OPTIMIZATION OF RURAL-URBAN
DEVELOPMENT AND MIGRATION

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Preface

Representatives from 132 nations assembled in Vancouver
in June of 1976 to convene HABITAT, the United Nations Confer-
ence on Human Settlements. The Conference was a global inquiry
into solutions of the critical and urgent problems of human
settlements created by the convergence of two historic events:
unprecedently high rates of population growth and massive rural
to urban migration.

Rapidly growing populations strain health and education
budgets, complicate efforts to utilize efficiently a nation's
manpower, and exacerbate problems connected with the provision
of adequate supplies of food, energy, water, housing and trans-
port and sanitary facilities. A better understanding of the
dynamics and consequences of population growth, particularly
with regard to resource and service demands, is therefore an
essential ingredient for informed policymaking.

The Human Settlements and Services Area at IIASA is de-
veloping a new research activity that is examining the principal
interrelationships between population, resources and growth. An
important ingredient in such an examination is the study of the
impact of urban-rural migration on agricultural and nonagricultural
production. The problem of the optimal allocation of labor be-
tween these two sectors is considered in this paper, and a formula
for determining an optimum migration policy is derived.

Andrei Rogers
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Papers of the Population, Resources and Growth Study

1. Nathan Keyfitz, "Understanding World Models," RM-77-18,
        April 1977.

2. Andrei Rogers, "Migration, Urbanization, Resources and
        Development," RR-77-14, July 1977.

3. Roman Kulikowski, "Optimization of Rural-Urban Development
        and Migration," RM-77- 41.

Abstract

The paper deals with rural-urban migration and the impact of migration on regional development. In order to derive the optimal migration strategy, the general problem of optimal allocation of production factors in time and space is introduced. Describing the regional economy by a generalized Cobb-Douglas production function, the general problem is decomposed into two levels. Using the generalized Hölder inequality at the first level, optimal allocation of factors in time is solved in an explicit form. At the second level, a spatial strategy is derived and the principle of spatial allocation of production factors is formulated. Using the optimal strategies, the simple two-sector (i.e. agriculture and the rest of the economy) model was investigated, and the labor surpluses in Polish agriculture and in an agricultural region in Poland were calculated.

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1. INTRODUCTION AND FORMULATION OF THE BASIC OPTIMIZATION PROBLEM

Rural-urban migration usually includes agricultural labor
which becomes employed in the non-agricultural sector of the econ-
omy. In this case, the migration processes have a direct effect
on production structure and regional development. It shall be as-
sumed in this paper that the total labor supply in the rural-urban
region to be analyzed is predetermined by demographic factors.
When mass migration starts (sometimes called the "Migration Revolu-
tion" [7]), the supply of labor is greater in rural areas than
urban centers, while the demand for labor is greater in industrial
centers, located mainly in towns.

The main question asked by regional development planners is:
what is the optimal allocation of labor to produce the greatest'
acceleration of regional growth? Planners feel that when they
discover the optimal allocation of labor, they can also deter-
mine the labor surplus in agriculture which can be regarded as
a labor reservoir for urban growth. The transfer of the labor
surplus to non-agricultural sectors is, however, difficult, and
it involves social costs. The social costs are due to additional
housing programs, creation of new jobs, change in the traditional
agricultural economy, increased environment protection programs,
etc. [3]

Due to high housing and urban development costs in many
countries, a large group of so-called "commuting" migrants exists,
who live in the country and work in the cities. They often spend
two to four hours a day commuting. The lost working time and the
increased transportation costs (aggravated by energy crises) re-
present a heavy burden to the economy.

In order to determine optimal policies regarding development
and migration in each particular region, cost-benefit analyses
should be carried out on the regional level. In particular, one
would like to know whether it is better to improve transportation,
which in turn increases the number of commuting migrants; or to
increase the number of commuting migrants; or to increase the

housing construction in the cities, encouraging outmigration; or to use the capital in the region to build factories, thereby employing the labor surplus.

To obtain an answer to this question, it is advisable to use macroeconomic concepts based on the allocation of production factors in the neoclassic two-sector models [1, 2, 6, 7]. In these models, migration depends on the labor employed and labor productivity, in agriculture and in the rest of the economy. We shall, however, try to extend these concepts in such a way that differences in all the factors, per worker, can be taken into account.

Since regional development is a complex process which takes place in time and space, we would like to find the optimal development and migration strategies as explicit functions of time and location. Such an approach requires a more general formulation of the classical problem of allocation of production factors. In the classical macroeconomy, the production function ϕ is usually assumed to be a concave (homogenous of degree one), differentiable function of production factors, say $F_\nu, \nu = 1 \ldots m,$[*] i.e., the output Y can be written

$$Y = \phi [F_1 \ldots F_m] . \tag{1}$$

The values of Y, F_ν, $\nu = 1 \ldots m$, describe the aggregated and averaged (usually within one year) processes which take place in time and space.

A more general expression for the production function,

$$y(s,t) = \phi [s,t,f_1(s,t), \ldots f_m(s,t)] , \tag{2}$$

includes the time (t) and location (s) variables. In the last case it is convenient to regard y, f_ν, as production and factor intensities. For example, the agricultural production intensity

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* The index $\nu = 1$ is usually reserved for labor, $\nu = 2$ for capital, while $\nu = 3 \ldots m$ may represent land, education, etc.

depends on the geographical location, s, and time of the year, t.

A typical example of (2) is the generalized Cobb-Douglas production function

$$y(s,t) = A(s,t) \prod_{\nu=1}^{m} [f_\nu(s,t)]^{\beta_\nu}, \tag{3}$$

where

$$\sum_{\nu=1}^{m} \beta_\nu = 1, \quad \beta_\nu > 0, \nu = 1, \ldots m, A(s,t) = A(s)e^{\mu t},$$

μ = positive coefficient representing the so-called neutral progress,

$A(s)$ = given positive function of location s.

Another, widely used, is the C.E.S. production function, which in our notation can be written:

$$y(s,t) = A(s,t) \left\{ \sum_{\nu=1}^{m} \beta_\nu [f_\nu(s,t)]^\varepsilon \right\}^{1/\varepsilon}, \tag{4}$$

where $\sum_{\nu=1}^{m} \beta_\nu = 1, \quad 0 \le \varepsilon \le 1$

In the class of n production factors we shall assume that at least one factor, say $f_1(s,t)$, is not mobile (exogenous) i.e. cannot be transferred between the individual production processes. Capital is usually regarded as mobile, while land is not. Some factors, e.g. labor and water, can also be regarded as mobile at additional (transportation) costs.

It is assumed that the total amount of production factors, integrated over the planning interval (O,T) and the region R, is bounded, i.e.

$$\int_R ds \int_0^T e^{-\lambda_\nu t} f_\nu(s,t) dt \le F_\nu \quad \nu = 1 \ldots m. \tag{5}$$

We shall also assume that R is a closed convex set in two-dimensional Euclidean space, and that F_ν, λ_ν, $\nu = 1 \ldots m$ are given. The

exponential functions $e^{-\lambda_\nu t}$, $0 \leq \lambda_\nu$, describe possible discounting of the initial values of production factors over time.

The basic regional optimization problem can be formulated as follows:

Find the non-negative strategies $f_\nu(s,t) = \hat{f}_\nu(s,t)$, $\nu = 2\ldots m$, $s \in R$, $t \in [0,T]$, which maximize the regional integrated product

$$Y = \int_R ds \int_0^T e^{-\lambda t} y(s,t) dt, \quad \lambda = \mu + \sum_{\nu=1}^m \lambda_\nu \beta_\nu, \quad (6)$$

subject to the total resources constraints (5), $\nu = 2\ldots m$.

