ECONOMIC EVALUATION OF WATER SUPPLY ALTERNATIVES: A MATHEMATICAL PROGRAMMING APPROACH

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Preface

Interest in water resources systems has been a critical part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design and operational decisions.

In the years of 1976 and 1977 IIASA has initiated a concentrated research effort focusing on *modelling and forecasting of water demands.* Our interest in water demands derived itself from the generally accepted realization that these fundamental aspects of water resources management have not been given due consideration in the past. However, integration of demand and supply considerations will always be the ultimate step towards efficient solutions in regional development of water resources.

This paper, the first in the IIASA water demand series, focuses on some aspects of demand-supply integration of water resources management. It presents a certain method for evaluation of water supply alternatives in a region, and for combining them in such a fashion as to meet projected water demands.

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Task Leader
Regional Water Demand
and Management

Abstract

The main task of this paper is to propose a method for deriving regional water supply functions, taking into account a variety of supply alternatives and some engineering and environmental aspects of each. The purpose is to provide a framework for decisions about the efficient use of a region's water resources. The first section deals with distinctions between engineering and economics. The notion of supply-demand equilibrium and the economic efficiency properties of this equilibrium are reviewed. The second section surveys the "State-of-the-Art" in regional water supply, describing a number of alternative sources of supply. The third section considers how, for a region having just two inputs, each point on a supply curve can be derived as the solution to a nonlinear program to minimize the cost of obtaining a given quantity of water. The procedure is however perfectly general, and in the fourth section an application is made to a hypothetical region with several sources of supply, each having several inputs, with constraints on their use, and so on. An interesting feature of the model is that it can--and does, in the application--reflect environmental constraints as well. For ease in computation the production relations are linearized in order to use a linear programming solution algorithm. Based on the assumed production relations and resource constraints, a well behaved regional water supply function is derived.

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I. INTRODUCTION

The objective of this study is to elaborate a method to evaluate water supply alternatives in a region, and combine them in some appropriate fashion to meet projected water demands. We think this may be useful for several reasons. First, people who have to make decisions about water supply ought to know whether it is in fact feasible to meet projected future demands. Second, they ought to know the cost of doing so. What are the sacrifices required to obtain specified additional quantities of water? Third, we assume they wish to obtain these quantities in an efficient, i.e., cost-minimizing, fashion. This is what we mean by combining supply alternatives in an "appropriate" fashion.

A typical approach in past studies of water supply (see Wollman and Bonem (1971)) has been to measure relevant physical system characteristics of a region, such as precipitation and runoff, plot these annually, and then draw some inferences about how much water will be available in the region over a given future period. Because of uncertainties in precipitation and stream flow, statements about availability must ordinarily be made in probabilistic terms, e.g., "minimum flow available 98 percent of the time" (Löf and Hardison (1966)). But in any event, an important feature of this approach is that it attempts to come up with a point estimate of water supply. That is, it attempts to say exactly how much water will be available (with probability p) at a given time and place.

A very useful extension of the physical system analysis has been the specification and estimation of what the economist calls water supply functions. Below we expand on the meaning and significance of supply functions. For now, it is enough to know that a supply function for water gives the amounts of water that could be made available (within a given time frame) at various cost increments, or that would presumably be made available at the corresponding prices under a regime of decentralized, profit-maximizing suppliers. Wollman and Bonem present some good examples of the incremental cost-output relationship for surface stream flow and storage in a number of water resource regions in the U.S. Costs (and benefits) of another supply alternative, interbasin transfers of water, are studied by Howe and Easter (1971) for the U.S. and by Cummings (1974) for Mexico. What we intend to do is to take this sort of supply analysis a step further by looking at a range of alternatives for a (hypothetical) region, and developing a method that combines them in cost-minimizing fashion to generate a regional water supply curve.

The remainder of the Introduction has two purposes:

(a) to provide a foundation for the supply analysis by relating supply to water demand and indicating the role of each in the efficient development of a region's water resources, and

(b) to provide an explanation of these terms - supply, demand, efficiency - as they are understood and used by economists.

The material is standard, and further references are given in footnotes. Those familiar with it may wish to skip to section II, which begins the discussion of water supply alternatives. But since this paper is addressed to engineers and others, besides economists, concerned with the management of water resources, we think a brief review here may be useful.

Supply, Demand, and Efficiency

We have already spoken of regional water demands, in particular of matching supplies to demands. Let us now fix the meaning of this term. Just as a supply function relates the quantity of water that will be made available (within a given time frame) by competitive producers, or a government agency that mimics their responses, to each of a set of hypothetical market prices for water, a demand function relates the quantity of water that will be purchased by users to each of a set of hypothetical prices. In principle, this definition includes the case in which water is not priced, or in other words, is given a zero price. Note that neither supply nor demand functions constitute predictions, in the ordinary sense of the word, about how much water will actually be available at a particular time or place, or how much a particular user, or all users, will actually take. Rather, these functions indicate the relationships between quantities that can be made available at various costs, or will be at various prices (supply), and quantities that will be purchased at various prices (demand). In order to determine the actual quantity supplied, or demanded, it is necessary first to specify something about costs or pirces.

We are now ready to address the main point of this Introduction, namely the relationship between supply and demand. Since both are functional relationships between quantity and cost or price, we can represent them in the same two-dimensional format, as in Figure 1. The supply curve generally slopes up, to reflect the higher incremental costs associated with increased quantities supplied, and the demand curve slopes down to reflect the reduced quantities that will be taken at higher prices.

What is the significance of the intersection of demand and supply, point E in the diagram? In a market system this represents the equilibrium price and output. At price P_E the quantities supplied and demanded are just equal, there is no pressure on price due to excess demand, hence no net tendency to change: in short, the system is in equilibrium.

The relationship of this point to the "welfare" produced by the system is an interesting and complicated one, and the subject of a vast literature.[1] Ignoring the many qualifications and subtleties, we can very briefly and loosely characterize the welfare implications of a competitive equilibrium in the following way. At the equilibrium point, the sacrifices required to obtain another unit of the good, as measured by the incremental cost, are just equal to the willingness of consumers to pay for it, as measured by the price.[2] At lower levels of output, the cost of expansion is less than the willingness to pay for it, so these

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[1]The relationship between equilibrium in an economic system and welfare criteria is the heart of theoretical welfare economics. A good idea of the range of issues here can be gotten from the American Economic Association volume, Readings in Welfare Economics, edited by Arrow and Scitovsky (1969).

[2]When we talk about the willingness of consumers to pay for something, we recognize that this depends on a given distribution of income among them. If the distribution changes, in general so would willingness-to-pay, and prices. But the resulting equilibrium would still have the desirable property noted in the text.

outputs are inefficient in the sense that it would be possible to make some people better off without harming others. There is some "slack" in the system: additional net benefits can be obtained by some reallocation of resources to production of the good in question. Of course, actual price and output changes typically do harm some people, and a very knotty problem in welfare economics is how to evaluate changes that harm some and benefit others.[3]

