prices	$(r/V)_{opt.}$	(%)	18.97	12.71	8.43	8.18	8.68
		$(r/V)_{act.}$	(%)	18.28	15.78	15.14	15.26	14.14
	Current prices	$(r/V)_{opt.}$	(%)	9.24	8.13	6.88	7.47	9.73
		$(r/V)_{act.}$	(%)	8.90	10.09	12.35	13.94	15.84
Electrical		η	(%)	8.87	8.93	7.30	5.95	4.62
Machinery		ε		1.69	1.69	1.69	1.65	0.57
		g		0.18	0.27	0.30	0.42	0.81
	1990 prices	$(r/V)_{opt.}$	(%)	29.16	19.57	14.40	8.59	10.01
		$(r/V)_{act.}$	(%)	33.93	23.70	20.59	17.35	12.95
	Current prices	$(r/V)_{opt.}$	(%)	8.02	8.92	9.03	7.54	13.23
		$(r/V)_{act.}$	(%)	9.33	10.80	12.91	15.24	17.12

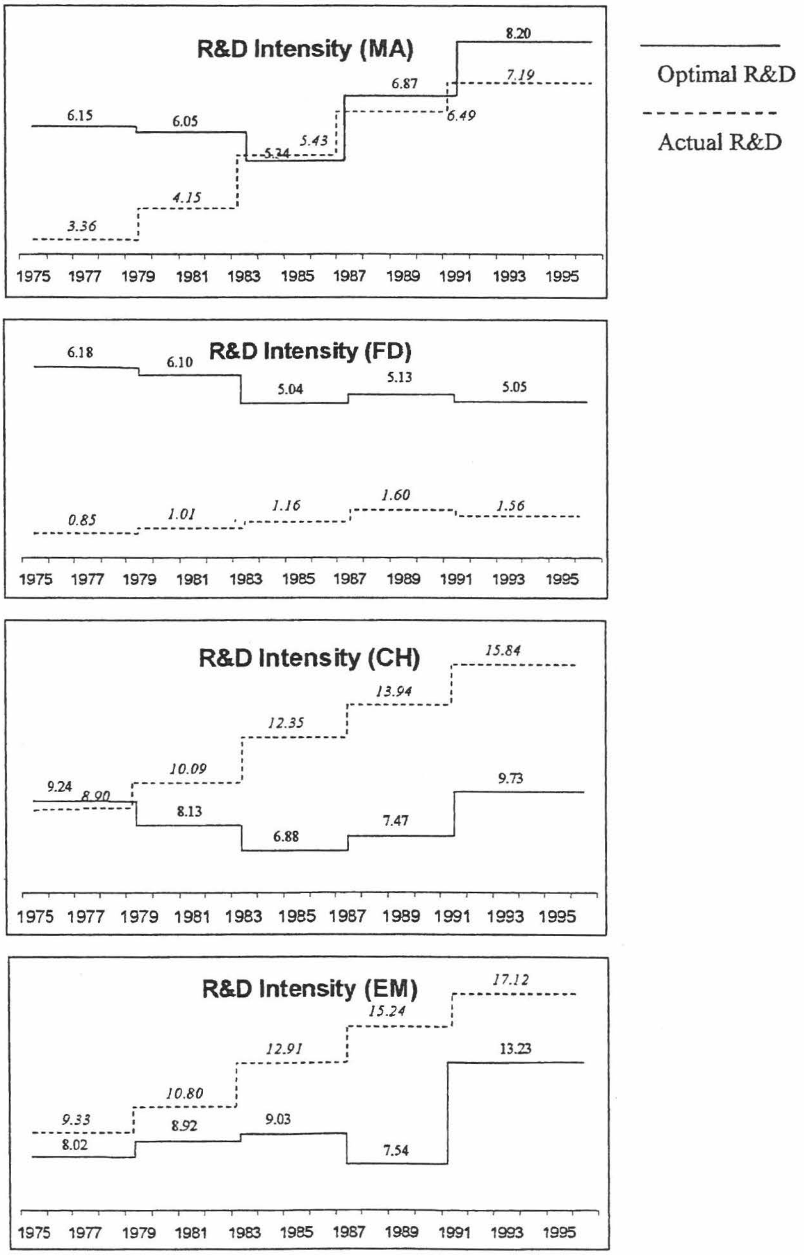
η: discount rate; ε: elasticity of substitution; g: discounted marginal productivity of technology; $(r/V)_{opt.}$: measured optimal R&D intensity; $(r/V)_{act.}$: actual R&D intensity.

