ADAPTIVITY AND STABILITY OF TIME SERIES MODELS

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June 1977

PREFACE

To forecast observations from a time series provides an important basis for planning and control. The problem of how to make good forecasts arises in many areas of application at IIASA, and is dealt within the System and Decision Sciences area.

Forecasters who use models which are specified and estimated from past data are concerned whether structure and parameters of the models change over time. The stability (or adaptivity) of forecasts from time series models with respect to interventions such as step changes or outliers is discussed in this paper. Furthermore a statistical test to assess parameter changes in the model is described.

ABSTRACT

The effect of interventions on economic variables in the presence of a time dependent noise structure is modelled in this paper. Forecasts from such models are derived and it is discussed whether forecasts from ARIMA time series models are adaptive with respect to interventions such as changes in the level or outliers.

An overall criterion to test the stability of the parameters in ARIMA models is derived and applied to three Austrian macroeconomic sequences.

ADAPTIVITY AND STABILITY OF TIME SERIES MODELS

Introduction and Summary

Economic forecasters who use models which are specified and estimated from past data are always concerned whether the structure and the parameters of their models change over time. Since predictions are usually obtained under the assumption of constant parameters, time changing parameters can seriously affect the forecasts.

Stability of the structure and parameters of the model over time is important both for forecasts from econometric models (simultaneous equation models which represent relationships among economic variables) as well as for forecasts which are derived from univariate time series models, commonly known under the name of Box-Jenkins (or ARIMA) models.

In this paper we focus on time series models and consider various aspects of parameter changes in such models. In particular we discuss how the effect of an intervention on a given economic variable can be modelled in the presence of a time dependent noise structure. Examples for intervention effects on economic variables are the effects of a preannounced introduction of an additional car sales tax on car sales, the effects of the introduction of value added tax on consumption, the effects of a recession on main economic indicators, the effects of a wage price freeze on the consumer price index, etc. We introduce difference equation models to represent the possible dynamic characteristics of the intervention and the noise. Furthermore we discuss the adaptivity (or sensitivity) of forecasts from time series models with respect to such interventions. It is investigated whether forecasts from ARIMA time series models can be expected to be adaptive with respect to an intervention such as a change in the level of the series.

The detection of parameter changes and a test for stability of the model is discussed in the last section of the paper. Such an analysis is only possible if observations after the supposed intervention are available. An overall criterion is given which uses the forecast errors to test whether the model has significantly changed after the intervention took place. This overall test criterion has the advantage of being not specific with respect to the feared discrepancy.

It is illustrated on three quarterly Austrian economic series (gross national product, total private consumption, total gross investment in fixed assets; all in real terms). It is tested whether the time series models which are estimated using data up to 1974/3 still adequately describe the period 1974/4 to 1976/4. Such an analysis is useful since it can indicate whether the last economic crisis changed the dynamics of the model.

If one concludes that a given model is not a valid description of the most recent data one has to examine the forecast errors and suggest theoretical explanations for the model change. Patterns in the forecast errors can be used to speculate on how the model has changed.

1. The Model

Let us denote the time series, which is observed at equal intervals, by $\ldots z_{t-1}, z_t, z_{t+1}, \ldots$. The model which is considered in this paper is given by

$$z_t = f(\underset{\sim}{\delta}, \underset{\sim}{\omega}, x, t) + n_t \qquad (1.1)$$

where

(a) the noise n_t follows a stochastic difference equation (ARIMA) model of the form

$$n_t = \frac{\theta(B)}{\varphi(B)} a_t \qquad (1.2)$$

$\{a_t\}$ is a sequence of independent normally distributed random variables with mean zero and constant variance. B is the back-shift operator i.e., $B^m a_t = a_{t-m}$. $\theta(B) = 1 - \theta_1 B - \ldots - \theta_q B^q$ is the moving average operator and it is assumed that its roots lie outside the unit circle (invertibility condition). $\varphi(B) = 1 - \varphi_1 B - \ldots - \varphi_p B^p$ is the autoregressive operator and its roots are assumed to lie on or outside the unit circle. Roots of $\varphi(B) = 0$ on the unit circle (i.e. terms like $1 - B$, $1 - B^4$, $1 - \sqrt{3}B + B^2$, etc.) are able to represent certain kinds of stable non stationary processes.

The class of autoregressive integrated moving average models (1.2) (integrated because of difference operators $1 - B$) was originally introduced by Box and Jenkins [1]. A detailed description and application of these models to Austrian economic data can be found in Ledolter, Schebeck and Thury [6].

