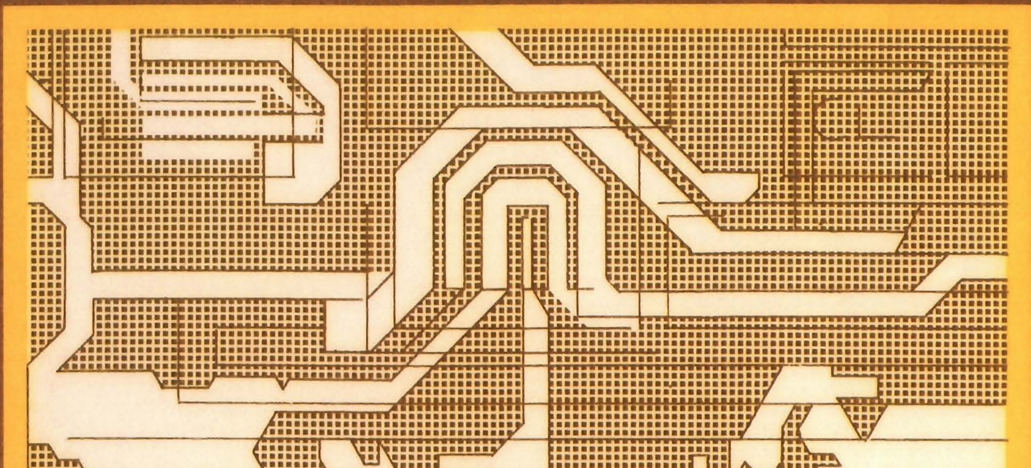
IIASA PROCEEDINGS SERIES

SARUM and MRI:
Description and Comparison of a World Model and a National Model

Gerhart Bruckmann, Editor



International Institute for Applied Systems Analysis

IIASA PROCEEDINGS SERIES

Volume 2

SARUM and MRI: Description and Comparison of a World Model and a National Model

IIASA PROCEEDINGS SERIES

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SARUM AND MRI: DESCRIPTION AND COMPARISON OF A WORLD MODEL AND A NATIONAL MODEL

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GERHART BRUCKMANN
Editor

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PREFACE

The IIASA conferences on global modelling are intended to serve as a forum for exchanging information among modelling groups and to bring these achievements to IIASA. The outcome of the conferences on global modelling will be synthesized, and the various modelling efforts and their potential will be evaluated with a view to understanding the possibilities and limits of global modelling in general, and of the major existing global models in particular.

Since 1974 IIASA has been monitoring developments elsewhere in the field of global modelling. Three conferences have been held, each focusing on one or two major global models approaching a state of completion.

This fourth conference was mainly devoted to a comparison of two models; one by the Department of the Environment of the United Kingdom (SARUM) and one by the Polish Academy of Sciences (MRI). Other models discussed at preceding IIASA conferences on global modelling have been:

Multilevel Computer Model of World Development Systems. Proceedings of the Symposium on Global Modelling, SP-74-1 to 6 and CP-74-1.

Latin American World Model Proceedings of the Second IIASA Symposium on Global Modelling, CP-76-8.

MOIRA: Food and Agriculture Model, Proceedings of the Third IIASA Symposium on Global Modelling, CP-77-1.

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WELCOMING ADDRESS

R. E. Levien

It gives me great pleasure to welcome you to the fourth global modelling conference organized by IIASA. As you know, IIASA has scheduled and sponsored global modelling conferences for the last three years. The first conference reviewed the work of the Pestel-Mesarovic model. The second conference examined the Bariloche Model. The third meeting reviewed the MOIRA Model constructed by Professor Linnemann's group and other food models. Today we are very pleased to begin the fourth in this series with a review of two models by groups led by Mr. Roberts at the Department of Environment in the UK and by Professor Kulikowski of the Computing Center of the Academy of Sciences, Warsaw.

I am sure that many of you have participated in these meetings before, so I do not think I have to review in very great detail IIASA or its role. Nevertheless, there are a few unfamiliar faces in the audience and IIASA continues to evolve from year to year as we learn more about international applied systems analysis. So I will take perhaps five more minutes to bring you up to date on IIASA.

The Institute, as all of you know, is a nongovernmental international applied research institution, sponsored by scientific institutions in, I am proud to say, 15, and by November 17, different countries. Since we met last year, a committee on applied systems analysis from Sweden has become a member and in November the Council is expected to accept organizations from the Netherlands and Finland as additional members. We began with 12 member organizations.

The purpose of the Institute is to carry out applied research on problems of international importance. We distinguish two such categories of problems. The first we call "global problems"; these are issues that inherently involve more than one nation and that cannot be resolved by the actions of individual nations, but require joint effort. Topics like the global climate and its protection, the oceans and their exploitation and development, are global problems in this sense. The second we call "universal problems". These are issues that lie within national boundaries, but which all nations share. Problems like the design of a health care system or transportation system or management of our cities are universal, because they are within the realm of individual governments. But every government has similar problems, and IIASA can help with these problems by the exchange of experience, particularly among countries having rather diverse social, economic, and political systems. So our goal is to work on major international problems, both global and universal.

We have focused our effort within the last year or so on two international problems. The first one is energy systems, the study of which is concerned with the global energy future in the medium or long term (fifteen to fifty years) and with the exploration of alternative sources of energy, the growth of demand, the constraints on energy choice and the strategies for getting from an economy based on oil or gas to one that relies on longer-term energy sources. Our universal problem is regional development. Here we are concerned with the integration of the aspects of regional development that are usually treated separately: water resource exploitation, human settlements development, industrial development, agricultural development, environmental concerns. This is a program that is becoming rather than in being, since we are not yet in the position to bring together the pieces that we are working on separately. But we hope to develop leadership and a strong crosscutting integration. At the same time, we are proposing to our council this year that we undertake a second global problem on food resources. Professor Rabar will be the leader of the program, which, if it is approved by our Council, will be concerned with the medium- and short-term aspects of the global food problem.

The second emphasis of our work is on the "systems analysis" part of our title. In order to study international applied problems in a comprehensive way, one needs to have a broad base of competence. We have identified four Areas where IIASA has to have continuing competence. The first is Resources and Environment. This is the concern for the earth's physical endowment. We must continue to have specialists in ecology, in water resources, in food and agriculture and eventually in mineral resources and other aspects of the physical endowment as well. Second, we are concerned with Human Settlements and Services: the human endowment of the earth--the people, their numbers, their distribution, the housing and other services necessary to provide for their needs. Currently, we have specialists in urban and regional development and planning and in the health care system aspects of human services. The third Area central to our base of competence is Management and Technology. That Area is concerned with organizations, with economic systems and with technologies, all the man-made contributions to the global endowment. In this group we have expertise in industrial systems and large organizations. The fourth Area of competence we need is in the System and Decision Sciences, the mathematical and computational tools for studying complex systems and for helping in the evaluation of complex decisions. In this Area, we have groups of mathematicians, computer scientists, and methodologists.

We have organized the research plan and the research structure of the Institute to enable us to gather the research Area talents to work on our crosscutting problems. We have what we call a "matrix structure" of research where the basic research staff resides in one of the four Areas: the Programs each have a small core group, but draw the remainder of their staff from the Areas.

In addition, we have a General Research Area that comprises topics that do not fit naturally into the four Areas and two Programs. I have mentioned that we are concerned with global problems, but we have studied them on a sectorial basis: energy and food separately. That was a choice that the Institute made--not to go ahead and try to construct yet another multisectorial global model, but rather to dig deeply into the individual sectors that comprise global models, at the same time as following, by means of conferences such as this, the various global modelling efforts underway. So, within General Research we have a small activity involving Professor Bruckmann and Professor Rabar in global modelling, and mainly concerned with the exchange of information and increasing understanding of the state of the art.

In that line we also have an activity called the survey project whose purpose is to publish a series of books on various aspects of systems analysis and eventually to prepare a handbook on the art of applied systems analysis.

This is the overall structure of our research plan. As you can see it is an ambitious plan; IIASA has a budget of about 6 million dollars per year, which is quite small to carry out all those activities. We can only succeed by drawing on resources that lie outside the Institute as well. We do this in a variety of ways. The IIASA core consists of about 75 scholars paid by our national member organizations' contributions. But around this core, we have another ten or so guest scholars, people who come here whose salaries are paid by their home institutions--IBM, Siemens, the National Center for Scientific Research in France, and so on. That enables us to enlarge the amount of effort underway. In addition, we have external funds: money that comes to us from the United Nations Environment Programme, from the Volkswagen Foundation, from the Ford Foundation, from the Federal Republic of Germany, etc. This contributes another million dollars a year or so to our research activities. These are things that all go on still within Schloss Laxenburg. But our capabilities here are limited. What really helps to enable us to carry out work going beyond the resources that are available to us is a network of collaborating institutions; organizations in each of our member organization countries, that in one way or another have linked their efforts to our research program. So we have here, for example a coal task force to which less than one man-year of IIASA effort is devoted, concerned with the exploration of the future of coal as an option for energy; but we draw on contributions from the National Coal Board in the UK, from Ruhrköhle in the FRG, and from Czech and Polish coal institutions. We formed a coal task force to mobilize the international effort in this study. And it is through this collaborative research that we are able to get the amplification that makes our ambitious research program reasonable.

In addition, we try to catalyze research, by which I mean to stimulate activity elsewhere that does not directly link to our research program, but should eventually contribute to the solution of the problem we are interested in. For example, in our Energy Program we have identified large CO_2 releases as a major potential impact on the environment. We are not able to explore that as fully as we like, but work is underway at the National Center for Atmospheric Research in the United States and elsewhere, to explore this within the framework of the meteorological community. And a final shell in this model of IIASA's activity is information exchange. I do not think I have to explain that to you because of your participating in it today and for the next four days. We hold not only this Conference, but a good 20 conferences a year. We have the state-of-the-art survey series as well. In fact, I feel that this is one of our more important functions.

So it is a pleasure for me to welcome you to the Institute. I hope you are successful in your activities over the next few days, and that you will view your participation here as just the beginning or the continuation of a long-term association with the Institute.

INTRODUCTION TO THE CONFERENCE

G. Bruckmann

I am grateful that, in his welcoming address, Dr. Levien has already given a comprehensive introduction to the Conference. This permits me, in my introduction, to limit myself to welcoming most cordially both the old habitués and the newcomers to our conference series, and to just mentioning a couple of points on global modelling and IIASA. As already mentioned by Dr. Levien, IIASA, in its early days, decided not to construct a global model on its own but rather to assume a monitoring role in the field—to provide a platform for the exchange of ideas and of results achieved. It is with some pride that, in retrospect, we may state that this idea has succeeded to such an extent that IIASA global modelling conferences have attained a definite place in the history of global modelling. As concerns this Conference, let me express a word of particular gratitude to the two main contributing groups who made available introductory papers six weeks ahead of time, enabling us to send them out to all participants well ahead of the Conference.

Whereas the models discussed at our first three conferences were to some extent known to a wider community, this is not the case for the two models to be analyzed at this Conference. For that reason, we had expected the number of participants this time to be rather small. To our surprise, the last two weeks brought so many additional registrations that the number of participants again almost exceeds the capacity of this hall.

As in our earlier conferences, the first three days will be devoted to a discussion of the main papers and the fourth day to a presentation and discussion of other modelling efforts. The best of success to our deliberations!

SARUM AND MRI

THE SARU MODEL 1976*

P. C. Roberts, D. Norse, W. G. B. Phillips, J. B. Jones, K. T. Parker, and M. R. Goodfellow

INTRODUCTION

Location and Function of the Systems Analysis Research Unit

The Systems Analysis Research Unit (SARU) is located in London at the Department of the Environment. The Unit was established in 1972 following a world conference at Stockholm that was the first to be concerned with the environment. The Stockholm conference also raised associated issues of resource use and population growth. The Government of the United Kingdom recognized that the separate problem areas were closely enough connected to warrant a broad investigation of interactive effects. The establishment of a research unit to advise on the value of global models and to discover the extent to which quantitative methods could illuminate world problems was therefore approved. However, a part of the Unit's work is directed at problems within the UK related to the supply of resources and other factors affecting the environment.

Discussion Paper *Future World Trends*

Within the UK administration there is an interdepartmental committee entirely concerned with problems of population increase, resource availability, climate change, and pollution. This committee under the aegis of the Cabinet Office has published a discussion paper** with the title *Future World Trends* which attempts a frank appraisal of the problems. The paper is based on inputs from research institutions, departments of government, and from SARU. Since it is an official document there is no commitment to policies intended to meet the challenge of the "problematique". This is consistent with the SARU view that careful preliminary analysis must precede policy decisions.

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*The views expressed are those of the authors and do not necessarily coincide with those of the Departments of the Environment and Transport.

***Future World Trends*, discussion paper, Her Majesty's Stationery Office, London, March 1976, price 60p.

Relation of SARUM 76 to other models

The SARU global model is referred to here as SARUM 76. The following remarks are to illustrate the philosophy that guides our working style. Human societies operate within a hierarchy of constraints. The physical limits are the best known and set wide bounds on the variety of futures that can exist. Within these limits at decreasing levels of certainty but placing narrower bounds on the state of the system are engineering, economic, and social constraints. These constraints delineate the area of choice that remains and this area expands as we move further into the future. Experiments can be performed by relaxing constraints and introducing uncertainty into the system. The results of such experiments are a set of numerical variables. They tell us nothing about political regimes or the experience of the people whose behaviour they describe. Though we might therefore be accused of irrelevance, we believe that by setting ourselves modest objectives we are more likely to achieve useful results.

Objective of Offering Tools to the Modelling Fraternity

The ability to construct world models will develop through sharing ideas and methods. We would prefer to explain the tools we have devised and the techniques we have developed rather than to urge acceptance of the futures we have simulated. In an area which arouses strong emotions we should not let our feelings upset our judgement of the soundness or otherwise of a modelling procedure.

Transparency of Structure

Though the complete SARUM 76 is elaborate it is built up from simple modules whose structure is transparent. These proceedings give only a brief survey and for a full account reference should be made to *SARUM 76; Global Modelling Project*, Research Report No. 19 published by Her Majesty's Stationery Office, London, UK.

Repeatability

In order that our results can be tested by others we are making the program of SARUM 76 available as widely as possible. Special input and output routines have been written so that the implementation of different scenarios is relatively simple. The program listing on magnetic tape and program documentation is available on application to our London office. The program will be or has been implemented by IIASA, the Science Policy Research Unit at Sussex University, the OECD Interfutures Project in Paris, the Department of the Environment, Community Development and Housing in Canberra, and others.

Evolutionary Structure

SARUM 76 as it stands at present represents a stage in an evolutionary development. We belive that our basic mechanisms are a good approximation to the way societies actually behave. Additional mechanisms that are observed in the world can be added to the basic structure when and if necessary. We would welcome contributions and criticisms from others working in this field.

SARUM 76 as Descriptive Rather than Prescriptive

SARUM 76 represents the basic causal linkages that are found in economic systems. Our work is directed at the first stage of postulating and testing these relationships. It is necessary to understand a system before attempts are made to change it. We must be certain that the results of our policy measures will indeed achieve the desired objectives.

Disaggregation

Disaggregation is largely determined by the objectives of a particular study. We believe that food is more critical than energy or minerals and that the imbalance of resource control is of more interest than the gross trajectory of an aggregated world. We have therefore modelled food production in some detail and divided the world into rich and poor. If we were examining other problems a different disaggregation would probably be appropriate.

PRINCIPLES OF SARUM 76

SARUM 76 is a dynamic simulation model of a disaggregated world economy.

Characteristic Features

As opposed to some earlier world models SARUM 76 is disaggregated. In order to study food problems the world was divided into three strata (Figure 1).

- the United States of America,
- the developed countries of the north and south temperate zones plus the oil rich countries, and
- the less developed countries of South America, Africa, and Asia.

Within each stratum we recognize thirteen sectors. Each sector buys goods and services from other sectors and sells its output either to intermediate sectors or final demand. For the food

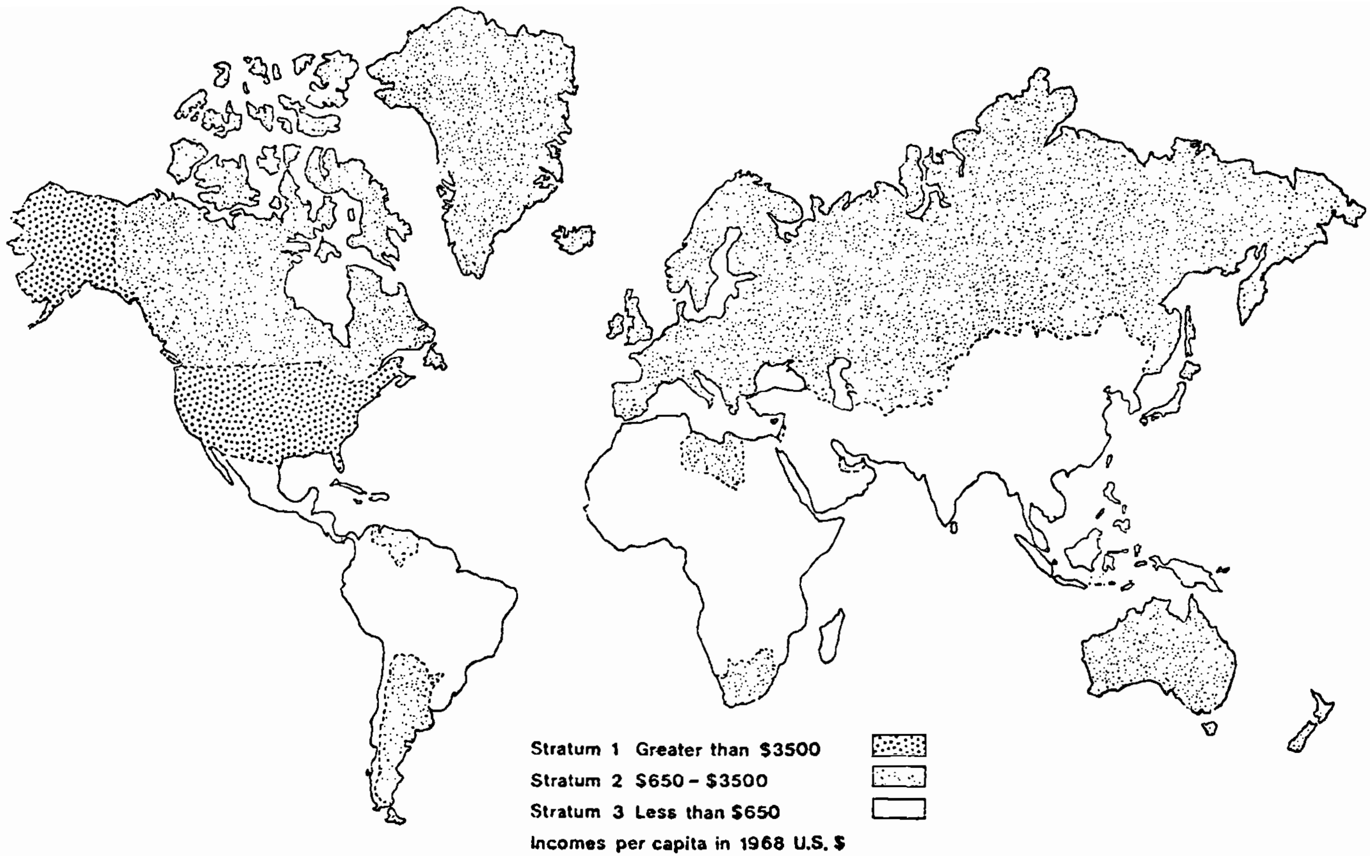


Figure 1. Composition of strata based on national disposable income.

study, nine of the thirteen sectors are concerned with agricultural production.

As far as possible the model is based on real world data as published by international agencies. Functional relationships specified in the model have been quantified by standard statistical methods. The causal linkages incorporated in the model are based on the principles of economic theory. We hope this will make our results intelligible both to simulation modellers and professional economists.

Economic Theory and Simulation Models

There are useful analogies between the theory of the firm in economics and the conditions for the viability of an organism in biology and ecology. In both cases we observe flows of input and output and find stocks of equipment and raw materials that must be maintained above a critical level. Competition, cooperation and negative feedback loops to maintain homeostasis are characteristic of a wide class of systems. We assert that such a paradigm is sufficiently general to describe the behaviour of most economic systems, regardless of political ideology. Different countries use different methods to secure the well-being of their members.

Our technique amounts to a synthesis of simulation methods and economic theory. This approach gives due emphasis to stocks (state variables, levels) and supplements the traditional economic preoccupation with rates of flow. The resulting models have dynamic properties that have suggestive parallels in real world economic behaviour.

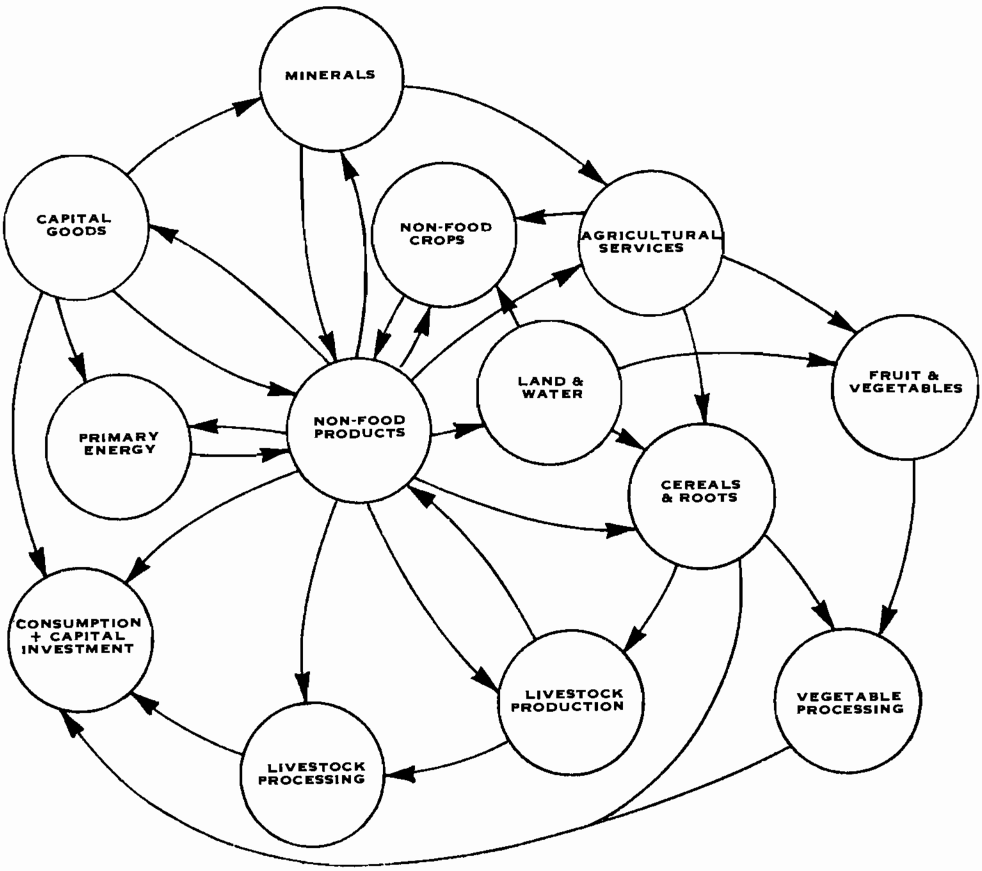
Economic Processes in SARUM 76

Consumption

The rate of consumption is taken to be dependent on per capita income and the relative price of the good. For the relation between income and consumption we have good cross-sectional data both between countries and between economic groups within countries. The relation between consumption and price is more speculative and depends on own and cross price elasticities derived from econometric studies and subject to consistency constraints.

Inventories of Stocks

Goods are produced by sectors and demanded by other sectors or consumers. Each sector's output is fed into a stock or inventory. If the size of the stock is equivalent to the desired coverage the price is set equal to the marginal cost. Movements of the stock level away from the desired coverage cause changes in price of opposite sign (i.e. prices are inversely proportional to stocks). A rise in price reduces demand via the price elasticities and simultaneously increases profits leading to an expansion of the sector producing the good. (Figure 2.)



Note 1. The sectors feed each other indirectly via stocks.

Note 2. "Capital investment" is distributed to all the sectors according to a profitability criterion.

Note 3. Minor transactions are omitted for simplicity.

Figure 2. Interrelationship of sectors for each stratum of the model.

Consumer Sovereignty

In equilibrium the sizes of the sectors match the pattern of demand imposed upon them. This pattern can change as income and costs vary. SARUM 76 is therefore a demand driven model and assumes consumer sovereignty. Production and demand by governments is not separately identified. It is tacitly assumed that the decisions made by governments reflect the wishes of the citizens.

The Role of Money

SARUM 76 does not represent financial institutions and there is no permanent stock of money. Money is however used to measure quantities for which there is no natural physical unit. Money is also used as a medium of exchange; the amount of money required to absorb the output in a given time interval is released during that time interval (allowance having been made for stock variation). This mechanism enforces the basic economic accounting identity:

$$\text{Cost of Output (wages, interest, profits)} = \begin{matrix}\text{Consumption} \\ \text{Purchases.}\end{matrix}$$

The model therefore operates with neither inflation or deflation and all values are expressed in base year dollars.

Production

The technology of the economy is summarized as a production function that describes the relationship between inputs and outputs. The simplest case is that of fixed coefficients and this is used for most of the intermediate flows in the model. Where capital/labour substitution is important the Cobb-Douglas production function is applied. A special two-part production function that allows water/fertilizer substitution has been developed for use by the agricultural production sectors.

As applied in SARUM 76 the output indicated by the production function is modified in two ways. Historical records show that the factors of production have become more productive and this is reflected in the model by multiplying the basic output by an exponential or logistic function of time. On the other hand one of the effects of the depletion of a resource is to make the factors of production less productive and to increase the quantities of raw materials required to produce a unit of output. Different procedures are implemented depending on whether the resource is limited by a rate of flow or a finite stock.

Allocation of Investment

The total amount of investment funds available is a function of per capita income and is based on empirical cross-sectional data. These funds are distributed in such a way as to make the marginal revenue per dollar of the sectors equal. This strategy maximizes total profits and matches the sizes of the sectors to the demand pattern at equilibrium.

Labour and Wages

Provisionally birth rates and death rates are regarded as exogenous to the system and for the base runs we have assumed either the United Nations "medium" projection or continued exponential growth. Every worker is assumed to be trained for work in one of the sectors. Whether he enters employment in the sector for which he is trained depends on the fraction of value added paid to labour. This relationship is based on empirical cross-sectional data. Workers are also free to move from one sector to another in response to wage differentials.

Profit Maximization

SARUM 76 assumes that sector managers will attempt to maximize profits. They will therefore choose a labour force that diminishes the difference between the marginal product of labour and the wage rate.

Testing Economic Relationships

It is difficult to test economic relationships by standard statistical techniques because of the difficulty of performing experiments and the problems raised by collinearity in the time series data. Econometric techniques can sometimes be used to overcome these difficulties and the additional problems raised by estimation of parameters in simultaneous equation systems. In some areas we can obtain clear-cut results and these add to the credibility of economic theory. An analysis of cross-sectional data on cereal farmers shows that they substitute labour for capital in response to relative cost so as to maximize their profits. An analysis of cross-sectional data on metals indicates that their price is largely dictated by the cost of mining and refining. This indicates that the market for metals is approximately competitive.

Indirect Tests of Economic Theory

A theory may be tested indirectly by incorporating economic relationships in a model and then observing whether the model is capable of generating behaviour found in the real world. In

SARUM 76 we find that the decline of a sector accompanied by the growth of a substitute sector has a characteristic logistic form. This is very similar to the manner in which many new products penetrate a market. We can adjust the time constants of the model to mimic the rates of penetration observed in the real world.

AGRICULTURAL RESOURCES

The Food Supply Problem

In contrast with other problems associated with finite resources widespread food supply problems already exist and are of major proportions. SARUM 76 was therefore structured to examine food production in the greatest detail. SARUM 76 is not however exclusively a food model and has a wide range of applications in other areas.

Sufficient food is grown at the present time to give the world population an adequate diet but the supply is not distributed evenly. Over 4×10^8 tonnes of grain are fed annually to livestock and many people consume more food, particularly more livestock products, than they need to satisfy their physiological requirements. Our model of food demand is therefore based on:

- Effective demand (i.e. ability to pay) rather than physiological demand, and

- Demand for dietary energy, on the grounds that if food energy requirements are met the diet will contain adequate protein and other nutrients.

The Agricultural Production Sectors

The system specification was designed to explore global food scenarios with the minimum number of sectors as follows:

Cereals and Starchy Roots

Cereal production dominates world agriculture. Cereals are grown on 71% of the cropped area and supply 53% of the total food energy. The other main staple is starchy roots which are grown on 5% of the area and supply 7% of the food energy.

Other Vegetable and Fruit Production

These products occupied 16% of the cropped area in 1968 and provide important nutrients.

Non-Food Crops

These occupy only 8% of the arable area but are important in economics and trade.

Livestock Production

Livestock provides only 11% of food energy on average, but in the developed countries as much as 40% of food energy is derived from livestock products. In some countries this sector accounts for up to 60% of agricultural GDP. Fisheries, which are relatively unimportant, are aggregated with this sector.

Livestock Processing and Vegetable Processing

These sectors permit the consideration of by-products and allow for the large increases in unit value that result from processing.

Agricultural Services

These manufacture fertilizers and pesticides.

Land Development

This clears land and prepares it for arable agriculture.

Irrigation Services

These maintain dams, wells, and irrigation networks and sell water.

Response Curves

More food can be obtained from the same area of land by the application of fertilizer and water. At low levels of fertilizer application there will be large returns but at higher levels the marginal return will fall to zero and may even become negative. Suitable functional forms fitted to cross-sectional data are used in SARUM 76. The additional return to fertilizer when irrigation is applied at the optimal level is represented by a simple multiplier of the rainfed response.

Depletion Functions

The stocks of land and water resources were split up into branches with greater or less constraints to development and appropriate development costs were assigned to them. It is assumed that within a stratum the resource will be developed in increasing order of cost. Analytical functions were fitted to the data relating costs to area of land developed and quantity of water produced.

Technological Progress in Agriculture

Technological progress affects the food producing sectors in two distinct ways.

- In common with the manufacturing sectors technological progress has improved the productivity of labour and increased the output from a given amount of capital and labour.

- Improved plant varieties have increased biological productivity so that they can benefit from higher rates of fertilizer application.

The second type of improvement is represented by an upward shift in the fertilizer response curve. The rate at which this shift occurs can be estimated by reference to historical data. There is a thermodynamic limit to fertilizer and land productivity so it is assumed that improvements in fertilizer productivity follow a logistic path from the current operating point to this limit.

Agricultural Production Sectors

Special production functions have been developed for the agricultural sectors to allow substitution between irrigation and fertilizer as well as capital and labour. The production functions consists of two parts: harvesting and yield which are regarded as complementary. Harvesting is carried out by labour and capital which can be substituted for each other depending on their relative marginal productivity and cost. Yield depends on fertilizer and irrigation water application and the optimum mixture again depends on return and cost. The outputs indicated by the harvesting and yield parts of the algorithm are not necessarily equal. In such cases the smaller of the two is regarded as the effective output. This production function is often referred to as the KALIF production function after the initials K, capital; A, area of land; L, labour; I, irrigation water; and F, fertilizer.

FOOD PRODUCTION AND CONSUMPTION

Demand Functions

The demand for food is determined by per capita income, relative price, and total population. The relation between per capita income and consumption of the various foodstuffs is based on empirical cross-sectional data. Expenditure on food increases as per capita income increases; however, the proportion of income spent on food falls. Expenditure on unprocessed cereals and starchy roots and unprocessed livestock products rises to a maximum at fairly low income levels and subsequently declines. In economic terms these are inferior goods. At higher income levels more is spent on increasing the variety of the diet, processing, and associated services.

The consumption of food is also affected by relative price by way of a system of own price and cross elasticities. Generally speaking a rise in price will depress consumption below the level indicated by the cross-sectional data. However very low income groups, spending a large proportion of their income on staple foods, may increase their demand for the staples when they rise in price because in effect their per capita income has fallen due to a rise in the cost of living. This effect may be important when the model is operating under conditions of extreme depletion. The price elasticities are themselves a function of per capita income. As per capita income rises the proportion of income spent on food falls and consumers become less sensitive to relative price changes therefore the price elasticities tend to zero.

It is necessary to assume in the model that a given good is homogeneous, i.e. it represents the same substance, however rich the purchaser. However, taken with the expenditure on food that we predict for the very rich, this implies that they are consuming an impossibly large amount of food. In reality, they are buying more expensive food, and also paying more for services such as retailing, preparation and catering (see Figure 3). This is dealt with by assuming that all expenditure on food that takes consumption over a specified physical level is really spent on non-food consumer goods and services. Also, for the very poor, it is important to ensure that the model of consumer behaviour does not represent them as consuming less food than that necessary to survive. Accordingly, a lower physical threshold is set so that if consumption falls below it, the consumption pattern is reorganized to restore the food intake, so far as is possible, to the threshold.

Retail Coefficients

Because of manufacturing and servicing costs the retail price of food may be more than four times greater than the farm gate price. SARUM 76 therefore distinguishes between unprocessed food having a low cost sold informally and processed food with a high

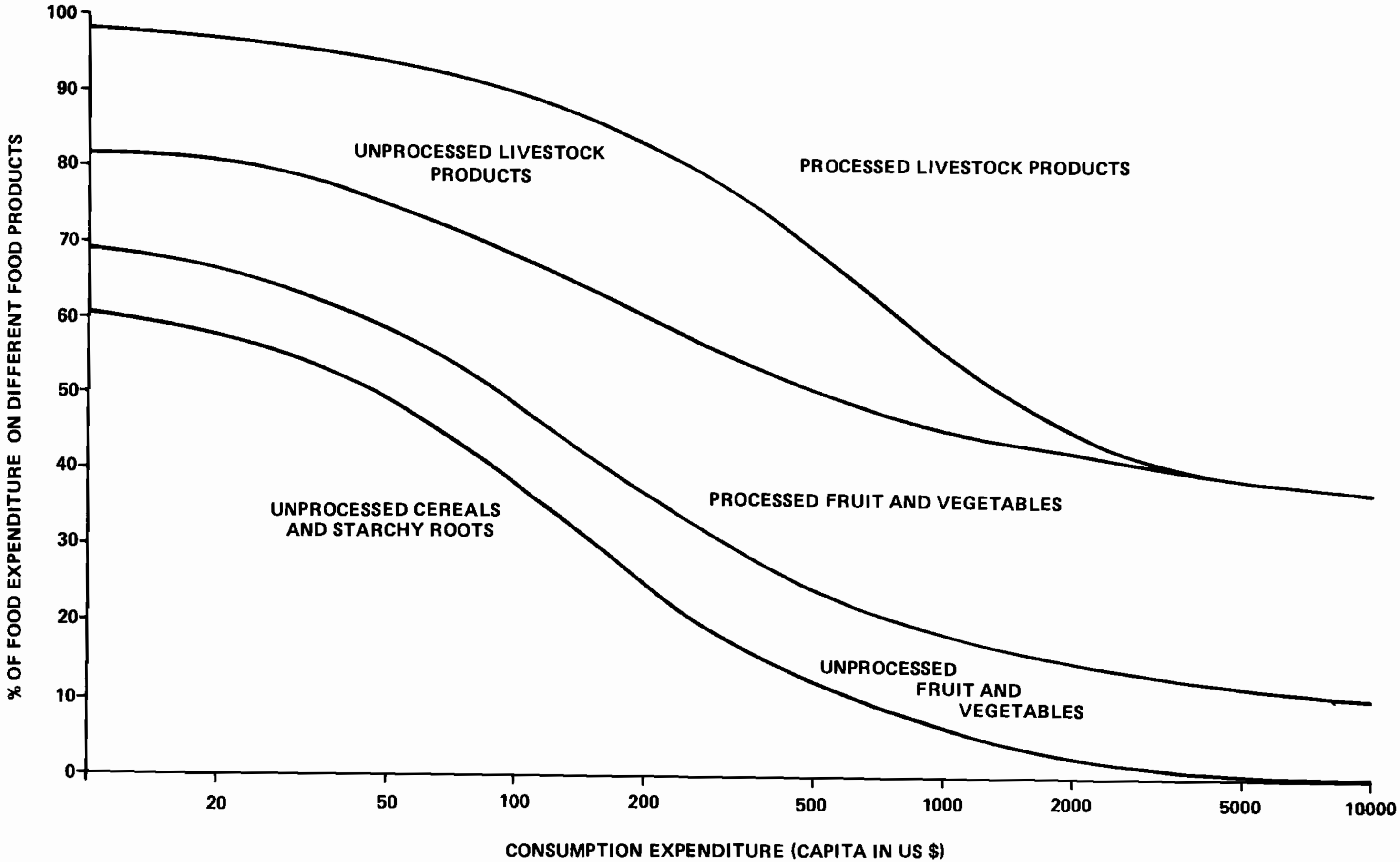


Figure 3. Proportion of food expenditure on different food products.

cost sold through shops and restaurants. The cost of retailing is represented by a coefficient that specifies the cost of packaging and distributing a kilogram of food. This coefficient may be made a function of per capita income.

Risk Aversion

There is evidence that farmers are not applying levels of fertilizer, pesticide, and irrigation water to the point where marginal costs equal marginal returns. This departure from optimality is most apparent in the less developed countries where the marginal return to fertilizer may be as much as $7 per $ spent. This behaviour is due to several factors.

- The investment in fertilizer is at risk to weather, disease, and pest attack while the crop matures.
- Change in the market price of the crop can occur between planting and harvest.
- Interest rates may be high and credit not available.
- Ignorance of the advantages of fertilizer and pest control.
- Supply and distribution difficulties due to a deficient infrastructure. Farmers must choose between the certainty of getting a small crop without fertilizer and the probability of getting a larger crop with fertilizer. If the crop fails in the latter case a peasant farmer may find that he has incurred a debt he cannot repay. It follows that in the less developed countries farmers often sacrifice short-term profit for long-term security making the perceived cost of fertilizer greater than the market price. In SARUM 76 a risk aversion coefficient is applied to the market price of fertilizer that raises the perceived price of fertilizer to the farmer when the yield per hectare is low. This coefficient declines as the yield increases to the levels found in Europe and Japan.

Seed Sectors

SARUM 76 models the growth of a new industry producing a commodity that is a substitute for another commodity by means of the "seed-sectors". The parameters of these sectors are input in the same way as conventional sectors. However the seed sector does not grow by attracting investment because it is not profitable. Under these conditions it is supplied with a subsidy that just balances the depreciation. If the conventional sector is made unprofitable by rising costs due to depletion the seed may become profitable and penetrate the market of the conventional sector.

Single cell protein (SCP) is a possible substitute for soyabean meal and fishmeal in the livestock production sector. Substitution can be modelled by setting up two livestock production sectors one of which purchases SCP and the other conventional feedstuffs. By running the model in the depletion mode the cost of conventional feedstuffs can be forced up and the price of SCP (which depends on a fossil fuel substrate) becomes competitive. In consequence SCP-fed meat products start to penetrate the market at a rate that depends on the relative costs.

Climatic Perturbation and Crop Production

A deterministic approach to forecasting the future climate is not possible because not enough is known about the key relationships. However, a probability can be assigned to the occurrence of a departure from the long-run average climate. Droughts cause more disruption to agricultural production than any other meteorological variable. In many arid and semi-arid areas the relation between yield and rainfall is nearly linear over the observed range. In some time series data, drought conditions appear to persist over a number of years and may coincide in major production regions on different continents. An adequate model must therefore include spatial and temporal autocorrelation effects. A synthetic sequence of rainfall data is generated with the same mean, variance, and autocorrelation as the empirical data. The relation between rainfall and yield is then used to modify the output from the cereal production sector. An ensemble of runs using different synthetic sequences can be analysed statistically to find the probability of the occurrence of famine.

EXPERIMENTAL CHARACTER OF SARUM 76

Uses of SARUM 76

SARUM 76 will not be used to make naive predictions of the future. Its value lies in its ability to work out the implications of policy options. By using the model to mimic "modes" of behaviour observed in the real world, our understanding of the factors that give rise to that behaviour is increased. In physics and engineering, models are built up from elementary relationships whose validity is established by experiment. In economics, however, relationships can change with time; behaviour can be modified by knowledge of the model output; entirely new technologies or social systems may come into being.

SARUM 76 will be used to study overall system behaviour and its response to change. Some of the relations and parameters that can be varied are:

- Production Factors: investment strategies, labour mobility, seed sectors of new technology.

- Demand Factors: population scenarios, consumer preferences and price elasticities.
- Distribution Factors: income distribution, trade biases, monopolies and cartels.
- Environmental Factors: depletion, pollution.

Anticipation

In the real world people react not only to the current state of the system but also to the rate of change. In consequence they have advance warning of an impending crisis and can take evasive action. If the model is operated in the depletion mode with substitute seed sectors, an economic depression may occur because a large investment is required in the substitute sector just at the time when overall output is dropping. This situation arises because investment is related to current profitability. In a modified version of the model a planning algorithm is used which extrapolates current trends to predict future demand. The rate of investment is then calculated so that the substitute sector grows smoothly to meet the target demand as the original sector is depleted. It is also possible to devise a hybrid strategy which is a weighted sum of the planned and unplanned investment. This weight is a measure of the discount rate. When the weight is equivalent to a low discount rate future events influence current decisions strongly. Under these conditions economic depressions caused by resource limitation can be ameliorated or eliminated. A high discount rate has the opposite effect (see Figure 4).

