RR-74-25

ON OPTIMUM CONTROL OF MULTI-RESERVOIR SYSTEMS

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December 1974

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1. Introduction

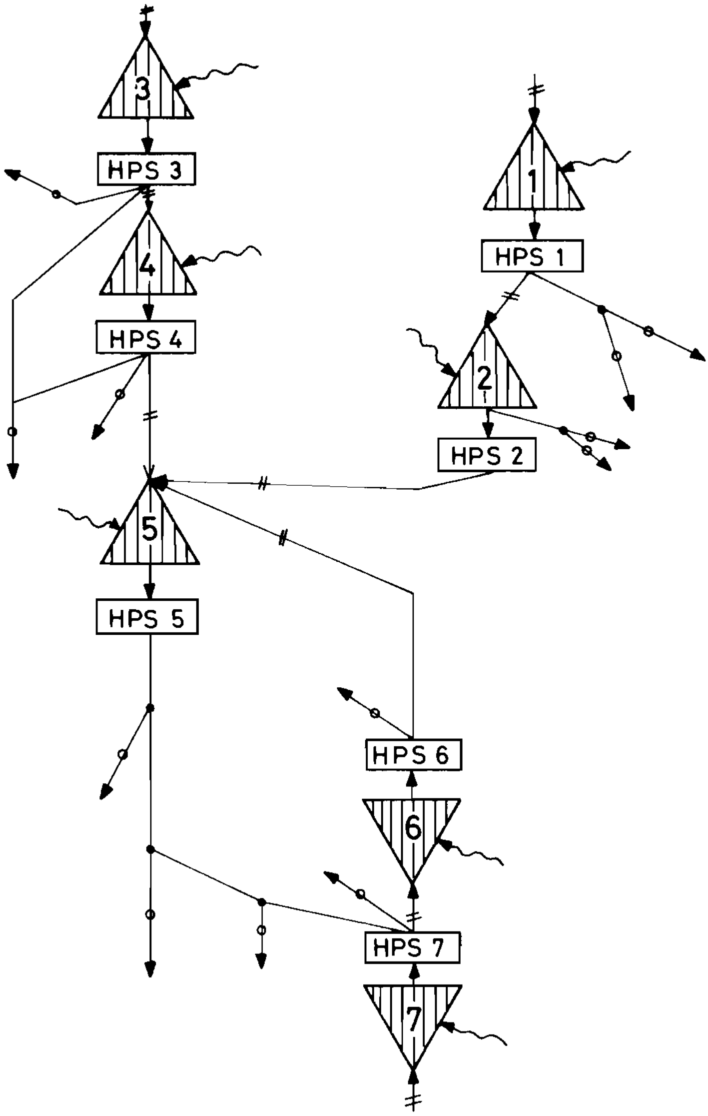
In this paper the multi-reservoir systems which are in operation are considered (e.g. see Figure 1). In recent years, many papers have been devoted to the analysis of these systems [1, 2, 3]. IIASA has also made a significant contribution to this field [4, 5]. Nevertheless, authors agree that there are many questions which are far from being settled. They pertain to the following problems:

i) development of mathematical models describing and predicting hydrological, ecological, economical and social processes,

ii) development of models for optimization of the controlled processes in multi-reservoir systems,

iii) development of decision making models which take into account the active part of people in the water resources systems.

The multi-reservoir systems combine large regions with different hydrological, economical and even political conditions. On that account, it is difficult to state that in these systems unequivocal solutions exist as far as optimum control is concerned. Therefore, optimum control of water resource systems, and particularly multi-reservoir systems, should be a dynamic iterative process



1,. . . . 7 - RESERVOIRS

HPS_i - HYDROPOWER STATION

- USERS

- INFLOWS IN THE RESERVOIRS

FIGURE 1. HYPOTHETICAL RESERVOIR SYSTEM

with the following phases: recognition of the system (including its verbal and quantitative description); simulation and optimization; decision making and its implementation; evaluation of the consequences and return to the recognition phase. This procedure is shown in Figure 2.

In this paper an approach is presented which involves the development of optimization models for control of the multi-reservoir system with arbitrary configurations. It is assumed that in this system the reservoirs are the basic controlled elements and their water resources are used for hydroelectric energy generation, irrigation, water supply of cities, recreation, fish and wildlife enhancement, etc. The objective is to determine the optimum release policy over the specified release periods in accordance with the predetermined criterion and set of constraints, which include for example storage reservations for flood control and for recreational use, mandatory releases to the users, etc.

2. Methodology

The description of the optimization model will be made, with no loss of generality, on the basis of the example shown in Figure 1. The generality assumptions which have been considered in this model are the following:

1. The main water controlled resources in the system are reservoirs. They have a restricted capacity and their resources are used both for satisfaction of the users'