(b) $f(\underset{\sim}{\omega},\underset{\sim}{\delta},x,t)$ is a dynamic difference equation model representing the additional effect of an exogenous variable x_t over the noise;

$$f(\underset{\sim}{\omega},\underset{\sim}{\delta},x,t) = y_t = \frac{\omega(B)}{\delta(B)} x_t \tag{1.3}$$

$\omega(B)$ and $\delta(B)$ are polynomials in B of degree r and s, respectively;

$$\omega(B) = \omega_0 - \omega_1 B - \ldots - \omega_r B^r$$

$$\delta(B) = 1 - \delta_1 B - \ldots - \delta_s B^s \quad .$$

The roots of $\delta(B) = 0$ are assumed to lie outside the unit circle. x_t is an exogenous time series whose effect needs to be taken into account. In this paper x_t will be an indicator variable taking the values 0 and 1 to describe the nonoccurrence and occurrence of an intervention.

Especially useful indicator variables are the *indicator variable representing a step*

$$(x_t^{(s)};\ x_t^{(s)} = 1 \text{ for } t \geq T \text{ and } x_t^{(s)} = 0 \text{ for } t < T)$$

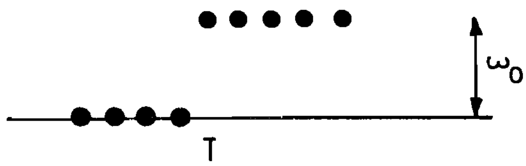
and the *indicator variable representing a pulse*

$$(x_t^{(p)};\ x_t^{(p)} = 1 \text{ for } t = T \text{ and } x_t^{(p)} = 0 \text{ otherwise}).$$

These simple indicator variables, together with the transfer function $\omega(B)/\delta(B)$, are capable of representing many different forms of interventions. Several simple cases are given below:

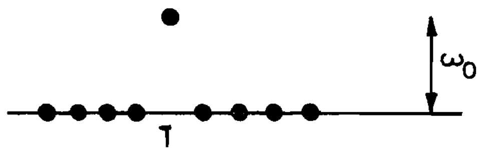
(i) change in level

$$y_t = \omega_0 x_t^{(s)}$$



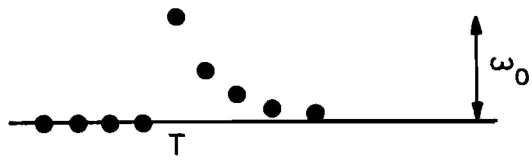
(ii) instantaneous change (outlier)

$$y_t = \omega_0 x_t^{(p)}$$



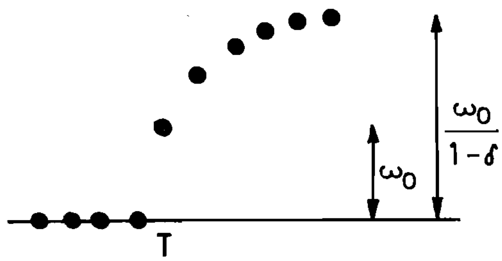
(iii) exponentially decreasing effect

$$y_t = \frac{\omega_0}{1 - \delta B} x_t^{(p)}$$



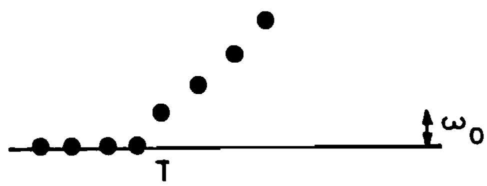
(iv) dynamic first order model effect (gradually increasing effect, converging to a constant)

$$y_t = \frac{\omega_0}{1 - \delta B} x_t^{(s)}$$



(v) linearly increasing effect

$$y_t = \frac{\omega_0}{1 - B} x_t^{(s)}$$

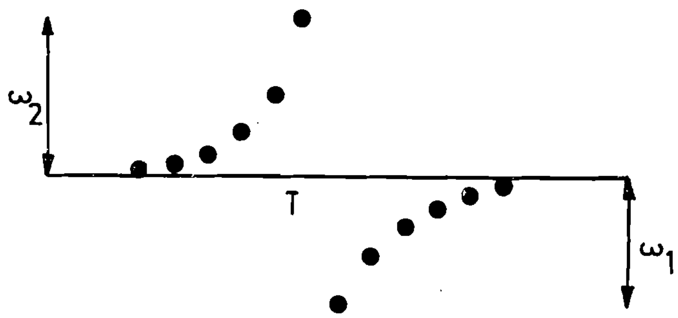


(vi) anticipatory effect

$$y_t = \left[\frac{-\omega_1 B}{1 - \delta_1 B} + \frac{\omega_2}{1 - \delta_2 F}\right] x_t^{(p)} \qquad (1.4)$$

$$= [\ldots - \omega_1 \delta_1^2 B^3 - \omega_1 \delta_1 B^2 - \omega_1 B + \omega_2 + \omega_2 \delta_2 F + \omega_2 \delta_2^2 F^2 + \ldots] x_t^{(p)}$$

where $F = B^{-1}$ is the forward shiftoperator, i.e., $B^m x_t^{(p)} = x_{t+m}^{(p)}$.



