# METHODS OF SYSTEMS ANALYSIS FOR LONG-TERM ENERGY DEVELOPMENT

YURI KONONOV, Editor

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## PREFACE

One of the cornerstones of the Energy Program is the identification and comparison of energy strategies for a period of transition from an economy based on scarce but cheap conventional fuels to an economy where the forms of energy come from resources which are plentiful but expensive.

Models in which the USSR is used as an example are described by A.A. Makarov et al. from the Siberian Power Institute of the Siberian Department of the USSR Academy of Sciences. They illustrate and link economic and technological developments of the transition.

## ABSTRACT

A constructive systems approach to long-term energy development forecasting (for 30 to 40 years) is elaborated and the main methods and mathematical models for implementing it are proposed. They are:

- An intersector model and method for long-term forecasting of basic indices of economic growth and national economic estimation of energy development strategies;
- An iterative scheme and optimization model for elaborating possible energy development strategies and comparison of tendencies in science and technology;
- A statistical model for forecasting final energy demand based on existing dynamics of the main indices of economic growth.

These models and methods are tested on preliminary information.

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# Problems and Methods of Long-Term Forecasting of the Structure of the USSR Rational Energy Supply System

A.A. Makarov

The energy supply system (ESS), being one of the most slow to adapt and fund-intensive spheres of the national economy, requires long-term forecasting under any circumstances, particularly for the forth coming transition in energy supplies when the following basically new and inescapable factors begin to act.

In the energy sector, a scientific and techological revolution is taking place because of the transition from fossil fuel to nuclear fuel and, possibly, to other new energy sources. This causes not only a change of the "resource base" of the energy sector, but also a change of ideas concerning the main aspects of production, concentration, centralization and distribution, capital investment/annual cost relationships--in fact, a change in the whole concept of energy supply.

Long-term reductions in energy price have been replaced by a sharp but not uniform increase (especially for high-grade fuels) throughout the world. This has greatly influenced the national economy by affecting export policy, fuel and energy supply for internal consumers, and efficiency of new science and technology.

A well-marked tendency towards growth in energy capital-intensiveness (resulting from the transition to low-grade and more distant fuel deposits, and the use of capital-intensive technologies like nuclear energy, etc.) may seriously, although not directly, restrain national economic development and directly affect both production structure and final consumption.

Rapid growth of energy production converts ecology from a local restraining factor to a global one, and makes us find new approaches to the solution of technological problems to energy production, including its large-scale removal from economically developed regions.

The above features of forthcoming developments in energy production result in a demand for a new long-term forecasting methodology for the ESS. Direct extrapolation of established trends in energy production and consumption is not suitable, since we are at a stage where a radical change is taking place. On the other hand, an extension of the five-year planning methodology to a longer-term perspective is also ineffective as no changes of this kind are expected for the majority of the other sectors of the national economy.*

The third possible approach, *systems analysis*, should be chosen as the basis for long-term ESS forecasting. The essence of this approach is to use the cause/effect relationships which form the main part of energy sector development as much as possible. Systems analysis evaluates these not by extrapolation of their past tendencies, but by a synthesis of many essential factors with certain assumptions about the future, and by including more or less complete modeling of actual relationships between these factors. It does not exclude, however, past experience, but emphasizes its use under actual conditions of the forecasted system operation. This approach is not purely academic: on the contrary, it is mainly heuristic and strengthens the forecasting ability of specialists based solely on computer simulation of different hypotheses of energy sector development.

The methods of systems forecasting for energy sector development are completely determined by the specific aims of the long-term forecast which for the energy supply system is not for any abstract knowledge of the future, but for the solution of specific and urgent problems: unsolved these would have grave and long-term consequences. Among these problems are:

1. To choose the most cost-effective scientific and technological progress in energy development in order to distribute best the funds available for research and development.

2. To determine the need for fuel resources recovery in extreme regions or conditions (e.g. the northern areas of East Siberia and the Far East, northern sea shelves, etc.) and to work out the complex measures necessary to develop these regions (railway building, geological exploration, new machinery construction, etc.).

3. To make clear the necessity of fuel resource preservation for the future and to correct export policy and internal energy consumption correspondingly in the medium term.

4. To study effects of variations in energy development on the rates and proportions of economic development as a whole; to evaluate the danger from lowered rates

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*The scientific and technological progress in such sectors as radio-engineering, instrument-making industry, etc. may result in even larger changes, but here their required lead-time is less than in the energy sector (due to the latters specific inertial and fund-intensive character).

of national economic development due to increased capital-intensiveness in the energy sector and its consequent adverse ecological effects; to try to eliminate this danger by changing total production structure and even final consumption towards a reduction in energy-intensiveness.

The first of these problems has been considered recently as the main (if not the only) one. To solve it, it is sufficient to compare every individual technical innovation with a marginal improvement in energy production separately. An example of such an approach is the study of strategies for a transition to breeder-reactors where different reactors are compared with each other and with electricity generation by conventional power stations which use coal.

The forthcoming depletion of high-grade fuel resources gives rise to the two problems of long-term forecasting numbered 2 and 3 above. To solve them it is important to study the development of the ESS as a whole.

The fourth problem is oriented to the evaluation of socio-economic effects of different strategies for energy sector development and should be covered by the long-term forecasting. Special techniques for long-term energy forecasting interrelated with national economic development macroforecasting are needed, and those adopted at the Siberian Power Institute are given below.

It is evident that each stage should be approached differently. Given that five-year planning is excluded, the remaining period may be divided into two stages, that of medium-term forecasting for 15 to 20 years ahead (in this work from 1980 to 1995) and that of long-term forecasting (from 1995 to 2010-2020).

## Medium-Term Forecasting

This covers the period when the ESS develops mainly by well-tested technological processes and types of equipment--in this case by already planned or adopted decisions on construction both in the energy sector and in related sectors of the national economy. With a sufficiently methodical approach, reliable medium-term energy development forecasts result. Two possibilities exist for their use: as the approximation to the fifteen year plan of national economic development, and as the initial state and statistical base for elaboration of the long-range ESS forecast.

Because of the double-oriented nature of the medium-term forecast, it should be fairly detailed and specific to allow preliminary assessment of the following:

Actual demand for the main types of fuel and energy within the 11-19 regions of the country and the main directions of their use.

Effective methods of export of fuel and energy resources in the amounts required.

Priority and funds for the development of the main fuel bases with the choice of effective methods of energy transportation.

Priority for construction, structure and ouput of large energy complexes (for example, the Kansk-Achinsk energy-technological complex, atomic centers, etc.).

Determination of optimal designs for specialized energy transport development and choice of the main parameters of interregional pipelines and transmission lines.

New energy equipment of promising type, exploration of deposits, fuel processing, etc.

Figure 1, part A shows the medium-term forecasting scheme elaborated at the Siberian Power Institute to solve these problems. In this scheme the following is assumed: (a) the aggregated hypothesis of national economic development by 1990 based on the long-term planned figures from the USSR Gosplan and by 1995 on the data of the economic institutes; (b) geological prediction for fuel reserves; and (c) new technologies and equipment types and their main technical and economic characteristics, i.e. capital investments, annual costs, manpower and material expenditures.

On the basis of this information, the fuel-energy demand for different regions of the country and the possible amounts and technological-economic indices for fuel extraction from existing and prospective fuel resources are calculated. At present, these calculations are made without models but by using a well-developed and tested technology. In so doing, the shadow prices for fuel and energy are made more accurate when the forecasting of ESS structure is used as necessary information (see dashed feedbacks in A of Figure 1).

The main problem of long-term forecasting is to determine real structural shifts in ESS development which depend on geological predictions and on whether the related sectors meet ESS requirements in materials, input and equipment (with due consideration for real dates and rates of growth of production capacity). The method of solution uses a dynamic optimization model which describes ESS development (for two zones of the country and for the main fuel and energy types) in relation to metallurgy, machine building (by production types), construction, transport and other sectors. Multivariant calculations with this model allow identification of real variants of ESS structure, and development of different hypotheses on the possible increment of high-grade fuel reserves, export policy and the development of non-energy sectors of the national economy.