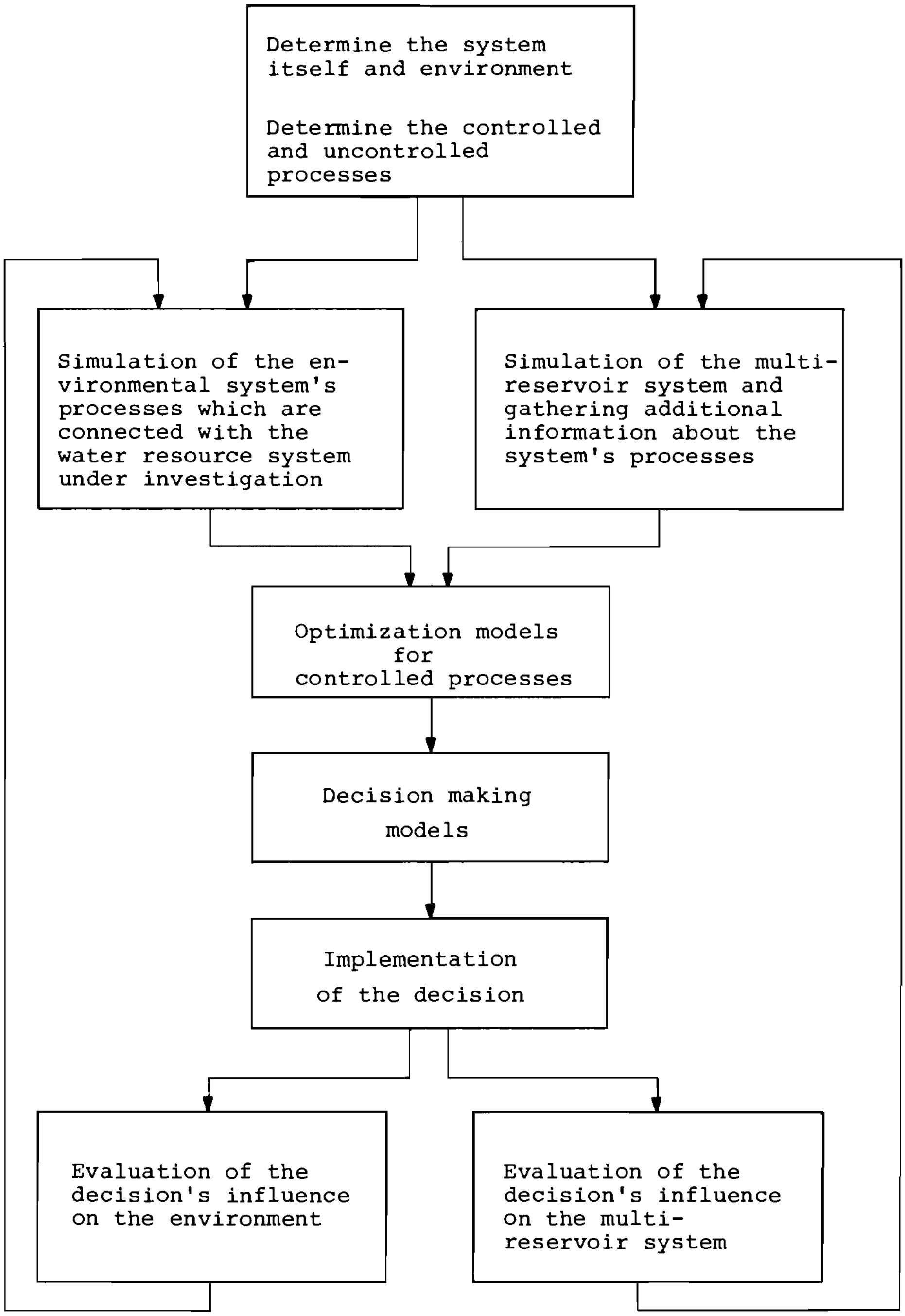


Figure 2. Iterative process of optimal control.

demand and for flood control.

2. The inflows and allocation processes are considered in the interval of time (t_1, t_k), divided into k subintervals (stages).

3. The function describing the relationship between the user's loss and the amount of water distributed to him is given for each user.

4. All social, political and other quality constraints can be expressed (at least roughly) in a quantity relationship.

5. Because of the finite volume of the reservoirs, the available amount of water in the system is controlled in a restricted manner.

6. It is assumed that all the processes in the system are predictable and can be evaluated in a quantitative manner in the interval (t_1, t_k). This assumption means that the model is deterministic.

Let us consider the system shown in Figure 3 under the above mentioned assumptions. This figure presents the same system as is shown in Figure 1, but the processes are divided into four stages. This means that both spatial and temporary linkage are shown.

The basic process in each reservoir is storage and allocation of water at every stage. Using the typical scheme of the i^{th} reservoir at the s^{th} stage, shown in Figure 4, the following equation can be written:

$$(1) \qquad Z_{is} = Z_{i,s-1} + \sum_{c \varepsilon C_{is}} I_c + \sum_{q \varepsilon Q_{is}} \xi_q + \sum_{p \varepsilon P_{is}} X_p - \sum_{h \varepsilon H_{is}} X_h - \sum_{r \varepsilon R_{is}} X_r - \delta is \quad ,^1 \qquad \forall i, s \ ,$$

where Z_{is} is state (amount of water in $[m^3]$) of the i^{th} reservoir at the s^{th} stage; $Z_{i,s-1}$ is state of the i^{th} reservoir at the $(s-1)^{th}$ stage; Z_{io} is initial state at $t = t_1$;

I_c is c^{th} main uncontrolled input of the i^{th} reservoir at the s^{th} stage,

$c \ \varepsilon \ C_{is}$, C_{is} is a set of numbers,
$C_{is} = \{1,2,...,C,...,\eta_{is}\}$, for all i,s,

ξ_q is q^{th} additional uncontrolled input,

$q \ \varepsilon \ Q_{is}$, $Q_{is} = \{1,2,...,q,...,W_{is}\}$, i^{th} reservoir, s^{th} stage,

X_p is p^{th} main controlled input,

$p \ \varepsilon \ P_{is}$, $P_{is} = \{u_{is} + 1, u_{is} + 2,...,p,...,V_{is}\}$, i^{th} reservoir, s^{th} stage,

X_h is h^{th} controlled output for allocating the amount of water X_h to the user h, $h \ \varepsilon \ H_{is}$, $H_{is} = \{1,2,...,h,...,\ell_{is}\}$, i^{th} reservoir, s^{th} stage,

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[1]This equation is written under the assumption that the amounts of water in the input and in the output of the hydropower station HPS_{is} are equal.