It should be observed that it is possible to formulate a discrete version (in time and space) of (5) and (6)· This is a matter of convenience·

In order to solve the (5) (6) problem in time and space, we shall apply the following decomposition method:

I. At the first (local) level, assume s to be fixed, and find the dynamic strategies

$$f_\nu(s,t) = \hat{f}_\nu(s,t), \quad \nu = 2\ldots m, \quad t \in [0,T], \text{ which maximize}$$

the product of $e^{-\lambda t}$ and $y(s,t)$, integrated over time,

$$\bar{Y}[s,\underline{f}] = \int_0^T e^{-\lambda t} y(s,t) dt, \quad \underline{f} \overset{df}{=} \{f_\nu(s,t)\}_2^m, \quad (7)$$

subject to time-integrated constraints

$$\int_0^T e^{-\lambda_\nu t} f_\nu(s,t) dt \leq F_\nu(s), \quad \nu = 2\ldots m,$$
$$\text{and} \quad (8)$$
$$f_\nu(s,t) \geq 0, \quad t \in [0,T], \quad s \in R.$$

II. Compute:

$$F_\nu(s) = \int_0^T e^{-\lambda_\nu t} \hat{f}_\nu(s,t) dt, \quad \nu = 2\ldots m, \text{ and}$$

$$\bar{Y}[s,\underline{\hat{f}}] = \hat{Y}[s,\underline{F}], \quad \underline{F} \overset{df}{=} \{F_\nu(s)\}_2^m.$$

III. Find at the second (spatial) level, the static strategies: $F_\nu(s) = \hat{F}_\nu(s)$, $\nu = 2 \ldots m$, which maximize:

$$Y = \int_R \hat{Y}[s,\underline{F}]ds, \tag{9}$$

subject to spatial constraints

$$\int_R F_\nu(s)ds \leq \bar{F}_\nu, \quad F_\nu(s) \geq 0 \qquad \nu = 2,\ldots, \quad s \in R. \tag{10}$$

In other words, we solve the problem of allocation of resources at the local level in planning interval [O,T]. Then using the local solution (in which the time variable has been depressed) we can concentrate on allocation of resources over the space R. From the formal point of view, the main question to be asked is the following: will the time-space decomposition method yield the solution which is equal to the solution of the original problem (5) (6)?

It can be shown (see Ref. [5]) that both solutions are equivalent when some regularity conditions are imposed on the functionals (5), (6). For that purpose, one can use the generalized (for Banach spaces) Weierstrass theorem. Since we shall deal with strictly concave, continuous functionals, (such as (6), where $\sum_{\nu=2}^{m} \beta_\nu < 1$) on the compact set defined by (5), the regularity conditions can be assumed to hold.

It should also be noted that factor levels $f_\nu(s,t)$ generally depend on factor endowment intensities $z_\nu(s,t)$ (usually given in monetary terms), which can be described by the integral operators (ref. [5])

$$f_\nu(s,t) = \int_{-\infty}^{t} e^{-\delta_\nu(t-\tau)} [z_\nu(s, t-T_\nu)]^\alpha dt, \quad \nu = 2\ldots m, \tag{11}$$

where $0 < \alpha \leq 1$, T_ν, δ_ν = positive constants.

A typical relation of the general form (11) is the relation between investment intensity $[z_2(\tau)]$ and capital stock $[f_2(t)]$; where T_2 = construction delay and δ_2 = capital depreciation rate.

It is necessary to notice that the maximization of GRP (6) can hardly be regarded as the universal objective of regional development. It shall be shown, however, that the optimal allocation of resources, which follows from the solution of the problem (3), (5) and (6) also yields maximum consumption per worker in the planning interval [0,T].

2. OPTIMAL DEVELOPMENT STRATEGIES: CONTINUOUS CASE

Starting with the production function (3), let us concentrate on solving the problem at the first level. To simplify the notation, we shall for now neglect the variable s in (7), (8), (3). We shall also assume that labor is not mobile so $f_1(t)$ will be regarded as a given exogenous variable.

Taking into account the analytical form of (7), (3), and (8), one can use the Hoelder inequality. This becomes an equality when the factors rise in constant proportion, i.e.:

$$f_\nu(t) = \hat{f}_\nu(t) = \frac{F_\nu}{F_1} e^{(\lambda_\nu - \lambda_1)t} f_1(t), \quad \nu = 2 \ldots m \quad (12)$$

(for details, see the appendix).

The relation (12) has been called the "_principle of factor coordination_". According to that principle capital, education, R & D etc. should change along with the exogenous factor, (e.g. labor), in fixed proportions. As shown in the appendix, the principle holds also for the C.E.S. function (4). Factor coordination can be used to derive the $\hat{z}_\nu(t)$ expenditures (at the national level),

$$\hat{z}_\nu(t-T_\nu) = \{\frac{\beta_\nu}{\beta_1} \frac{\omega_1}{\omega_\nu} [\delta_\nu f_1(t) + f_1'(t)]\}^{\frac{1}{\alpha}}, \quad \nu = 2 \ldots m, \quad (13)$$

where $\omega_\nu, \nu = 1 \ldots m$ are prices attached to factors F_ν (see appendix).

It is assumed that the prices, ω_ν, satisfy the monetary balance

$$\sum_{\nu=1}^{m} z_\nu(t-T_\nu) \leq y(t).$$

It is also interesting to observe that, due to delays T_ν, $\nu = 2\ldots m$, the expenditures connected with investments, educations, etc., $z_\nu(t)$, $\nu = 2\ldots m$, should precede the employment $f_1(t)$. This is shown in Fig. 1 for $f_1(t) = a_0 + a_1 t$, where a_0, a_1 are given constants, $\alpha = 1$, and

$$\hat{z}_\nu(t-T_n) = \frac{\beta_\nu \omega_1}{\beta_1 \omega_\nu} [a_0 \delta_\nu + a_1 + \delta_\nu a_1 t].$$

As shown in the appendix (Theorem I), the optimal factor allocation strategy, which maximizes Y, is equivalent to the strategy of maximizing consumption per head.

Now we can solve the problem at the second level, (9), (10). It can be shown (compare A,2) that under the strategy $\underline{f} = \underline{\hat{f}}$, the output $\hat{Y}(s,\underline{F})$ becomes:

$$Y(s,\underline{F}) = [G(s)]^{\beta_1} \prod_{\nu=2}^{m} [F_\nu(s)]^{\beta_\nu}, \quad G(s) = [A(s)]^{\beta_1^{-1}} F_1(s).$$

$$(14)$$

According to (9), (10) we have to derive $F_\nu(s) = \hat{F}_\nu(s)$, $\nu = 2\ldots m$ which maximize

$$Y = \int_R [G(s)]^{\beta_1} \prod_{\nu=2}^{m} [F_\nu(s)]^{\beta_\nu} ds$$

subject to

$$\int_R F_\nu(s) ds \leq F_\nu \qquad (15)$$

$$F_\nu(s) \geq 0, \quad \nu = 2\ldots m$$

Using the Hoelder inequality, one gets:

$$Y \leq \left\{\int_R G(s) ds\right\}^{\beta_1} \prod_{\nu=2}^{m} \left\{\int_R |F_\nu(s)| ds\right\}^{\beta_\nu} =$$

$$= \left\{\int_R G(s) ds\right\}^{\beta_1} \prod_{\nu=2}^{m} F_\nu^{\beta_\nu},$$

where the equality sign appears iff

$$F_\nu(s) = C_\nu \, G(s) , \quad C_\nu > 0 , \quad \nu = 2 \ldots m$$

Finding the unknown coefficients C_ν by (15) one obtains

$$C_\nu = \frac{F_\nu}{\int\limits_R G(s) ds} , \quad \nu = 2 \ldots m$$

Now we can formulate the following theorem, which may be called the "principle of spatial-factors coordination".