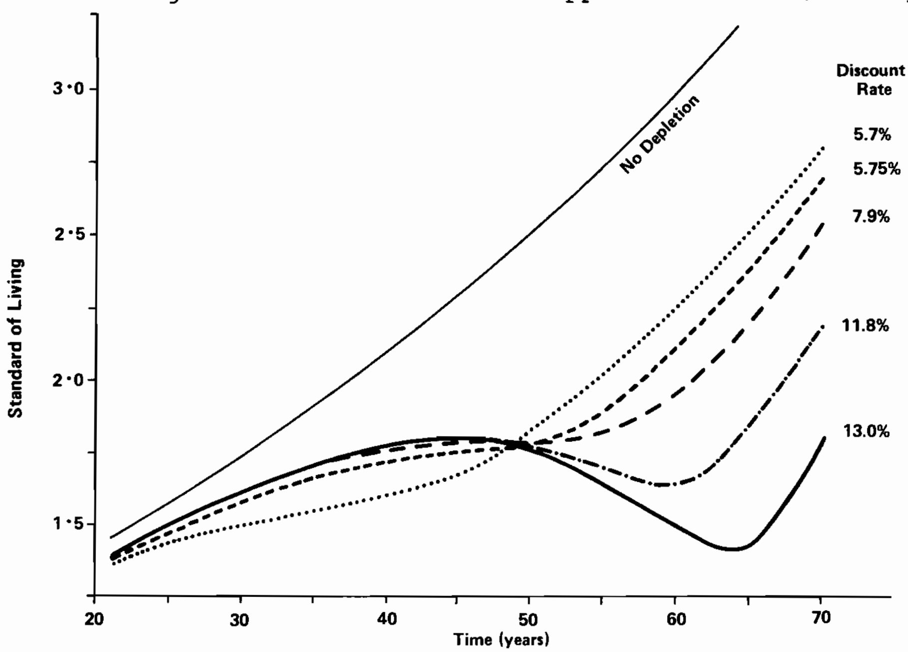


Figure 4. Variation of standard of living through a depletion/substitution transition.

Trade Bias Changes

The pattern of trade as observed in the real world cannot be explained simply on the basis of the difference between the price of a good bought at home and the same good imported from abroad. A number of other factors affect the buying decision apart from price; these include tariffs, embargoes, quotas, quarantine regulations, transport costs, and other barriers to trade of an economic and political nature. In SARUM 76 these factors are summarized in a matrix of coefficients known as trade biases. These biases modify the prices perceived by the consumer and alter the way in which he allocates expenditure between purchases at home and overseas. Consequently he tends to buy not from the cheapest source but from the source perceived as the cheapest.

These trade biases are calculated at the beginning of a run so that trade flows in the model simulate real world data. However there is no reason to believe that these coefficients remain constant. Historical data show that they change in a random fashion but that the rate of change of bias is strictly limited. This is due to the massive physical infrastructure on which trade depends and a certain inertia in the forming and breaking of alliances.

Of course future trading relationships cannot be predicted but a type of stochastic forecast can be made. An autoregessive process can be generated having the same statistical properties as the observed trade bias time-series. An ensemble of runs, using this synthetic time series data, can be analysed to find the probability that a given level of trade restriction will occur. A general bias interrupting trade and preventing the third world from buying food when necessary would have a strong influence on model behaviour.

Monopoly Action

The importance of these experiments is underlined by recent experience in the market for crude oil. The basic SARUM 76 algorithm can be simply modified to maintain a given sector at an abnormal level of profitability. The price of a good is kept above the free market price by adjusting the apparent marginal profitability so that labour and capital are not attracted into the sector. This is equivalent to restricting output by erecting barriers against the entry of new firms into an industry.

Energy Depletion

The sensitivity of agricultural prices to the cost of energy has been tested using SARUM 76. The results suggest that world food prices have a low sensitivity to likely energy price changes. This is because food production is less energy intensive than many manufacturing processes. It follows that the *relative* cost of producing food falls when energy prices rise.

Further experiments have been conducted by disaggregating the energy sector into fossil fuels, bio-fuels and electricity. A simple hyperbolic depletion function is used for the bio-fuel sector in which the upper asymptote of production is calculated from the current land area devoted to wood growing for fuel multiplied by the maximum forest productivity.

A depletion curve for fossil fuels is based on the cost of extracting available carbon from various types of carbon bearing rocks (e.g. oil and gas trap rocks, coal measures, tar sands, oil shales) using the assumption that carbon is distributed log-normally in the sedimentary rocks. This curve suggests a relatively slow rise in fossil fuel costs, however, it may conceal the problems associated with the timing of investment when switching to the new carbon sources. It would be necessary to disaggregate further in order to study these problems.

STRUCTURE AND CALIBRATION

Program Structure

The basic building block of the model is the autonomous sector. The program structure allows these sectors to be assembled in any desired configuration and facilitates sector aggregation and breakdown. Because the basic algorithms are repeated many times subscripted variables are essential for economy. The program was written in FORTRAN partly for this reason and partly because it can be used on almost any machine.

In order to aid user comprehension the program is split into four basic blocks which are themselves divided into subroutines.

Preparing Input Data

The data are presented in a descriptive way using key words. These data are transformed into numerical data suitable for the simulation routine. After the data for the base run have been set up only modifications need be entered for subsequent runs.

Running a Simulation

This has been written in a form suitable for integrating any set of differential equations using any numerical integration method. The relation between the state variables and their derivatives are handled by separate subroutines and the order of calculation is fixed.

Tabulating and Plotting Results

In principle only the values of the state variables need be retained but in practice nearly every variable is logged and put on a mass storage device. A subset of results is picked off and presented in graphical or tabular form. Points of interest can be examined further by extracting additional output from mass storage.

Dumping the Dictionary

At the end of a run every variable in the dictionary can be dumped to the line printer with its value at the last time step. This option is selected by default if any error, such as a negative stock, is detected.

Choice of Time Interval

The model is framed in terms of non-linear differential equations. These may be solved by a number of numerical integration methods, but Euler's method is the simplest and quite adequate for our purposes. There is a trade-off between the length of the time step and accuracy. At four steps per year there is a danger that the run will not survive the transient. More than six steps per year improves the accuracy very little. The program runs presented have been carried out with Euler integration and six steps per year.

Cost of a Run

Program development was mainly carried out on a CDC 6600. The main component of cost is the central processor time used in solving the differential equations. For a 3-stratum, 13-sector, 6-final demand goods per stratum model a good rule of thumb is 2 simulation years per central processor second. For this size of program 2 CP seconds cost £1. For a small number of sectors SARUM 76 is cheap to run and this makes exhaustive sensitivity tests and Monte Carlo analysis feasible.

Calibration

This difficult topic has two phases, matching the mode of model behaviour to real world phenomena, and matching actual numbers from the program output with real world data. Throughout the process it is essential to keep the objectives of the study clearly in view. Beyond a certain point, improving the data, relationships, and structure may add little to the usefulness of the model.

Data differs in reliability and may conflict if taken from different sources. Some items required are not available or incomplete. There may be conflicts between the data and the model structure. A hierarchy of the data and the hypotheses on which the model structure is based must be set up depending on the degree of confidence and on relevance to the objectives of the model.

The first runs of the program will show whether the model behaviour accords with the basic objectives. Fine-tuning can be used at a later stage to generate the desired numbers. In order to achieve the required results it may be necessary to modify items that rank low in the data/structure hierarchy. This is an iterative time-consuming process. SARUM 76 is intended to model rational behaviour over a wide range of income levels and under severe depletion. The model structure has therefore been amended until this objective has been achieved. The most important output variables are the sector production figures in physical and money terms. The model is initialized to make the level of production and the rate of growth of production for each sector accurate. Final consumption is known with considerable certainty; supply and demand can be reconciled by adjustment of the input-output coefficients. In equilibrium growth the rate of return must be the same in all sectors: this makes it impossible to use the published capital figures which are known to be unreliable. Labour force data are usually quoted as the total wage bill; the number of people employed is derived from the wage bill and the wage rate.

Initialization of the model depends on generating a consistent set of values for all the state variables of the model in the first time interval. This calculation is based on the values of output in the first time interval, value added, wage bill as a fraction of value added, and depreciation rate of capital for each sector; and on population, growth rate of stratum economy, growth rate of labour force, price of capital goods, and total value of investment in new capital for each stratum. From these data all the other variables can be calculated so that model generates a transient-free trajectory with a growth rate close to the observed figure.

UNCERTAINTY

The Concept of Uncertainty

The empirical data used in a particular model structure do not, in general, consist of a set of point values but of a set of probability distributions. As a consequence the projected model variables belong to a set of distributions. There are further components of error associated with different aggregations, different subjective assessments of what constitutes an adequate model algorithm, and the possibility that the system being modelled may change its nature in the future, invalidating any initially valid structure.

Monte Carlo Methods

Because SARUM 76 is relatively cheap to run the Monte Carlo method for uncertainty analysis was adopted. The technique depends on repeated model runs with randomly perturbed inputs. Because the inputs are varied simultaneously any synergistic effects are included in the dispersion of the outputs. An estimate of the probability distribution function (PDF) of the outputs is obtained rather than simple confidence intervals.

An early version of the model without depletion was used in the tests. The runs were performed in batches in which variation was introduced into a progressively larger set of parameters. The marginal contribution of each source of uncertainty could then be examined directly. The parameters used in this study were assumed fixed through time. For changing parameters, they would be allowed to evolve by pseudo-random processes based on observed behaviour. Each Monte Carlo replication uses a different realization of the random process. The model outputs at some future time can be regarded as functions of the inputs and this relation can be approximated by the general linear model by using regression techniques. The estimated coefficients are equivalent to conventional sensitivities. If the linear approximation is inadequate the variances are not additive indicating that the model exhibits synergistic behaviour. The output was analysed by using a standard regression package. The results may be expressed in dimensionless form as amplifications.

Income Distribution

For some purposes it may be sufficient to regard the populations of the strata as homogeneous with the same per capita income. However, when examining resource limitations, different demand patterns resulting from a spread of incomes affect the pattern of economic activity and therefore resource usage. For example a redistribution of income from the rich to the poor may increase the demand for food and increase the rate at which land resources are consumed.

A useful measure of the inequality of income is the Gini coefficient which is defined in terms of the Lorentz diagram. If the underlying PDF is assumed to be lognormal there is a simple relation between the Gini coefficient and the standard deviation of the lognormal variate.

National Gini coefficients usually lie between 0.3 and 0.6 and change slowly with time in a random rather than a systematic manner. Centrally planned economies have a somewhat lower than average Gini coefficient. High income countries show a slight tendency to greater equality. Variation of income in a stratum is made up of international and intranational effects which have a roughly equal effect on the distribution of income within a stratum. The GDP per capita growth rates of individual countries differ widely and strongly affect the evolution of the income distribution in a stratum. It is assumed that changes in national income distribution are random and that evolution from the situation in the base year is due to differential growth. Some countries are observed to have growth rates persistently higher or lower than average for several years. Such persistence can cause rapid divergence of national per capita incomes within strata.

To model these effects the stratum is divided into equal population groups. Each group is assigned a fraction of the total stratum income as indicated by the observed base year income distribution. The incomes of the groups grow at a rate that fluctuates about the mean stratum per capita growth rate. The fluctuations are drawn from a first order autoregressive process with positive autocorrelation to simulate the persistence effect. The trajectories of the group incomes can then be aggregated to give a synthetic time series for the stratum Gini coefficient. Repeated simulations with different pseudo-random number sequences yield an envelope of likely Gini coefficients. The mean trend and size of the envelope are strongly influenced by the time constant associated with persistent deviations from the mean growth rate. An estimate of the time constant can be obtained from empirical data.

The Gini coefficient time series was fed to the model which calculated the fraction of total income paid to each income group. The demand pattern for each group was calculated from the standard demand functions. The indicated consumption by each group for each good was summed to give total demand.

The results of these experiments suggest that:

- The group most strongly affected by income inequality is the poorest section of stratum 3 whose food energy intake is very sensitive to the Gini coefficient.

- Consumer demand for cereals and starchy roots reaches a maximum and declines if the Gini coefficient is low.

- Consumer demand for processed meat is relatively insensitive to income distribution because the bulk of demand arises from income groups at or near saturation.

- Greater equality leads to increased pressure on land as the poorer groups move more quickly to foods that provide fewer calories per hectare.

There is some evidence that higher incomes are correlated with reduced fertility so that greater equality would lower the population pressure. However this relationship has not yet been incorporated in SARUM 76.

FEEDBACK LOOP STRUCTURE OF SARUM 76

SARUM 76 attempts to simulate the viability characteristics of living systems which can adapt themselves to a changing environment. Viable systems display homeostasis and can persist in a steady state. In terms of the model this implies that at least one of the state variables must remain constant on average through time. In order to achieve this objective any change in a state variable must induce a change in the system which brings the state variable back to its original value. Such a system can be simulated by including closed negative feedback loops in the structure (see Figure 5).

In each sector of SARUM 76 there is a state variable representing "stocks". A fall in "stocks" causes a rise in "price" and "value added". This leads to an increase in "marginal profit" and the "marginal productivity of labour". The former attracts additional "investment" to the sector which increases "capital" and therefore "output", and the latter attracts additional "labour" to the sector by increasing "wages" and this also increases "output". The increases in "output" will tend to replenish "stocks" thereby bringing them back to their original value. The negative feedback loops confer great stability on the model and it can survive large transients due to inconsistent data or deliberately imposed step changes in exogenous variables.

Response to exogenous changes can be illustrated by experiments carried out on a simplified version of the model. For example a sudden influx of population into a stratum has the immediate effect of reducing percentage employment and expenditure per capita. However because wages are driven down the desired labour force rises. Simultaneously profits rise so that investment is increased and the larger working force is balanced by a bigger stock of capital. Expenditure per capita quickly recovers to its former level.

For another example expenditure could be switched suddenly from investments to consumer goods. Under these conditions the sizes of the capital and consumer goods sectors rapidly adjust themselves to meet the new demand pattern. The capital goods sector becomes temporarily unprofitable and the size of its capital stock and labour force shrinks. The situation is reversed in the consumer goods sector. The response of the model to sinusoidal inputs may also be tested by modulating one of the exogenous variables. Frequency response curves show well marked maxima indicating a resonant frequency between 5 and 7 years. The frequency at which resonance occurs may be adjusted by varying the time constants of the model. It is therefore possible to make the model simulate the dynamic and cyclic behaviour observed in the real world.

The time constants control the dynamic behaviour of the model and affect the rate at which a state variable approaches its target value. Long time constants imply that the standard of living

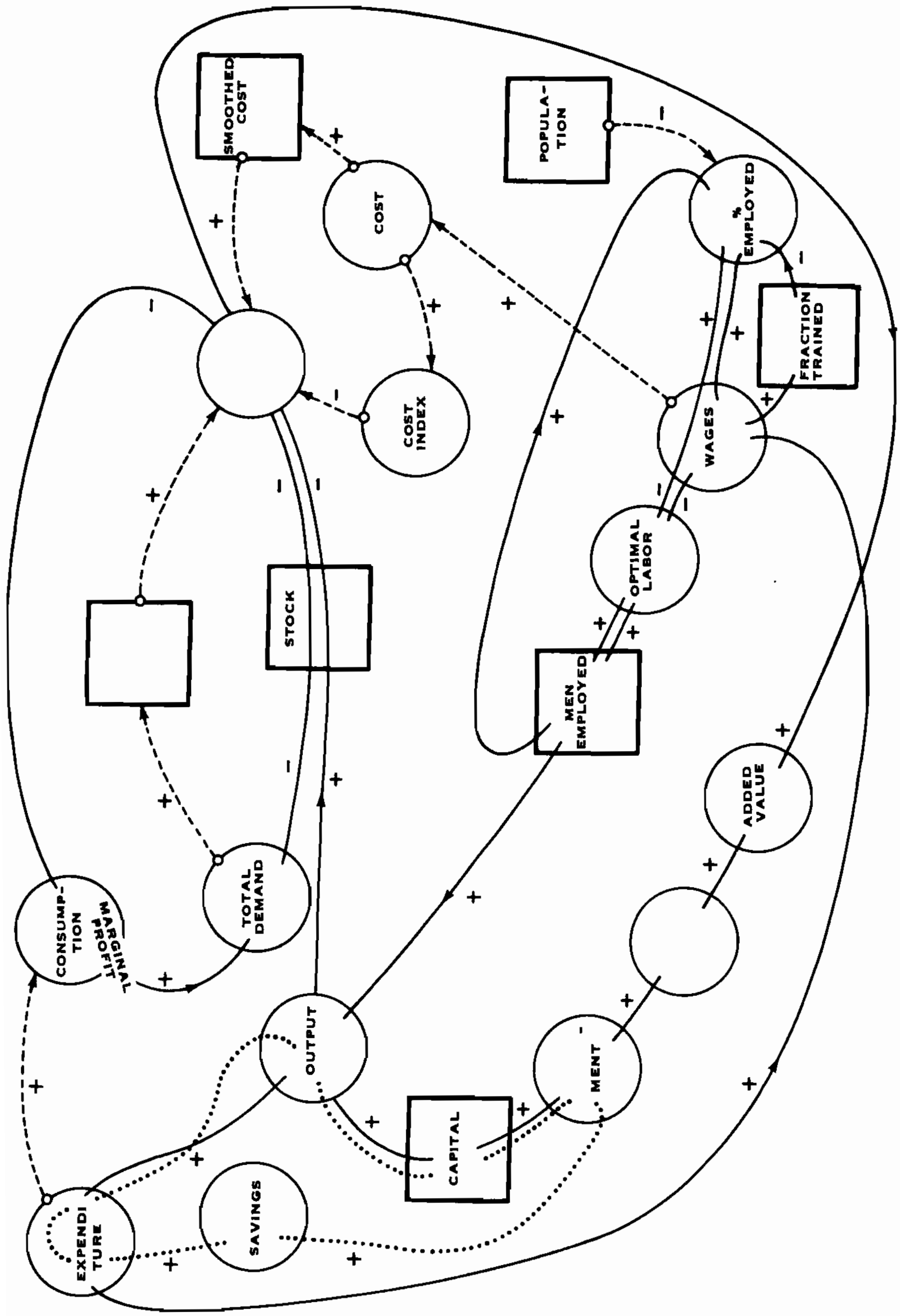


Figure 5. Dominant feedback loops SARUM, April 1976.

is persistently lower than the standard attainable at full employment. The time constants also determine the rate at which seed sectors take over the market of a conventional product.

During the last 200 years the standard of living has greatly improved in most parts of the world. This improvement is associated with the development of new sources of energy and the substitution of capital for labour. Part of the increase in the standard of living can be attributed to the growth in the capital stock but there is a residual that can only be explained by an intrinsic improvement in capital goods and production methods. This factor is known as technical progress and model behaviour is very sensitive to the assumptions made about it. For the base runs it is assumed either that technical progress continues at the rates observed in the past or that it follows a logistic trajectory being constrained by some physical limit.

It would be preferable to make technical progress endogenous to the system. For products that require bulk handling it seems reasonable to associate technical progress with economies of scale. In these cases total costs are related to capacity by a two-thirds power law and unit cost is proportional to the inverse of the cube root. This is because cost is related to surface area and capacity is related to volume. In a growing industry plant size tends to increase and it is often found that cumulative production is correlated with falling costs related to economies of scale. Persuasive technical progress algorithms might also be based on vintage capital and production functions with increasing returns to scale.

DEVELOPMENT

Population Projections

The most useful way to consider population trajectories is to quantify the implications of different assumptions rather than to build in mechanisms that are supposed to explain the underlying causes of fertility change. However, to choose suitable exploratory trajectories some limits must be found which apply to the speed of potential fertility change.

During the very long period of time before the development of agriculture, the human population was almost stationary which implies that the birth rate was on average equal to the death rate. During the last 150 years advances in hygiene and medicine have lowered the death rate and this has been followed by a fall in the birth rate. Temporary increases in the death rate due to the two world wars have been accompanied in each case by a temporary increase in the birth rate. An analysis of the data from 40 countries suggests that the birth rate does tend to follow the death rate but that the associated time constant that determines the adjustment rate can vary widely. Very small time constants are not observed and this places the upper bound on the rate of population adjustment. The UN population projections imply a range of values of the time constant but this range is much narrower than that observed in the real world.

It is possible to include a relationship in the model that drives up the death rate as starvation occurs due to a deterioration in the food supply. Starvation-induced high death rates will probably be indicated if the model is run with the higher population trajectories but these tests have not yet been made.

Scope of SARUM 76 Development

SARUM 76 has a simple basic structure but it can be elaborated and modified in many ways. It is possible to simulate all the departures from the economic paradigm that critics have been able to suggest. As far as possible the variables used in the model are those actually measured and recorded. Where appropriate, variables are measured in physical units (kilograms, hectares, man-hours, joules, etc.). The relationships have been estimated by standard statistical methods on empirical data.

SARUM 76 is capable of a wide range of behavioural modes and this has been achieved in a simple model that is economical to run. Different conceptions of the future can be incorporated in the model and their implications worked out in detail. For example it has been held that the technical possibilities exist for supporting large populations in affluence and only need implementation. The feasibility of such a strategy can be tested by setting up seed-sectors in the model representing industries which desalinate sea-water, discard excess heat into the oceans and so forth. Model runs will indicate whether such scenarios are economically viable and whether the speed of development required is likely in the light of past experience. The format of SARUM 76 compels attention to the economic and technical details and displays the full implications of each policy option.

SARUM 76 can also handle the costs of pollution abatement. These costs fall on producers but are passed on to consumers and this may affect the pattern of demand. If people are prepared to bear these higher costs they will approve strict legislation that protects the environment.

A simple modification of SARUM 76 will allow for competition for land between urban development, agriculture, and amenity space.

Availability of Program to Modelling Groups

Open discussion of world models and free availability of programs is very desirable. Accordingly the program of SARUM 76 is available either on cards or magnetic tape to those who wish to make use of it. A detailed description of the model will be published by HMSO in mid-1977 as a research report entitled

SARUM 76; Global Modelling Project. Program documentation can also be provided. Successful use of the program requires familiarization by using simple structures in the first instance. Some assistance can be given in setting up initial tests.

DISCUSSION

Mottek opened the discussion of the SARU model by stating that neoclassical theory is not adequate to describe disequilibria. If the purpose of a model is to predict catastrophes or crises, it should not use neoclassical theory. As a long-range model, SARU should use neither neoclassical theory nor prices. Phillips replied that they have tried to achieve a synthesis between the simulation model and economic theory; economic theory defines the target, the simulation model the trajectory.

Herrera warned against stressing analogies between biological and economic systems too strongly; the dissimilarities are often more important than the similarities. He went on to say that it would be fallacious to believe that SARUM is equally well applicable for developing countries as it is for industrial nations. The treatment of the price mechanisms and the absense of "government" to some extent make the model unfit to depict problems of developing countries. Phillips replied that features depicting better the mechanisms at work in developing countries could, in principal, be incorporated into the model without any basic difficulties.

Lord Kennet stressed that economic theory, as incorporated into models is increasingly incapable of depicting the economic and social interrelationships really at work, both in less developed countries and industrialized nations.

Panov asked whether it is intended to incorporate into the model the influence of different social policies on food consumption, e.g. in different strata. Norse replied that this is not intended.

Richardson wanted to be reminded of how supply and demand are linked. Norse and Parker replied that the linkage basically occurs via stocks, not by solving simultaneous equations at every time-step.

Parikh pointed out that at a very low income level consumption of cereals can be what may be called "inferior cereals". If one were to look at a selection of data of consumption quantities in a country such as India, one would find that cereal consumption at the very low level is not all that low. Norse replied that this was not yet included, but it can and will be included quite easily. The calorie coefficient pattern, how many calories one gets for a dollar at different levels of income, can be specified appropriately to take care of this aspect.

Keyzer asked how the production choice is made between different types of crops if constant returns to scale are assumed. Norse replied that the cropping pattern is assumed fixed. Keyzer asked how the transition is made from cross-sectional equations at the country level to the stratum level. Parker admitted that they have no ready solution to this problem. Batteke pointed out that in many cases social habits may impede the realization of solutions suggested by equations that are based on the market mechanisms.

Lord Kennet asked for more information concerning the backward runs. Parker replied that, although the system of differential equations may be stable if run forward, it need not necessarily yield the 1900 values if run backwards from 1970.

Panov asked whether the model is used mainly to prove or illustrate certain economic hypotheses. Parker replied that the model certainly reflects many aspects of accepted economic theory, but in addition many dynamic relationships based on common sense.

Kaya asked whether a model that is continuous in time can comply with quarterly or annual data. Parker replied that differential equations depict reality better than difference equations, insofar as decisions are also being taken continuously rather than once a year.

Some discussion arose about the growth rate which is in part chosen exogenously and partly yielded by the model.

Page asked whether the use of generalized data based management techniques had been considered. Parker replied that, so far, they have found it more convenient to prepare their own data.

Bruckmann remarked that a comparison of Gini coefficients from different countries requires much caution. As a rule, Gini coefficients are computed from income tax data rather than from income data, and the Gini coefficient reacts very sensitively to any truncation on the lower end of the distribution. Goodfellow replied that these data were used mainly to obtain some general conclusions.

Pestel wondered to what extent the model can actually be used as a planning aid for the political decision maker. For instance, does the model help to determine what decisions could be considered "robust" (in the sense that they do not prevent further corrections)? Hopkins underlined the importance of this question; data on income distribution are often very poor, if not misleading. Works by Kuznets, Tinbergen, and others have shown that income distribution is rather equal at low per capita values, tends to become more unequal when per capita income rises, and levels off again when the average income becomes very high. Hence income distribution cannot be considered stable. He also believes that, if differences were measured in purchasing power, the result would be that inequalities within countries would be more marked. Goodfellow replied that not all demand curves were based on national data, but some on surveys for certain income groups.

Preining asked whether the sensitivity of particular variables had been tested. Goodfellow replied that the Monte Carlo approach gives good insight into this problem; it turned out that the model reacts particularly sensitively to changes in prices.

Pestel pointed out that the rapidly changing age structure (increase of proportion of people above 65) may have a much stronger impact upon the demand pattern than income data. Norse replied that his investigations have not yielded dramatic changes.

Keyzer asked whether the results of the parameter estimation procedure were used as input for the sensitivity analysis, and whether the results obtained from the model concerning agricultural versus nonagricultural portion of GDP are fed back to the income distribution. Goodfellow replied that the model is not calibrated by a genuine estimation procedure, but rather by "guesstimates". The same holds for the second question; a real urban/rural split is among the plans for the near future.

Herrera pointed out that constancy of income distribution tacitly assumes constancy of the social system. Any major change of a social system gravely changes both the income distribution and the pattern of consumption. This may be of particular importance for new nations. Goodfellow replied that he and his colleagues were increasingly dissatisfied with the kind of disaggregation they used.

In reply to a question by Batteke, Goodfellow asserted that the agricultural technological improvement function is of a logistic type, which leads to a certain leveling off after 30 years.

Parikh referred to the fact that, when the multiple regression technique was applied, only a small subset of the basic parameters explained the bulk of the variations, and asked whether it had been checked if, by applying another Monte Carlo simulation excluding the variations of these significant parameters, the variations in the final output were small. Goodfellow replied that no test rerunning of the Monte Carlo simulation had been carried out, but the conclusions should be pretty robust.

M'Pherson asked whether the use of factor analysis had been considered when it turned out that multiple regression techniques did not yield any trend patterns in income distributions. Goodfellow replied, that, mainly due to lack of time, this had not yet been done. Roberts stated that the programs they used are open to the scientific public to be used by whoever wishes to use them. He warned, however, that it may take two to three months before one had acquired sufficient familiarity with them to do so.

Richardson asked whether the basic differences between SARUM and earlier global models could be summarized in a few sentences. Roberts replied that SARUM is similar in structure to the Forrester/Meadows model in as far as decision-making is programmed into the model but that SARUM is descriptive rather than prescriptive. Tempting as a constantly interactive model may be, any interaction implies tacit assumptions about all other possible interactors; we have neither world government nor world agreement As to the optimal degree of disaggregation, this is not only a question of maneuverability but also one of cost (in money or in time). Pestel defended the interactive approach, if properly mapped. If a model were to be of any use to a decision maker, he must find himself or his role explicitly reflected in the model.

Osterrieth asked whether the long-term goal of the model was not somewhat obscured by short-term results and what the nature of the linkages between regions was. Roberts replied that short-term effects are only observed at the service of long-term development and the linkages between regions are via trade only. Other linkages could have consisted in capital movements, migration, or direct aid. Test runs of these have resulted in surprisingly small effects.

Lord Kennet asked whether SARUM has a main point or main paradigm and whether or not the model has been or will be used by British authorities. Roberts replied that the team undertook its work in a rather open-ended way, hoping that the results would be useful. SARU does not report directly to a minister, but to a committee advising a minister.

McLean asked whether one model suffices to give advice to the decision maker, or whether the outcomes of one particular model can only be validated if compared to the outcomes of other models. Roberts agreed that such comparisons would be valuable, although in practice not often attainable. Hopkins stressed that, to meet the needs of this world, prescriptive (normative) models are more useful than descriptive models. Roberts replied that explicitly prescriptive models may suffer from wishful thinking; solutions given by a descriptive model are, in this sense, more valid (e.g. the solutions proposed by MOIRA).

Gelovani pointed out that the descriptive approach used does not reflect the mechanisms at work in a centrally planned economy. Roberts agreed in principle, but added that as soon as both systems are demand driven, their black-box behavior shows distinct similarities.

Slesser asked if the model provides an answer to whether an elitist or an egalitarian society is better fit to achieve the greatest spread of welfare. In reply, Roberts said that the question should not be phrased in this way: The real problems lie in the terrible impact of trade-offs; to give more food to a growing population in poor countries makes them dependent on imports they cannot afford, to increase their own cultivated land may result in ecological trade-offs, the more interconnected a society, the more vulnerable it becomes etc.

Parikh warned against considering robustness of a model value to be strived for as such from the beginning. Robustness can only be determined by a sensitivity analysis after the model has been completed. Roberts agreed.

M'Pherson opined that what has been expressed as a fault of SARUM is in fact an advantage, namely of reflecting the market mechanisms in international relations and not being prescriptive. Herrera claimed that for long-term models the differences between the normative and the descriptive approach begin to vanish; they all become normative. On the other hand, it is impossible to build an "objective" model. Roberts contested this statement; in natural science, is it not possible to investigate free of ideology? Herrera replied that any application of science is determined by ideology. Roberts agreed that the choice of the topic to be investigated is culture-dependent and therefore not ideology-free, but the methodology used in the descriptive work is not culture-dependent; there is no capitalist or marxist thermodynamics or econometrics.

LONG-TERM NORMATIVE MODEL OF DEVELOPMENT, MRI

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CONTENTS

FOREWORD

Research on the construction of long-term, normative models of national development was started at the Polish Academy of Sciences (Institute for Organization and Management and Computation Center) in 1972. At that time our main objective was to construct a normative model that could be used for the long-term planning and forecasting of national development.

We have constructed and investigated a number of different models based on different methodologies and varying degrees of aggregation. They mainly dealt with specific (sectorial) aspects of general complex modelling problems. In particular, we spent a lot of effort trying to incorporate normative aspects into the development models, described by nonlinear dynamic production functions. Some of the models investigated seemed very attractive from the theoretical point of view but needed statistical data that were not available and in order to use the data available we had to abandon certain rigorous concepts and theories.

In order to have a good model of national development we had to take into account the complex interactions between production, consumption, demography, and environment. The model also had to take into account the existing hierarchical management system, the interactions and cooperation between regions of the country and with other nations, including foreign trade, credits, scientific cooperation, etc. It became obvious that we could not model these spheres of activity successfully without introducing the concept of decomposition (i.e. decentralization) and aggregation. As a result the concept of a decentralized system of models, in cooperation with a core model (called MRI), was adopted. The concept was that a relatively simple core model generating the basic macro-data used as inputs to the sectorial and regional (more detailed) submodels should be constructed. The sectorial submodels generate data that are fed back to the core. As a result an exchange of information between core and submodels takes place and the general accuracy of modelling (which can be checked in historical runs) increases. The concept of aggregation also enables the linkage of different national models in order to construct a normative model of global development. Following the concept described, we have constructed several core models with different degrees of aggregation (labeled MRI, I = 0,1,...,5), as well as a number of specialized, sectorial submodels (such as energy, agriculture, regional, and environment submodels).

Research on the development of the existing and new MRI models continues. The modelling is a continuous process of approximation to constantly changing reality. Each particular model is a step in that approximation. The constant progress in science, modelling technique, and collection of data enables us to construct a better model at each step that offers a new, improved tool for planning and forecasting purposes.

This paper reflects the present state of our modelling methodology. As the core and data base, the MR4 version of MRI has been used. In order to simplify the exposition we are trying to concentrate on the methodological aspects and avoiding many details (mostly in numerical and economic analysis).

The papers are grouped in three parts. Part A deals with general methodological aspects; Part B deals with the production subsystem and consists of three papers describing the construction of MR4, prices, and technological change and the effect of foreign credits on national development; Part C deals with personal and aggregate consumption, including in particular the health service and education models.

The research and construction of MRI models would have been impossible without the support and encouragement we have obtained from many institutions and individuals. In particular, it is our pleasant duty to acknowledge the support we got from the Institutes and Scientific Committees of the Polish Academy of Sciences, the Planning Commission, Central Statistical Office, and many specialists, representing the economic, technical, and environmental sciences.

We would like to convey special thanks to the Soviet scientists from the Central Economic and Mathematical Institute (CEMI) and Institute of Management Problems (IPU) who invited us to Moscow to present the MR4 model. The discussions we had in Moscow were very helpful and stimulating for our future research programs.

R. Kulikowski

A. METHODOLOGICAL ASPECTS

R. Kulikowski

INTRODUCTION

Few academic discussions in recent years have become as controversial as those regarding the application of computer-operated mathematical models (see Reference [7, 10]) which forecast the future of mankind.

It has been argued, for example, that the modelling methodology usually takes a descriptive, i.e. passive, attitude with regard to global development processes. As a result, even slight tendencies towards a crisis may appear as if they had been determined, in which case the whole future development appears predestined and "doomsday" seems inevitable. It has also been argued that descriptive models do not take into account the changes in the system of socio-economic values and development goals, which have a direct effect on the consumption structure, allocation of resources, prices, fertility, and the growth of population, etc. Since, in descriptive models, decisions cannot be introduced explicitly, one cannot find out what can be done when the crisis is in sight and what chances one has in trying to avoid or reduce the effects of the crisis. It was proposed that, in order to have a realistic global development model, the normative rather than descriptive approach should be used.

First of all, that approach requires the existing system of national development goals and the decision system, which is capable of implementing these goals, should be investigated. It is also necessary to investigate to what extent the system of development goals can be realized when one takes into account the constraints imposed by shortages of resources, protection of the environment, etc.

Since most of the decisions regarding the allocation of resources take place at the national or regional level, it is convenient to start global modelling efforts with relatively simple national models. The next feasible step is to see whether the national development policies could be coordinated in order to make the best contribution to the welfare of global development. It is possible to observe that, using the above approach, one arrives at the global model by a "bottom-up" process of construction and linkage of national submodels. Another possible approach employs the "top-down", or decomposition technique. The advantage of the "bottom-up" technique is that one can use the original data base, whereas the "top-down" technique uses mostly aggregated data (from international organizations) which must again be decomposed when the model is extended.

An additional advantage of the "bottom-up" approach is that it enables many questions to be answered regarding the perspectives of national development. In particular, it is possible to learn how global development affects the national plans of development; what the possible national specialization in production, trade, research and development, etc., could be; what the best policy is in population growth, specialization in education, science, etc. Such an approach is also helpful when trying to answer the basic question: How much can a country benefit from international cooperation? Should it follow the strategy of complex isolation and autarky or engage in any of the various forms of international cooperation?

Using the optimization theory, we shall demonstrate that the strategy of broad international cooperation yields a faster socio-economic growth. The strategy of international cooperation (e.g. The Council for Mutual Economic Assistance (CMEA) created by the socialist countries) has already proven to be beneficial to each member country.

The approach used in this paper can be called "optimistic" (compared to the approaches used in the so called "Doomsday Models") in the sense that it relys on the realistic assumption that mankind will choose the policy of international cooperation and will optimize the allocation of scarce resources rather than follow the passive attitude and apathy when confronted with a crisis or catastrophy. The increase in international cooperation, especially after the Helsinki Peace Conference, indicates that such a strategy is feasible.

Following the "bottom-up" approach, outlined above, research on the construction of a long-term, normative model of national development was begun in 1972 at the Polish Academy of Sciences. As a result of that research, a number of models (labelled MRI, I = 0,1,2,3,...) were constructed. The purpose of the present paper is to describe the general methodology which was used for the construction of MRI models.

The most difficult problem the modellers faced was the necessity to formalize the development goals which were formulated in a descriptive form. The next problem was to describe the system under consideration in mathematical language; i.e. find the appropriate models of production, consumption and environmental subsystems, as well as the set of development constraints. In order to do that we have been trying to use, as much as possible, the existing methodology in macro-economics, environmental studies, system analysis and computer sciences. In this respect, papers written by scientists from socialist countries [1, 3, 9, 13, 17, 38, 42, 43, 44] were of particular help. However, many sacrifices were inevitable, especially in the cases where existing theories did not fit well together. Some extensions of the concepts commonly used were also necessary.

As far as the development goal was concerned, the classical utility theory seemed appropriate but an extension enabling us to deal with dynamic processes was necessary. The Polish national development goals and policy objectives are clearly outlined in the Constitution of the Polish People's Republic. According to the Constitution, "national policy should contribute to the country's full political, socio-economical and cultural development. It should contribute to the national strength and independence, realization of socialist ideology, strengthing of friendship and cooperation with allied countries and all peaceful nations in order to consolidate and secure global peace". The Constitution also states that the main socio-economic goal of Polish internal policy is "systematic improvement of the material, social and cultural living conditions and the constant development of production factors". Realizing these goals, the Polish government submits the budgets to the Polish parliament at the end of each year for approval. In other words, the nation decides each year how much of the GNP generated should be spent on individual consumption, productive investments and government expenditure, i.e. the aggregated consumption including education, medical care, research and development, etc.

According to our methodology, the 'ex post' data for the allocation of GNP can be used for the construction of a national utility functional. That functional can be used 'ex ante' to derive the future strategies of resources allocation which will also be called here, "the development factors".

We have assumed that all the development factors contribute to the development, i.e. the GNP growth and a generalized Cobb-Douglas function has been used to describe the corresponding development functional. The functional takes into account all the inertial effects, caused by the delays in investment, education, research and development, etc. processes. Then the development problem can be formulated in terms of the optimum allocation of development factors. One should obviously follow such an allocation strategy that yields the maximum value of the GNP, generated within the planning interval, and is subject to the constraints imposed by the quality of environment, shortages of natural resources, foreign trade balances, etc.

Since the utility and development functional parameters are closely related, the maximization of the integrated GNP is equivalent to the maximization of the utility functional. As a result, the development goal takes into account all the socio-economic subgoals which are represented in the utility functional by the corresponding factor endowments.

It should be observed that the optimization of development plays an important role in the policy of socialist countries, which use an effective system of allocation of development factors. The importance of mathematical methods in the planning practice of these countries has been fully recognized [1, 9, 13, 43].

From a mathematical point of view, our optimization problem boils down to the maximimization of an integral, nonlinear functional, subject to a number of integral and/or amplitude type constraints. The solution to multi-variable problems of that type is, generally speaking, not easy. As shown in References [19 - 31], the method based on the generalized Hölder and Minkowski inequalities is, in that respect, very effective. A solution to the optimization problem exists and it is unique. It can be derived in an explicit form, independent of the number of variables involved. Another advantage of the approach used is that under optimum strategy, the sectorial development functionals can be aggregated to yield a simple resulting development function. In other words, the optimization process can be regarded as a useful aggregation device. Using this device, it is possible to allocate the development factors in the optimum manner along the hierarchical, multilevel structure, which corresponds to the decentralized decision system of Polish economy. According to that system, productive investments, labour and other development factors can be allocated by the decision centre (which is the Planning Commission) among the production sectors and services. The sectors, in turn, allocate the factors among the corresponding subsystems or individual factories. The factors are also allocated among different regions of the country. In addition, the sectors decide on how much of the commodities (produced by the other sectors) to purchase in order to have the next sector profit maximized. The model incudes a system of prices, controlled by special government agencies in such a way as to achieve the equilibrium between the supply and demand. The prices are also supposed to realize a number of further requirements which will be discussed in Section 2.3.

From the point of view of control theory, the model discussed can be regarded as a complex, nonlinear system with inertial feedbacks controlled by the hierarchical decision system. The system parameters (i.e. the technological coefficients) change slowly in time. In order to control such a system the methods known as "adaptive control" can be applied. As will be shown later, an "adaptive control" model of the technological change can be constructed. Using this approach, the model parameters are "adapted" to the changing reality, and a high degree of accuracy can be achieved. An interesting feature of the model discussed is the slow change of the goal (i.e. utility) coefficients. This kind of system is unknown in classical control theory. In order to solve the problem effectively, a moving time horizon technique has been used. This technique is based on the consecutive solutions of the optimization problems in the planning intervals $[k, T + k]$, $k = 0,1,2...$ and adjustments of the resulting strategies. It should be noted that the methodology used for the construction of MRI models, can also be applied to modelling the development of other countries, primarily all the CMEA countries. The exogenous variables, such as foreign trade price indices, then become endogenous; i.e. they can be derived from the aggregated multinational model.

Bearing in mind the general philosophy outlined above, the present paper has been divided into three parts. The first part, consisting of five sections, deals with the single-sector model. In the first section the development factors and development functions are introduced. In the simple case of two development factors (capital and labour), the development function coincides with the known production function used in macroeconomics. The next section deals with optimum development problems. Here, the important "principle of factor coordination" has been formulated, and optimization strategies derived in the explicit form. The third section, where the optimization of population policy has been formulated, deals with long-term development planning problems. Then the pollution impact on environment and development was investigated. It was shown that an increase in the pollution abatement level decreases the growth of GNP. The last section deals with development objectives and utility functionals where an extension of the "golden rule" of development has been formulated.

The second part of the paper -- the multi-sector case -- deals with an extension of the results obtained in Part I. In the first section it is shown that with optimum strategy the sector contributions to GNP do not depend on the intersector flows so that decomposition of optimization strategy is possible. In the second section, the optimum strategies have been derived by the explicit form and the sector coordination principle has been formulated. The third section deals with the model of sector prices and the last section deals with the "adaptive" model of technological and structural change. The third part of the paper deals with the regional and international cooperation models. Further details concerning the methodology used and models constructed can be found in References [19 - 37].

The author appreciates receiving the invitations to visit the International Institute for Applied Systems Analysis, where he has been able to study the global modelling problems. Discussions with Prof. T. Koopmans, Prof. F. Rabar and other scientists have been very helpful.

Special thanks should also be given to the Soviet scientists from the Central Economic and Mathematical Institute and the Institute of Management Problems in Moscow, for the valuable comments on the modelling problems which have been presented in the paper.

I. Single-Sector Model

1.1. Development factors and functions

By the development process here we understand a complex dynamic process described by the vector $\underline{X} \equiv \{X_1(t), X_2(t), \ldots, X_n(t)\}$ which depends on the vector $\underline{Y} \equiv \{Y_1(t), \ldots, Y_m(t)\}$ of the development factors $Y_i(t)$, $i = 1, \ldots, m$.