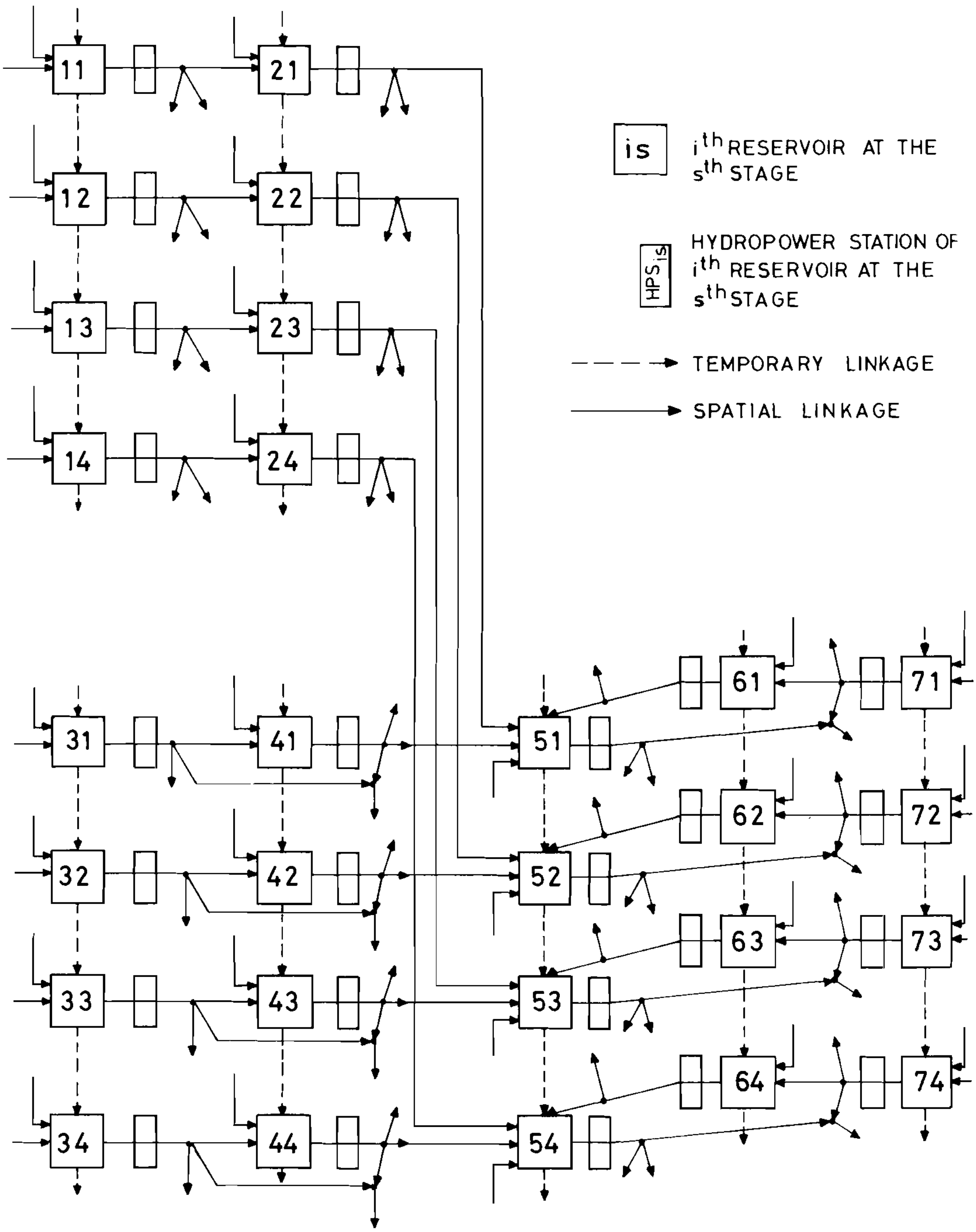
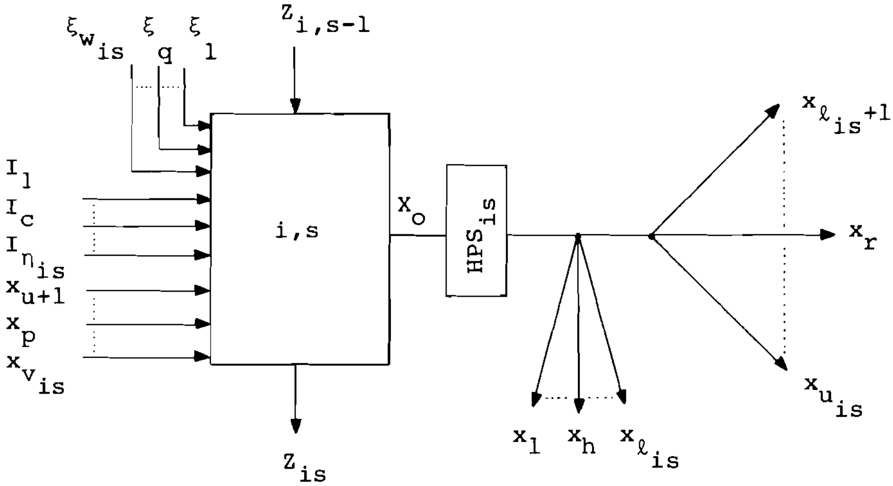


FIGURE 3. SPATIAL AND TEMPORARY LINKAGE BETWEEN THE RESERVOIRS



I_c, $c \in C_{is} = \{1,2,\ldots,c,\ldots,n_{is}\}$ - main uncontrolled inputs,

ξ_q, $q \in Q_{is} = \{1,2,\ldots,q,\ldots,w_{is}\}$ - additional uncontrolled inputs,

X_h, $h \in H_{is} = \{1,2,\ldots,h,\ldots,\ell_{is}\}$ - controlled outputs,

X_r, $r \in R_{is} = \{\ell_{is}+1,\ell_{is}+2,\ldots,r,\ldots,u_{is}\}$ - additional controlled outputs,

X_p, $p \in P_{is} = \{u_{is}+1,u_{is}+2,\ldots,p,\ldots,V_{is}\}$ - main controlled inputs.