Theorem 2: Let $G(s)$ be a given, integrable function. The optimum spatial allocation of production factors (for the problem (9), (10), (14) exists:

$$\hat{F}_\nu(s) = G(s) \frac{F_\nu}{\tilde{F}_1} , \quad \tilde{F}_1 = \int\limits_R G(s) ds, \quad \nu = 2 \ldots m \qquad (16)$$

and is unique.

Using (12) one can also find $\hat{f}_\nu(s,t)$ strategies

$$\hat{f}_\nu(s,t) = [A(s)]^{\beta_1^{-1}} e^{(\lambda_\nu - \lambda_1) t} f_1(s,t) \frac{F_\nu}{\tilde{F}_1} , \quad \nu = 2 \ldots m \qquad (17)$$

Formula (16) can also be used for the case when labor is regarded as mobile, while another factor (e.g. land) is immobile (exogenous). In that case, the optimum spatial allocation of labor should follow the exogeneous factor change (e.g. the land productive efficiency).

It should be observed that solution (16) can also be used for the model which shown how the density of population, $D(s)$, is related to the environmental factors specified by $G(s)$. Assuming that the utility function U, representing social preferences of inhabitants with respect to environment, has a form similar to (14), i.e.:

$$U = \int\limits_R G^q(s) \, D^{1-q}(s) ds, \qquad (18)$$

and assuming the total population in area R to be bounded, i.e.,

$$\int_R D(s)\,ds \leq P, \qquad (19)$$

one gets by (16) the optimum strategy of population allocation:

$$\hat{D}(s) = \frac{\tilde{G}(s)}{\int_R \tilde{G}(s)\,ds}\,P. \qquad (20)$$

When, as usually happens, $G(s) \neq \tilde{G}_s$, the existing density of population does not coincide with the best allocation of population (from the economic point of view). An important problem for regional planning is to create environmental and living conditions such that $G(s) = \tilde{G}(s)$, that is the utility-maximizing strategy should coincide with the production-maximizing strategy. In general this involves many expenses, so other solutions should be considered as alternatives, e.g. (rapid transportation, better housing, industrial pollution abatement programs, etc.)

3. OPTIMAL REGIONAL ALLOCATION OF PRODUCTION FACTORS: STATIC, DISCRETE CASE.

Instead of dealing with continuous strategies, it has become customary (at the regional level) to deal with models which are static in time and discrete in space. We shall, therefore, discuss the discrete version of strategy (16). We shall also regard labor as mobile at some additional transfer costs.

Generally speaking, the supply and demand for production factors in different regions may be different and may change in time. It is possible, however, to transfer some production factors between regions at the expense of additional costs. As a typical example, consider a two-sector, two-region system. The first region, R_1, represents the rural part of the country with agricultural production, while the second region, R_2, represents the urban part of the country and the rest of the economy. During the industrialization period, the demand for labor at R_2 is greater than at R_1, while the supply at R_1 is greater than at R_2. At the

same time, the labor efficiency at R_2 is greater than at R_1. The migration of labor from R_1 to R_2 is hampered by high costs of housing, urbanization, etc., and in many countries an antimigration policy is adopted. In order to find out what the best government policy in migration should be, one should take into account the losses due to non-efficient allocation of labor, and migration costs. The best migration policy corresponds to the minimum value of the resulting loss function.

In order to derive the optimal allocation strategy, assume at the moment that all factors are mobile and that interregional transfer costs are not involved. The production function of a plant belonging to sector i, location in the region j, can be written in the monetary form (A.10),

$$\Delta Y_{ij} = G_{ij}^q \prod_{\nu=1}^{m} Z_{ij\nu}^{\delta_\nu} \ , \quad q = 1 - \sum_{\nu=1}^{m} \delta_\nu, \quad \delta_\nu = \alpha\beta_\nu. \tag{21}$$

In dealing with the allocation of production factors, it is convenient to introduce a three-level optimization structure. At Optimization Level I resources are allocated among sectors and it is necessary to find strategies $Z_{ij\nu} = \hat{Z}_{ij\nu}$, $i = 1,\ldots,n$, $\nu = 1,\ldots,m$, which maximize the regional (R_j) production

$$\Delta Y_j = \sum_{i=1}^{n} \Delta Y_{ij} \ ,$$

subject to

$$\sum_{i=1}^{n} Z_{ij\nu} \leq Z_{j\nu}, \quad \nu = 1,\ldots,m$$

Using the discrete version of (16), it can be shown that

$$\hat{Z}_{ij\nu} = (G_{ij}/G_j)Z_{j\nu} \ , \quad \nu = 1,\ldots,m, \tag{22}$$

where

$$G_j = \sum_{i=1}^{n} G_{ij}$$

and

$$\Delta Y_j(\hat{\underline{z}}) = G_j^q \prod_{\nu=1}^{m} z_{j\nu}^{\delta_\nu} \tag{23}$$

At Optimization Level II, resources are allocated among regions and it is necessary to find $z_{j\nu} = \hat{z}_{j\nu}$, $j = 1,\ldots,r$, $\nu,\ldots,m$, which maximize

$$\Delta Y = \sum_{j=1}^{r} \Delta Y_j, \tag{24}$$

subject to $\quad \sum_{j=1}^{r} z_{j\nu} \leq Z_\nu$, $\nu = 1,\ldots,m.$ $\tag{25}$

The optimal strategies according to the discrete version of (16), become

$$\hat{z}_{j\nu} = (G_j/G) Z_\nu, \quad \nu = 1,\ldots,m, \qquad j = 1,\ldots,r \tag{26}$$

and

$$\Delta Y(\hat{\underline{z}}) = G^q \prod_{\nu=1}^{m} z_\nu^{\delta_\nu}, \qquad G = \sum_{j=1}^{r} G_j \tag{27}$$

At Optimization Level III, the allocation of resources is as follows: $\hat{z}_\nu = \beta_\nu Y$, where Y can be determined from (A.11).

The formula in (27) expresses the G.N.P. gain under the assumption of full factor mobility.

In order to find the optimal allocation of labor in a two-sector, two-region system, assume the total labor supply L to be given:

$$L = L_1 + L_2,$$

and find by (26) the optimum labor allocation

$$\hat{L}_1 / \hat{L}_2 = {}^{G}1 / {}_{G_2} \tag{28}$$

Assume also that

$$Y_i = G_i^q \ L_i^{\beta 1} \ \prod_{\nu=2}^{m} \ z_{\nu i}^{\ \beta_{\nu i}}, \ q = 1-\beta \ -\beta_1, \ \beta = \sum_{\nu=2}^{m} \ \beta_{\nu 1} =$$

$$= \sum_{\nu=2}^{m} \ \beta_{\nu 2}, \ i = 1,2$$

Using (28) one finds

$$\frac{\hat{L}_1}{L_1} : \frac{\hat{L}_2}{L_2} = \left\{ \frac{Y_1 / L_1}{Y_2 / L_2} \cdot \frac{\prod\limits_{\nu=2}^{m} \left(\dfrac{z_{\nu 2}}{L_2}\right)^{\beta_{\nu 2}}}{\prod\limits_{\nu=2}^{m} \left(\dfrac{z_{\nu 1}}{L_1}\right)^{\beta_{\nu 1}}} \right\}^{1/q} \tag{29}$$

Assuming sector $i = 1$ represents agriculture, and sector $i = 2$ the rest of the economy, so that

$$\hat{L}_1 = L_1 - L_s, \quad \hat{L}_2 = L_2 + L_s \tag{30}$$

where L_s = surplus of labor in the agriculture, equal to the shortage of labor in non-agricultural sectors, one can write

$$\frac{\hat{L}_1}{L_1} : \frac{\hat{L}_2}{L_2} = \frac{1-x}{1+ax} ,$$

where $\quad x = \dfrac{L_s}{L_1} , \quad a = \dfrac{L_1}{L_2} .$

In the simple case , $m = 1$, $q = 0.5$, one gets from (29), (31):

$$\frac{1-x}{1+ax} = y^2 , \tag{31}$$

where $y = Y_1/L_1 : Y_2/L_2$, is the ratio of labor productivities (in agriculture to the rest of the economy).