The effects of interventions such as the preannounced introduction of new taxes (for example the special sales tax on new cars in Austria in 1969, or the value added tax in 1973) can be modelled by (1.4).

(vii) The intervention model can be extended to cover the additional effect of several exogenous variables (or indicators) $\underset{\sim}{x}_t' = (x_{1t} x_{2t} \ldots x_{mt})$

$$f(\underset{\sim}{\omega}, \underset{\sim}{\delta}, \underset{\sim}{x}, t) = \sum_{i=1}^{m} \frac{\omega_i(B)}{\delta_i(B)} x_{it} \qquad (1.5)$$

where $\omega_i(B)$ and $\delta_i(B)$ $(1 \leq i \leq m)$ are polynomials in B as discussed before. Detailed description of intervention models and their application to environmental problems is given by Box and Tiao ([2,3]), Tiao, Box and Hamming [7].

2. Effect of interventions on time series forecasts

In this section optimal forecasts for future observations from model (1.1) are derived. Furthermore the bias introduced by ignoring the deterministic intervention part of the model is computed. The results are interpreted and the adaptivity (sensitivity) of forecasts from ARIMA models to step changes and outliers is discussed.

Theorem: Assume that the observations follow the model discussed in the previous section

$$z_t = v(B)x_t + \psi(B)a_t \tag{2.1}$$

where

$$v(B) = \omega(B)/\delta(B)$$

and

$$\psi(B) = \theta(B)/\varphi(B) \quad .$$

$\{x_t\}$ is a deterministic indicator sequence known for all t (such as step or pulse indicator), $\{a_t\}$ is a white noise sequence.

Then it can be shown that the ℓ-step ahead minimum mean square error (MMSE) forecast of $z_{n+\ell}$ is given by

$$\hat{z}_n(\ell) = \sum_{j\geq 0} \pi_{j+1}^{(\ell)} z_{n-j} + v(B)\left[x_{n+\ell} - \sum_{j\geq 0} \pi_{j+1}^{(\ell)} x_{n-j}\right] \tag{2.2}$$

where

$$\pi_j^{(\ell)} = \pi_{j+\ell-1} + \sum_{k=1}^{\ell-1} \pi_k \pi_j^{(\ell-k)} \tag{2.3}$$

and the π_j - weights are the coefficients in

$$\pi(B) = 1 - \sum_{j\geq 1} \pi_j B^j$$

and given by

$$\pi(B) = \varphi(B)/\theta(B) = [\psi(B)]^{-1} \quad . \tag{2.4}$$

Proof: Model (2.1) can be written as

$$\pi(B)\, z_{n+\ell} = v(B)\pi(B)\, x_{n+\ell} + a_{n+\ell} \quad . \tag{2.5}$$

It is easily shown (see for example Box and Jenkins [1], page 126) that the minimum mean square error forecast of $z_{n+\ell}$, given observations up to time n, is the conditional expectation $E(z_{n+\ell}|z_n,z_{n-1},\ldots)$. For $\ell=1$, (2.5) is given by

$$z_{n+1} = \sum_{j\geq 0} \pi_{j+1}\, z_{n-j} + v(B)\left[x_{n+1} - \sum_{j\geq 0} \pi_{j+1}\, x_{n-j}\right] + a_{n+1}.$$

Since the expectation of a future shock is zero, the one step ahead forecast is given by

$$\hat{z}_n(1) = \sum_{j\geq 0} \pi_{j+1}\, z_{n-j} + v(B)\left[x_{n+1} - \sum_{j\geq 0} \pi_{j+1}\, x_{n-j}\right]. \tag{2.6}$$

The proof of the theorem is completed by induction. Assuming that (2.2) holds for $\ell-1$, we show it is true for ℓ.