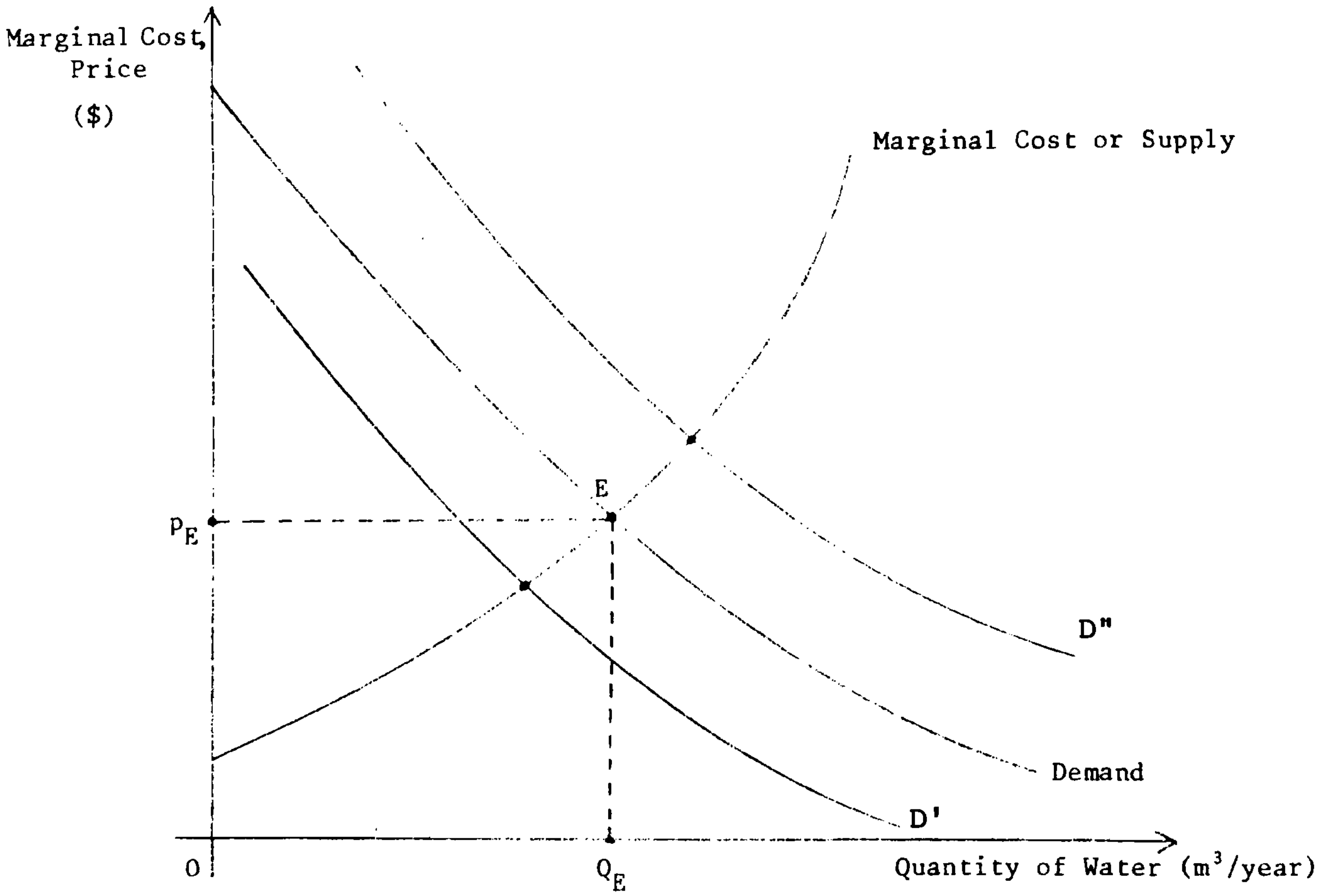

Figure 1

[3] Important contributions to the debate about a solution to this problem can be found in the Readings volume cited in footnote 2. In particular, see Kaldor, Hicks, and Scitovsky.

But the weaker efficiency condition that is satisfied by a market equilibrium says only that an allocation is efficient if it is not possible to make a change that harms no one (while benefitting some), as might be accomplished through income transfers from the gainers to the losers. On this definition higher levels of output (than at E), as well as lower, are seen to be inefficient, since the incremental cost of obtaining them exceeds the willingness to pay. Only the equilibrium point, E, is efficient.[4]

What are the implications of efficiency, in the sense we have defined it, of a market equilibrium for a nonmarket economy, or for that matter for the nonmarket provision of water supplies typical of most market economies? One way of characterizing the equilibrium point is to say that it represents an output for which price equals incremental or marginal cost. This condition, namely that price equals marginal cost, has in turn been proposed as a guide to resource allocation in centrally planned economies.[5] The proposal is simply that the planning agency give the firm or plant manager a price for his product, along with instructions to produce up to the point where marginal cost equals price. The idea is presumably that this can achieve efficiency in resource allocation, as would a perfectly competitive market system, but in a manner that is not inconsistent with other planning objectives.

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[4]Although we have promised to ignore the many qualifications to this proposition, one that is often particularly important where water and other natural resources are concerned really must be mentioned. It is the possible deviation of private from social costs of obtaining the resource. If, for example, the diversion of water by upstream users results in an increase in salinity - or other pollution - in the water available to downstream users, the upstream users' marginal cost curve will be "too low", and the market allocation of water to them too great. What is required for social efficiency, as a number of the contributions to the Readings volume point out, is that the external costs of upstream use be internalized to the users, perhaps through some sort of government policy to accomplish this, such as a tax on pollution or water use.

[5]The classic work here is by Lange (1952).

Here, by the way, is the explanation of the equivalence of marginal cost and supply that we have assumed all along. The marginal cost of producing any given output, say n $\frac{m^3}{year}$ of water, is just the extra cost involved in going from (n-1) to n units of output. But in a competitive equilibrium, as we have just seen, price will be equal to marginal cost. So the supply curve, which relates output to price, coincides with the marginal cost curve.

The demand-supply equilibrium can be characterized in another way, that leads to the efficiency criterion employed in water resource and other public sector benefit-cost analysis in market economies. We have defined demand as a function relating quantity purchased to price. But we have also spoken of price as the consumer's willingness-to-pay for or marginal valuation of the good or service in question. Thus we can write price (P) as a function of quantity (Q):

(1) $$P = P(Q) \quad .$$

The area under this marginal valuation curve between zero and the quantity consumed, $\bar{Q}$, is then the total valuation of, or benefit from, the good. Analytically, it is represented as

(2) $$\int_0^{\bar{Q}} P(Q)dQ \quad .$$

Let us represent the marginal cost (MC) curve as

(3) $$MC = MC(Q) \quad ,$$

and total cost as the area under it, or

(4) $$\int_0^{\bar{Q}} MC(Q)dQ \quad .$$

Once again ignoring all sorts of complications and subtleties, the idea of benefit-cost analysis is simply to compare (2) and (4); if (2) > (4), the project in question yields net benefits and on efficiency grounds ought to be undertaken. The significance of the equilibrium point in this analysis is that it represents the most profitable size or output level for the project, i.e., the one for which net benefits are maximum. If the shapes of the curves are known, and there is no resource or budgetary constraint that prevents it, this is the output that, again on efficiency grounds, ought to be chosen.

Identification of Supply

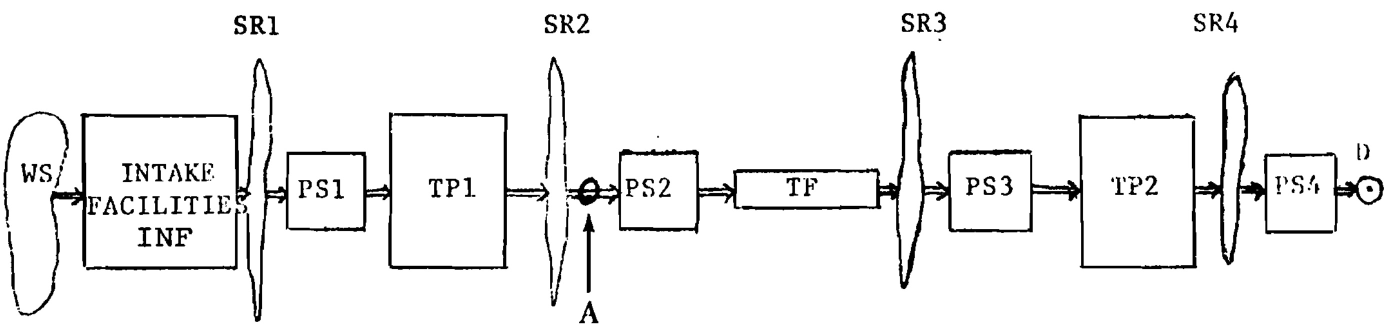
We have now reviewed some of the distinctions between engineering and economic interpretations of "water supply", with an emphasis on the economic, which we shall be using in our study. In order to motivate the derivation of a regional water supply function, the particular object of the study, we have also reviewed some relationships between supply and demand. Information about both - supply and demand - turns out to be important to an efficient use of a region's water resources. Before proceeding to sketch out (in the next section) some of the features of actual supply alternatives, such as reservoir construction or groundwater pumping, let us briefly indicate here how we propose to identify, in the econometric sense, a regional water supply curve. Like the elements of welfare economics presented just above, this material is standard, and further details may be found in any econometrics text.

In econometric estimation of a supply relationship, such as that in Figure 1, we are ordinarily confronted with a scatter of observed (price, quantity) points. The problem is to determine whether they trace out the supply curve, or the demand, or some mixture of both. Now, if only the demand varies, from point to point, because only some influence on demand varies, the scatter traces out the supply, For example, consumer income would be expected to influence demand and not supply, whereas plant capacity would influence supply and not demand. If only the former varies across the sample of observations, then the demand-price relationship is shifted along the stable supply curve, and supply is "identified". This is represented in Figure 1 by the intersections of the supply curve with the additional demand curves D' and D".

We do not carry out this sort of statistical estimation here. Instead, in the analytical sections III and IV we simply specify a shifting demand. This demand may be assumed to be perfectly price-inelastic, i.e., invariant with respect to water price; it is a "requirement". But it is also parametric, in that we allow it to vary, in order to trace out points on the water supply curve.