According to statistical data on the Polish economy for 1970-75 (see Ref [8], the ratio of labor productivities, y, was decreasing from 0.33 to 0.24, while L_1/L_2 was (0.625÷0.529). In Fig. 2, the graph of the function $x(y)$ for $q = 0.5$, $a = 0.529$ has been shown. The surplus of labor in 1970 was around 4.5×10^6 people.

Using formula (31) it is also possible to find the surplus of labor at the regional level. Since labor can migrate over the whole country, it may be assumed that $a_r = a$ ($a_r = L_1^r/L_2^r$) i.e. that a_r at the regional level is the same as that for the whole country. Using this assumption, and data regarding the labor productivities at the micro-level, labor surplus can be determined in a relatively simple way. In particular, such an approach has been used to determine the labor surplus in Drobin County, a typical rural area in the central part of Poland. The total number of inhabitants was 9990. The labor productivity ratios were around 0,9 ÷ 0.85, for the individual villages , this corresponds to $x = 0.15 ÷ 0.85$ and is illustrated by the map shown in Fig. 3.* Differences in productivity depend largely on the size of privately owned farms. In Drobin, the great surplus of labor can be explained by the extremely small farms. The farmers, however, take advantage of bus transportation which enables them to commute to work in the industrial towns of Plock, Warsaw, etc.

It should be observed that the total migration from rural to urban areas per year in Poland is around $150.10^3 ÷ 245.10^3$, so it will take 20 - 25 years to transfer the whole of surplus labor. The explanation for the small migration figure is simple. Mass migration involves costs connected with housing, urban services, training, environmental protection costs, etc. Some of these costs are connected with additional nonproductive investments.

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*The calculations were performed by W. Kulikowski.

Therefore, the optimal migration should be chosen in such a way that the loss function Δ, which takes into account the losses due to non-efficient allocation of labor and migration costs, $\bar{c}L_s$, is minimal. In the simple model $m = 1$, $q = 1-\beta$:

$$\Delta = G^q L^\beta - G_1^q (L_1 - L_s)^\beta - G_2^q (L_2 + L_s)^\beta + \bar{c}L_s =$$

$$= G^q L^\beta \left[1 - (\frac{G_1}{G})^q (\frac{L_1}{L} - x)^\beta - (\frac{G_2}{G})^q (\frac{L_2}{L} + x)^\beta + cx \right]$$

$$(32)$$

where $G^q L^\beta = \hat{Y}$, $c = \bar{c}x$, $x = L_s/L$.

The loss function can be easily constructed using statistical data for the Polish economy (Ref. [8]). Assuming $\beta = 0.5$, one gets (for 1970):

$$L_1/L = 0.3848 \qquad L_2/L = 0.6152$$

$$G_1/G_2 = (Y_1/Y_2)^{0.5} (L_2/L_1) = 0.0687322$$

$$G_2/G = (1 + G_1/G_2)^{-1} = 0.935688 = \hat{L}_2/L$$

$$G_1/G = 0.0643118 = \hat{L}_1/L .$$

Then $\Delta/\hat{Y} = 1 - 0.2535977(0.3843 - x)^{0.5} +$
$$- 0.9673096(0.6157 + x)^{0.5} + cx.$$

Unfortunately, there is not much statistical information available regarding the cost, c. The investment costs which were estimated in [3] should, in this case, be discounted and averaged over the planning interval and assigned to the operating costs. The cost function for $c = o$, and $c = 0.15$, which yields the optimum, $\hat{x} = 0.25$, has been drawn in Fig. 4.

It is possible to show that in the case of commuting migration the cost function is nonlinear. A simple model can be constructed in which time lost and cost of transportation can be derived in an explicit form. Assume for this purpose that city C uses a transportation system which delivers commuting labor, spread with density D over sector ABC, with radius R (see Fig. 5).

The number of commuters X within ABC becomes

$$X = \int_0^R D\,ds = \int_0^R D\,\theta\,r\,dr = aR^2, \quad a = \frac{\theta}{2}\,D\ .$$

Assuming the transportation cost to be c_t (per km, per person), the transportation cost C_t of X becomes:

$$C_t = \int_0^R c_t r D\,ds = b_t R^3, \quad b_t = \frac{c_t \theta D}{3}\ .$$

Assuming the transport velocity to be v km/h, the time taken to travel v km is $\frac{r}{v}$. Let the cost of 1h per worker be c_h. Then the cost C_h of time lost during transportation is

$$C_h = \int_0^R \frac{r}{v} c_h D\,ds = b_h R^3, \quad b_h = \frac{c_h \theta D}{3}\ .$$

Since $R = \sqrt{X/a}$ one gets

$$C = C_t + C_h = AX^{3/2}, \quad A = b/a^{3/2}, \quad b = b_t + b_h\ .$$

In the case that commuting migration prevails, which may happen for small X, one should deal with convex cost functions. A value X_o exists such that for $X > X_o$ the commuting costs are greater than migration costs, and it is more profitable to let the migrants settle in urban centers.

The present statistical model does not tell us what the best migration strategy is, as a function of time. We shall study that problem in the next section, assuming that labor resources, integrated over [O,T], are given.

It follows from (29) that the migration necessary for maximum gain depends largely on the ratio of labor productivities $Y_1/L_1 : Y_2/L_2$ and $(K_1/L_1) : (K_2/L_2)$, i.e. on capital allocation.

In general the allocation model of governmental expenditures on education, services, housing, environment (characterized by Z_ν, ν = 3,...m) also plays an important role. Data on transfer costs of corresponding services are, however, seldom available. When the relation (29) is used ex-ante in the planning interval [0,T], one can also find the best labor allocation, on condition that the remaining factors are allocated in a optimal fashion, i.e. according to the strategy (26).

4. OPTIMUM REGIONAL ALLOCATION OF RESOURCES, THE DYNAMICAL, CONTINUOUS CASE

Since the labor surplus in a particular region can be determined (by the method described in Sec. 3), one can concentrate on dynamic optimization with given labor and capital resources. In that case, one would like to achieve the fastest possible regional industrial growth, assuming the labor supply intensity $x(t)$, to be constrained in the integral sense within the planning interval [0,T]:

$$\int_0^T w_1(t)\ x(t)\ dt \leq Z_1 \tag{33}$$

Where Z_1 = total labor cost (wages), $w_1(t)$ = given weight or wage function. We shall be interested in two production factors only, labor and capital stock, assuming that the investment $z(t)$ is constrained by

$$\int_0^T w_2(t)\ z(t-T_2)dt \leq Z_2 \tag{34}$$

where Z_2 = total investment costs, $w_2(t)$ = given weight function. When new investments are financed out of a bank loan, with

interest rate η, it is natural to assume $w_2(t) = (1 + \cdot)^{T-t}$.
Since the migration cost, c, is partly connected with new invest-
ments, i.e. $c = z_\mu$ (in particular, housing can be regarded as ac-
companying the productive investments z), one can write $z_\mu = az$,
$a = \text{const}$. The production function for the model investigated can
be written in the form (3), (11):

$$\Delta y(t) = A[x(t)]^\beta \left\{ \int_0^t \exp[-\delta(t-\tau)] \, [z(\tau-T_2)]^\alpha \, d\tau \right\}^{1-\beta} \tag{35}$$

The problem of dynamic optimization of regional development
which faces us can be formulated as follows:

Find the negative strategies $x(t) = \hat{x}(t)$, $z(t) = \hat{z}(t)$, $t\epsilon[0,T]$,
such that the regional contribution to G.N.P.,

$$\Delta Y(x,z) = \int_0^T \exp(-\lambda t) \, \Delta y(t) \, dt \tag{36}$$

attains a maximum, subject to the constraints (33), (34).