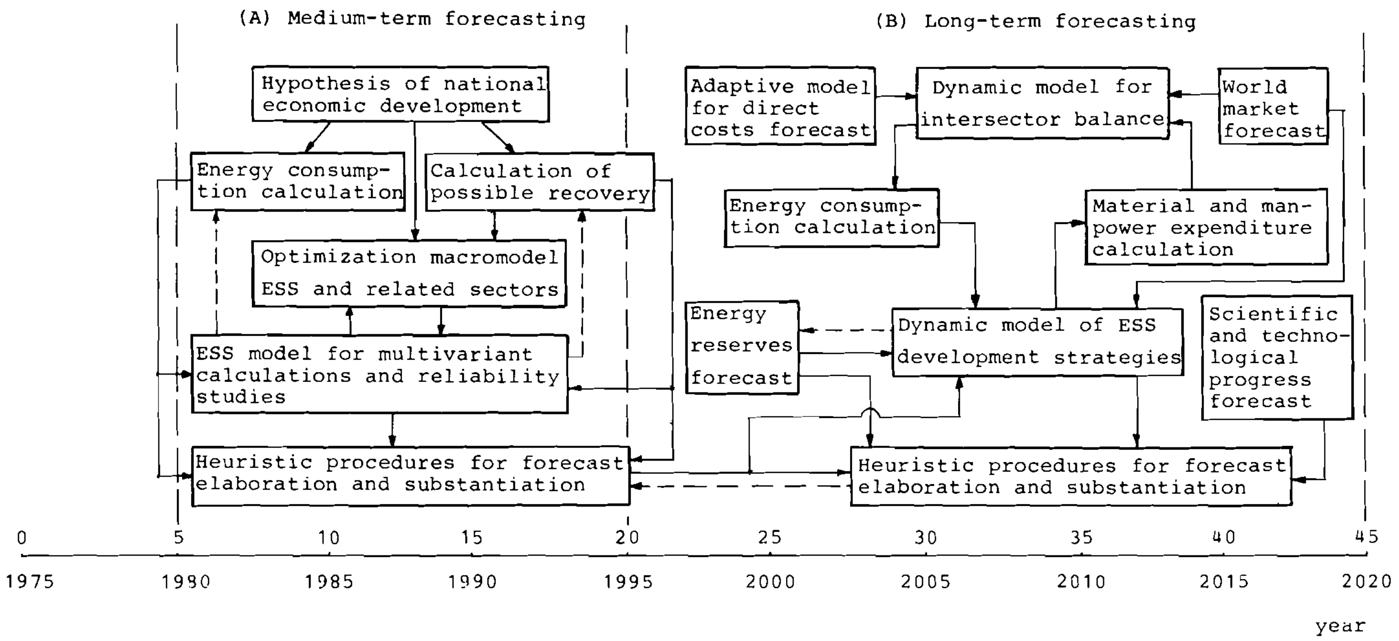
(A) Medium-term forecasting
(B) Long-term forecasting
Hypothesis of national economic development
Energy consumption calculation
Calculation of possible recovery
Optimization macromodel ESS and related sectors
ESS model for multivariant calculations and reliability studies
Heuristic procedures for forecast elaboration and substantiation
Adaptive model for direct costs forecast
Dynamic model for intersector balance
World market forecast
Energy consumption calculation
Material and manpower expenditure calculation
Energy reserves forecast
Dynamic model of ESS development strategies
Scientific and technological progress forecast
Heuristic procedures for forecast elaboration and substantiation
0
5
10
15
20
25
30
35
40
45
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
year

Figure 1.

The variants selected from the detailed optimization model are decoded enough to allow solution of the above problems of medium-term energy forecasting. The same model is also used in so-called adaptive conditions to check to what extent the decisions made (for the first decade of the studied period as a rule) can be adjusted to subsequent ESS development. This check enables the choice of the most reliable and economically effective variants capable of adapting to future conditions of ESS development at lowest possible cost.

Model calculations cannot of course cover all factors and conditions of ESS development (for example, the medium-term forecasting models consider ecological factors only by including those energy production and distribution modes evaluated by experts as ecologically acceptable ones). Therefore, the main forecasting should be carried out by skillful specialists who are capable not only of choosing heuristically the best trends in ESS development, but of formulating and substantiating the main advantages of the variant chosen.

## Long-Term ESS Development Forecasting

This should be based on results of the medium-term forecast and elaborated for a period of time into the future when the direct cause/effect relations with present decisions on energy development become less close, but when things are not so uncertain as to eliminate any possibility of formation, assessment and comparison of definite strategies for energy development. Preliminary estimates show the long-term forecasting horizon to be about 40 years. By this time (2020) the practical realization of all the most important technical innovations, such as thermonuclear energy, superconductivity, wave-guide transportation of energy, large-scale coal processing into high-grade fuel, production of hydrogen, etc. is expected. However, this is also the time by when, according to some forecasts, economic oil and natural gas resources will have been depleted. All this will cause a radical change in the energy sector. It is obvious that long-term forecasting techniques should provide both an analysis of possible changes, and the evaluation of the most preferable.

A simplified long-term forecasting scheme which is under way at the Siberian Power Institute is shown in B of Figure 1. The long-term forecast should not be based on assumptions used at the medium-term stage where the hypothesis of economy development, fuel reserves and technological-economic characteristics of new technologies are known. Here each of these factors is to be forecast and dependent on the strategy chosen.

The main problem of long-term forecasting is to interrelate the energy forecast and the economy macroforecast. The latter should meet the following three requirements: to lay a foundation for the energy demand forecast, to compare different strategies for energy development in terms of their economic importance, and to recognize the danger of the rate of national economic development being restrained by the energy sector and to avoid this.

To this end, a macroeconomic model with special orientation to long-term energy forecasting has been constructed. It is an aggregated dynamic model of intersector balance which defines the structure and rates of economic growth (for 6 to 8 sectors and with the manpower constraints) required for maximum growth of final consumption. The model is done as a stochastic one enabling results to be obtained as ranges of values dependent on the initial data variance.

The main problem of using this model is to determine the coefficients of direct material expenditures, labor- and fund-intensiveness and the final consumption. For this purpose a special adaptive model is offered. It is expected to be useful in adjusting the dynamic model coefficients to the real macroeconomic data used in the summary reports (from 1965 to 1975) and in planning (to 1990-1995), and thus in getting their dynamics for 25 to 30 years. This statistical basis may be used as a sufficiently reliable forecast of direct cost coefficients (with a proper aggregation) for the subsequent 20 years. This formal statistical forecast should naturally be corrected heuristically by such important facts as foreign trade (see the corresponding block in part B of Figure 1) and scientific and technological progress. The latter is taken into consideration for the energy sector as an interface between the economy macromodel and the forecast of energy development strategies.

The resultant dynamics of national income, gross product, and production for 6 to 8 aggregated sectors are used for the final energy consumption forecasting. The results are put into the dynamic model of energy development strategy evaluation comprising the central part of the forecasting method offered.

The dynamic ESS model and the ways of using it in long-term energy forecasting are discussed in detail elsewhere. The main goal of the model is to correlate the above considered variants of trends and rates in scientific and technological progress (elaboration of such variants is a final stage of a large-scale work on scientific and technological forecasting in the energy sector) as well as those of high-grade fuel reserve decrease (obtained from special geological predictions) to ESS development.

The main difficulty is that during the long-term forecasting period the USSR energy sector will be rather more than twice as large as at present. Therefore, the fuel-energy basis brought into being by the beginning of that period (1995) will be of primary importance for the whole period and particularly for the first decade. The changes caused by principally new technologies should obviously be introduced in the ESS with due regard for avoiding disproportions in its existing parts. At the same time, the above variants should be represented as being as productive as possible, enabling their economic efficiency to fall within the limits of the macroeconomic model.

To meet these requirements the dynamic model of energy development strategies is constructed as a conventionally optimizational one with a sharp fixation of two opposite dynamic tendencies, that of actual productive capacity removal and that of new technologies brought into action, with a variation in solid fuel recovery and processing as a "marginal layer".

Fairly different ESS development strategies identified and balanced by using the model are estimated by means of the macroeconomic model. The coefficients of material-, labor-, energy- and fund-intensiveness are calculated by each strategy and substituted for statistically forecasted indices in the macromodel. As these coefficient values are different for each of the energy strategies, the national economy dynamics, i.e. growth rates of the national income, gross product, and final consumption, will change accordingly. These values will be indicative of both the influence of the energy sector on national economic development as a whole and the comparative efficiency of different strategies for ESS development.

An experimental test (on reliable data) of certain models of the long-term forecasting scheme discussed is being completed. Much work has to be done to couple these models and to master numerous procedures for the results to be prepared and analyzed heuristically.

In conclusion, it should be emphasized that the present forecasting system is considered only as a helpful tool, from the computational standpoint, in the creative activity of people dealing with long-term energy development forecasting.

# Intersector Models for Long-Term Energy Forecasting

M.A. Gershenson

This paper discusses the problems of building the model for long-term energy forecasting described in the previous paper. The model is used for studying the influence of different variants of energy development on the rates and proportions of economic growth and conversely, the influence of concrete strategies for economic development on the dynamics of the most important energy characteristics. The time horizon for the forecast is limited to 40 to 50 years and is divided into two intervals, namely, middle-term forecasting (1975 to 1990) and long-term forecasting (1990 to 2010-2020).

We have more information for the first period, especially with regard to hypotheses for national economic development produced by the planning organizations and the economic institutes. It is desirable to have a special adaptive model for calculations at this time period, capable of adjusting to given national-economic indices: for example, national income, output volume of the most important sectors, etc. These calculations, in turn, provide the necessary information for the second period forecast. The second forecasting model used is *stochastic* and is based on results of calculations from the adaptive model and uses the maximum aggregated information from non-energy economy sectors.

## THE ADAPTIVE ECONOMY MODEL

When building this model it is necessary to take into account: intersector interrelations in dynamics; the uncertainty of future economic development; and the ability to adapt optimally to change.

The adaptive model is built on the deterministic intersector model which calculates a set of balanced indices for national economic development over the years (output by sectors, size and structure of capital investments, consumption funds, etc.). In the deterministic model three groups of sectors are stressed: the sectors--the product of which is used for needs of the current productive consumption and accumulation of circulating productive funds; fund-forming sectors; and sectors that produce consumer products.

Correspondingly, there are three groups of model equations, characterizing the distribution of sector products (Table 1). The balance equations for the group I sector are formed on the basis of the norms of prime material costs of the ith product for

Table 1. Relationships of the deterministic dynamic intersector model.

