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Interim Report **IR-03-023**

Integrated Modeling of Spatial and Temporal Heterogeneities and Decisions Induced by Catastrophic Events

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June, 2003

Contents

List of Figures

Abstract

This paper discusses an integrated model capable of dealing with spatial and temporal heterogeneities induced by extreme events, in particular weather related catastrophes. The model can be used for quite different problems which take explicitly into account the specifics of catastrophic risks: highly mutually dependent losses, inherent scarcity of information, the need for long-term perspectives (temporal heterogeneity) and geographically explicit analyses (spatial heterogeneity) with respect to losses and gains of various agents such as individuals, governments, farmers, producers, consumers, insurers, investors, and their decisions on coping with risks. We illustrate emerging challenging decision-making problems with a case study of severe floods in a pilot region of the Upper Tisza River. Special attention is given to the evaluation of a flood loss-spreading program taking explicitly into account location specific distributions of agricultural and structural losses. This enables us to evaluate premiums, insurance coverage, and governmental compensation schemes minimizing, in a sense, the risk of locations to overpay actual losses, risks of bankruptcy/insolvency for insurers, and overcompensation of losses by the government. GIS-based catastrophe models and stochastic optimization methods are used to guide policy analysis with respect to location-specific risk exposures. We use special risk functions in order to convexity discontinuous insolvency constraints.

(JEL G22, G28, C61)

Keywords: catastrophic risks, integrated catastrophe modeling, adaptation and mitigation measures, insurance, stochastic optimization, insolvency, contingent credit, CVaR.

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1. Introduction

Rising global temperature due to anthropogenic activities are expected to cause sea level rise and changes in the region-specific precipitation patterns and other local climate conditions. Changing local climate could alter forests, crop yields, and hydrological cycles. It could also affect human health, animals, increase occurrences of epidemic outbreaks, and disturb many types of ecosystems. A major concern in the assessment of consequences of global climate change relates to the possibility of increased frequency, severity, and duration of extreme natural events, such as heat waves, precipitation events, droughts, etc. In 1998, natural catastrophes claimed 50,000 lives and cost US$90 billion throughout the world, according to a press release issued by Munich Re, a leading international re-insurer (this dollar cost is the second highest total ever). The international insurance industry paid US$15 billion of the total, the fourth highest amount ever. Hurricane Mitch killed an estimated 11,000 people and displaced or injured 3,000,000 more in Honduras and Nicaragua. Elsewhere, Hurricane George in the Caribbean and the Gulf of Mexico, Typhoon Vicki in Japan, the heat waves in the United States, forest fires in Southern Asia and Russia, and the ice storm in southeastern Canada caused grave suffering and losses. According to Munich Re, these phenomena may in part be explained by the natural climate fluctuations of El Niño and La Niña. The losses from natural and human-made catastrophes, Munich Re-insurance reports, will become even more devastating: within the last three decades the direct damages from natural disasters alone have increased nine-fold [8], [27], [29].

One of the main reasons for this is the current land use practices, which often do not account for risks: the ignorance of risks leads to the clustering of people and capital in hazard-prone areas as well as the creation of new hazard-prone areas. For instance, it is estimated [28] that within the next 50 years more than a third of the world population will live in seismically and volcanically active zones.

This alarming human-induced tendency creates challenges and new scientific problems requiring integrated approaches, new concepts and tools for risk-based land use planning and catastrophic risk management. The existing approaches often ignore rare disasters of high consequences, which require a variety of management strategies [3], [5], [7], [34], [37] and strong cooperation of the burden sharing agents. Impacts of catastrophes cannot be properly evaluated on aggregate levels. For example, aggregate worldwide economic impacts of global change may even be considered as beneficial whereas some regions and even countries may be wiped out. The modeling of spatial and temporal heterogeneity becomes a critical issue. For different world regions extreme events have different loss and vulnerability scales. While the developed world has the financial and technological means to cope with these events, for many developing countries natural catastrophes may cause major shocks and disruptions meaning increased poverty, health problems, decreased water quality and supply, etc. After the Millennium Summit in 2000 when the heads of government from around the world agreed to commit time-bound goals regarding poverty, hunger, water, education, and health, the adequate treatment of catastrophic event became especially evident.

Following this discussion, the studies conducted by LUC project at IIASA structure the issues related to Climate Change and Agricultural Vulnerability to integrate ecological-economic analysis of climate change on the world food system. The analysis, in particular, includes quantification of scales and location of hunger, international agricultural trade, prices, production, land use, etc. It emphases that the climate change issue is global, long-term, and involves complex interactions between climatic, environmental, economic, political, institutional, social, and technological processes. “Climate change will impact on social, economic, and environmental systems and shape prospects for sustainable agricultural and rural development. Adaptation to climate change is essential to complement climate-change mitigation, and both have to be central to an integrated strategy to reduce risks and impacts of climate change,” say the authors of [18].

A recent IPCC Workshop on Changes in Extreme Weather and Climate Events (Beijing, China, 11-13 June, 2002, [21]) has identified urgent research and modeling recommendations on coping with climate change related catastrophes. They emphasized necessity of consistent methods and software to encourage region–specific analyses of these extremes, development of regional vulnerability indices, to produce coordinated approach to their management. In particular, this concerns the sustained agricultural and rural development.

The main question in connection with catastrophes is management of the losses. Until recently, losses were mainly absorbed by the immediate victims and their governments, [19], [24], [25]. The insurance industry and its premium payers (and investors) also absorb a portion of catastrophic losses, but even in the wealthy countries this share is relatively small. As currently losses increase, governments are concerned with escalating costs for disaster prevention, response, compensation to victims, and public infrastructure repair. It is important to increase the responsibility of individuals and local governments for the risks of extreme events and their consequences. Local governments may be more effective in the evaluation and enforcement of loss-reduction and loss-spreading measures, but this is possible only through location-specific analysis of potential losses, of the mutual interdependencies of these losses, and of the sensitivity of location-specific losses to new land use and other risk management strategies. A number of crucial questions arise within this analysis, such as: What are optimal ways to alter location-specific risk profiles by modification or/and reinforcement of structural measures such as dikes, reservoirs, irrigation systems, etc? How to adapt the existing situation, in particular, land use practices, reallocation of capital, etc., to the existing risk profile? What are optimal financial strategies for mitigation and adaptation? Where is the balance between ex-ante "here-and-now" and ex-post "wait-and-see" decisions? Reallocation of properties away from the risk prone areas is an important option, which however for many regions may be practically infeasible (or feasible only within the long-term horizon).