II. WATER SUPPLY ALTERNATIVES

We first consider the problem of developing a general scheme for water supply in a particular region. By a general scheme we mean one that abstracts from considerations of the location of sources, the topographical determination of stream flow, etc. Such a general scheme is represented in Figure 2.



D = demand point
WS = water source
INF = intake facilities
TP = treatment plant

PS = pumping station
SR = small reservoir
TF = transfer facilities (channels, rivers)

Figure 2.

In this scheme a given point, D, in region R is to be supplied with water from some water source WS. The latter requires intake facilities INF, and eventually a small (auxiliary) reservoir SR1.

In this scheme a given point, D, in region R is to be supplied with water from some water source WS. The latter requires intake facilities INF, and eventually a small (auxiliary) reservoir SR1. Before being transferred to D, the water has to be purified by the treatment plant TP1. Treatment might be disirable if, for example, at point A other users are supplied or if transfer facilities TF are used also for other purposes, such as recreation, that would require water of a standard quality. Of course, the specific location of these various facilities, and their size, will depend on the region's available water sources, its topography, and the quality and quantity of water being transferred to point D.

To derive a supply function for D we have to identify all of the feasible water sources or supply alternatives, which could be represented as in Figure 2. In contemporary water supply the following alternatives are employed:

1. River Water (RIV WAT)

This is probably the least cost alternative and is ordinarily the first one which is employed in a given river basin. However, there are two difficulties which prevent wider utilization of this water source: pollution (there is typically a need for intensive treatment of the water), and low dependability of flow.

2. Reservoir Water (RES WAT)

This alternative is an improvement over the first in both respects. Pollution may be less due to the sedimentation of solids in the reservoirs, and the dependability of supplies increases substantially due to the possibilities for regulating the stream flow.

3. Groundwater (GRD WAT)

"All water that exists below the surface of the earth in the interstices of soil and rock may be called subsurface water; that part of subsurface water in interstices completely saturated with water is called groundwater" [Water Policies for the Future (1973)]. As an alternative source of water it is readily accessible in many regions, often where surface supplies are becoming difficult and costly to expand. Groundwater also has two very important characteristics: it does not require construction of dams, and it is often of good quality. However, it should be noted that overuse can lead to a deterioration in the quality of the ground water and can also lower the water table.

4. Inter-Basin Transfer (INT BAS)

This alternative provides for a substantial augmenting of supply by transferring water from one watershed to another. The region receiving water gains while the region that donates water loses. This means that in studying this alternative one should take into account problems which pertain to both regions, unless the donating region has an excess supply at a zero price for the foreseeable future.

5. Desalting of Sea Water (DESALIN)

This alternative has always been a challenge to scientists and practitioners but until recently, it was not technically feasible to convert meaningful amounts of either sea water or brackish water into fresh water. Today, the technology for large scale desalting is at hand. In fact, as of 1971, there were some 745 plants in operation in various parts of the world, producing over 300 million gallons/day ($\approx$ 1. 136 million m^3/day) of water [Water Policies for the Future (1973)]. There are problems, however. Costs are still relatively high and the environmental impact can be substantial. Further cost reduction will probably come from reduction in the cost of energy used in the process, or more likely from more efficient use of the energy. One possibility here would be to combine power generation with desalination. The environmental problem is that the volume of brine effluent from a sea water conversion plant is about 50 per cent of the total volume treated. As indicated in [Water Policies for the Future (1973)], "the effluent from a 10 m.g.d. (37854 m^3/day) plant will contain 2000 tons of salt residue daily".

These are the alternatives considered in our illustrative example of a regional water supply function in section IV below. There are however a number of others which might be noted here.

6. Reclamation of Waste Water Effluent

This alternative is very close to the previous one. The main differences are that the amount of water to be treated is more limited than for the desalination alternative, more sophisticated treatment plants are needed due to the variety of ingredients in the waste water effluent, and environmental problems concerned with disposing of outputs from the treatment process could be more severe.

7. Land Management

It is well known that the manner in which a watershed is managed can affect the quantity and quality of water available for use. There are four land management techniques for increasing the supply of water [Water Policies for the Future (1973)]:

a) vegetation management in forest and brush areas,
b) phreatophyte control along river banks,
c) snowpack management in forest and alpine areas,
d) water harvesting by treatment of soil surface to increase the collection of precipitation.

All these techniques increase water supply either by reducing evapotranspiration or by delaying or stretching out run-off.

8. Modification of Precipitation

Although criticism and controversy still surround this alternative for water supply, in recent years the prospects have begun to look quite promising. The most common basis for modification (augmentation) of precipitation is cloud seeding. The theory behind cloud seeding is that "under certain conditions air containing a great deal of moisture will not yield precipitation, or as much precipitation as might possibly occur, because of the absence of nuclei--microscopically small particles of dust, crystal, or chemical droplets. By implanting such particles artifically in supersaturated clouds, rainfall can be stimulated" [Water Policies for the Future (1973)].

Experiments have shown a spectrum of results, from precipitation increases as high as 200 per cent for some storms, to slight decreases in the amount of precipitation which otherwise would have been expected. Although ecological research to date indicates that catastrophic impacts are not expected there is speculation that precipitation augmentation could bring about some alteration in the structure of plant and animal communities.

III. A DERIVED SUPPLY FUNCTION: STRUCTURE AND DESCRIPTION OF THE MODEL

The key idea in deriving a supply function for point D in region R is that different supply alternatives, and the resource inputs required for each, can be substituted for each other until the least cost combination for producing any desired amount of water is found. In this section we indicate formally how the process ought to work. For ease in exposition, we consider just two alternatives, each using just two inputs. But the model is perfectly general, and in the next section, where we work through an application to a hypothetical region, it is extended to include a more realistic range of supply alternatives and inputs.

There are however a number of simplifications adopted throughout. First, we importantly abstract from time. In the real world there are time lags in developing water resources; a dam may take several years to build, a reservoir or pipeline months to fill, and so on. Also, time enters in a significant way in the exploitation of a natural resource like a groundwater aquifer. Especially if recharge is slow, efficient use of the resource requires attention to its value over the entire planning period. Water pumped today has an opportunity cost; it is unavailable for use in the future. In the static analysis of this paper however all time is compressed into a single period. Some of the relevant dynamics are addressed in a subsequent study. There is a substantial literature on reservoir management, to which we shall not try to add. For a rigorous analysis of groundwater use over time, the reader is referred to the work of Oscar Burt (1967, 1970) in particular.

A second simplification in the present study is the neglect of uncertainity. As noted in the Introduction, water supply is often properly viewed in probabilistic terms: a quantity available with, say, a 98 percent probability. This uncertainity may be regarded as implicit in the water supply variable of the analysis which follows. That is, the quantity of water supplied may be thought of as having attached to it a particular probability figure, but we are not explicit about it. Again, this is further considered in the follow-on.

A third simplification has to do with water quality. In the analysis of this and the next section, we speak of quantities: for each of a set of demand "requirements" (inelastic demand curves), how can the required quantity be supplied at least cost? But recall that in the previous section's discussion of supply alternatives (see also Figure 2), water quality was mentioned. There we spoke of treatment plants, desalination, reclamation of waste water and so on. Except for desalination, though, water quality is not explicitly considered in the analysis. This is not because we think the environment is unimportant. On the contrary, a number of environmental quality constraints are specified in the programming model of section IV. But as with the probability or reliability of supply, discussed just above, the quality of the water may be regarded as implicitly specified. Some of the inputs - such as the chemicals in the example of section IV - presumably would be employed to bring the quality of the water produced up to the specified standard.

Model Structure and Assumptions

We assume the regional water supply agency wishes to minimize the cost of making available a given quantity of water, Y_D, say to meet projected demand at the prevailing price. Water can be supplied from either of two sources, X_1 and X_2, where $X_1 + X_2 = Y_D$. To get water from either source requires two production inputs, L_1 and K_1 for X_1, and L_2 and K_2 for X_2.

The inputs L_1 and K_1 can be combined to yield a given quantity of X_1 according to the production function $f_1(L_1, K_1) = X_1$, and L_2 and K_2 combined to produce X_2 according to $f_2(L_2, K_2) = X_2$.

As we shall indicate in the next section's application, environmental quality considerations are readily incorporated in this format. For example, we might represent the waste assimilative capacity of a watercourse as a scarce input, like L or K. But for now we stick with the simple two-input two-source model.