Since the period between 1975-86 is relatively homogeneous in comparison to the other two periods examined, in order to use the elasticity of substitution for as long a period as possible (see [24]) the elasticity of substitution for three periods is used for this evaluation analysis.

4.4 Comparative analyses between optimal and actual R&D intensity

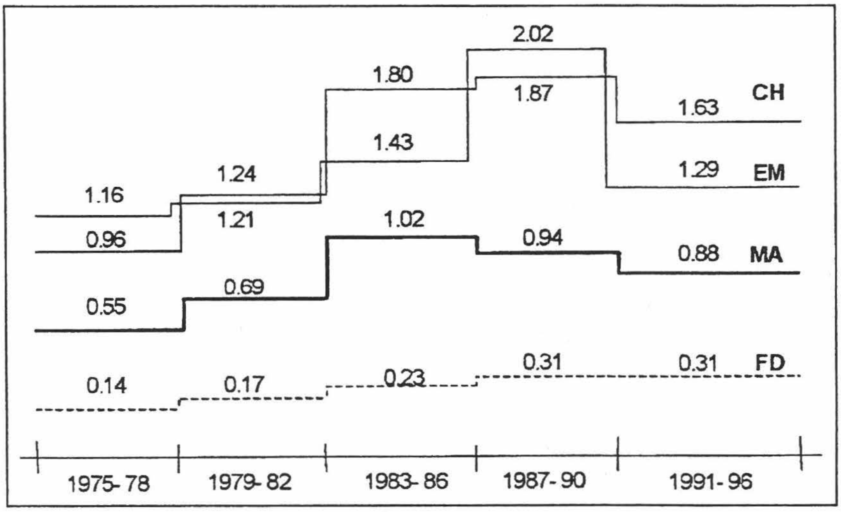
Based on the calculation results summarised in Table 1, comparative analyses between optimal and actual R&D intensity for secors MA, FD, CH and EM in the Japanese manufacturing industry over the period 1975-1996 are illustrated in Figure 1.

Figure 1 Comparison of optimal R&D intensity and actual R&D intensity – comparison of leading sectors in the Japanese manufacturing industry (1975-1996): %



First of all, let us look at the case of manufacturing average (MA). The Japanese manufacturing industry's R&D intensity was far behind the optimal level until the middle of the 1980s. This imbalance decreased due to consistent efforts to strengthen R&D investment. Consequently, Japan's manufacturing industry reached a reasonable level of R&D intensity, 5.4% of GDP and a little bit higher than the optimal level, making the balance reversed. However, this balance reversed again during the period of the bubble economy when the actual R&D intensity changed to 5.5% lower than the optimal level. This imbalance grew after the bursting of the bubble economy resulting in the manufacturing industry's R&D intensity dropping to 12.3% lower than the optimal level (see Figure 2). This low level of R&D intensity might be the source of the current 'vicious cycle' between R&D and economic growth resulting in Japan's decrease in international competitiveness [25].

Figure 2 Trends in the ratio between actual and optimal R&D intensity



By looking at the trends in R&D intensities of optimal and actual level, quite different observations can be found between FD and other two sectors; while similar observations can be found between CH and EM.

In the case of FD, the R&D intensity is always far behind the optimal level over the whole period examined. It increased slightly before the bubble economy while the optimal level changed from 6.2% of GDP to 5.0%. After the bursting of the bubble economy, both actual and optimal R&D intensity decreased slightly. However, the optimal level is still much higher than the actual one.