The scarcity of historical data is an inherent feature and a main challenge in dealing with rare catastrophes and new strategies. Purely adaptive, learning-by-doing type of approaches may be very expensive and dangerous. The role of models enabling the simulation of possible catastrophes for designing mitigation and adaptation programs becomes a key task.

In what follows we discuss an integrated framework that enables to analyze spatial and temporal heterogeneity of various agents (stakeholders) induced by mutually dependent losses from extreme events. The model explicitly addresses the specifics of catastrophic risks: the lack of information, the need for long-term perspectives and geographically explicit models, the involvement of multiple "actors" such as individuals, governments, farmers, producers, consumers, insurers, re-insurers, and investors. The model combines geographically explicit data on distribution of capital stocks and economic values in infrastructure and agriculture in a region with a stochastic model generating magnitudes, occurrences, and locations of catastrophes. Using advanced stochastic optimization techniques, the model allows to analyze robust optimal portfolios of ex-ante (land use, structural mitigation, insurance) and ex-post (adaptation, rehabilitation, borrowing) measures for decreasing regional vulnerability measured in terms of economic, financial, human losses as well as in terms of selected welfare growth indicators.The approach is applied in a case study of severe floods in a region of the Upper Tisza River, Hungary. A Monte Carlo type catastrophe model generates location specific random flood losses from an infinite variety of catastrophic flood scenarios. Embedded in this model-based stochastic optimization analyses are the spatial and temporal heterogeneities of locations (individuals), insurers, and the government to give insights into the feasibility of fair flood management programs by taking explicitly into account the burden sharing of these agents. Discussion of similar issues with respect to catastrophic risks other than floods can be found in studies discussed in [1-2], [11-16]. As it is shown in these studies, the technique implied in the model is becoming increasingly important to governments, central and local, as they can better negotiate risks and make decisions on allocation of properties, agricultural units, implementation of mitigation and adaptation strategies. Also, it is important for insurance companies to make decisions on contracts, premiums, and reinsurance agreements. It is also useful for disaster planning agencies, by providing a more realistic evaluation of the spatial and temporal distribution of the potential losses.

The analysis of possible gains and losses from different arrangements of the program is a multi-disciplinary task, which takes into account the frequency and intensity of hazards, the stock of capital at risk, its structural characteristics, and different measures (in particular, engineering, financial) of vulnerability. It requires the development of so-called catastrophe models [38]. Section 2 discusses the main features of a GIS-based catastrophe model developed for the Upper Tisza pilot region that, in the absence of historical data, simulates samples of dependent potential losses. Traditionally insurance and finance quantify extreme events in monetary units [10]. The catastrophe model deals with events, which are non-quantifiable in this sense, and with multivariate distributions of extreme values, i.e., with cases that are not treated within the conventional extreme value theory. Section 3 discusses the shortcomings of the standard "if-then" scenario analyses for catastrophic risk management and outlines general ideas of Adaptive Monte Carlo Optimization (AMCO) proposed in [12], [13] to overcome these difficulties. This rather general optimization technique is compared with AMC simulation as proposed in [32] to improve the efficiency of the sampling. Section 4 describes a spatial and dynamic stochastic optimization model for evaluation of the flood loss-spreading program in the Upper Tisza region [16]. Similar ideas of Adaptive Monte Carlo Optimization are used in [1], [2], [36]. The model results emphasize the temporal and spatial heterogeneity of different winners and losers and the need for the cooperation of agents in dealing with catastrophes. The point to make is that catastrophes do not exist "on average". We cannot treat a 50-year crash of an airplane as a sequence of discounted flows of losses: the crash of one wheel in the first year, another wheel in the second year and so on. Catastrophic losses come as a "spike" in time and space, which calls for multi-agent actions. In particular, the solution to catastrophic risk management, especially for small economies with limited risk absorption capacity, cannot be accomplished (see discussion in [30] and, e.g., in [1], [9]) without pooling of risk exposures. The analyzed in this paper scheme involves pooling of risks through mandatory flood insurance based on location-specific exposures, partial compensation to the flood victims by the central government, and a contingent credit to the pool. This scheme encourages accumulation of regional capital to better "buffer" against the volatilities of international reinsurance markets. In order to stabilize the insurance program we use economically sound risk indicators such as expected overpayments by "individuals" (cells of flood-prone areas) and an expected shortfall of the mandatory insurance similar to our analysis of seismic risk programs (see [1], [2], [11], [14]). These indicators are used together with so-called stopping times to orient the analysis towards the most destructive scenarios. The explicit introduction of ex-post borrowing as a measure against insolvency permits approximating the insolvency constraint by a convex optimization problem, whereas the use of the contingent credit leads to the Conditional-Value-at-Risk (CVaR) type of risk measures. Section 5 specifies this model further. Numerical experiments are based on real and modified data from the flood risk studies in the Upper Tisza region, Hungary, [6], [16]. [20]. They indicate a strong dependence of demand for contingent credit on the composition of other risk management measures. The importance of such an integrated analysis was emphasized in [13], [23], [26]. This section also illustrates that "if-then" type of analyses based just on the intuition (opinions) of "stakeholders" may easily fail to produce robust strategies in the case of highly interdependent multivariate distributions of catastrophic losses.

2. The Integrated Catastrophe Management Model

As shown in Figures 1 and 2, the integrated catastrophe management model consists of two major components: a flood catastrophe model and an economic multi-agent model. The economic multi-agent model is, in general, a multiregional stochastic dynamic welfare growth model. The two main components integrate five modules: the "River" module, the "Inundation" module, the "Vulnerability" module, the "Multi-Agent Accounting System", and the "Variability" module.

The River module calculates the volume of discharged water to the pilot region from different river sections for given heights of dikes, given scenarios of their failures or removals, and of rainfall runoff. The latter is modeled by upstream discharge curves, which can be significantly affected by land-use practices. Thus, formally, the River module maps an upstream discharge curve into the volume of water released to the region from various sections. The underlying sub-model is able to estimate the volume of water discharged into the region under different conditions, for example, if the rain patterns change, if the dikes are heightened, or if they are strengthened or removed.

The Inundation module is a spatial GIS-based Inundation sub-model. For the pilot region it contains 1500 by 1500 grid cells. This module maps water released from the river into levels of standing water in each grid cell and thus it can estimate the area of the region affected by different decisions.