The conditional expectation of $z_{n+\ell}$ is given by

$$\hat{z}_n(\ell) = \sum_{k=1}^{\ell-1} \pi_k\, \hat{z}_n(\ell-k) + \sum_{j\geq 0} \pi_{\ell+j}\, z_{n-j} + v(B)\pi(B)\, x_{n+\ell} \quad . \tag{2.7}$$

Substitution of (2.2) for $\hat{z}_n(1),\ldots,\hat{z}_n(\ell-1)$ into equation (2.7) gives

$$\hat{z}_n(\ell) = \sum_{k=1}^{\ell-1} \pi_k \left\{ \sum_{j\geq 0} \pi_{j+1}^{(\ell-k)} z_{n-j} + v(B)\left[x_{n+\ell-k} - \sum_{j\geq 0} \pi_{j+1}^{(\ell-k)}\, x_{n-j}\right]\right\}$$
$$+ \sum_{j\geq 0} \pi_{\ell+j}\, z_{n-j} + v(B)\pi(B)x_{n+\ell} \tag{2.8}$$

rearranging terms

$$\hat{z}_n(\ell) = \sum_{j\geq 0}\left[\pi_{j+\ell} + \sum_{k=1}^{\ell-1} \pi_k \pi_{j+1}^{(\ell-k)}\right] z_{n-j}$$
$$+ v(B)\left\{x_{n+\ell} - \sum_{j\geq 0}\left[\pi_{j+\ell} + \sum_{k=1}^{\ell-1}\pi_k \pi_{j+1}^{(\ell-k)}\right] x_{n-j}\right\} \quad . \tag{2.9}$$

Using the relation in (2.3) leads to

$$\hat{z}_n(\ell) = \sum_{j\geq 0} \pi_{j+1}^{(\ell)} z_{n-j} + v(B)\left[x_{n+\ell} - \sum_{j\geq 0} \pi_{j+1}^{(\ell)} x_{n-j}\right] \quad . \qquad (2.10)$$

q.e.d.

Bias introduced by ignoring the intervention

If the intervention part of the model is ignored future observations are predicted according to

$$\hat{z}_n^*(\ell) = \sum_{j\geq 0} \pi_{j+1}^{(\ell)} z_{n-j}$$

and the bias in the prediction is given by

$$B_n(\ell) = \hat{z}_n(\ell) - \hat{z}_n^*(\ell) = v(B)\left[x_{n+\ell} - \sum_{j\geq 0} \pi_{j+1}^{(\ell)} x_{n-j}\right] \quad . \qquad (2.11)$$

Expression (2.11) will help to illustrate the adaptivity of forecasts from stochastic difference equation (ARIMA) models with respect to interventions. First, we consider a

Change in level

at time T (i.e. $v(B) = \omega_0$ and $x_t = 1$ for $t \geq T$ and $x_t = 0$ for $t < T$). Predictions are derived from time n (i.e. $n - T \geq 0$ periods after the change in level took place). We are interested how the intervention $n - T$ time periods ago affects the current predictions.

From (2.11) it follows that

$$B_n(\ell) = \omega_0[1 - \pi_1^{(\ell)} - \pi_2^{(\ell)} - \ldots - \pi_{n-T+1}^{(\ell)}] \quad . \qquad (2.12)$$

The following lemma will help in the interpretation of this result.

Lemma: For nonstationary models (i.e. models which have at least one root of $\varphi(B) = 0$ on the unit circle) it can be shown that

$$1 - \sum_{j\geq 1}^{\infty} \pi_j^{(\ell)} = 0 \quad .$$

Proof: By induction

$$\text{for } \ell = 1, \qquad \pi(B) = 1 - \sum_{j\geq 1} \pi_j B^j = \varphi(B)/\theta(B) \quad .$$

Since at least one root of $\varphi(B) = 0$ is on the unit circle, it follows that

$$\pi(1) = 1 - \sum_{j\geq 1} \pi_j = 0 \quad .$$

Assume the lemma is proved for $\ell-1$; then

$$1 - \sum_{j\geq 1} \pi_j^{(\ell)} = 1 - \sum_{j\geq 1}\left[\pi_{j+\ell-1} + \sum_{k=1}^{\ell-1} \pi_k \pi_j^{(\ell-k)}\right]$$

$$= 1 - \sum_{j\geq 1} \pi_{j+\ell-1} - \sum_{k=1}^{\ell-1} \pi_k \sum_{j\geq 1} \pi_j^{(\ell-k)}$$

$$= 1 - \sum_{j\geq 1} \pi_{j+\ell-1} - \sum_{k=1}^{\ell-1} \pi_k = 1 - \sum_{j\geq 1} \pi_j = 0 \quad .$$

q.e.d.

Equation (2.12) and the result of the lemma show that nonstationary difference equation (ARIMA) models adapt themselves with respect to changes in the level; in the sense that even if the intervention is ignored the forecast bias will eventually approach zero if the intervention occurred some time before the prediction is made.

Since the common model form for macroeconomic quarterly series includes regular and seasonal differences (i.e. operators

$(1-B)$ and $(1-B^4)$) the predictions from such models will be adaptive to changes in the level; the speed with which the forecasts adapt to the new level depends on the π-weights of the model.

If the model is stationary (i.e. $\varphi(B) = 0$ has all its roots outside the unit circle and therefore $\pi(1) = 1 - \sum_{j\geq 1} \pi_j \neq 0$) ignoring the intervention will lead to a forecast bias which even in the limit is non zero.