A typical example of such a process is a n-sector economic system with the outputs $\underline{X}$ depending on the production factors: labour and capital stock.

In a more general case (illustrated by Figure 1), one can assume that $\underline{X}$ depends as well on social and environmental factors such as health, education, research and development level, housing, transport, quality of air, water etc. The factors Y_i, $i = 1, \ldots, m$, are regarded as external (exogenous) while X_i, $i = 1, \ldots, n$, are dependent (endogenous) variables.

We shall assume that the $\underline{X}(\underline{Y})$ relation can be described by n development functions

$$F_i[\underline{X}, \underline{Y}, t] = 0 \quad , \qquad i = 1, \ldots, n \quad , \tag{1}$$

Since the direct solution of (1) is usually not easy, a linearization technique may be applied. Assume for that purpose that around the point $(\underline{X}_0, \underline{Y}_0, t_0)$ the functions (1) are continuously differentiable so that (1) can be replaced by

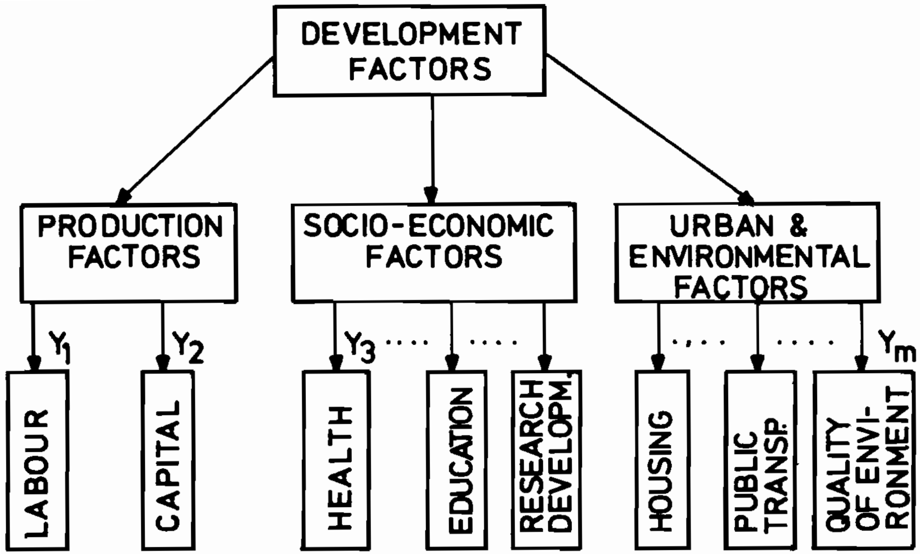

Figure 1. Development factors structure.

$$\sum_{j=i}^{n} F'_{ix_j} \dot{X}_j + \sum_{\nu=1}^{m} F'_{iy_\nu} \dot{Y}_\nu + F'_{it} = 0 \qquad (2)$$

$$i = 1,\dots,n \quad ,$$

where

F'_{ix_j}, F'_{iy_ν}, F'_{it} are partial derivatives of F_i.

The next assumption is that around $(\underline{X}_0, \underline{Y}_0, t_0)$

$$F'_{ixj} X_j \triangleq a_{ji} \quad , \quad F'_{iy_\nu} Y_\nu \triangleq b_{\nu i} \quad ,$$

$$i,j = 1,\dots,n \quad , \quad \nu = 1,\dots,m \quad ,$$

do not change much so that (1) can be approximated by the linear equations:

$$\sum_{j=1}^{n} a_{ji}\rho_{xj} + \sum_{\nu=1}^{m} b_{\nu i}\rho_{y} + F'_{it} = 0 \quad , \qquad (3)$$

$$i = 1,\dots,n \quad ,$$

where

$\rho_{xj} = \frac{\dot{X}_j}{X_j}$, $\rho_{y\nu} = \frac{\dot{Y}_\nu}{Y_\nu}$ are called the growth coefficients.

When the determinant of $[\partial_{ji}]$ is not zero, the system (3) can be solved with respect to ρ_{xi}, $i = 1,\dots,n$, i.e.

$$\rho_{xi} = \frac{\dot{X}_i}{X_i} = \mu_i(t) + \sum_{\nu=1}^{m} \beta_{\nu i} \frac{\dot{Y}_\nu}{Y_\nu} \quad , \quad i = 1,\dots,n \quad , \qquad (4)$$

where

$\beta_{\nu i}$ = given numbers;

$\mu_i(t)$ = given functions.

Assuming $x_i(0)$ to be given, one can integrate the system (4) and obtain (see [36]):

$$X_i(t) = K_i e^{\int_0^t \mu_i(\tau)d\tau} \prod_{\nu=1}^{m} [Y_\nu(t)]^{\beta_{\nu i}} dt \quad , \qquad (5)$$

$$K_i = X_i(0) \prod_{\nu=1}^{m} [Y_\nu(0)]^{-\beta_{\nu i}} \quad , \qquad i = 1,\ldots,n \quad .$$

The functions (5) should be regarded as an approximation of the more general relations (1). They express however the "input-output" relation in the explicit form. The functions (5) shall be called the (explicit) development functions. In the simple case of a single sector and m = 2, and when $\mu_i(t) = \mu = \text{const.}$ and Y_1, Y_2 represent the labour and capital respectively, the function

$$X = A\, e^{\mu t}\, Y_1^{\beta_1}\, Y_2^{\beta_2} \quad , \qquad \beta_1 + \beta_2 \leq 1 \quad , \qquad (6)$$

is the classical Cobb-Douglas production function with the neutral technical progress represented by μ.

The development factors obviously depend on the expenditure intensity $z_\nu(t)$, $\nu = 1,\ldots,m$. For example, capital stock depends upon the investments; the education level, R & D, medical care level, etc., depend on government expenditures. These relations can generally be described by the strictly concave, integral operators [19]:

$$Y_\nu(z_\nu) = Y_\nu(t) = \int_{-\infty}^{t} K_\nu(t-\tau)\ [z_\nu(\tau)]^\alpha d\tau \qquad (7)$$

where

$K_\nu(t)$ is a non-negative function, which can be approximated by:

$$\tilde{K}_\nu(t) = \begin{cases} K_\nu e^{-\delta_\nu(t-T_\nu)} \quad , & t > T_\nu \\ 0 \quad , & t < T_\nu \end{cases} \qquad (8)$$

K_ν, δ_ν, T_ν = given positive constants; and $0 < \alpha < 1$.

There is a simple interpretation of $Y_\nu(z_\nu)$ in the case of $\nu = 2$, when Y_2 represents the capital stock and z_2 is the investment intensity. Here the capital represents the accumulated investments $z_2(\tau)$ for $\tau \leq t$; δ_2 represents the depreciation (aging, wear and tear) of capital stock in time while T_2 is the construction delay, i.e. the time required for investment funds to materialise in the form of new production capacity. The inertial effect of investments on the plant capacity $Y_2(z_2)$ is illustrated by Figure 2, for the case $z_2(\tau)$ = const. $0 \leq \tau \leq T_2$. It is possible to observe that the plant capacity decreases for $t > 2T_2$ if no investments are being made after $t > T_2$. The construction delay T_ν for different branches of economy, ranges from one to four years, while δ_ν is usually assumed 0.1 - 0.05.

A similar interpretation (except labour) can be given for the rest of $Y_\nu(z_\nu)$, $\nu = 1,\ldots,m$, factors. In the case of the health service ($\nu = 3$ according to Figure 1) one can assume, as a first approximation, that most government expenditure is used for new facilities (hospitals, medical equipment, etc.) so that $Y_3(z_3)$ behaves in a similar way to $Y_2(z_2)$. The same assumption can be made with respect to the rest of the development factors. However, that approach neglects the effects of current expenditures and qualitative factors, such as knowledge and qualified services. For example, the health-level depends greatly on the training of medical and scientific staff, i.e. on the "investment" in employed specialists. The depreciation of Y_ν ($\nu \geq 3$) in time, results not only from aging of equipment, but also from the depreciation of knowledge acquired in the past. The "production" delay T_ν in the case of education is the basic tuition time period, which, at present in Poland, is 10 years.

The observation made above indicates that the possible development-factor-parameters may vary in a wide range of values.

The general expression (7) simplifies in the case of labour force ($\nu = 1$). It can be assumed that labour does not depend on past salaries, so

$$Y_1(z_1) = K_1[z_1(t)]^\alpha \quad . \tag{9}$$

The same effect can be obtained assuming in (7):

$$\tilde{K}_1(t) = K_1\delta(t) \quad ,$$

where

$\delta(t)$ = Dirac's pulse.

In the production model under consideration it is important to deal with strictly concave operators so the numbers $\beta_{\nu i}$, α, satisfy the conditions:

$$\sum_{\nu=1}^{m} \beta_{\nu i} = 1 \quad , \qquad i = 1,\dots,n \quad , \qquad 0 < \alpha < 1 \quad ,$$

which are called the "decreasing return to scale".

It should be noted that (7) can be written alternatively as

$$Y_\nu(t) = \bar{Y}_\nu(t) + \int_0^t K_\nu(t - \tau)[z_\nu(\tau)]^\alpha d\tau \quad , \tag{10}$$

where

$\bar{Y}_\nu(t) = \int_{-\infty}^{0} K_\nu(t - \tau)[z_\nu(\tau)]^\alpha d\tau$ represents the effect of past expenditures (i.e. $z_\nu(\tau)$, $t < 0$).

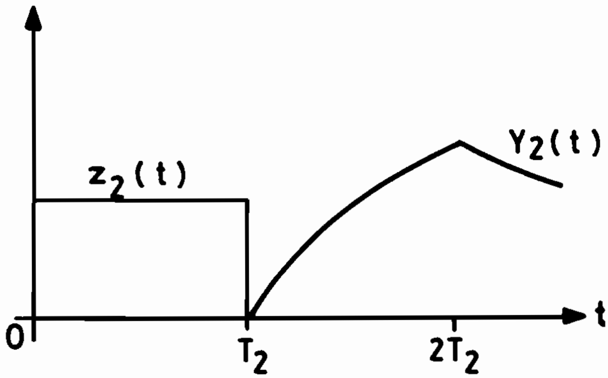

Figure 2.

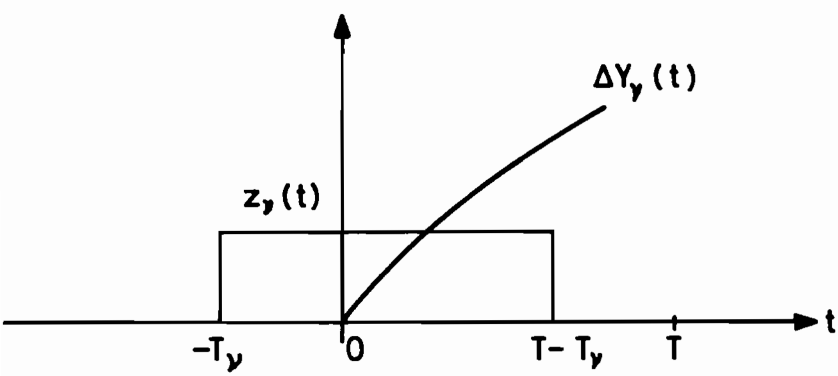

Figure 3.

In the case of (8) one can also write

$$Y_\nu(t) = e^{-\delta_\nu t}\left\{Y_\nu(0) + \int_0^t K_\nu e^{\delta_\nu \tau}[z_\nu(\tau - T_\nu)]^\alpha d\tau\right\} \quad (11)$$

which can be regarded as the solution to the differential equation

$$\dot{Y}_\nu(t) + \delta_\nu Y_\nu(t) = K_\nu[z_\nu(t - T_\nu)]^\alpha \quad ,$$

with the given initial condition $Y_\nu(0)$.

It should be observed that the function

$$\Delta Y_\nu(t) = Y_\nu(t) - e^{-\delta_\nu t} Y_\nu(0)$$
$$= \int_0^t K_\nu e^{-\delta_\nu(t-\tau)}[z_\nu(\tau - T_\nu)]^\alpha d\tau$$

in (11), can be regarded as a present contribution to the development resulting from previous expenditures (i.e. the expenditures shifted by T_ν as shown in Figure 3).

Since the statistical data regarding the production effects and expenditures are usually collected periodically (e.g. once a year). It is convenient to deal with the discrete (instead of continuous) development functions (7) - (11). In particular (11) can be written

$$Y_\nu(t) = e^{-\delta_\nu t}\left\{Y_\nu(0) + \sum_{\tau=1}^{t} K_\nu e^{\delta_\nu \tau}[z_\nu(\tau - T_\nu]^\alpha\right\} \quad .$$
$$t = 0,1,2,\ldots$$

Using the discrete form of development factors, the output is represented by development capacities belonging to different generations, For example, capital stock can be regarded as being composed of equipment acquired in different years. In the subsequent computations we shall use the continuous form of $Y_\nu(t)$, which is a matter of convenience rather than general methodology.

It should also be observed that in dealing with the development function of the form

$$X = A\, e^{\mu_0 t} \prod_{\nu=1}^{m} Y_\nu^{\beta_\nu} \quad , \qquad \sum_{\nu=1}^{m} \beta_\nu = 1 \tag{12}$$

instead of (6), it is possible to express the contribution to development which results from government expenditure in different areas. In order to do that it is convenient to introduce the notion of the growth coefficient $\rho_x = \frac{\dot{x}}{x}$, of the differentiable function x(t). It is easy to show that the growth coefficients derived for the functions (6),(12) become

$$\rho_x = \mu + \beta_1 \rho_{y_1} + \beta_2 \rho_{y_2} \quad ,$$

$$\rho_x = \mu_0 + \sum_{\nu=1}^{m} \beta_\nu \rho_{y_\nu} \quad ,$$

respectively. In the case where $\beta_1 + \beta_2 = \beta < 1$, the terms $\sum_{\nu=3}^{m} \beta_\nu \rho_{Y_\nu}$ can be regarded as a contribution of Y_ν - factors, $(\nu > 2)$, to the development. Such an approach has already been suggested by many authors (e.g. Ref.[8]). The difficulty is, that the data regarding ρ_{Y_ν} are usually not available. One usually has, however, data for the expenditures growth, i.e. $\rho_{z_\nu} = \frac{\dot{z}_\nu}{z_\nu}$, $\nu = 1,\ldots,m$, which are related to Y_ν by the integral equations (7).

As will be shown in the following section, it is possible to express the contribution to growth which results from expenditures $z_\nu(t)$, $\nu = 1,\ldots,m$, but the contribution is expressed in the integrated form (25) rather than the discrete one (in terms of ρ_{z_ν}).

It should also be noted that, when $\mu_o = 0$, the value of $\sum_{\nu=3}^{m} \beta_\nu \rho_{Y_\nu}$ can be interpreted as the neutral technical progress in Hicks' sense. It incorporates the parts which magnify the labour as well as the capital stock level. The relation to government expenditure in this case, however, is explicit.

1.2. Development feed-backs and optimum development problems

Development in the model analysed is a result of positive feedbacks which exist between the output $Y = pX$ (where p = price attached to the production X) and the expenditures z_ν, $\nu = 1,\ldots,m$, which constitute the given parts of Y -- the GNP generated by the economy (Figure 4). The decision center (D.C.) allocates the GNP between different development factors, including labour (i.e. private consumption) and government expenditures (health, education, R & D, etc.) in such a way that the given development goal attains its maximum.

The aggregated production function can be written, according to (5) - (11) as

$$Y(t) = K e^{\mu t} \prod_{\nu=1}^{m} [Y_\nu(t)]^{\beta_\nu} , \qquad (13)$$

$$Y_\nu(t) = \int_{-\infty}^{t} e^{-\delta_\nu(t-\tau)} [z_\nu(\tau - T_\nu)]^{\alpha} d\tau , \qquad (14)$$

$$\sum_{\nu=1}^{m} \beta_\nu = 1 , \quad 0 < \alpha < 1 , \quad \mu, \delta_\nu > 0 ,$$

where $z_\nu(t)$ should satisfy, generally speaking, one of the following two sets of constraints:

a) the amplitude type of constraints:

$$\sum_{\nu=1}^{m} z_\nu(t - T_\nu) \leq Z(t) , \qquad (15)$$

$$z_\nu(t - T_\nu) \geq 0 , \quad t \in [0,T] , \quad \nu = 1,\dots,m , \qquad (16)$$

b) the integral type of constraints:

$$\int_{0}^{T} W_\nu(\tau) z_\nu(\tau - T_\nu) d\tau \leq Z_\nu , \quad \nu = 1,\dots,m , \qquad (17)$$

$$\sum_{\nu=1}^{m} Z_\nu \leq Z , \quad Z_\nu \geq 0 , \quad \nu = 1,\dots,m , \qquad (18)$$

where

T is the given planning horizon and $W_\nu(t)$, $\nu = 1,\ldots,m$, the given weight functions.

In the most simple case $T_\nu = 0$, $\nu = 1,\ldots,m$ it is possible to replace Z(t) by Y(t - 1). In that case the constraint (15) has the following meaning. The GNP generated at the end of the year t - 1 is allocated at the year t[†] among m development factors, i.e.

$$z_\nu(t) = \gamma_\nu(t)Y(t-1) \quad , \qquad \nu = 1,\ldots,m \quad ,$$

where

$$\sum_{\nu=1}^{m} \gamma_\nu(t) = 1 \quad , \qquad t = 1,2,\ldots,$$

$\gamma_\nu(t)$ are decision variables.

In the case when some of the government expenditures, say $z_o = \gamma_o Y(t-1)$, have no productive effects, one should write

$$Z(t) = (1-\gamma_o)Y(t-1) \quad .$$

In the case when $T_\nu = T_o$, $\nu = 1,\ldots,m$, one obtains

$$z_\nu(t-T_o) = \gamma_\nu(t-T_o)Y(t-T_o-1) \quad , \qquad \nu = 1,\ldots,m \quad .$$

When $T_\nu > T$ the corresponding $z_\nu(t - T_\nu)$, $t \in [0,T]$ strategy shifts outside the planning interval (see Figure 3). In that case $z_\nu(t - T_\nu)$ should be regarded as an exogenous (given) function.

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† In economic literature the delay taking place between the output y and the imput z_ν, is called the Robertson's postponement [2].

In the case when T_ν is different but $T_\nu < T$, one obtains

$$Z(t) = \sum_{\nu=1}^{m} \gamma_\nu(t-T_\nu)Y(t-T_\nu-1) \quad .$$

Assuming that $Y(t-T_\nu-1) = Y(t)e^{-\rho(T_\nu+1)}$, where ρ is the average annual growth of GNP in $[t-T_\nu,t]$, the above relation can be also witten as

$$Z(t) = \sum_{\nu=1}^{m} Z_\nu(t) \quad , \quad Z_\nu(t) = \tilde{\gamma}_\nu(t)Y(t) \quad , \tag{19}$$

where

$$\tilde{\gamma}_\nu(t) = \gamma_\nu(t-T_\nu)e^{-\rho(T_\nu+1)} \quad .$$

It should be observed that, usually

$$\sum_{\nu=1}^{m} \tilde{\gamma}_\nu(t) < 1 \quad .$$

The number Z in (18) can be assumed equal to

$$Z = \int_0^T w(t)\ Z(t)\ dt \quad , \quad w(t) = \text{given discount function} \tag{20}$$

The amplitude constraints are characteristic of the development in a closed economy (autarky) when expenditure used for factor endowments (i.e. investments, labour, education, R & D, etc.) are strictly limited by the GNP currently achieved.

In the case of integral constraints, it is possible to make use of international cooperation by taking foreign credits, exchange in skilled labour, expertise, etc. The only requirement is that foreign credits should be paid back together with the interest, described by the functions $W_\nu(t)$, of the type

$$W_\nu(t) = (1 + \varepsilon_\nu)^{T-t} \quad , \quad \nu = 1,\ldots,m \quad .$$

As will be shown later in the case of cooperative development (i.e. development subject to (17) and (18) only) growth is faster than in the case of complete autarky (i.e. development which is subject to (15) and (16) only).

Before we formulate the corresponding optimization problems, the optimization goals should be introduced.

In the present section, the discounted and integrated GNP (within the planning interval)

$$Y = \int_0^T w(t)\; Y(t)\; dt, \tag{21}$$

where

$$w(t) = (1 + \varepsilon)^{-t} = e^{-\lambda t} \quad , \qquad (\lambda = \ln(1 + \varepsilon)),$$

ε = the given discount rate ;

will be used.

The problem of optimization of development can be formulated as follows. Find the strategies $z_\nu(t) = \hat{z}_\nu(t)$, $\nu = 1,\ldots,m$, which maximize (21) subject to the integral (17), (18) and/or amplitude (15), (16) constraints.

From the mathematical point of view, the optimization problem formulated boils down to the maximization of the strictly concave functional subject to a number of functional (17), (18) and operator type (15), (16) of constraint. There are many well known techniques suitable for attacking that problem. We shall use here, and in the next sections, a relatively simple technique which makes use of some well known integral inequalities (Hölder and Minkowski inequalities, mainly). As shown in Reference [20], that technique enables the expression of optimum strategies in a simple and explicit form.

First of all it should be observed that in the planning practice instead of dealing with time variables $Y_\nu(t)$, $\nu = 1,\ldots,m$, the integrated discounted values

$$\int_0^T e^{-\lambda_\nu t} Y_\nu(t)dt \leq \bar{Y}_\nu \quad , \nu = 1,\ldots,m, \tag{22}$$

where λ_ν = discount rates, are being used.

In the case when the amounts of factors $\bar{Y}_\nu$, $\nu = 1,\ldots,m$, within the planning interval [O,T] are given one would like to know how the factor intensities $Y_\nu(t)$ should change in time in order to maximize (21) subject to (22).

Using the generalized Hölder inequality

$$Y = K\int_0^T \prod_{\nu=1}^m f_\nu(t)dt \leq K \prod_{\nu=1}^m \left\{ \int_0^T |f_\nu(t)|^{1/\beta_\nu} dt \right\}^{\beta_\nu} \tag{23}$$

where

$$f_\nu(t) = \left[e^{-\lambda_\nu t} \; Y_\nu(z_\nu) \right]^{\beta_\nu} \quad , \quad (\lambda \overset{\Delta}{=} \mu + \sum_{\nu=1}^m \lambda_\nu \beta_\nu) \quad ;$$

one finds that (23) becomes an equality if (almost everywhere):

$$c_\nu |f_\nu(t)|^{1/\beta_\nu} = |f_1(t)|^{1/\beta_1} \quad , \quad c_\nu = \text{const.}, \; \nu = 2,\ldots,m \tag{24}$$

and c_ν , according to (22) becomes equal $\bar{Y}_1/\bar{Y}_\nu$, i.e.

$$Y_\nu(z_\nu) = \frac{\bar{Y}_\nu}{\bar{Y}_1} e^{-\vartheta_\nu t} Y_1(z_1), \; \vartheta_\nu = \lambda_1 - \lambda_\nu \quad , \quad \nu = 2,\ldots,m \tag{25}$$

One finds also that under optimum strategy $z_\nu = \hat{z}_\nu$, $\nu = 2,\ldots,m$, which satisfies (25), the output

$$\hat{Y} = Y(\underline{\hat{z}}) = K \prod_{\nu=1}^m \bar{Y}_\nu^{\beta_\nu}$$

while

$$\rho = \dot{Y}(t)/Y(t) = \sum_{\nu=1}^{m} \lambda_\nu \beta_\nu + \mu = \lambda \quad .$$

The obtained result indicates that Y attains maximum value $\hat{Y}$, when the factors $f_\nu(t)$, $\nu = 1,\ldots,m$, change in coordinated fashion. The relation (25) will be called, therefore, the principle of factor coordination. According to that principle the capital, education, research and development etc. should change along with employment in fixed proportions.

When the coordinated growth strategy is used it is possible to separate the contribution of $\hat{\underline{z}}$ to growth, i.e.

$$Y = K \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \int_0^T dt \; e^{\vartheta_1 t} \int_{-\infty}^{t} e^{-\delta_1 (t-\tau)} [z_1(\tau - T_1)]^\alpha d\tau$$

$$= \bar{Y} + \Delta Y \quad , \qquad \vartheta_1 = \mu - \lambda_1$$

where $\bar{Y}$ is the contribution to GNP resulting from past decisions:

$$z_\nu(t) \quad , \quad t < -T_\nu \quad , \quad \nu = 1,\ldots,m, \text{ and}$$

$$\Delta Y = K \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \int_0^T dt \; e^{\vartheta_1 t} \int_0^t e^{-\delta_1 (t-\tau)} [z_1(\tau - T_1)]^\alpha d\tau \qquad (26)$$

represents the contribution to the GNP in the planning interval $[0,T]$, resulting from the expenditures $z_\nu(t)$ $t \in [-T_\nu, T - T_\nu]$, $\nu = 1,\ldots, m$, as shown in Figure 3.

First of all we shall solve the optimization problem subject to the integral constraints (17) only. Changing the integration order in (26) one can express (26) in the form:

$$\Delta Y = \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \int_0^T [w_1(\tau) z_1(\tau - T_1)]^\alpha [w_1(\tau) \mathcal{C}_1(t)]^q d\tau \ , \ q = 1-\alpha \quad , \qquad (27)$$

where

$$\mathcal{C}_1(\tau) = \left\{ K W_1(\tau)^{-1} \int_\tau^T e^{\vartheta_1 t - \delta_1 (t-\tau)} dt \right\}^{\frac{1}{q}}$$

$$= \left\{ K \frac{w_1(\tau)^{-1}}{\delta_1 - \vartheta_1} e^{\delta_1 \tau} \left[e^{(\vartheta_1 - \delta_1)\tau} - e^{(\vartheta_1 - \delta_1) T} \right] \right\}^{\frac{1}{q}}$$

Applying the Hölder inequality to (27) one gets

$$\Delta Y \leq \prod_{\nu=2}^{m} c_\nu^{-\beta_\nu} \left\{ \int_0^T |w_1(\tau) z_1(\tau - T_1)| \, d\tau \right\}^{\alpha} \left\{ \int_0^T |w_1(\tau)\varphi_1(\tau)| \, d\tau \right\}^{q} ,$$

where the equality sign holds when

$$\hat{z}_1(\tau - T_1) = c_1 \varphi_1(\tau) \ , \quad c_1 = \text{const}, \quad \tau \in [0,T] \ .$$

The constant c_1 can be derived by (17) yielding

$$\hat{z}_1(\tau - T_1) = \frac{\varphi_1(t)}{\int_0^T w_1(\tau)\varphi_1(\tau) \, d\tau} Z_1 \ . \tag{28}$$

In order to find the remaining strategies $\hat{z}_\nu(t)$, $\nu = 2,\dots,m$, it is necessary to solve the set of integral equations (24):

$$c_\nu e^{\vartheta_\nu t} \int_0^t e^{-\delta_\nu (t-\tau)} [z_\nu(\tau - T_\nu)]^{\alpha} \, d\tau$$

$$= \int_0^t e^{-\delta_1 (t-\tau)} [z_1 (\tau - T_1)]^{\alpha} \, d\tau \ , \quad \nu = 2,\dots,m \ . \tag{29}$$

For that purpose it is convenient to use the Laplace transforms of (29) [24]. For example, in the case when $\vartheta_\nu = 0$, $\nu = 2,\dots,m$, one gets †

$$\hat{z}_\nu (\tau - T_\nu) = c_\nu \varphi_\nu(\tau) \ , \quad \nu = 2,\dots,m$$

where

$$\varphi_\nu(\tau) = [\hat{z}_1^{\alpha}(\tau - T_1) + (\delta_\nu - \delta_1) \int_0^\tau e^{-\delta_1(\tau-t)} \hat{z}_1^{\alpha}(t - T_1) \, dt]^{\frac{1}{\alpha}} ,$$

and the constants c_ν can be derived by (17) yielding

$$\hat{z}_\nu(\tau - T_\nu) = \frac{\varphi_\nu(\tau) \ Z_\nu}{\int_0^T w_\nu(\tau) \ \varphi_\nu(\tau) \, d\tau} \ , \quad \nu = 1,\dots,m \ . \tag{30}$$

---

† When $\vartheta_\nu \neq 0$, $\nu=2,\dots m$, one finds easily by differentiating both sides of (29), that (30) holds but φ_ν becomes

$$\varphi_\nu(\tau) = e^{-\frac{\vartheta_\nu}{\alpha}\tau} \left\{ z_1^{\alpha}(\tau-T_1) + \left[(\delta_\nu-\delta_1)-\vartheta_\nu\right] \int_0^T e^{-\delta_1(\tau-t)} \hat{z}_1^{\alpha}(t-T_1) \, dt \right\}^{1/\alpha}$$

As follows from (30) in order to have the strategies $\hat{z}_\nu$, $\nu = 1,\ldots,m$, which satisfy the conditions (16) it is necessary to enumerate the development factors in such a way that $\delta_\nu \geq \delta_1$ $\nu = 2,\ldots,m$, i.e. $\nu = 1$, should be assigned to that factor which has the smallest depreciation in time.

Since $\hat{z}_\nu(t)$, depend in linear manner on Z_ν it is possible to see that the GNP value (21) under optimum strategy can be written in the form

$$Y(\underline{z}) = \bar{Y} + \Delta Y(\underline{\hat{z}}) \quad ,$$

where

$$\Delta Y(\underline{\hat{z}}) = G^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad , \qquad \gamma_\nu = \alpha\beta_\nu \quad , \tag{31}$$

G = a number depending on K, T, T_ν, δ_ν, w_ν parameters. Now it is possible to derive the optimum values of expenditures $Z_\nu = \hat{Z}_\nu$, $\nu = 1,\ldots,m$, which maximize (31) subject to the constraint (18). Since (31) is strictly concave in the compact set (18), a unique optimum solution exists and can be derived by the following formula:

$$\hat{Z}_\nu = \frac{\gamma_\nu}{\sum_{\nu=1}^{m} \gamma_\nu} Z = \beta_\nu Z \quad , \qquad \nu = 1,\ldots,m \tag{32}$$

When the optimum strategy (32) is set in (31) one obtains

$$\Delta Y = \Delta\bar{Y} = G^q \prod_{\nu=1}^{m} \beta_\nu^{\gamma_\nu} \quad Z^\alpha \quad ,$$

which represents the maximum possible development increase under integral constraints. The value of Z here, represents the total resources which have been produced and used in the planning interval [0,T].

As follows from relations (19), (20) one can assume γ_ν equal to the average of $\tilde{\gamma}_\nu(t)$, $\nu = 1,\ldots,m$, so that $\sum_{\nu=1}^{m} \gamma_\nu = \alpha$, and $Z = \alpha Y$. Then

$$Y = \bar{Y} + G^q \prod_{\nu=1}^{m} \gamma_\nu^{\gamma_\nu} Y^\alpha \quad . \tag{33}$$

As shown in Figure 5, a unique solution $Y = Y^*$ of the equation (33) exists, which determines the GNP generated within [0,T] under optimum strategy.

Since $\alpha < 1$, the contraction property of the right side of (33) takes place for any given T or G(T). It should also be noted that when α approaches unity $q \to 0$ and the function $\varphi_1(\tau)$, which determines $\hat{z}_1(\tau)$, goes at $t = 0$ to infinity, i.e. it approximates the Dirac's $\delta(t)$ function.

It is also possible to show that the optimum strategy, derived for the amplitude constraints (15), (16) degenerates (for $\alpha \to 1$) in the so called "bang-bang" strategy, which requires maximum expenditures at the starting subinterval of [0,T] and expenditures equal zero for the rest of the planning interval. Such strategies cannot be implemented in the real systems. That was one of the reasons why the complex nonlinear production functions (14) have been used here instead of relatively simple linear ($\alpha = 1$) relations.

It should also be observed that when one sets $Z = \alpha Y$ the open loop solutions (30) and (32) become the "closed loop" solutions. Such a procedure is in agreement with the planning practice, which makes projections of GNP rise in order to determine the amount of resources which can be spent on investments and other government expenditures.

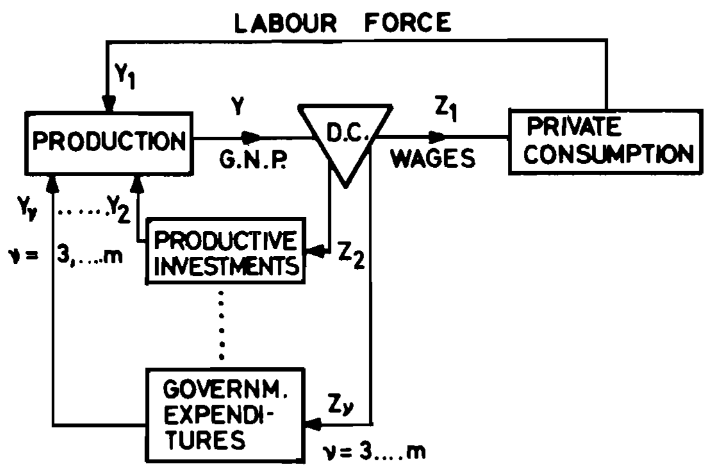


Figure 4.

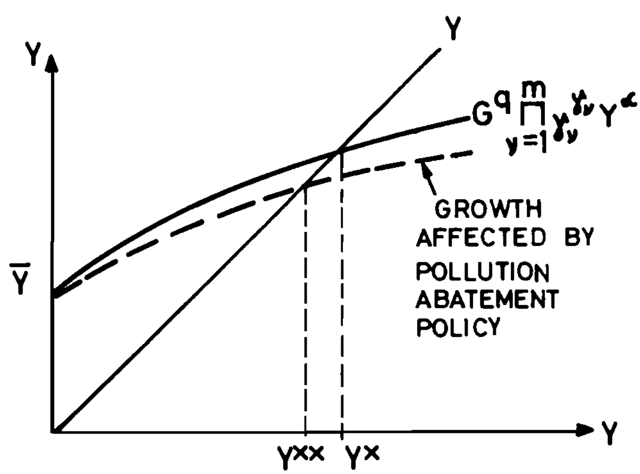


Figure 5.

Using the general solution method described above, it is possible to investigate some important, special cases. For example, consider the problem of optimization of functional:

$$\Delta Y = \int_0^T \left\{ \int_0^t e^{-\delta(t-\tau)} [z_2(\tau)]^{\alpha} d\tau \right\}^{\beta} [z_1(t)]^{\alpha(1-\beta)} dt , \qquad (34)$$

subject to

$$\int_0^T z_1(\tau) \, d\tau \leq Z_1 , \qquad z_1(\tau) \geq 0 , \qquad (35)$$

$$\int_0^T z_2(\tau) \, d\tau \leq Z_2 , \qquad z_2(\tau) \geq 0 , \qquad t \in [0,T] . \qquad (36)$$

The variable $z_2(\tau)$ can be interpreted as investment intensity, while $z_1(\tau)$ is the labour employed. The total amount of capital and labour in the planning interval [0,T] is limited according to (36) and (35). The problem consists of finding the optimum strategy of investments $\hat{z}_2(t)$ and employment $\hat{z}_1(t)$.

The strategy $\hat{z}_2(t)$ becomes, by (28):

$$\hat{z}_2 = \frac{\varphi_2(\tau)}{\int_0^T \varphi_2(\tau) \, d\tau} Z_2 ,$$

where

$$\varphi_2(\tau) = \left\{ \int_\tau^T e^{-\delta(t-\tau)} dt \right\}^{\frac{1}{q}} = \left\{ \frac{1}{\delta} [1 - e^{-\delta(T-\tau)}] \right\}^{\frac{1}{q}} .$$

The equation (29) becomes

$$c_1 \, z_1(t)^{\alpha} = \int_0^t e^{-\delta(t-\tau)} [\hat{z}_2(\tau)]^{\alpha} d\tau . \qquad (29')$$

Then

$$\hat{z}_1(t) = \frac{\varphi_1(t)}{\int_0^T \varphi_1(t) \, d\tau} Z_1 ,$$

where

$$\varphi_1(t) = \left\{ \int_0^t e^{-\delta(t-\tau)} [\hat{z}_2(\tau)] \, d\tau \right\}^{\frac{1}{\alpha}} .$$

The functions $\hat{z}_1(t)$, $\hat{z}_2(t)$, $\varphi_1(t)$, for $\alpha = \frac{1}{2}$, $\delta T = 4$, and with accuracy to the degree of constant multipliers, have been shown in Figure 6.

The optimum strategy of investments $\hat{z}_2(t)$ decreases to zero when $t \to T$, while the labour employed $\hat{z}_1(t)$ increases along with t, according to the factor coordination principle, in such a way that it is proportional to the capital stock $\varphi_1(t)$. It is also possible to solve the problem when $z_1(t)$ is given exogenously, while $\hat{z}_2(t)$ is chosen in such a way as to satisfy (29').

Since the optimum investment strategy $z_2(t)$, for integral constraints may violate the admissible values $\gamma_2 Y(t)$, γ_2 = given number, at the beginning of the planning interval, as shown in Figure 7, it is also necessary to find the optimum strategy for the amplitude constraints (15). Since no harm can be done when one assumes the equality sign in (15), the unknown $\hat{z}_1(t)$ can be derived from the equation

$$z(t) = \sum_{\nu=1}^{m} \hat{z}_\nu(t - T_\nu) = \hat{z}_1(t - T_1)$$

$$+ \sum_{\nu=2}^{n} c_\nu \left[\hat{z}_1^{\alpha}(t - T_1) + (\delta_\nu - \delta_1) \int_0^t e^{-\delta_1(t-\tau)} \hat{z}_1^{\alpha}(\tau - T_1)\, d\tau\right]^{\frac{1}{\alpha}} .$$

In the case when $\alpha \approx 1$, one can solve that equation by means of Laplace Transforms:

$$\hat{z}_1(t - T_1) \cong \mathscr{L}^{-1}\left\{\frac{\overline{Z}(p)}{\sum_{\nu=2}^{m} c_\nu \frac{p + \delta_1}{p + \delta_\nu} + 1}\right\} , \tag{37}$$

where

$$\overline{Z}(p) = \mathscr{L}\{Z(t)\} .$$

The numbers c_ν, $\nu = 2,\dots,m$ in (37) can be chosen in such a way that:

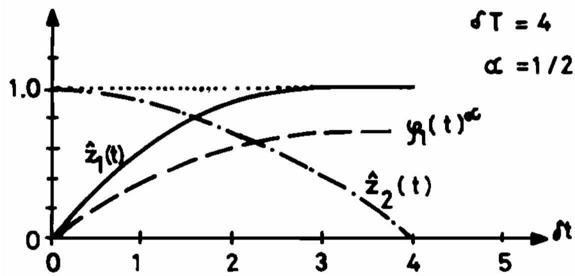


Figure 6.

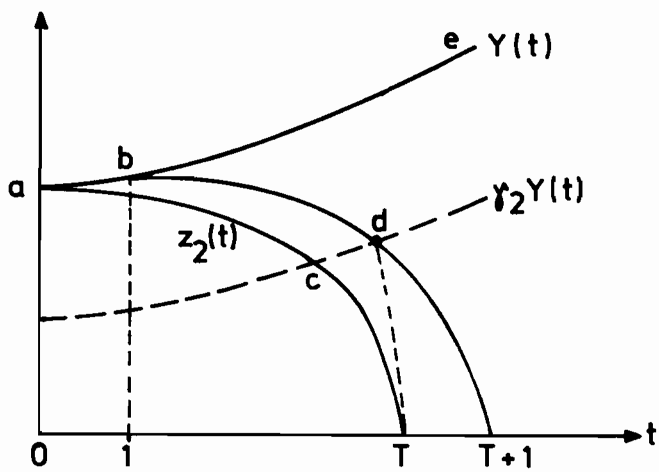


Figure 7.

$$\frac{\int_0^T z_\nu(\tau - T_\nu)\, d\tau}{\sum_{\nu=1}^{m} \int_0^T z_\nu(\tau - T_\nu)\, d\tau} = \gamma_\nu \quad , \qquad \nu = 1,\ldots,m \quad ,$$

where

$$\gamma_\nu \text{ - given numbers} \quad , \qquad \sum_{\nu=1}^{m} \gamma_\nu = 1 \quad .$$

It should also be noted that the optimization method described in the present section can be used to deal with the situation when the development factors and the expenditures z_ν , $\nu = 1,\ldots,m$, are subdivided into different, hierarchically ordered categories, as shown in Figure 8. For example, the productive investments can be split among m subcategories (e.g., the investment goods supplied from the different sectors, imported investment goods, foreign credits, etc.). Education can be subdivided into elementary, secondary and higher education, etc.

For example, consider z_1 in (26) as being composed of $z_\nu^{\gamma_\nu}$, $\nu = 1,\ldots,m$ factors, so that the production function can be written:

$$\Delta Y = K \int_0^T dt\, e^{\vartheta t} \int_0^t e^{-\delta(t-\tau)} \prod_{\nu=1}^{m} [z_\nu(\tau - T_\nu)]^{\gamma_\nu}\, d\tau \quad , \quad (38)$$

where

$$\int_0^T w_\nu(\tau)\, z_\nu\, (\tau - T_\nu)\, d\tau \leq Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \qquad (39)$$

$$\sum_{\nu=1}^{m} Z_\nu \leq Z \quad , \qquad (40)$$

$$z_\nu(t - T_\nu) \geq 0 \; , \quad Z_\nu \geq 0 \; , \quad \nu = 1,\ldots,m \; , \quad t \in [0,T] \; , \quad (41)$$

$$\sum_{\nu=1}^{m} \gamma_\nu = \alpha < 1 \quad .$$

In order to find the strategies $z_\nu = \hat{z}_\nu$, $\nu = 1,\ldots,m$, which maximize (38), subject to the constraints (39) - (41), change the integration order in (38) and use the generalized Hölder inequality:

$$\Delta Y = \int_0^T \prod_{\nu=1}^{m} [z_\nu(\tau - T_\nu)\ w_\nu(\tau)]^{\gamma_\nu}\ [\varphi(\tau)]^q\ d\tau \quad , \quad q = 1 - \alpha$$

$$\varphi(\tau) = \left\{K \prod_{\nu=1}^{m} w_\nu(\tau)^{-\gamma_\nu} \int_\tau^T e^{\vartheta t-\delta(t-\tau)}\ dt\right\}^{\frac{1}{q}} \quad ;$$

$$\Delta Y \leq \prod_{\nu=1}^{m} \left\{\int_0^T |z_\nu(\tau - T_\nu)\ w_\nu(\tau)|\ d\tau\right\}^{\gamma_\nu} \left\{\int_0^T |\varphi(\tau)|\ d\tau\right\}^q \quad . \tag{42}$$

The equality sign in (42) appears when

$$\hat{z}_\nu(\tau - T_\nu)\ w_\nu(\tau) = c_\nu\, \varphi(\tau) \quad , \quad \nu = 1,\ldots,m \ , \quad \tau \in [0,T] \quad .$$

The constants c_ν can be derived by (39), yielding

$$\hat{z}_\nu(\tau - T_\nu) = \frac{\varphi(\tau)\ Z_\nu}{w_\nu(\tau)\ G} \quad , \quad G = \int_0^T \varphi(\tau)\ d\tau \quad , \quad \nu = 1,\ldots,m \tag{43}$$

One gets, therefore

$$\Delta Y\ (\underline{\hat{z}}) = G^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad . \tag{44}$$

Now it is possible to derive $Z_\nu = \hat{Z}_\nu$, $\nu = 1,\ldots,m$, which maximize (44) subject to (40):

$$\hat{Z}_\nu = \frac{\gamma_\nu}{\sum_\nu \gamma_\nu} Z = \beta_\nu Z \quad , \quad \nu = 1,\ldots,m \tag{45}$$

It should also be observed that the government expenditures can generally be subdivided into "inertial and noninertial" categories. For example, the part of these expenditures which goes into capital investments in R & D, education, health, etc., behaves in the same way as the "productive" investments in the general expression (13), (14). In a similar way, the labour employed in services (i.e. education, health, R & D, etc.) has the same (noninertial) effect on the growth of GNP as the productive labour. The corresponding elasticities (β_ν in (14)) are, of course, different.