The Vulnerability module maps spatial patterns of released water into economic losses. The module calculates direct losses, but may include indirect ones due to possible cascading effects, e.g., floods causing fires and their consequences. It also takes into account investments into loss reduction measures, e.g., altered land use and flood preparedness measures. Thus it models sensitivity of economic losses to changes (investments) in risk reduction measures.

The Multi-Agent Accounting System (MAAS) module maps spatial economic losses into gains and losses of heterogeneous agents. These agents include the central government, a mandatory catastrophe insurance (pool), an investor, and “individuals” (cells). This module plays a critical role in the integrated catastrophe management model since gains and losses, which are the determinants of the regional welfare, and the way they are distributed, have a crucial impact on the sustainable development and the overall growth of the region.

In conclusions, the five sub-models generate scenarios of spatially and temporally heterogeneous losses and gains at different locations for specific scenarios of weather, dike failures, risk reduction measures, and risk spreading schemes. Simulation shows that there are significant uncertainties and a considerable variability in generated losses and gains. A 50-year flood may occur in 5 days or in 70 years. Insurers are especially concerned about variability since they may not have the capacity to cover very large losses. In an attempt to maintain their solvency, they may charge higher premiums, which may result in overpayments by the insured. Alternatively, insurers may undercharge contracts. Insurers are also concerned about loss-reduction measures. A higher dike may fail and cause more damages in comparison to a dike without modification. To reflect these possible outcomes, the Variability module, a Monte Carlo model, transforms spatial probabilistic scenarios of rains, dike failures, risk reduction measures and risk spreading schemes into histograms (probability distributions) of gains and losses, underpayments and overpayments of agents (see Section 5). For example, it derives histograms of direct losses at a location or a sub-region. In fact, the AMCO does not use explicitly these distributions. It proceeds with its search for robust strategies in an evolutionary manner by adjusting decisions at each step of an iterative procedure based on the randomly sampled gains and losses.

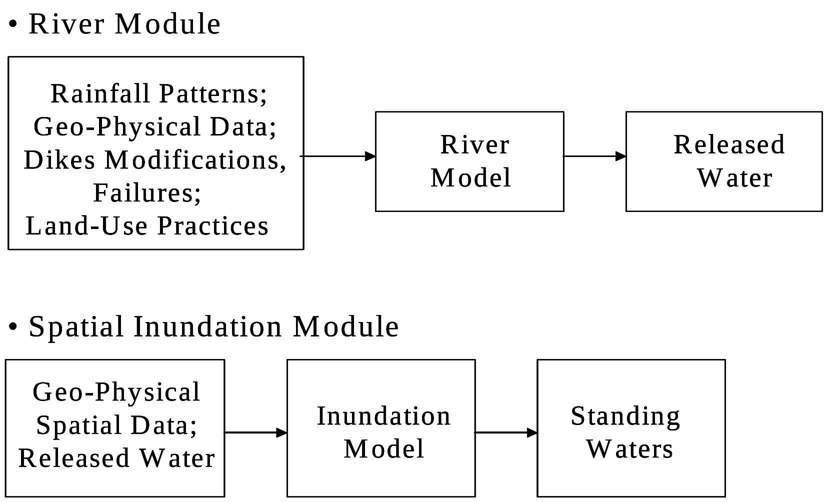

Figure 1. Flood Catastrophe Model.

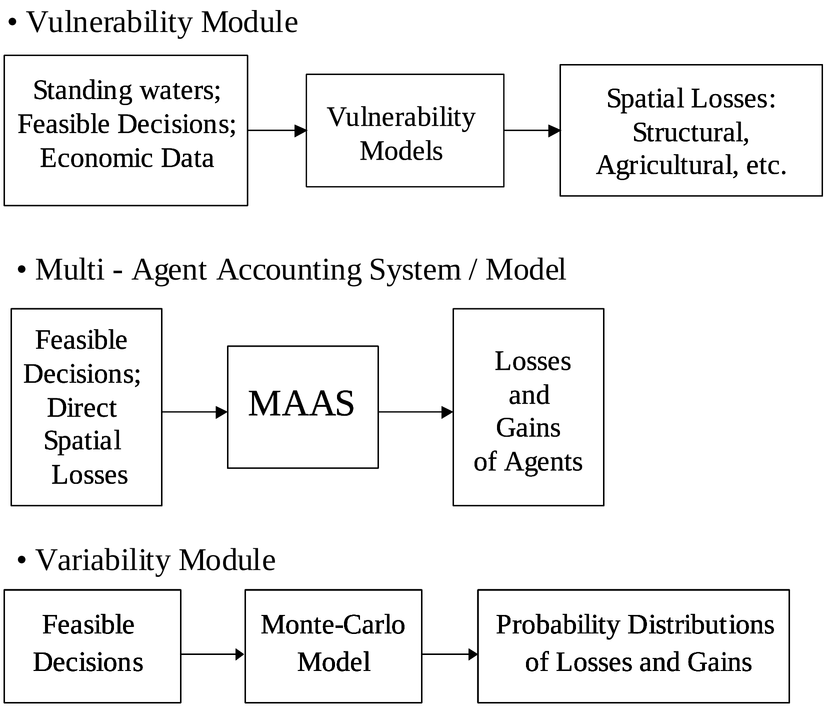

Figure 2. Economic Multi-Agent Model.

3. Adaptive Monte Carlo Optimization

Combining spatial/temporal catastrophe modeling with economic evaluation of heterogeneous agents opens up possibility for "if - then" analyses, which allows the evaluation of a finite number of policy alternatives. It is possible in a straightforward manner to evaluate quite different policy options on the type and the level of different loss-reduction measures, say, land-use modifications, location specific coverage by the insurance, arrangements of pools, governmental compensation schemes, or various other financial instruments to spread losses at the national and international level. However, such analyses may run quickly into huge and intractable number of possible combinations. For example, an insurer in the region can have different policies regarding the offered extent of coverage, say 0%, 10%, 20%, …, 100%, i.e., altogether 11 alternatives. For only 10 locations the number of possible combinations is already astronomical, namely 10^{11}. If running the sequence of modules would only take one second per instance, the computer time required for complete evaluation would approach 100 years. Therefore, with 100 locations the straightforward “if–then” analysis already runs into “eternity”. The same computational complexity arises in dealing with location-specific land-use modifications, premiums or investments in different segments of dikes.