Sensitivity of model forecasts with respect to outliers

If we consider the model for an outlier ($z_t = \omega_0 x_t + n_t$, where $x_t = 1$ for $t = T$ and zero otherwise) the forecast bias (2.11) for forecasts derived from time n ($n - T \geq 0$ periods after the outlier occurred) is given by

$$B_n(\ell) = -\omega_0 \pi^{(\ell)}_{n-T+1} \quad . \tag{2.13}$$

The effect of the outlier on the forecasts depends on how fast the $\pi^{(\ell)}$-weights approach zero. Since $\pi(B) = \varphi(B)/\theta(B)$, the convergence of the π-weights depends on how close the roots of $\theta(B) = 0$ are to the unit circle. For invertible models (roots of $\theta(B) = 0$ outside the unit circle) the π-weights converge fairly rapidly.

3. Stability analysis for selected Austrian economic variables

In this section we consider the following quarterly series:

gross national product (QWSSMR ... Bruttonationalprodukt zu Marktpreisen)

total private consumption (CINSGR ... privater Konsum, insgesamt)

total gross investment in fixed assets (ITSSGR ... Bruttoanlageinvestitionen, insgesamt).*

Time series models for these series are given in Ledolter, Schebeck and Thury [5].

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*QWSSMR, CINSGR, ITSSGR correspond to databank labels of the Austrian Institute of Economic Research.

In this part of the paper we discuss the stability of the fitted time series models. In particular we address the question if and how the last recession (from the fourth quarter 1974 onwards) affected the form of the models as estimated from empirical data over the period 1954/1 to 1974/3.

To detect model changes we compare the forecasts made from the model built on data prior to the suspected change (in this case data up to and including 1974/3) with data actually occurring. An overall test criterion which can be used to assess the statistical significance of the differences between actual data and forecasts is described below.

The ℓ-step ahead forecast error from ARIMA models $\varphi(B)z_t = \theta(B)a_t$ is given by

$$e_n(\ell) = z_{n+\ell} - \hat{z}_n(\ell) = a_{n+\ell} + \psi_1 a_{n+\ell-1} + \dots + \psi_{\ell-1}a_{n+1} \quad (3.1)$$

where the ψ-weights are given by the expansion $\psi(B) = \theta(B)/\varphi(B)$ and $\{a_t\}$ is the white noise sequence (sequence of random shocks) with variance σ^2. The forecast errors made by predicting $z_{n+1}, z_{n+2}, \dots, z_{n+k}$ with information up to time n, are denoted by $\underset{\sim}{e}' = (e_n(1)e_n(2) \dots e_n(k))$ and are given by

$$\underset{\sim}{e} = \Psi \underset{\sim}{a} \quad (3.2)$$

where Ψ is the lower triangular matrix

$$\Psi = \begin{bmatrix} 1 & & & \\ \psi_1 & 1 & & \\ \psi_2 & \psi_1 & 1 & \\ \vdots & \vdots & & \ddots \\ \psi_{k-1} & \psi_{k-2} & \cdots & 1 \end{bmatrix}$$

and

$$\underset{\sim}{a}' = (a_{n+1}a_{n+2} \dots a_{n+k}) \quad .$$

Under the usual assumption of normally distributed shocks and under the null hypothesis that the model up to time n is still valid for $t > n$, the distribution of $\underset{\sim}{e}$ is a multivariate normal with mean zero and covariance matrix $\sigma^2 \Psi \Psi'$. Thus, the statistic

$$Q = \frac{1}{\sigma^2} \underset{\sim}{e}'(\psi\psi')^{-1}\underset{\sim}{e} = \frac{1}{\sigma^2} \underset{\sim}{e}'(\psi')^{-1}\psi^{-1}\underset{\sim}{e} = \frac{1}{\sigma^2} \underset{\sim}{a}'\underset{\sim}{a} = \sum_{j=1}^{k} [a_{n+j}/\sigma]^2 \quad (3.3)$$

follows a χ^2 distribution with k degrees of freedom. Expression (3.3) shows that Q is the standardized sum of squares of the one step ahead forecast errors $a_{n+j} = z_{n+j} - \hat{z}_{n+j-1}(1)$.

The test criterion in (3.3) which has been used by Box and Tiao [4], Tiao, Box and Hamming [7] is an overall test criterion. It is a "catch all" criterion which looks at discrepancies in a general way and is not specific about the nature of the feared discrepancy (alternative hypothesis).