The agency's planning problem can be stated formally as,

minimize

$$(5) \qquad C = P_L(L_1 + L_2) + P_K(K_1 + K_2)$$

subject to the constraint

(6) $$f_1(L_1,K_1) + f_2(L_2,K_2) \geq Y_D$$

and the nonnegativity restrictions

(7) $$L_1,L_2,K_1,K_2 \geq 0$$

where P_L is the price of input L, P_K is the price of input K.

The Lagrange function is

(8) $$Z = C + \lambda[Y_D - f_1(L_1,K_1) - f_2(L_2,K_2)]$$

Assuming the production functions $f_1(L_1,K_1)$ and $f_2(L_2,K_2)$ are concave in both arguments, the Kuhn-Tucker (K-T) conditions for this program are necessary and sufficient for a minimum. Further assuming positive values for all the solution variables, the K-T conditions can be written

(9) $$\frac{\partial Z}{\partial L_1} = P_L - \lambda \frac{\partial f_1}{\partial L_1} = 0$$

(10) $$\frac{\partial Z}{\partial K_1} = P_K - \lambda \frac{\partial f_1}{\partial K_1} = 0$$

and similarly for L_2 and K_2.

Input Demand and Marginal Cost

From these conditions we may deduce the standard formulae for input demand, for example $P_L = \lambda \frac{\partial f_1}{\partial L_1}$, or $P_K = \lambda \frac{\partial f_1}{\partial K_1}$. These indicate simply that an input will be purchased up to the point where its price, P_L for L, equals the value of its marginal product, $\lambda \frac{\partial f_1}{\partial L_1}$. This expression is in turn the product of the shadow price of water, λ, and the marginal product of L, $\frac{\partial f_1}{\partial L_1}$.

Note also that, in an optimal, or cost-minimizing program, the value of an input's marginal product must be the same in both alternatives, because it is used in both to the point where its value is equal to the common input price. That is, we have, for L, $P_L = \lambda \frac{\partial f_1}{\partial L_1} = \lambda \frac{\partial f_2}{\partial L_2}$. Further, since the shadow price of water, λ, is obviously the same, we have $\frac{\partial f_1}{\partial L_1} = \frac{\partial f_2}{\partial L_2}$; the marginal product is the same in both supply alternatives. This result will be useful in deriving the marginal cost, or supply function.

The marginal cost of supplying water from alternative 1 is $\frac{P_L}{\frac{\partial f_1}{\partial L_1}}$ or $\frac{P_K}{\frac{\partial f_1}{\partial K_1}}$ (both = λ). Similarly, the marginal cost of supplying water from alternative 2 is $\frac{P_L}{\frac{\partial f_2}{\partial L_2}}$ or $\frac{P_K}{\frac{\partial f_2}{\partial K_2}}$. What are the relationships between the marginal costs of the two alternatives, to each other, and to the marginal cost of water? The answer is easy. The two marginal costs must be the same, for if they are not, the cost of supplying a given quantity of water can be reduced by shifting inputs from the higher cost alternative to the lower. The marginal cost of water supply is then just the marginal cost of either of the alternatives - at the total cost-minimizing solution, of course. To show that the alternative marginal costs are the same, we observe that $P_L \equiv P_L$ (working with L) and $\frac{\partial f_1}{\partial L_1} = \frac{\partial f_2}{\partial L_2}$, so that $P_L \Big/ \frac{\partial f_1}{\partial L_1} = P_L \Big/ \frac{\partial f_2}{\partial L_2}$. This is the marginal cost associated with a given quantity of water, say Y_D.

What has all of this to do with the derivation of a marginal cost or supply function, which is the point of this section? As explained in the Introduction, we calculate the marginal cost associated with any given level of output, Y_D, by treating Y_D as a parameter, i.e., by varying it and calculating the marginal cost at the new levels of the solution variables. This is in fact just what we do in the numerical application in the next section. Of course, this procedure yields only a scatter of points, each representing an output, cost pair. But it is still possible to calculate slopes and elasticities, for example, at each point, as we shall demonstrate.

The Linear Case: Specification and Economic Implications[6]

Before proceeding with the application, there is just one more point we should address. The application is in the form of a linear programming (LP) problem, which represents a special case of the problem we have just worked through. Although our main reason for adopting this technique is its advantage in computation, note that the objective function, equation (5), is already linear. The only remaining simplifying assumption, to convert the problem described by equations (5) - (7) to an LP one, is that the production constraints should also be linear. But to represent these constraints in linear form, it will be helpful to view them slightly differently.

Thus far we have considered how two different inputs, L and K, are combined to produce water in a particular process, like X_1, according to the production relation $f_1(L_1,K_1) = X_1$. But it is also possible to consider how a single (scarce) input, say L, is used to produce water in two different ways, X_1 and X_2. In general nonlinear form, the constraint might be written $g(L_1,L_2) \leq L'$, where L' is the limited amount of L available to the regional water supply agency. Of course, the agency may be able to purchase as much L as it wants, but the constraint would still be written in much the same way, as $g(L_1,L_2) = L''$, where L" is the amount of L actually purchased.

In linear form, the constraint function $g(L_1,L_2)$ becomes $g(L_1,L_2) = a_{11}X_1 + a_{12}X_2$, where a_{11} is the amount of L used in the production of one unit of X_1 and a_{12} is the amount of L used in the production of one unit of X_2. Then for constraint (6) we might substitute something like

(6a) $$a_{11}X_1 + a_{12}X_2 \leq L' ,$$

(6b) $$a_{21}X_1 + a_{22}X_2 \leq K' ,$$

[6]The linear programming model described in this section was suggested to us by the linear programming models for water demand developed by Thompson and his collaborators (see in particular Thompson and Young (1973) and Calloway and Thompson (1976)).

which is in fact the way the resource input and environmental constraints are specified in the application, and

(6c) $$X_1 + X_2 \geq Y_D \ .$$

There we also specify the objective function a bit differently, in terms of the costs of the alternative processes, instead of the process inputs. That is, assuming just two alternatives, X_1 and X_2, we wish to minimize

(5') $$C = C_1X_1 + C_2X_2$$

where C_1 is the unit cost of X_1 and C_2 is the unit cost of X_2, subject to constraints (6a) and (6b) on inputs, (6c) on outputs, and the usual nonnegativity restrictions.

Of course, it doesn't really matter whether we read the constraints "down" column activities, as before, or "across" row inputs, as in (6a) and (6b). But the assumption of linearity in production does matter. In economic terms, linearity means that production is subject to constant returns to scale. That is, if each input is increased by k percent, output is also increased by k percent, regardless of the size of k. This may be a realistic description of some processes, but then again it may not. In particular, some limiting factors, often overlooked in the specification of the production technology, like managerial input, will typically prevent the indefinite realization of constant returns to scale. This suggests that the way to interpret the linear format which we adopt for ease in computation is to recognize that it may be a good approximation to the workings of a process for producing water only up to some point. This is one reason, though not the most immediate one, for our specification in the next section's application of "less than or equal to" constraints on the operation of each of the water supply alternatives.

Another property of the production structure specified in (6a) and (6b) is that the inputs L and K are combined in fixed proportions to produce water in a given alternative. This is obviously more restrictive than the production function we earlier specified, which allows for varying input proportions. But the apparent restriction need not cause any difficulties in practice, because different proportions, and even different production techniques, that might be used to supply water from a given source, say groundwater, are easily represented as separate alternatives. This is not done in our particular application, but clearly it could be where relevant.

IV. An Illustrative Example of a Regional Water Supply Function

In this section we elaborate a somewhat more realistic system of regional water supply alternatives (drawing on the discussion in Section II) and resource and environmental constraints than in the previous section's stripped-down, schematic derivation of a supply function. We also present some hypothetical data on the costs of the alternatives, and on the constraints, and then solve the cost-minimizing program for a range of water outputs.

Let us begin by considering the column vector $a_j = (a_{1j}, a_{2j}, \ldots, a_{ij}, a_{mj})$ associated with some supply alternative j. Each element in this vector represents the amount of good i (material, labour, etc) which is input to or output from alternative j being run at the unit level. The following example can clarify the essence of the vector a_j. Consider the second alternative, supplying point D with reservoir water. For such an alternative the following version of the general scheme might be appropriate.

