Modeling Endogenous Technological Change with Heterogeneous Agents

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ABSTRACT

Based on earlier, pioneering work done at IIASA, this paper presents a model of endogenous technological change under the three most important "stylized facts": increasing returns to adoption, uncertainty, and heterogeneous agents following diverse technology development and adoption strategies. As an intermediary step towards the final, long-term research objective of developing a multi-agent model, this paper deals with two heterogeneous agents, a risk-taking agent and a risk-aversion agent. Interactions between the two agents include trade on resource and good, and technological spillover ("free-riding" and technology trade). With the two agents, we run Pareto optimization to minimize the total system's cost. The simulations show how agent heterogeneity — different risk attitudes and sizes, trade between agents and technological spillover effect influence the technological change process. Finally this paper plots and analyzes emission paths as results of different technological change process.

Keywords: endogenous technological change, uncertainty, heterogeneous agents

1. INTRODUCTION

The development and diffusion of new technologies is the most important source of long-run productivity and economic growth. Technological change is both costly and highly uncertain. For example, the importance of technological uncertainty has been recognized and explored ever since the earliest days of global environmental modeling [1, 2].

In most of traditional models, technological change has to date largely been treated as exogenous, i.e. technological change, typically in form of improvements in engineering and economic characteristics of individual or aggregate technologies is a free good and also known with perfect foresight within a given scenario of technological "expectations". This is both the case for models developed within the tradition of growth theory and associated production function models (so-called "top-down" models), as well as those developed within a systems engineering perspective (e.g., detailed sectorial "bottom-up" optimization models). In both modeling traditions, technological change is either reduced to an aggregate exogenous trend parameter (the "residual" of the growth accounts), or introduced in form of numerous (exogenous) assumptions on costs and performance of future technologies. Common to both modeling traditions is that the only endogenous mechanism of technological change is that of progressive resource depletion and resulting cost increases, which also explains that the inevitable outcome of imposing additional (e.g. environmental) constraints on the model: rising costs due to the forced adoption of more costly capital vintages that remain unaffected by endogenous policy variables in the model. Such constraints which are at odds with historical experience [3] trigger both substitutions of factor inputs as well as the penetration of otherwise uneconomic technologies. These are either represented generically as aggregates in form of so-called "backstops" [1], or through detailed assumptions on numerous technologies individually.

Traditional technology change models assumed the existence of a global social planner with perfect foresight, which have been criticized (e.g. see [4]) for being overly naive and optimistic on the feasibility of meeting (e.g. environmental) constraints, as availability and adoption of new technologies will be much slower and discontinuous due to agent heterogeneity and uncertainty than suggested in traditional policy models. However, traditional models can also be technologically too "pessimistic", as missing out on spillover effects and adaptive, innovative behavior that arises precisely because of agent heterogeneity and interaction.

Based on earlier, pioneering work done at IIASA [5-8], the model presented in this paper deals with uncertain increasing return and two heterogeneous agents -- a risk-taking agent and a risk-aversion one. Interactions between the two agents include trade on resource and good, and technological spillover effects ("free-riding" and technology trade). With the heterogeneous agents, we run Pareto optimization.

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The model presented here is not intended to be by any means a "realistic" model in the sense of technological or sectorial detail. Rather, the main objective of the model is for exploratory modeling purposes and as a heuristic research device to examine in depth the impacts of alternative model formulations on the endogenous technology transition dynamics.

The rest of this paper is organized as the following. Section 2 briefly introduces the technological change model with uncertain learning and two heterogeneous agents. Section 3 analyzes different simulation results, focusing on how agent heterogeneity and their interactions affect technological change process; Section 4 plots and analyzes emission paths which are the results of different technological change process; and Section 5 gives concluding remarks.

2. THE MODEL

2.1 Technology Change with Increasing Return and Uncertainty

Our optimization model of technology choice is highly stylized. We suppose one primary resource, whose extraction costs increase over time as a function of resource depletion. The economic demands one homogeneous good and the exogenous demand increases over time.

There are three kinds of technology, namely "Existing", "Incremental", and "Revolutionary", which can be used to produce the good. The "Existing" and "Incremental" technologies need consuming primary resource for producing the good, while the "Revolutionary" hardly need no resource input.