It should be observed that the present problem differs from
the problem discussed in Section 2, eqs. (12), (13), where the
labor supply was given. The main idea in solving the present pro-
blem is to use the Hoelder inequality twice. Introduce the de-
fining equations:

$$A \exp(-\lambda t) \, [w_1(t)]^{-\beta} \int_0^t \exp[-\wedge(t-\tau)] \, (z(\tau-T_2))^\alpha \, d\tau \stackrel{df}{=}$$

$$\stackrel{df}{=} f(t),$$

$$[w_1(t) \, x(t)]^\beta \stackrel{df}{=} \phi(t)$$

and observe that

$$.Y - \lambda_1 \left\{ \int_0^T |f(t)|^{1/\cdot} \, dt \right\}, \quad = 1 - \cdots \tag{37}$$

where the equality sign appears iff

$$\phi(t) = c_1 [f(t)]^{\beta|\gamma}, \quad t\epsilon[0,T], \tag{38}$$

$$c_1 = \text{const.}, \quad c_1 > 0.$$

Changing the integration order and again using the Hoelder inequality, gives:

$$\int_0^T [f(t)]^{1/\gamma} dt =$$

$$= \int_0^T \left\{ A \exp(-\lambda t) [w_1(t)]^{-\beta} \right\}^{1/\gamma} \int_0^t \exp[-\delta(t-\tau)]^{\alpha} d\tau dt =$$

$$= \int_0^T [w_2(\tau) z(\tau-T_2)]^{\alpha} [w_2(\tau)]^{-\alpha} \int_\tau^T w(\tau) \exp[-\delta(t-\tau)] dt \, d\tau \leq$$

$$\leq \left\{ \int_0^T w_2(\tau) z(\tau-T_2) \, d\tau \right\}^{\alpha} \left\{ \int_0^T [w_2^{-\alpha}(\tau) \int_\tau^T w(t) \exp[-\delta(t-\tau)]^{\delta} d\tau \right\}^{1-\alpha}$$

$$= z_2^{\alpha} F^{1-\alpha}, \qquad\qquad \delta = 1/(1-\alpha) \tag{39}$$

where

$$F = \int_0^T [w_2^{-\alpha}(\tau) \int_\tau^T w(t) \exp[-\delta(t-\tau)] \, dt]^{\delta} \, d\tau,$$

$$\dot{w}(t) = A[\exp(-\lambda t) w_1^{-\beta}(t)]^{1|\gamma}$$

The equality sign in (39) appears iff

$$z(t) = \hat{z}(t) = c_2 \, g(t), \quad c_2 = \text{const.}, \quad c_2 > 0,$$

where

$$g(\tau) = \left\{ w_2^{-1}(\tau) \int_\tau^T w(t) \exp[-\delta(t-\tau)] dt \right\}^{\delta}$$

Then,

$$\Delta Y(x,z) \leq F^{(1-\beta)(1-\alpha)} z_1^{\beta} z_2^{\alpha(1-\beta)} \tag{40}$$

Finding c_1, c_2 by (33), (34), the optimum strategies can be written

$$\hat{z}(t) = \frac{g(t)}{\int_0^T w_2(t) \ g(t) \ dt} z_2, \tag{41}$$

$$\hat{x}(t) = \frac{h(t)}{\int_0^T w_1(t) \ h(t) \ dt} z_1, \tag{42}$$

where

$$h(t) = \left\{ \frac{A \exp(-\lambda t)}{w_1(t)} \int_0^t \exp[-\delta(t-\tau)] \ [\hat{z}(\tau-T_2)]^{\alpha} \ d\tau \right\}^{1|\gamma}.$$

Assuming that $A = w_1(t) = w_2(t) = 1$, $\lambda = 0$, $\alpha = 1/2$, $T_2 = 0$ one gets

$$g(t) = \left\{ 1 - \exp[-\delta(T-t)] \right\}^2 \tag{43}$$

$$h(t) = H \left\{ 1 - \exp(-\delta t) + \delta t \ \exp[-\delta(t+T)] \right\} \tag{44}$$

$$H = \sqrt{Z_2/G} \ , \quad G = \int_0^T g(t) \ dt$$

In Figure 6, the form of optimum strategies (43), (44) for $\delta T = 4$ have been shown.

It is interesting to observe that as $t \to T$ the investment intensity $z(t)$ goes down. At the same time, productive capital stock

$$f_2(t) = \int_0^t \exp[-\delta(t-\tau)] \ [\hat{z}(\tau)]^{1/2} \ d\tau$$

increases (as shown by the dotted line). Employment, i.e. migration intensity $\hat{x}(t)$, increases in constant proportion to capital stock, according to the factor-coordination principle (new jobs and housing available).

It can be shown that the maximal regional growth, specified by the right side of (40), with the integral constraints (33), (34), is not smaller than any exogenous strategy $x(t)$, $z(t)$ satisfying (33), (34), can produce. However, that strategy can be exercised only when the labor and capital stock resources exist and can be effectively used.

When Z_2 in (40) includes the migration investment cost, the productive investment $\hat{z}$ is reduced by factor $(1-a)$. Output (40) is reduced by $(1-a)^{\alpha(1-\beta)}$.

The present dynamic model of regional growth can be easily extended to the general case with m-production factors. From the point of view of optimization of migration policy, the most interesting case concerns the situation when skilled labor is needed and part of the regional budget should be spent on education, research, development, services and environment. Obviously, the factor-coordination principle can be applied here, and the corresponding strategies can be derived in an explicit form.

5. CONCLUSION

From the analysis carried out in Sec. 3, the surpluses (or deficits) of labor X_j, for each subregion R_j within the planning interval, can be estimated. It was shown in Sec. 4, that for the given subregion R_j and integrated costs of labor and capital (z_1^j, z_2^j), one can find the optimal allocation of labor (x_j) and investments (z_j) over time.

Local labor and capital resources can, in general, be assigned to three alternative development strategies:

1. out (or in) migration of labor (x_j^1), involving the investment z_j^1;

2. commuting of labor (x_j^2), involving the investments z_j^2;

3. in (or out) transfer of capital (z_j^3), which yields employment for x_j^3, $\sum_{i=1}^{3} x_j^i = X_j$, $j = 1...r$.

Each strategy involves different operating (or maintenance) costs, such as e.g. (33):

$$c_j^i (x_j^i) = z_{1j}^i, \quad i = 1,2,3, \quad j = 1...r$$

and capital costs, such as e.g. (34):

$$c_j^i (z_j^i) = z_{2j}^i, \quad i = 1,2,3, \quad j = 1...r.$$

To the c_j^i costs one should assign also the costs of social, technological and environmental changes.

Using the cost-benefit approach, one can investigate the ratios

$$\eta_j^i = \frac{z_{1j}^i + z_{2j}^i}{\Delta Y_j^i (x_j^i, z_j^i)}, \quad i = 1,2,3, \quad j = 1...r,$$

and find the j, i indices which render the smallest value of η_j^i. Another possible approach is to find (by the method used in Sec. 4) the strategies: $x_j^i = \hat{x}_j^i$, $z_j^i = \hat{z}_j^i$, $i = 1,2,3$, $j = 1...r$, which maximize

$$\sum_{j=1}^{r} \sum_{i=1}^{3} \Delta y_j^i (x_j^i, z_j^i),$$

subject to

$$\sum_{j=1}^{r} \sum_{i=1}^{3} C_j^i (x_j^i) \leq z_1,$$

$$\sum_{j=1}^{r} \sum_{i=1}^{3} \ddot{C}_j^i (z_j^i) \leq z_2.$$

The solutions to these problems can be used to determine optimal regional development and optimal interregional migration policies.