(All upper variable indexes referring to the i^{th} reservoir at the s^{th} stage are omitted.)

Figure 4. Typical scheme of the i^{th} reservoir at the s^{th} stage.

X_r is r^{th} additional controlled output of the i^{th} reservoir at the s^{th} stage, $r \varepsilon R_{is}$, $R_{is} = \{\ell_{is} + 1, \ell_{is} + 2, \ldots, r, \ldots, u_{is}\}$,

δ_{is} is the minimum admissible amount of water in the i^{th} reservoir at the s^{th} stage

Taking into consideration that the set $P_{is} = \emptyset$ both for all initial reservoirs and for all corresponding stages (e.g., for reservoirs 1, 3 and 7 at stages 1, 2, 3 and 4 in Figure 3) and that the main controlled inputs of the i^{th} reservoir are additional controlled outputs of the previous reservoirs, the variables X_p for all $p \varepsilon \bigcup_{\forall i,s} P_{is}$ can be eliminated. For example, if the r^{th} output of the $(i-k)^{th}$ reservoir is linked with the p^{th} input of the i^{th} reservoir (see Figure 5), then

(2) $$X_p = (1 - \alpha^i_{i-k})\, X_r;\ p \varepsilon P_{is};\ \ r \varepsilon R_{i-k,s}\ .$$

The coefficient α^i_{i-k}, $0 < \alpha^i_{i-k} < 1$, reflects the losses along the additional controlled output (called canal below), linking the $(i-k)^{th}$ and i^{th} reservoirs.

After transformation (2) has been made, the controlled outputs for every reservoir are reduced to those belonging to the sets H_{is} and R_{is} for all i and s.

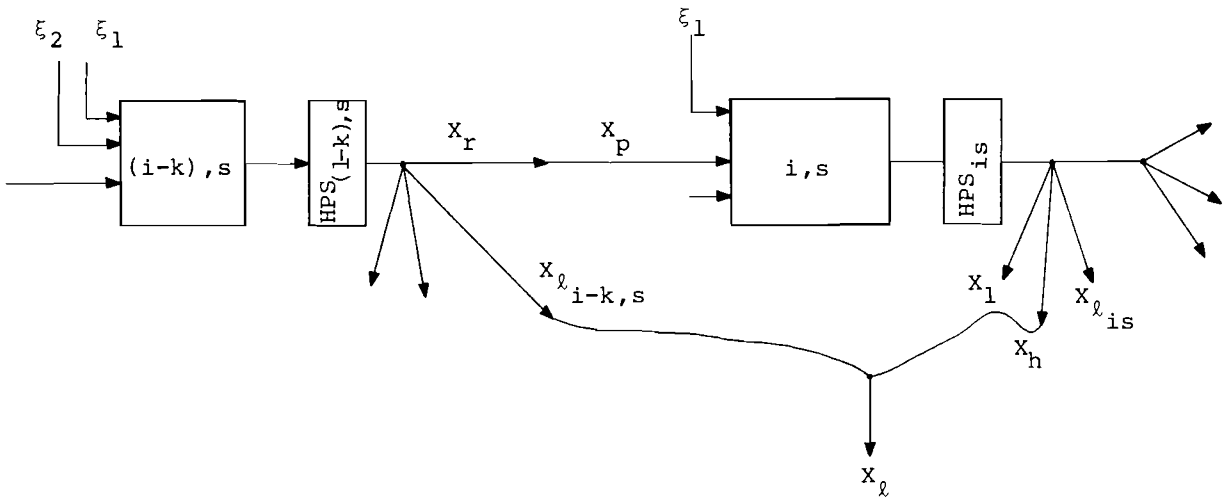


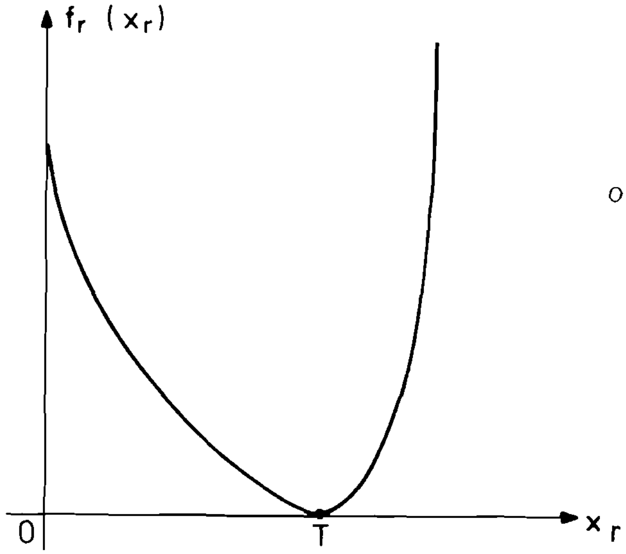
Figure 5. Obtaining water for ℓ^{th} user from both the reservoirs i^{th} and $(i-k)^{th}$.