In the case of CH, the R&D intensity is almost always higher than the optimal level except for the period 1975-1978. Due to consistent efforts to strengthen R&D investment, the R&D intensity continued to increase over the whole period while the optimal level decreased by the middle of the 1980s before changed to an increasing trend. Consequently, the discrepancy between actual and optimal R&D intensity levels continued to increase.

In the case of EM, similar to the behaviour of CH, the R&D intensity is always higher than the optimal level over the whole period examined. Also due to consistent efforts to strengthen R&D investment, the R&D intensity increased while the optimal level increased slightly before the bubble economy, changed to a decrease slightly during the period of the bubble economy and again increased dramatically after the bursting of the bubble economy. The discrepancy between actual and optimal R&D intensity levels increased until the period of the bubble economy and changed to a decrease after the bursting of the bubble economy.

4.5 Allowance of R&D intensity

It is generally accepted that, similar to the safety allowance of the machine, in order to secure a sustainable development trajectory, a certain allowance between actual R&D intensity and optimal R&D intensity level is necessary. Figure 2 illustrates the trends in the allowance of R&D intenisty (the ratio of actual and optimal R&D intensity). Looking at this Figure, we note the following observations:

1 Allowance of R&D intensity has been increasing steadily until 1987 in all sectors examined.

2 However, this ratio changed to a decreasing trend from 1987 in manufacturing average and chemicals. While the ratio of electrical machinery continued to increase until 1990, it changed to a dramatic decrease from 1991.

3 Contrary to the above trends, the ratio of food continued to increase and maintained the same level after 1987.

Noteworth trends depicted in 2, demonstrate the hypothetical view of the concern about a vicious cycle between R&D and growth.

4.6 Interpretation of the results

In this empirical analysis, three leading manufacturing sectors, FD, CH and EM are examined. FD is one of the typical biological resources dependent industries, while EM is a knowledge intensified industry. CH can be classified between FD and EM as it encompasses such a nature as resources dependency and knowledge intensified. FD is generally classified in the low-tech sector while encompassing some high-tech facets such as depending on advanced biotechnology. Contrary to FD, CH and EM are generally classified in the high-tech sector as they depend on a high level of R&D intensity. Pavitt [26] contrasted these three sectors with scale-intensive characteristics (FD) and with science-based characteristics (CH and EM).

As summarised by Pavitt, the nature of technological opportunities and threats facing firms varies considerably as a function of their principle activity. Rich technological opportunities are associated with science-based and specialised suppliers, with relatively many opportunities for innovations and high outside threats from others diversifying horizontally and from technologically active users. Supplier-dominated firms have fewer technological opportunities and are under threat of entry from suppliers. Scale-intensive firms focus on improving complex and interdependent product technologies. Together with specialised suppliers, they can exploit opportunities for 'fusion' with radical breakthrough technologies.

Figure 1 suggests the following interesting observations with respect to a clear contrast in the three sectors examined:

1 Optimal R&D intensity in EM and CH (typical high-tech sectors) are higher than FD.

2 Actual R&D intensity of EM and CH is much higher than the optimal level, while FD demonstrates the opposite. This is considered to be due to the following:

   - Among priority strategies for EM and CH firms to seek maximum profit including market strategies (e.g. sales promotion, propaganda, etc.), process management/control and strategic alliances, the decision for the investment option is one of the most crucial issues.

   - The results demonstrate that in order to achieve maximum return, R&D investment plays a more significant role than manufacturing investment for EM and CH in which speed of innovation is crucial; while manufacturing investment plays a significant role for FD in which mass volume production of variety of goods is crucial. Another reason for the lower level of R&D intensity in FD is because it mainly depends on other sectors' R&D rather its own.

3 The high-level of R&D intensity in CH and EM is due to the typical nature of high-tech sectors under severe competition, encompassing not only really essential R&D intensity but also Pseudo R&D intensity ('Pseudo innovation,' see [27]), similar to the safety allowance in a machine.

4 However, these allowances have dramatically decreased after the bursting of the bubble economy in 1991 leading to the vicious cycle between R&D and growth.