1.3. The long term development planning problems

Planning in the socialist economy is mostly concerned with an optimum allocation of resources. In the present section we shall investigate that problem from the point of view of allocation of GNP among the development factors. Since planning involves information about the future state of the system concerned and the system parameters change slowly, and usually in an unpredictable fashion, the standard procedure is to use the "moving planning horizon approach". According to that approach, the planning process is repeated each year with new statistical information regarding the changes of system parameters, constraints and development objectives. The strategy derived at each new planning interval should replace the previous strategies in such a way that the continuation of a general development strategy is possible. For example, expenditures in investments should cover the expenditures connected with the continuation of construction processes of factories originated in the past as well as they should cover the expenditures connected with the present and future constructions. The optimum investment strategy for the integral constraints decreases when time approaches the end of the planning horizon T, as shown in Figure 7 by acT-curve. The optimum strategy derived for the planning interval [1, T + 1] shown in Figure 7 by bdT + 1 curve also decreases in time when $t \to T + 1$. However, the effectively spent investments at each consecutive year represent the abe-curve which is rising in given proportion to the GNP growth. In other words, the optimum investment strategy does not terminate in time. The same statement is true for the strategy with amplitude constraints, as shown in Figure 7. The optimum investment strategy moves along with the curve fcd, which is described by $\gamma_2 Y(t)$, where γ_2 is a given coefficient.

As stated in Section 1.2., the fastest growth, in term of GNP, can be obtained when the expenditure strategy follows the "coordinated growth principle" (24). In order to implement that strategy in the case of development factors with long delays (T_ν) a prediction of future development strategy is needed. Consider, for example, the education system in Poland which has $T_\nu \cong 10$ years. In order to get the maximum benefits out of education, it is necessary to teach now the young generation the skills required, not by the present but by the expected socio-economic needs in the year 1986. It is therefore important to predict the future professional qualification structure, specialization, etc. The same arguments can be used to prove that the prediction of future development strategy is required when planning the other inertial factors of development -- first of all, R & D, environment protection, demographic policy. In the short range planning problems (e.g. T = 5 years) many of the factor endowments have a negligible effect on the development, as shown in Figure 3 for $T_\nu > T$. Since one cannot affect the development by changing these factors, it has become a standard practice to replace them by the exogenous term $e^{\mu t}$, as shown in Section 1.1., and deal with the single production function of the form (6). That approach constitutes the main concern of macroeconomic planning. The extension of the planning interval necessitates, however, the inclusion of all the other important socio-economic and environmental factors in the planning process.

One of the most important factors is the labour force. In short range planning the labour is usually regarded as an exogenous factor. That can be explained by taking into account that it takes almost eighteen years before there is any effect which results from a change in the demographic policy. When, for example, the system of social benefits for families with many children is changed, it may affect the female fertility factor instantly, but the labour force will change only when the new born children reach the age of adolescence. In other words, in order to have an effective planning system for demographic policy, it is necessary to deal with the planning horizon T longer than the demographic delay $T_d \approx 18$ years.

It is well known that in many developed countries the female fertility F decreases in time. That can be explained by assuming that F depends on such factors as the value of GNP per capita, health service level, family planning level, social care program, traditions, religions, etc. In particular, it is believed that a strong correlation of F to the GNP per capita exists and as a result F decreases along with the rise of GNP/capita. Figure 9 shows the change in recent years of female fertility in Poland and the GNP/capita. Fertility decreases up to 1970. A slow down and a slow rise of fertility (starting from 1970) can probably be attributed to a constant increase in the social benefits for families with many children. In 1960, about 1.85% of GNP was being spent on the additional monthly allowances which rises in proportion to the number of children. The present system of social benefits in Poland (supplemented with Acts of Law 14, 1972 and December 17, 1974) includes in addition:

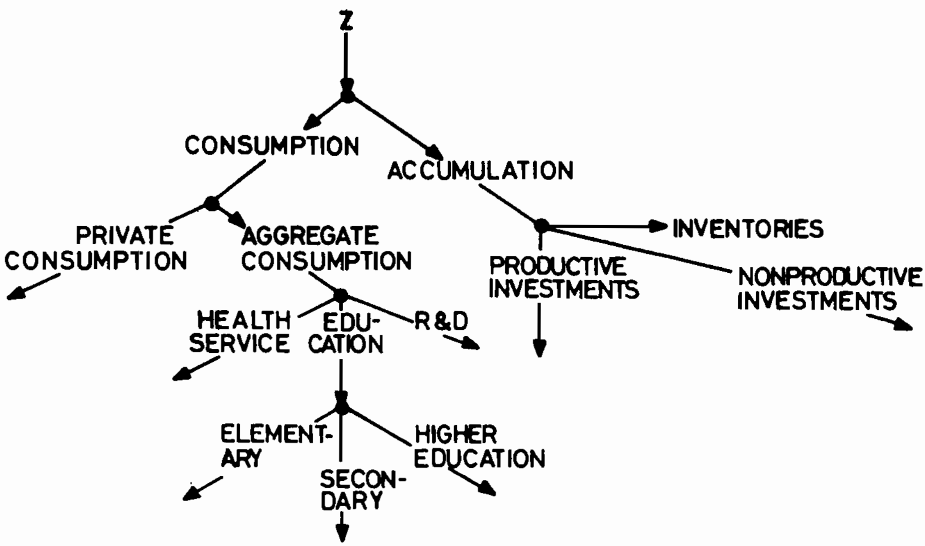


Figure 8.

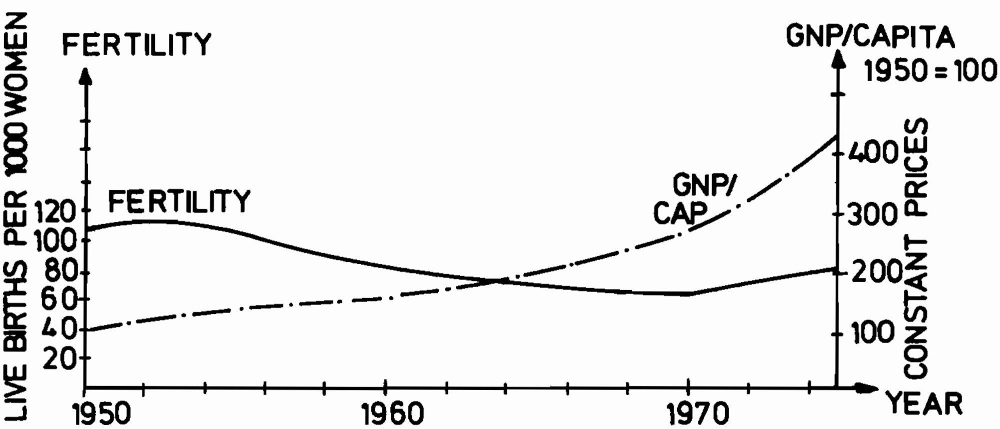


Figure 9.

- 16-18 weeks paid maternity leave;
- 60 days of paid leave per annum for taking care of a sick child;
- protection against dismissal from the job during maternity;
- leave with the option of transfer to suitable employment during pregnancy;
- health insurance benefits and many others.

It is expected that the above program will be further developed and extended.

The second group of social policy measures is aimed at reducing the cost of child upbringing. It is believed that the successful implementation of the above programmes, serving the purpose of social justice and welfare, will help in preparing the ground of a policy stimulating population growth, should the need arise [48].

From the point of view of system analysis, it is important to know the elasticity α_d of fertility with respect to the social care expenditures z_d:

$$\alpha_d = \frac{dF}{F} : \frac{dz_d}{z_d}$$

Then one can try to construct a model of the general form:

$$F = F\,[\underline{x}, z_d] = \bar{F}\,[\underline{x}]\, z_d^{\alpha_d} \quad , \tag{46}$$

where $\underline{x}$ = the vector of exogenous variables including such factors as GNP/capita, health-service level, etc.

The next step is to find the relation between the fertility and total labour force L, which enters as a development factor in (13). In order to do that, it is necessary to employ a model of population growth. Keeping in mind the general form of development factor levels (7), it is convenient here to use a continuous (Lotka) version of population processes. Following Ref. [15], assume that the births of the community concerned have gone through a certain trajectory, described by $B(t)$ -- the density of births -- and a fixed life table gives the number of surviving to age a on radix unity, $p(a)^{\dagger}$. Then the number of persons at each age a of time t is determinate and equal to $B(t - a)\, p(a)$ and by integration, the total population at time t must be

$$N(t) = \int_0^{\infty} B(t - a)\, p(a)\, da \quad , \tag{47}$$

where $p(a) = 0$ for $a > w$ = last age of life table.

In order to get the amount of people in the productive age, one has to set

$$\bar{p}(a) = p(a) \quad , \quad a \geq T_d \quad ,$$
$$= 0 \quad , \quad a < T_d \quad ,$$

T_d = the age of adolescence.

Assuming that a part $\xi (0<\xi<1)$ of the total population in the productive age group can be employed and introducing the new variable $\tau = t - a + T_d$ in the integral (47) one gets the labour employed:

$$L(t) = \xi \int_{-\infty}^{t} \bar{p}(t - \tau + T_d)\, B\, (\tau - T_d)\, d\tau \quad . \tag{48}$$

Since $B(\tau - T_d)$ can be regarded as the product of total (female) population in reproductive age $\bar{N}(t)$, which is a part of $N(t)$, and fertility $F(\tau - T_d)$ one can write (48) in the form

† $p(a)$ can also be interpreted as a chance of living a years by a person born at $t = 0$, i.e.

$$p(a) = \exp[-\int_0^a \mu(t)\, dt] \quad ,$$

where $\mu(t)$ is the instantaneous death rate.

$$L(t) = \int_{-\infty}^{t} k_d(t, \tau)\ z_d^{\alpha_d}\ (\tau - T_d)\ d\tau \quad , \tag{49}$$

where

$$k_d(t, \tau) = \xi\ \bar{p}(t-\tau+T_d)\ \bar{F}\ [\underline{x}]\ \bar{N}\ (\tau - T_d) \quad . \tag{50}$$

It can be observed that (49) does not differ much when compared to (7), (14). The only difference is that (50) is generally not stationary in time, i.e. $k_d(t, \tau) \neq k_d(t - \tau)$. However, as shown in Ref. [22], the general optimization technique, which has been used so far, can be easily extended to the non stationary systems described by (49). The same remark concerns α_d which may be, generally speaking, different from α. Apart from that, the optimization of development methods, previously described, may be easily extended to the case which includes the expenditures spent on social benefits connected with labour and population policy. The impact of the expenditures z_d change on the labour level change is (in the present model) similar to the impact of investment change on the capital stock level. One can "invest" here in the population sector out of the present resources (i.e. GNP) in order to increase the labour force, which is the main production factor, for future development.

It is possible to observe, also, that L(t) depends directly on the ξ parameter, which represents the employment share in the total productive population. In the case where one changes the duration of working time, employment of women, etc., the value of $\xi(t)$ can be regarded as the decision parameters. The shortening of working time can be regarded as contribution to the social benefits programme. The leisure time increased in that way can be used effectively for relaxation, better education, wider participation of the whole population in cultural and social life, etc. On the other hand, the factor $\bar{F}[\underline{x}]$ depends on the exogenous parameters, or on the parameters which are endogenous, but shifted in time by T_d so they can be regarded as given (i.e. determined at the previous optimization intervals).

It should be noted that the problem of optimization of population policy involves many different factors and aspects. The model analyzed here takes into account the "productive aspects" of population change mainly in development.

In order to use that model for optimization of long-term development, it is also necessary to investigate the influence of all the factors affecting fertility and to construct the corresponding explicit relationship $\bar{F}[\underline{x}]$.

1.4. Pollution impact on environment and development

The productive activity is usually accompanied by side production of waste materials which are generally harmful to the human environment. In the present state of science and technology, most of the waste materials can be purified, utilized or recycled. However, the cost of purifying waste materials increases rapidly when a high degree of purity is required. Since the environment has an ability of clearing itself with the waste decay ratio, (depending on the waste ingredients) the following approach to the pollution problem has been proposed: Minimize the cost of waste and pollution treatment subject to the conditions that the degree of environment pollution is less than a given value. Following that approach, consider the pollution control model shown in Figure 10.

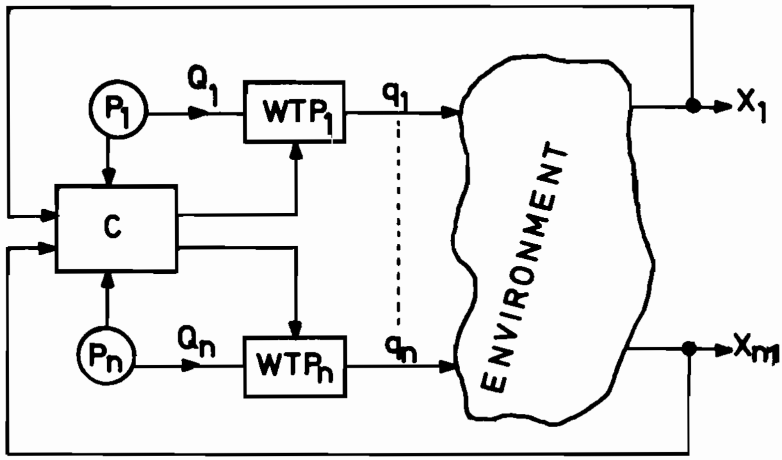


Figure 10.

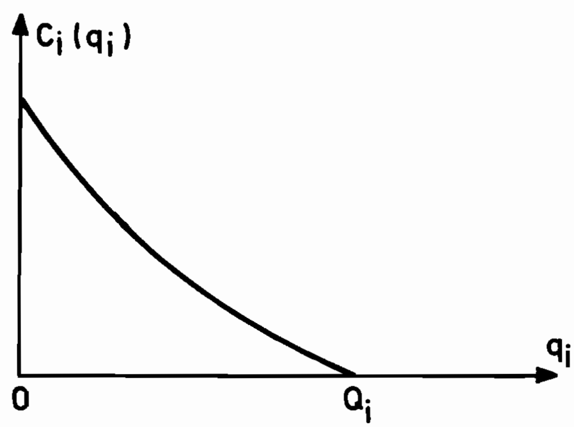


Figure 11.

Assume that the waste with intensity $Q_i(t)$, $i = 1,\ldots,n$, generated by n given polluters P_i (such as factories, power plants, urban centers, etc.) is being treated by the waste treatment plants WTP_i and with the intensity q_i is discharged into the environment (i.e. into air, water or soil). Q_i in turn may depend on the sectors production. The degree of environment contamination (expressed by such factors as pollutant fallout, dissolved oxygen (D.O.) concentration, or the biological oxygen demand (B.O.D.)) $x_i(t)$ can be observed by the pollution sensitive devices in the m given points or areas. The information obtained in that way together with the information regarding the weather forecast etc. is being used by the controller C to optimize the decision variables q_i, $i = 1,\ldots,n$.

The performance of pollution control can be measured by the functional

$$\Phi = \sum_{i=1}^{n} \int_0^T w_i(t) x_i(t) dt \quad , \tag{51}$$

where

$w_i(t)$ = given non-negative continuous weight functions,

T = optimization horizon.

The input-output dynamical properties of the environment according to the theoretical and experimental data can be approximated by the Volterra operator [27].

$$x_j(t) = \sum_{i=1}^{n} \int_0^T K_{ij}(t,\tau) q_i(\tau) d\tau \quad , \qquad j = 1,\ldots,m \quad , \tag{52}$$

where

$K_{ij}(t,\tau)$ = given non-negative continuous functions which satisfy the causality condition $K_{ij}(t,\tau) = 0$ for $t < \tau$.

A typical example of the cost function of the waste treatment plant has been shown in Figure 11. It can be approximated by the function

$$C_i(q_i) = k_i^{1-\alpha}(Q_i - q_i)^{\alpha} \quad , \qquad \alpha > 1 \quad , \qquad k_i > 0 \quad . \tag{53}$$

It is also assumed that the total waste treatment cost is limited, i.e.

$$\sum_{i=1}^{n} \int_0^T C_i(q_i)dt \leq C \quad , \tag{54}$$

where

C = given positive number.

The pollution treatment optimization problem can be formulated as follows. Find the non-negative strategy $c_i = \hat{c}_i$, $i = 1,\ldots,n$, such that

$$\Phi(c) = \sum_{j=1}^{m} \int_0^T w_j(t) \sum_{i=1}^{n} \int_0^T K_{ij}(t,\tau) \; [Q_i(\tau)+k_i^{1-\beta}c_i^{\beta}(\tau)]d\tau \, dt \quad ,$$

where

$\beta = 1/\alpha$ attains for $c = \hat{c}$ the minimum value subject to the constraints (54), and

$$Q_i(\tau) - k_i^{1-\beta} c_i^{\beta}(\tau) \geq 0 \quad , \quad t \in [0,T] \quad , \tag{55}$$

$$c_i(\tau) \geq 0 \quad . \tag{56}$$

Since the term

$$\bar{\Phi} = \sum_{j=1}^{m} \sum_{i=1}^{n} \int_0^T w_j(t) \int_0^t K_{ij}(t,\tau) Q_i(\tau) d\tau \, dt$$

is a constant, the problem boils down to the maximization of

$$F(c) = \sum_{i=1}^{n} \int_0^T f_i^q(\tau) c_i^{\beta}(\tau) d\tau \quad ,$$

where

$$f_i(\tau) = \left[\int_{\tau}^{T} k_i^{1-\beta} \sum_{j=1}^{m} w_j(t) K_{ij}(t,\tau) dt \right]^{1/q} \quad , \quad q = 1 - \beta$$

subject to the constraints (54) - (56).

It is obvious that when the constraint (55) is not active, the optimum control strategy can be derived by using the method of Section 1.3. When (55) is active the optimum strategy can be derived from the equation:

$$Q_i(t) - k_i^{1-\beta} \hat{c}_i^{\beta}(t) = 0 \quad .$$

Then the following theorem can be proved (for details see Ref. [27]).

<u>An optimum pollution strategy</u>

$$c_i(t) = \hat{c}_i(t) = \tilde{f}_i(t) \frac{C}{F} \quad , \quad i = 1,\ldots,n, \quad t \in [0,T] \quad , \tag{57}$$

where

$$\tilde{f}_i(t) = \begin{cases} f_i(t) & \text{for } t \notin S_i \\ \frac{F}{C} k_i^{1-1/\beta} Q_i^{1/\beta}(t) & \text{for } t \in S_i \end{cases} ,$$

$$S_i = \left\{ t : f_i(t) > \frac{F}{C} k_i^{1-1/\beta} Q_i^{1/\beta}(t) \quad , \qquad t \in [0,T], \quad i=1,\ldots,n \right\} ,$$

$$F = \sum_{i=1}^{n} \int_0^T \tilde{f}_i(t)\, dt$$

exists, such that

$$\Phi(\hat{C}) = \min_{c=\Omega} [\bar{\Phi} - F(C)] = \bar{\Phi} - F^{1-\beta} C^{\beta} \tag{58}$$

and Ω is the admissible control set defined by (54) - (56).

The optimum waste discharge strategies become

$$\hat{q}_i(t) = k_i^{1-\beta} \hat{C}_i^{\beta}(t) \quad , \qquad i = 1,\ldots,n \quad .$$

Using that theorem, it is possible for a given admissible pollution level ε to find the corresponding minimum waste treatment cost $\hat{C}$ (by solving the equation $\bar{\Phi} - \varepsilon = F^{1-\beta} C^{\beta}$)

$$\tilde{C} = \left[\frac{\bar{\Phi} - \varepsilon}{F^{1-\beta}} \right]^{1/\beta} \tag{59}$$

When the function $\bar{\Phi}(Y)$, which expresses the pollution level in terms of production Y, increases rapidly it may happen that the corresponding waste treatment cost becomes greater than the production income. In that case a new technology of production or new waste treatment plant should be developed.

That requires capital investment which can be optimized by the methods described in Section 1.2 and 2.2.

Using the aggregation formula (58) it is also possible to optimize a complex hierarchical system of environment pollution control (for details see Ref. [27]).

On the macro-level, the pollution cost C impact on the development can be analysed starting with formula (33). Obviously the pollution cost should be subtracted from the GNP generated in the planning interval Y so that Z = Y - C(Y) and (33) can be written

$$Y = \bar{\bar{Y}} + G^{q} \prod_{y=1}^{m} \gamma_{\nu}^{\gamma_{\nu}} \quad \{Y - [\frac{\bar{\Phi}(Y) - \varepsilon}{F^{q}}]^{\frac{1}{\beta}}\}^{\alpha} \qquad (60)$$

Since $\bar{\Phi}$ depends on Q_i in a linear fashion, it is natural to assume that $\bar{\Phi}(Y) = aY$, where a is a constant. Then it is possible to see that the right side of (60) decreases along with the admissible pollution level ε. As a result the solution Y^{**} of equation (60) decreases along with ε as shown in Figure 5. In other words the higher the standards are set, with respect to the pollution abatement policy, the lower the integrated GNP can be obtained. The admissible value of ε depends of course on many factors and the feedbacks exist between ε and government expenditures in health, housing and urban quality areas. In the present model ε will be regarded as a given (exogenous) parameter representing the existing or planned standards in pollution abatement policy. The optimum value $\hat{\varepsilon}$ of ε can be derived by minimizing the cost function $K(\varepsilon) = \hat{C}(\varepsilon) + D(\varepsilon)$, where $D(\varepsilon)$ is an increasing, strictly convex function, representing the cost of damages done to the environment. Since $K(\varepsilon)$ is strictly convex an optimum, unique value $\hat{\varepsilon}$ exists.

1.5 Development objectives, utility functionals

When one solves a complex optimum development problem the main concern is to choose the appropriate development goal. So far the integrated GNP has been used for that purpose. Doubts have frequently been expressed that it can hardly be used as the universal goal for development. It has been argued that the maximization of consumption per capita might be an alternative. Assuming the latter as a goal, one begins to wonder whether he really needs to spend much on development factors. However, rigorous analysis shows that it is impossible to consume, without financing, the development factors. We shall show that in the simplified situation, when the development is described by the function

$$Y(t) = K \prod_{\nu=1}^{m} [Y_\nu(t)]^{\beta_\nu} \quad , \qquad \sum_{\nu=1}^{m} \beta_\nu = 1 \quad , \tag{61}$$

$$Y_\nu(t) = \int_{-\infty}^{t} z_\nu(\tau) dt \quad ,$$

i.e. $\delta_\nu = T_\nu = 0$, $\alpha = K_\nu = 1$, $\nu = 1,\dots,m$.

We shall also assume that labour is exogenously introduced and is growing in an exponential fashion:

$$L(t) = \bar{L}\, e^{\lambda t} \quad , \qquad \lambda > 0 \quad .$$

The GNP generated per capita can be written in the form

$$Y/L = K \prod_{\nu=1}^{m} W_\nu^{\beta_\nu} \quad ,$$

where

$$W_\nu = Y_\nu/L = \text{development factor level/capita}$$

The GNP is allocated in such a way that

$$Y_1(t) = Y(t) - \sum_{\nu=2}^{m} z_\nu(t) = Y(t) - \sum_{\nu=2}^{m} \dot{Y}_\nu(t)$$

represents the consumption.

Consider the steady-state of development

$$\dot{W}_\nu(t) = \frac{\dot{Y}_\nu}{L} - \frac{Y_\nu}{L^2}\dot{L} = \frac{\dot{Y}_\nu}{L} - \lambda W_\nu = 0 \quad , \qquad \nu = 2,\ldots,m,$$

$t = 1,\ldots$.

The consumption per capita becomes

$$C = \frac{Y_1}{L} = \frac{Y}{L} - \sum_{\nu=2}^{m} W_\nu \lambda = K \prod_{\nu=2}^{m} W_\nu^{\beta\nu} - \sum_{\nu=2}^{m} W_\nu \lambda \quad . \tag{62}$$

<u>The following optimization problem can be formulated: find the values $W_\nu = \hat{W}_\nu$, $\nu = 2,\ldots,m$ which maximize the consumption per capita C</u>. Since (62) is concave the unique optimum strategy exists and it can be derived as the solution of the equations

$$\beta_\nu W_\nu^{\beta_\nu - 1} \prod_{\substack{i=2 \\ i \neq \nu}}^{m} W_i^{\beta_i} = \lambda / K \quad , \qquad \nu = 2,\ldots,m \quad ,$$

or

$$\begin{array}{l} (\beta_2 - 1)\ln W_2 + \cdots + \beta_m \ln W_m = \ln \dfrac{\lambda}{K\beta_2} \quad , \\ \vdots \qquad\qquad \cdots \qquad\qquad \cdots \\ \beta_2 \ln W_2 + \cdots + (\beta_m - 1)\ln W_m = \ln \dfrac{\lambda}{K\beta_m} \quad . \end{array} \tag{63}$$

When

$$D = \begin{vmatrix} \beta_2 - 1 \ , & \beta_3 \cdots \beta_m \\ \beta_2 \ , & \beta_3 \cdots \beta_m - 1 \end{vmatrix} \neq 0 \quad ,$$

there exists a unique positive solution of (63) $W_\nu = \hat{W}_\nu$, $\nu = 2,\dots,m$.

The result obtained can be formulated as follows. Among the systems having different development factor levels; W_ν, $\nu = 2,\dots,m$, and $D \neq 0$, the highest consumption per capita can be achieved for $W_\nu = \hat{W}_\nu$, $\nu = 2,\dots,m$. One can regard that result as an extension of the "classical golden rule of development" formulated originally for the simple-two factor production function (6).

What the present version of the golden rule says is, that in the steady state the maximum consumption per capita can be enjoyed if the development factors have a predetermined level. When one is determined to reach that level, the expenditures in development factors are inevitable. One can also consider the development factors Y_ν, $\nu = 1,\dots,m$, as valuable assets and regard the function (61) as a utility function. The last interpretation suggests that the capital stock, health, R & D, education facilities, etc. are public property. That assumption is fully justified in the socialist countries.

A known approach in the utility theory consists in finding the decision maker's strategy $Y_\nu = Y_\nu$, $\nu = 1,\dots,m$, which maximizes the utility

$$U = K \prod_{\nu=1}^{m} Y_\nu^{\beta_\nu} , \qquad (64)$$

subject to the budget constraints

$$\sum_{\nu=1}^{m} \omega_\nu Y_\nu \leq Z \qquad (65)$$

where

ω_ν - prices attached to Y_ν , $\nu = 1,\ldots,m$.

The optimum solution (in steady state) becomes

$$\hat{Y}_\nu = \frac{\beta_\nu}{\omega_\nu} Z \quad , \qquad \nu = 1,\ldots,m \quad . \tag{66}$$

Since the prices ω_ν (excluding the average wage) are generally unknown and $Y_\nu(t)$ depend in an inertial way on $z_\nu(\tau)$ the strategy (66) cannot be derived in the effective way.

One can, however, formulate a problem of maximization of the functional (21), which represents the integrated GNP, in terms of utility function approach. Since the factor levels Y_ν in (13) depend in the inertial fashion on the expenditures z_ν, and the $Y_\nu(t)$ change in time we have here a problem of preferences in time. Then it is natural to use the functional (21) with the budgetary constraints (15) - (18), as the expected utility in the planning interval [0,T]. Assuming that the central decision maker allocates the resources in the optimum manner, we have seen that the utility satisfies the additive property (25), i.e. one can deal with the increase of utility functional $\Delta U = \Delta Y(\hat{\underline{Z}})$ resulting out of the "present" time optimum expenditures strategy. Under that strategy one arrives at the "static" function (31):

$$\Delta U = G^q \prod_{\nu=1}^{m} z_\nu^{\gamma_\nu} \quad , \qquad \gamma_\nu = \alpha\beta_\nu \quad , \tag{67}$$

with the budget constraint

$$\sum_{\nu=1}^{m} z_\nu \le Z \quad . \tag{68}$$

The values of Z_ν represent here, however, the total (i.e. integrated within [0,T]) expenditures i.e. the values of factors or services acquired in [0,T]†. When the prices ω_ν do not change in [0,T] it is also possible to write (67) and (68) in the form similar to (64) and (65):

$$\Delta U = \bar{G}^q \prod_{\nu=1}^{m} \bar{Y}_\nu^{\gamma_\nu} , \tag{69}$$

$$\sum_{\nu=1}^{m} \omega_\nu \bar{Y}_\nu \leq Z , \tag{70}$$

where

$$\bar{Y}_\nu = Z_\nu/\omega_\nu \quad , \qquad \bar{G}^q = G^q \prod_\nu \omega_\nu^{-\gamma_\nu} \quad .$$

One of the important problems in modelling practice is the experimental determination of the utility function parameters. One possible approach is to learn about the utility from the decisions already taken in the past. For example, the optimum decisions which maximize (67) subject to (68), according to (32), become

$$\hat{Z}_\nu = \beta_\nu Z \quad , \qquad \nu = 1,\ldots,m \quad . \tag{71}$$

Assume that the "moving horizon" technique has been used for the past sequence of planning intervals $[-\tau,\ T - \tau]$, $\tau = 1,2,\ldots$ and the corresponding statistical data: $Z(\tau)$, $\hat{Z}_\nu(\tau)$, $\tau = 1,2,\ldots$ $\nu = 1,\ldots,m$, are available. Then it is possible to estimate the values of β_ν, $\nu = 1,\ldots,m$, from (71)

$\dagger$ It is also possible to start with the dynamic utility functional (23), which attains a maximum under the factor coordination strategy, subject to the constraints imposed on the value of production factors (22).

by known statistical methods. The estimated values $\hat{\beta}_\nu$ of β_ν can be used 'ex ante' for the planning interval [0,T]. When $\hat{\beta}_\nu$ change slowly in time, a prediction technique (as will be shown later) can be also applied.

It should be observed that the proposed estimation technique is based on the assumption that the decision regarding the allocation of Z_ν expenditures are optimum with respect to the goal function (67). The β_ν parameters, which were regarded so far as inherent to the development mechanisms (including the technology of production), are generally unknown 'ex ante' to the decision maker but one is inclined to treat them as "stable" or slightly changing in time. On the other hand the observed, from the statistical data values $\tilde{\beta}_\nu(\tau) = \frac{Z_\nu(\tau)}{Z(\tau)}$, $\nu = 1,\ldots,m$, $\tau = 0,-1,\ldots$ change usually randomly in time, which can hardly be attributed to the change of development mechanisms. Rather, these changes should be attributed to the slight variations in the allocation strategy: $\tilde{Z}_\nu = \tilde{\beta}_\nu Z$, $\nu = 1,\ldots,m$, as a result of which the utility decreases by the factor δ, i.e. $\tilde{\Delta U} = \delta \Delta U$, where

$$\delta = \prod_{\nu=1}^{m} \left(\frac{\tilde{\beta}_\nu}{\beta_\nu}\right)^{\gamma_\nu} .$$

One can assume, however, that the values estimated by known statistical methods (e.g. the mean squares) $\hat{\beta}_\nu$, $\nu = 1,\ldots,m$, are close to β_ν , $\nu = 1,\ldots,m$. Otherwise the utility $\tilde{\Delta U}$ would be much less than the potentially possible value ΔU.

Much that has been said about the allocation of government expenditures concerns the private (personal) consumption. In particular, the dynamic model of consumption expenditures can be used here. As shown in Ref. [23] the consumers change their preferences with age; they spend, for instance, less out of the income on durable goods at the end of their life. Since the consumer's preference structure also depends on the price structure, while the prices are determined by the supply-demand relations with respect to the commodities generated by the productive sectors, we shall study that model later.

When the prices are known it is also possible to express the integrated GNP (21) and the utility (67) in constant prices per capita. The maximization of that functional will be regarded as the main optimization goal which, as is believed, is close enough to the notion of maximum personal and social satisfaction and well being. It enables, as it will be seen later, the effective planning of long term development subject to different development opportunities and constraints.

It should be noted that by virtue of our neoclassic formulations and (62):

$$\frac{dC(\underline{\hat{W}})}{d\lambda} = - \sum_{\nu=2}^{n} \hat{W}_{\nu} < 0 \quad ,$$

one is inclined to accept the widespread wisdom that rapid population growth is harmful to economic welfare. Recently, however, Samuelson has shown (International Economic Review Oct. 1975, vol. 16, No. 3) that when the consumption per capita C is divided between two groups of population (productive and economical dependent), which consume C^1, C^2 respectively, the golden rule favours population growth (a large number of descendants means better support for retired). The golden rule should be supplemented in that case by the "Biological interest rate" relation $U_1' \left[C^1,C^2\right]/U_2'\left[C^1,C^2\right]=$

$1 + \lambda$, where $U\left[C^1, C^2\right]$ = given utility function. Then by maximizing $U(\lambda)$ one can find the optimum value of λ, which determines the "goldenest golden-rule" of development.

II. Multi-sector Model

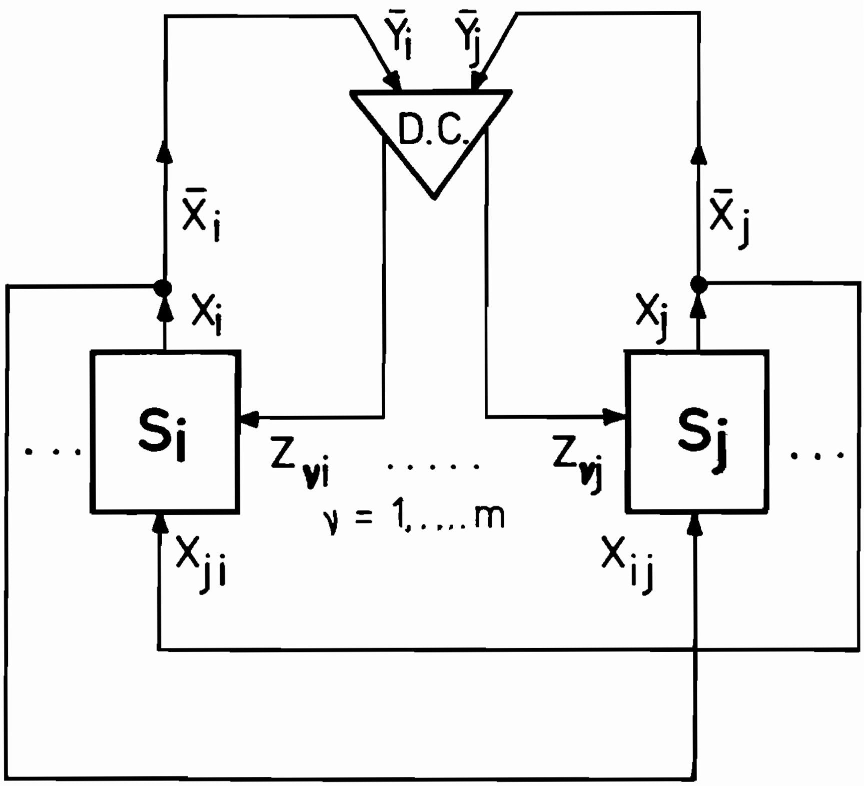
2.1 Optimization of sectorial strategies

In order to describe in a more general way the production processes it is necessary to investigate the n-sector system shown in Fig. 12, where X_i represents the output production of sector S_i and X_{ji} - the amount of commodities which S_j is selling to S_i, $i,j = 1,\dots,n$. The net outputs $\bar{X}_i$, $i = 1,\dots,n$ contribute to the GNP, which is allocated by the decision center (DC) among the S_i sectors. Adopting the structure of Fig.12 the next step is to describe the input-output relation for each sector (i.e. the sector production functions). At that step one is tempted to use the linear model (e.g. the Leontief input-output model), which is attractive from the point of view of analytic simplicity and an easy way of estimating the technological coefficients. However, the linear models are not so attractive from the point of view of macro-economic growth theory and can hardly be used effectively to describe the long-range development. In order to describe the technological change and substitution among the production factors, one would rather adopt the nonlinear production function of the Cobb-Douglas or C.E.S. type.

In the present section we shall show that a sort of reconciliation between these two approaches is also possible.

We shall start with the simple Cobb-Douglas production functions:

$$X_i = F_i^{q_i} \prod_{j=1}^{n} X_{ji}^{\alpha_{ji}} \quad , \qquad i = 1,\dots,n \quad , \tag{1}$$



$\bar{Y}_i$
$\bar{Y}_j$
D.C.
$\bar{X}_i$
$\bar{X}_j$
X_i
X_j
S_i
S_j
$Z_{\nu i}$
$Z_{\nu j}$
$\nu = 1, \ldots, m$
X_{ji}
X_{ij}

Figure 12.

$$q_i = 1 - \sum_{j=1}^{n} \alpha_{ji} > 0 \quad , \qquad \alpha_{ji} \geq 0 \quad , \qquad i,j = 1,\ldots,n \tag{2}$$

where α_{ji}, i,j = 1,...,n, given positive numbers while F_i depend on the vector $\underline{Z}_i$, of development factors allocated by DC, i.e. $F_i = F_i(\underline{Z}_i)$, i = 1,...,n.

We shall also assume that a set of sector prices p_i, i = 1,...,n, is given so the model of Fig.12 can be described in monetary terms by the Eqs.

$$Y_i - \sum_{j=1}^{n} Y_{ij} = \bar{Y}_i \quad , \qquad i = 1,\ldots,n \quad , \tag{3}$$

$$Y_i = K_i \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} \quad , \tag{4}$$

where $Y_{ji} = p_j X_{ji}$, $K_i = p_i F_i^{q_i} \prod_{j=1}^{n} p_j^{-\alpha_{ji}}$, i,j = 1,...,n .

Now it is possible to introduce the decision structure. We shall assume that each sector S_i, j = 1,...,n, can decide how much of input $\underline{Y}_{ji}$, i = 1,...,n to buy in order to maximate the net profit (value added)

$$D_i = Y_i - \sum_{j=1}^{n} Y_{ji} \quad , \qquad i = 1,\ldots,n \quad . \tag{5}$$

It should be noted that the profit maximizing strategy is a micro-economic concept. Since the sector consists of a given number of factories and firms, it is natural to extend that concept to the macro-model.

Since, by virtue of (2), D_i is strictly a concave function, a unique set of strategies $Y_{ji} = \hat{Y}_{ji}$, i,j = 1,...,n exists such that $D_i(\hat{Y}_{ji},\ j,i = 1,\dots,n) = \hat{D}_i$ is maximum.

That strategy, as shown in Ref.[30], becomes

$$\hat{Y}_{ji} = \alpha_{ji}\hat{Y}_i \quad , \qquad j,i = 1,\dots,n \tag{6}$$

where

$$\hat{Y}_i = F_i \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \quad , \qquad i = 1,\dots,n \ . \tag{7}$$

When one uses that strategy

$$\hat{D}_i = q_i\hat{Y}_i \quad , \qquad i = 1,\dots,n \tag{8}$$

and the gross product becomes

$$Y = \sum_{i=1}^{n} \bar{Y}_i = \sum_{i=1}^{n} \hat{D}_i \quad . \tag{9}$$

The two important results follow from the relations (6)÷(9):

I. As follows from (6) the normative n-sector nonlinear model (3)-(5) behaves under optimum strategy in a similar way to the linear Leontief model with the technological coefficients α_{ji}, $i,j = 1,\dots,n$. However, the outputs $\hat{Y}_i$, $i = 1,\dots,n$ in the nonlinear model are specified in an unique manner by prices p_j, $j = 1,\dots,n$ and F_i coefficients.

II. When prices are fixed the sector net productions $\bar{Y}_i$ $i = 1,\dots,n$, do not depend on sector interactions (in terms of Y_{ji}, $j,i = 1,\dots,n$), and the gross product is a linear function of F_i, $i = 1,\dots,n$.

Result I can be used for a simple estimation procedure of α_{ji}, $j,i = 1,\dots,n$, coefficients.

Assume for that purpose that the input-output tables for an n-sector economy be given. Assume also that each sector optimizes the net profit (5) so that relation (6) is valid. Then it is possible to derive a sequence of numbers

$$\tilde{\alpha}_{ji}(t) = \frac{Y_{ji}(t)}{Y_i(t)} = \frac{\hat{Y}_{ji}(t)}{\hat{Y}_i(t)} \quad , \qquad j,i = 1,\dots,n \qquad t = -1,-2,\dots \tag{10}$$

As follows from real statistical data, for each fixed j,i, the sequence $\tilde{\alpha}_{ji}(t)$, $t = -1,-2,\dots$ of past data is generally a random sequence.

Since

$$\tilde{\alpha}_{ji} = \frac{p_j X_{ji}}{p_i X_i} \tag{11}$$

the variation of $\tilde{\alpha}_{ji}$ can be attributed to the technological change, i.e. the change of $\frac{X_{ji}}{X_i}$, or the variation of sectorial prices p_{j/p_i} .