Equations of input-output (for year t):

(I) $$X_{it} = \sum_{j=1}^{n} a_{ijt}X_{jt} \quad , \qquad i = 1,2,\dots,n_1$$

(II) $$X_{kt} = \sum_{j=1}^{n} a_{kjt}X_{jt} + \sum_{j=1}^{n} Y_{kjt} \quad , \qquad k = n+1,\dots,n_2$$

$$Y_{kjt} = \max\left\{\frac{f_{kjt}X_{jt} - F_{kjt}}{l_{kjt}\beta_{kjt}} \ , \ a_{kjt}t_{kjt}X_{jt}\right\}$$

(III) $$X_{lt} = S_{l(t)}X_{l(t-1)} \quad , \qquad l = n_2+1,\dots,n$$

$$\begin{pmatrix} X_{l+1t} \\ X_{l+2t} \\ \vdots \\ X_{nt} \end{pmatrix} = Z_t \ .$$

Manpower constraints:

$$\sum_{j=1}^{n} C_{jt}X_{jt} = L_t$$

Transition to the next year:

$$F_{kj(t+1)} = F_{kjt} + l_{kjt}Y_{kjt} \quad , \qquad j = 1,2,\dots,n$$

$$k = n_1+1,\dots,n_2$$

Table 1. Relationships of the deterministic dynamic intersector model (concluded).

Where:

$C_j$ is the coefficient of labor-intensiveness of sector;

$a_{ij}$ is the coefficient of input-output;

$f_{kj}$ is the coefficient of fund-intensiveness of sectors;

$\beta_{kj}$ is the coefficient of uniformity of fund-production;

$l_{kj}$ is the coefficient of change in incomplete building;

$S_l$ is the growth rate of non-productive consumption;

$t_{kj}$ is the coefficient limiting the possibility of "transfer" of the kth funds from the jth sector;

L is the manpower volume; and

$F_{kj}$ is the funds at the beginning of the year in the jth sector.

Note: Export-import relations are omitted for simplification.

the output of product of the jth sector. The relationships for the fund-forming sectors (group II) that determine product distribution for compensation and increment of basic productive funds are formed from the coefficient of fund-intensiveness of the product, $f_{kj}$, the coefficient of uniformity of fund production, $\beta_{kj}$, and the coeffecent of change in incomplete building, $l_{kj}$. The balance equation for group III sectors is more simple:

$$X_{lt} = S_{lt}X_{l(t-1)} \quad , \tag{1}$$

where $X_{lt}$ is the production volume of the sector l in the year t of the studied period; and $S_l$ is the growth rate of non-productive consumption.

The permissible values for estimated indices are determined by the growth rates given in the model. The latter is a matrix whose number of rows is equal to the number of sectors in the studied group ($n_3$), and whose number of columns is equal to the levels of the scale (r) (Table 2).

Table 2. Growth rates of non-productive consumption.

| Sectors of output of consumer products | Scale levels | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | . . . | r |
| $n_2 + 1$ | 1.00 | 1.01 | 1.02 | . . . | 1.10 |
| $n_2 + 2$ | 1.01 | 1.03 | 1.04 | . . . | 1.15 |
| $n_2 + 3$ | 1.01 | 1.02 | 1.03 | . . . | 1.07 |
| . | . | . | . | . . . | . |
| . | . | . | . | . . . | . |
| . | . | . | . | . . . | . |
| n | 1.02 | 1.04 | 1.05 | . . . | 1.11 |

Each level (column) of the scale defines a set of possible growth rates. The required rates $\bar{S}_t = (S_{(n_2+1)t},...,S_{nt})$ are found by linear combination of any two adjacent levels of the scale. The considered scale is convenient in practice as it provides a simple and natural method for including indices for change in consumption in the model depending on the realized level of production and consumption.

Besides the balance equations for product distribution mentioned, the model takes into account manpower constraints and describes the transfer from indices for one year to indices for the next. As a result of calculation by means of the model, basic indices of dynamic intersector balance are obtained and these are coordinated with hypotheses of change in norms of material and labor expenditure, and available manpower consumption fund structure. The choice of growth rate, $S_{1t}$, within allowable limits gives the maximum possible growth rate of consumption fund.

In using the model, errors in determination of the gross product volume of the basic year, $X_o$ (observation errors) and of the volume and structure or the consumption fund, $Z_t$ (forecast errors) are inevitable. Errors in the determination of product volumes of sectors $X_t, t = 1,\ldots,T$, and other indices related to $X_t$ result from the previous errors.

The initial values of $X_o$ and of the "controls", $Z_t$, are not defined exactly, but are given with a range of possible values, based on expert assessment.

The uncertainty in the determination of $X_o, Z_t$ can easily be described in probabilistic terms. The principle of maximum entropy can be used to choose the law of distribution of $X_o$ and $Z_t$, which can be conditionally regarded as random vectors, determined within fixed intervals with known average values.

The probabilistic description of observation and forecast error results in the stochastic intersector model, in which possible trends of sector development correspond to trends in normative indices of material resource use.

With such assumptions we can define the mathematical expectation and error of the main estimated model indices; the correlation coefficients between dynamic sectors; and the forecast entropy, which can be interpreted as an indication of its reliability.

In order to pass from the stochastic model to the adaptive one, it is necessary to know the dynamics of some aggregated characteristics, $U_t$, of economic development, such as gross social product, national income, material- and capital-intensiveness of the social product, labor productivity, etc.

Formally, the adaptation is an optimum assessment of the estimated indices of the stochastic model, $X_t$, for given values of the vectors $U_1, U_2, \ldots, U_t$ and with the use of the relation

$$U_t = H_t X_t + W_t \quad , \tag{2}$$

where $W_t$ is a random vector simulating errors in the determination of $U_t$. The matrix structure H (of dimension $k \times n$, where k is the number of components of the vector U) is defined by the set of characteristics with respect to which the adaptation is carried out. So, for the case of adaptation to a reference index (national income), we have

$$H = (1 - \sum_{i=1}^{n} a_{i1}, 1 - \sum_{i=1}^{n} a_{i2}, \ldots, 1 - \sum_{i=1}^{n} a_{in}) \quad ,$$

and, in the case of adaptation to national income and gross social product, we have

$$H = \begin{pmatrix} 1 - \sum_{i=1}^{n} a_{i1}, 1 - \sum_{i=1}^{n} a_{i2}, \ldots, 1 - \sum_{i=1}^{n} a_{in} \\ 1 \quad , \quad 1 \quad , \ldots, \quad 1 \end{pmatrix} .$$

Using the results of filtering theory and the prediction of random processes, we can show (Gershenson, 1975) that $\hat{X}_{tT} = M(X_t/U_1,U_2,\ldots,U_T)$ is a linear solution (with respect to $U_t$) of the problem, i.e. it is a conditional mathematical expectation for all $X_t$ known realizations of the aggregated characteristics $U_t, t = \overline{1,T}$. The solution $\hat{X}_{tT}$ can easily be written in terms of the Kalman filter for t = T. The solution for t < T (usually called the smoothing problem solution) is based on calculating the vector $\hat{X}_{tt}$ and its subsequent refinement. There is no need for this refinement when $\hat{X}_{tt}$ provides a sufficiently good adaptation to the reference indices.

The adaptive solution $\hat{X}_{tt}$ differs from the deterministic model solution $X_t^d$ by

$$K_t[H_t\hat{X}_{tt} - H_tX_t^d] \quad ,$$

where the expression in square brackets is the discrepancy between the prescribed values of the reference indices and their calculated ones, and the filter $k_t$ is a matrix of regression factors $X_t$ from $U_t$. Adaptation within the studied model is essentially reduced to an inverse relation, whose coefficients are determined by the Kalman filter.

The stability of the deterministic dynamic model is determined by the value of the time step. Usually a year step is considered, but this is smaller than the time step for which the model becomes stable (Gershenson, 1975). From Miller's work (Miller, 1971) it is easy to show that variation in the filter parameters of the adaptive model by the introduction of weighting functions in the covariance error matrix of aggregated characteristics) can result in stability for any time step. This considerably improves the reliability of calculations with the model.

EXPERIMENTS ON THE ADAPTIVE MODEL

The adaptation scheme was carried out on the dynamic model for the 29 sectors that describe the development of the economy for 1965-1990.* We shall analyze adaptation to one reference index--national income. The annual rate of growth was taken to be 7%.

Experimental computations were carried out to estimate the effect of capital-intensive energy growth. Comparison was made first on the basis of an optimistic variant of economic growth, corresponding to an almost stable energy capital-intensiveness over time (the level of 1965). The second variant corresponded to a growth in energy capital-intensiveness (both for machines and equipment, and buildings and constructions) so that by 1990 capital-intensiveness of energy production was almost double that of 1965. This variant is supposed to characterize the existing tendencies in capital-intensive variations that are caused by transition to nuclear energy and by deterioration of fuel extraction and processing conditions.

Some comparative results of these forecast variants are given in Figure 1. A more capital-intensive variant of energy development results in a substantial reduction in growth rates of the economy's sectors producing consumer goods. The growth of agricultural output is reduced especially quickly--an absolute reduction in agricultural production is even possible in the 1980s.