5 Motivations for Pseudo innovation include the following:

   - 'feint,' 'decoy' to rivals,

   - posturing as a really high-tech firm demanding customers,

   - 'cheaper propaganda,'

   - 'cannot stop,' and

   - 'innovation hungry' ……

These observations remind us of a 'pseudo innovation' postulated by Mensch [27]. He pointed out a source of this pseudo innovation, particularly in high-tech sectors, the time discrepancy of customers' reaction to such characteristics of high-tech products as functionality of the product, safety for the user and durability of the product. Due to this discrepancy in high-tech products he pointed out that "It is important to note that in advanced stage of brand growth, important innovations are replaced with the increased frequency by pseudo-innovation."

5 Conclusions

Conclusions obtained from this analysis can be summarised as follows:

1 Theoretical analyses based on the optimal control theory are applied to this R&D investment decision making aiming at maximising a utility function.

2 The maximum principle of Pontryagin is applied for designing optimal nonlinear dynamics. Optimality principles are interlinked with equilibrium properties of the Hamilton system. Suboptimal feedbacks of the rational type for balancing the dynamic system are constructed. Properties of suboptimal feedbacks and techno-economic trajectories are indicated for different slopes of R&D intensities.

3 Methodologies for the measurement of core factors essential for the practical application of the optimal control model are developed.

4 The empirical analyses demonstrate the practical significance of this approach. Evaluations of the R&D intensity level in major sectors were conducted by comparing the optimal and actual level. 'Pseudo innovation' in certain high-tech sectors and its sources are identified.

Acknowledgement

Alexander M. Tarasyev was supported by the Russian Fund for Fundamental Research under grants 99-01-00146, 00-15-96057, and Fujitsu Research Institute (FRI) under IIASA-FRI contract 01-117.

References and Notes

1 Arrow, K.J. (1985) *Production and Capital. Collected Papers*, The Belknap Press of Harvard University Press, Cambridge, Massachusetts, London.

2 Borisov, V.F., Hutschenreiter, G. and Kryazhimskii, A.V. (1999) 'Asymptotic growth rates in knowledge-exchanging economies', *Annals of Operations Research*, Vol. 89, pp.61–73.

3 Grossman, G.M. and Helpman, E. (1991) *Innovation and Growth in the Global Economy*, The MIT Press, Cambridge, Massachusetts, London.

4 Leitmann, G. and Lee, C.S. (1999) 'On one aspect of science policy based on an uncertain model', *Annals of Operations Research*, Vol. 88, pp.199–214.

5 Romer, P.M. (1990) 'Endogenous technological change,' *Journal of Political Economy*, Vol. 98, pp.71–102.

6 Watanabe, C. (1992) 'Trends in the substitution of production factors to technology – empirical analysis of the inducing impact of the energy crisis on Japanese industrial technology', *Research Policy*, Vol. 21, pp.481–505.

7 Intriligator, M. (1971) *Mathematical Optimization and Economic Theory*, Prentice-Hall, N.Y.

8 Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V. and Mishchenko, E.F. (1962) 'The mathematical theory of optimal processes', *Interscience*, New York.

9 Crandall, M.G. and Lions, P-L. (1983) 'Viscosity solutions of Hamilton-Jacobi equations', *Trans. Amer. Math. Soc.*, Vol. 277, No. 4, pp.1–42.

10 Subbotin, A.I. (1995) *Generalized Solutions for First-order PDE*, Birkhauser, Boston.

11 Krasovskii, A.N. and Krasovskii, N.N. (1995) *Control Under Lack of Information*, Birkhauser, Boston.

12 Dolcetta, I.C. (1983) 'On a discrete approximation of the Hamilton-Jacobi equation of dynamic programming', *Appl. Math. Optimiz.*, Vol. 10, No. 4, pp.367–377.

13 Tarasyev, A.M. (1999) 'Control synthesis in grid schemes for Hamilton-Jacobi equations', *Annals of Operations Research*, Vol. 88, pp.337–359.

14 Feichtinger, G. and Kopel, M. (1993) 'Chaos in nonlinear dynamical systems exemplified by an R&D model', *European Journal of Operational Research* , Vol. 68, pp.145–159.