As far as the interpretation of α_{ji} is concerned, it is also possible to regard α_{ji} as a characteristic of a technological relation independent of market phenomena like prices. L.R. Klein in the paper [16] has shown for example that the relation (6) can also be obtained by an assumption connecting the pricing system to the technology. He assumes for the competitive market

$$\frac{\partial F_i}{\partial X_i} = -\lambda_i p_i \quad , \qquad \frac{\partial F_i}{\partial X_{ji}} = \lambda_i p_j \quad , \qquad i,j = 1,\dots,n \quad ,$$

where $F_i(X_i, X_{ji}, j=1,\dots,n) = 0$, is the production function and for the ratio in (11) obtains the system of partial differential eqs:

$$-\frac{\frac{\partial F_i}{\partial X_{ji}} X_{ji}}{\frac{\partial F_i}{\partial X_i} X_i} = \alpha_{ji} \quad , \qquad j = 1,\ldots, i-1, \quad i+1,\ldots,n \quad ,$$

which as a solution have F_i functions. For the case of Cobb-Douglas functions, one gets from here the formulae equivalent to the (obtained in a different way) relations (6).

Since the statistical data regarding X_i, X_{ji}, $i,j = 1,\ldots,n$ are usually not available the only way to learn about α_{ji} is from $\tilde{\alpha}_{ji}(t)$ series. In Saito's model [41], the input-output table for a chosen year has been used for that purpose.
In the MRI models, constructed in Poland, the statistical estimates $\bar{\alpha}_{ji}$ of α_{ji} derived by using the ex post data $\tilde{\alpha}_{ji}(t)$, $t = -1,-2$, have mostly been used. That approach enables us to neglect the price-variation effect on possible suboptimal strategies in a sectorial policy.

Then the estimates $\bar{\alpha}_{ji}$ can be used ex ante for modeling the future development. In the long-term planning, the model of technological changes (for $\tilde{\alpha}_{ji}$ coefficients) can be used (as shown in Sec. 2.4).

It should also be noted that the decomposition method, which yields the important results (6)-(9), can also be used effectively in the model described by C.E.S. production function (instead of (1)) [20].

Another possible extension concerns the labor force. In the input-output tables, it is customary, for example, to regard labor as additional inputs to the productive sectors. As shown in Ref. [30], it is also possible to regard labor as the output of an additional sector S_o, which cooperates with the production system. The production function of that sector

$$Y_o = K_o \prod_{j=1}^{n} Y_{jo}^{\alpha_{jo}} , \quad q_o = 1 - \sum_{j=1}^{n} \alpha_{jo} > 0$$

where

$$K_o = p_o F_o^{q_o} \prod_{j=1}^{n} p_j^{-\alpha_{jo}}$$

Y_{jo} - cost of goods produced by sectors S_j and consumed by S_o,

$Y_o = p_o X_o + \bar{Y}_o$ - total value of employment in monetary units, p_o - average net wage,

X_o - number of employees, $\bar{Y}_o$ - other means of income.

The sector S_o can be treated as a "productive" sector with the value added (savings)

$$D_o = Y_o - \sum_{j=1}^{n} Y_{jo} .$$

One can assume that the S_o objective is to maximize D_o (i.e. to maximize wages subject to the given consumption; the savings earned in this way can be used for purchases of durable goods). Then according to (6)-(8) one gets

$$\hat{Y}_o = F_o \prod_{j=1}^{n} \left(\frac{\alpha_{jo}}{p_j}\right)^{\alpha_{jo}/q_o} p_o^{\frac{1}{q_o}} ,$$

$$\hat{Y}_{jo} = \alpha_{jo} \hat{Y}_o ,$$

$$\hat{D}_o = q_o \hat{Y}_o .$$

In that model each sector employs $\hat{Y}_{oi}$ value of labor, $i = 1,\ldots,n$, while the rest goes to nonproductive sectors.

2.2 Decomposition and optimization of decision strategies

Using the result II of section 2.1, it is possible to decompose the decision structure. The higher level decision center (D.C.) is concerned mainly with an optimum allocation of GNP generated (at the year t-1) among the sectors S_i, $i = 1,\ldots,n$. The allocation strategy $z_{\nu i}(t)$ in the general case may concern all the development factors ($\nu = 1,\ldots,m$) and sectors ($i = 1,\ldots,n$), or may be limited to a given number of factors (e.g. the investments or investment and labor only).

The production factors endowments received by sectors S_i, according to the decision of D.C. are regarded as exogenous, by the sectorial decision units, which are concerned with optimization of input mix Y_{ji}, $i,j = 1,\ldots,n$, mainly. On the other hand, the decision center, according to the result II, does not need to worry about choosing the best intersector flows $\hat{Y}_{ji}$, $j,i = 1,\ldots,n$, which are subject to sectorial policy.

In other words, a decentralization of management structure, which corresponds to the existing structure of management of the planned economies, is possible.

When prices and technological coefficients are fixed the sector contribution to GNP: $D_i = q_i\hat{Y}_i$, where $\hat{Y}_i$ is defined by (7), becomes proportional to $F_i(\underline{z}_i)$. Taking into account the development models which were developed in Sec. 1.1, 1.2, it is natural to assume that sectorial production functions are of the form (12)(14) or (38) while the optimization goal is to maximize

$$Y = \sum_{i=1}^{n} \int_{-\infty}^{T} e^{-\lambda t} Y_i(t)\,dt \triangleq \bar{Y} + \Delta Y \quad .$$

We shall start with the model which is an n-sector extension of (38)/(41). The problem consists of finding the strategy $z_{\nu i}(\tau) = \hat{z}_{\nu i}(\tau)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$, $\tau\varepsilon[0,T)$, which maximizes:

$$\Delta Y = \sum_{i=1}^{n} \Delta Y_i = \sum_{i=1}^{n} \int_{0}^{T} dt\; e^{\vartheta_i t} \int_{0}^{t} K_i e^{-\delta_i (t-\tau)}$$

$$\prod_{\nu=1}^{m} [z_{\nu i}(\tau - T_{\nu i})]^{\gamma_\nu}\, d\tau \quad , \tag{12}$$

where K_i, ϑ_i, δ_i, γ_ν, $T_{\nu i}$ = given constants. $(\vartheta_i = \lambda_i - \lambda)$,

subject to the constraints:

$$\sum_{i=1}^{n} \int_{0}^{T} w(\tau) z_{\nu i}(\tau - T_{\nu i})\, d\tau \leq Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \tag{13}$$

$$\sum_{\nu=1}^{m} Z_\nu \leq Z \quad , \tag{14}$$

$$z_{\nu i}(\tau - T_\nu) \geq 0 \quad , \qquad Z_\nu \geq 0 \quad , \qquad \nu = 1,\ldots,m, \quad i = 1,\ldots,n \quad , \tag{15}$$

$t\varepsilon[0,T]$, $\sum_{\nu=1}^{m} \gamma_\nu = \alpha < 1$, $w(t)$ = given weight function.

The expression (12) can be written in the form (27) of Sec. 1.2:

$$\Delta Y = \sum_{i=1}^{n} \int_{0}^{T} \prod_{\nu=1}^{m} [w(\tau) z_{\nu i}(\tau - T_{\nu i})]^{\gamma_\nu} [w(\tau)\varphi_i(\tau)]^q \, d\tau \quad , \quad q = 1 - \alpha \; , \tag{16}$$

where

$$\varphi_i(\tau) = \left\{ K_i w(\tau)^{-q} \int_{\tau}^{T} e^{\vartheta_i t - \delta_i (t-\tau)} dt \right\}^{1/q} .$$

Then the problem of maximizing (16) subject to (13) can be solved. Using the Holder and Minkovski inequalities [20], one obtains:

$$\hat{z}_{\nu i}(\tau - T_{\nu i}) = \frac{\varphi_i(\tau)}{G} Z_\nu \quad , \qquad i = 1,\ldots,n \quad , \quad \nu = 1,\ldots,m \tag{17}$$

where

$$G = \int_{0}^{T} \sum_{i=1}^{n} \varphi_i(\tau) w(\tau) d\tau \quad .$$

The value of $\Delta Y(\hat{\underline{z}})$ under optimum decisions becomes

$$\Delta Y(\hat{\underline{z}}) = G^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad . \tag{18}$$

The optimum strategies $Z_\nu = \hat{Z}_\nu$, $\nu = 1,\ldots,m$, which maximize (18) subject to (14) can be derived by (32) of Sec.1.2. Then by virtue of (33) one can write

$$Y = \bar{Y} + G^q \prod_{\nu=1}^{m} \gamma_\nu^{\gamma_\nu} Y^\alpha \quad , \tag{19}$$

where

$$\gamma_\nu = \beta_\nu \alpha \quad .$$

Obviously, for $0 < \alpha < 1$ a positive solution $Y = Y^*$ of (19) exists and the numbers $Z = Y^*$, $\hat{Z}_\nu = \beta_\nu Z$, $\nu = 1,\dots,m$ and strategies (17), are completely determined in the "closed loop" conditions.

The relation (18) can be regarded as the aggregated development function of the n-sector model (12).

In the case when the integral constraint is replaced by the amplitude constraints:

$$\sum_{i=1}^{n} z_{\nu i}(t-T_{\nu i}) \le Z_\nu(t) \quad , \qquad t \in [0,T] \quad , \tag{20}$$

$$Z_\nu(t) = \text{given functions} \quad , \qquad \nu = 1,\dots,m \quad ;$$

the corresponding solution becomes [20]:

$$\hat{z}_{\nu i}(\tau - T_{\nu i}) = \frac{\varphi_i(\tau)}{\varphi(\tau)} Z_\nu(\tau) \quad , \qquad i = 1,\dots,n \quad , \qquad \nu = 1,\dots,m \quad , \tag{21}$$

where

$$\varphi(\tau) = \sum_{i=1}^{n} \varphi_i(\tau) \quad ,$$

and

$$\Delta Y(\hat{\underline{z}}) = \int_0^T \varphi^q(\tau) \prod_{\nu=1}^{m} [Z_\nu(\tau)]^{\gamma_\nu} d\tau \quad . \tag{22}$$

Since $\sum_{\nu=1}^{m} Z_\nu(t) = Z(t)$, $\tau\varepsilon[O,T]$, and $Z(t)$ by virtue of the relation (19) of Sec. 1.2 depends on the past value of GNP, the solution (21) should be regarded as the closed loop solution.

One should observe that according to (17),(21), <u>in order to get maximum growth, the sectorial strategies $z_{\nu i}(\tau)$ should be chosen in such a way that they are proportional to $\varphi_i(\tau)$ functions</u>. We can call that strategy the"<u>sectors coordination principle</u>". It should be noted that using the sectors and factors coordination principles [(24) of Sec. 1.2] one can derive a unique optimum allocation strategy $\hat{z}_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots n$, $\tau\varepsilon[O,T]$, for the n-sector system, described by the equations:

$$\Delta Y = \sum_{i=1}^{n} \int_0^T \prod_{\nu=1}^{m} f_{\nu i}(t)dt \quad , \tag{23}$$

$$f_{\nu i}(t) = \left\{ e^{\vartheta_i \frac{t}{\beta_{\nu i}}} \int_0^t K_i e^{-\delta_{\nu i}(t-\tau)} [z_{\nu i}(\tau - T_{\nu i})]^\alpha d\tau \right\}^{\beta_{\nu i}}$$

$$\sum_{\nu=1}^{m} \beta_{\nu i} = 1 \quad , \qquad i = 1,\ldots,n \quad ,$$

which can be regarded as an extension of (21)-(23) of the Sec. 1.2 formula. Indeed, using the factor coordination principle for each sector S_i one finds $\hat{z}_{\nu i}$, $\nu \geq 2$ and can represent (23) as the sum of ΔY_i components ($\sum_{i=1}^{n} \Delta Y_i = \Delta Y$), where according to (26) of Sec.1.2:

$$\Delta Y_i = \prod_{\nu=2}^{m} C_{\nu i}^{-\beta_{\nu i}} \int_0^T dt\, e^{\vartheta_i t} \int_0^t K_i e^{-\delta_{\nu i}(t-\tau)} [z_{1i}(\tau - T_{1i})]^{\alpha} d\tau \quad ,$$

$$i = 1,\dots,n \quad , \tag{24}$$

and $C_{\nu i}$ can be determined from the corresponding constraints.

Since the problem of maximizing $\sum_{i=1}^{n} \Delta Y_i(Z_{1i})$ subject to the integral:

$$\sum_{i=1}^{n} \int_0^T w(\tau) z_{1i}(\tau - T_{1i}) d\tau \leq Z_1 \quad ,$$

or amplitude

$$\sum_{i=1}^{n} z_{1i}(t - T_{1i}) \leq Z_1(t) \quad , \quad \tau \varepsilon [0,T] \quad ,$$

constraint is a special case of the problem (12)-(15) it is obvious that the optimum strategy $\hat{z}_{\nu i}(\tau)$, $\nu = 1,\dots,m$, $i = 1,\dots,m$, which maximizes the integrated output (23) subject to the amplitude or integral constraints exists and can be derived in an explicit manner. When one does not count the different possible formulations of constraints (i.e. amplitude and/or integral constraints) the strategy derived is unique.

One should observe that the solution of the general optimization problem, with n-sector nonlinear development functions (23) degenerates when $\alpha \to 1$ (i.e. $q \to 0$) and the optimum strategies described by (17),(21) degenerate into Dirac's $\delta(t)$ functions or "bang-bang" solutions. These strategies are not unique and the analytic derivation of explicit solutions in the general case of the n-sector system is not easy. In other words, the linearization of our problem does not simplify it.

It should be also noted that in the model (23) the sectorial development factors $f_{\nu i}$, $\nu = 1,\ldots,m$, depend on the "directed" expenditures $z_{\nu i}$. In the real systems part of the general Z_ν expenditures goes to the sectors for financing the specialized education, health service, research and development, etc. The rest (z_ν) has a "universal" character, e.g. the expenditures on general education, health care, science, etc. In the more general model, one can represent $f_{\nu i}$ in the form

$$f_{\nu i}(t) = \left\{ e^{\vartheta_i \frac{t}{\beta_{\nu i}}} \int_0^t K_i e^{-\delta_{\nu i}(t-\tau)} [z_{\nu i}(\tau - T_{\nu i})]^{\alpha_1} [z_\nu(\tau - T_\nu)]^{\alpha_2} d\tau \right\}^{\beta_{\nu i}} \tag{25}$$

where $\alpha_1 + \alpha_2 = \alpha$, and replace (13)(20) by

$$\sum_{i=1}^{n} \int_0^T w(\tau)[z_{\nu i}(\tau - T_{\nu i}) + Z_\nu(\tau - T\ ]d\tau \leq Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \tag{26}$$

$$\sum_{i=1}^{n} z_{\nu i}(\tau - T_{\nu i}) + z_\nu(\tau - T\ ) \leq Z_\nu(t) \quad , \qquad \tau \varepsilon [0,T] \quad , \quad \nu = 1,\ldots,m$$

respectively. It is possible to show that the present problem can also be solved effectively by the methodology used so far. The main advantage of that methodology is that it constitutes a useful aggregation device at the same time. For example, the complicated n-sector production functions in (12) aggregate under optimum strategy to the simple formula (18). At the next aggregation step (18) aggregates to (19), etc.

On the other hand it is possible to decompose (18) into the given number of specialized productive or regional subsystems. To show that possibility, consider N subsystems, each described by the development function:

$$\Delta Y_j = F_j^q \prod_{\nu=1}^{m} Z_{\nu j}^{\gamma_\nu} \quad , \qquad j = 1,\ldots,N \quad , \tag{27}$$

The problem consists of finding the nonnegative strategy $Z_{\nu j} = \hat{Z}_{\nu j}$, $\nu = 1,\ldots,m$, $j = 1,\ldots,N$, such that $\Delta Y = \sum_{j=1}^{N} \Delta Y_j$, attains maximum subject to the constraints

$$\sum_{j=1}^{N} Z_{\nu j} \leq Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad . \tag{28}$$

It is easy to show that the optimum strategy becomes

$$\hat{Z}_{\nu j} = \frac{F_j}{F} Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \quad j = 1,\ldots,N \quad , \tag{29}$$

where

$$F = \sum_{i=1}^{N} F_i \quad ,$$

and the value of ΔY under optimum strategy is

$$\Delta Y(\hat{\underline{Z}}) = F^q \prod_{\nu=1}^{m} Z_\nu^{\gamma_\nu} \quad . \tag{30}$$

The results obtained so far indicate that it is possible to use the proposed methodology for optimization of the decentralized multi-level decision structure shown in Fig.13.

At the second level the expenditures Z_ν , $\nu = 1,\ldots,m$, are allocated among sectors according to (17),(21).

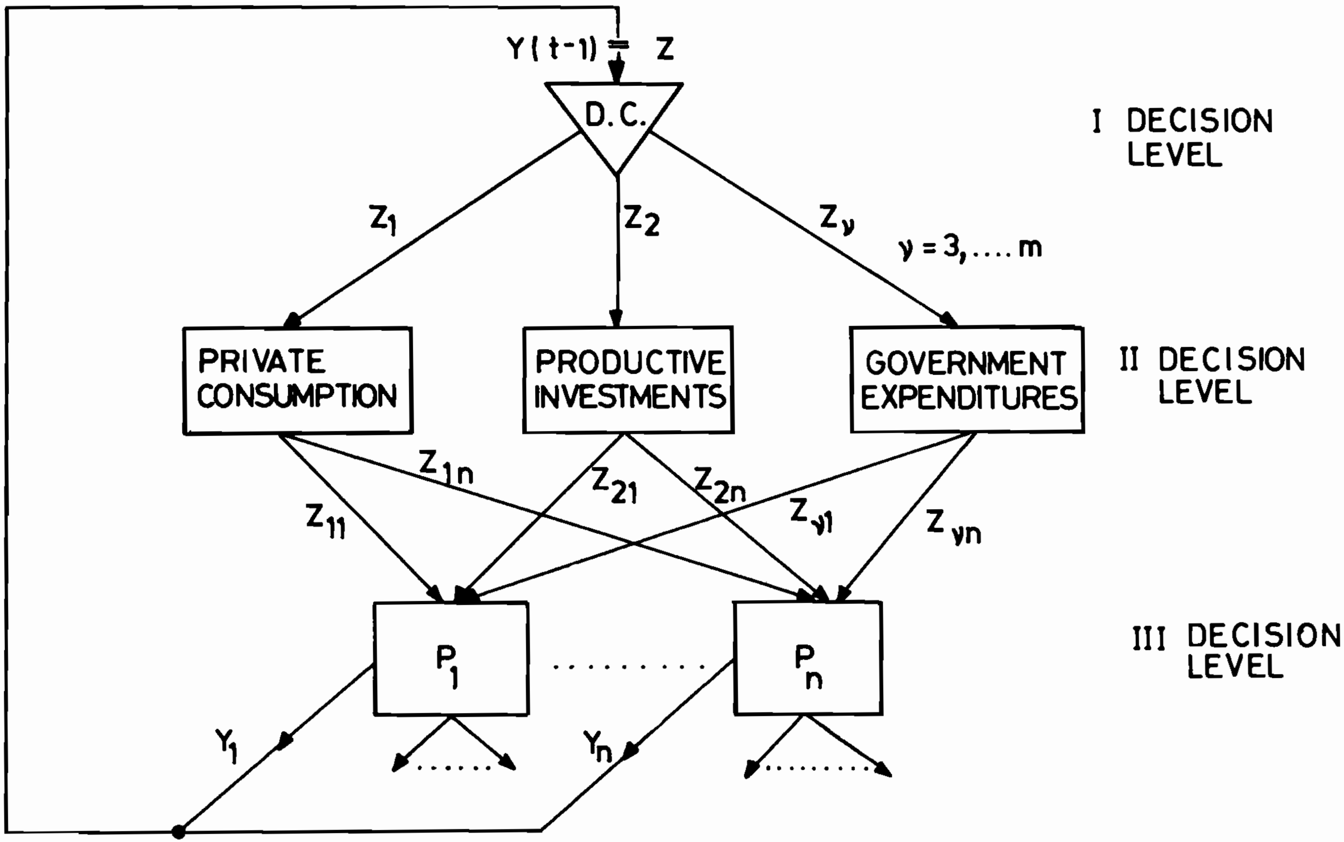

Y(t-1) = Z
D.C.
I DECISION LEVEL
Z1
Z2
Zν
ν = 3, ... m
PRIVATE CONSUMPTION
PRODUCTIVE INVESTMENTS
GOVERNMENT EXPENDITURES
II DECISION LEVEL
Z1n
Z11
Z21
Z2n
Zν1
Zνn
P1
Pn
III DECISION LEVEL
Y1
Yn

Figure 13.

At the third (and possible lower-levels) the expenditures can be allocated in more specialized or regional ways.

The outputs generated by Sectors $S_1,\ldots,S_n$ constitute the GNP

$$Y(t) = \sum_{i=1}^{n} \bar{Y}_i(t) \quad ,$$

allocated at the next year.

In Refs. [19-37] more detailed aspects of optimum allocation of resources in production, consumption, environment and regional subsystems have been studied, using the present methodology. It should also be noted that the methodology presented can be extended in different directions. One can, for example, solve the problem when the integral and amplitude constraints are acting at the same time, or when $w(\tau)$ in (13) is different for different ν-factors. Another possible extension concerns the prices. As follows from (7) of Sec. 2.1, the prices' effect on the output production can be represented by the factors

$$\Pi_i = \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \quad , \qquad i = 1,\ldots,n \quad ,$$

which change in time, along with $p_j(t)$, $j = 1,\ldots,n$.

One does not know the future $\Pi_i(t)$ change, so it was tacitly assumed that the factors $K_i e^{\mu_i t}$ in (23) will take care of that change. Since the parameters K_i, μ_i in the model under consideration are estimated ex post from statistical data, regarding $\hat{Y}_i(t)$ and $\hat{Z}_{\nu i}(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$, $t = -1,-2,\ldots,$ one cannot be certain that they are accurately describing $\Pi_i(t)$ "ex ante". The strategies $\hat{z}_{\nu i}(t)$, derived at the present section should be therefore regarded as the first approximation $\hat{z}_{\nu i}^{(o)}$

of the optimum strategy. However, it is possible to construct an iterative process $\hat{z}_{\nu i}^{(k)}$, k = 0,1,... of approximations converging for $k \to \infty$ to the optimum strategy.

The main idea behind that process consists in computation of the prices resulting from the $\hat{z}_{\nu i}^{(k)}$ strategy and finding the corresponding $\Pi_i^{(k)}$, which can be used for computation of the next iteration $\hat{z}_{\nu i}^{(k+1)}$ etc.

2.3 The model of prices

The prices play an important role in the optimally planned, socialist economy and they are supposed to fulfil many functions. In particular, it is important that prices reflect the social expenses and results of production efficiency. They should stimulate the incentives of producers and influence the consumption structure in such a way that a maximum social utility is obtained. The prices should also guarantee the market equilibrium. In addition, by a system of indirect taxation of output and government dotation, it is possible to change the prices in such a way that the widely used consumption goods are cheap, compared to some luxury goods. In that way, a just social policy, which serves the purpose of fair income distribution among different wage classes, can be implemented.

From those introductory remarks, one can see that the construction of a good mathematical price-model for a socialist economy is not easy and a number of simplifying assumptions is inevitable. First of all it is necessary to observe that we are interested in the highly aggregated sector prices. These prices, say p_i, are usually defined as an arithmetic weighted mean of a number M of commodities X_{ji} with prices p_{ji}, i.e.

$$p_i \triangleq \sum_{j=1}^{M} \frac{X_{ji} p_{ji}}{X_i} \quad , \qquad X_i = \sum_{j=1}^{M} X_{ji} \quad . \tag{31}$$

Since p_i change in time in the statistical tables, the relative prices, or price indices

$$p_i^t = p_i(t) / p_i(t-1) \quad , \qquad t = 0,1,...$$

are usually given.

The price indices can be used to express the value of product $Y_i(t)$, at the basic year (t=0) prices. Obviously, for that purpose it is necessary to divide $Y_i(t)$ by $\prod_{\tau=1}^{t} p_i^{\tau}$. Since in the model discussed it is important to express the GNP in constant (i.e. basic year) prices, our first objective is to derive the aggregated sector price indices p_i^t , t = 1,2,...

So far (in Sec.2.2) the supply part of the model has been studied.

On the demand side of the model, one faces in each year t the expenditures

$$Z_\nu(t) = \gamma_\nu Z(t) \quad , \quad \sum_{\nu=1}^{m} \gamma_\nu = 1 \quad , \quad t = 0,1,... \tag{32}$$

which are spent on the purchase of goods $\bar{Y}_i$, i = 1,...,n , produced by the S_i-sectors, i.e.

$$\bar{Y}_i(t) = \sum_{\nu=1}^{m} \lambda_{\nu i} Z_\nu(t) = Z(t) \sum_{\nu=1}^{m} \lambda_{\nu i} \gamma_\nu \quad , \tag{33}$$

where $\lambda_{\nu i}$ = given non-negative coefficients determining the ν-th expenditure contribution to the demand confronting the i-th production sector, and

$$\sum_{i=1}^{m} \lambda_{\nu i} = 1 \quad , \qquad \nu = 1,\ldots,m \quad . \tag{34}$$

The coefficients $\lambda_{\nu i}$, γ_ν , $\nu = 1,\ldots,m$, $i = 1,\ldots,n$ depend on prices p_i, $i = 1,\ldots,n$, GNP per capita in constant prices $(\tilde{Y}/N)$ etc. In rather simple models one can assume that

$$\gamma_\nu = a_\nu (\tilde{Y}/N)^{\varepsilon_\nu} \quad , \tag{35}$$

$$\lambda_{\nu i} = b_{\nu i}(p_i)^{-E_{\nu i}} \quad , \qquad \nu = 1,\ldots,m \quad , \quad i = 1,\ldots,n$$

where the parameters a_ν, ε_ν, $b_{\nu i}$, $E_{\nu i}$ can be estimated ex post by known statistical methods. Care should be taken that γ_ν , $\lambda_{\nu i}$ satisfy the (32),(34) conditions. Since $\bar{\underline{Y}} = [1-\underline{A}]\hat{\underline{Y}}$, $\underline{A} \triangleq [\alpha_{ij}]$, the equilibrium conditions (33) can be written in the form

$$\hat{\underline{Y}} = [1 - \underline{A}]^{-1}[\underline{\lambda}]\underline{\gamma}Z \triangleq \underline{\ell}Z \quad . \tag{36}$$

Since Z(t) is determined by the GNP generated at t - 1, the last relation can also be written as

$$\hat{Y}_i(t) = \ell_i[Y(t-1),\underline{p}(t-1)]Y(t-1) \quad , \qquad i = 1,\ldots,n \tag{37}$$

In order to satisfy (37) it is necessary to choose the sector prices in the proper way. For that purpose, consider the i-th sector output, which according to (1) can be written

$$X_i = \left[F_i({}^{Y_{1i}}/_{\omega_1}, {}^{Y_{2i}}/_{\omega_2}, \ldots, {}^{Y_{mi}}/_{\omega_m})\right]^{q_i} \prod_{j=1}^{n} X_{ji}^{\alpha_{ji}} \ , \quad i = 1,\ldots,n$$

where $Y_{\nu i}/\omega_\nu$ = development factors used in production by the i-th sector. The production cost C_i consists, generally speaking, of two parts; the first connected with the materials cost $(\sum_{j=1}^{n} Y_{ji})$, and the second with the cost of production factors $(\sum_{\nu=1}^{m} Y_{\nu i})$† so that $C_i = \sum_{j=1}^{n} Y_{ji} + \sum_{\nu=1}^{m} Y_{\nu i}$. At the sectorial level it is natural to assume that the GNP, which consists of the sum of values added by sectors, is spent on the production factors endowments.

The average sectorial price

$$\tilde{p}_i = \frac{C_i}{X_i} \quad ,$$

depends on the allocation of Y_{ji}, $Y_{\nu i}$, $j,i = 1,\ldots,n$, $\nu = 1,\ldots,m$. As already noted, the planned socialist economy is very effective in the efficient allocation of resources, i.e. the development factors and materials used in production. The efficiency conditions obviously require that $Y_{ji}, Y_{\nu i}$, be chosen in such a way that $\tilde{p}_i$ attains a minimum equal to p_i , for $Y_{ji} = \hat{Y}_{ji}$, $Y_{\nu i} = \hat{Y}_{\nu i}$, $j,i = 1,\ldots,n$, $\nu = 1,\ldots,m$. In that case, the consumer's utility attains maximum, and from the condition

$$\frac{\partial \tilde{p}_i}{\partial X_i} = \left[\frac{\partial C_i}{\partial X_i} X_i - C_i\right] X_i^{-2} = 0 \quad ,$$

†After the reforms which took place in socialist countries in 1961-1971, and in Poland in 1971, it became a common practice to take into account the cost of production factors, and first of all the cost of labor and capital, when calculating prices at the micro-level.

one gets

$$\frac{\partial C_i}{\partial X_i} = \frac{C_i[\hat{Y}_{ji},\hat{Y}_{\nu i}, j,i = 1,\ldots,n,\nu = 1,\ldots,m]}{X_i[\hat{Y}_{ji},\hat{Y}_{\nu i}, j,i = 1,\ldots,n,\nu = 1,\ldots,m]} = p_i \quad ,$$

$$i = 1,\ldots,n \quad ,$$

i.e. the equilibrium price p_i is equal to the marginal production costs $\partial C_i/\partial X_i$.

Since

$$\hat{X}_i = \left[F_i(\hat{Y}_{1j},\ldots,\hat{Y}_{mi})\right]^{q_i} \prod_{\nu=1}^{m} \omega_\nu^{-\beta_{\nu i}q_i} \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_i}\right)^{\alpha_{ji}} \hat{Y}_i^{1-q_i}$$

and

$$\hat{C}_i = \sum_{j=1}^{n} \alpha_{ji}\hat{Y}_i + q_i \prod_{\nu=1}^{m} \beta_{\nu i}\hat{Y}_i = \hat{Y}_i \quad , \qquad i = 1,\ldots,n \quad ,$$

one gets

$$p_i = \frac{\hat{C}_i}{\hat{X}_i} = \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{-\alpha_{ji}} \prod_{\nu=1}^{m} \omega_\nu^{\beta_{\nu i}q_i} \hat{Y}_i^{q_i} F_i^{-q_i} \quad , \quad i = 1,\ldots,n \quad .$$

Introducing variables $p_i^t = p_i(t)/p_i(t-1)$, $i = 1,\ldots,n$, $\omega_\nu^t = \omega_\nu(t)/\omega_\nu(t-1)$, $\nu = 1,\ldots,m$, $\hat{Y}_i^t = \hat{Y}_i(t)/\hat{Y}_i(t-1)$, $F_i^t = F_i(t)/F_i(t-1)$, and taking logarithms from both sides of the last equation, one gets

$$\ell n p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ell n p_j^t = q_i \left[\ell n \frac{\hat{Y}_i^t}{F_i^t} + \sum_{\nu=1}^{m} \beta_{\nu i} \ell n\, \omega_\nu^t\right] , \quad i = 1,\ldots,n \quad , \qquad (38)$$

where according to (37)

$$\hat{Y}_i^t = \ell_i^{t-1} \, Y^{t-1} = \ell_i^{t-1} \sum_{j=1}^{n} \hat{Y}_j^{t-1} \quad ,$$

and

$$\ell_i^{t-1} = \frac{\ell_i[Y(t-1),\underline{P}(t-1)]}{\ell_i[Y(t-2),\underline{P}(t-2)]} \quad .$$

Since for the real economy the determinant

$$D = \begin{vmatrix} 1, & -\alpha_{11}, & \cdots & -\alpha_{n1} \\ \vdots & & & \\ -\alpha_{1n}, & -\alpha_{2n} & \cdots & 1 \end{vmatrix} \neq 0$$

there exists a unique solution to the set of eqs. (38). It yields a set of positive price indices p_i^t , $i = 1,\ldots,n$.

It will be shown now that the formulae (38) can be used to find p_i^t , $i = 1,\ldots,n$, $t = 1,2,\ldots,T$, in an iterative way, starting with the known base year $t = 0$ prices.

The term

$$\frac{\hat{Y}_i^t}{F_i^t} = \frac{\ell_i^{t-1} Y^{t-1}}{\hat{Y}_i^t} \; \Pi_i^t \quad ,$$

in (38), where $\Pi_i^t = \Pi_i(t) / \Pi_i(t-1)$, and

$$\Pi_i(t) = \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \prod_{\nu=1}^{m} \omega_\nu^{-\beta_{\nu i}} \quad ,$$

determines the demand to supply ratio.

For a given strategy $\underline{z}^k \triangleq \{\hat{z}_{\nu i}^{(k)}(t),\ \nu = 1,\ldots,m,$ $i = 1,\ldots,n,\ \tau\varepsilon[0,T]\},\ k = 0,1,\ldots$, the supply $\hat{Y}_i^t(\underline{z}^k) \triangleq S_i^k$ is a known function of time, while the term Π_i^t has been derived by using price indices $\underline{p}^{k-1} \triangleq \{p_i^t,\ i = 1,\ldots,n,\ k-1\}$ (from the previous step of computations) so $\Pi_i^t(\underline{p}^{k-1}) \triangleq \Pi_i^{k-1}$.

The demand term $\ell_i^{t-1}\ Y^{t-1}$ depends in turn on $\underline{z}^k$ and $\underline{p}^{t-1}$, so one can write

$$\ell_i^{t-1}\ Y^{t-1} \triangleq D_i^k \quad ,$$

and

$$\frac{\hat{Y}_i^t}{F_i^t} = \frac{D_i^k}{S_i^k}\ \Pi_i^{k-1} \quad , \qquad i = 1,\ldots,n \quad . \tag{39}$$

The process of computations can be briefly described as follows. One starts at the initial step k = 1, with the $\Pi_i^o = a_i e^{\mu_i f}$ and derives by formulae of Sec.2.2 the $\underline{z}^1$ strategy which yields the supplies S_i^1, $i = 1,\ldots,n$. At the same time, the corresponding prices $\underline{p}^1$ and demands D_i^1, $i = 1,\ldots,n$ by formulae (38) are being derived. When $\frac{D_i^1}{S_i^1} = 1$, $i = 1,\ldots,n$, it means that our assumption regarding Π_i^o, $i = 1,\ldots,n$, was correct; so the process terminates.

When $\frac{D_i^1}{S_i^1} \neq 1$, at least for one index i, the prices computed by (38): $\underline{p}^1$ should be used for computation of Π_i^1 and $\underline{z}^2$ can be computed in order to derive $\frac{D_i^2}{S_i^2}$ etc. It should be observed that in the case when Π_i^o for a particular "i" has been chosen too large, the optimum strategy $\hat{z}_{i\nu}^1$, $\nu = 1,\ldots,m$, increases the S_i term and as a result the equilibrium price p_i^1 decreases. That in turn decreases Π_i^1 , i.e. $\Pi_i^1 < \Pi_i^o$. In other words, the iterational process has a tendency to correct the unjustified projections of Π_i^o . When, after a number of iterations one arrives at the situation where $D_i^k \approx S_i^k$, $i = 1,\ldots,n$ the values of $\underline{p}^k$ can be regarded as the prices corresponding to the optimum planning strategy in $[0,T]$, i.e. $\underline{z}^k = \hat{\underline{z}}$.

The proposed methodology can also be used in the case when one wants to maximize the GNP in constant (e.g. the base year) prices. In that case, the optimization goal is to maximize the functional:

$$\bar{Y} = \sum_{i=1}^{n} \int_0^T e^{-\lambda t}[p_i(o)/p_i(t)]Y_i(t)dt \quad ,$$

and the problem boils down to using the functions

$$\bar{\Pi}_i^t = \Pi_i^t/p_i^t \quad , \qquad i = 1,\ldots,n \quad ,$$

instead of Π_i^t , in the computation process already described.

The next problem which should be clarified is the computation of $\sum_{\nu=1}^{m} \beta_{\nu i} \ln \omega_\nu^t$ in (38). The prices ω_ν , of the development factors (beside labor) are in general not included in the statistical data available.

The average salary $\omega_1 = \frac{Z_1}{L} = \gamma_1 \frac{Z}{L}$ depends on the GNP $(Z(t) = Y(t-1))$ and the employment L which is exogenous, or derived from the population-employment submodel. In the simple model therefore, it is possible to regard $\bar{\mu}_i = \sum_{\nu=2}^{m} \beta_{\nu i} \ln\omega_\nu^t$, as exogenous parameters which can be estimated by fitting $\bar{\mu}_i$, ex post to the eqs. (38). These parameters should also be regarded as part of Π_i^t , responsible for the technical progress resulting from the government expenditure, $Z_{\nu i}$, $\nu \geq 3$.

In a more detailed employment model, it is possible to introduce the sectorial employments L_i and sectorial wages $\omega_{1i} = \frac{Z_{1i}}{L_i}$, $i = 1,\ldots,n$, where $\sum_{i=1}^{n} L_i = L$, $\sum_{i=1}^{n} Z_{1i} = Z_1$. The average sectorial wage ω_{1i}^t may change as a result of labor efficiency change in the productive sectors.

In the price model discussed so far, we did not take into account the taxation of the outputs of some of the sectors and corresponding government subsidies to the other sectors. The general requirement is that the taxes $(t_i>0)$ and subsidies $(t_i<0)$ should satisfy the balance equation:

$$\sum_{i=1}^{n} p_i t_i X_i = o \quad , i = 1,\ldots,n \tag{40}$$

where X_i - sector production.

The result of taxation is the change in values added (5):

$$\tilde{D}_i = Y_i(1-t_i) - \sum_{j=1}^{m} Y_{ji} \quad , \quad i = 1,\ldots,n \quad . \tag{41}$$

It is possible to see that (due to (40)) the GNP

$$Y = \sum_{i=1}^{n} \tilde{D}_i = \sum_{i=1}^{n} D_i \quad ,$$

so the taxation-dotation process does not change the GNP.

Since according to (8) and (41)

$$\hat{D}_i = \hat{Y}_i(1-t_i) - \sum_{j=1}^{n} \alpha_{ji}\hat{Y}_i = (1-t-\sum_{j=1}^{n} \alpha_{ji})\hat{Y}_i$$

the result of taxation is equivalent to change of α_{ii} , i.e. $\tilde{D}_i$ can be written

$$\tilde{D}_i = \tilde{q}_i\tilde{Y}_i \quad , \qquad \tilde{q}_i = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_{ji} - \tilde{\alpha}_{ii} \quad , \qquad (42)$$

where $\tilde{\alpha}_{ii} = \alpha_{ii} + t_i$.

As a result of taxation the price indices $\tilde{p}_i^t$ derived by (38) (with α_{ii} replaced by $\tilde{\alpha}_{ii}$) change. By using (38), it is also possible to derive the tax-dotation system, which keeps the given sectorial price $\tilde{p}_j^t$ constant. For that purpose it is necessary to regard t_j as the unknown parameter. It should also be observed that the introduction of taxation-dotation changes the consumption strategy, expressed in terms of the utility function parameters $\lambda_{\nu i}\gamma_\nu$. It is possible to show that the taxation-dotation process decreases the utility. As already mentioned, that process is motivated by the social policy aspects, which have not been introduced explicitly in our simple utility function (67) of Sec. 1.5. Therefore, we have to regard t_i parameter as exogenous variables rather than decision variables. These parameters for the Polish economy do not change much. The subsidies for the market commodities are around 1.8 percent of the GNP (30 percent of that sum goes to the grain products, and 12 percent to the milk products);around 8 percent of the GNP goes for subsidizing the agriculture (fertilizers, pesticides, machinery, fodders, etc.). As a result of that policy, the basic food products did not change much. However, in recent years, a growing concern with regard to food-prices stability is frequently expressed. The problem obviously boils down to choosing the proper values for t_i. By using different values of t_i in relation (42) and by using the price model proposed, it is possible to investigate the taxation policy impacts on the development goal values.

2.4 Adaptive model of technological and structural change

By the technological change, we shall understand here the change of production technology, motivated mainly by possible substitutions of expensive inputs by their cheaper equivalents. As mentioned previously in Sec.2.1, the technological change is the main factor influencing the $\tilde{\alpha}_{ji}(t)$ coefficients derived ex post from the statistical data by formula (10). As follows from the analysis carried out for the Polish economy, the $\tilde{\alpha}_{ji}(t)$ functions have a trend (mainly decreasing) in time. It is usually assumed that there are three main factors responsible for that trend: new capital investments and modernization, technical progress, and change of aggregated sector prices.

Consider first of all the change of technological coefficients due to new investments. Assume that in the year $t = t_1$ there exists a production sector S^e with the output $Y^e(t_1)$, which continues the production at $t = t_2$, so that

$$Y^e(t_2) = Y^e(t_1) \quad .$$

The sector input Y_i^e (supplied by another sector) becomes

$$Y_i^e(t_2) = Y_i^e(t_1) = \alpha^e(t_1)Y^e(t_1)^{\dagger} \quad .$$

Assume also that at $t = t_2$ a new factory (which is classified as belonging to the same sector S^e) with the output $Y^n(t_2)$ starts to operate so that the resulting production increases from $Y(t_1) = Y^e(t_1)$ to

$$Y(t_2) = Y^e(t_2) + Y^n(t_2) \quad .$$

As a result, the sector technological coefficient changes from the value

$$\alpha(t_1) = \frac{Y_i^e(t_1)}{Y^e(t_1)} \quad ,$$

to the value of

$$\alpha(t_2) = \frac{Y_i^e(t_2)+Y_i^n(t_2)}{Y(t_2)} = \alpha^e(t_1)\,\frac{Y^e(t_2)}{Y(t_2)} + \alpha^n(t_2)\,\frac{Y^n(t_2)}{Y(t_2)}$$

$$= \alpha^e(t_1)\,\frac{Y(t_1)}{Y(t_2)} + \alpha^n(t_2)\,\frac{Y(t_2)-Y(t_1)}{Y(t_2)} \quad . \qquad (43)$$

Assume the value of $\alpha^n(t_2)$ to differ from $\alpha^e(t_1)$ by a

---

†The indices j,i at α_{ji} have been dropped for the sake of simplicity in notation.

constant multiplier:

$$\alpha^{n}(t_2) = b\ \alpha^{e}(t_1) \quad , \tag{44}$$

where b is a positive number.

The assumption (44) expresses the fact that the level of new technology is proportional to the technical level achieved in the old (existing) technology. Using (44) and assuming that $t_1 \to t_2$ we get by (43)

$$\frac{d\alpha}{\alpha} = (b - 1)\ \frac{dY}{Y} \quad . \tag{45}$$

Integrating (45) one gets

$$\alpha(t) = \alpha_o [Y(t)]^{b-1} \quad , \qquad \alpha_o = \alpha^{e}(t_1)[Y(t_1)]^{1-b} \quad . \tag{46}$$

When $b < 1$, the $\alpha(t)$ decreases along with an increase of output production $Y(t)$.