The function $f_h(X_h)$, $h \varepsilon H_{is}$, all i,s, can be put in accordance with every controlled output. This function measures the losses which the user will suffer if the water allocated to him is X_h. This function reflects some economical, social and political aspects of the water allocation problem.

In the cases where one user receives water from two or more reservoirs the problem of obtaining the function $f_h(X_h)$ arises. For example, Figure 5 shows that the amount of water X_ℓ for the ℓ^{th} user comes from both the i^{th} and $(i-k)^{th}$ reservoirs. In such cases the determination of the functions $f_h(X_h)$ and $f(X_{\ell_{i-k,s}})$ can be made on the basis of: the costs connected with transferring the amount of water X_h and $X_{\ell_{i-k,s}}$, and the preferences of the users for the value of X_h and $X_{\ell_{i-k,s}}$. The latter presuppose that the utility function of X_h and $X_{\ell_{i-k,s}}$ should be involved.

In a similar way, the function $f_r(X_r)$ can be put in accordance with every additional controlled output (canal) $r \varepsilon R_{is}$, all i and s. This function usually depends on the following factors:

i) the amount of water needed for keeping up the ecological equilibrium in the canal, for fish and wildlife enhancement, etc.

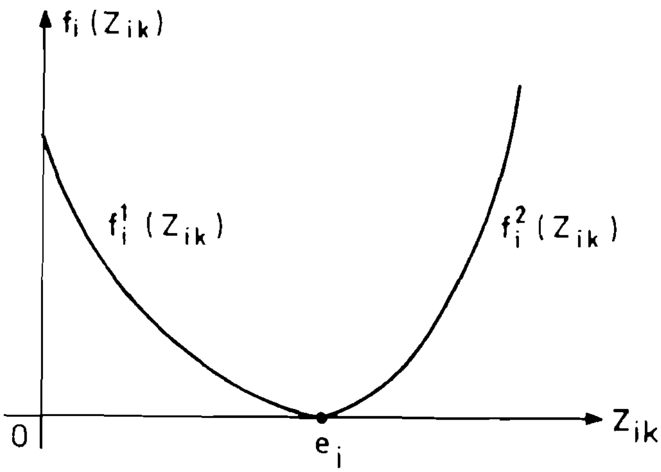


$x_r = T$ - target

$0 \leq x_r \leq T$ - region in which losses from disturbance of the ecological equilibrium, recreational conditions and navigation occur.

$x_r \geq T$ - region in which losses from flooding mainly occur.

Figure 6. Canal's goal function. (not to scale)



g_i and e_i are fixed (see equation 3).

Figure 7. Terminal function for i^{th} reservoir at k^{th} stage. (not to scale)

ii) the amount of water needed for recreational use and navigation,

iii) the maximum value of X_r, $r \varepsilon R_{is}$ which leads to flooding and other destruction.

The function shown in Figure 6 reflects some of these requirements.

In the model under investigation, the 3^{rd} type of the function is involved. By means of this function (let us call it terminal function) the final state of the i^{th} reservoir can be determined.

Let us denote with g_i the expected quantity of inflow in the time interval (t_k, t_m), $m > k$ and with e_i the expected demand of water in the same interval from the i^{th} reservoir. Then, the terminal function $f_i(Z_{ik})$ can be expressed as

$$(3) \qquad f_i(Z_{ik}) = \begin{cases} f_i^1(Z_{ik}), & \text{if } g_i + Z_{ik} \leq e_i \\ f_i^2(Z_{ik}), & \text{if } g_i + Z_{ik} > e_i \end{cases}$$

An example of this function is shown in Figure 7.

The main goal of systems control is to determine a release policy from the reservoirs (variables X_o^{is}), and an allocation of the released water between the users (variables X_h) and additional controlled outputs (variables X_r), so that the total loss, expressed by the equation (4), will be minimum.

$$(4) \qquad F(X) = \sum_{i=1}^{n} \sum_{s=1}^{k} \left[\sum_{h \varepsilon H_{is}} f_h(X_h) + \sum_{r \varepsilon R_{is}} f_r(X_r) + f_o^{is}(X_o^{is}) \right] + \sum_{i=1}^{n} f_i(Z_{ik})$$

where

$$X = \{X_h, X_r, Z_{ik}, X_o^{is}, h \varepsilon H_{is}, r \varepsilon R_{is}, \text{for all } i,s\}$$

When one minimizes (4) the following physical and other constraints are included:

a) The total release from the reservoir cannot be in excess of the available water and the expected inflow.

b) Each reservoir $i = 1,\ldots,n$ at every stage $s = 1,\ldots,k$ has a constraint capacity.

c) The controlled variables X_r and X_h have a lower and an upper boundary, respectively. The lower boundary usually reflects the mandatory demand of the user, while the upper boundary quantifies the goal of the user or represents the physical restrictions on the corresponding canal.

d) Reservoir storage constraints corresponding to recreational use, flood control, etc.

e) Constraints which take into account the possibility of one user obtaining water from two or more reservoirs.

The five types of constraints mentioned above can be quantified in the following way.