APPENDIX

As shown in paper [4], for $A(t) = A \exp \mu t$, the solution to problem (7), (8) can be easily derived by using the generalized Hoelder inequality:

$$\int_0^T \prod_{\nu=1}^m \phi_\nu(t)\,dt \leq \prod_{\nu=1}^m \left\{ \int_0^T |\phi_\nu(t)|^{1/\beta_\nu}dt \right\}^{\beta_\nu}$$

$$\phi_\nu(t) = \left\{ \exp(-\lambda_\nu t)\, f_\nu(t) \right\}^{\beta_\nu}$$

which becomes an equality when (almost everywhere)

$$c_\nu |\phi_\nu(t)|^{1/\beta_\nu} = |\phi_1(t)|^{1/\beta}, \quad c_\nu = \text{const.}, \quad \nu = 2,\ldots,m$$

Determining the values of c_ν by (8), one gets

$$f_\nu(t) = \hat{f}_\nu(t) = (F_\nu/F_1) \exp[(\lambda_\nu - \lambda_1)t]\, f_1(t), \qquad (A.1)$$
$$\nu = 2,\ldots m.$$

The output, (7), under optimum strategy (A1) becomes

$$\hat{Y} = Y(\hat{\underline{f}}) = A \prod_{\nu=1}^m F_\nu^{\beta_\nu} \qquad (A.2)$$

From (A1), (A2), Y attains its maximum, $\hat{Y}$, when factors $f_\nu(t)$ change in coordinated fashion.

The variables Y, F_ν, $\nu = 1,\ldots m$, have, until now, been expressed in natural units. Introducing the price p of output Y and the prices $\omega_\nu, \nu = 1,\ldots m$, for rental of factors, one can express (7), (8) in monetary units as

$$\bar{Y} = pY, \quad \bar{F}_\nu = \omega_\nu F_\nu, \quad \nu = 1,\ldots m.$$

Since the income $\bar{Y}$ should be in balance with the expenditures $\bar{F}_\nu$, $\nu = 1,\ldots,m$, one obtains

$$\sum_{\nu=1}^{m} \bar{F}_\nu = \sum_{\nu=1}^{m} \omega_\nu F_\nu = \bar{Y} \qquad (A.3)$$

i.e. the producers pay, out of their income, for renting the production factors. Such an approach is used in socialist economics. Now it is possible to find $F_\nu = \hat{F}_\nu$, $\nu = 1,\ldots m$, to maximize

$$\hat{Y} = A \prod_{\nu=1}^{m} F_\nu^{\beta_\nu} \qquad (A.4)$$

subject to (A.3); It can be easily shown that

$$\hat{F}_\nu = (\beta_\nu/\omega_\nu)\bar{Y}, \quad \nu = 1,\ldots,m , \qquad (A.5)$$

and it is possible to derive the optimum factor ratios in (A.1):

$$\hat{W}_\nu = \hat{F}_\nu/\hat{F}_1 = (\beta_\nu/\beta_1)(\omega_1/\omega_\nu), \quad \nu = 2,\ldots m \qquad (A.6)$$

Then, by (A.1) and (11), one can derive the necessary expenditure intensities

$$\hat{z}_\nu(t - T_\nu) = \{\hat{W}_\nu [\delta_\nu f_1(t) + f_1'(t)]\}^{1/q}$$
$$= 2,\ldots,m . \qquad (A.7)$$

From the point of view of national accounting and planning systems, it is also important to know how expenditure intensities, $z_\nu(t)$, are related to $\bar{Y}$ and to know the contribution, ΔY, of new plants (constructed within $[0,T]$) to the G.N.P. It, therefore, is assumed that the integrated expenditures Z_ν, $\nu = 1,\ldots,m$, for the planning period are given, i.e.

$$\int_0^T w_\nu(t) z_\nu(t - T_\nu)dt \leq Z_\nu, \nu = 1,\ldots,m *. \qquad (A.8)$$

---

* The weights $w_\nu(t)$, $\nu = 1,\ldots,m$, can be regarded as given discount functions.

The balance equation,

$$\sum_{\nu=1}^{m} z_\nu \leq \bar{Y} \tag{A.9}$$

should be observed.

It can be shown that the optimal strategies $\hat{z} \underset{=}{df} \{\hat{z}_1(t), \ldots, \ldots, \hat{z}_m(t)\}$, which maximize the functional $\bar{Y}(\underline{z})$ subject to the constraints (A.8) exist and can be derived in an explicit form. Besides,

$$\bar{Y}(\hat{\underline{z}}) = \tilde{Y} + \Delta Y ,$$

where

$$\Delta Y = G^q \prod_{\nu=1}^{m} z_\nu^{\alpha \beta_\nu} \quad ,q = 1 - \alpha \sum_{\nu=1}^{m} \beta_\nu , \tag{A.10}$$

G = constant multiplier, $\tilde{Y}$ = part of G.N.P. generated by plants constructed before the planning interval.

Maximizing (A.10) with respect to $z_\nu, \nu = 1, \ldots, m$, subject to (A.9), one gets $z_\nu = \hat{z}_\nu = \beta_\nu \bar{Y}, \nu = 1, \ldots, m$, and

$$\bar{Y} = \tilde{Y} + G^q \prod_{\nu=1}^{m} \beta_\nu^{\alpha \beta_\nu} \bar{Y}^\alpha \tag{A.11}$$

A unique solution, $\bar{Y} = \hat{Y}$, of equation (A.11) exists, which determines the optimal strategy of allocation of resources.

It should be observed that the factor-coordination principle can be generalized for an arbitrary A(t) function. For the purpose, it is necessary to replace $f_1(t)$ by the function $[A(t)]^{1/\beta} f_1(t)$. The principle can also be used for the C.E.S. production function, (4). For that purpose, one uses the generalized Minkowski inequality,

$$\int_0^T \left| \sum_{\nu=1}^{m} \phi_\nu(t) \right|^{1/\varepsilon} dt \leq \left\{ \sum_{\nu=1}^{m} \left[\int_0^T \left| \phi_\nu(t) \right|^{1/\varepsilon} dt \right]^\varepsilon \right\}^{1/\varepsilon} ,$$

which becomes an equality iff (almost everywhere)

$$\phi_\nu(t) = c_\nu \, \phi_1(t), \quad c_\nu = \text{a positive constant} \qquad (A.12)$$

$$\nu = 2,\ldots m$$

Assuming

$$\phi_\nu(t) = \beta_\nu [f_\nu(t) \, A(t)]^\varepsilon$$

and determining c_ν by (8), one obtains a result identical to (A.1)

<u>Theorem 1.</u> <u>Under factor coordination, the income-maximizing strategy, (A.5), is equivalent to the strategy which maximizes consumption per head.</u>

    <u>Proof.</u> In order to prove the theorem, observe that consumption, in monetary terms, becomes

$$\bar{F}_1 = \bar{Y} - \sum_{\nu=2}^{m} F_\nu \omega_\nu$$

The consumption per head in constant prices C can be written (using (A.2)) as

$$C = \frac{\bar{F}_1}{pF_1} = \frac{\omega_1}{p} = A \prod_{\nu=2}^{m} W_\nu^{\beta_\nu} - \sum_{\nu=2}^{m} \frac{\omega_\nu}{p} W_\nu \qquad (A.13)$$