15 In the empirical analysis part of this research, we measured the production factors as the following:

- y Production, = (gross cost at 1990 fixed prices)
- V Value added, = (gross domestic product at 1990 fixed prices)
- L Labour, = (number of employed persons) x (working hours)
- K Capital, = (capital stock) x (operating rate)
- M Materials: intermediate inputs except energy, = (intermediate input at 1990 fixed prices) (gross energy cost at 1990 fixed prices)
- E Energy, = (final energy consumption)
- T Technology, $T_t = r_{t-m} + (1-\rho)\, T_{t-1}$, where r_{t-m}: R&D expenditure in time t-m; m: time lag between R&D and commercialisation; and ρ: rate of obsolescence of technology.

16 Tarasyev, A.M. and Watanabe, C. (2001) 'Optimal dynamics of innovation in models of economic growth', *Journal of Optimization Theory and Applications*, Vol. 108, No. 1.

17 Hartman, P.H. (1964) *Ordinary Differential Equations*, J. Wiley and Sons, N.Y., London, Sydney.

18 (a) Empirical analysis on the invention of innovative goods in the Japanese manufacturing industry over the period 1975-1996 demonstrates the following structure of the function of the quantity of invented products $n(s)$:

$PAT = 33.78\, r^{0.34}\, T^{0.62}$ adj.R^2 0.992 DW 1.22

(5.41) (5.25) (5.79)

(where PAT: quantity of invention of innovative goods measured by number of patent applications by the Japanese manufacturing industry)

This suggests that κ (in equation 6) could be approximated to 0.

(b) $T_t \approx \dfrac{r_{t-(m-1)}}{\theta+\sigma}$ Provided that $t >> m$-1, $T_t \approx \dfrac{r_t}{\theta+\sigma}$

where θ: average increase rate of r.

19 (a) In the case where $n(s)$ satisfies the condition of the constant returns to scale with respect to r and T, theoretically we can have $\dfrac{\partial \ln n}{\partial \ln r}+\dfrac{\partial \ln n}{\partial \ln T}=1$ which means sum of β_1 and β_2 is 1.

(b) According to the empirical analysis in [18] we can approximate that the sum of β_1 and β_2 is 1. Therefore equation (55) can be used.

20 Here we used the production function $V = V(t, L(T), K(T))$.

21 In L' or K', factor input directing to R&D has been deducted.

22 Watanabe, C. and Wakabayashi, K. (1997) 'The perspective of techno-metabolism and its insight into national strategies,' *Research Evaluation*, No. 2, pp.69–76.

23 Saito, T. (1997) 'Significance of the capital cost for firm's strategy', *Finance Review*, Vol. 23, pp.13–23.

24 Griffin, J.M. and Gregory, R. (1976) 'An intercountry translog model of energy substitution responses', *American Economic Review*, Vol. 66, pp.845–57.

25 Griffy-Brown C., Watanabe, C. and Zhu, B. (2000) 'Japan struggles to overcome high-technology failures and prolonged recession: implications for both policy and business practice', *The Graziadio Business Report*, No. 2.

26 Pavitt, K., Robson, M. and Townend, J. (1989) 'Accumulation, diversification and organisation of technological activities in UK companies, 1945-1983', in M. Dodgson (Ed.), *Technology Strategy and the Firm: Management and Public Policy*, Longman, pp.38–67.

27 Mensch, G.O. (1979) *Stalemate in Technology – Innovation Overcome the Depression*, Ballinger Publishing Company, Cambridge, Massachusetts, (Original title; *Das Technologische Patt*, Umschau Vertag, Frankfurt, 1975)].

Bibliography

Isaacs, R. (1965) *Differential Games*, John Wiley and Sons, New York.

Ivanov, V.A., Tarasyev, A.M., Ushakov, V.N. and Khripunov, A.P. (1993) 'The toreador problem', *Journal of Applied Mathematics and Mechanics*, Vol. 57, No. 3, pp.419–425. .