The "Existing" technology is assumed to be entirely mature, and its costs and efficiency do not change over time.
The "Incremental" technology has a slight efficiency advantage. With a higher initial cost than that of the "Existing" technology (by a factor 2 higher than the "Existing" technology), it has potential for technological learning (we assume a mean learning rate of 10%).
The "Revolutionary" technology requires no resource input. Its initial cost is much higher than the "Incremental" one (by a factor 40 higher than the "Existing" technology), but its learning potential is also higher (we assume a mean rate of 30%).

We assume in the initial year, all demand can be satisfied by the "Existing" technology, and the "Incremental" and "Revolutionary" has no installation in the initial year. Technology learning is based on RD & D (research, development and demonstration) [7-9]. In our model, the input and effort in RD & D is quantified by the cumulative installed capacity, and future investment cost is a function of cumulative installed capacity. The learning rates of the "Incremental" and "Revolutionary" technology are treated as random values. The probabilistic characteristics of these random values can be derived from lognormal distribution functions of the corresponding learning rates. In our model, we use a simultaneous approximation of these random future cost values by N sample functions of the learning rate, where N is the sample size. The model is solved for a sufficiently large sample N, where the size of N has been determined through successive experiments. Several successive model runs with the same sample size N are compared. If no major change in the solution structure and the objective function can be observed then N is considered sufficiently large.

For more details of the model, please refer to the appendix of this paper.

2.2 Heterogeneous Agents and their Interactions

The model deals with two heterogeneous agents (agent 1 and agent 2) operating simultaneously in the technological change process. The agents' heterogeneities which this paper will focus on are agents' different risk attitudes to uncertainty in learning and weights.

The interaction between the two agents includes trade on resource and good and technology spillover. Trade on resource and good means that one agent can buy resource and good from the other. By means of Pareto optimization, our model does not treat the price of resource and good, instead it includes the cost of the trade. This cost can be viewed as cost for transportation, distributions and any other additional cost caused by moving and using resource and good from the other agent. The quantity of trade flow at each time step is treated as decision variables.

We distinguish two kinds of technology spillover effects: technological "free-riding" and technology trade. Technological free-riding means that one agent can benefit from the other's learning effect without cost, but most of time with some delay. There are no additional decision variables for free-riding. Technology trade means that one agent can benefit from the other's experience (cumulative installed capacity) with some cost. Technology trade is different from resource trade and good trade in sense that the bargainer agent does not lose the experience, unlike in the case of resource and good trade. It just shares the experience with the purchaser agent. Again, here we don't consider the price of technology. And we let the quantity of technology trade at each time step be decision variables

We assume a 100-year scale (from 1990 to 2090) problem, with a 10-year interval between decision times. The objective function of the Pareto optimization can be simply denoted as

$$\begin{aligned}&\min A_{dc}^{1}+A_{dc}^{2}\\&+\rho_1\varpi_1+\rho_2\varpi_2\\&+\sum_{t=1}^{100}\left[(1-\delta)^t\left(\alpha_1\left|r^t\right|+\alpha_2\left|g^t\right|\right)\right],\\&+\sum_{t=1}^{100}\left[(1-\delta)^t\left(\alpha_3\left|s^t\right|\right)\right]\end{aligned}$$

where A_{dc}^{1} and A_{dc}^{2} in the first part of the objective function denotes discounted deterministic cost of agent 1 and agent 2, including both R&D and investment cost and O+M (Operation and Management) cost; ϖ_1 and ϖ_2 in the second part denotes agent 1's and agent 2's discounted cost resulting from overestimating learning rate, with ρ_1 (ρ_2) denoting agent 1's (agent 2's) risk factor; the third part is the cost of trading on resource and good and the forth part is the cost of technology trade, with δ denoting the discount rate, α_1 denoting the unit cost of resource trade, α_2 denoting the unit cost of good trade, α_3 denoting the unit cost of technology trade, r^t denoting trade quantity of resource at time t, g^t denoting trade quantity of good, and s^t denoting the trade quantity of experience (cumulative installed capacity) at time t. The r^t, g^t and s^t can be negative, depending on the direction of the trade.

Each agent is subject to two constraints. Its total annual production of all three technologies must satisfy given demand, and its annual production for each technology does not exceed installed capacity.