We illustrate the use of this test statistic on the following three economic series:

(a) QWSSMR

The model for the period 1954/1 to 1974/3 is given by

$$(1 - B)(1 - B^4) \log z_t = (1 - .25B)(1 - .43 B^4) a_t; \quad \sigma = .0163 \quad (3.4)$$

The predictions for the period 1974/4 to 1976/4 (9 predictions), using the data up to and including 1974/3 and the model (3.4) are given in Figure 1. The Q statistic is given by 18.24 which compared to a χ^2 distribution with 9 degrees of freedom is significant at the $\alpha = .05$ level (χ^2 table value for $\alpha = .05$ is 16.92). Both Figure 1 and the Q statistic show that model (3.4) does not adequately represent the observations over the last 9 quarters.

(b) CINSGR

The model for the period 1954/1 to 1974/3 is given by

$$(1-B)(1-B^4)\log z_t = (1-.60B)(1+.24\,B^3)(1-.56\,B^4)\,a_t \qquad (3.5)$$

$$\sigma = .0206 \quad .$$

The Q statistic which incorporates the information from the next 9 predictions is 6.87. Compared to a χ^2 distribution with 9 degrees of freedom there is no reason to reject the null hypothesis (i.e. to doubt the validity of model (3.5) for the last 9 observations).

(c) ITSSGR

The model for the period 1954/1 to 1974/3 is given by

$$(1-B)(1-B^4)\log z_t = (1-.66B+.33B^3-.70\,B^4)\,a_t \qquad (3.6)$$

$$\sigma = .0579 \quad .$$

The Q statistic is 3.74. Again there is no reason to doubt the validity of the model for the period 1974/4 - 1976/4.

The conclusion from the overall test criterion is that the model for QWSSMR has changed while the models for CINSGR and ITSSGR remained the same for the period 1974/4 to 1976/4.

In speculating how the model for QWSSMR has changed we have to examine the data, in particular the forecast errors. The standardized one step ahead forecast errors for the observations 1974/4 to 1976/4 using model (3.4) are given in Figure 2a.

A possible explanation for consistently higher ℓ step ahead forecast errors (Figure 1) is an intervention occurring after 1974/3. In this case the model for $t \geq 1974/4$ would be given by

$$z_t = v(B)\,x_t + \frac{(1-\theta_1 B)(1-\theta_4 B^4)}{(1-B)(1-B^4)}\,a_t \qquad (3.7)$$

with $\theta_1 = .25$, $\theta_4 = .43$ and $v(B)x_t$ of the form (1.3). Equation (3.7) can equivalently be written as

$$\pi(B)z_t = v(B)\pi(B)x_t + a_t \tag{3.8}$$

where

$$\pi(B) = (1-B)(1-B^4)/(1-\theta_1 B)(1-\theta_4 B^4)$$

Furthermore, $\pi(B)z_t = a_t^0$ where a_t^0 are the one step ahead forecast errors for model (3.4) given in Figure 2a.

Thus, equation (3.8) can be written

$$a_t^0 = v(B)\pi(B)x_t + a_t \qquad t \geq 1974/4 \tag{3.9}$$

(3.9) is in the form of a regression model. Parsimonious models for the intervention effect have to be specified.

(a) A simple model describing the effect of the recession is a change (decrease) in the level at period 1974/4 (compare with case (i) of Section 1; $\omega < 0$); i.e., $v(B)x_t = \omega_0 x_t^{(s)}$ where $x_t^{(s)} = 1$ for $t \geq 1974/4$ and zero otherwise. The equation

$$a_t^0 = \omega_0[\pi(B)x_t^{(s)}] + a_t \qquad t = 1974/4,\ldots,1976/4 \tag{3.10}$$

is fitted by least squares and the estimate of ω_0 is given by

$$\hat{\omega}_0 = -.0433 \quad .$$

A comparison of the fitted values $\hat{\omega}_0\pi(B)x_t^{(s)}$ with a_t^0 indicates, however, that the change in the level model (3.10) does not adequately describe the pattern of the one step ahead forecast errors.

(b) Model (3.10) implies that the impact of the recession on gross national product is felt immediately in its full strength. A more plausible assumption, however, is that the change in the level is not sudden, but follows a first order dynamic model; thus decreasing exponentially at first and then reaching a new equilibrium level after a few steps (compare with case (iv) of Section 1; $\omega < 0$); i.e.,

$v(B)x_t = \frac{\omega_0}{1-\delta B}x_t^{(s)}$ where $x_t^{(s)} = 1$ for $t \geq 1974/4$ and zero otherwise.

The equation

$$a_t^0 = \frac{\omega_0}{1 - \delta B} [\pi(B) x_t^{(s)}] + a_t \qquad t = 1974/4, \ldots, 1976/4 \tag{3.11}$$

is fitted by nonlinear least squares and the estimates of ω_0 and δ are given by

$$\hat{\omega}_0 = -.0416 \quad ; \qquad \hat{\delta} = .57$$

Comparing the fitted values $\frac{\hat{\omega}_0}{1 - \hat{\delta} B} [\pi(B) x_t^{(s)}]$ with the one step ahead forecast errors shows good agreement.