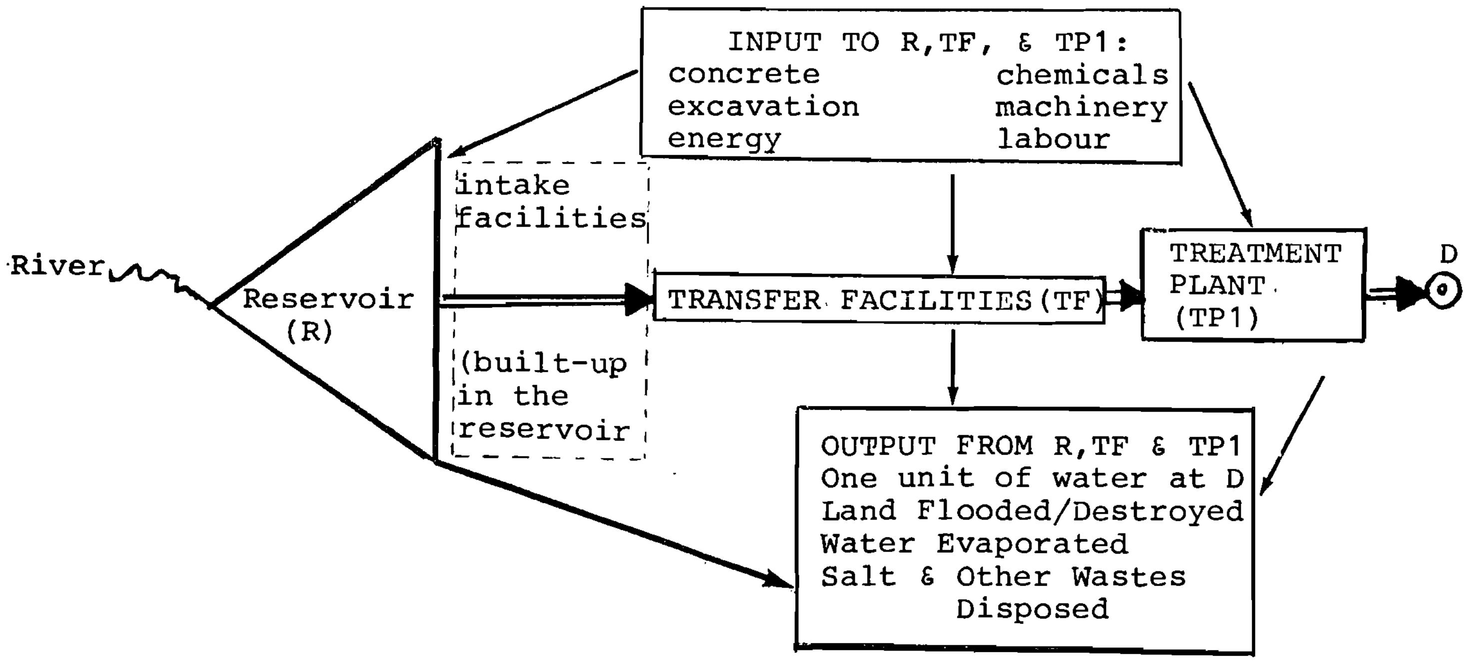


Figure 3

It follows from Figure 3 that there are 3 major elements that need to be constructed and operated: T, TF, and TP1. To supply one unit of water in a given period of time at point D certain inputs to all three elements are needed. These might consist of: concrete, excavation, energy, chemicals, machinery, and labour. As an output one can consider: water supplied, land flooded/destroyed, water evaporated, salt & other wastes disposed. Therefore, the components of the vector a_2 would be:

input a_{12} - total amount of concrete needed to supply one unit of water.

input a_{22} - total amount of excavation needed to supply one unit of water.

input a_{32} - total amount of energy needed to supply one unit of water.

input a_{42} - total amount of chemicals needed to supply one unit of water.

input a_{52} - total amount of machinery needed to supply one unit of water.

input a_{62} - total amount of labour needed to supply one unit of water.

output a_{72} - one unit of water supplied at point D; $a_{72} = 1$

output a_{82} - amount of land flooded/destroyed to supply one unit of water.

output a_{92} - amount of water evaporated to supply one unit of water.

output $a_{10,2}$ - amount of salt and other wastes disposed to supply one unit of water.

Having specified vectors a_j for all supply alternatives $j = 1,...N$, one obtains the matrix $A = \left\{a_{ij}\right\}$. The coefficients of this matrix are either inputs or outputs from a given supply alternative j.

The next step in defining the linear programming problem is to organize proper constraints out of the coefficients a_{ij}. The constraints reflect generally the availability of materials and labor as well as the economically justified scale of each alternative's development. And as we noted earlier, constraints can also be defined to reflect environmental impacts connected with water supply. One possible representation of these various constraints is given by the tableau shown in Table 1.

Columns / Rows	SUPPLY ALTERNATIVES									RIGHT HAND SIDES
	RIV WAT	RES WAT	GRD WAT	INTBAS	DESALIN	RECL WASWAT	LAN MANG	MOD PREC		
RESOURCE AVAILABILITY CONSTRAINTS	RESOURCE REQUIREMENTS TO SUPPLY ONE UNIT OF WATER: MATERIALS LABOUR								≤	AVAILABILITY OF MATERIALS AND LABOUR
SCALE CONSTRAINTS	WATER SUPPLIED								≤	ECONOMICALLY JUSTIFIED SCALE OF EACH ALTERNATIVE'S DEVELOPMENT
WATER QUANTITY REQUIREMENTS	WATER SUPPLIED								≥	WATER REQUIRED (y_D)
ENVIRONMENTAL CONSTRAINTS	LAND FLOODED/DESTROYED WATER EVAPORATED SALT AND WASTES DISPOSED								≤	PREFERENCES ABOUT ENVIRONMENT'S STATE
OBJECTIVE FUNCTION	(NET) COST OF WATER SUPPLY								=	CS

Table 1

At the bottom of Table 1 the linear objective function is shown. The supply function MCS = F(y_D) can be obtained by solving the LP problem shown in Table 1 for a number of values of y_D. It should also be mentioned that by varying the right hand sides, one could obtain various supply functions. Thus the sensitivity of the supply function to different constraint parameters could be determined.

The methodology discussed above can be illustrated by the following hypothetical example. In region R there is a point D to be supplied with water (Fig. 4). For this purpose five supply alternatives are available: river water (RIV-WAT), ground water (GRD-WAT), reservoir water (RES-WAT), interbasin transfer (INTBAS) from region K, and desalination (DESALIN). For each of these alternatives, various materials and types of labour are needed. The economic problem is that there are constraints on the availability of each. All of the relevant data are given in Table 2. The objective function is also specified. Note again that to obtain the supply function MCS = F(y_D), the variable y_D is taken as a parameter.

Constraints No. 9, 10, 11, 12, and 13 in Table 2 reflect scale considerations. While the first two are firmly rooted in the physical characteristics of the water resource (we cannot take more water than there is in an underground pool, for example), the last three are somewhat artificial in that they derive more fundamentally from input limits.

As we noted in the preceeding section, the scale of operations of a particular alternative may be limited by an inability to expand the supply of some input not explicitly represented in the illustrative application, such as managerial ability, or perhaps the amount of investment that can be generated by the national economy. In this case, the constraints on the alternatives can be interpreted as proxies for these implicit input limits.