(5) $$\sum_{h\varepsilon H_{is}} X_h + \sum_{r\varepsilon R_{is}} X_r \leq Z_{i,s-1} + \sum_{c\varepsilon C_{is}} I_c + \sum_{p\varepsilon P_{is}} X_p + \sum_{q\varepsilon Q_{is}} \xi_q - \delta_{is} \quad , \quad \text{for all } i \text{ and } s = 1,\ldots,k-1 \quad ,$$

(6) $$\sum_{h\varepsilon H_{is}} X_h + \sum_{r\varepsilon R_{is}} X_r + Z_{ik} \leq Z_{i,k-1} + \sum_{c\varepsilon C_{is}} I_c + \sum_{p\varepsilon P_{is}} X_p + \sum_{q\varepsilon Q_{is}} \xi_q - \delta_{ik} \quad , \quad \text{for all } i \text{ and } s = k \quad ,$$

(7) $$\sum_{h\varepsilon H_{is}} X_h + \sum_{r\varepsilon R_{is}} X_r \geq Z_{i,s-1} + \sum_{c\varepsilon C_{is}} I_c + \sum_{p\varepsilon P_{is}} X_p + \sum_{q\varepsilon Q_{is}} \xi_q - (M_{is} - \gamma_{is}) \quad , \quad \text{for all } i \text{ and } s \quad ,$$

where

M_{is} is the maximum utilized storage in the i^{th} reservoir at the s^{th} stage (usually $M_{i1} = \cdots = M_{is} = \cdots = M_{ik}$), γ_{is} is the empty volume in the i^{th} reservoir at the s^{th} stage required for flood control,

(8) $$N\{X_\ell - X_h - X_{\ell_{i-k,s}} = 0\} \quad ,$$

if the amount of water X_ℓ for the ℓ^{th} user is obtained from both the i^{th} and $(i-k)^{th}$ reservoirs.

The expression $N\{\cdot\}$ means that there is a set of such constraints depending on the structure of the system. It is assumed that indexes of all the users in the system obtaining water from more than one reservoir belong to the set H'_{is}.

(9) $\underline{\nu}_h \leq X_h \leq \bar{\nu}_n, \bar{\nu}_h \geq 0,\ \underline{\nu}_h \geq 0$, for all $h \varepsilon H_{is}$ and all i and s ,

(10) $\underline{\nu}_r \leq X_r \leq \bar{\nu}_r,\ \bar{\nu}_r \geq 0,\ \underline{\nu}_r \geq 0$, for all $r \varepsilon R_{is}$ and all i and s ,

(11) $X_o^{is} - \sum_{h \varepsilon H_{is}} X_h \geq 0$, for all i and s ,

(12) $\underline{X}_o^{is} \leq X_o^{is} \leq \bar{X}_o^{is}$,

(13) $\delta_{ik} \leq Z_{ik} \leq M_{ik}$,

where

$\underline{\nu}_h$, $\underline{\nu}_r$ and $\underline{X}_o^{is}$ are the lower boundaries of the ν_h, ν_r and X_o^{is} respectively,

$\bar{\nu}_h$, $\bar{\nu}_r$ and $\bar{X}_o^{is}$ are the upper boundaries of the ν_h, ν_r and X_o^{is} respectively.

The variables X_p, $p \varepsilon P_{is}$, all i and s, and the variables Z_{is}, all i and s = 1,...,k - 1 can be eliminated using the recurrent equation (1) and equation (2).

Hence, the set of linear inequalities defined by (5), (6), (7), (8), (9), (10), (11), (12) and (13) can be expressed in the following more compact abbreviated form

(14) $A\ X \leq B$

where

$$X = \{X_h, X_r, Z_{ik}, X_o^{is}, h\varepsilon H_{is}, r\varepsilon R_{is}, \text{ for all } i,s\} \quad .$$

The matrix A contents L columns and D rows,

where

$$L = \sum_{i=1}^{n} \sum_{s=1}^{k} (u_{is} + X_o^{is}) + \sum_{i=1}^{n} Z_{ik} \ ,$$

$D = 3nk + 2L + a$; a is the number of

constraints of the type (8),

B is a vector column having D components.

Putting together (4) and (14) the following nonlinear optimization problem is derived: to find

$$\min \ F(X)$$

subject to

(15) $$A\,X \leq B \quad .$$

3. Some Computational Procedures

The main property of the problem (14) is its separable function F(X) and the set of linear constraints. That means that for solving the problem, some routine programming procedures could be applied (especially the separable programming technique [6]). Nevertheless, this problem can comprise a huge number of variables even if the system is not so large. For example, for the system shown in

Figure 3, the matrix A has 338 rows (246 of the rows represent constraints of type (9), (10), (12) and (13)) and 123 columns when only four stages are considered and if for solving the problem nonlinear programming is used.

Because of this, in this paper an idea for a specific procedure for the problem (15) is given. This procedure consists of the following:

a) The variables $G_{is} = \sum_{h \in H''_{is}} X_h$, each having θ_{is} values $G^1_{is}, G^2_{is}, \ldots, G^{\rho}_{is}, \ldots, G^{\theta_{is}}_{is}$, are introduced;

$$\sum_{h \in H''_{is}} \underline{\upsilon}_h \leq G_{is} \leq \sum_{h \in H''_{is}} \overline{\upsilon}_h ,$$

where

$$H''_{is} = H_{is} \setminus H'_{is} .$$

b) The optimum allocation for all the variables belonging to the set H''_{is} for every reservoir i at each stage s and for all values of the parameter $\rho = 1, \ldots, \theta_{is}$ is found. After that, for every value of this parameter the value of the function

$$F^{\rho}_{is}(G_{is}) = \sum_{h \in H''_{is}} f(\tilde{X}_h)$$

is determined.

c) The function $F_{is}(G_{is}) = F(F^{1}_{is}(G^{1}_{is}),\ldots,F^{\rho}_{is}(G^{\rho}_{is}),\ldots,$ $F^{\theta_{is}}_{is}(G^{\theta_{is}}_{is}),\ i = 1,\ldots,n;\ s = 1,\ldots,k$ is derived by interpolation between the values of the functions $F^{\rho}_{is}(G^{\rho}_{is}),\ \rho = 1,\ldots,\theta_{is};$ all i and s.

d) The optimization problem (15) is solved under the condition that for every reservoir at each stage only the variables X^{is}_{o}, G_{is} and those belonging to the set H'_{is} exist. As a result, the optimum values of the variables X^{is}_{o}, G_{is}, X_h, X_r, $h \varepsilon H'_{is}$, $r \varepsilon R_{is}$ for all i and s are obtained.

e) The optimum value of the variables X_h, $h \varepsilon H''_{is}$ is obtained after we come back to item b with the optimum value $\tilde{G}_{is}$.