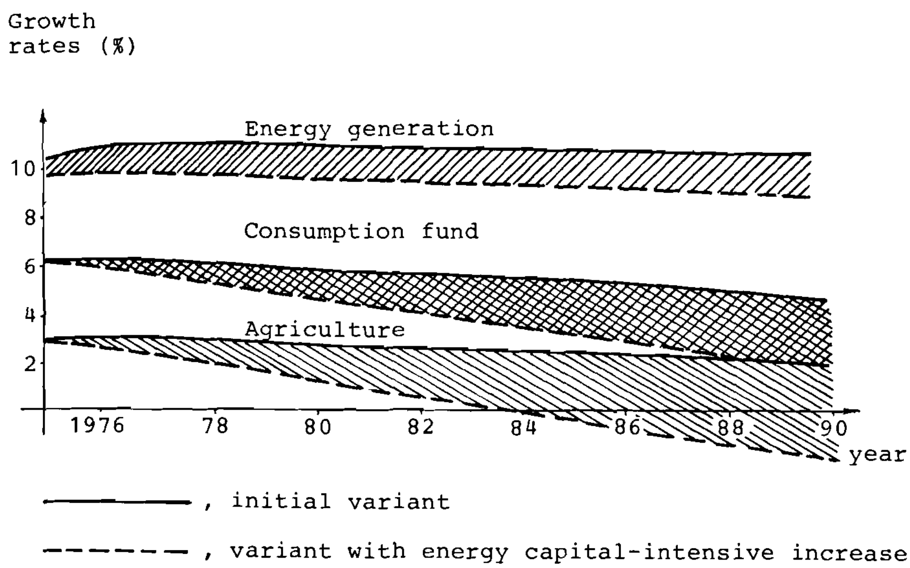


Figure 1. Effect of energy capital-intensive increase.

*All data given are used for illustration only as they are based on provisional information.

Growth of basic elements of the consumption fund slows down at sufficiently high and stable energy growth rates, proving once again that stable economic growth rates are provided by sufficiently high and stable energy growth rates. A high energy growth rate in the second variant is possible thanks to "escalation" of capital investments in the energy and related sectors, leading to the lack of production means in those sectors providing non-productive consumption. However, product growth rates in fund-forming sectors in the second variant are higher by 2 to 3%.

Secondly, structural analysis of the filter coefficients $k_t = (k_{1t}, k_{2t}, \ldots, k_{nt})$ helps to specify our knowledge of the adaptive properties of the economy (within the given model description).

If one reference index is taken into account, then

$$k_{it} = \left[h_{it}\sigma^2_{it} + \sum_{j \neq i} k_{ijt}\sigma_{it}\sigma_{jt}h_{jt}\right]\frac{1}{\sigma^2} \quad ,$$

where

$h_i$ is the coefficient of the relation $U = \sum_i h_i X_i + W$ (with adaptation to the national income dynamics $h_i = (1 - \sum_j a_{ji})$,

$\sigma^2_i$ is the variance in the determination of production volumes of the ith sector,

$k_{ij}$ is the correlation coefficient between the sectors i and j, and

$\sigma^2$ is the variance in determining the reference index.

The first addend is naturally interpreted as adaptation to variation within sector i, the second one $\sum_{i \neq j} k_{ij}\sigma_i\sigma_j h_j$ as adaptation caused by its relations with other sectors. Computations show that in the forecast period the proportion of the first addend for the energy sector increases from 17 to 49%. It confirms the feasibility of the intersector approach to energy forecasting inasmuch as the majority of variations in this sector are caused by relations with other sectors. In addition, increase in the proportion of $h_i\sigma^2_i$ in the total amount of filter coefficients enables one to suggest that at some time in the future the energy sector use will be forecast by means of a model with more aggregated representation of non-energy sectors.

Information received in the process of filter realization can be used to assess the accuracy of the main estimated indices of the adaptive model.

Thus, one can estimate the range of possible production volumes in sectors by the values of diagonal coefficients of the covariance matrix $\sigma^2_{it}$. If $X_{it}$ is approximated by a Gaussian random process (or by some other process satisfying "rule 3σ"), the error of the adaptive calculation is determined as follows

$$\Delta X_{it} = \pm \frac{3\sigma_{it}}{\hat{X}_{it}} \; 100\% \quad .$$

In this calculation (for 1990) production volumes of electrical and heat energy are given to within ± 4.3%, fuel industry ± 5%, agriculture ± 9%, and food industry ± 10%.

THE INTERSECTOR MODEL FOR LONG-TERM ECONOMIC FORECASTING

The development of the economy in the second time period (1990-2020) is forecast on the basis of the adaptive model calculations. In so doing, it is reasonable to aggregate the model over 7 to 8 sectors (for example, generation of electrical and heat energy, fuel industry, production of machines and equipment, raw materials, sectors of industry, building, agriculture, transport, communication, and material production) and to simplify description of the model normatives as much as possible. Thus, it is possible to modify the model so that it retains all normative coefficients of branches within the energy sector but aggregates normative coefficients of other sectors. The technique used is illustrated by the example of the coefficient of prime material expenditures, $a_{ij}$. For simplicity, suppose that in balance nomenclature the energy sector is represented by just one branch, the first. In the modified model the coefficients $a_{1j}$ and $a_{j1} (j = \overline{1,n})$ are retained and more aggregated normatives are introduced instead of the remaining coefficients, namely

$$\alpha = \frac{\sum_{j=2}^{n} X_{ij}}{\sum_{j=2}^{n} X_j} \, , \qquad i = 2,3,\ldots,n$$

($X_{ij}$ is the product flow from the sector i into the sector j).

Here the model remains the same and variation takes place only in the content of the normative matrix. Instead of the model with the normative matrix $\| a_{ij} \|_1^n$ we analyze the model with the following matrix:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \hline a_{21} & \alpha_2 & \alpha_2 & \cdots & \alpha_2 \\ a_{31} & \alpha_3 & \alpha_3 & \cdots & \alpha_3 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{n1} & \alpha_n & \alpha_n & \cdots & \alpha_n \end{pmatrix}$$

The latter compresses only (3n - 2) distinguishing coefficients, whereas in the initial model there are $n^2$ of them.

Formally, the problem of forecasting during the second time period can be formulated as the problem of determination

$$\hat{X}_{tT} \;, \qquad t > T \;,$$

where [0,T] is the adaptation interval.

Forecasting normative coefficients necessary for determining $\hat{X}_{tT}$ can be made on the basis of: extrapolation models that use the values of normative coefficients obtained by the adaptive model as initial information; or expert estimates of future technologies.

Subsequently, the main problem of developing the intersector model for long-term economic forecasting is to determine an admissible time horizon.

REFERENCES


Gershenson, M.A. (1975), *Analysis of Simplified Dynamic Models of Intersector Balance*, Nauka, Novosibirsk.

Miller, R.W. (1971), Asymptotic Behavior of the Kalman Filter with Exponential Aging, *AIAA Journal*, 3, 537-539.

Shatilov, N.F., and M.A. Gershenson (1971), *Methodical Aspects of Using Dynamic Model of Intersector Balance in the Summary of Long-Term National-Economic Planning*, Nauka, Novosibirsk.

## The Method for Elaborating Long-Term Energy Development Strategies

L.D. Krivorutsky, A.S. Makarova and A.A. Papin

The long-term energy development concept shown in Figure 1 of the paper by Makarov envisages a comprehensive economic assessment of different energy development strategies. For the period forecast (20 to 40 years) the intersector model of economic growth (the "economy model") considered in the previous paper is coordinated with the model for energy development described below.

A diagram of such coordination (Figure 1) shows two iterations.

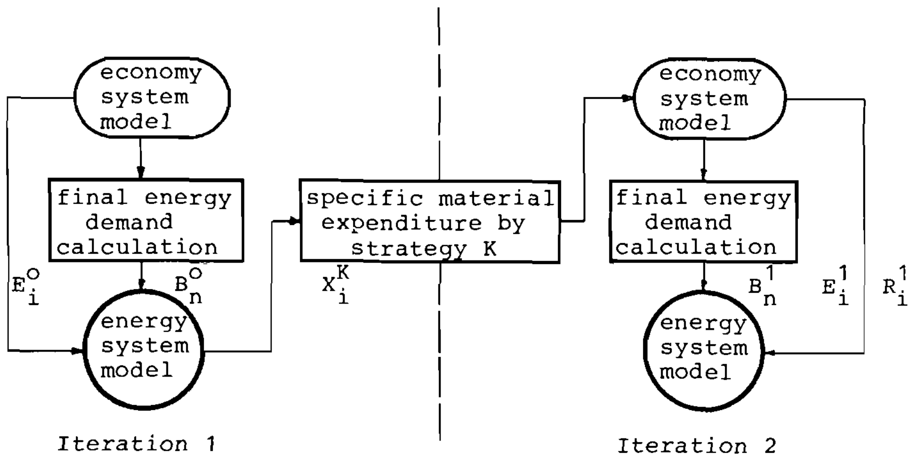


Figure 1.

<u>At the first iteration</u>, the economy model forecasts the national demand for final energy ($B_n^o$) and forms a set of normatives ($E_i^o$) that allow matching of the economic significance of different components in the energy sector expenditures, i.e. materials and manpower. It then forms a limited number of different development

strategies for the energy supply system (ESS) and relates energy sector requirements to those of the national economy corresponding to each strategy K. These requirements are the minimum amount of capital investment, material and manpower ($R_i^{KO}$).

At the second iteration (Figure 1), coordinated data on the updated demand for energy ($B_n^1$), on the set of matching coefficients ($E_i^1$), and on the possible ranges of materials and manpower used in the ESS ($\underline{R}_i, \overline{R}_i$) are transmitted from the economy model to the energy model.

These limit the region of ESS development to those actually possible for the national economy. A set of efficient variants of ESS development is determined in this relatively narrow region of the energy model, and then energy development is forecast from these variants by special analysis.

## OPTIMIZATION MODEL FOR ENERGY SECTOR DEVELOPMENT

The energy model is intended to realize the two most important goals of long-term forecasting: finding out the main trends of scientific and technological progress in the energy sector; and determining the need for extracting fuel and energy resources from extreme regions e.g. the northern areas of East Siberia and the Far East, in the sea shelves on the North of the country, etc.). Thus, the energy model has to simulate different trends for ESS development for the next 20 to 40 years and to choose the most preferable.