The standardized residual sum of squares

$$\frac{1}{\sigma^2} \sum_{t=1974/4}^{1976/4} \left[a_t^0 - \frac{\hat{\omega}_0 \pi(B)}{1 - \hat{\delta} B} x_t^{(s)} \right]^2 = 1.824 \quad .$$

Thus, the intervention model (3.11) accounts for (18.24 - 1.824)/ 18.24 = 90 per cent of the Q statistic for QWSSMR.

4. Concluding remarks

The conclusions of this paper can be summarized:

(1) Simple difference equations and indicator variables such as step or pulse indicators can be used to represent a wide variety of different intervention effects.

(2) It is shown that the predictions from non-stationary time series models, which usually provide a good description of economic time series, will eventually adapt themselves to the new level. The speed with which the forecasts adapt themselves depends on the π-weights of the model.

(3) If data after the supposed intervention is available, an overall criterion can be given which tests whether the model has changed after the intervention took place. This

criterion is applied to Austrian data and it is tested whether the last recession changed the dynamics of the estimated models. For total private consumption and total gross investment in fixed assets no such change can be found. For gross national product the hypothesis of model stability, however, has to be rejected. A simple first order intervention model can be shown to explain most of the bias in the forecast errors.

(4) If observations after the intervention are not available one can sometimes use theoretical knowledge and incorporate it into the model. Forecasts for different scenarios (i.e. assumptions about the expected effect of the intervention) can be derived.

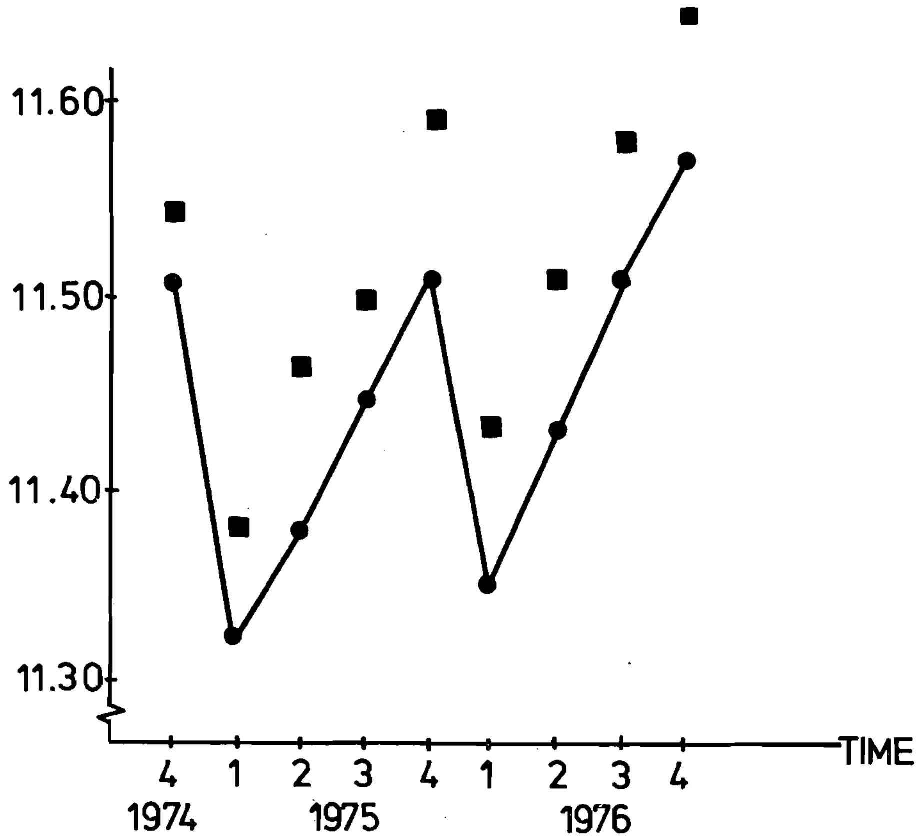

Figure 1: ■ ℓ-step ahead forecasts of QWSSMR from 74/3 for next 9 quarters (logarithm)

● observations (logarithm)