As follows from the $Y(t)$ and $\alpha(t)$ trends analysis, these functions can be approximated by the exponential functions:

$$\hat{\alpha}(t) = A\ e^{-\bar{a}t} \ , \qquad \hat{Y}(t) = B\ e^{\rho t} \quad ,$$

where A, B, $\bar{a}$, ρ - given numbers.

Taking into account (45) one obtains

$$b = 1 - \frac{\bar{a}}{\rho} \quad . \tag{47}$$

Then it is possible to express the estimated value of $\hat{\alpha}(t_2)$ by the estimated value $\hat{\alpha}(t_1)$ and $\tilde{\alpha}(t_1)$, which is taken from the last (ex post) observation:

$$\hat{\alpha}(t_2) = \hat{\alpha}(t_1)\ e^{-\rho(t_2-t_1)} + b\tilde{\alpha}(t_1)\left[1 - e^{-\rho(t_2-t_1)}\right] .$$

Assuming $t_2 - t_1 = 1$, it is also possible to construct an iterative process

$$\hat{\alpha}(t_{i+1}) = \hat{\alpha}(t_i)e^{-\rho} + b\tilde{\alpha}(t_i)\left[1 - e^{-\rho}\right] , \quad i = 1,2,... \tag{48}$$

The estimation $\hat{\alpha}(t_1)$ of $\alpha(t_1)$ can be obtained by taking the mean value of N observations $\tilde{\alpha}(t)$, in the identification interval $[T_1,T_0]$ (Fig.14), i.e.

$$\hat{\alpha}(t_1) = \frac{1}{N}\sum_{t=T_1}^{t=T_0} \tilde{\alpha}(t) \quad , \tag{49}$$

Since $\bar{a}$ is a small number, in the case of short $[T_1,T_0]$ interval, $\hat{\alpha}(t)$ is almost linear and one can assume

$$t_1 \approx \frac{T_0 + T_1}{2} .$$

The formula (48) is similar to the exponential smoothing form of adaptive forecasting methods (see Ref.[6]) with the weights $e^{-\rho(t_2-t_1)}$, $b\left[1-e^{-\rho(t_2-t_1)}\right]$, attached to the estimate $\hat{\alpha}(t_1)$ and the observed value $\tilde{\alpha}(t_1)$.

It should be observed that in standard adaptive algorithms the weights attached to $\hat{\alpha}(t_1)$ and $\tilde{\alpha}(t_1)$ are unknown and are chosen experimentally to yield the best results in historical runs. Since in the practical situations one is dealing with the short

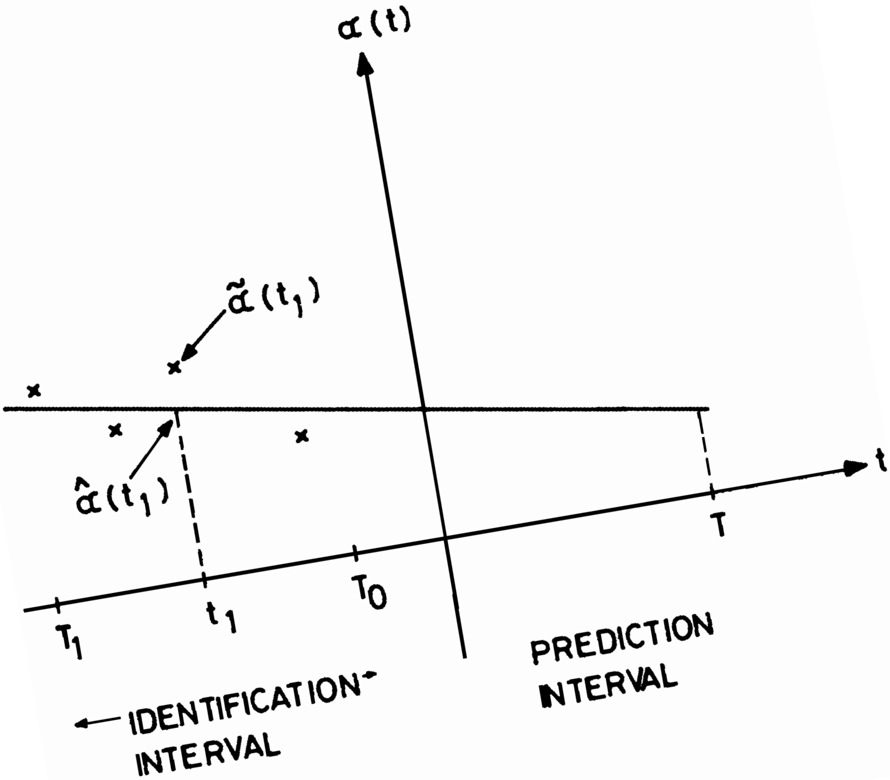

α(t)
$\tilde{\alpha}(t_1)$
$\hat{\alpha}(t_1)$
t
T
T_1
t_1
T_0
PREDICTION
INTERVAL
IDENTIFICATION
INTERVAL

Figure 14.

time sequences of $\alpha(t)$ the expressions with different weights are not effective and we prefer to choose the weights by using a macroeconomic model of technological change.

The forecasting of $\alpha(t)$ trend outside $[T_1, T_0]$ can be obtained by a linear or exponential extrapolation of trend function. In the last case the ratio

$$r = \hat{\alpha}(T_0 - 1)/\hat{\alpha}(T_0)$$

can be used for evaluation of the exponent a in the prediction formula

$$\hat{\alpha}(t) = \hat{\alpha}(T_0)e^{-a(t-T_0)} \quad , \ t \geq T_0 \quad , \tag{50}$$

i.e.

$$a = \ln r \quad .$$

It should also be observed that the model (43)-(46) does not take into account the influence of the neutral technical progress. A possible way to avoid that drawback is to replace (45) by

$$\frac{d\alpha}{\alpha} = (b - 1) \frac{dY}{Y} + \mu \quad , \tag{51}$$

and obtain instead of (46)

$$\alpha(t) = \alpha_0 \ e^{\mu t}[Y(t)]^{b-1} \tag{52}$$

In the present model the change of α also depends on the exogenous variable μ representing the new achievements in science and technology. It is possible to show that when the data regarding the time series $\ln \alpha(t)/\alpha(t-1)$ and $\ln Y(t)/Y(t-1)$, $t = 1,2,...$

are available, the unknown parameters μ, b, can easily be estimated by means of the least squares method.

Now we should investigate the effect of price changes on the estimation of technological coefficients. Since the sector generally consists of factories with different technological coefficients, the observed sector technological coefficients $\tilde{\alpha}_{ji}(t)$, may differ from a particular factory (which started to produce at $t = t_0$) coefficients $\bar{\alpha}_{ji}$. Assume that $\bar{\alpha}_{ji}$ have been chosen (by proper design and construction) in such a way that $\bar{\alpha}_{ji}$ are the "best" values at the existing prices $p_i(t_0) = \bar{p}_i$, $i = 1,\dots,n$, so that

$$\bar{\alpha}_{ji} = \hat{Y}_{ji}(t_0)/\hat{Y}_i(t_0) = \frac{X_{ji}(t_0)\bar{p}_j}{X_i(t_0)\bar{p}_i} , \qquad j,i, = 1,\dots,n \quad .$$

It is also assumed that the factory under consideration is "small enough" so its production has negligible effect on the prices and they can be regarded as exogenous variables. We shall also assume that the "input compositions" of goods sold by sector S_j to S_i do not change in time and as a result, we can deal with p_j instead of p_{ji} .

The output and net profit of the factory becomes

$$\hat{Y}_i = F_i \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{\bar{p}_j}\right)^{\bar{\alpha}_{ji}/\bar{q}_i} \bar{p}_i^{1/\bar{q}_i} \quad , \quad \bar{D}_i = \bar{q}_i\hat{Y}_i \quad . \qquad (53)$$

Suppose now that at t the prices have changed but the technology of our factory requires that $\frac{X_{ji}(t)}{X_i(t)} = \frac{X_{ji}(t_0)}{X_i(t_0)}$. Then technological coefficients α_{ji} have to change and become

$$\tilde{\alpha}_{ji}(t) = \frac{p_j(t)X_{ji}(t)}{p_i(t)X_i(t)} = \frac{p_j^t}{p_i^t}\bar{\alpha}_{ji} \quad . \tag{54}$$

where $p_i^t = p_i(t)/p_i(t_0)$, $i = 1,\ldots,n$.

As a result, the value of net profit of the factory becomes

$$\tilde{D}_i = R_i\bar{D}_i \quad , \tag{55}$$

where R_i is the "profitability" coefficient

$$R_i = \frac{1-\sum_{j=1}^{n}\tilde{\alpha}_{ji}}{1-\sum_{j=1}^{n}\bar{\alpha}_{ji}} \prod_{j=1}^{n}\left(\frac{\tilde{\alpha}_{ji}}{\bar{\alpha}_{ji}}\right)^{\bar{\alpha}_{ji}/\overline{q_i}} \quad . \tag{56}$$

Since $\tilde{\alpha}_{ji}(t)$ are changing in time, it is interesting to find that value of $\tilde{\alpha}_{ji}$ which produces the highest profit of a particular factory. Since the value added by the factory is usually calculated by taking into account the value of productive factors (i.e. the exogenous factors in our analysis) we shall assume that

$$\sum_{j=1}^{n}\tilde{\alpha}_{ji} = \sum_{j=1}^{n}\bar{\alpha}_{ji} = \alpha_i \tag{57}$$

where α_i is given.

Then the problem consists of finding

$$\max_{\tilde{\alpha}_{ji}} R_i(\tilde{\alpha}_{ji})$$

subject to (57) . It can easily be shown that the maximum value $R_i = 1$ is obtained if

$$\tilde{\alpha}_{ji} = \frac{\bar{\alpha}_{ji}}{\sum_{j=1}^{n} \bar{\alpha}_{ji}} \alpha_i = \bar{\alpha}_{ji} \quad , \qquad j,i = 1,\ldots,n \quad . \tag{58}$$

When a factory employs an obsolete technology and as a result $\bar{\alpha}_{ji} \neq \tilde{\alpha}_{ji}$, $j,i = 1,\ldots,n$, it suffers losses. Most of the factories will try to change $\bar{\alpha}_{ji}$ by modernization of technology in order to achieve at least the average sectorial level of technology, characterized by $\bar{\alpha}_{ji} = \tilde{\alpha}_{ji}$, $j,i = 1,\ldots,n$.

As follows from statistical data, the value of α_i is usually decreasing in time. In that case, one should replace the coefficient α_i in (58) by $\tilde{\alpha}_i < \alpha_i$, and as a result

$$\tilde{\alpha}_{ji} = \delta_i \bar{\alpha}_{ji} \quad ,$$

where

$$\delta_i = \tilde{\alpha}_i \Big/ \sum_{j=1}^{n} \bar{\alpha}_{ji} < 1 \quad ,$$

In that case the R_i coefficient in (56) should be multiplied by

$$r_i = \frac{1/\bar{\alpha}_i - \delta_i}{1/\bar{\alpha}_i - 1} \, \delta_i^{\bar{\alpha}_i/1-\bar{\alpha}_i} < 1 \quad .$$

In order to maximize profit, the factory should in addition increase δ_i , i.e. increase the value added.

In the hypothetical situation when no investments and modernization takes place, so that $\alpha_{ji}(t_0) = a_{ji}(t) = \bar{\alpha}_{ji}$, the observed $\tilde{\alpha}_{ji}$ change should be assigned solely to p_j^t/p_i^t , which can be estimated from the obvious relation

$$p_j^t/p_i^t = \tilde{\alpha}_{ji}(t)/\tilde{\alpha}_{ji}(t_0) \quad . \tag{59}$$

On the other hand, when the prices do not change in $[t_0,t]$ (e.g. the prices for coal, steel, electric energy in Poland are quite stable), $\tilde{\alpha}_{ji}$ change can be assigned solely to the technological change.

When there are data on sector price indices p_j^t , $j = 1,\ldots,n$, available, it is possible to compare them with (59) in order to determine the "pure" technological change.

It can also be demonstrated that the technological change resulting from modernization is generally lagging behind the price change.

Since data regarding α_{ji} at the sectorial level are usually not available, there is a tendency to use $\tilde{\alpha}_{ji}(t)$ for that purpose (as an approximation) estimated from input-output tables of national economy.

As follows from (55), the change of α_{ji} in the production function results in decreasing of the net output by a factor $R_i \leq 1$. Since in the model under consideration the K_i parameters in (12) are being identified by least squares, the accurate identification of α_{ji} is not so important and one can use the estimated by (50) $\hat{\alpha}_{ji}(t)$ as proxy for α_{ji} .

Now let us turn our attention to the construction of adaptive models for the consumption-structure change. The γ_ν coefficients which express the social preferences, regarding the allocation of expenditures among the different spheres of activity (production and development factors), can be estimated ex post by the following relations:

$$\tilde{\gamma}_\nu(t) = \tilde{Z}_\nu(t)/Z(t) \quad , \quad [Z(t) = Y(t-1)] \; , \; \nu = 1,\ldots,m \; ,$$

where $\tilde{Z}_\nu(t)$ - the government expenditures in fiscal t year known from statistical data.

The γ_ν coefficients should satisfy the following relations:

$$\sum_{\nu=0}^{m} \gamma_\nu(t) = 1 \quad . \tag{60}$$

The change of $\gamma_\nu(t)$ in time can be regarded as due to an additional social goal $\gamma_\nu^n(t_2) = g_\nu \gamma_\nu(t_1)$ and exogenous influence μ_ν . Then by the similar arguments and relations (43)-(51), one arrives at the formulae equivalent to (51),(48):

$$\gamma_\nu(t) = \gamma_{\nu 0} \, e^{\mu_\nu t} [Z_\nu(t)]^{g_\nu - 1} \quad , \tag{61}$$

$$\hat{\gamma}_\nu(t_{i+1}) = \hat{\gamma}_\nu(t_i) e^{-\rho_\nu} + g_\nu \tilde{\gamma}_\nu(t_i) [1 - e^{-\rho_\nu}] \quad . \tag{62}$$

However, care should be taken while g_ν are being estimated, in order to satisfy condition (60).

It should be noted that the adaptive approach to the modeling of technological and structural change has several advantages. First of all, it should be observed that only the short time sequences of $\tilde{\alpha}_{ji}(t)$, $j,i = 1,\ldots,n$, $t = -1,-2,\ldots,T_1$ are available. For example, the reliable input-output tables for Polish economy are available only for the last eight years. The a priori knowledge, regarding the statistical properties of coefficients estimated, and the random factors involved, is limited. In that situation the application of standard econometric methods (such as least squares) would hardly give the positive results. On the other hand, the application of the adaptive forecasting models , in such situations, have been shown to be effective [6].

The adaptive algorithms (48) are simple enough so the estimation of the α_{ji} coefficients for each planning interval is easy.

When the "moving-horizon" planning technique is used, it is necessary to derive the projected values of $\alpha_{ji}(t_k)$ for the [k,k+T], (k=0,1,2,...) subinterval. Using the methodology described, one should derive the projection time intervals by

$$t_k = k + \frac{T}{2} , \qquad k = 0,1,...$$

and then use the prediction formula (50):

$$\hat{\alpha}_{ji}(k+ \frac{T}{2}) = \hat{\alpha}_{ji}(k) e^{-a_{ji}(k)\frac{T}{2}} , \qquad k = 0,1,... \tag{63}$$

In each planning interval, the α_{ji} are kept constant and equal $\hat{\alpha}_{ji}(t_k)$. These values are changed when one passes to the next planning interval. At the same time, the new estimates of $\hat{\alpha}_{ji}(k+1)$ are derived by (48), so that

$$a_{ji}(k+1) = \ln \frac{\hat{\alpha}_{ji}(k)}{\hat{\alpha}_{ji}(k+1)} , \qquad k = 0,1,... \tag{64}$$

As a result, at each planning interval [k,k+T], one deals with a new steady-state set of technological coefficients, which are approximating the expected change of technology, as shown in Fig.14. In other words, we approximate the continuously changing technology by a set of stationary (within each planning interval) models, or - the model is adapting to the changing technology. It should also be noted that an adaptive approach enables us to deal with the optimization and price submodels, which require that α_{ji} do not change within the optimization interval.

The adaptive approach in estimation of consumption structure change has many advantages as well. Besides what has already been said with regard to the technological change, it should be noticed that the utility function parameters γ_ν can hardly be regarded as determined completely by GNP and prices only (as the model (35) may suggest). Instead of taking into account a number of factors, social values and goals which can hardly be explicitly expressed, it is better to learn the effects resulting from these goals in terms of expenditures policy, i.e. $\gamma_\nu(t)$, $\nu = 1,\ldots,m$. The adaptive forecasting algorithms for $\gamma_\nu(t)$ change seem to be best suited for that purpose.

III. Interregional and International Cooperation Models

3.1 Optimization of regional development

The national model of development can be regarded as the aggregation of submodels, which have been constructed for all the regions of the country. It is important that the methodology used for construction of regional submodels enables the aggregation, which yields the consistent relations between the regional and national (core) model variables and model equations.

So far, we have been dealing with the methodology, which is useful for modeling and planning of long-term development in n-sector production systems. The purpose of the present section is to show how that methodology could be used for modeling of regional development and regional planning.

First of all, it can be observed that the national model can be regarded as the sum of regional submodels if all the statistical data are available. One can consider also a particular regional model S_r cooperating with the rest of the country S_c (Fig.15). All the submodels' technological (and other) coefficients should be estimated or chosen in such a way that the aggregated submodels give the same set of basic relations as the core national model. Then the sector's strategies, regarding the allocation of production factors among the set of regions, can be analyzed. If we consider, e.g. a particular production sector S_i and N regional production functions of the general type (13) of Sec. 1.2, the contribution of j-th region to the regional production $y_{ij}(t)$ can be written as

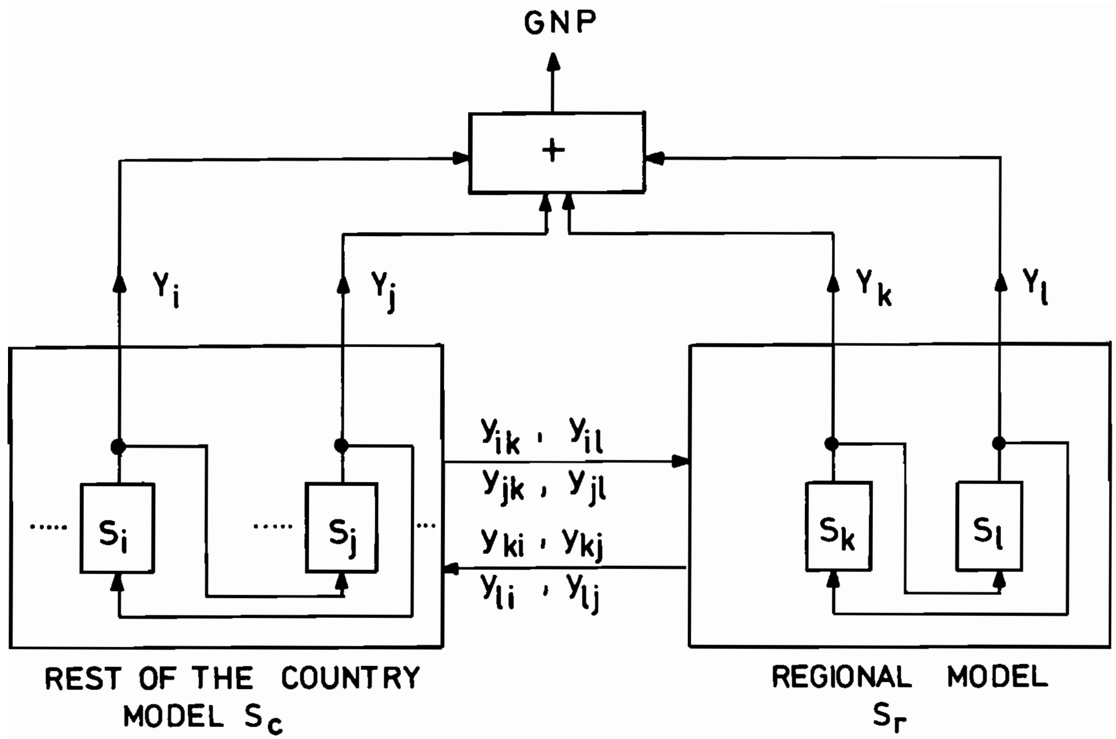

GNP
+
Yi
Yj
Yk
Yl
Si
Sj
Sk
Sl
yik , yil
yjk , yjl
yki , ykj
yli , ylj
REST OF THE COUNTRY
MODEL Sc
REGIONAL MODEL
Sr

Figure 15.

$$y_{ij}(t) = \prod_{\nu=1}^{m} \left\{ f_{\nu ij}(t) \right\}^{\beta_\nu} \tag{1}$$

where

$$f_{\nu ij}(t) = \int_{-\infty}^{t} k_{\nu ij}(t-\tau)\,[z_{\nu ij}(\tau)]^{\alpha}\, dt \quad ,$$

$$k_{\nu ij}(t) = K_{\nu ij} e^{-\delta_{\nu i}(t - T_{\nu i})} \quad , \qquad t > T_{\nu i}$$

$$0 \quad , \qquad t < T_{\nu i} \quad .$$

We shall also assume, for the moment, that the total regional resources $Z_{\nu j}$, $\nu = 1,\ldots,m$, $j = 1,\ldots,N$ be given. Then it is possible to find the regional optimum development strategy $z_{\nu ij}(t) = \hat{z}_{\nu ij}(t)$, $\nu = 1,\ldots,m$, $i = 1,\ldots,n$, $j = 1,\ldots,N$, $\tau \in [0,T]$, such that

$$Y_j = \int_0^T (1+\varepsilon)^{-t} \sum_{i=1}^{n} y_{ij}(t)\, dt \tag{2}$$

is maximum subject to

$$\sum_{i=1}^{n} \int_0^T z_{\nu ij}(t)\, dt \le Z_{\nu j} \quad , \qquad \nu = 1,\ldots,m$$

$$, \qquad j = 1,\ldots,N \tag{3}$$

$$z_{\nu ij}(t) \ge 0 \quad , \qquad i = 1,\ldots,n \quad , \qquad \tau \in [0,T] \quad .$$

The optimum strategies can be used to derive the value of $\Delta Y_j = \Delta\hat{Y}_j$, which takes the form (31) of Sec. 1.2.

$$\Delta\hat{Y}_j = G_j^q \prod_{\nu=1}^{m} Z_{\nu j}^{\delta_\nu} , \quad \gamma_\nu = \alpha\beta_\nu , \quad j = 1,\ldots,N . \tag{4}$$

The problem which presently faces us is to derive the optimum values of $Z_{\nu j} = \hat{Z}_{\nu j}$, $\nu = 1,\ldots,m$, $j = 1,\ldots,N$, which would maximize

$$\Delta Y = \sum_{j=1}^{N} G_j^q \prod_{\nu=1}^{m} Z_{\nu j}^{\gamma_\nu} , \tag{5}$$

subject to

$$\sum_{j=1}^{N} Z_{\nu j} \leq Z_\nu , \quad \nu = 1,\ldots,m , \tag{6}$$

$$Z_{\nu j} \geq 0 , \quad \nu = 1,\ldots,m , \quad j = 1,\ldots,N . \tag{7}$$

It can easily be verified that a unique optimum strategy exists and it can be derived by the formulae:

$$\hat{Z}_{\nu j} = \frac{G_j}{G} Z_\nu , \quad j = 1,\ldots,N , \quad \nu = 1,\ldots,m , \tag{8}$$

where

$$G = \sum_{j=1}^{N} G_j .$$

Using the present method, we can derive the optimum allocation of production factors and government expenditures among different regions and production sectors within the planning interval. There is, however, an obvious drawback to the present approach: it is very much production oriented, i.e. it takes into consideration, first of all, the efficient allocation of resources. The government expenditures in education and health services are treated here as complementary (i.e. supporting) production factors. A possible way to avoid that drawback is to assume that a part of the government budget is used for an increased financing of these regions which are behind the average country's figures. In that case, we can use the production function (25) of Sec. 2.2, where $z_{\nu j}(\tau)$ represent that part of government expenditures which has a universal (with respect to a particular technology) production effect. In order to allocate that part of government expenditure in an explicit form, a method described in [26] can be applied. According to that method, a regional dissatisfaction function can be constructed of the general form:

$$D_j(\underline{Z}) = d_j \prod_{\nu} |\bar{Z}_{\nu j} - Z''_{\nu j}|^{\beta_\nu} \quad , \qquad j = 1,\ldots,N$$

where d_j , β_ν - given positive numbers,

$\bar{Z}_{\nu j}$ - given country's average (per capita) of government expenditure level.

The problem consists in finding $Z_{\nu j} = \hat{Z}''_{\nu j}$, $j = 1,\ldots,N$, $\nu = 1,\ldots,m$, such that

$$D = \sum_j D_j$$

is minimum, subject to

$$\sum_j Z''_{\nu j} \leq a''_\nu Z_\nu \quad , \qquad \nu = 1,\ldots,m \quad , \qquad 0 < a''_\nu < 1 \quad ,$$

$$Z''_{\nu j} \geq 0 \quad , \qquad j = 1,\ldots,N \quad .$$

The numerical values of a''_ν , $\nu = 1,\ldots,m$, which represent the share of universal expenditures in Z_ν , can be estimated from past (historical) data, or considered as decision variables.

Using that approach, the regional benefit (utility) function (4) can be written as

$$\Delta Y_j = G_j^q \prod_{\nu=1}^{m} (Z'_{\nu j})^{\gamma'_\nu} (Z''_{\nu j})^{\gamma''_\nu} \quad , \qquad \gamma'_\nu + \gamma''_\nu = \gamma_\nu \quad ,$$
$$j = 1,\ldots,N \quad , \qquad (9)$$

which shows the contribution of all government expenditures to the regional welfare. That contribution can be regarded in two possible ways. The direct way in the form of salaries (Z'_{1j}) , education, medical and social care organized by production sectors ($Z'_{\nu j}$) and the indirect way (expressed by $Z''_{\nu j}$) in the form of public education, social and medical care, and environment protection organized by regional and government institutions. The main factor, determining the regional growth in terms of Y_j is, of course, G_j , which depends on the $K_{\nu ij}$, $i = 1,\ldots,n$, $\nu = 1,\ldots,m$ factors. Since the numerical values of $K_{\nu ij}$ in the model under consideration are being determined ex post from statistical data, the model has a tendency to maintain the existing development trends. However, it is a rather common situation that the regional growth depends as well on new geological discoveries, for example, which change the existing regional production structure. For that reason, a more detailed location analysis and optimization is needed. In particular, it is necessary to analyze the change of model technological coefficients, resulting from the change of location of production sectors.

Consider a simple model, shown in Figure 16, where the national core model cooperates with a new production sector S_r being planned at the given region r. It is assumed that the core model projections of the total investment intensity ($Z_2(t)$), labor cost ($Z_1(t)$) and other government expenditures ($Z_\nu(t)$, $\nu = 3,\dots,m$) in the planning interval [0,T] are given. The expenditure intensities connected with the regional project $C_i(t)$, $i = 1,\dots,m$ are assumed to be known. It is assumed that the central planning unit considers a number (M) of different regional projects characterized by given cost functions $C_i^j(t)$, $i = 1,\dots,m$, $j = 1,\dots,M$, where generally

$$C_i^j(T) \le Z_i(t) \quad , \quad i = 1,\dots,m \quad , \quad j = 1,\dots,M \quad ,$$

$$\tau \in [0,T] \quad ,$$

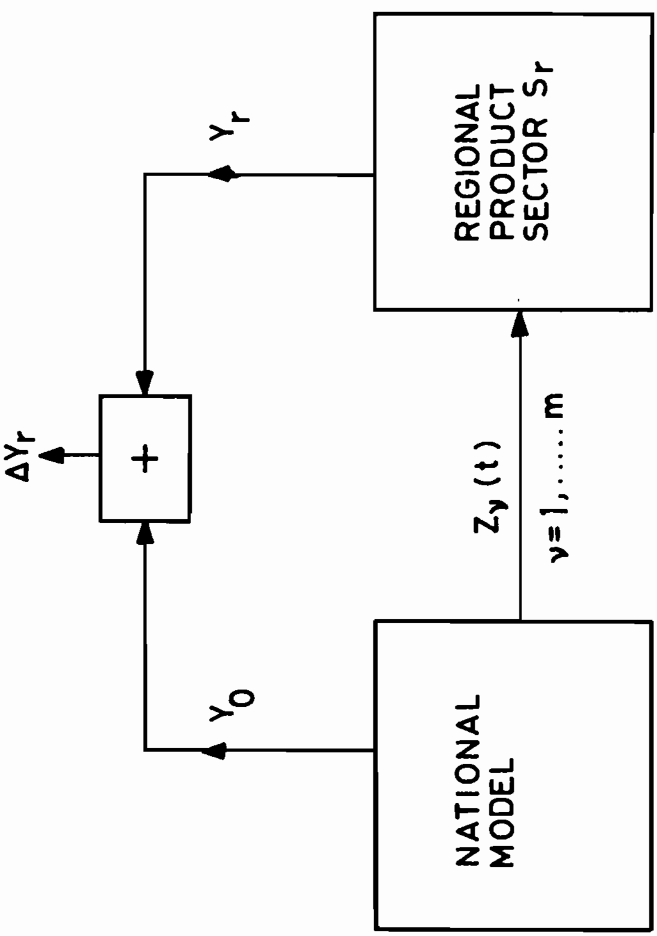

NATIONAL MODEL
REGIONAL PRODUCT SECTOR S_r
Y_0
Y_r
+
ΔY_r
$Z_\nu(t)$
$\nu = 1, \ldots, m$

Figure 16.

but

$$\sum_{j=1}^{M} c_i^j(t) > z_i(t) \quad ,$$

at least for some $i \in [1,\ldots,m]$, $t \in [0,T]$. Then it is necessary to choose a subset $M' \in M$ of these projects which are most effective for national and regional development. Generally speaking, the projects can be realized at N different regions yielding different values of expected GNP increases:

$$\Delta Y_j = \hat{Y}_j - Y_0 \quad , \qquad j \in N \quad ,$$

where Y_0 = the GNP generated within the planning interval [0,T] by the core model when all the resources are allocated in optimal manner, but no specific regional project is indicated,

Y_j = the GNP generated within the planning interval, by the core model and regional project, when the cost of regional project resources is shifted from core to regional project.

Since, generally speaking, the change of project location will induce the corresponding change of transport costs and prices for S_r output and other sectors' outputs, it is necessary to derive ΔY_j , $j = 1,\ldots,N$, in constant prices. In that way, one takes into account the direct economic effects of regional location as well as the indirect effects resulting from price changes within the whole socio-economic systems. Some of these changes can be regarded as beneficial (for example, an increase of regional production may decrease the product price and increase the consumption), while at the same time the industrial growth may induce more pollution, decrease the agriculture productivity, etc. Another reason is that dealing with output expressed in constant prices, it is possible to neglect the inflationary effects on the economic growth.

Suppose that at the first stage of regional planning each project has been checked for an optimum location. To do that, it is necessary to find $j = r$, such that $\Delta Y_r = \max\{\Delta Y_j\}$ $j \in N$. When the project inputs and outputs are traded with the core mainly (at least during the planning interval), that process gives us the optimum location of individual projects among the possible regions.

The next step is to choose the best portfolio of projects satisfying the constraints on the available resources generated by the core model. In order to solve that problem, one can use the well-known integer programming method. In order to do that, introduce the discrete variables $X_j \in [0,1]$, $j = 1,\ldots,M$. The problem consists in finding the strategy $X_j = \hat{X}_j$, $k = 1,\ldots,M$, such that

$$\Delta Y = \sum_{j=1}^{M} X_j \Delta Y_j \quad , \tag{10}$$

attains maximum subject to the constraints

$$\sum_{j=1}^{M} C_i^j(t)\, X_j \le Z_i(t) \quad , \quad i = 1,\ldots,m \quad , \quad t = 0,\ldots,T \quad . \tag{11}$$

The present method can easily be extended to the case when the regional project involves a complex of n' sectors S_{ri} , $i = 1,\dots,n' \leq n$, which exchange the products with core as well as among themselves. A typical example is an energy complex which involves the coal mine, electric power station, which consumes coal and generates electricity, utilized together with coal to produce chemicals, etc. In the last case, it is necessary to coordinate the core expenditures assigned to different production sectors.

In order to use the proposed methodology for optimization of regional allocation of resources, it is necessary to introduce the regional aspects in the regional (S_r) production function. The main factor which should be taken into account is the change of technological coefficients and prices resulting from the transport cost changes. Consider as an example the core sector production function (4) of Sec. 2.1, which corresponds to a fixed location. For the optimum sector production strategy one gets

$$\alpha_{ji} = \frac{\hat{Y}_{ji}}{\overset{*}{Y}_i} = \frac{p_j \hat{X}_{ji}}{p_i \overset{*}{X}_i} \quad , \qquad j,i = 1,\dots,n \quad . \tag{12}$$

Suppose that the project under consideration has been located at the same place as the core production sector and the same technology (requiring the given ratios of $\hat{X}_{ji}/\hat{X}_i$, $j,i = 1,\dots,n$) has been adopted. In that case, the project technological coefficients are determined by (12).

Suppose now that the location of the project S_r has been changed (with respect to core sector location) and the cost $\hat{Y}_{jr}$ of the inputs $\hat{X}_{jr}$ has changed to become

$$\tilde{Y}_{jr} = \hat{Y}_{jr}(1 + t_{jr}) \tag{13}$$

where t_{jr} - an increasing function of distance between the old and new location. The effect on the economy is the same as if the α_{jr} of S_r had changed to become:

$$\tilde{\alpha}_{jr} = \alpha_{jr}(1 + t_{jr}) \quad . \tag{14}$$

Besides the transport costs which depend on S_r location a new production project may also use more advanced technology, which changes $\hat{X}_{jr}/\hat{X}_r$, $j = 1,\ldots,n$. That process is, however, neutral with respect to location of the project. In a similar way, the change of S_r location affects the α_{ri} and ℓ_γ coefficients in equations (36)-(38) of Sec. 2.3. The final result of these changes is a change of price indices p_i^t , $i = 1,\ldots,n$ and the corresponding change of ΔY_j (in constant prices).

In order to derive the effect of $\tilde{\alpha}_{jr}$, $\tilde{\alpha}_{ri}$ on the resulting national model output, one can also consider S_r as an independent sector with the given $\tilde{\alpha}_{jr}$, $i = 1,\ldots,n$ technological coefficient and the price index p_r^t , which can be derived from the extended set of equations (38) of Sec. 2.3:

$$\ln p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ln p_j^t - \tilde{\alpha}_{ri} \ln p_r^t = q_i \left[\ln \frac{\ell_i^t Y^{t-1}}{F_i^t} + \sum_{\nu=1}^{m} \beta_\nu \ln \omega_\nu^t \right], \quad i = 1,\ldots,n$$

$$\ln p_r^t - \sum_{j=1}^{n} \tilde{\alpha}_{jr} \ln p_j^t = q_r \left[\ln \frac{\ell_r^t Y^{t-1}}{F_r^t} + \sum_{\nu=1}^{m} \beta_\nu \ln \omega_\nu^t \right] \quad . \tag{15}$$

The next step is an aggregation of sector S_r with the corresponding sector in the core model. As shown in [33], such an aggregation results in a new set of aggregated technological coefficients and a new sector price index. It can be observed that a regional location process has an important effect on the technological change and development on the regional, as well as the national level.

It should be observed that in the regional models discussed, the labor force, as well as all the other development factors, is regarded as mobile so that it can be transferred from the higher to the lower populated areas if necessary. That, however, involves additional costs (mainly in housing), which are usually assigned to the new investment costs. As a result, the effectiveness of regional investment may decrease when there is a shortage of labor present.

Another important factor which decreases the expected (within the planning interval) gross regional product (GRP) is the cost of protection of the environment. As already mentioned, the harm done to the environment depends much on the regional location of new production and consumption centers. For example, in a heavily industrialized or populated area, the construction of a new factory is generally more harmful than in the desert. The pollution control cost necessary to satisfy the regional standards depends, therefore, on the location of each new factory. As a result, the GRP generated in the planning interval according to (60) of Sec. 1.4 depends on the location and the technology used.

In order to achieve the fastest GRP growth, it is necessary to choose not only the location, but the area of productive specialization as well. It is also important that the chosen location ensures the availability of natural resources. It happens, for example, that industry competes with agriculture for supplies of fresh water. As a result, the price of water depends on the location, and changes with time.

3.2 International cooperation models

As already mentioned, the main idea behind the normative modeling of development is to find the best allocation of development factors strategy (which yield the maximum integrated output) subject to the constraints imposed on the flow of resources. At the regional level, as already mentioned, the availability of the labor force, natural resources (e.g. water, energy, etc.) and environment protection, represent the typical constraints of development, while the amount of financial resources depends on the higher level (i.e. government) decisions. At the national level, the constraints take the additional form of monetary constraints. For example, the GNP generated by the economy should be in balance with the government expenditures, as shown in Sec. 1.2. Since in an open economy the foreign trade influences the GNP level, the foreign trade-balance should also be observed. In addition, since the international cooperation also involves crediting of investments, exchange of "know-how", patents, skilled labor, scientific and technical cooperation, etc., the general international cooperation model takes a complex form as shown in Fig.17. The general idea behind the model is that all the factors contributing to the development can be subject to international cooperation. That usually involves international agreements on a government level. Government decisions are based on the system of values and development goals which again are subject to international exchange in values and ideology. In the case of Poland, for example, close cooperation with the Soviet Union and socialist countries is very important in that respect. It stimulates international cooperation at government and even lower levels. The creation of international organizations such as the Council for Mutual Economic Assistance (CMEA), involving nine socialist countries, has had a great impact on the rapid development of all of the member countries. The creation of the International Bank of Economic Assistance (in 1963) and the International Investment Bank of CMEA (in 1970) stimulated foreign trade, mutual assistance in the development of new branches of industry, and utilization of scarce resources among the socialist countries. The close cooperation among the CMEA countries also

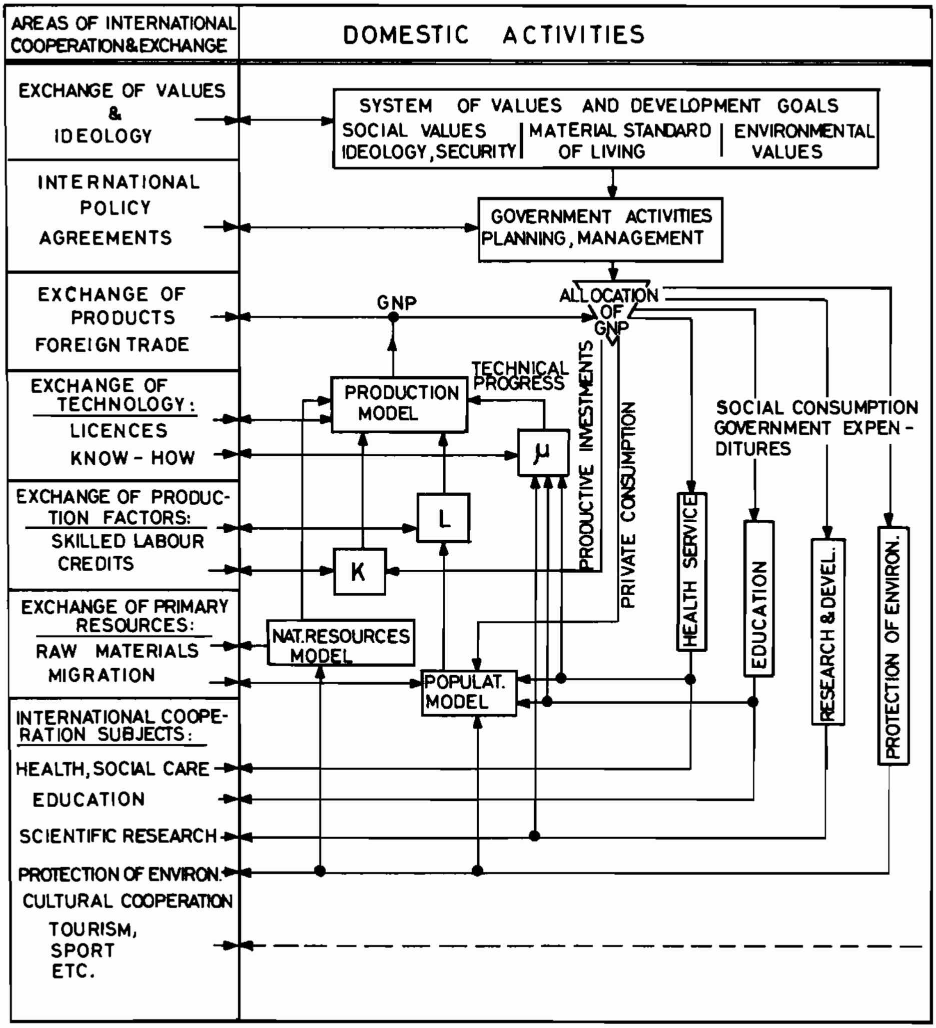


Figure 17. Model of international cooperation.

exists in science and technology, education, and cultural life.

The cooperation of CMEA countries with the rest of the world, especially in trade, science and technology, is rapidly developing as well.

The benefits which result from foreign trade development for each participating country are well-known and are analyzed in the "pure theory of foreign trade" [14]. Little comprehensive theory exists, however, on the benefits resulting from general international cooperation, according to the activities shown in Fig.17. Our first objective in the present section is, therefore, to find the impact of trade, as well as of mutual assistance in crediting the national investments, exchange in science and technology, education and other national activities.

It is assumed that international organizations and banks have certain stocks of initial resources (created by contributions of participants according to the mutual agreements), which are used as aid in financing the development plans proposed by individual countries. The loans are given, generally speaking, with interest rates, which help the less-developed countries to grow at a faster rate. In that way, the socio-economic development of different countries may rise to predetermined levels should the need arise.

Since the loans obtained should usually be paid (including interest) back, from the national and the long-term development points of view, it is important to know how many benefits one can expect, using foreign credits.