Energy models* constructed hitherto have simulated development of the productive part of the ESS, i.e. have included primary resource extraction, processing, conversion and transport. This allowed the optimization of (a) production from primary fuel and energy resources (FER), i.e. the size of fossil (oil, gas, coal) and mineral (uranium) fuel extraction, and nuclear and solar energy production; (b) conventional and new methods for extracting primary FER and for their transport to the consumers; (c) proportions of different energy carriers, namely electricity, steam and hot water, liquid, solid and gaseous fuel; and (d) volumes of primary FER processing and conversion into energy carriers by conventional and new methods.

All possible methods of extraction (production) and conversion of the FER are shown in Figure 2.

The proposed model describes both the production and consumption of energy. Analysis shows that large changes in either of

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*See for example Häfele, Manne (1974).

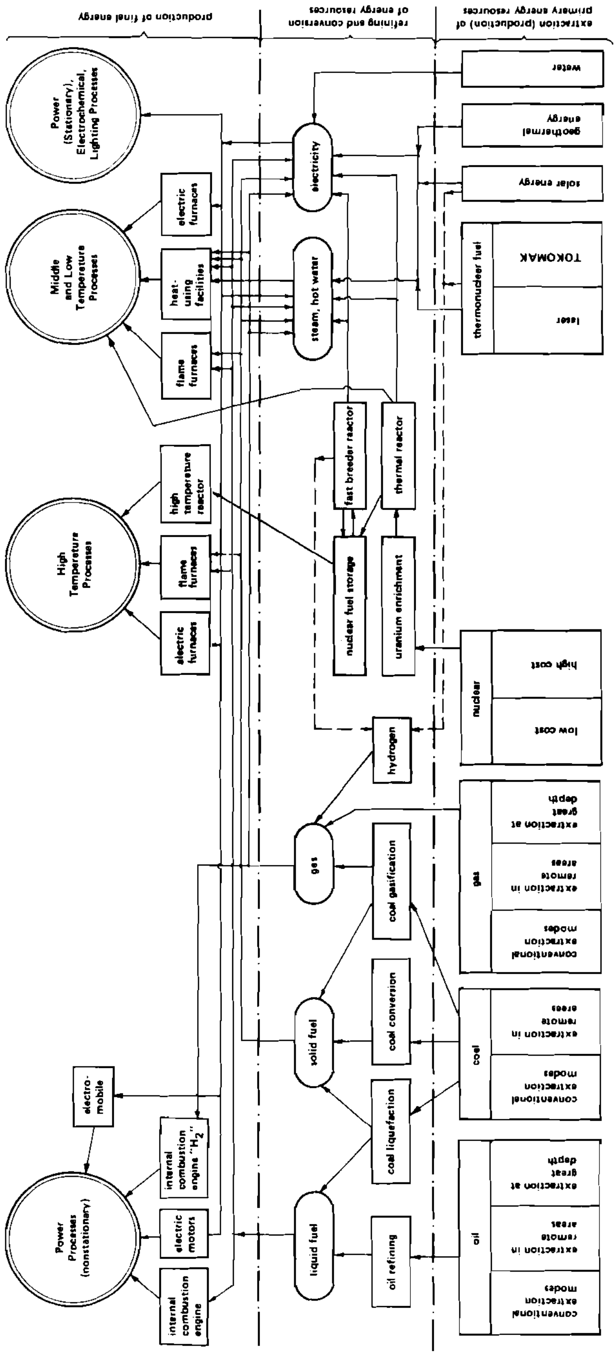
production of final energy
refining and conversion of energy resources
extraction (production) of primary energy resources
Power (Stationary), Electrochemical, Lighting Processes
Middle and Low Temperature Processes
High Temperature Processes
Power Processes (nonstationary)
electric furnaces
heat-using facilities
flame furnaces
high temperature reactor
flame furnaces
electric furnaces
electro-mobile
internal combustion engine "H2"
electric motors
internal combustion engine
electricity
steam, hot water
fast breeder reactor
thermal reactor
nuclear fuel storage
uranium enrichment
hydrogen
gas
coal gasification
coal conversion
solid fuel
coal liquefaction
liquid fuel
oil refining
water
geothermal energy
solar energy
thermonuclear fuel
TOKOMAK
laser
nuclear
high cost
low cost
gas
extraction at great depth
extraction in remote areas
conventional extraction modes
coal
extraction in remote areas
conventional extraction modes
oil
extraction at great depth
extraction in remote areas
conventional extraction modes

Figure 2.

these are equally important in affecting the national economy. For example, a sharp increase in the share of liquid and gaseous fuel as a primary energy source results in a variation in expenditure in materials, manpower and funds, which can be matched with the variations in the corresponding indices for the consumption part of the ESS (thanks to the increase in electrification).

The stage of final energy production in the model substantially improves long-term forecasting reliability. The structure of final energy consumption by processes depends basically on major national-economic indices (economic structure, growth of national income, etc.), whereas the demand for energy carriers depends, in addition, on the "inner" characteristics of the ESS (structure of the primary FER, comparative efficiency, etc.). Figure 3 shows approximate dynamics of energy consumption by low- and middle-temperature, high-temperature and power processes and

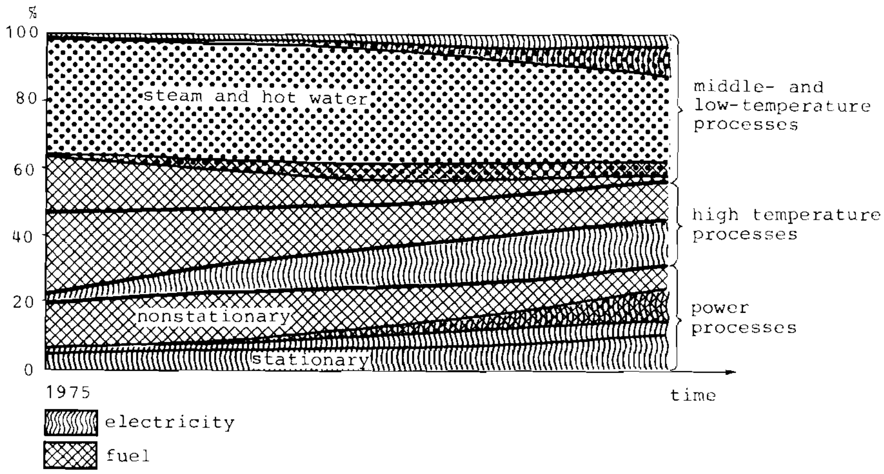


Figure 3.

gives an approximate structure of the meeting of this demand by different energy carriers. For example, power processes are divided into stationary and nonstationary ones, with choice of energy carrier for the first group being electricity while for the second group the advent of electric cars gives a choice between liquid fuel and electricity. The same situation can be observed for middle- and low-temperature processes where the basic heating of towns, the technological needs of industry, etc. will be effected by steam and hot water as previously; remaining consumers have a choice between fuel and electricity. Under some conditions (for example, cottages built in areas of relatively warm climate) hot water can be substituted for electricity.

Thus, each group of processes is characterized by having an obligatory part of the demand met by one type of energy carriers, and by a variable part which can reasonably be met by different ones. Figure 4 shows an approximate dynamic structure of energy

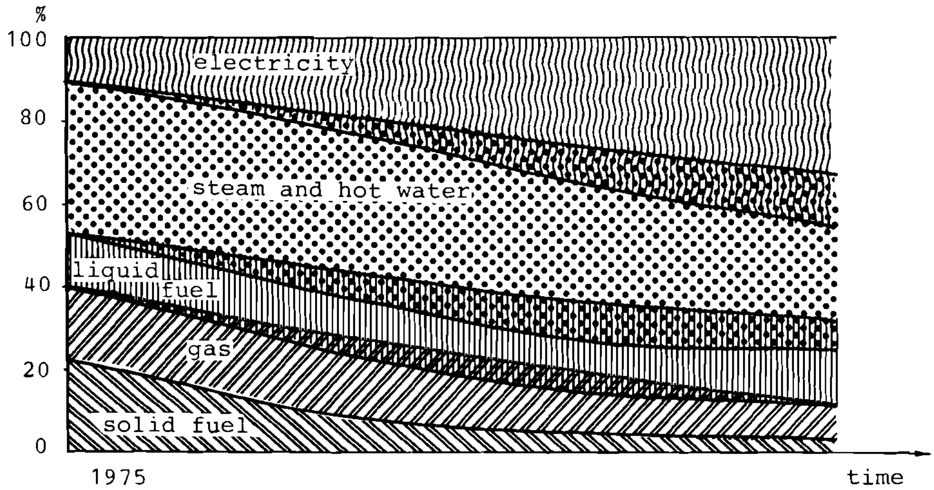


Figure 4.

carriers, obtained by summation according to their process of origin (see Figure 3). The obligatory part of demand is shaded by just one type of line, while the variable part has an overlapping of different types. For example, the upper zone of Figure 4 represents an obligatory demand for electricity; immediately below is the variable part of demand for electricity. Figure 4 shows that a sufficiently large part of demand appears to be variable during the forecast period, and can be covered by different energy carriers. Their comparative efficiency greatly depends on the inner structure of the ESS, and the optimization energy model should directly consider different ways of meeting the variable part of demand for energy ($Q_f$) (Figure 2).

A peculiarity of the model is the considerable modification of the energy development optimization criterion. It is rather dangerous to use only the criterion of minimum total discounted cost for medium-term forecasting in view of the uncertainty in prices for non-energy production. For long-term forecasting the use of a set of parameters for estimation and comparison of different energy variants becomes inevitable.