Hence, the fundamental question concerns the identification and evaluation of a desirable policy without the simulation of all possible options. The complexity of this task is due to analytical intractability of stochastic catastrophe models, often precluding the use of standard optimization methods, e.g., genetic algorithms. Therefore, in general, we have to rely on stochastic optimization methods [16], in particular, on the so-called Adaptive Monte Carlo Optimization (AMCO) [12], [13]. “Adaptive Monte Carlo” [32] means a technique that makes on-line use of sampling information to sequentially improve the efficiency of the sampling itself. We use AMCO in a rather broad sense, i.e. the efficiency of the sampling procedure is considered as part of more general improvements with respect to different decisions and goals. The AMCO model for the Upper Tisza region consists of three interacting blocks: (i) Feasible decisions, (ii) the Monte Carlo Catastrophe model, and (iii) Performance indicators (Figure 3).

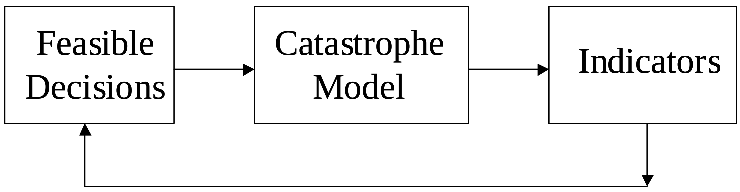

Figure 3. The Adaptive Monte Carlo Optimization Model

The block “Feasible Decisions” represents the set of all feasible policies, i.e., all technically, legally and financially possible options for coping with floods. In general, they may include altering heights of dikes, different levels of insurance coverage, land use modifications, crop structure portfolio, etc. These variables affect performance indicators such as profits of farmers, producers, consumers, insurers, underpayments or overpayments by the insured, costs, insolvency and stability indicators.

The essential feature is the feedback mechanism updating, based on evaluation of performance indicators, decisions towards specific goals of agents. The updating procedure relies on stochastic optimization techniques as is discussed in Section 5. Gains and losses are simulated by the catastrophe model, causing an iterative revision of the decision variables after each simulation run. In a sense, the AMCO procedure simulates in a remarkably simple and evolutionary manner the learning and adaptation process on the basis of the simulated reversible history of catastrophic events. This technique is unavoidable when the outcomes of the catastrophe model do not have a well-defined analytical structure.

4. The Stochastic Optimization Model

Stochastic optimization provides a framework for the iterative revision of decisions embedded in the catastrophe evaluation model. These decisions influence the magnitude and distribution of location-specific risks and contributions to the overall catastrophe losses. In the model for the Upper Tisza region we use approaches similar to those adopted in [1-2], [13-15] for seismic risks. The main idea is based on subdividing the study region into risk-homogeneous land units, represented by grid-cells $j = 1,2,...,m$. These cells may correspond to a collection of households at a certain site, a collection of locations (zones) with similar land-use structure, an administrative district, or a grid with a segment of a gas pipeline. The choice of cells provides a desirable representation of losses. In our case, the cells consist of the value of the physical structures. Catastrophes, which are simulated by the catastrophe model, randomly affect different cells and produce mutually dependent losses L_j^t at time t. These losses depend on various decision variables. Some decisions reduce losses, say a dike, whereas others spread them at a regional, national, and international level, e.g., insurance contracts, catastrophe securities, credits, and financial aid. If $x = (x_1, x_2, ..., x_n)$ is the vector of decision variables, then losses L_j^t in a cell j at time t are transformed into $L_j^t(x)$. In the case of the Tisza river, for example, we can think of $L_j^t(x)$ as L_j^t being affected by the decisions of the insurance to cover losses from an interval $[x_{j1}, x_{j2}]$ for a cell j in the case of a flood disaster at time t:

$$L_j^t(x) = L_j^t - \max\{x_{j1}, \min[x_{j2}, L_j^t]\} + x_{j1} + \pi_j^t, \tag{1}$$

where $\mathbf{max}\{x_{j1}, \mathbf{min}[x_{j2}, L_j^t]\} - x_{j1}$ are retained by insurance losses, and π_j^t is a premium function. The variable x_{j1} defines the deductible part ("trigger") of the contract and x_{j2} defines its "cap".

In the most general case, vector x comprises decision variables of different agents, including government decisions, such as the height of a new dike or a public compensation scheme defined by a fraction of total losses $\sum_{j=1}^{m} L_j^t$. The insurance decisions concern premiums paid by individuals and the payments of claims in the case of catastrophe. There are complex interdependencies among these decisions, which call for the cooperation of agents. For example, the partial compensation of catastrophe losses by the government enforces decisions on loss reductions by individuals and, hence, increases the insurability of risks, and helps the insurance to avoid insolvency. On the other hand, the insurance combined with individual and governmental risk-reduction measures can reduce losses, compensations and government debt, and can stabilize the economic growth of the region and the wealth of individuals.

Let us now turn to considering a potential insurance system for Hungary and introduce some important indicators. In the following we do not, for simplicity of notation, consider the most general situation, i.e., we consider only a proportional compensation scheme by the government, proportional insurance coverages, and we do not use discount factors.

In this application the system is modeled until a first catastrophic flood, which occurs within a given time horizon. We define this time point as the *stopping time*. For the Upper Tisza region this event is associated with the breaking of a dike, which may occur only after a 100-year, 150- or 1000-year flood. Floods are characterized by upstream discharge curves and the probability of breaking each of the three dikes. The occurrence of the first catastrophic flood significantly affects the accumulation of risk reserves by the insurance, and total payments of individuals; for example, a 100-year flood, with the break of a dike, may occur in two years, and this may lead to significant underpayments to insurance.

Let τ be a random (stopping) time to a first catastrophe within a time interval $[0,T]$, where T is some planning horizon, say, of 10 or 50 years. If no catastrophe occurs, then $\tau = T$. Since τ is associated with the break of a dike, the probability distribution of τ is, in general, affected by some components of vector x, e.g., by decisions on dike modifications, land use changes, construction of reservoirs, etc. Here, we discuss only the case when τ does not depend on x .