Agents' weights denote their sizes or their share in the total system. The weight for the agent 1 is $w_1 \in (0,1)$, and the weight for agent 2 is $w_2 \in (0,1)$, the two weights satisfy the formulation: $w_1 + w_2 = 1$.

The two agents' weights don't appear in the objective function, instead it appears in constrains related to demand. Suppose D^t is the demand in whole market at time step t, then agent 1's demand at time step t is $D_1^t = w_1 D^t$, and for agent 2, it is $D_2^t = w_2 D^t = (1-w_1)D^t$.

Obviously, we can generate infinite future scenarios and stories with different combinations of those parameters. And also with some specification value, the model can be used for some practical analysis. But before that, we would show the behaviors of the model, which is the main purpose of this paper.

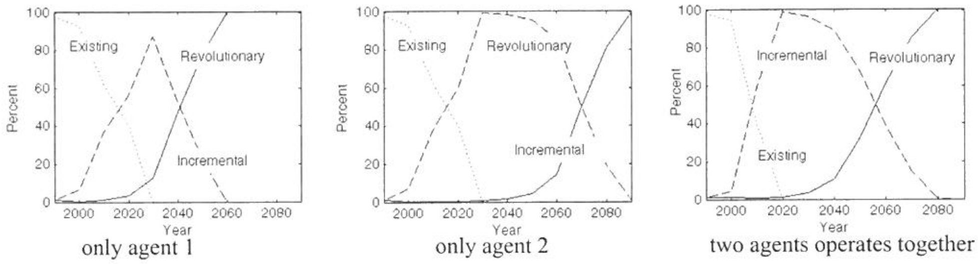
3. SIMULATION

3.1 Pareto Optimization without Technology Spillover

In our basic case (BC in short) simulation, we assume the two agents have the same weight ($w_1 = w_2 = 0.5$), agent 1 is risk-taking ($\rho_1 = 0.1$) while agent 2 is a risk-aversion one ($\rho_2 = 1$), and the unit trade costs of good and resource are the same and they are 100 ($\alpha_1 = \alpha_2 = 100$).

The third part of Fig. 1 shows the result -- market share (percent) of each technology -- of the basic case simulation, comparing with the result of assuming a single global social planner agent (the first and second part of Fig. 1). We can see, in the two-agent model, the diffusion time of the "Revolutionary" technology is longer, and the break-even point of it is later than the case with a risk-taking agent 1 and is earlier than that with a risk-aversion agent 2.

In the basic case simulation, we found that agent 2 did not develop the "Revolutionary" technology at all, it only developed the "Incremental" technology, with its total installed capacity decreased to zero at 2080. The reason is that from 2050, it becomes an economic way for agent 2 to import good from agent 1 because of the continuous diffusion of the "Revolutionary" technology in agent 1 making the cost of the good produced by agent 1 continuously decrease. Before 2040, the demand of agent 2 is totally satisfied by its own production; from 2040 to 2080, it is satisfied by both its own production and the import from agent 2; and from 2080, it is completely satisfied by the import from agent 2, i.e., finally, agent 1 dominated the whole market.



only agent 1

only agent 2

two agents operates together

Figure 1 a global social planner VS two agents interacting with each other
-- Market share (percent) of each technology with the time axis

We varied the values of α_1 and α_2 to see what will happen in the simulation. We assumed $\alpha_1 = \alpha_2 = 10$, which means the cost for trade is very low. We found with the low trade cost, agent 1 dominates the whole market of good, and agent 2 does not produce any good. It imports good from agent 1 and exports resource to agent 1. The reason is very obvious, for agent 1, its adventure character makes it develop advanced technology early and result in low production cost; while for agent 2, since the trade cost is cheap, it is a rational decision to buy cheap good from agent 1 rather than to execute expensive RD & D investment and production. Then we assumed the unit trade cost is expensive, $\alpha_1 = \alpha_2 = 1000$, and in this case, it was not economic for agents to do business with each other, and agent 2 did not develop and adopt the "revolutionary" technology within the simulation period.