In view of this, the model uses a synthetic national-economic criterion which is a combination of expenditure for different material resources and manpower. To match these expenditures, instead of prices one can use a system of coefficients ($E_i$), obtained from the economy model by statistical analysis. These coefficients describe a specific impact of the energy sector on economic growth caused by the exhaustion of each type (i) of material and manpower.

In addition, the energy model is intended to consider ecological constraints such as the maximum permissible pollution of the environment by combustion products, water utilization, etc.

The proposed model is shown in Figure 5. It reflects (equations 1 to 3): (a) dynamics of ESS development by stages (t) of the studied period; (b) territorial interrelations; (c) the most important technological relations of the energy sector at different stages of production (extraction, processing conversion, transport) and consumption (primary energy resources, energy carriers and final energy) of energy. Simultaneously, the model takes into consideration the limited supply of such non-renewable energy resources as oil, gas, cheap uranium (equation (6)), ecological constraints (equation (4)); export-import interrelations, fixed as amounts of input and output of corresponding energy types; national-economic constraints on the volumes of materials and manpower used (equations (5) and (7)).

The functional of the model is derived as follows:

$$\sum_{i}^{I} E_i X_i \rightarrow \min$$

where $E_i$ is the coefficient that matches the value of the ith national-economic resource with the remaining resources, and $X_i$ is the total demand of the ESS for the appropriate resource (i).

The following degree of detail for dynamic, territorial and technological interrelations are used for problems solved by the energy model. The whole forecast period (20 to 40 years) which is divided into four time stages (t), each equal to five years. The territory of the country is divided into two zones: Siberia and the remaining regions of the USSR. Here Siberia is considered solely as a source of cheap fuel and energy transported in one direction from Siberia. The model indirectly takes account of transport within each of these zones--important in considering new production modes which require removal of enterprises from developed areas. Production methods shown in Figure 2 are considered at every technological stage. The energy model includes approximately 200 equations and 500 variables. It is formed as a general linear programming problem.

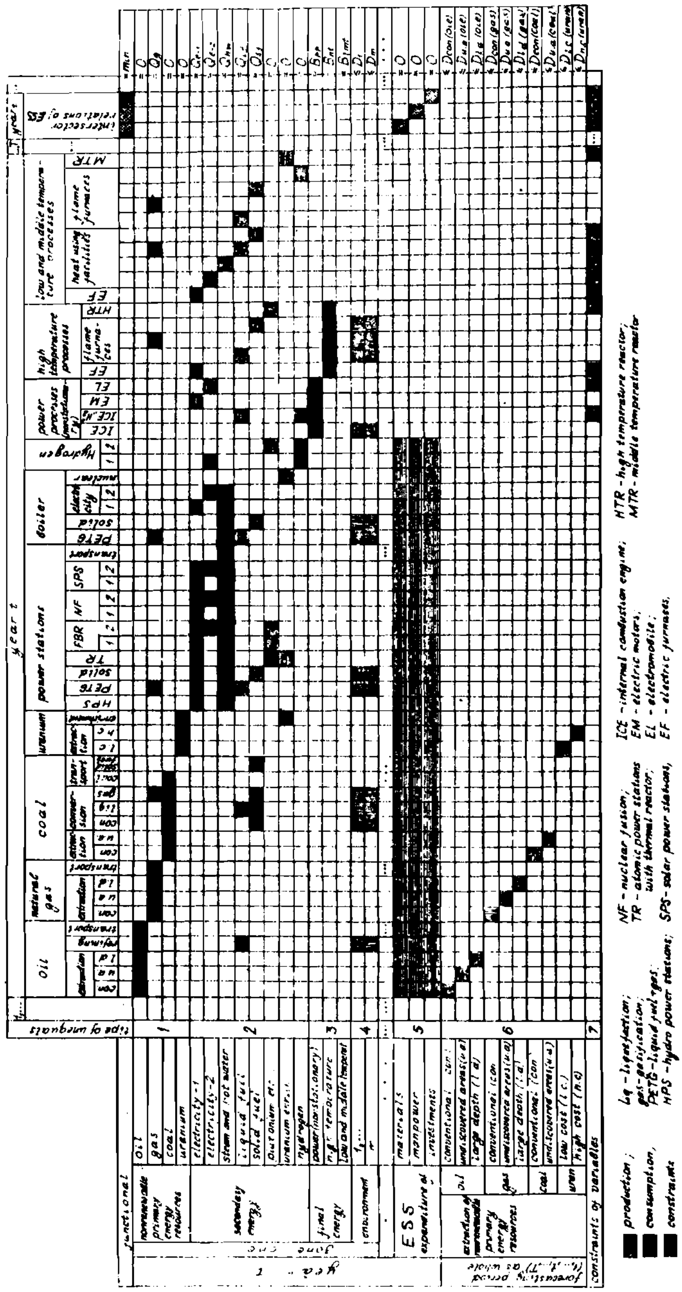

Figure 5.

## THE MODEL USE FOR THE FORMATION AND COMPARISON OF ENERGY DEVELOPMENT STRATEGIES

Energy carriers (Figure 2) are a main part of the technological chain of energy production and conversion, and it is feasible to proceed from the production structure of energy carriers to form different energy development strategies. For this, all possible combinations of the most efficient production methods for energy carriers must be established and proper development levels of the remaining parts of the technological energy production chain must be determined. Obviously, less efficient production methods for energy carriers will be considered only after the possibilities of the first group have been exhausted, i.e. they will be considered as marginal.

It should be emphasized that for the forecast period the USSR energy sector is characterized by a relatively small number of efficient production modes of energy carriers. Liquid and gaseous fuel is reasonably expected, as earlier, to be produced from oil and natural gas but not artificially (for example, by gasification, hydrogenation and hydrogen production); oil processing methods will be basically the same as at present. Therefore, efficient methods of liquid and gaseous fuel production will differ mainly in the method of extraction of the corresponding primary FER, namely by conventional methods, from undeveloped areas, or at large depth.

Without doubt, solid fuel can be considered as a marginal energy carrier in our country, and its method of production will depend on prevailing energy development conditions.

The largest changes are expected in the production technology of electrical and heat energy (steam and hot water), for example, development of nuclear power stations with fast breeder reactors, thermonuclear and solar power stations.

Thus, the number of combinations of efficient production methods for energy carriers will be relatively small. Table 1 gives different combinations obtained from minimum and maximum values of production volume for the appropriate energy carriers. For example, combination 1 corresponds to the maximum production of liquid and gaseous fuel, with preference being given to the development of fast breeder reactors while the share of thermonuclear and solar stations remains minimal. Marginal (minimum and maximum) production sizes of every energy carrier are estimated tentatively by forecasting the trends of technical progress in the energy sector and machine building, as well as by geological forecasts. Figure 6 gives tentative ranges in production volumes of liquid and gaseous fuel, nuclear and thermonuclear energy, expected in the USSR energy sector for the forecast period.

Although the number of combinations of efficient production methods for energy carriers is small (Table 1), each of them is affected by the volume of consumed material resources and manpower which depends on the method of extraction (production) of the

Table 1. Possible combinations of efficient methods for energy carrier production as a basis for forming energy development strategies.

| Production volumes | | | Combination of energy carrier production modes | | | | | |
|---|---|---|---|---|---|---|---|---|
| natural liquid and gaseous fuel | electricity, steam and hot water | | 1 | 2 | 3 | 4 | 5 | 6 |
| much | nuclear power stations | much | X | | | | | |
| | | little | | X | X | | | |
| | thermonuclear power stations | much | | X | | | | |
| | | little | X | | X | | | |
| | solar power stations | much | X | X | X | | | |
| | | little | X | X | | | | |
| little | nuclear power stations | much | | | | X | | |
| | | little | | | | | X | X |
| | thermonuclear power stations | much | | | | | X | |
| | | little | | | | X | | X |
| | solar stations | much | | | | X | | X |
| | | little | | | | X | X | |

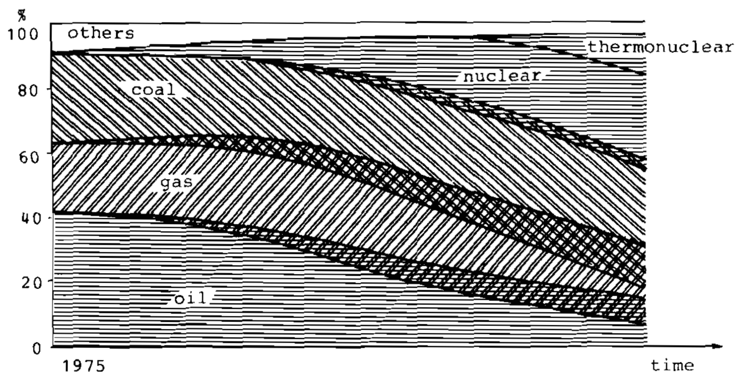


Figure 6.

corresponding primary FER. For example, material and manpower expenditures for the ESS will distinctly differ in combination 2 (Table 1) depending on the mode of thermonuclear energy production (by lasers or by TOKOMAK) and the type of national-economic expenditure for each. Therefore, the energy model forms strategies, the number of which can considerably exceed the number of combinations of efficient production methods for energy carriers. To form appropriate strategies, the energy model gives both the production volumes of liquid and gaseous fuel and generation volumes of electricity, steam and hot water by three new types of electric power stations (nuclear power stations with fast breeder reactors, thermonuclear and solar stations), corresponding to the combinations 1-6 in Table 1, and the expected technological-economic indices of the main methods of extraction (production) of the corresponding primary FER. Simultaneously, it automatically solves other problems of ESS development: (a) determination of volumes and methods of solid fuel production; (b) production volumes of all energy carriers by inefficient ways; and (c) trends in energy carriers for the different production processes (for the variable part of demand); etc.