The necessary (and sufficient) conditions for (A.13) to be a maximum $(dC/dW_i) = 0$, $i = 2,\ldots,m$, yield the equations

$$\frac{\beta_i}{W_i} A \prod_{\nu=2}^{m} W_\nu^{\beta_\nu} - \frac{\omega_i}{p} = 0, \quad i = 2,\ldots,m \qquad (A.14)$$

    On the other hand, p can be expressed as the marginal cost to product ratio

$$p = \frac{\displaystyle\sum_{\nu=1}^{m} F_\nu \omega_\nu}{A \displaystyle\prod_{\nu=1}^{m} F_\nu^{\beta_\nu}} = \frac{\omega_1 + \displaystyle\sum_{\nu=2}^{m} \omega_\nu W_\nu}{A \displaystyle\prod_{\nu=2}^{m} W_\nu^{\beta_\nu}} \, .$$

Then

$$\frac{\omega_i}{p} = \frac{A \prod_{\nu=2}^{m} W_\nu^{\beta_\nu}}{\dfrac{\omega_1}{\omega_i} + \sum_{\nu=2}^{m} \dfrac{\omega_\nu}{\omega_i} W_\nu}, \quad i = 2,\ldots,m. \tag{A.15}$$

Comparing (A.14) and (A.15), one gets

$$\frac{\omega_1}{\omega_i} = \frac{W_i}{\beta_i} - \sum_{\nu=2}^{m} \frac{\omega_\nu}{\omega_i} W_\nu, \quad i = 2,\ldots,m.$$

The last relations can be written

$$\beta_i \sum_{\nu=1}^{m} \omega_\nu F_\nu = \omega_i F_i, \quad i = 2,\ldots,m,$$

or

$$F_i = (\beta_i/\omega_i)\bar{Y}, \quad i = 2,\ldots,m,$$

which is equivalent to (A.5). Q.E.D.

FIGURE 1

$$q = 0.6$$
$$a = 0.529$$
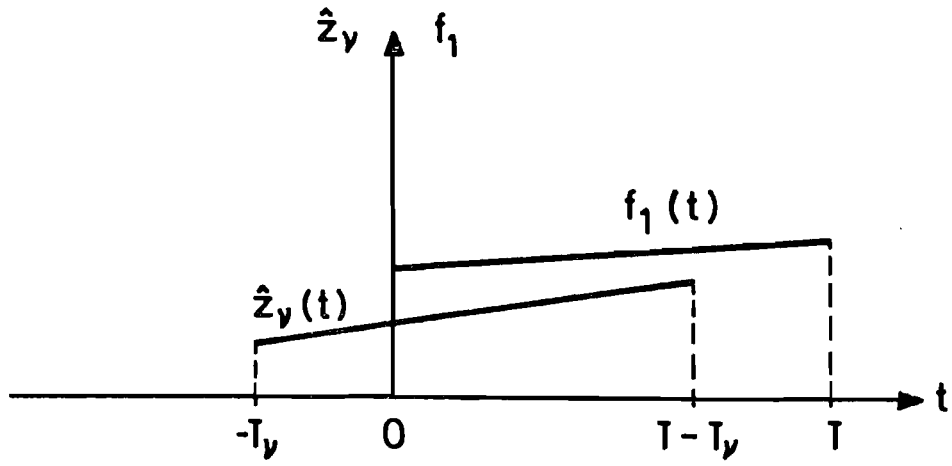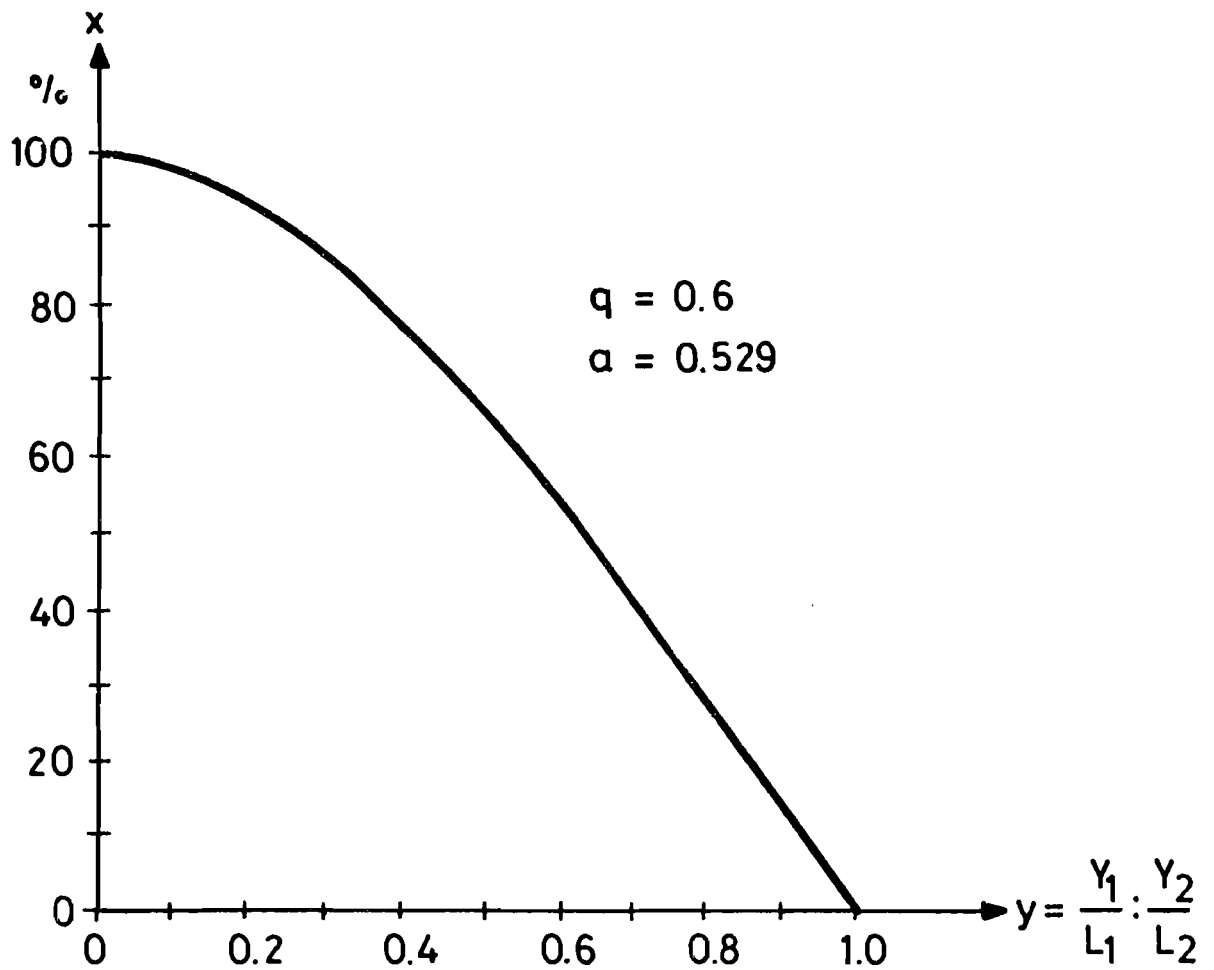
FIGURE 2