From the systems analysis point of view, that problem boils down (as mentioned in Sec. 1.2) to the comparison of relative advantages of optimal development strategies, which are subject to the amplitude (i.e. full autarky) or integral (i.e. international credits available) constraints. In order to solve that problem, consider first of all the model of international trade shown in Fig.18. The model consists of national production-sectors S_i, $i = 1,\dots,n$, which export part Y_{iz} of their products to the foreign market S_z. On the other hand, S_z is exporting $\bar{Y}_z\,(1-q_2)$ part of the total export $\bar{Y}_z$, to the national system, consisting of production and consumption subsystems. The "production function" of S_z (in market S_z currency) can be written in similar form to the S_i - sectors functions, i.e.

$$\bar{Y}_z = F_z^{q_z} \prod_{i=1}^{n} \bar{Y}_{iz}^{\alpha_{iz}}\, \bar{p}_z \prod_{i=1}^{n} (\bar{p}_{iz})^{-\alpha_{iz}} , \tag{16}$$

where

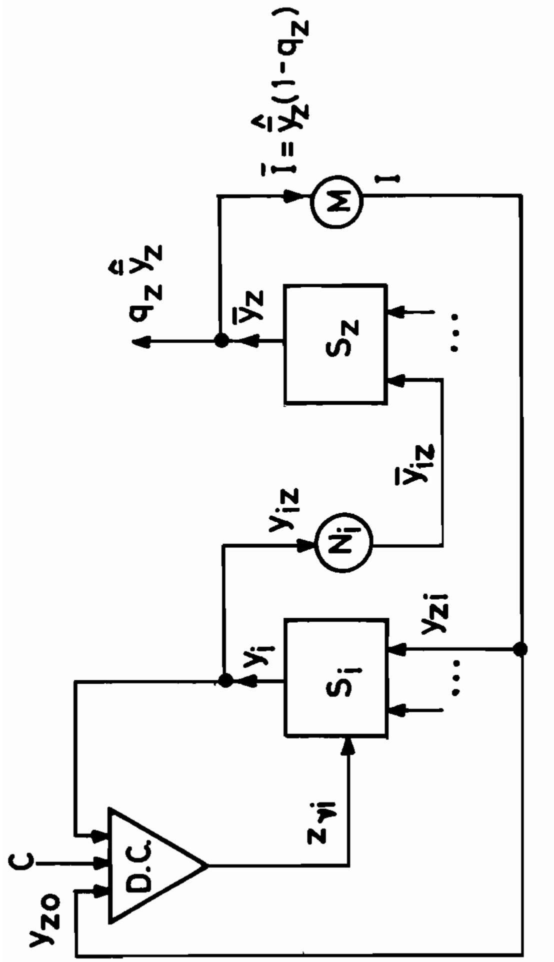
$$q_z = 1 - \sum_{i=1}^{n} \alpha_{iz} > 0 ,$$

$\bar{p}_{iz}$ = prices of exported sectorial commodities in S_z currency.

The production functions of S_i, $i = 1,\dots,n$, sectors are supplemented with $Y_{zi}^{\alpha_{zi}}$ terms, i.e.

$$Y_i = F_i^{q_i} \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} Y_{zi}^{\alpha_{zi}} \cdot P_i \prod_{j=1}^{n} P_j^{-\alpha_{ji}} P_{zi}^{-\alpha_{zi}} ,$$

$$q_i = 1 - \sum_j \alpha_{ji} - \alpha_{zi} \tag{17}$$



C
y_{zo}
D.C.
z_{yi}
S_i
y_i
y_{iz}
N_i
$\bar{y}_{iz}$
y_{zi}
S_z
$\bar{y}_z$
$q_z \hat{y}_z$
M
$\bar{I} = \hat{y}_z(1-q_z)$
I

Figure 18.

where

p_{zi} = the price of imported commodities in the domestic currency.

Since Y_{iz} , $i = 1,\ldots,n$, are measured in domestic currency it is necessary to introduce the following exchange rates:

a) the export exchange rate

$$N_i = {}^{\bar{Y}_{iz}}/Y_{iz} = {}^{\bar{P}_{iz}}/P_i \quad , \qquad i = 1,\ldots,n \quad , \tag{18}$$

where

p_i - sectorial prices in domestic currency.

b) the import exchange rate

$$M = {}^{I}/_{\bar{I}} \quad . \tag{19}$$

It is assumed that sector S_z maximizes the net output

$$\bar{D}_z = \bar{Y}_z - \sum_{i=1}^{n} \bar{Y}_{iz} \quad ,$$

in the same way as the domestic sectors S_i , $i = 1,\ldots,n$, do .

As a result of that strategy the corresponding trade flows (according to (6)(7) of Sec. 2.1) become

$$\hat{Y}_{zi} = \alpha_{zi}\hat{Y}_i \quad , \qquad i = 1,\ldots,n \quad , \tag{20}$$

$$\hat{\bar{Y}}_{iz} = \alpha_{iz}\,\hat{\bar{Y}}_z \quad , \qquad i = 1,\ldots,n \quad , \tag{21}$$

where

$$\hat{Y}_i = F_i \prod_{j=1}^{n} \left(\frac{\alpha_{ji}}{p_j}\right)^{\alpha_{ji}/q_i} p_i^{1/q_i} \cdot \left(\frac{\alpha_{zi}}{p_{zi}}\right)^{\alpha_{zi}/q_i} ,$$

$$q_i = 1 - \sum_{i=1}^{n} \alpha_{ji} - \alpha_{zi} > 0 ,$$

p_{zi} = the price of imported commodities in domestic currency,

$$\hat{\bar{Y}}_z = \bar{F}_z \prod_{i=1}^{n} \left(\frac{\alpha_{iz}}{\bar{p}_{iz}}\right)^{\alpha_{iz}/q_z} \bar{p}_z^{\ 1/q_z} .$$

Since

$$I = \sum_{i=1}^{n} \hat{Y}_{zi} + Y_{zo} = \sum_{i=1}^{n} \alpha_{zi}\hat{Y}_i + Y_{zo}$$

where

Y_{zo} = the part of import used for factor endowments (mainly the productive investments, some government expenditures, and part of private consumption) and, on the other hand

$$\bar{I} = \hat{\bar{Y}}_z(1 - q_z) , \tag{22}$$

one gets

$$M = \frac{I}{\bar{I}} = \frac{\sum_{i=1}^{n} \alpha_{zi}\hat{Y}_i + Y_{zo}}{\hat{\bar{Y}}_z(1-q_z)} \quad . \tag{23}$$

According to (23) the import exchange rate M increases along with the increase of demands for imports of production inputs $\alpha_{zi}\hat{Y}_i$ (claimed by sectors) and demands for development factors Y_{zo}.M decreases when the supply of import $\hat{\bar{Y}}_z(1-q_z)$ increases. From the domestic point of view,the lower M is, the greater the benefits.

The value of export under optimum strategy becomes

$$E = \sum_{i=1}^{n} \hat{Y}_{iz} = \sum_{i=1}^{n} \hat{\bar{Y}}_{iz/N_i} = \hat{\bar{Y}}_z \sum_{i=1}^{n} \frac{\alpha_{iz}}{N_i} \quad . \tag{24}$$

When the balance of payment: I - E = 0 should be observed one gets by (22)(24)

$$M = \frac{E}{\bar{I}} = \frac{1}{1-q_z} \sum_{i=1}^{n} \frac{\alpha_{iz}}{N_i} = \sum_{i=1}^{n} \bar{\alpha}_{iz/N_i} \quad , \tag{25}$$

where

$$\bar{\alpha}_{iz} = \frac{\hat{Y}_{iz}}{\sum_{i=1}^{n} Y_{iz}}$$

is the production of the S_i share in the total export E.

As follows from (25) the resulting import exchange rate M, under the balance of payment condition $I - E = 0$, is determined by shares $\bar{\alpha}_{iz}$ and export exchange rates N_i. In order to decrease M, the domestic system should increase N_i by decreasing p_i prices with respect to $\bar{p}_{iz}$. That can be done by investments and technical and organizational progress in these sectors S_i which yield the greatest values of N_i. In the case when $N_i = N =$ const. for all $i = 1,\dots,n$, one gets by (25) $M = {}^1/_N$. The last condition should be regarded as an approximation. In the foreign trade practice, the commodities offered for export are usually classified according to the values of $\bar{p}_{iz}/p_i$. There is a tendency to export first of all these commodities which have the greatest values of $\bar{p}_{iz}/p_i$. That tendency can be interpreted as a willingness to keep $N_i \approx \frac{1}{M}$, $i = 1,\dots,n$. Assuming that the condition $N_i = \frac{1}{M}$, $i = 1,\dots,n$ holds, it is possible to express $p_{zi} = M\bar{p}_{zi}$ in terms of export (fob) to import (cif) price ratios $T_{zi}(t) = \bar{p}_{iz}(t)/\bar{p}_{zi}(t)$, i.e.

$$p_{zi}(t) = p_i(t)\,\frac{\bar{p}_{zi}(t)}{\bar{p}_{iz}(t)} = p_i(t)/T_{zi}(t) \quad , \quad i = 1,\dots,n \quad .$$

Now we can investigate the impact of $T_{zi}(t)$ change on the domestic price system. For that purpose, we can use the eq.(38) of Sec. 2.3. Since the domestic production function (17) includes in addition the $Y_{zi}^{\alpha_{zi}}\ p_{zi}^{-\alpha_{zi}}$ factor, the term

$$\alpha_{zi} \ln p_{zi}^t(t) = \alpha_{zi} \ln p_i^t(t)/T_{zi}^t(t) \quad ,$$

where

$$T_{zi}^{t}(t) = T_{zi}(t)/T_{zi}(t-1) \quad ;$$

should be added to the left side of eq.(38), Sec.2.3.

In the general case, the foreign trade system consists of many markets S_z , $z = 1,\ldots,s$, and it is necessary to have the balance of payment with each of the markets (though some transfers of payments between markets are sometimes possible). Then supplementing (38)(Sec. 2.3)with the additional factor

$$\prod_{z=1}^{s} Y_{zi}^{\alpha_{zi}} p_{zi}^{-\alpha_{zi}}$$

one obtains

$$(1 - \sum_{z=1}^{s} \alpha_{zi}) \ln p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ln p_j^t = q_i \left[\frac{\hat{Y}_i^t l_i^t}{F_i^t} + \sum_{\nu=1}^{m} \beta_{\nu i} \ln \omega_\nu^t \right.$$

$$\left. - \sum_{z=1}^{s} \alpha_{zi} \ln T_{zi}^t \right] ,$$

$$i = 1,\ldots,n \qquad (26)$$

where

$$q_i = 1 - \sum_j \alpha_{ji} - \sum_z \alpha_{zi} > 0 \quad .$$

It should be observed that the increase of T_{zi}^{t} decreases the domestic prices p_i^t , $i = 1,\ldots,n$. Since we have no foreign trade price model, the T_{zi}^{t} should be regarded as exogenous variables. For forecasting purposes, one can extrapolate the $\bar{p}_{zi}$, $\bar{p}_{iz}$ prices by exponential functions:

$$\bar{p}_{zi}(t) = a_i\, e^{\alpha_i t} \quad , \qquad \bar{p}_{iz}(t) = b_i\, e^{\beta_i t} \quad ,$$

and find

$$\ln T_{zi}^{t} = \beta_i - \alpha_i \quad , \qquad i = 1,\ldots,n \quad .$$

It should be observed that the assumption of exponential price trends for the depletable natural resources (i.e. coal and oil) can be justified by theoretical arguments as shown in Ref. [47].

The derivation of T_{zi}^{t} is especially simple when the sector trade is homogenous. For example, the sector of energy production of the Polish economy exports coal and imports oil, so $\ln T_{zi}^{t}$ is the difference of annual rates of price-change for these two commodities. Since oil price rises at a faster rate than coal price, $\ln T_{zi}^{t}$ for energy is negative, and as a result, the domestic prices p_i^t , $i = 1,\ldots,n$ increase. The impact of T_{zi}^{t} change on domestic prices depends also on the α_{zi} , i.e. the share of imported commodities in the total sector input, characterized by $1 - q_i$. Since the optimization goals , which is the integrated GNP in constant prices, depends on the sectorial prices, the optimum allocation of $z_{\nu i}$ strategy depends on T_{zi}^{t} change.

It is possible to show that when $\ln T_{zi}^{t}$ change (e.g. from positive to negative values) for a number of sectors a reorientation of existing development strategy may follow which gives, in the planning interval [0,T], preferences to the development of these sectors of national economy which contribute most to the general development goal.

It should be observed that the proposed methodology of planning the optimum development depends a great deal on the accuracy of world prices forecasts. Obviously, the best results would be obtained if one had a world trade model including all the markets under consideration and trading the commodities which correspond to the sectorial aggregates.

In order to show how such a model can be constructed by using the present methodology, consider a simple model consisting of two submodels (representing two countries), each consisting of a number of production sectors S_i , $i = 1,\dots,n$, S_z , $z = 1,\dots,N$. Suppose two sectors S_i , S_z have engaged in trade and it is necessary to find out what terms of trade can be obtained, when the countries have agreed to keep the constant balance of payment (e.g. equal zero).

The production function of S_i can be written as

$$Y_i = F_i^{q_i} \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}} Y_{zi}^{\alpha_{zi}} = \tilde{F}_i^{\tilde{q}_i} Y_{zi}^{\alpha_{zi}} ,$$

$$\tilde{q}_i = 1 - \alpha_{zi} = q_i + \sum_j \alpha_{ji} .$$

In the same way for the S_z sector one gets

$$\bar{Y}_z = \tilde{F}_z^{\tilde{q}_z} \bar{Y}_{iz}^{\alpha_{iz}} \quad ,$$

where $\bar{Y}_{iz} = M^{-1}Y_{iz}$, Y_{iz} - export from S_i to S_z ,

M = exchange rate of S_z currency for S_i currency.

The optimum cooperation requires [see (6) of Sec. 2.1]:

$$\hat{\bar{Y}}_{iz} = \alpha_{iz}\hat{\bar{Y}}_z \quad , \qquad \hat{Y}_{zi} = \alpha_{zi}\hat{Y}_i \quad ,$$

where

$$\hat{\bar{Y}}_z = \tilde{F}_z\left(\frac{\bar{p}_{iz}}{\alpha_{iz}}\right)^{\alpha_{iz}/\tilde{q}_z} \bar{p}_z^{\frac{1}{\tilde{q}_z}} \quad ,$$

$$\hat{Y}_i = \tilde{F}_i\left(\frac{p_{zi}}{\alpha_{zi}}\right)^{\alpha_{zi}/\tilde{q}_i} (p_i)^{\frac{1}{\tilde{q}_i}} \quad .$$

and

$$\bar{p}_{iz} = M^{-1}p_i \quad , \qquad p_{zi} = M\bar{p}_z \quad .$$

Introducing the term of trade $T_{zi} = \bar{p}_{iz}/\bar{p}_z$ one gets

$$M = \frac{p_i}{\bar{p}_z T_{zi}} \quad .$$

From the balance of payment condition $M\hat{\hat{Y}}_{iz} = \hat{Y}_{zi}$ one obtains

$$\frac{p_i}{\bar{p}_z T_{zi}} \alpha_{iz} \hat{\hat{Y}}_z = \alpha_{zi} \hat{Y}_i \quad ,$$

which can be written in the form similar to (26):

$$(\alpha_{iz/\tilde{q}_z} + \alpha_{zi/\tilde{q}_i} - 1) \ln T_{zi}^t - 2\alpha_{zi/\tilde{q}_i} \ln p_i^t + 2\alpha_{iz/\tilde{q}_z} \ln \bar{p}_z^t$$

$$= \ln \left[\alpha_{zi}^{1-\alpha_{zi/\tilde{q}_i}} \alpha_{iz}^{\alpha_{zi/\tilde{q}_z}-1} \tilde{F}_{i/\tilde{F}_z} \right]^t = \ln \frac{\tilde{F}_i^t}{\tilde{F}_z^t} \qquad (27)$$

The equation (27) combines the two systems of linear equations of the type (26), which refer to the two countries and the respective systems of prices p_i^t , $i = 1,\dots,n$, $\bar{p}_z^t$, $z = 1,\dots,N$. In order to find the development strategies which are beneficial for both countries, it is necessary that the investment-price adjustment technique, described in Sec. 2.3, is extended and coordinated by taking into account the interactions specified by (27). The resulting terms of trade T_{zi} , $T_{iz} = T_{zi}^{-1}$ are then derived endogenously. The coordination of the development process can be extended to the general case of all the countries wishing to cooperate in the given areas (sectors) of foreign trade. Using the present approach, it is possible to find these areas of international specialization which give the highest possible comparative advantages.

It should be observed that in the literature on the comparative advantages resulting from different growth policies and corresponding terms of trade, much confusion exists. The problem has been especially controversial since a paper was written by J.R. Hicks (1953), in which he had shown that export-biased growth would turn the terms of trade against a country, and that import-biased growth would improve a country's terms of trade. The so-called British school held that the terms of trade would go against the developed countries, while another school (represented by Singer and Prebish) maintained that the terms of trade would more likely have gone against the less-developed countries.

The resulting confusion is presumably due to the complex nature of international growth equations. In the situation when "everything depends on something else", it is possible to obscure and hopelessly entangle the whole issue when one starts to speculate on the possible solutions of nonlinear, many-variables equations in qualitative terms. It is probably better to leave the whole thing to the normative mathematical models and computers, which can tell us in monetary terms who will benefit and how much they will benefit, given the concrete values and constraints.

It can be shown that using a model of trading countries with the production function (which use as inputs the exports from the remaining countries) and observing the balance of payment conditions, it is possible to find a unique system of exchange ratios. That means that M in (23) should be regarded as given exogenously (from the world monetary model) and as a result, the value Y_{zo} is bounded from above. In other words, it is not economically justifiable to decrease M (e.g. by enforcing export) in order to get additional Y_{zo} for factor endowments. Then the only possible way to increase the factor endowments (and the rate of growth) is to take foreign credits. Of course the credits should be paid back (including interest) in the planning interval [0,T].

In order to derive the comparative advantages when using credits, one can apply the aggregated model of Sec.1.2, described by equations (13)-(20), or the simple version (38)-(41). In the last case, the computations are much simpler. Since the credits are usually and most often used for financing the productive investments, we shall assume also m = 2. The optimum strategy of investments, according to (43) of Sec.1.2†

$$\hat{z}(\tau - T) = \frac{\varphi(\tau)}{w(\tau)} \cdot \frac{Y^*}{G} \quad ,$$

where

$$\varphi(\tau) = \left\{ K\omega(\tau)^{-1} \int_{\tau}^{T} e^{\vartheta t - \delta(t-\tau)} \, dt \right\}^{\frac{1}{q}} \quad ,$$

$$G = \int_{0}^{T} \varphi(\tau) \, d\tau \quad , \qquad q = 1 - \alpha \quad ,$$

and Y^* is the solution of the equation:

$$Y = \bar{Y} + G^{q} \left[\beta^{\beta} \, (1-\beta)^{1-\beta} \, \alpha \right]^{\alpha} Y^{\alpha} \quad .$$

† The index $\nu = 2$ can be dropped here.

In Fig. 19 the typical forms of the functions $\hat{z}(t)$, $Y(\hat{z}) = \hat{Y}(t)$ have been shown. It is possible to show that $\hat{Y}(t)$ increases at a faster rate than $Y(\bar{z})$, which corresponds to the strategy $\bar{z}(t) = \gamma Y(t-1)$, i.e. the amplitude constraints or autarky. That phenomenon occurs even in the case when the total amounts of capital investments (i.e. the areas under $\hat{z}(t)$ and $\bar{z}(t)$) are equal, and it can be explained by the fact that in the inertial system it pays to allocate most of the resources at the beginning of the planning interval [0,T]. That can be done, of course, with the mobile part of capital investments only when foreign credit is used for purchasing investment goods abroad. The credit, together with the interest, should be repaid at $t = T$. That requires that foreign and domestic mobile investment goods can be traded without any restrictions. However, when the GNP increases according to $\hat{Y}(t)$, the part $\tilde{z}(t) = \gamma\hat{Y}(t-1)$ can be used for investments and only the shaded area between the curves $\hat{z}(t) - \tilde{z}(t)$, $t \in [0,T_1]$, as shown in Fig.19, represents the necessary total credits required. The corresponding area for $t \in [T_1 T]$ represents the domestic "savings" of mobile capital investments which can be traded or used for further investment purposes. Then the net credit value C, which is necessary for optimum development within [0,T], can be written as

$$C = \int_0^{T_1} [\hat{z}(t)-\tilde{z}(t)]w\ (t)dt - \int_{T_1}^{T} [\tilde{z}(t)-\hat{z}(t)]w\ (t)dt \quad . \tag{28}$$

Since the function w (t) increases along with interest rate ε , as shown in Fig.20, the first term in (28) increases at a

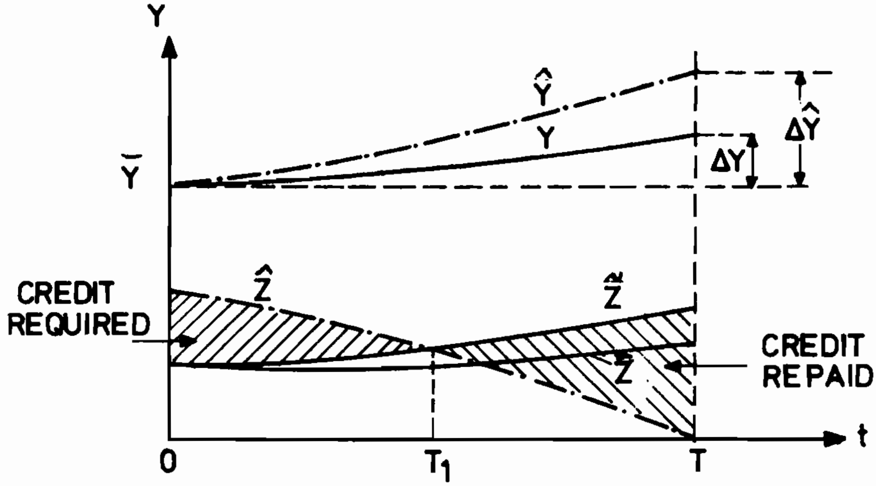


Figure 19.

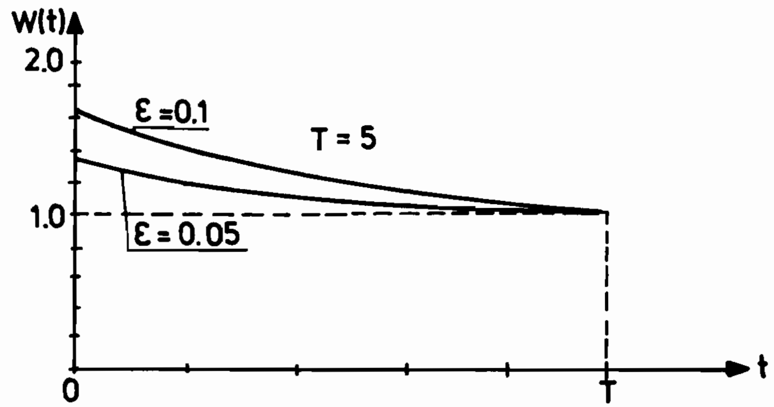


Figure 20.

faster rate than the second term. As a result, C is an increasing function of ε and the generated within [0,T] increase of GNP (ΔY_C), which takes into account the repayments of credits, becomes

$$\Delta Y_C = \Delta\hat{Y} - C \quad . \tag{29}$$

That value (which is a decreasing function of ε) should be compared with ΔY, corresponding to the development in autarky. For a given planning interval T, there exists a value of credit rate $\bar{\varepsilon}$, such that $\Delta Y_C \leq \Delta Y$ for $\varepsilon \geq \bar{\varepsilon}$ and it does not pay to take credits with $\varepsilon \geq \bar{\varepsilon}$. The value of $\bar{\varepsilon}$ depends on the effectiveness of investments, which in turn depends on the model K, δ, β parameters and prices.

In the planning practice, the following optimization problem is also formulated. What is the minimum value of time $T = T_m$ and credits $C = C_m$, which will guarantee the given development rate $r = \Delta Y/_Y$?

That problem can be solved by finding the required GNP (Y^*) and T_m from the equations:

$$Y = \frac{G^q(T)}{r^{1/q}} \left[\beta^{\beta} \ (1-\beta)^{1-\beta} \ \alpha \right]^{\alpha} \tag{30}$$

$$r = \frac{Y - \bar{Y}(T)}{Y} \quad , \tag{31}$$

Then from equation (28) one can find the necessary minimum value of $C = C_m$, and the optimum crediting strategy can be derived

$$C(t) = \hat{Z}(t) - \tilde{Z}(t) \quad , \qquad t \in [0,T] \quad .$$

When instead of r the required development rate is defined as $\zeta = \Delta Y/\bar{Y}$ one easily finds $r = \zeta/(1 + \zeta)$.

It is necessary to observe that in the present model, r cannot be an arbitrarily great number. The increase of r increases $\hat{Z}(t) - \tilde{z}(t)$, which represents the mobile part of the investments. In the real economy, the mobile part cannot exceed a certain fraction of the whole investment volume, and as a result, r is bounded from above. The mobile part can be financed out of the credits available at different ε rates at corresponding markets. Since the change of investment strategy also affects the sector price indices, it is necessary to express ΔY, ΔY_C in constant prices. Then the crediting strategy depends on the expected terms of trade T_{zi}^{t} , $t \in [0,T]$.

The present model can easily be extended to the general model of international cooperation of Fig.17, which involves all the development factors. It can easily be shown that the rate of development can be increased when one follows a broad international cooperation program in the factor endowments policy. To what extent that program can be used depends, of course, on the international agreements, which depend in turn on the admissable ratios of mobile to immobile parts of development factors available. Since the general trend in development is a national specialization in production areas and the development plans do not coincide in time, it is often possible to coordinate the plans of exchange of scarce development factors. For example, when a particular country is planning to develop a new branch of industry it can "import" the technology, specialists, etc. at the initial part $[0,T_1]$ of the planning interval in order to "export" them in the interval $[T_1T]$, when the development reaches a sufficiently high level.

IV. Conclusions

As already stated, the main motivation behind the present research was to develop a consistent methodology enabling the construction of long-term, normative development models. That methodology enables in particular the construction of national models, starting with a data base regarding the sectorial or regional activity, and the construction of global models, starting with a national data base.

The normative approach also enables the derivation of national development strategies, which maximize the national goals and satisfy the national aspirations, subject to the constraints which result from the shortages of natural resources and an equilibrium in international relations.

The model then enables the derivation of the system of price indices, which revaluate the resources available, and help to relocate the development factors in an optimum manner. The technological and structural changes take place, and as a result, the utility functional of the optimally planned country increases along with the time horizon. In that sense, no crisis is in sight in the model investigated.

As shown in the present paper, the construction of national and then global models based on the methodology proposed is feasible.

As shown in Ref.[37], a number of MRI models, constructed at the Polish Academy of Sciences, have proved to be exact enough (in the historical runs) to be used for projections of development in Poland.

The methodology used can also be applied for the construction of a development model for C.M.E.A. countries which have similar development objectives and management structure, and which use a similar long-term planning method for allocation of development factors.

The methodology proposed should, however, be extended in the case of countries which use a higher degree of decentralization in their production factor endowments. As shown in Ref. [18], in cases where the sectors (rather than the government) decide how much capital to invest, one can easily extend the proposed methodology. In the case of underdeveloped countries, it is necessary to take into account the additional constraints resulting from shortages in food supply, population explosions, etc.

Since the proposed methodology uses a "bottom-up" technique, the problem of who should construct the national submodels and how the national submodels could be linked together arises. Before answering those questions, it is necessary to observe that in most of the countries, short-term, descriptive econometric models already exist. International cooperation in the area of econometric models resulted in the creation of the project LINK (Ref.[4]), which enables the linkage of national econometric models.

The present problem, then, involves extending the modeling efforts, in order to create a long-term, normative and complex global development model.

It seems that what should be done first is to develop a unified long-term, normative linkage methodology which would be accepted by national model builders. Then the linkage of national models in order to yield a global development model would be comparatively easy.

The role of international institutions such as IIASA is especially important in that respect.

It should also be noted that the aggregation methodology proposed in the present paper enables the decomposition and extension of national models to the sectorial and regional submodels. That approach has already been used in the system of models constructed by the Polish Academy of Sciences, where the aggregated national MRI models are regarded as the "core" models. The core cooperates with the global model (by means of foreign trade, crediting, etc.), and with a number of regional and sectorial submodels, as shown in Fig.21. The structure of sectorial and regional submodels is more complex. For example, in MR1 the agriculture is represented by one sector only, while the extended food and agriculture submodel consists of many sectors which are responsible for the production of grain, meat, etc., and for the food industry. The extended food model cooperates with the core by receiving information regarding the GNP and investment endowments, as well as the intersector flow of fertilizers and other chemicals used in food production, agricultural machinery, fodders, etc. It sends information to the core

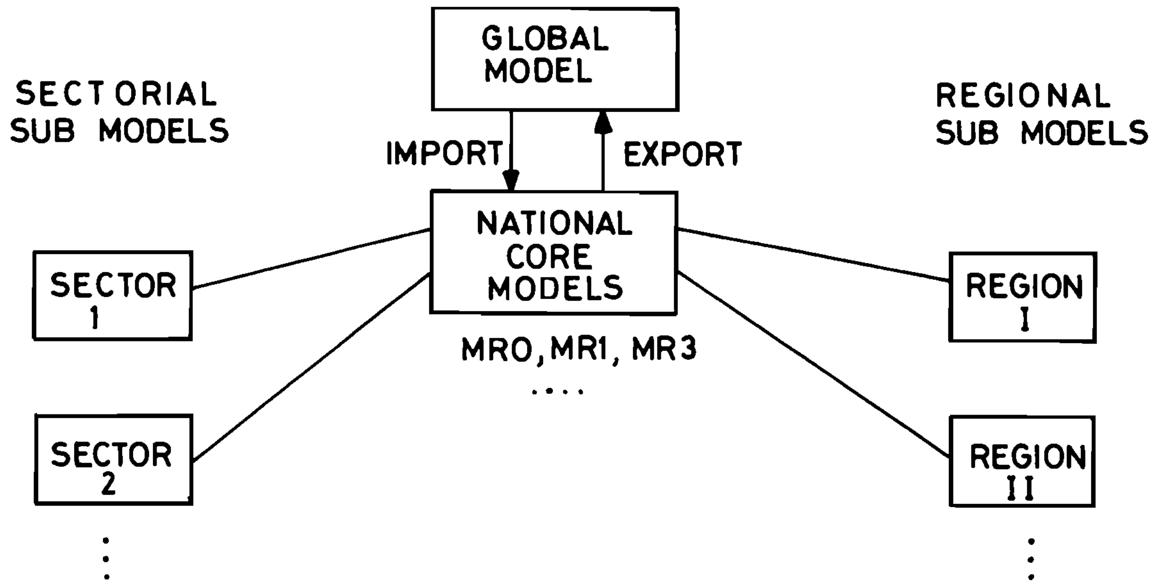


Figure 21. The system of models.

regarding the food supply structure, export and import demands, etc.

By using regional and sectorial models, and cooperating with the core model, it is possible to extend the main planning and decision area to the needs of low-level or local decision units. However, the central decision system obtains more information regarding local problems and aspirations.

As a closing observation, it is necessary to notice that the modeling of complex national and global development is a long process of successive approximations of the constantly changing reality. The approximations take place in time and space. Scientists all over the world are trying to contribute to the final result, which is a clear path of development leading to global welfare.

The present paper should be regarded, therefore, as a small step in the long approximation process. There is still much to be done before we have a good model of national and global development.

V. Summary of basic objectives, structure, variables and MRI-project organization

The main purpose of the present section is to summarize the basic objectives, structure, variables and operation modes used in MRI models, as well as the general MRI-project organization.

As already stated, the MRI models should be regarded as computerized tools, which are supposed to help the decision makers in long-term planning and forecasting of national development. They are therefore normative. We understand by that term the models in which the impact of different government decisions and policies on the given development goals (utilities) can be investigated. The normative model should enable, first of all, the investigation of optimal decisions and policies. The models should in particular:

a. consist at least of four main, cooperating submodels:
   - production,
   - personal and aggregate consumption,
   - environment;
b. be opened and the impact of world crises in energy, food etc. on national development should be investigated;
c. enable the computation (in the explicit form) of optimum strategies in allocation of production factors (i.e. investments and labor) and other government expenditures;
d. enable the computation of optimum socio-economic policies in the field of demography, migrations, wages, taxes, subsidies, environment pollution abatement, etc.;
e. enable the disaggregation of the model structure in order to investigate the more detailed regional and sectorial submodels;
f. enable the comparison of different development strategies (e.g. the investment strategy which uses the foreign trade and credits or follows the policy of autarky) and the impact of these strategies on the development.

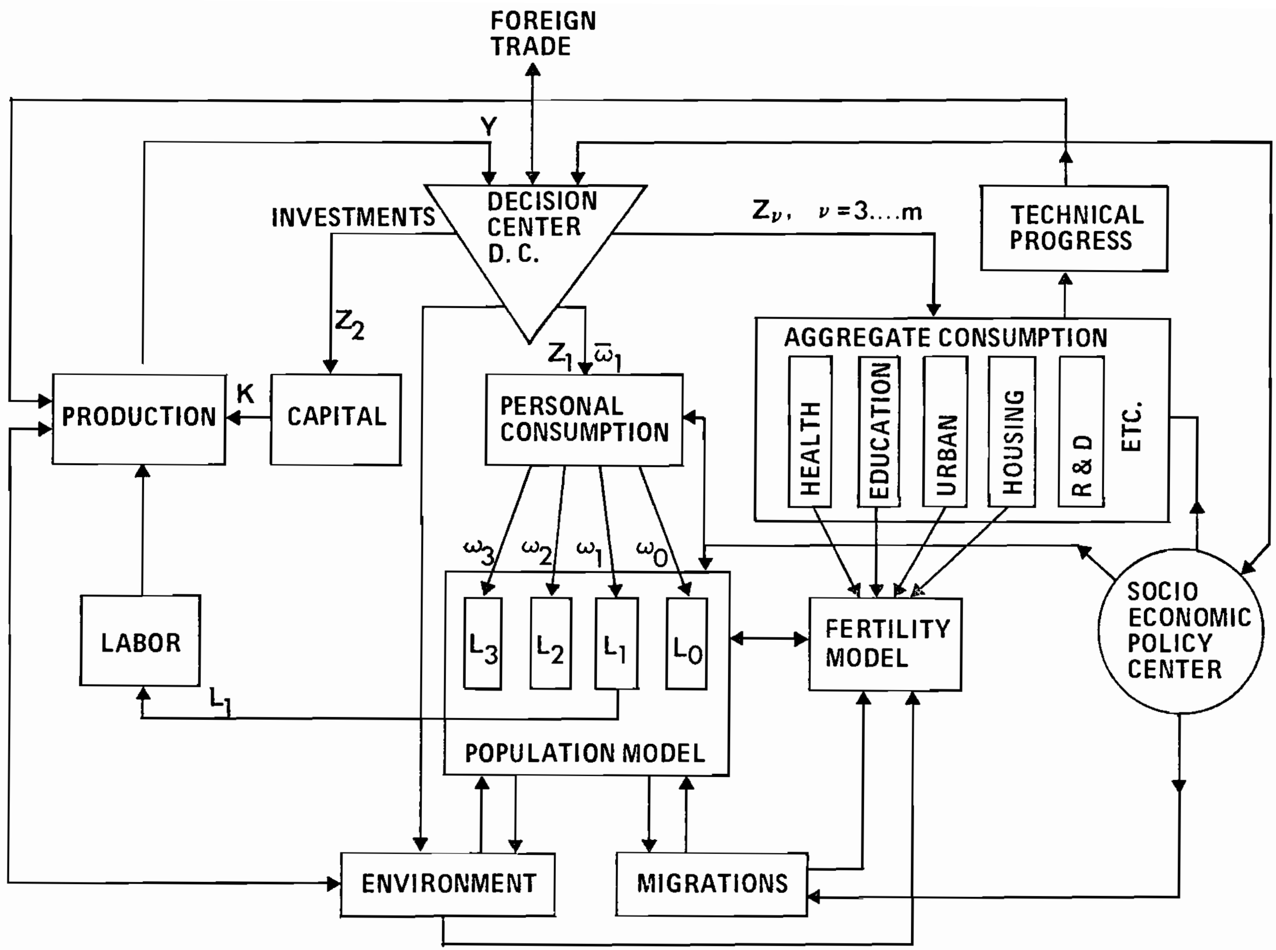


Figure 22. Socio-economic model of development.

The general (simplified) structure of MRI models is shown in Fig. 22. There are two main decision centers in MRI. The first one is concerned with the allocation of production factors (capital and labor) and other government expenditures. The second is concerned with the socio-economic policy, which includes in particular: wages, prices, subsidies, demographic policy, migration policy and environment protection policy.

These two decision centers cooperate closely in order to optimize the given development goals. As the development goals we use, in particular,

a. maximization of GNP (in current or constant prices),
b. maximization of GNP per capita (in current or constant prices),
c. maximization of consumption per capita (in current and constant prices).

The mode of operation which enable the investigation of development resulting out of the stragegy of continuation of trends estimated ex post is also possible. Another mode of operation possible concerns the investigation of effects of strategy, which follows a prescriptive scenario, e.g. the energy or food crisis in the world market.

The decision maker can investigate in the model the effect of different development strategies on the national development (using different objectives) and the impact of these strategies on socio-economic development and environment.

The main feedbacks affecting the development in the model of Fig. 22 can be classified as follows:

1. investment (Z_2) creates the capital K which contributes to the production (Y),
2. personal consumption (Z_1) [paid to the population in different social groups:

L_0- part of population in preworking age,

L_1- the working part of population in working age,

L_2- the nonworking part of population in working age (e.g. house-wives),

L_3- part of the population in post-working age]

supports the labor employed L_1 which contributes to the production (Y);

3. the government expenditures (Z_ν, $\nu = 3,\ldots,m$), which contribute to the aggregate consumption, affect the technical progress, which in turn contributes to the production (Y);
4. the personal and aggregate consumption affects the fertility, which in turn determines the demographic processes and labor supply (L_1).

In the model of Fig. 22 several interactions take place:

a. the productive activity pollutes the environment and at the same time the natural resources are used for production activity,
b. the consumption is accompanied by pollution of environment, and at the same time the natural resources are being consumed,
c. there is a number of interactions between environment, fertility, migration and population sub-models,
d. the interaction with global model by the way of foreign trade, credits etc. also exist.

The basic variables and operation modes used in MRI at different forecasting runs are shown in Table 1.

In the first versions of MRI we have used as exogeneous variables: population, labor and sectorial terms of trade. At the present time an effort is being made to endogenize these variables. That requires first of all that a better submodel of population and fertility, as well as foreign trade model should be constructed.

The general MRI-project organization is shown in Fig. 23.

The main part of our activity is concerned with the modelling methodology. We are trying to use here as much as possible the help, support and criticism coming from the scientific community (scientific committees of Polish Academy of Sciences etc.).

When a concrete version of the model (labelled MRO, MR1,...) is proposed the identification of model parameters should be carried out. As a data base we use mainly the annual publications of Central Statistical Office (G.U.S.).

Table 1. Basic variables and operation modes used in MRI in forecasting runs.

Exogeneous variables	Endogeneous variables	Decisions, Policies, Modes
Population N Labor L Sectorial terms of trade T_i, $i = 1,\dots,n$	Sectorial productions: gross Y_i, net $\overline{Y}_i$, $i = 1,\dots,n$ Sectorial inputs Y_{ji}, $j,i = 1,\dots,n$ Price indices P_j^t, $j = 1,\dots,n$ Wages w_{1i}^t, $i = 1,\dots,n$	<u>I-level</u>: O,C,T,U,S investment, personal and aggr. consumption: $\gamma_\nu(t)$, $\nu=1,\dots,m$ <u>II-level</u>: O,C,U,S Sectorial investments, employments, etc. $z_{\nu i}(t)$, $\nu=1,\dots,m$, $i = 1,\dots,n$ $t\varepsilon[0,T]$ <u>Intern. cooperation</u>: O,S,C Credits, Interest rate, Migrations <u>Environment</u>: T,S Pollution abatement ε <u>Price & wage policy</u>: O,S Taxes t_i, $i = 1,\dots,n$ Subsidies <u>Technological change</u>: O,T,S Technological coeff. $\alpha_{ji}(t)$ $j,i = 1,\dots,n$

<u>Basic Modes of Operation:</u>

O - open economy (int. cooperation)

C - closed economy (autarky)

T - continuation of trends estimated ex post

U - optimization of given utility:

U_1 - max GNP

U_2 - max GNP/cap. in constant prices

U_3 - max consumption/cap.

S - scenarios (crises)

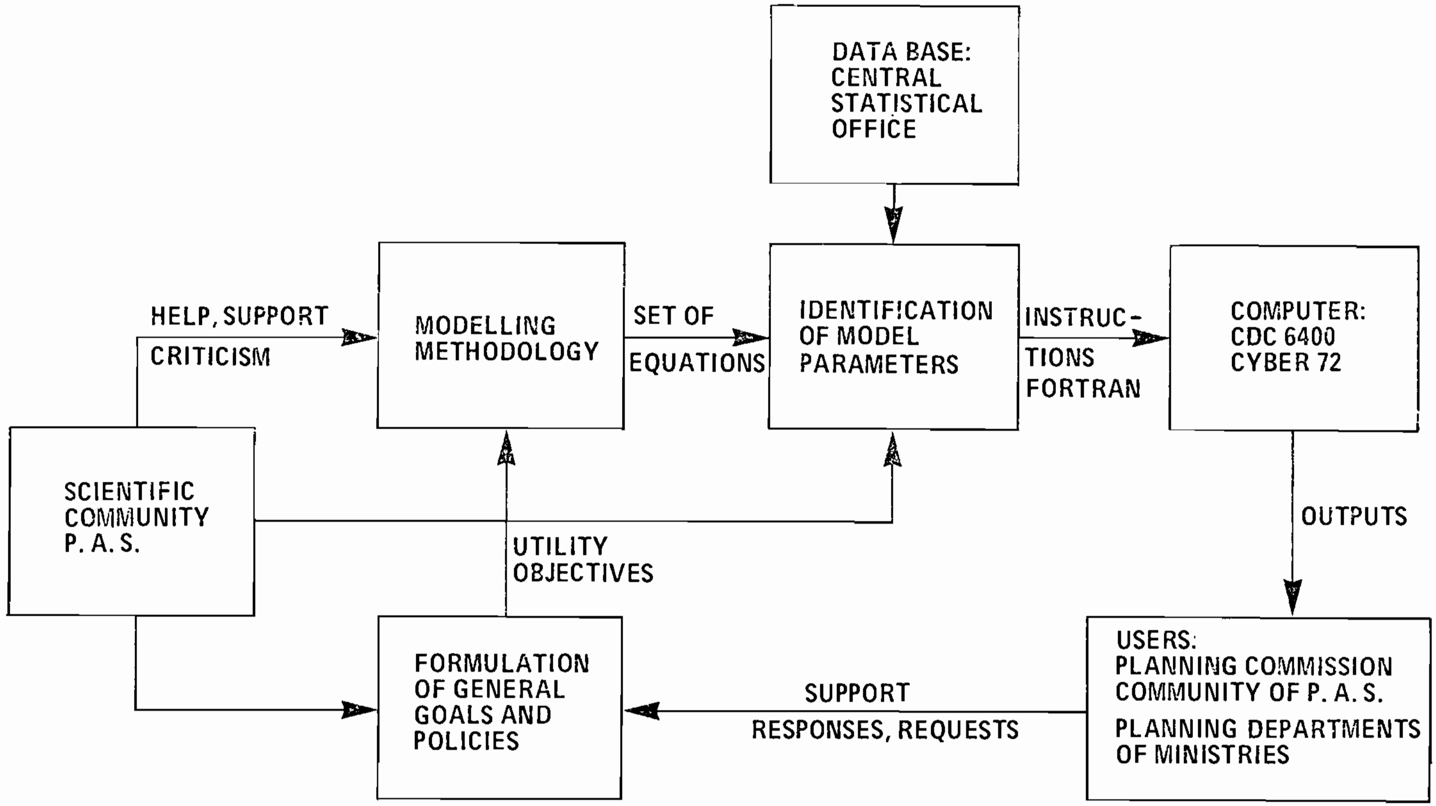


Figure 23. General MRI- project organization.