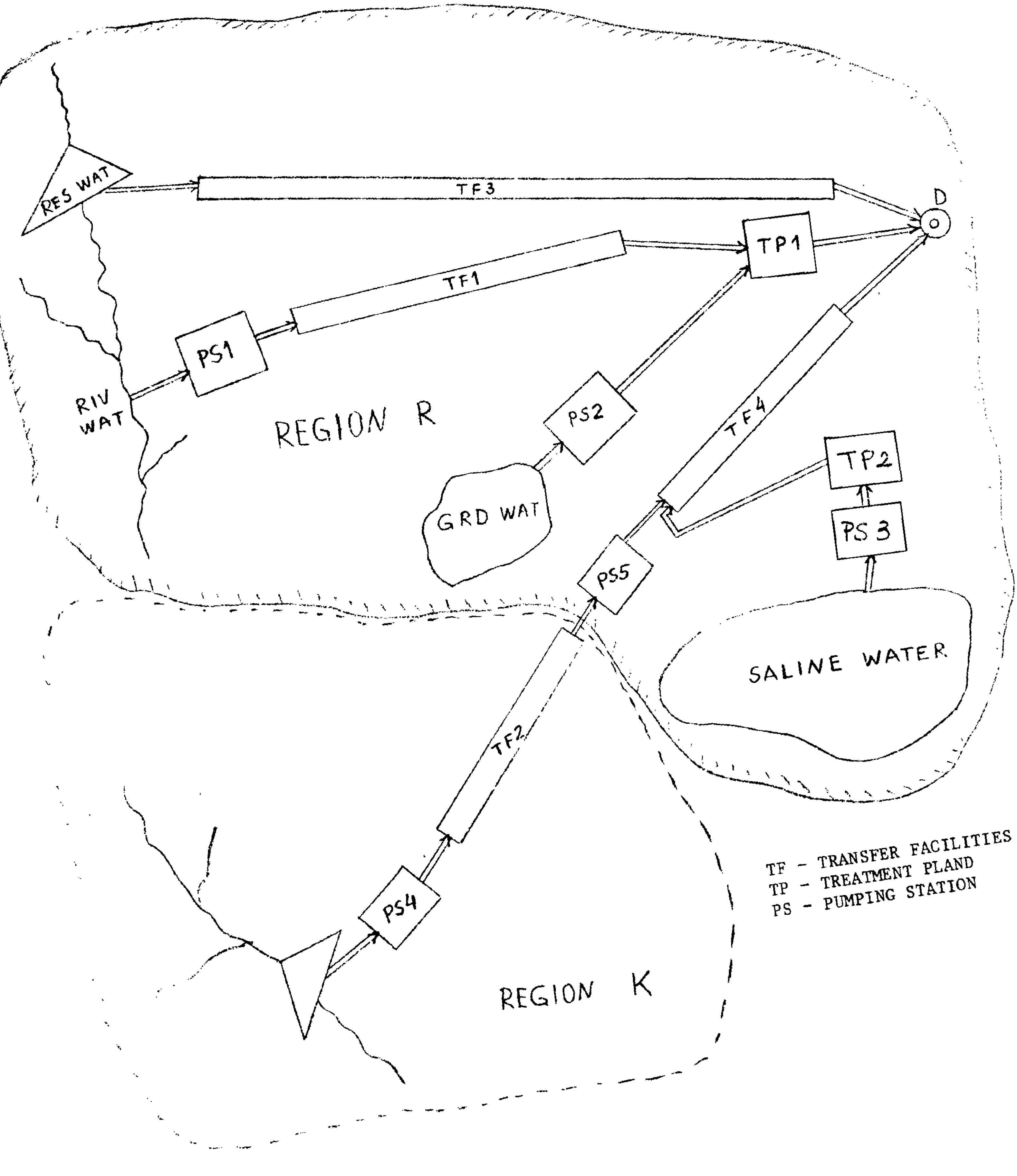

RES WAT
TF3
D
TP1
TF1
PS1
RIV WAT
REGION R
PS2
TF4
TP2
PS 3
GRD WAT
PS5
SALINE WATER
TF2
PS4
REGION K
TF - TRANSFER FACILITIES
TP - TREATMENT PLAND
PS - PUMPING STATION

Figure 4

	ROWS \ COLUMNS		RIV WAT	GRD WAT	RES WAT	INTBAS	DESALIN		RIGHT HAND SIDES
1	Concrete	m^3/unit	2200	1800	7840.2	53200	2200	$\leq$	94×10^4
2	Excavation	m^3/unit	3400	1500	24407.4	17600	1500	$\leq$	24×10^5
3	Pumping stations	kw/unit	900	2340	0	5708	1010	$\leq$	6×10^4
4	Energy	kwh/unit	18000	24000	841.7	50000	69950	$\leq$	15×10^5
5	Chemicals	t/unit	8500	10000	0	0	98000	$\leq$	63×10^4
6	Other machinery	t/unit	84.2	245	199.2	985	384.5	$\leq$	27×10^3
7	Labour	people/unit	79.5	123.4	420.9	1240	210.5	$\leq$	18×10^3
8	Water required		1	1	1	1	1	$\geq$	$y_D ; 0 \leq y_D \leq 30$
9	RIV WAT		1	0	0	0	0	$\leq$	2.4
0	GRD WAT		0	1	0	0	0	$\leq$	8.3
1	RES WAT		0	0	1	0	0	$\leq$	5.94
2	INTBAS		0	0	0	1	0	$\leq$	25
3	DESALIN		0	0	0	0	1	$\leq$	7.3
	Land flooded/ destroyed	ha/unit	0.01	0.01	158.7	0.46	0.01	$\leq$	800
	Water evaporated	m^3/unit	80×10^3	45×10^3	100×10^3	20×10^3	45×10^3	$\leq$	14×10^6
	Salt disposed	t/unit	9900	10400	0	0	96000	$\leq$	300000

Objective function	\$/unit	0.45×10^6	0.78×10^6	1.56×10^6	35.2×10^6	9.24×10^6	min	CS

1 unit of water = 10^8 m^3/year

Table 2

Results

The results obtained for the supply function MCS = $F(y_D)$, using a standard linear programming code, are shown in Table 3. Analysing the results shown in part A of Table 3, we observe various patterns of meeting the required supply y_D.

If $0 \leq y_D \leq 2\times10^8$ m^3/year just RIV WAT is used. For $2\times10^8 < y_D \leq 12\times10^8$ m^3/year the second alternative GRD WAT is introduced into the solution. In this interval RIV WAT has already reached its upper limit.

If the amount of water y_D to be supplied is more than $12\times10^8 m^3$/yr then the third source of water, RES WAT, enters the solution. There is an interesting phenomenon here associated with this source of supply. RES WAT reaches the value of 5.0387×10^8 m^3/year (the upper limit is 5.94×10^8 m^3/year) and then follows a pattern of slight decrease. The reason for this is that constraint No. 14 on land flooded is becoming active.

The amount of water supplied by desalination (DESALIN) reaches a level of 1.9783×10^6 m^3/year. It does not go beyond this level because constraint No. 16 on the amount of salt disposed is becoming active.

For values of $y_D > 24\times10^8$ m^3/year there is no feasible solution since constraint No. 3 is violated, i.e., no more pumping stations are available.

The contribution of all alternatives to water supply of the point D is also displayed graphically in Figure 5

The last three columns of Table 3 can help in clarifying three additional economic properties of water supply: total cost of supply CS, marginal cost of supply MCS, and elasticity of supply E_S. The first property is rather clear. It indicates the value of the objective function, the total cost of supplying y_D units of water. The second characteristic MCS = f (y_D) is the derived supply function itself. As can be seen in Figure 6, this function follows a pattern of monotonic increase of the marginal cost with an increase in the amount of water y_D to be supplied. The most interesting part of the supply function is in the range of y_D between 10×10^8 m^3/year and 20×10^8 m^3/year. In this interval substantial increase occurs in the marginal cost of supply.