Let L_j^τ be random losses at location j at time $t = \tau$. In our analysis we assess the capacity of a catastrophe insurance scheme in the upper Tisza region only with respect to financial loss-spreading decisions. Let us use a special notation for the elements of the scheme such as π_j , φ_j , ν , q , y . Let π_j denote the premium rate paid by location j to the mandatory insurance, then the accumulated mutual catastrophe fund at time τ including the proportional compensation $\nu\sum_j L_j^\tau$ by the government amounts to $\tau\sum_j \pi_j + \nu\sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau$, where $0 \le \varphi_j \le 1$, is the insurance coverage for cell j. Thus, in this model we assume that the compensation to flood victims by the government is paid through the mandatory insurance.

The stability of the insurance program depends on whether the accumulated mutual fund together with the governmental compensation is able to cover claims, i.e., on the probability of event:

$$e_1 = \tau\sum_j \pi_j + \nu\sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau \ge 0 \ . \tag{2}$$

The stability also depends on the willingness of individuals to accept premiums, i.e., with the probability of overpayments:

$$e_2 = \tau\pi_j - \varphi_j L_j^\tau \ge 0, \quad j = 1,\ldots,m \ . \tag{3}$$

Apart from the compensation $\nu\sum_j L_j^\tau(x)$ the government arranges a contingent credit y with a fee q to improve the stability of the mandatory insurance (pool) by transforming event (2) into (4):

$$e_3 = \tau\sum_j \pi_j + \nu\sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau + y - \tau q y \ge 0 . \tag{4}$$

Constraints (3), (4) describe the burden sharing within the program. Here we assume that the mandatory insurance pays the fee $\tau q y$ and receives a credit y, whereas the government pays back the credit with the interest rate γy, $\gamma > 1$.

The difference between compensation $\nu\sum_j L_j^\tau$ and contingent credit y is significant: the outflow of fees is smooth, whereas the compensation of claims has a sudden impact at time τ, and without y it may require a higher government compensation (greater ν) possibly exceeding the available budget. Therefore, without ex-ante contingent injections of capital y the diversion of capital from other government expenditures may occur.

Let us note that the budget constraint, which raises a general question on the optimal dynamic management of the available budget in order to increase the stability of the mandatory insurance and its efficiency. For example, besides the contingent credit, a reasonable option may also be to invest some money in liquid assets. The main aim of our analysis is narrower: the evaluation of the mandatory insurance capacity and the demand for contingent credit.

Inequalities (3)–(4) define important events, constraining the choice of decision variables, which determine the insurance program, i.e., the compensation rate ν by the government, coverage rates by the insurance company φ_j, premiums π_j, and credit y with fee q. The probability of events (3)-(4), i.e., overpayments by individuals and underpayments to the pool, determine the stability (resilience) of the scheme. This can be expressed in terms of the probabilistic constraint

$$P[e_2 > 0, e_3 < 0] \le p, \tag{5}$$

where p is a desirable probability of the program's failure (default), that occurs, say, only once in 100 years. Constraint (5) is similar to insolvency constraint [35], a standard for regulations of the insurance business. In stochastic optimization [17] constraint (5) is known as the so-called chance constraint. Note, however, that this constraint does not account for the attained values of e_2 and e_3, what is important for the government, since it cannot walk away from the region in a distress. The main goal in setting up the insurance scheme can now be formulated as the minimization of expected total losses $F(x) = E\sum_j (1-\varphi_j)L_j^\tau + \gamma y$ including uncovered (uninsured) losses by the insurance scheme and the cost of credit γy, subject to chance constraint (5), where vector x consists of the components π_j, φ_j, y.

Constraint (5) imposes significant methodological challenges even in cases when $\tau(x)$ does not depend on x and events (3)-(4) are defined by linear functions of decision variables (see discussion in [17], p. 8, and in [13-15]). This constraint is of "black-and-white" character, i.e., it accounts only for a violation of (3)-(4) but not for the magnitude of violation. There are important connections between the minimization of $F(x)$ subject to highly non-linear and possibly discontinuous chance constraints (5) and the minimization of convex functions, which have important economic interpretations. Consider the following function

$$G(x) = F(x) + \alpha E \max\left\{0, \sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j - y + \tau q y\right\} + \beta E \sum_j \max\left\{0, \tau\pi_j - \varphi_j L_j^\tau\right\}, \tag{6}$$

where α, β are positive parameters.

It is possible to proof (see chapter 2 in [17] and more general results in [13-15]) that for large enough α, β the minimization of function $G(x)$ generates solutions x with $F(x)$ approaching the minimum of $F(x)$ subject to (5) for any given level p.

The minimization of $G(x)$ as defined by (6) has a simple economic interpretation. Function $F(x)$ comprises expected direct losses associated with the insurance program. The second term quantifies the expected shortfall of the program to fulfill its obligations; it can be viewed as the expected amount of ex-post borrowing with a fee α needed for this purpose. Similarly, the third term can be interpreted as the expected ex-post borrowing with a fee β needed to compensate overpayments. Obviously, large enough fees α, β will tend to preclude the violation of (3)-(4). Thus, ex-post borrowing with large enough fees allows for a control of the insolvency constraints (5). It is easy to see that the use of the ex-post borrowing (expected shortfall) in the second term of $G(x)$ in combination with the optimal ex-ante contingent credit y controls the CVaR type risk measures. Indeed, the minimization of $G(x)$ is an example of stochastic minimax problems (see [17], chapter 22). By using standard optimality conditions for these problems we can derive the optimality conditions for the contingent credit y. For example, assuming continuous differentiability of $G(x)$ which follows in particular from the continuity of underlying probability distributions, it is easy to see that the optimal level of the credit $y > 0$ must satisfy the equation

$$\frac{\partial G}{\partial y} = \gamma - \alpha P\left[\sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j > y\right] = 0\,. \tag{7}$$

Thus, the optimal amount of the contingent credit is defined as a fraction of the random variable $\sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j$ specified by the ratio γ/α, which has to be less or equal 1. Hence, the expectation in the second term of $G(x)$ for optimal y is taken under the condition that

y is a quantile of $\sum_j \varphi_j L_j^\tau - \nu \sum_j L_j^\tau - \tau \sum_j \pi_j$. This is in accordance with the definition of CVaR [4], [28]. More general risk measures emerge from the optimality conditions of $G(x)$ with respect to premiums π_j, φ_j.