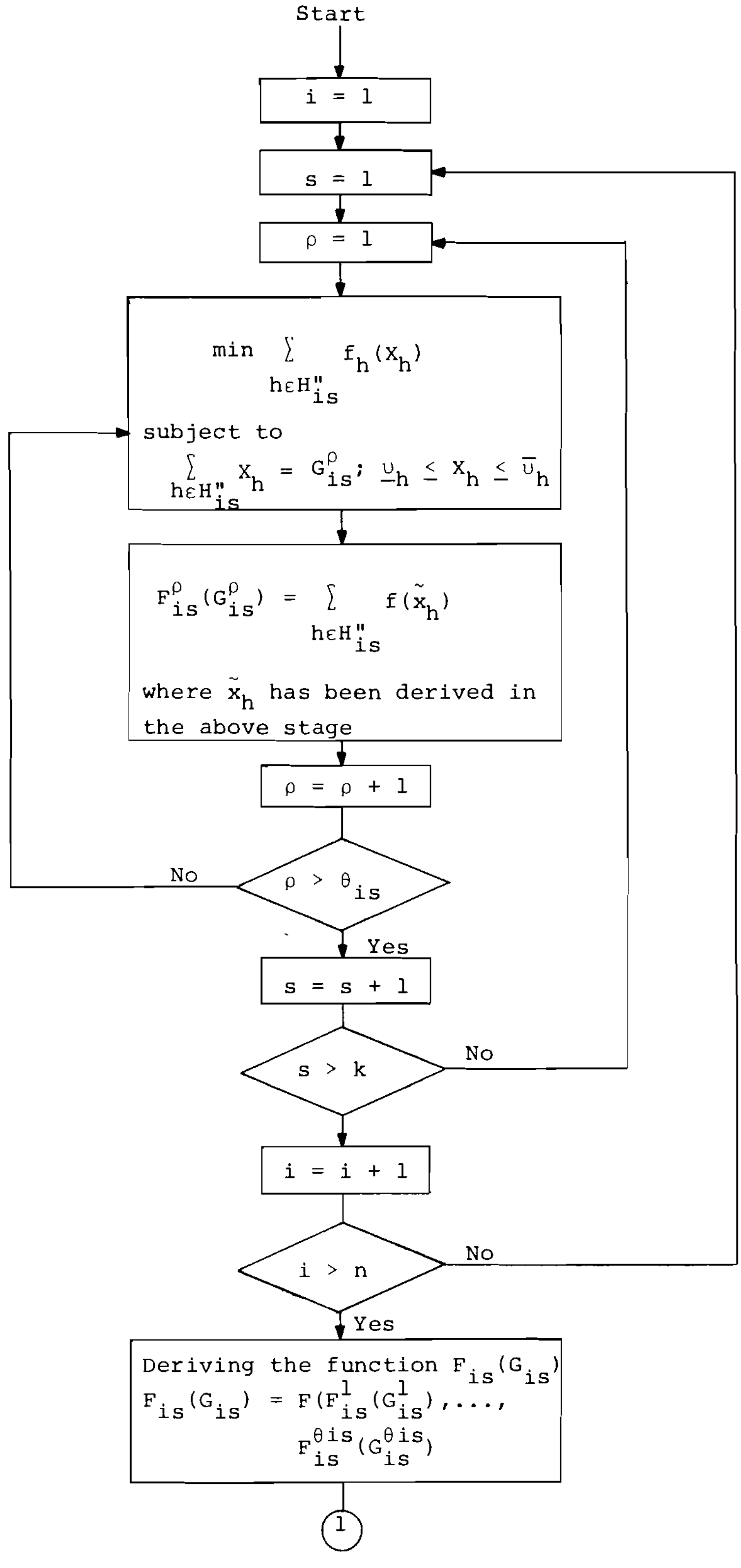
This procedure is described in detail in the block- diagram shown in Figure 8.

4. Possibilities of the Model

Using the model the decision maker (DM) can:

1. Find the optimal allocation of water resources in the system between both users and reservoirs at predetermined stages in time.

2. Simulate the parameters' influence on the optimal allocation. The model parameters which should be changed are: I_c; γ_{is}; δ_{is}; $f_h(X_h)$, $h \varepsilon H_{is}$; $f_r(X_r)$, $r \varepsilon R_{is}$; mandatory and demand releases. From this simulation one can:


Start
$i = 1$
$s = 1$
$\rho = 1$
$\min \sum_{h \in H''_{is}} f_h(X_h)$
subject to
$\sum_{h \in H''_{is}} X_h = G^{\rho}_{is};\ \underline{\upsilon}_h \leq X_h \leq \overline{\upsilon}_h$
$F^{\rho}_{is}(G^{\rho}_{is}) = \sum_{h \in H''_{is}} f(\tilde{x}_h)$
where $\tilde{x}_h$ has been derived in the above stage
$\rho = \rho + 1$
$\rho > \theta_{is}$
No
Yes
$s = s + 1$
$s > k$
No
$i = i + 1$
$i > n$
No
Yes
Deriving the function $F_{is}(G_{is})$
$F_{is}(G_{is}) = F(F^1_{is}(G^1_{is}), \ldots, F^{\theta is}_{is}(G^{\theta is}_{is})$
1