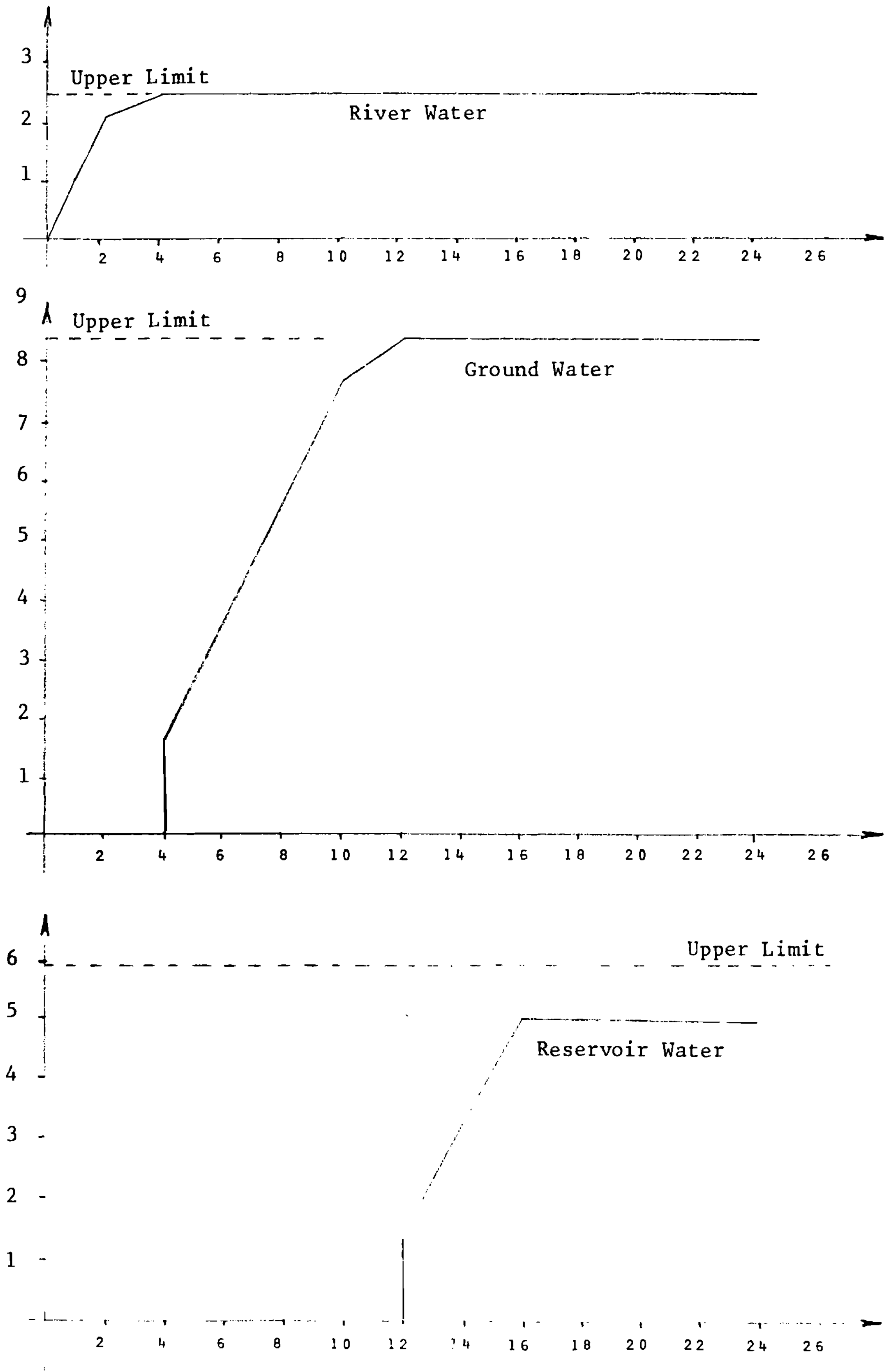
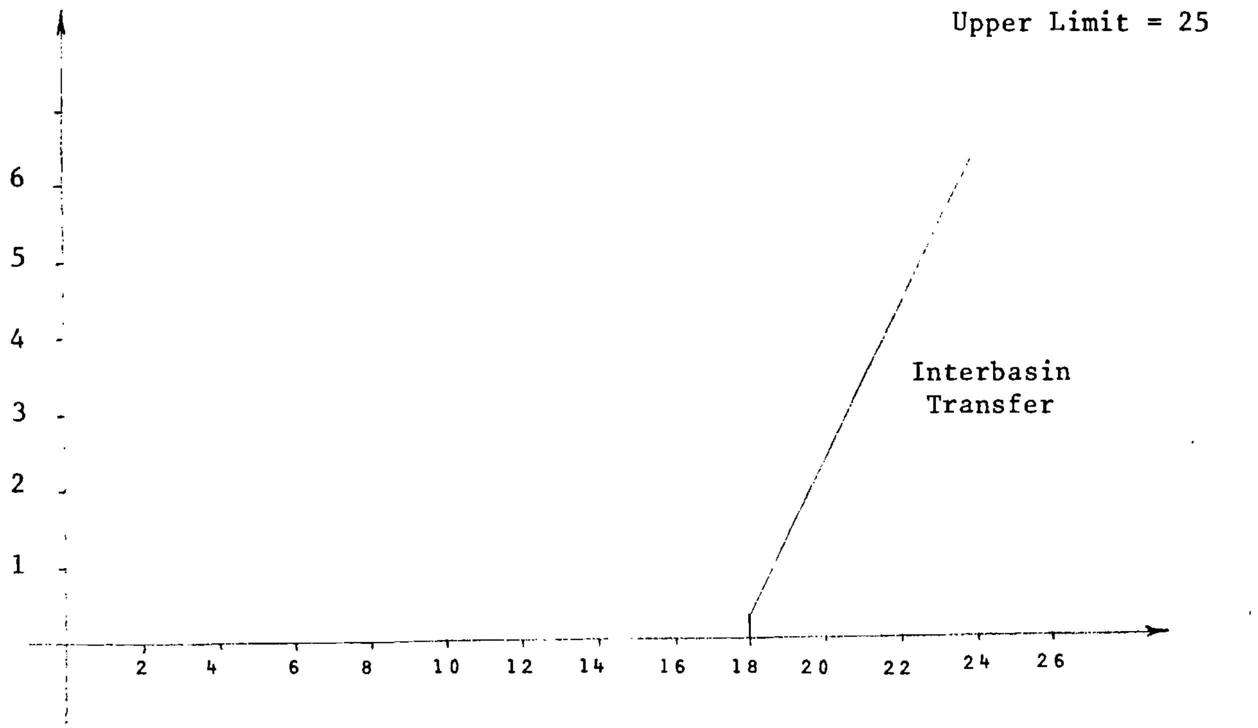

Upper Limit
River Water
Upper Limit
Ground Water
Upper Limit
Reservoir Water
3
2
1
9
8
7
6
5
4
3
2
1
6
5
4
3
2
1
2 4 6 8 10 12 14 16 18 20 22 24 26

Figure 5



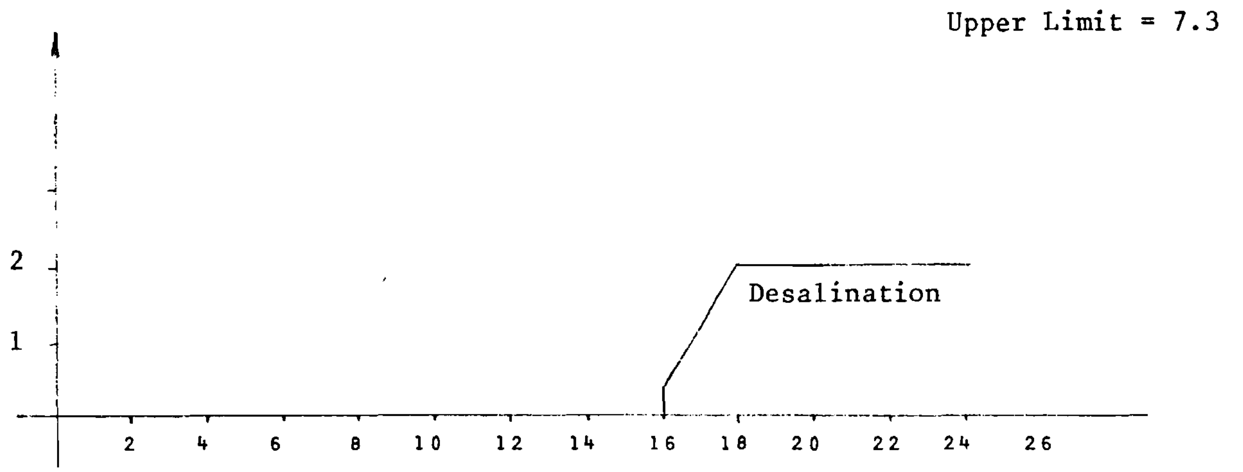


FIGURE 5
(continued)

A						
Amount of Water y_D Supplied at Point D $x10^8 [m^3/year]$	OPTIMAL COMBINATION OF THE ALTERNATIVES:					No. of the Active Constraints (No. refer to those in Table 2)
	RIV WAT	GRD WAT	RES WAT	INT BAS	DESAL	
	X 10^8 $[m^3/year]$					
1	1.0000	0	0	0	0	8
2	2.0000	0	0	0	0	8
4	2.4000	1.6000	0	0	0	8,9
6	2.4000	3.6000	0	0	0	8,9
8	2.4000	5.6000	0	0	0	8,9
10	2.4000	7.6000	0	0	0	8,9
12	2.4000	8.3000	1.3000	0	0	8,9,10
14	2.4000	8.3000	3.3000	0	0	8,9,10
16	2.4000	8.3000	5.0387	0	0.2613	8,9,10,15
18	2.4000	8.3000	5.0377	0.2839	1.9783	8,9,10,15,17
20	2.4000	8.3000	5.0319	2.2897	1.9783	8,9,10,15,17
22	2.4000	8.3000	5.0261	4.2955	1.9783	8,9,10,15,17
24	2.4000	8.3000	5.0203	6.3013	1.9783	8,9,10,15,17
25	NO FEASIBLE SOLUTION					constraint No. 3 is violated

Table 3

B			
Total Cost of Supply CS $\times 10^6$ [$]	Marginal Cost of Supply MCS $MCS = \frac{dCS}{dy_D}$ $\times 10^{-2}$ [$/m^3]	ELASTICITY OF SUPPLY E_S*	
		$E_{S-} = \left(\frac{dy_D}{dMCS} \cdot \frac{MCS}{y_D}\right)_-$	$E_{S+} = \left(\frac{dy_D}{dMCS} \cdot \frac{MCS}{y_D}\right)_+$
0.4500	0.4500	∞	∞
0.9000	0.4500	∞	1.7045
2.3280	0.7140	1.3523	5.4091
3.8880	0.7800	3.9394	∞
5.4480	0.7800	∞	∞
7.0080	0.7800	∞	0.3077
9.5820	1.2870	0.4231	0.7857
12.7020	1.5600	0.8163	0.2221
17.8289	2.5634	0.3193	0.0309
43.6866	12.9288	0.1386	0.0642
114.2821	35.2977	0.1578	∞
184.8776	35.2977	∞	∞
255.4731	35.2977	∞	-

*The signs (·) - and (·) + indicate the left and the right derivatives, respectively.