The importance of an economically sound risk measure, such as expected shortfall, was emphasized by many authors (see [4], [10], [22], [33]). Important connections of CVaR with the linear programs were discussed in [4], [25]. Let us note that $G(x)$ is a convex function in the case when τ and L_j^τ do not depend on x. In this case the stochastic minimax problem (6) can be approximately solved by linear programming methods (see general discussion in [11]). The main challenge is concerned with the case when τ and L^τ are implicit functions of x. Then we can only use the Adaptive Monte Carlo optimization. Let us outline only the main idea of this technique. More details and further references can be found in [12-15].

Assume that vector x incorporates not only risk management decision variables but also includes components affecting the efficiency of the sampling itself (for more detail see [12], [13], [32]). An Adaptive Monte Carlo procedure searching for a solution minimizing $G(x)$ of type (6) starts at any reasonable initial guess x^0. It updates the solution iteratively at steps $k = 0,1,...,$ by the rule $x^{k+1} = x^k - \rho_k \xi^k$, where numbers $\rho_k > 0$ are predetermined step-sizes satisfying the condition $\sum_{k=0}^{\infty} \rho_k = \infty$, $\sum_{k=0}^{\infty} \rho_k^2 = \infty$. For example, the specification $\rho_k = 1/(k+1)$ would suit. Random vector ξ^k is an estimate of the gradient $G_x(x)$ or its analogs for the non-smooth function $G(x)$. This vector is easily computed from random observations of $G(x)$. For example, let G^k be a random observation of $G(x)$ at $x = x^k$ and $\tilde{G}^k$ be a random observation of $G(x)$ at $x = x^k + \delta_k h^k$. The numbers δ_k are positive, $\delta_k \to 0$, $k \to \infty$, and h^k is an independent observation of the vector h with elements independent and uniformly distributed on [- 1,1] components. Then ξ^k can be chosen as $\xi^k = \left[\left(\tilde{G}^k - G^k\right)/\delta_k\right] h^k$. The formal analysis of this method, in particular for discontinuous goal functions, is based on general ideas of the stochastic quasigradient methods (see [17] and further references in [13-15]).

5. Numerical Experiments

In this section we discuss some numerical experiments using data collected in the Upper Tisza region. The main purpose of the following discussion is the illustration of the proposed model rather than the numerical results of our findings. Therefore, in what follows we use some simplified assumptions.

The case study region consists of 1500 x 1500 grid-cells. For each grid there are data on the property value and vulnerability of its content. For policy analysis these grid-cells are further aggregated into 40 locations/municipalities.

In the numerical experiments we analyzed outcomes under different assumptions suggested by various stakeholders involved in these studies, in particular, policies on premiums π_j, $j = 1,...,m$, where m is the number of locations, in our case $m = 40$. These assumptions reflect different views of stakeholders on potential flood loss sharing programs in the region. We show that they significantly affect spatial and temporal heterogeneity of individuals, the insurance, and the government. In these experiments the demand for government intervention is modeled as the demand for the contingent credit, whereas the burden sharing by individuals and the insurance pool are defined by the distribution of constraint vector e and the distribution of the left hand side of stability constraint (5). Here we discuss only the following two options for premium calculation:

1. Premiums calculated according to the actuarial principle based on location-specific average losses.
2. "Fair" robust premiums calculated by minimization of function (6). This takes into account the distribution of losses for each location, and considerations of fair prices defined by (3), (4).

Flood occurrences are modeled according to specified probabilities [31] of heavy rainfall events and dike breaks. There are three dikes located along the pilot river branch. Each of them may break with some probability in 100-year, 150-year, and 1000-year flood. Here we only take into account structural losses. The simulation time horizon is 50 years. The number of random simulations (scenarios) per experiment is 10000. A contingent credit in our model is introduced to stabilize equation (2) in correspondence with equation (4). The demand for credit (the burden of the government) is, therefore, defined by negative values of indicator e_1 or e_3 for optimal solutions φ_j, π_j minimizing (6) for $y = 0$ and given ν. This defines also the lack of capacity (the burden sharing) for the mandatory insurance. Figures 4 and 5 illustrate the results of the experiments with $\nu = 0.25$, $y = 0$. The horizontal axis shows the total demand for contingent credit, negative e_1, whereas the vertical axis shows the number of simulations and the cumulative probability.

In practical calculations (see [1], [2], [11], [12] and Section 5) histograms for constraints (5) calculated simultaneously with the minimization of (6) provide a signal for increasing or decreasing “penalties” (risk factors) α, β to achieve a satisfactory level p. Intuitively, greater values α, β lead to constraints (5) with smaller p. On the other hand, this may considerably reduce insurance coverage of catastrophic exposures. The trade-off between these two effects can be resolved by using some additional criteria, e.g., political considerations or purely visual character of histograms, which cannot be formalized in general.

According to our experiments, the premium for the first option equals on average (per location and year) 0.87 million HF (Hungarian Forint) (exchange rate: one HF equals 0.003302 US dollars). As we can see from Figure 4, the inflow of premiums is not enough to compensate the losses, since e_1 is often negative, which defines a certain safety (solvency) level p for constraint (5). It is clear that in more than 2000 scenarios out of 10000 of simulated catastrophic events the mandatory insurance lacks the capacity to cover losses. This calls for a more significant intervention by the government (burden sharing) through either increasing the level of compensations ν, and/or through the contingent credit. These premiums lead to considerable overpayment by locations (Figure 6).

Location-specific optimal premiums improve the situation. Figure 5 illustrates the changes in the total demand for contingent credit by using the optimal premiums calculated from the minimization of (6) for the same $\nu = 0.25$, $y = 0$, α and β. The model suggests a premium rate on average (per location and year) equal to 0.83 million HF, which is lower than in the first case. Figure 5 shows that the demand for contingent credit is significantly reduced (fewer negative values on the horizontal axis).

Optimal premiums improve also overpayments per year (Figure 6, 7), i.e., the distribution of $\sum_j \max\{0, \tau\pi_j - \varphi_j L_j^\tau\}/\tau$ computed from the third term of $G(x)$. Figures 8, 9 show the distribution of uninsured losses computed from the first term of $G(x)$. It is evident (Figure 9) that the optimal premiums adjust location-specific coverage of flood losses in order to reduce insolvencies (negative e_1 in Figure 4) and overpayments. In the numerical experiments we used $\alpha = \beta = 0.1$ as parameter values. Figures 10, 11 show further reductions of overpayments and the demand for contingent credit for $\alpha = \beta = 0.05$, which are, in fact, due to further adjustment of location-specific coverages and premiums.