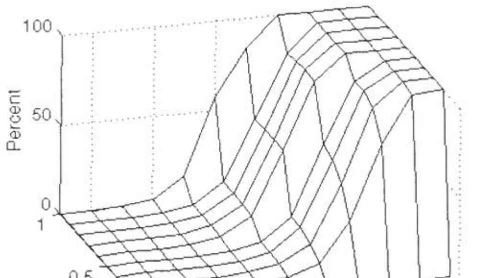


Figure 2 Diffusion of the "Revolutionary" technology in whole market with different w_1

The above simulations assumed that the two agents had the same size. Now we give sensitive analysis on the parameter w_1. Fig. 2 shows the diffusion of the "Revolutionary" technology – its share at each time step -- in the whole market with different w_1. We can see that the entry and mature time of the "Revolutionary" technology was brought forward with the increasing of the risk-taking agent's weight.

With the above Pareto optimization, we did not consider the price of good and resource, that is to say, in fact, we optimize the total system's cost instead of maximizing each agent's profit.

3.2 Pareto Optimization with Technological "Free-Riding"

Now we consider the situation that there is free-riding between agents. That is to say, although agent 2 does not have R&D investment in the "Revolutionary" technology, it can benefit from agent 1's learning effect. We assume that agent 2's future investment cost on the "Revolutionary" technology relies on agent 1's cumulative installed capacity, but with one-decade delay.

With the parameter values set in the basic case simulation, we found the "free-riding" made agent 2 develop the "Revolutionary" technology from 2070, and the diffusion time of it was very short -- only one decade from introduction to mature. While with low trade cost, i.e., $\alpha_1 = \alpha_2 = 10$, the "free-riding" did not show its effect at all; and from 2030, agent 2's consumption completely depends on the import from agent 1. With high trade cost, i.e., $\alpha_1 = \alpha_2 = 1000$, with "free-riding", agent 2 started R&D investment on "Revolutionary" technology from 2060, and "Revolutionary" technology finally occupies all of the market in 2090. We would imagine with higher trade cost, for example, 1000 rather than 500, trade between two agents will be less, and thus technological "free-riding" will show stronger effect. But our simulation results show this is not the truth, the "revolutionary" technology occupy the market earlier with $\alpha_1 = \alpha_2 = 500$ than that with $\alpha_1 = \alpha_2 = 1000$.

This is because with $\alpha_1 = \alpha_2 = 1000$, the "incremental" technology in agent 2 has a wider application, thus it takes more time for the "revolutionary" technology to takeover.

The above result was based on the assumption that the two agents have the same weight, i.e. $w_1 = w_2 = 0.5$. We varied both weights and the trade cost to see what will happen. The simulations showed that, in the whole market (the sum of agent 1 and agent 2's market), when agent 1's weight is bigger and the unit trade cost is lower, the entry time and mature time of the "revolutionary" technology is earlier; and the same happened in agent 1's local market[2]. For agent 2, the "free-riding" showed its effect (which means that agent 2 develops the "revolutionary" technology) when agent 2's weight is big and when the unit trade cost is high, otherwise agent 2 imports good from agent 1 instead of developing the "revolutionary" technology.

3.3 Pareto Optimization with Technology Trade instead of "Free-Riding"

In the above, "free-riding" means one agent can benefit from the other's learning effect without any cost, but with some delay (eg. one decade). In terms of technology trade, we allow an agent to decide whether it need buy technology from the other and when to buy. Technology trade is different from resource trade and good trade in the sense that the bargainer agent does not lose the experience, unlike in the case of resource and good trade. It just shares the experience with the purchaser agent. In the following simulation, we assume that agent 2 will buy the "Revolutionary" technology from agent 1. But not all the bought experience can be internalized. We assume 80% of the bought experience from agent 1 can be internalized by agent 2. For making the technology trade show its effect, we assume a low unit cost for technology trade, i.e., $\alpha_3 = 10$.

Fig. 3 shows the result of the diffusion and trade of "revolutionary" technology with different weight of agent 1. We can see when agent 1 is very big, i.e., $w_1 = 0.9$, agent 1 developed the "revolutionary" technology a little earlier than other occasions. This is because agent 1's large weight denotes that it faces a large market. Varying w_1 from 0.2 to 0.8 does not show obvious impact on agent 1's behavior. What is interesting here is that when agent 1's weight is very small, i.e., $w_1 = 0.1$, agent 1 developed the "revolutionary" technology slightly earlier. This can be explained as: with a small weight, agent 1's learning effect (accumulative installed capacity) becomes relatively weak for agent 2 who will buy agent 1's experience, and thus agent 1 developed the "revolutionary" technology slightly earlier as compensation.