The energy development strategies obtained are assessed from a national economy standpoint by the economy model where all inefficient strategies are eliminated (Figure 1). The ESS development forecast is made at the second iteration on the basis of the remaining strategies. For this purpose, the energy model first of all considers updated values of matching coefficients ($E_i^1$) and corresponding constraints on material resources and manpower ($\underline{R}_i^1, \bar{R}_i^1$) obtained from the economy model. The latter limit the possible domain of ESS development to a great extent, and so, the main task of the energy model at the first iteration is a complete and detailed investigation of this.

It is necessary to choose a limited number of sufficiently representative conditions of energy development, for example, the assumed volumes of material resources and manpower used, the expected reserves of such high-grade fuel types (of different economic efficiency) as oil and natural gas. For different combinations* of values of these factors "uncontrolled" within the energy sector, optimal trends of ESS development are determined by using the optimization model. The set of solutions obtained can easily be grouped in a relatively small number of ESS development variants, as they have a sufficiently large difference in composition and dynamics of such important technical innovations as: nuclear power stations with fast breeder reactors; thermonuclear power stations; solar power stations; production of synthetic oil; production of synthetic gas; and production of hydrogen.

*See Makarov, Melentiev (1973).

This set of development variants is then used as a basis for forecasting the future ESS. However, the long-term energy forecast itself requires a versatile and informal analysis along the following lines.

First, a preliminary estimate of each variant preference is made by informal anlysis. Taking into account the necessity of adjusting variants to any combination of external ("uncontrolled") conditions, we give preference to the two or three variants of energy development that can be optimal for the greatest number of conditions (for example, variants 1 to 3 in Table 2). Those variants that are efficient at very rare combinations of "uncontrolled" factors, such as reserves of natural resources and the possibilities for the national economy to single out material resources and manpower for the energy sector, are rejected as unpractical (for example, variants 4-N in Table 2). Obviously, all efforts

Table 2. The method for preference assessment of each ESS development variant.

| Variant | combinations of "uncontrolled" factors, simulating ESS development conditions | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | x | | 0 | | x | 0 | | | x | 0 | | 0 | 0 | | 0 | x |
| 2 | 0 | | x | | 0 | | x | | | x | 0 | | | 0 | | |
| 3 | | 0 | | 0 | | X | | x | | 0 | x | | | x | 0 | |
| 4 | | x | | 0 | | | | 0 | | | | | 0 | | | |
| 5 | 0 | | | x | | | | | | 0 | | | | | | 0 |
| . | | | | | | | | | | | | | | | | |
| . | | | | | | | | | | | | | | | | |
| . | | | | | | | | | | | | | | | | |
| N | | | | | | | | | | 0 | | 0 | x | | | |

Note: x, conditions when the corresponding variants are optimal; 0, conditions of rather easy adaptation of the corresponding variants.

should now be directed at scientific and technological improvement of the innovations included in the two or three variants chosen. Similarly, preparations for developing those fuel deposits (including those in extreme regions) that are needed for these variants should be made.

Secondly, the main similarities in ESS development for the period under consideration are determined for the remaining variants. They are:

(a) The absolutely efficient or inefficient technical innovations that make up (or do not make up) a sufficiently large part in all considered variants;

(b) Typical substitutions of the remaining technical innovations that can be efficient only under certain conditions of ESS development;

(c) The variation in marginal technological-economic indices for every technical innovation, where competitive with conventional modes of energy production.

REFERENCES

Häfele, W., and A.A. Manne (1974), *Strategies for a Transaction from Fossil to Nuclear Fuels*, RR-74-7, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Makarov, A.A., and L.A. Melentiev (1973), *Methods of Energy Economy Study and Optimization*, Nauka, Novosibirsk.

# A Non-Linear Regression Model for Energy Consumption Forecasting

V.R. Elokhin

According to the long-term energy forecast (see Figure 1 in the paper by Makarov), the main link between the intersector economy development model and the optimization energy development model is the energy consumption forecasting model. This model has to formalize the objective regularities of energy consumption increase with accepted strategies of national economy development, that is, to approximate functional relations between the dynamics of the main macroeconomic indices and indices used in energy consumption forecasting.

The composition of these macroeconomic indices is defined by Gershenson in his paper on the intersector economy growth model. Besides the dynamics of national income ($X_1$), it includes gross national product dynamics of the most energy-consuming sectors of the national economy, namely, metallurgy ($X_2$), machine building($X_3$), transport ($X_4$) and possibly agriculture ($X_5$).

The characteristics of energy consumption are forecast by the optimization model of energy development. As shown by Gershenon, it is necessary to know the dynamics of final energy demand broken down into the main energy-consuming processes and directions of energy use in accordance with Table 1.

For long-term forecasting the following values of energy consumption indices are assumed: (a) accounting and planned values of the macroeconomic indices ($X_1,X_2,...,X_5$) and energy consumption characteristics ($Y_1,Y_2,...,Y_{13}$) from which numerical values for the functional relations between them are determined; (b) forecast values of the macroeconomic indices from which the energy consumption forecast is obtained by functional relations. Here the number of value realizations {X} and {Y}, that is, the "training" period, should be considerably longer than the duration of the forecast period itself. For instance, for the period 2001 to 2020 we take realizations {X} and {Y} to determine the required relation parameters for a period from 1960 to 1975, and their values in 1976 to 2000 are determined by quite exact calculations.

A distinguishing feature of this forecasting method is its systems character. The task is not to forecast each of the values

Table 1

| | Energy-consuming processes | | | |
|---|---|---|---|---|
| | Middle- and low-temperature | High temperature | Non-stationary power | Stationary power, electrochemical and lighting |
| Directions of energy use | $Y_1$-total | $Y_5$-total | $Y_9$ -total | $Y_{13}$-total |
| | $Y_2$-heating | $Y_6$-metal melting | $Y_{10}$-rail and road transport | |
| | $Y_3$-food cooking | $Y_7$-metal and product heating | $Y_{11}$-motor transport | |
| | $Y_4$-material and manufacture drying | $Y_8$-product burning | $Y_{12}$-building and agricultural mechanisms | |

$Y_1, Y_2, \ldots, Y_{13}$ separately, but as a system of interrelated (correlated) variables. In addition, the dependent variables of other equations are used as independent variables of equations that stipulate the interdependency equations. Thus, the given model represents not a choice but a system of interdependent regression equations.

The relations between variables {X} and {Y} have a complicated non-linear character, that is, every equation of the model is non-linear and is of the form:

$$Y_{ij} = X_o\theta^i_o + X_{j1}\theta^i_1 + \ldots + X_{jm}\theta^i_m + X_{j1}X_{j2}\theta^i_{12} + \ldots$$

$$+ X_{jm-1}X_{jm}\theta^i_{(m-1)m} + Y_{jK_1}\theta^i_{mm} + \ldots$$

$$+ Y_{jK_{l-1}}\theta^i_p = \sum_{\gamma=o}^{m} X_{j\gamma}\theta^i_\gamma + \sum_{\gamma\leq K}^{m} X_{j\gamma}X_{jK}\theta^i_{\gamma K}$$

$$+ \sum_{\gamma\neq i}^{l-1} Y_{j\gamma}\theta^i_\gamma \quad , \qquad (1)$$

where

$i = 1,2,\ldots,l$ is the number of values of the forecast variable Y;

$j = 1,2,\ldots,n$ is the number of realizations of the values $\{X\}$ and $\{Y\}$;

m is the quantity of macroeconomic indices X;

$p = C_{m+2} - m + 1 - 1$ is the number of terms in equation (1); and

$\theta$ is an estimation of parameters in equation (1).

The type of polynomial (1) was chosen: (a) as practice has shown that second order polynomials describe many complex economic processes with sufficient precision, and (b) with a view for a more objective analysis of economic phenomena and forecast results. In this case it is necessary to have an efficiency estimate only of paired interactions between macroeconomic indices (variables X), since higher order interactions cannot be described.

The method of building mathematical models, which can be used for long-term forecasting of energy consumption, is now considered. Mathematically, the problem of forecasting consists in the search for relations over time between dependent (endogenous) and independent (exogenous) variables. These can be described in general by some function

$$Y/X = \eta(X) \quad , \qquad (2)$$

where $Y^T = \| Y_1, Y_2, \ldots, Y_l \|$ is a vector of correlated endogenous variables with the values of exogenous variables determined by the vector coordinates X; the function $\eta(X)$ depends on the unknown parameters $\theta_1, \theta_2, \ldots, \theta_p$.

Furthermore, every vector $Y_t$ and $Y \in T$ of the set is uniquely compared with the $X_t (X \in \chi)$, namely:

$$X_t \sim Y_t \quad , \qquad (t = 1,2,\ldots,n) \qquad (3)$$

where $\sim$ is the conformity sign.

A procedure for searching for the best linear estimations of equations of the type (1) is now proposed: it is distinct from any known in the statistical literature.*

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*See for example Fedorov (1971); and Nalemov, Techernova (1965).