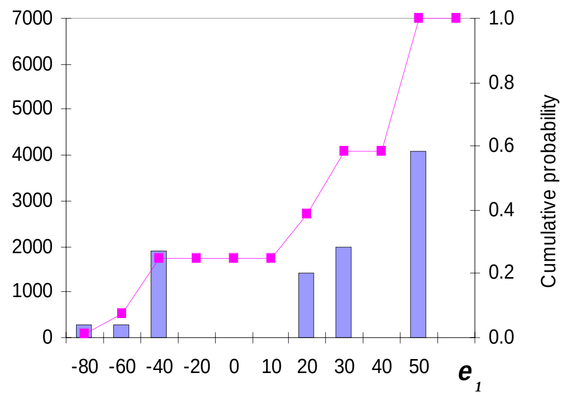

Figure 4. Actuarial premiums.

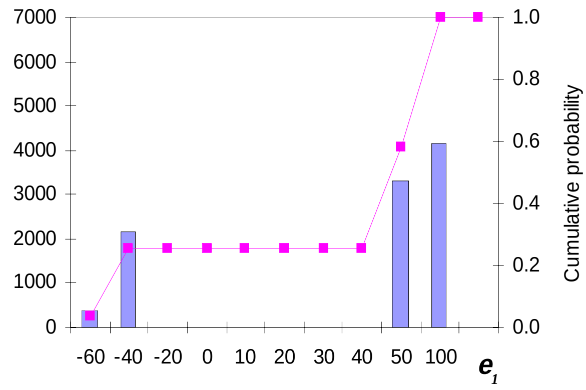

Figure 5. Optimal premiums.

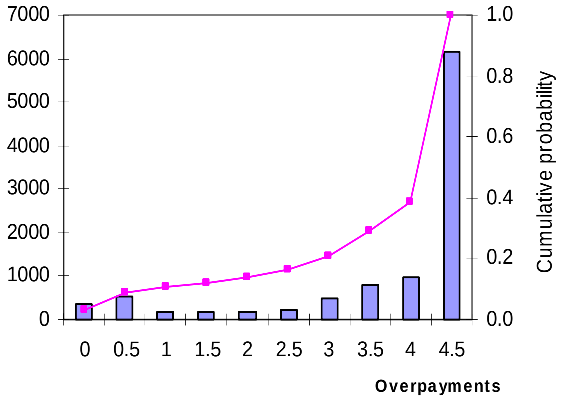

Figure 6. Actuarial premiums.

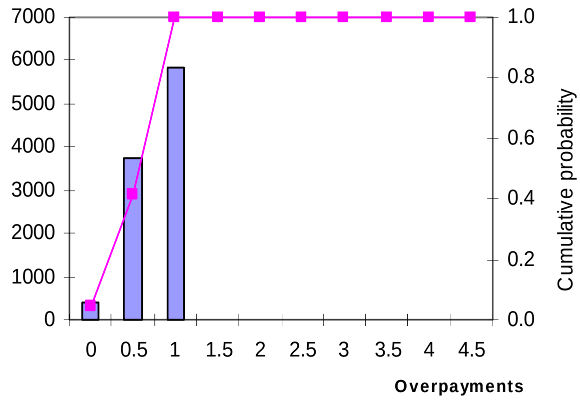

Figure 7. Optimal premiums.

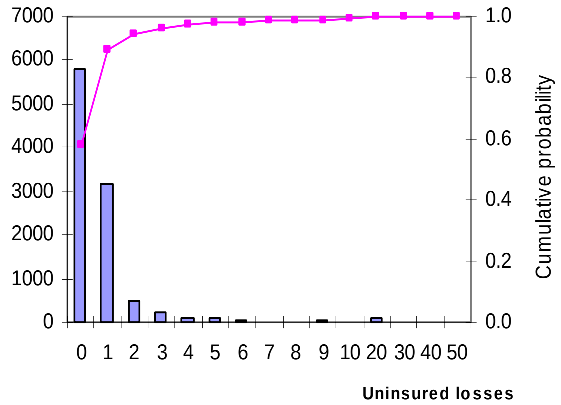

Figure 8. Actuarial premiums.

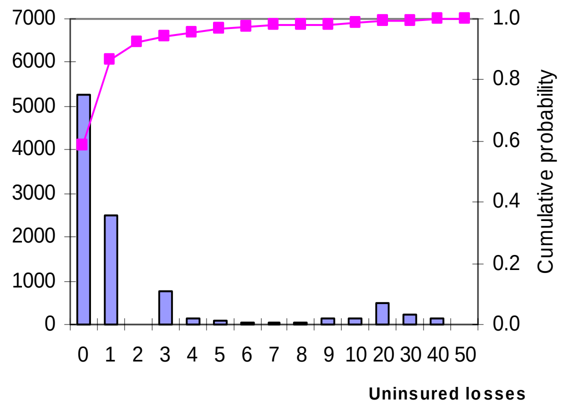

Figure 9. Optimal premiums.

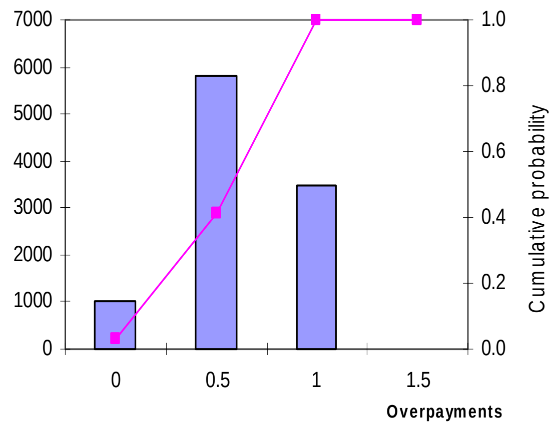

Figure 10. Risk coefficient $\alpha = \beta = 0.05$.

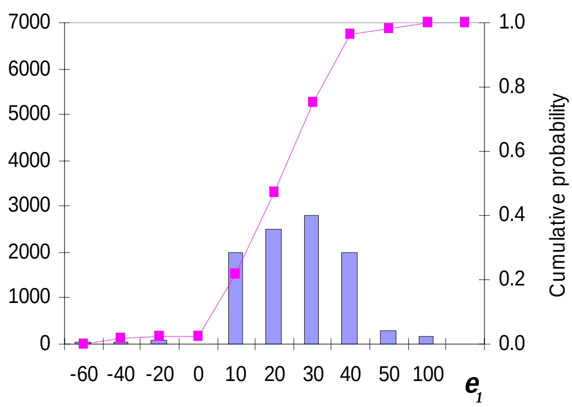

Figure 11. Risk coefficient $\alpha = \beta = 0.05$.