For agent 2, with its risk-aversion attitude, it adopted the "revolutionary" technology a little later than agent 1 by buying technology from agent 1. It made no obvious impact on agent 2's behavior when varying w_1 from 0.1 to 0.7, and the diffusion of the "revolutionary" technology in agent 2 started from about 2050 and occupied all the market in about 2080. The diffusion time (about 3 decades) was shorter than that (about 5 decades) in agent 1. This accords with the historical observation that the later developer of a new technology can obtain a shorter diffusion period [5]. And we can see that the technology trade happened during the period from 2030 to 2060. This suggest a policy for those risk-aversion actors that it is not a good decision to wait until new technology has been developed to a very mature-level by others, e.g., after 2060 in the simulation, instead it is better to learn from technology pioneers (those risk-taking agents who develop new technology earlier) at the earlier diffusion stage (niche market stage according to the life-cycle model) of the new technology, and most of time it is not a right decision to buy new technology forever. After some periods, the later adopters should do RD&D by themselves. When agent 2 is very small ($w_1 \geq 0.8$), agent 2 never developed the "revolutionary" technology. It imported good instead of technology from agent 1.

From the whole market level, with the increase of agent 1's weight, the entry time and mature time of the "revolutionary" technology was brought forward.

4. CARBON EMISSION PATHS AS RESULTS OF DIFFERENT TECHNOLOGY CHANGE

Our model is very idealized in terms of carbon emission. It assumes that the "Revolutionary" technology has little emission which can be negligible.

[2] When agent 1's weight is very small, i.e., $w_1 = 0.1$, and the trade cost of resource and goods is not big (e.g. $\alpha_1 = \alpha_2 < 600$), agent 1 developed the "revolutionary" technology slightly earlier to let agent 2 benefit from "free-riding" on agent 1's learning effect.

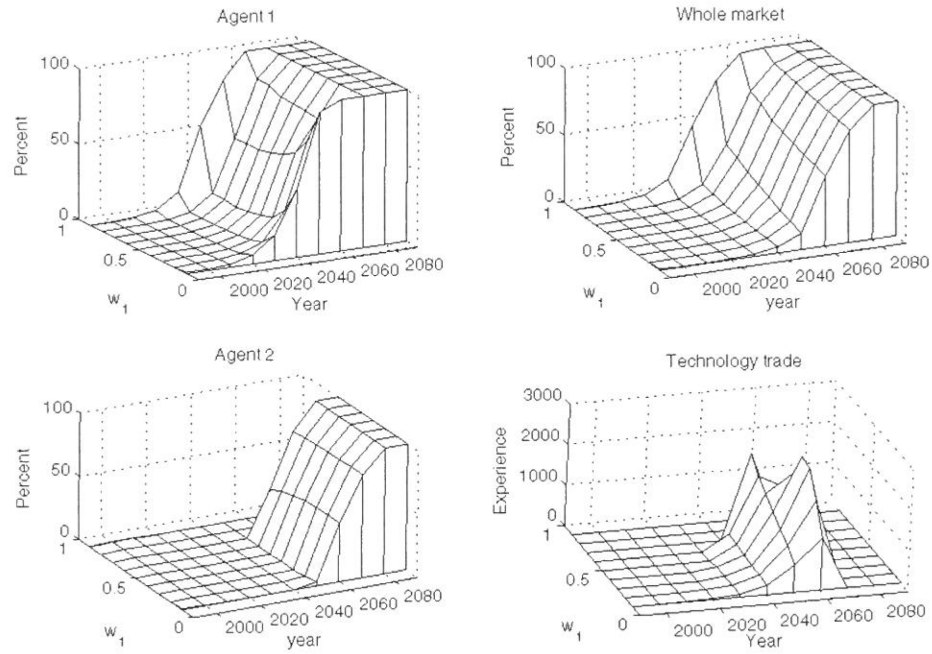


Figure 3 Pareto optimization with technology trade and different weight of agents