Suppose that the relation between endogenous and exogenous variables can be described by some function

$$Y/X = \eta(X,\theta) = \| v_1^T f(X),\dots,v_1^T f(X) \|^T \quad . \tag{4}$$

We determine the error matrix for endogenous variables $Y_1$, $Y_2,\dots,Y_1$

$$D = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \cdots & \sigma_{11} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{21} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{11} & \sigma_{12} & \cdots & \sigma_{11}^2 \end{bmatrix} \tag{5}$$

We consider the so-called structural matrix of dimension $n \times p$

$$A = \begin{bmatrix} 1 & z_{11} & z_{12} & \cdots & z_{1P} \\ 1 & z_{21} & z_{22} & \cdots & z_{2P} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & z_{n1} & z_{n2} & \cdots & z_{np} \end{bmatrix} \tag{6}$$

the elements of which are exogenous variables $X_1, X_2, \dots$ and their paired interactions $X_1X_2, X_1X_3, \dots$ (see equation (1)), reduced to the normalized form.

We assume we have a block-diagonal matrix S of dimension $(n \times 1) \times (p \times 1)$:

$$S = \begin{bmatrix} A_1 & & 0 \\ & A_2 \ddots & \\ 0 & & A_1 \end{bmatrix} \tag{7}$$

and matrices L of dimension $(n \times 1) \times (p \times 1)$

$$L = \begin{bmatrix} \sigma_{11}^2 I & \sigma_{21} I & \cdots & \sigma_{21} I \\ \sigma_{21} I & \sigma_{22}^2 I & \cdots & \sigma_{21} I \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{11} I & \sigma_{12} I & \cdots & \sigma_{11}^2 I \end{bmatrix} \tag{8}$$

where I is the unit matrix of dimension (n × n), and $\sigma_{ij}$ are elements of the matrix D.

If the introduced notation and structural properties of the matrices S and L are taken into account, approximate estimates of the coefficients in equations of type (I) can be obtained by the formula:

$$\Theta = M^{-1}U \quad , \tag{9}$$

where the matrix $M^{-1}$ is non-singular:

$$M^{-1} = \| S^T L^{-1} S \|^{-1} \tag{10}$$

and

$$U = \| S^T L^{-1} Y \| \quad . \tag{11}$$

In order to obtain the best estimates, an iterative process of specifying their values until they are consistent, unbiased, and efficient, is organized. For example, unbiased estimators $\tilde{\Theta}$ are effective in the case of the inequality

$$D(\tilde{\Theta}) \leq D(\tilde{\tilde{\Theta}}) \quad , \tag{12}$$

where $D(\tilde{\Theta})$ is a variance matrix of the estimators $\tilde{\Theta}$, and $D(\tilde{\tilde{\Theta}})$ is a variance matrix of any other unbiased estimators $\tilde{\tilde{\Theta}}$. In expression (12), the variance matrix of the estimators $\Theta$ is

$$D(\Theta) = M^{-1} \tag{13}$$

Condition (12) can be satisfied if instead of the matrix D at the first iteration we use its estimation:

$$d(\Theta) = n^{-1} \sum_{i=1}^{n} [Y_i - v_i^T f(X)][Y_i - v_i^T f(x)]^T \quad . \qquad (14)$$

All subsequent iterations are also realized, with account taken of expression (14). Experiments show that the best linear estimations $\Theta$ can be obtained in 2 or 3 iterations.

The following experiment (Figure 1) was carried out in order to check the efficiency of the proposed forecasting method and the algorithm for obtaining the best linear estimates.

For three macroeconomic indices, namely, national income, gross production of industry and of agriculture, accounting and planned values (solid lines in Figure 1a) were received, and estimates for their change during the forecast period accepted (dashed lines). Corresponding finite energy requirements, namely, total and the three main types of processes (power, high- and low-temperature), are also presented in Figure 1b. The accounting and planned values are again shown here by solid lines and energy consumption dynamics, obtained by a direct calculation from specific energy consumption standards and production forecasts of 70 types of national economy products and services, are shown by dashed lines.

In this way, from accounting and planned values of macroeconomic indices and from energy consumption, the coefficients (parameters $\theta$) of the system of non-linear regression equations that describe a functional relation between given values were determined. Then two forecast values for macroeconomic indices were substituted and the forecast values of energy consumption characteristics under consideration were computed.

As in any stochastic model, the functional relation parameters and, consequently, the values of forecast indices can be obtained as confidence regression intervals. In Figure 1b they are presented as shaded regions, in which the "true" average value of each (endogenous variable) is within the range

$$\hat{Y} \pm t_\alpha S \sqrt{X_p'(X^T X)^{-1} X_p} \quad , \qquad (15)$$

where

$X_p^T = \| X_1, X_2, \ldots, X_m \|$ is the vector of forecast values of macroeconomic indices with their paired interactions;

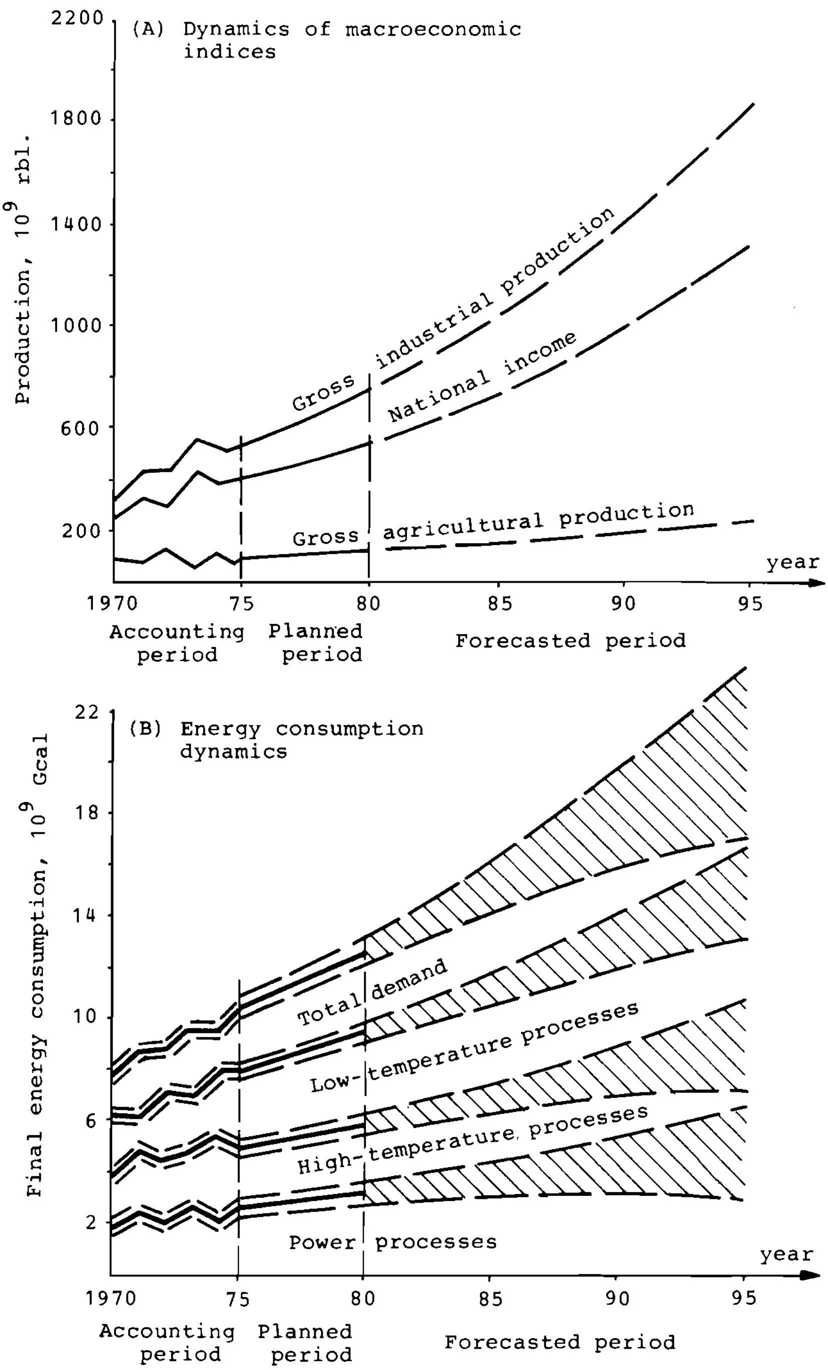


**Figure 1. Presentation of energy consumption forecast with the help of nonlinear regression model.**

$t_\alpha$ is the table value of the t-criterion for the given confidence probability (generally equal to 95%);

S are the diagonal elements of the matrix $d(\Theta)$.

The results obtained show a good accuracy for the proposed energy consumption forecast method: it enables functional relations to be taken with a precision of up to 1% by restriction to a limited set of realizations (about twenty) sufficient for forecasting purposes; the ranges of values of forecast indices that were obtained by the method agreed with results obtained by more detailed and laborious methods of direct calculation of demand.

Note. It is obvious that with the large number of exogenous and endogenous variables under consideration, say $l > 10$ and $m > 10$ ($p = 56$), the matrix sizes considerably exceed the computing possibilities of the working memory of a high class digital computer. This problem can be solved in two ways:

- Forming the inverse matrix M by dividing it into sub-matrices (Frobenious formula 13).*

- Reducing the size of this matrix by using the random balance method,** which eliminates a small number of significant factors and their interactions (paired in our case) in a noise field.

## REFERENCES

Federov, V.V. (1971), *Optimal Experiment Theory*, Nauka, Moscow.

Nalimov, V.V., and N.A. Tchernova (1965), *Statistical Method of Planning Extremum Experiments*, Nauka, Moscow.

Sirl, S., and U. Gosman (1974), *Matrix Algebra in Economy*, Statistika, Moscow.

*See for example Sirl, Gosman (1974).
**See for example Nalimov, Tchernova (1965).