Table 3 (continued)

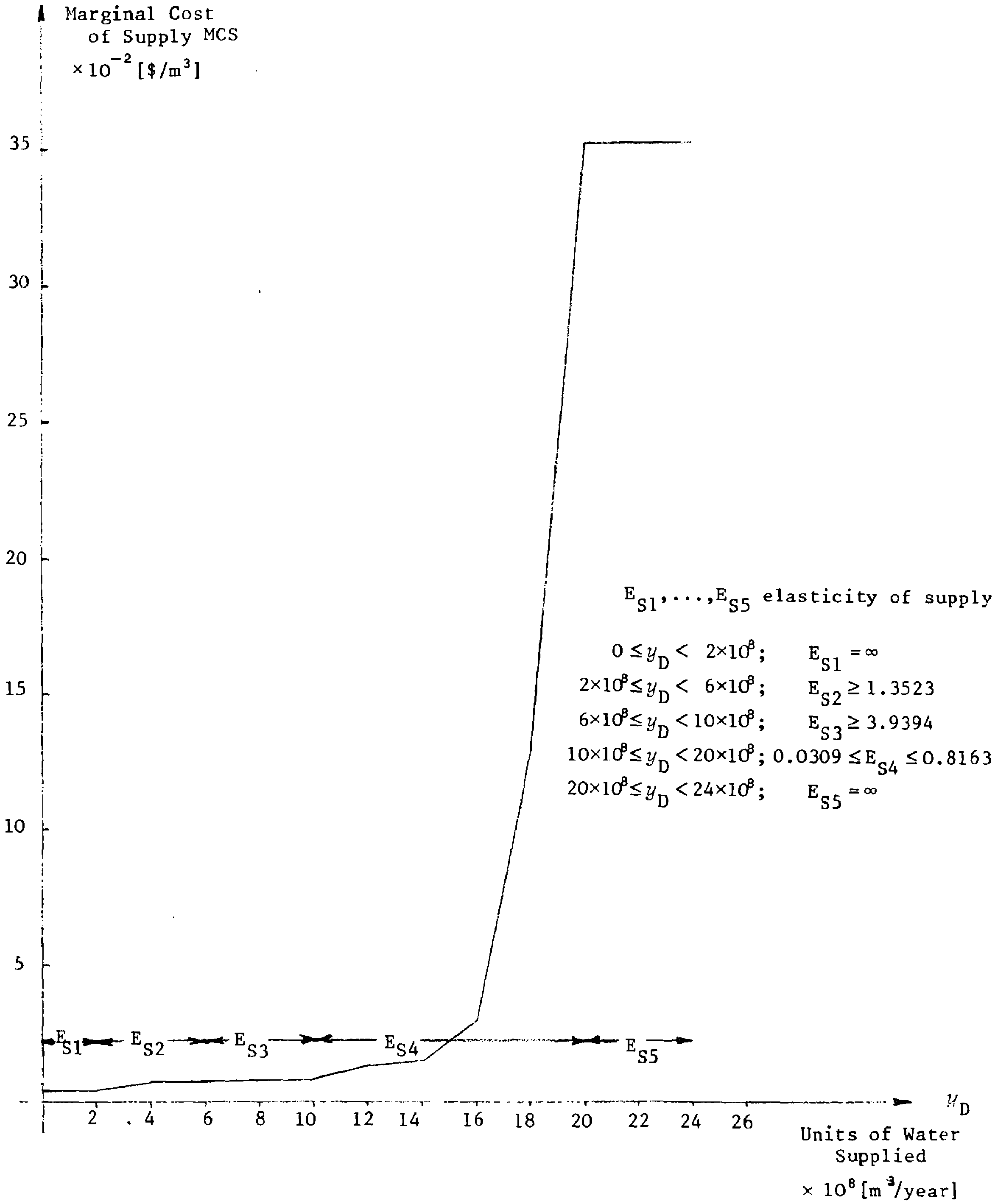

Marginal Cost
of Supply MCS
×10^-2 [$/m³]
35
30
25
20
15
10
5
E_S1,...,E_S5 elasticity of supply
0 ≤ y_D < 2×10^8; E_S1 = ∞
2×10^8 ≤ y_D < 6×10^8; E_S2 ≥ 1.3523
6×10^8 ≤ y_D < 10×10^8; E_S3 ≥ 3.9394
10×10^8 ≤ y_D < 20×10^8; 0.0309 ≤ E_S4 ≤ 0.8163
20×10^8 ≤ y_D < 24×10^8; E_S5 = ∞
E_S1
E_S2
E_S3
E_S4
E_S5
2
4
6
8
10
12
14
16
18
20
22
24
26
y_D
Units of Water
Supplied
× 10^8 [m³/year]

Figure 6.

Increasing costs are reflected also in the behaviour of the elasticity of supply E_s, shown in the last column of Table 3. Note that elasticity is computed for both left and right derivatives $(dy_D/dMCS)_-$ and $(dy_D/dMCS)_+$, respectively, The reason is that the supply function $MCS = f(y_D)$ is piece-wise linear, hence, left and right derivatives are not equal. Figure 6 indicates also the general behaviour of the elasticity of supply. For example, for all values of y_D, $0 \leq y_D < 2\times10^8$ m^3/year the elasticity $E_1 = \infty$. An infinitely elastic supply curve is just another way of describing the constant returns to scale, which are experienced in this range because only one (linear) production process, namely RIV WAT, is employed. Again, the most interesting interval for y_D is probably $10\times10^8 \leq y_D < 20\times10^8 m^3$/yr. in which the elasticity falls to less than 1. In other words, in this interval price increases would have relatively little effect on the quantity of water supplied. This sort of result can be especially useful in directing the attention of the water resource planners to management of demand, rather than supply. That is, if it will be very costly to increase the production of water beyond some point, then measures to restrict demand, rather than augment supply, might be warranted.

Finally, in discussing these results it would be interesting to know how sensitive they are to variations in resource availabilities, costs, and so on. This sort of sensitivity analysis is easily carried out in the framework of the model. For example, one could relax or tighten by some specified amount the constraint on land flooded, or on labor available, or whatever, and calculate new solution values, including the incremental cost of supply for each quantity supplied.

V. CONCLUDING REMARKS

The main task of this paper has been to propose a method for deriving regional water supply functions, taking into account a variety of supply alternatives and some engineering and environmental aspects of each. The purpose of this exercise is, as suggested in the Introduction, to provide a framework for decisions about the efficient use of a region's water resources.

In the Introduction we first discuss some distinctions between engineering and economic concepts of water supply, and provide definitions of supply and demand as they are used in economics. We then review the notion of supply-demand equilibrium and, most importantly, the economic efficiency properties of this equilibrium and their relevance for planning investments in water resources.

In the second section we briefly survey the "State-of-the-Art" in regional water supply, describing a number of alternative sources of supply. These include, ranging from more to less conventional, surface streams, reservoirs, groundwater, interbasin transfers, desalination, land use controls, and modification of precipitation. The third section retreats from this brush with reality to consider how, for a region having just two sources of supply, each having just two inputs, each point on a supply curve can be derived as the solution to a nonlinear program to minimize the cost of obtaining a given quantity of water. The procedure is however perfectly general, and in the fourth section we return to the more complex reality by working through an application to a hypothetical region with several sources of supply, each having several inputs, with constraints on their use, and so on. An interesting feature of the model is that it can--and does, in our application--reflect environmental constraints as well. For example, the use of desalination is limited by a constraint on the quantity of salt that can be disposed.

In dealing with a realistic range of alternative sources and constraints, however, computational difficulties multiply. For ease in computation, we have in the application linearized the production relations, in order to use a linear programming solution algorithm. We recognize that this reintroduces a degree of unreality into the approach, as well as some difficulties, described in the preceding section, in interpreting results.

It appears to us that future work in this area could usefully consider how to introduce nonlinearities in as painless a fashion as possible. The water quality dimension might also be explicitly introduced, for example through several different quality output requirements, or additional environmental constraints. Finally, the dynamics of water supply ought to be considered. Withdrawals from reservoirs or groundwater pools necessarily involved dynamic considerations, and the construction of supply facilities takes time. This is a question--how to incorporate the relevant dynamics--to which we hope in particular to return.

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