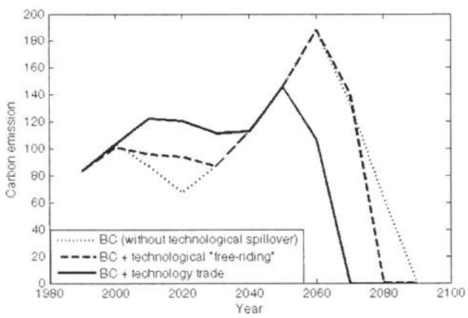
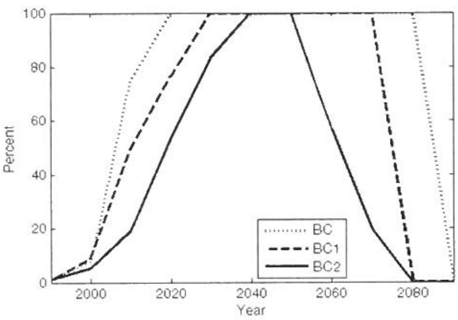


Figure 4 Different carbon emission paths

Fig. 4 shows different emission paths (sum of agent 1 and agent 2) with different technology change, from which we can see that from 2000 to 2040, the basic case (BC in short) simulation (without any technological spillover effect) has the most obvious carbon abatement, while BC plus "free-riding" ranks the second and BC plus technology trade shows the weakest abatement. But after 2050, the rank order is inversed. According to our intuition, technological spillover should be helpful for carbon abatement. The reason why the emission paths from 2000to 2040 are contrary to this intuition is that, during that period, with technological spillover effect, agent 2 developed less "Incremental" technology than in the case without spillover, as shown in Fig. 5, thus more "existing" technology was used in this period which resulted in weaker carbon abatement.



BC --without technological spillover
BC1: BC + technological free-riding
BC2: BC + technology trade

Figure 5 Agent 2's "Incremental" technology in different simulations

5. CONCLUSIONS

With increasing returns and uncertainty, this paper presented a model of endogenous technological change with two heterogeneous agents, a risk-taking pioneer agent and a risk-aversion follower agent. The model showed the following behaviors.

- With the two heterogeneous agents, the diffusion of the "revolutionary" technology is longer than that with a single global social planner. And diffusion time in the follower agent is shorter than that in the pioneer agent.
- Technological spillover effects between the two agents, technological "free-riding" and technology trade in this paper, will encourage the risk-aversion agent to develop and adopt the "Revolutionary" technology, at the same time, it will discourage the wide diffusion of the "Incremental" technology in the risk-aversion agent.
- Concerning the environmental issue, technological spillover effects between the two agents can enhance carbon abatement in a long-term, but in a short or middle-term, they can also weaken carbon abatement.

The stylized and also idealized two-agent model and simulations introduced in this paper can enhance people's imagination about how agent heterogeneity (different weights and risk attitudes) and technological spillover among agents ("free-riding" and technology trade) impact technological change processes. In addition, the simulation results can give some policy implications for both risk-taking and risk-aversion decision makers, e.g., for risk-aversion agent, it is better to import a new technology from risk-taking agent at the niche market stage of the new technology, instead of waiting until the new technology being mature.

APPENDIX

Here we give more mathematic expressions of the model. The demand is exogenous and it increases over time as shown in Eq. (1).

$$D^t = 100(1+2.6\%)^t \tag{1}$$

Here, D^t denotes the demand at decision time t, which increases by 2.6% annually. Each agent has a share of the demand based on its weight. The following expression is for an individual agent.

Let x_i^t $(i=1,2,3)$ denotes the annual production of technology i at time t, and let η_i denotes technology i's efficiency, then the annual extraction R^t is the sum of resources consumed by each technology, as shown in Eq. (2)

$$R^t = \sum_{i=1}^{3} \frac{1}{\eta_i} x_i^t. \tag{2}$$

Thus the cumulative extraction by time t is:

$$\overline{R}^t = \sum_{j=1}^{t} R^j, \tag{3}$$

The extraction cost of the resource increases over time as a linear function of resource depletion, as shown in Eq. (4)

$$c_E^t = c_E^0 + k_E \overline{R}^t \tag{4}$$

where c_E^t denotes the extraction cost per resource unit at time t, c_E^0 is the initial extraction cost, $\overline{R}^t$ is the total extraction by decision time t, and k_E is a constant coefficient.