SURPLUS OF LABOR IN DROBIN COUNTY IN 1975

LEGEND
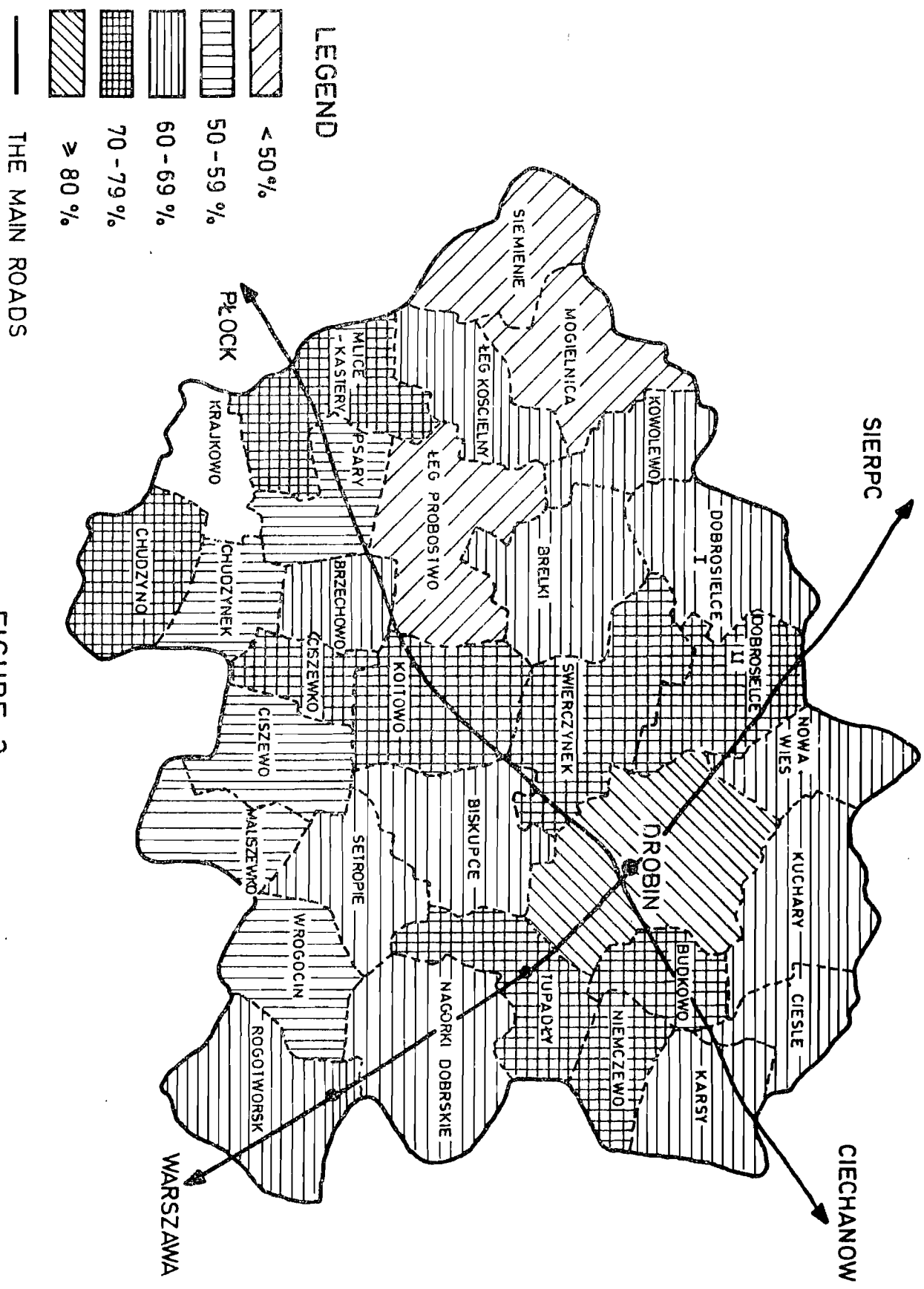
< 50 %

50 - 59 %

60 - 69 %
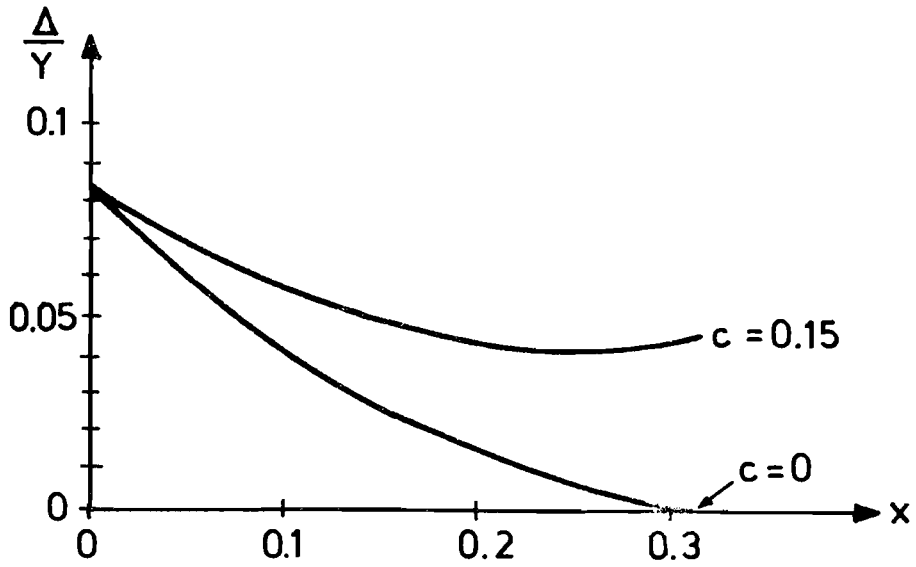
70 - 79 %

≥ 80 %

THE MAIN ROADS
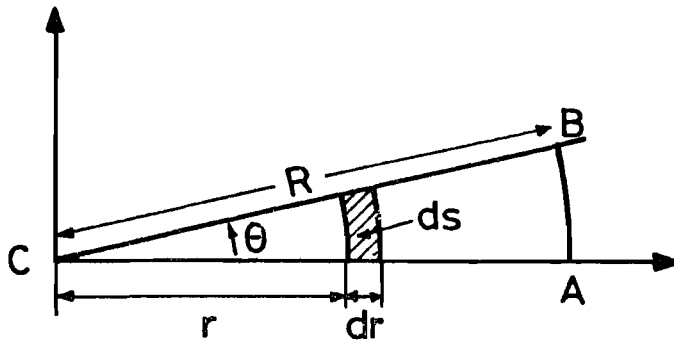
FIGURE 3
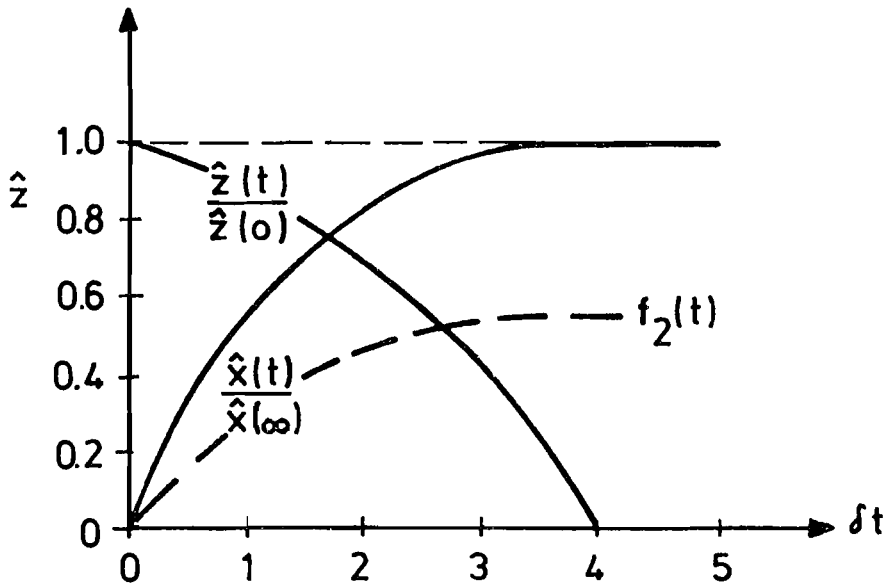
FIGURE 4

FIGURE 5

FIGURE 6

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