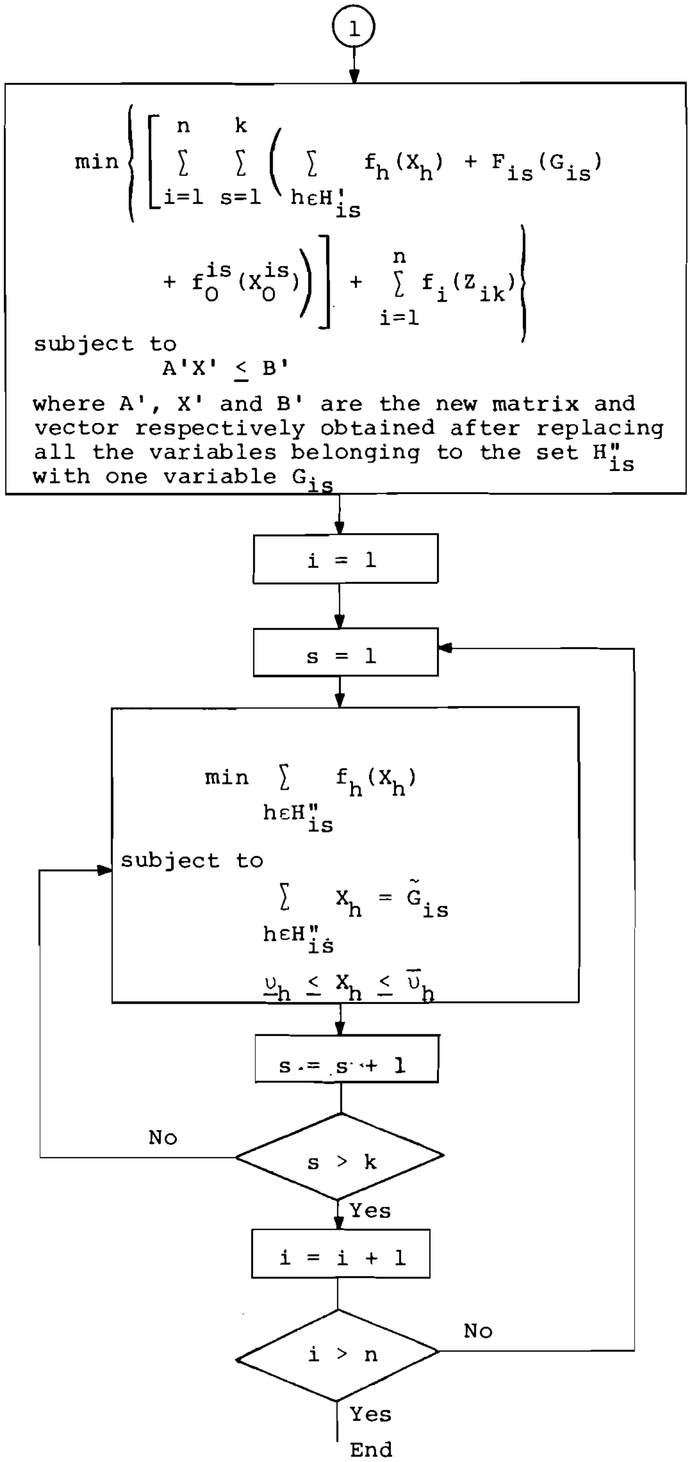


Figure 8. Computational decomposition procedure.

a) Evaluate the sensitivity of the optimum processes in the system. By means of this, the accuracy of the data for these parameters can be evaluated.

b) Trace the change of the optimal values of the variables X_r (amount of water in canals linking the reservoirs) and obtain the distribution function of X_r. With this function one can evaluate the places where the flood is expected, and accordingly:

- plan and create new dikes and reservoirs
- change the optimum allocation by means of decreasing the γ_{is} and increasing the δ_{is} of higher reservoirs so that the variables X_r are in predetermined intervals.

5. <u>Using the Model for Control of Multi-Reservoir System</u>

In the beginning of this paper, it had been mentioned that the optimization model would be an auxiliary one when the decision making model of the multi-reservoir system is developed. In general terms the decision making model can be presented in the following procedure:

1. The optimization problem (15) is solved following the algorithm shown in Figure 8 (when the number of variables is large) or by other (direct) methods when the number of variables allows the use of conventional procedures. The obtained results for optimum allocation of water are called optimal program.

2. The relationship between the optimal program and the changed parameters of equation (15) is obtained, i.e. a set of optimal programs is generated.

3. The DM evaluates the obtained optimal programs by the set of predetermined criteria and select the best program. This is called the "rational program" and it is valid for the first step of control of the water resource system.

4. The procedure described in items 1 to 4 is repeated after the first step, by taking into consideration the new information for the processes in the system.

6. Some Problems for Future Investigation

The improvement in the model's adequacy should be accomplished after its refinement with respect to:

1. Development of stochastic optimization model. A distinctive feature of the model (15) is that the values of vector B and the functions $f_h(X_h)$, $f_r(X_r)$ can be considered as a stochastic variable. Some results and investigations on this type of optimization problems are given for example in [7].

2. Development of the specific models for obtaining the functions $f_h(X_h)$ and $f_r(X_r)$ taking into account economical, social and physical processes in the system under investigation.

3. Special attention should be paid to the active part of man, and to the closely related problems of decision making.

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