Let y_i^t $(i=1,2,3)$ denotes the annual new installation of technology i at time t, then the total installed capacity (TIC) of technology i at time t, denoted by C_i^t $(i=1,2,3)$ can be calculated according to Eq. (5).

$$C_i^t = \sum_{j=t-\tau_i}^{t} y_i^j, \tag{5}$$

where τ_i denotes the plant life of technology i.

The cumulative installed capacity (CIC) $\overline{C}_i^t$ of technology i by time t is calculated as:

$$\overline{C}_i^t = \sum_{j=-\infty}^{t} C_i^j = \sum_{j=1}^{t} C_i^j + \overline{C}_i^0. \tag{6}$$

Technology learning is based on experience which is quantified by the cumulative installed capacity, thus future investment cost is a function of cumulative installed capacity, as shown in Eq. (7)

$$c_{Fi}^{\ t} = c_{Fi}^{\ 0} \times (\overline{C}_i^t)^{-b_i}, \tag{7}$$

where b_i is the progress ratio ($1-2^{-b_i}$ is the learning rate) of technology i, and c_{Fi}^0 is the initial cost of technology i.

The deterministic cost, which includes investment on plants, extraction cost, and O+M (operation and maintenance) cost, is

$$\begin{aligned} &\mathrm{E}\left\{\sum_{i=1}^{3}\sum_{t=1}^{100}(1-\delta)^t c_{Fi}^{\ t} y_i^t\right\} \\ &+\sum_{t=1}^{100}(1-\delta)^t c_E^t R^t + \sum_{i=1}^{3}\sum_{t=1}^{100}(1-\delta)^t c_{OMi} x_i^t \end{aligned}, \tag{8}$$

and the cost resulted from overestimating learning rates is

$$\rho\left\{\mathrm{E}\left\{\sum_{t=1}^{100}\sum_{i=1}^{3}(1-\delta)^t\left\{\max[0, c_{Fi}^{\,t}(\psi)-c_{Fi}^{\,t}]\right\}y_i^t\right\}\right\}, \quad (9)$$

where δ denotes the discount rate, c_{OMi} denotes the operating and maintenance (O+M) cost of technology i, ρ is a risk factor ($\rho > 1$ emphasizing the risk and $\rho < 1$ reflecting a tendency toward risk neutrality), $c_{Fi}^{\,t}(\psi)$ is a random variable with ψ denoting an element from a probability space that is characterized by a lognormal distribution, and **E** denotes expectation.

Constrains are

$$\begin{cases} D^t \le \sum_{i=1}^{3} x_i^t & (t=1,\cdots 100) \\ x_i^t \le C_i^t & (t=1,\cdots 100) \quad (i=1,\cdots 3) \\ -x_i^t \le 0 & (t=1,\cdots 100) \quad (i=1,\cdots 3) \\ -y_i^t \le 0 & (t=1,\cdots 100) \quad (i=1,\cdots 3) \end{cases} \quad (10)$$

Table 1. gives the initial values of parameters.

Table 1. Initial values of parameters

Parameters related to the three technologies			
	Existing Tech.	Incremental Tech.	Revolutionary Tech.
Initial cost	$c_{F1}^{\,0} = 1000$	$c_{F2}^{\,0} = 2000$	$c_{F3}^{\,0} = 40000$
Efficiency	$\eta_1 = 30\%$	$\eta_2 = 40\%$	Hardly no resource input
Mean Learning Rate and the variances	$b_1 = 0$ $(1-2^{-b_1} = 0)$	$(1-2^{-b_2} = 10\%)$ $Var(b_2) = (0.1\ \mathrm{E}\ b_2)^2$	$(1-2^{-b_3} = 30\%)$ $Var(b_3) = (0.1\ \mathrm{E}\ b_3)^2$
Plant life	$\tau_1 = 30$	$\tau_2 = 30$	$\tau_3 = 30$
Initial Total Installed Capacity	$C_1^0 = 50$	$C_2^0 = 0$	$C_3^0 = 0$
Initial Cumulative Installed Capacity		$\bar{C}_2^0 = 0.5$	$\bar{C}_3^0 = 0.5$
O+M cost	$c_{OM1} = 30$	$c_{OM2} = 50$	$c_{OM3} = 50$
Other Parameters			
Initial extraction cost	$c_E^0 = 200$	Extraction cost coefficient	$K_E^0 = 0.01$
Discount rate	$\delta = 5\%$	Risk factor	Different for the two agents

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