In these experiments, apart from varying premiums, we considered also other flood loss sharing and mitigation strategies, e.g., heightening of the dikes, alternative shares of insurance coverages, etc. Here, we do not discuss these scenarios in detail. Nevertheless, they are worth mentioning. Similarly to premiums, for each alternative set of policies, the model estimated (in the form of histograms) gains and losses of all agents. For example, Figure 12 shows the insurer's reserve when the dikes were heightened (and actuarial premiums). Figure 13 shows the outcomes for the insurer when premiums where assumed actuarial and the insurer's coverage decreased from 90 to 60 percent. The number of possible alternative scenarios can be very large.

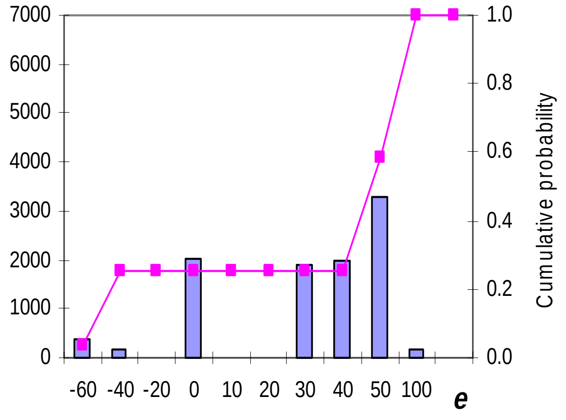

Figure 12. Heightening of the dikes, actuarial premiums.

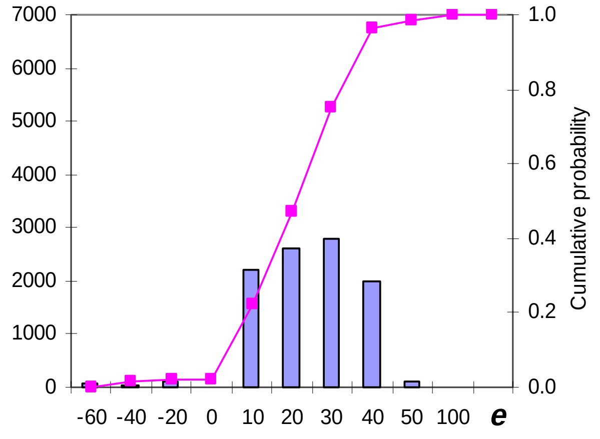

Figure 13. Decreased insurance coverage, actuarial premiums.

A main limitation of such scenario analysis, as discussed in Section 3, is that the evaluation of alternative policies does not take into account goals and constraints of agents, e.g., insolvency constraints of insurers, incomes of individuals, etc. Thus, the approach does not answer the question of policies' robustness.

The computer program (optimization part) was implemented using the mathematical software package MATLAB on a DELL GX240 personal computer. The solution time for 40 land units and 10000 iterations (scenarios) is about 10 minutes. The optimization procedure is easily restarted from different initial solutions, for new compositions of units, and distributions of random parameters. The solution time slightly changes with the number of decision variables and random parameters. It may increase with the increase of N (unreasonably large N may cause degeneracy of G - level sets) and it also depends on the scarcity of catastrophes. Important is that the computation time can be further reduced by implementing the so-called fast versions of the Monte Carlo simulations.

6. Concluding Remarks

In the pilot study of flood risk management in the upper Tisza region, we analyzed spatial and temporal heterogeneities that are typically induced by catastrophes. Main features of these heterogeneities are: spatial and temporal distributions and dependencies between losses; involvement of various agents in coping with catastrophes - local and central governments, households, farmers, investors, insurers, financial markets; multiple goals, constraints, and perceptions of these agents with respect to catastrophes. The temporal heterogeneity in our experiments was modeled with respect to random time occurrence of the first catastrophic flood (the stopping time). We used a spatially explicit GIS-based model for policy-oriented treatment of catastrophes in which we accounted for spatial and temporal distribution of structural and agricultural values in the studied region, regional vulnerability to catastrophic floods, stability constraints and loss indicators for different stakeholders with regard to catastrophes. The main challenge addressed within the stochastic model addressed was the identification of robust combinations of location specific ex-ante and ex-post mitigation and adaptation strategies decreasing regional vulnerability measured in terms of structural and agricultural damages and, thus, sustained welfare growth of the region.

In particular, we illustrated the applicability of the proposed model to analyze and improve spatial and temporal outcomes for individuals (locations), for the insurance pool and the government with respect to assumptions on location specific insurance premiums. We demonstrated that heterogeneous optimal premiums can decrease location-specific overpayments and also increase the stability of the insurance pool. In the experiments, the demand for a contingent credit indicates the need for additional government intervention, i.e., additional burden sharing by taxpayers to cover losses induced by catastrophic floods. As experiments show, another important issue is to perform the flood risks analysis with respect to spatial patterns of land-use decisions. For instance, deforestation or afforestation affect discharge curves of the catastrophe model, i.e., the frequencies of catastrophic floods, and their severity. Similar effects can result from other land use decisions, e.g., building of reservoirs (flood retention areas), the transformation of agricultural land to industrial complexes, the sealing of land surfaces, or changes in the allocation of capital values, in particular, settlements in risk prone areas.

Changing flood frequencies, as discussed in this paper, could also result from regional or global climate change. A future research challenge will be to extend the proposed AMCO approach to other risk-related fields of global change policy analysis.

Global change is the expression of a broad set of cumulative and interlocking changes in technological, economic, social and environmental conditions [39]. These changes alter locally varied profiles of risks and vulnerabilities. They appear to create more intense and more frequent episodes of extreme events, and they may change and intensify exposure to various forms of risk, e.g., disease vectors and occurrence of epidemic outbreaks. A characteristic feature of global change impacts and adaptation/mitigation efforts is their spatial diversity and dependency. The main message here is that heterogeneity and uncertainty are abundant and essential. For adequate treatment of the resulting complex decision problems, spatially explicit and dynamic stochastic modeling approaches, as discussed here, can provide innovative and flexible tools for identifying robust and fair (with regard to multiple stakeholders) policy responses.

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