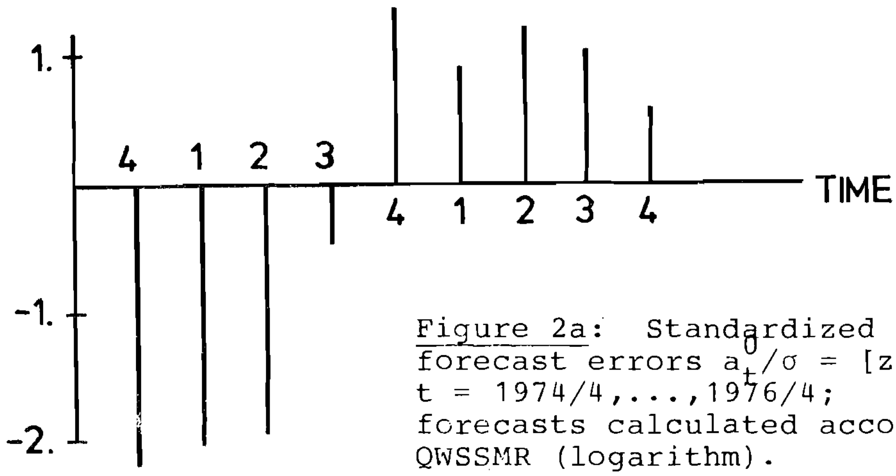

Figure 2a: Standardized one step ahead forecast errors $a_t^0/\sigma = [z_t - \hat{z}_{t-1}(1)]/\sigma$; $t = 1974/4,\ldots,1976/4$; forecasts calculated according to model (3.4); QWSSMR (logarithm).

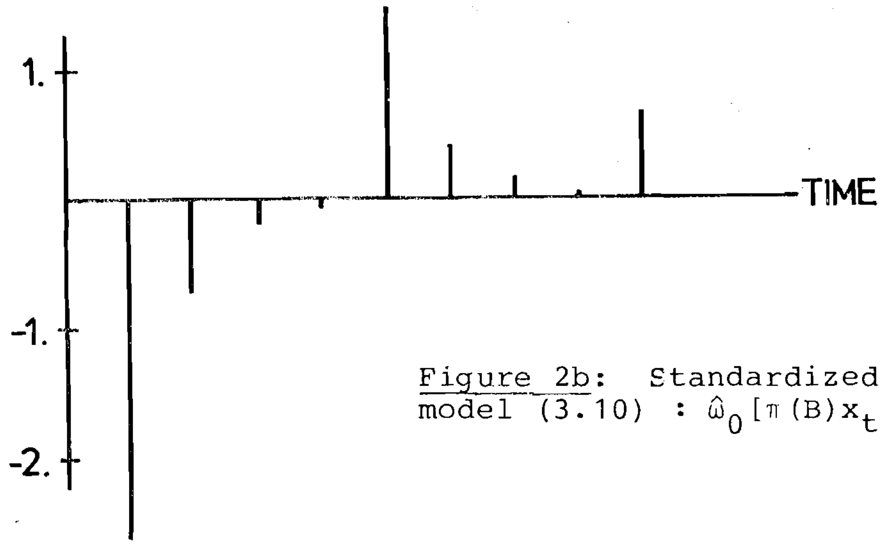

Figure 2b: Standardized fitted values of model (3.10) : $\hat{\omega}_0[\pi(B)x_t]/\sigma$.

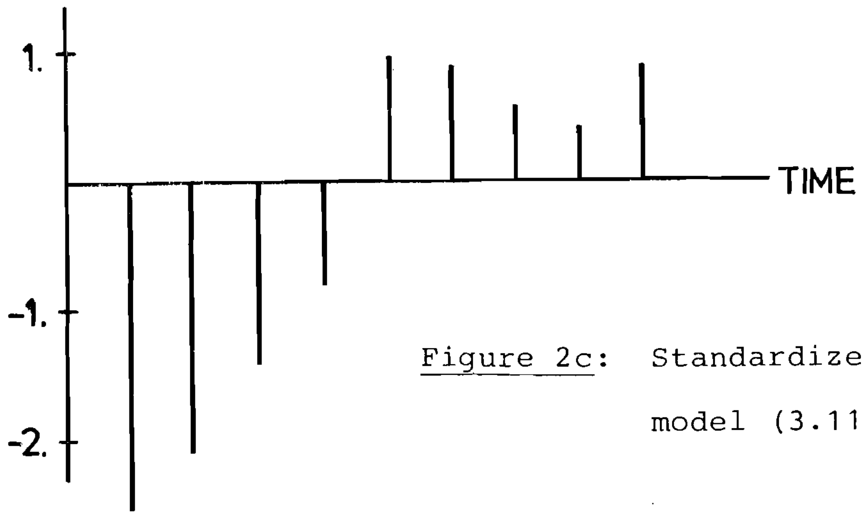

Figure 2c: Standardized fitted values of model (3.11): $\frac{\hat{\omega}_0}{1-\hat{\delta}B}[\pi(B)x_t]/\sigma$.

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