Then a set of instructions, in FORTRAN, is being written for the CDC 6400 Cyber 72 computer.

The printed outputs from the computer are being evaluated. We are trying to use here as much as possible the cooperation with the Planning Commission, Committees of P.A.S., Planning Dept. of Ministries etc. The responses and requests obtained from them are being used for reformulation of model goals (utilities) and policy objectives.

As can be observed, the MRI modelling is a long iterative process which creates a series of models. At each stage we compare the model constructed with the reality. For that purpose we use the historical runs. We are trying also to incorporate at each stage the new concepts achieved in macroeconomic, environmental and social sciences.

Since it is impossible to present in a short conference paper the complete version of our modelling efforts in the next chapters only the main parts of version MR5 have been described.

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B. PRODUCTION SUBSYSTEM

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1. Introduction

The paper deals with the model of production subsystem which is a part of the normative development model of the MRI type described in [A]. The present modelling effort of the national economy was started in 1972 at the Polish Academy of Sciences. Within the MRI series the following models were constructed:

MRO - singe sector model
MR1 - 9 sectors model
MR3 - 15 sectors model
MR4 - 15 sectors model (modified version)
MR5 - 33 sectors model.

This presentation deals with the production subsystem for the MR4 model, and the following problems are considered:

1) mathematical description of a production subsystem,
2) structure of the management system,
3) optimal allocation of resources,
4) evaluation of the model parameters,
5) algorithms of the economic forecast and optimal policy.

2. Multisector Production Model - A General Description

Consider the n production sectors S_i, $i = 1,\dots,n$. Each sector produces Y_i goods annually, and cooperates with the remaining sectors shown in Fig. 1.

The intersector flows are labeled Y_{ij}, $j = 1,\dots,n$, $i = 1, 2,\dots,n$.

There are two exogeneous inputs for each sector: $Y_{n+1,i}$ is the import from the countries of The Council of Mutual Economic Assistance (CMEA), and $Y_{n+2,i}$ is the import from the other countries.

Let us consider labour Z_{0i}, $i = 1,\dots,n$, as additional inputs to the productive sectors.

The net product of sector $\overline{Y}_i$ contributes to the national income, which in turn is used for factors endowments.

In the present paper the investment allocation for the production sectors is considered only.

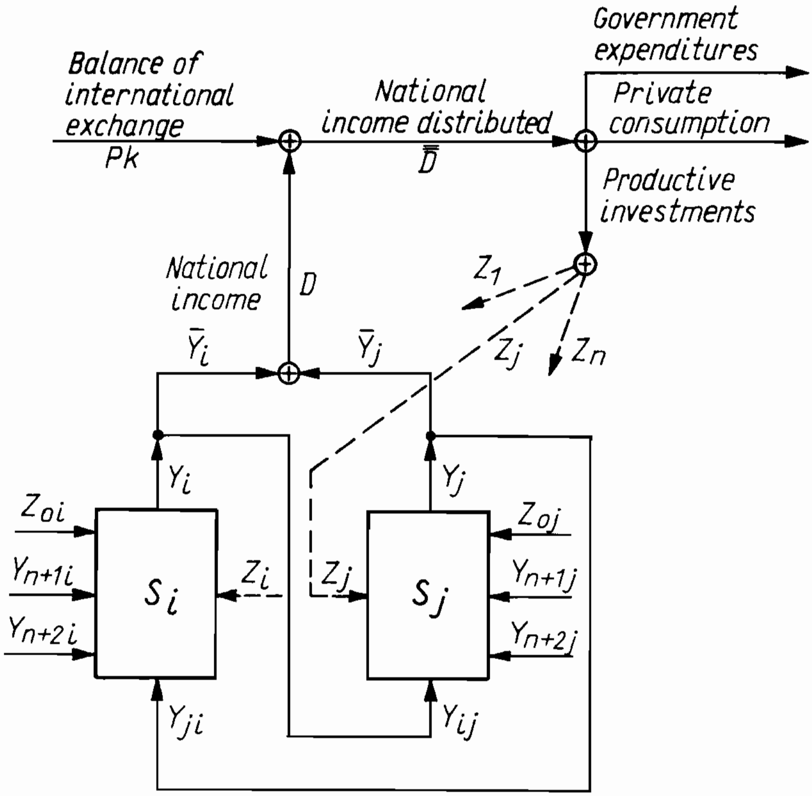

Figure 1. Production subsystem.

Assume that each sector is described by the generalized Cobb-Douglas production function

$$Y_i(t) = \left[F_i(t) Z_{0i}(t)^{\gamma_{0i}} Z_{1i}(t)^{\gamma_{1i}}\right]^{q_i} \prod_{j=1}^{n+2} Y_{ji}(t)^{\alpha_{ji}} \qquad (2.1)$$

$$i = 1, \ldots, n$$

where

$$q_i = 1 - \sum_{j=1}^{n+2} \alpha_{ji} > 0, \quad \alpha_{ji} \geqslant 0, \; j = 1, \ldots, n+2, \; i = 1, \ldots, n \qquad (2.2)$$

$$\gamma_{0i} + \gamma_{1i} = 1, \quad \gamma_{ji} > 0, \tag{2.3}$$
$$j = 0,\ 1, \quad i = 1,\dots,n$$

In (2.1)-(2.3) α_{ji} are the elasticities of output with respect to the current production factors, $\gamma_{0i}q_i$ is the elasticity with respect to employment Z_{0i}, and $\gamma_{1i}q_i$ is the elasticity with respect to capital stock Z_{1i}.

All sectorial inputs and output are expressed in monetary units.

The positive function $\bar{F}_i(t)$ combines technical progress, and change of prices.

By virtue of (2.2) and (2.3) there exist a constant return to scale in all sectors, i.e. for each sector production functions are linear homogeneous in all the inputs and capital stock.

The relation between the investments and the capital stock can be written in the form of a difference equation (in the other versions of the model, the direct production- investments relations were used):

$$Z_{1i}(t) = Z_{1i}(t-1)\ \exp\ (-\delta_i) + c_i\left[z_i(t-T_{0i})\right]^{\beta_i} \tag{2.4}$$
$$i = 1,\dots,n$$

$$Z_{1i}(T_p - 1) = \hat{Z}_{1i}(T_p - 1) \tag{2.5}$$

where $\hat{Z}_{1i}(T_p - 1)$ is the given initial capital stock at the end of period $T_p - 1$.

The relation (2.4) takes into account the delay of plant construction T_{0i} and the exponential depreciation of capital (δ_i is the depreciation rate).

The $c_i > 0$ and $0 < \beta_i < 1$ coefficients take into account the nonlinear effects of the investment process.

Solving the equation (2.4) one gets an alternative form of relation between investments and capital stock

$$Z_{1i}(t) = Z_{1i}(T_p - 1)\ \exp\ (-\delta_i\ (t - T_p + 1)\ +$$

$$+ \sum_{\tau=T_p}^{t} c_i \exp\left[(-\delta_i)(t-\tau)\right]\left[z_i(\tau - T_{0i})\right]^{\beta_i} \qquad i = 1,\dots,n \tag{2.6}$$

3. Optimization Problem

3.1. Structure of the Management System

The decision structure is introduced in the model.

Let us assume that there exist three decision levels in the economy. The national income generated at the end of the period (t - 1) is divided, at the first level, among the productive investments, private consumption, and government expenditures.The second decision level is concerned with allocation of productive investments among the sectors S_i, $i = 1,\dots,n$. At the third level the sectors select the technique of production that maximize their net profit.

3.2. Optimization of Sectorial Strategies

Let us assume that the sector capital stock is fixed by the second decision level. The sectors maximize the net profit in each period t.

If the time index is neglected then the net sector profits are described as following:

$$D_i = Y_i - \sum_{j=1}^{n+2} Y_{ji} - Z_{0i}, \quad i = 1,\dots,n \tag{3.1}$$

Substituting (2.1) into (3.1) and denoting

$$Y_{0i} = Z_{0i}, \quad i = 1,\dots,n \tag{3.2}$$

$$\alpha_{0i} = \gamma_{0i} q_i, \quad i = 1,\dots,n \tag{3.3}$$

$$\bar{q}_i = q_i - \alpha_{0i}, \quad i = 1,\dots,n \tag{3.4}$$

we get the following problem:

For each sector i find the nonnegative strategies $Y_{ji} = \hat{Y}_{ji}$, $j = 0,\dots,n+2$, such that the net profit

$$D_i = (F_i^{q_i/\bar{q}_i} Z_{1i}^{\gamma_i \cdot q_i/\bar{q}_i})^{\bar{q}_i} \prod_{j=0}^{n+2} Y_{ji}^{\alpha_{ji}} - \sum_{j=0}^{n+2} Y_{ji} \qquad (3.5)$$

attains the maximum value.

The strategy as shown in [A] becomes

$$\hat{Y}_{ji} = \alpha_{ji}\hat{Y}_i, \quad j = 0,\dots,n+2, \quad i = 1,\dots,n \qquad (3.6)$$

where

$$\hat{Y}_i = (F_i^{q_i/\bar{q}_i} \prod_{j=0}^{n+2} \alpha_{ji}^{\alpha_{ji}/\bar{q}_i}) Z_{1i}^{\gamma_{1i} \cdot q_i/\bar{q}_i}, \quad i=1,\dots,n \qquad (3.7)$$

and the sectorial profit

$$\hat{D}_i = \bar{q}_i \hat{Y}_i, \quad i = 1,\dots,n \qquad (3.8)$$

The expression (3.6) implies that the sectorial production process with selected techniques of production corresponds to the Leontief model with the technological coefficients

$$\alpha_{ji} = Y_{ji}/Y_j \qquad (3.9)$$

The expression (3.7) shows that the sector production depends on the capital stock only.

Having assumed the constant returns to scale, the relation

$$\gamma_{1i} q_i/\bar{q}_i = 1, \quad i = 1,\dots,n \qquad (3.10)$$

holds.

It should be observed that the expression (3.7) and (3.10) gives linear relation between the sectorial production and the capital stock.

In the model the first part of relation (3.7) is approximated by $K_i \exp \mu_i t$

$$\bar{F}_i(t)^{q_i/\bar{q}_i} \prod_{j=0}^{n+2} \frac{\alpha_{ji}^{\alpha_{ji}/\bar{q}_i}}{\alpha_{ji}} = K_i \exp \mu_i t \qquad i = 1,\dots,n \tag{3.11}$$

where parameters K_i and μ_i are evaluated from statistical data.

Then, from the point of view of the second decision level, each sector is described by one factor-production function

$$Y_i(t) = K_i \exp(\mu_i t) \, Z_{1i}(t) \tag{3.12}$$

3.3. Optimal Investment Allocation

In this section the problem of dynamic investments allocation among the sectors is considered.

The discounted cumulative national income over the time interval $[T_p, T_k]$ can be defined as follows:

$$\bar{D} = \sum_{t=T_p}^{T_k} \sum_{i=1}^{n} w_i(t) \, \bar{D}_i(t) \tag{3.13}$$

where:

$$w_i(t) = \text{given discount function}$$
$$\bar{D}_i(t) = q_i(t) Y_i(t) = \text{net sector profit} \tag{3.14}$$

As a discount function we take $w_i(t) = (1+\varepsilon)^{T_p - t}$, where ε = = discount rate.

Using the relation between the sector production $Y_i(t)$ and the capital stock $Z_{1i}(t)$ one gets expression (3.13) in terms of the capital. On the other hand, the relation between the sector investment $z_i(t)$ and the capital stock $Z_{1i}(t)$ is written in the form of the difference equation (2.4).

Let us assume that the investment $z_i(t)$ satisfies the so called amplitude constraints. The problem of investment allocation can be formulated as follows:

Find the nonnegative strategies $z_i(t) = \hat{z}_i(t)$, $i = 1,\dots,n$, $t = T_p,\dots,T_k$, such that the discounted cumulative national income

$$\bar{D} = \sum_{t=T_p}^{T_k} \sum_{i=1}^{n} w_i(t)\, q_i(t)\, K_i \exp\,(u_i(t-T_p))Z_{1i}(t) \qquad (3.15)$$

attains the maximum value subject to

$$Z_{1i}(t) = Z_{1i}(t-1) \exp\,(-\delta_i) + c_i\left[z_i(t - T_{0i})\right]^{\beta i} \qquad (3.16)$$

$$i = 1,\dots,n, \quad t = T_p,\dots,T_k$$

$$Z_{1i}(T_p - 1) = \hat{Z}_{1i}(T_p - 1) \qquad (3.17)$$

and

$$\sum_{i=1}^{n} z_i(t) \leqslant Z(t), \quad t = T_p,\dots,T_k \qquad (3.18)$$

$$z_i(t) \geqslant 0, \quad i = 1,\dots,n, \quad t = T_p,\dots,T_k \qquad (3.19)$$

The above formulation can be interpreted as the discrete optimal control problem. It is possible to reformulate that problem in the form of a nonlinear programming problem.When one solves (3.16) for $Z_{1i}(t)$ one gets (2.6).Substituting (2.6) into (3.15), and changing the summation order one gets the equivalent formulation:

Maximize

$$\sum_{i=1}^{n} \sum_{t=T_p-T_{0i}}^{T_p-1} \varphi_i(t) z_i(t)^{\beta i} + \sum_{i=1}^{n} \sum_{t=T_p}^{T_k} \varphi_i(t) z_i(t)^{\beta i} \qquad (3.20)$$

subject to (3.18) and (3.19), where

$$\varphi_i(t) = \sum_{\tau=t+T_{0i}}^{T_k} \exp(\delta_i(t+T_{0i}))\, q_i(t) K_i \cdot \exp\left[(\mu_i - \delta_i)\tau\right] w_i(\tau) \tag{3.21}$$

The first component of (3.20) does not depend on the investment in the interval $[T_p, T_k]$. Thus the optimization problem can be formulated as follows:

Maximize

$$\sum_{t=T_p}^{T_k} \sum_{i=1}^{n} \varphi_i(t)\, z_i(t)^{\beta_i} \tag{3.22}$$

subject to

$$\sum_{i=1}^{n} z_i(t) \leqslant Z(t), \quad t = T_p, \ldots, T_k \tag{3.23}$$

$$z_i(t) \geqslant 0, \quad i = 1, \ldots, n, \quad t = T_p, \ldots, T_k \tag{3.24}$$

Obviously problem (3.22)-(3.24) can be decomposed in time, and we can solve $(T_k - T_p + 1)$ static nonlinear problems. Thus, it is necessary to find an allocation strategy for the following reduced problem:

Find the strategies z_i, $i = 1, \ldots, n$, such that

$$I = \sum_{i=1}^{n} \varphi_i\, z_i^{\beta_i} \tag{3.25}$$

attains maximum value in the set

$$\mathfrak{R} = \left\{ (z_1, \ldots, z_n) : \sum_{i=1}^{n} z_i \leqslant Z,\ z_i \geqslant 0,\ i = 1, \ldots, n \right\} \tag{3.26}$$

According to the assumption one has $0<\beta_i<1$, $\varphi_i>0$, $i = 1,\dots,n$.

Since the function (3.25) is strictly concave continuous and monotonically increasing on the set $\mathfrak{R}$, the maximum of I is obtained on the boundary of $\mathfrak{R}$ determined by

$$\sum_{i=1}^{n} z_i = Z \tag{3.27}$$

For the maximization problem (3.25) subject to constraint (3.27) we form the Lagrangean $L(.,.)$ on $E^n \times E^1$ defined by

$$L(z,\lambda) = \sum_{i=1}^{n} \varphi_i z_i^{\beta_i} + \lambda\left(Z - \sum_{i=1}^{n} z_i\right) \tag{3.28}$$

where λ is a Lagrange multiplier, and compute the stationary points $z = \hat{z}$ and $\lambda = \hat{\lambda}$ by solving the equations

$$\frac{L(z,\lambda)}{z_i} = \varphi_i \beta_i z_i^{\beta_i - 1} - \lambda = 0, \quad i = 1,\dots,n \tag{3.29}$$

$$\frac{\partial L(z,\lambda)}{\partial \lambda} = Z - \sum_{i=1}^{n} z_i = 0 \tag{3.30}$$

From (3.29) we obtain

$$Z_i = \frac{1}{(\varphi_i\beta_i)^{\frac{1}{\beta_i - 1}}}\,\lambda^{1/(\beta_i - 1)}, \quad i = 1,\dots,n \tag{3.31}$$

Substituting (3.31) into (3.30) we get a nonlinear equation

$$f(\lambda) = Z - \sum_{i=1}^{n} \frac{1}{(\varphi_i\beta_i)^{1/(\beta_i - 1)}}\,\lambda^{1/(\beta_i - 1)} = 0 \tag{3.32}$$

That equation has a unique solution and the optimum value $\lambda = \hat{\lambda}$ can be derived.

The algorithm for solving the reduced problem of the resources optimal allocation is presented below.

Algorithm

Step 1: Choose an initial Lagrange multiplier λ^0 satisfying $f(\lambda^0) > 0$

Comment: From step 2 to step 5 the equation (3.32) is solved by the Newton method

Step 2: Set $j = 0$

Step 3: Compute a derivative at λ^j

$$f'(\lambda^j) = \sum_{i=1}^{n} \frac{1}{\varphi_i \beta_i (\beta_i - 1)} (\lambda^j)^{(2-\beta_i)/(\beta_i - 1)} \qquad (3.33)$$

Step 4: Compute

$$\lambda^{j+1} = \lambda^j - f'(\lambda^j)^{-1} f(\lambda^j)$$

Step 5: If $f(\lambda^{j+1}) = 0$ set $\lambda = \lambda^{j+1}$ and go to step 6. Otherwise set $j = j+1$ and go to step 3

Step 6: Compute the optimal allocation of resources

$$z_i = \frac{1}{(\varphi_i \beta_i)^{1/(\beta_i - 1)}} \lambda^{1/(\beta_i - 1)}, \quad i = 1, \ldots, n$$

and compute the coefficient of allocation

$$r_i = z_i / Z, \quad i = 1, \ldots, n \qquad (3.34)$$

The above algorithm is used to solve the dynamic allocation problem (3.22)-(3.24) for each period.

At the right hand side of the constraint (3.18) it is possible to replace $Z(t)$ by $\gamma(t) \cdot \bar{\bar{D}}(t-1)$, where $\bar{\bar{D}}(t-1)$ is the national income distributed at the end of the year t-1, and $\gamma(t)$ is the share of productive investments in national income (a decision variable on the first level)

The algorithm is also used to maximize the accumulated global income when the constraints (3.18) are replaced by a cumulative investment constraint as follows:

$$\sum_{t=T_p}^{T_k} \sum_{i=1}^{n} w_i(t)\ z_i(t) \leqslant Z \qquad (3.35)$$

where $w_i(t)$ = discount function.

The above type of constraint is advantageous for the analysis of a credit strategy [B.II].

In this case one finds a Lagrange multiplier by solving the following equation

$$f(\lambda) = Z - \sum_{i=1}^{n}\left(\sum_{t=T_p}^{T_k} \frac{w_i(t)^{\beta_i/(\beta_i-1)}}{\left[\varphi_i(t)\beta_i\right]^{1/(\beta_i-1)}}\right)\lambda^{1/(\beta_i-1)} = 0 \qquad (3.36)$$

and optimal solution is expressed in the form

$$z_i(t) = \left[\frac{w_i(t)\lambda}{\varphi_i(t)\beta_i}\right]^{1/(\beta_i-1)} \qquad (3.37)$$

4. Evaluation of the Model Parameters

Using the data from statistical yearbooks published by the Central Statistical Office it is possible to evaluate the model parameters for the Polish economy. The MR4 development model has been constructed using the aggregation (fifteen sectors) as follows:

1 - Fuels and energy
2 - Metallurgy
3 - Machinery, electric equipment
4 - Chemical industry
5 - Minerals
6 - Wood and paper
7 - Light industry

8 - Food industry
9 - Other industrial branches
10 - Building industry
11 - Agriculture
12 - Forestry
13 - Transport and communications
14 - Trade
15 - Rest of production

Assume that at the time interval $[T_p, T_k]$ the following values have been observed and recorded:

$$\tilde{Y}_{ji}(t),\ j = 0,\dots,n+2,\ i = 1,\dots,n$$

$$\tilde{Y}_i(t),\ i = 1,\dots,n$$

$$\tilde{Z}_{1i}(t),\ i = 1,\dots,n$$

$$\tilde{z}_i(t),\ i = 1,\dots,n.$$

The adaptive model here is used for estimation of the elasticities $\alpha_{ji}(t)$ [B.I]. For the estimation of these parameters the input-output tables have been employed.

4.1. Estimation Parameters of the Production Function

Using the reduced form of the production function

$$Y_i(t) = K_i \exp(\mu_i(t - T_p))Z_{1i}(t),\ i = 1,\dots,n \tag{4.1}$$

parameters μ_i and K_i have been evaluated in the following manner:

From statistical data the indices of global production and capital stock have been obtained

$$\tilde{Y}_i^t = \frac{\tilde{Y}_i(t)}{\tilde{Y}_i(t - 1)},\ i = 1,\dots,n,\ t = T_p+1,\dots,T_k \tag{4.2}$$

$$\tilde{Z}_{1i}^t = \frac{\tilde{Z}_{1i}(t)}{\tilde{Z}_{1i}(t - 1)},\ i = 1,\dots,n,\ t = T_p+1,\dots,T_k \tag{4.3}$$

Since by virtue of (4.) one has

$$\tilde{Y}_i^t = \frac{K_i \exp(\mu_i(t-T_p))\,\tilde{Z}_{1i}(t)}{K_i \exp(\mu_i(t-T_p-1))\tilde{Z}_{1i}(t-1)}, \quad i = 1,\ldots,n \quad t = T_p+1,\ldots,T_k$$

the μ_i become

$$\mu_i = \frac{1}{(T_k - T_p)} \sum_{t=T_p+1}^{T_k} \ln \frac{\tilde{Y}_i^t}{\tilde{Z}_{1i}^t} \tag{4.4}$$

Then it remains to determine the numbers K_i.

For that purpose one can apply the least square method minimalizing the expression

$$\sum_{t=T_p}^{T_k} \left[\tilde{Y}_i(t) - K_i \exp(\mu_i(t-T_p))\tilde{Z}_{1i}(t) \right]^2 \tag{4.5}$$

with respect to K_i. The solution takes the form

$$K_i = \frac{\sum_{t=T_p+1}^{T_k} \exp(\mu_i(t-T_p))\tilde{Z}_{1i}(t)\tilde{Y}_i(t)}{\sum_{t=T_p+1}^{T_k} \left[\exp(\mu_i(t-T_p))\tilde{Z}_{1i}(t) \right]^2} \tag{4.6}$$

4.2. Estimation Parameters of the Capital Stock Equation

The relation between the investment and capital stock takes the following form

$$Z_{1i}(t) = Z_{1i}(t-1) \exp(-\delta_i) + c_i \left[z_i(t-T_{0i}) \right]^{\beta_i} \tag{4.7}$$

$$t = T_p+1,\ldots,T_k, \quad i = 1,\ldots,n$$

The depreciation rate δ_i and the time delay T_{0i} have been obtained from statistical yearbooks. The least squares method has been applied here again, to find the estimates $c_i = \hat{c}_i$, and $\beta_i = \hat{\beta}_i$, which minimize the following expression

$$\sum_{t=T_p+1}^{T_k} \left\{ Z_{1i}(t) - Z_{1i}(t-1) \exp(-\delta_i) - c_i \left[z_i(t-T_{0i}) \right]^{\beta_i} \right\}^2 \tag{4.8}$$

The conjugate-gradient method has been used to minimize the above expression.

4.3. Evaluation Criteria

To compare the data ex post with the values received from the model the following evaluation criteria have been used:
relative annual error:

$$\varepsilon_i(t) = \left[X_i(t) - \tilde{X}_i(t) \right] / \tilde{X}_i(t), \quad t = T_p, \ldots, T_k, \quad i = 1, \ldots, n \tag{4.9}$$

and relative mean square error

$$\eta_i = \frac{1}{(T_k - T_p + 1)\tilde{X}_i(T_p)} \sqrt{\sum_{t=T_p}^{T_k} \left[X_i(t) - \tilde{X}_i(t) \right]^2}, \quad i = 1, \ldots, n \tag{4.10}$$

where: $X_i(t)$ is the value derived from the model; $\tilde{X}_i(t)$ is that from statistical data.

5. Algorithm for the Economic Forecasts and the Optimal Policy

There are two main purposes which were taken into account during the construction of the algorithm. The first one is the examination of impact of decisions taken at I and II level (see [A] Fig. 13 and Fig. 1) on national economy development. The investments among all sectors are assumed as the decisions variables at the first and second level, respectively. The second purpose of the algorithm presented is the computation of optimal investments strategies, according to the goal function (3.13). However, there exists a possibility to examine the national economy development according to given investments strategies, in order to compare them with the optimum strategies.The share of investments in national income can be either computed with the help of the algorithm, according to the previous trend, or it can be treated as given value.

In the second part of the algorithm the economic vôlume indicators are computed for the whole economy as well as for each sector.

5.1. The Description of the Algorithm

1. Data used in the algorithms:

1.1. Forecasting parameters:

n - number of sectors

T_p - initial year

T_h - optimization horizon

$T_k = T_p + T_h$

1.2. Parameters of the model

α_{ji} - technological coefficients; $j = 1,\dots,n+2$, $i = 1,\dots,n$

K_i, μ_i - parameters of the production function, $i = 1,\dots, N$ $\dots, N$

C_i, T_{0i}, δ_i, β_i - parameters of capital stock equation, $i = 1,\dots,n$

ε_i - discount rate, $i = 1,\dots,n$

1.3. Initial values which describe the current state of economy

$Z_{1i}(T_p-1)$ - initial capital stock, $i = 1,\dots,n$

$z_i(t)$ - investment in previous period, $i = 1,\dots,n$, $t = T_p - T_{0i},\dots,T_p-1$

$Y_i(T_p-1)$ - sectors production, $i = 1,\dots,n$

$D(T_p-1)$ - national income

1.4. Exogeneous variables

$PK(t)$ - balance of foreign trade, $t = T_p-1,\dots,T_k$

1.5. Following decision variables have to be given (unless they are computed by the algorithm):

$\gamma(t)$ - investments share in national income

$r_i(t)$ - rate of investments in each sector, $i = 1,\ldots,n$, $t = T_p,\ldots,T_k$

The Algorithm:

1. Compute the income parameters

$$q_i = 1 - \sum_{j=1}^{n+2} \alpha_{ji}, \quad \bar{q}_i = q_i - \alpha_{0i}, \quad i = 1,\ldots,n$$

2. Set $t = T_p$
3. Compute the capital stock

$$Z_{1i}(t) = Z_{1i}(t-1) \exp\left(-\delta_i\right) + c_i\left[z_i(t-T_{0i})\right]^{\beta_i}, \quad i = 1,\ldots,n$$

4. Compute the sector production

$$Y_i(t) = K_i \exp\left(\mu_i(t-T_p)\right) Z_{1i}(t), \quad i = 1,\ldots,n$$

5. Compute the current inputs

$$Y_{ji}(t) = Y_i(t)\alpha_{ji}, \quad j = 1,\ldots,n+2, \quad i = 1,\ldots,n$$

6. Compute the sector wage fund

$$Y_{0i}(t) = Y_i(t)\alpha_{0i}, \quad i = 1,\ldots,n$$

7. Compute the sector income

$$\bar{D}_i(t) = Y_i(t)q_i, \quad i = 1,\ldots,n$$

and the sector profit

$$D_i(t) = Y_i(t)\,\bar{q}_i$$

8. Compute the national income

$$D(t) = \sum_{i=1}^{n} \bar{D}_i(t)$$

and the national income distributed

$$\bar{\bar{D}}(t) = D(t) + PK(t)$$

9. If the productive investments share in national income $\gamma(t)$ is not given compute

$$\gamma(t) = C_p \left[\frac{\bar{\bar{D}}(t-1)}{L(t)}\right]^{a_p(t-T_p)}$$

where C_p, a_p - parameters evaluated from statistical data; $L(t)$ - population

10. Compute the productive investment

$$Z(t) = \gamma(t)\, \bar{\bar{D}}(t-1)$$

11. If the coefficients of investment allocation among the sectors are not given solve the optimal investment allocation problem using the method described in the Sec. 3 (the subprogram OPTIM)
12. Compute the personal income of the population Y_0 (the subprogram KONSUM)
13. Compute the supply of the consumption goods

$$Y_{j0}(t) = Y_0(t)\,\alpha_{j0},\ j = 1,\dots,n$$

14. Set t = t + 1
15. If $t \leqslant T_p+T_h$ go to step 3

Comment: Below the volume indicators are being derived.

16. Compute average wage if the number of workers $ZS_i(t)$ in each sector is given, otherwise go to step 18

$$\bar{P}_i(t) = Y_{0i}(t)/ZS_i(t),\ i = 1,\dots,n,\ t = T_p,\dots,T_k$$

17. Compute average annual growth of wages

$\bar{D}P_i(t) = 100 \cdot \bar{P}_i(t)/\bar{P}_i(t-1)$
and go to 20

18. Compute the current average wages

$$\bar{P}_i(t) = \bar{P}_i(t-1)\ (1 + \bar{DP}_i(t)/100)$$

19. Compute number of workers in each sector

$$ZS_i(t) = Y_{0i}/(12\bar{P}_i(t))$$

20. Compute capital stock per worker

$$T_{ki}(t) = Z_{1i}(t)/ZS_i(t)$$

21. Compute labour productivity

$$W_{pi}(t) = Y_i(t)/ZS_i(t)$$

22. Compute productivity of the capital

$$P_{ri}(t) = Y_i(t)/Z_{1i}(t)$$

23. Print results

5.2. Comments to Algorithm Run

According to (3.21) the values $\varphi_i(t)$ become zero for $t > T_k - T_{0i}$ and as a result no investments are allotted to these sectors in period $t \in (T_k - T_{0i}, T_k)$. That is due to the finite optimization horizon. Since such a situation is unacceptable in real economy it is assumed that when $\varphi_i(t) = 0$ we set $\varphi_i(t) = \varphi_i(t-1)/(1+\varepsilon_i)$.

The algorithm described was implemented in Fortran Extended on the CDC 6400 computer Cyber 72. The present version of the algorithm was used for forecasting the economic development of Poland by MR4 model for 10 year horizon. The present version of the algorithm requires about 70000 (in octal) words of memory and 11 second of central processor time for one execution run.

6. Results of Identification Run

For the identification the statistical data provided by Central Statistical Office (C.S.O. - Główny Urząd Statystyczny)for period 1967-1973 were used. According to Secs. 4.2 and 4.3 parameters of production function and capital stock equation were identified. Results are given in Table 1, where: BETA - β_i, MI - μ_i, C - c_i, K - K_i.

Table 1. Parameters of production function and capital stock equation

SEKTOR	BETA	MI	C	K
1	.34779	.01798	.10319E+04	.49415E+00
2	.47035	.03334	.16443E+03	.11632E+01
3	.31505	.02092	.94767E+03	.17792E+01
4	.40306	-.00398	.28280E+03	.11054E+01
5	.63761	.03203	.24154E+02	.66006E+00
6	.35818	.00599	.23096E+03	.15923E+01
7	.15092	.01799	.14433E+04	.28220E+01
8	.54771	.00930	.89951E+02	.28160E+01
9	.32789	-.05166	.67415E+02	.18762E+01
10	.17373	.01430	.18864E+04	.34231E+01
11	.85893	.03613	.45553E+01	.67052E+00
12	.11877	-.02279	.13019E+04	.85430E+00
13	.82785	.05805	.68453E+01	.25477E+00
14	.10711	.02864	.17359E+04	.17783E+01
15	.45985	.07840	.14408E+03	.37278E+00

For the sake of illustration of the results of identification the parameters for sector No. 3 (machinery and electric equipment) are given in Table 2, where the following notation is used:

ROK - year

MAJ.PROD.MOD. - capital stock according to model

MAJ.PROD.GUS. - capital stock according to C.S.O. data

PROD.GLOB.MOD. - production value according to model

PROD.GLOB.GUS. - production value according to C.S.O. data

BLAD WZGLEDNY - relative annual error

BLAD SREDNIOKWADRATOWY - mean square error

IDENTYFIKACJA NA PODSTAWIE MAJATKU PRODUKCYJNEGO - identification based on capital stock

GUS - C.S.O.

MODELU - model

Table 2. Value of parameters for sector No. 3

SEKTOR NR 3

ROK	MAJ.PROD.MOD.	MAJ.PROD.GUS.	BLAD WZGLEDNY
1968	.130417E+06	.128469E+06	.151598E-01
1969	.143675E+06	.140708E+06	.210879E-01
1970	.157170E+06	.155317E+06	.119289E-01
1971	.170459E+06	.163762E+06	.408960E-01
1972	.184345E+06	.182232E+06	.115942E-01
1973	.200084E+06	.208429E+06	-.400365E-01

BLAD SREDNIOKWADRATOWY = .15073E-01

ROK	PROD.GLOB.MOD.	PROD.GLOB.GUS.	BLAD WZGLEDNY
1968	.229395E+06	.229123E+06	.127455E-01
1969	.256561E+06	.254768E+06	.246138E-01
1970	.289186E+06	.281045E+06	.375360E-01
1971	.311357E+06	.320863E+06	.646285E-02
1972	.353799E+06	.356621E+06	.181693E-04
1973	.413216E+06	.410599E+06	-.373561E-01

BLAD SREDNIOKWADRATOWY = .14525E-01

IDENTYFIKACJA NA PODSTAWIE MAJATKU TRWALEGO MODELU
K = .17792E+01

ROK	PROD.GLOB.MOD.	PROD.GLOB.GUS.	BLAD WZGLEDNY
1968	.229395E+06	.229123E+06	.118525E-02
1969	.256561E+06	.254768E+06	.703737E+02
1970	.289186E+06	.281045E+06	.289676E-01
1971	.311357E+06	.320863E+06	-.296265E-01
1972	.353799E+06	.356621E+06	-.791230E-02
1973	.413216E+06	.410599E+06	.637417E-02

Table 2 - continued

BLAD SREDNIOKWADRATOWY = .96157E-02

IDENTYFIKACJA NA PODSTAWIE MAJATKU TRWALEGO GUS K = .17856E+01

The values of the mean square error were in the range 0.07% -5.7% for capital stock and 1.0%-8.0% for production value.

The results which are given in Table 2 are also shown in Figs. 2 and 3.

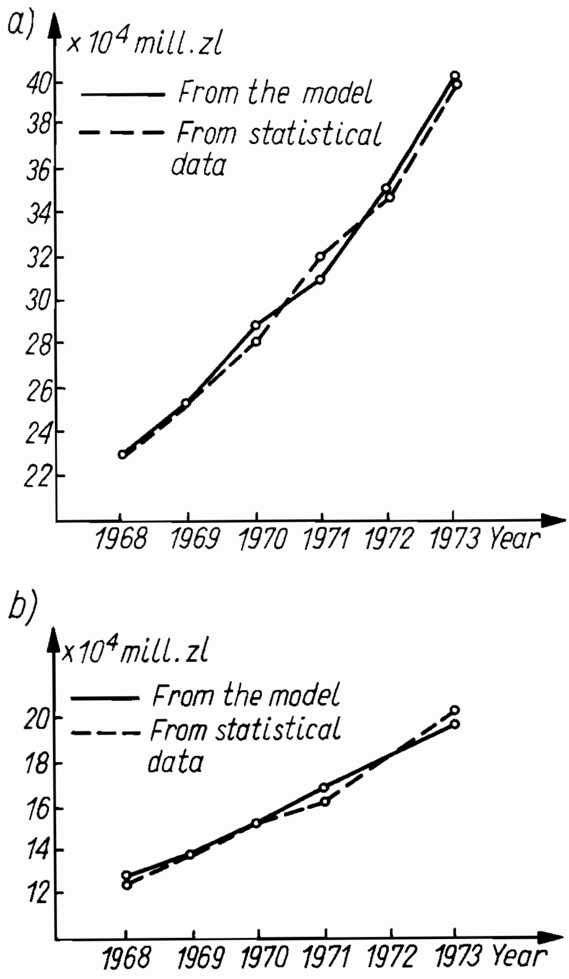

Figure 2. Gross product (a) and capital stock (b) in sector 3 (machinery, electric equipment).

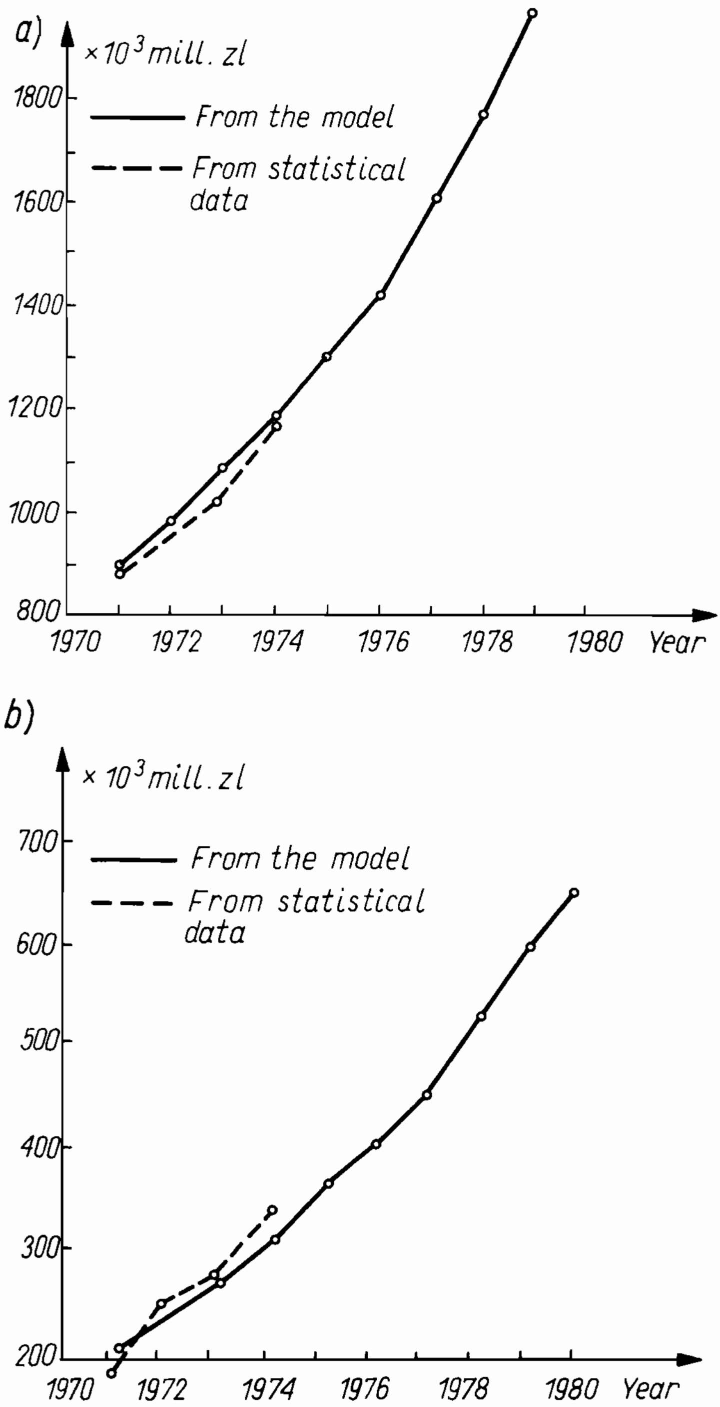

Figure 3. National income (a) and investments in economy (b).

7. Results of Optimization and Forecasting Run

The run described was performed for MR4 model with 10 years optimization horizon, starting with 1971. The results obtained are shown in the following tables.

Table 3. Forecasting parameters and elasticity coefficients

MODEL MR4 OPTYMALIZACJA I PROGNOZA

DANE

LICZBA SEKTOROW N = 16

ROK POCZATKOWY TP = 1971 HORYZONT T = 10

PARAMETRY MODELU

WSPOŁCZYNNIKI ELASTYCZNOSCI STALE

NR SEKTOROW	1	2	3	4	5
1	.105484	.004390	.068859	.009146	.009783
2	.054376	.397719	.056352	.009666	.014940
3	.017398	.114503	.209757	.034643	.006338
4	.037859	.026657	.036397	.182676	.005631
5	.086862	.018472	.054553	.027293	.077943
6	.023029	.010770	.047482	.032429	.011515
7	.009099	.000643	.011504	.026191	.001038
8	.009712	.002203	.013130	.002887	.010663
9	.029338	.077679	.038440	.060732	.004476
10	.012681	.054471	.101977	.018681	.068128
11	.018242	.006972	.028938	.033003	.002972
12	.009724	.002441	.023749	.006352	.012166
13	.074170	.001891	.041837	.027715	.011440
14	.021083	.001518	.026450	.007193	.006568
15	.211061	.039905	.086289	.020247	.023541

NR SEKTOROW	6	7	8	9	10
1	.013643	.008523	.000594	.000667	.041792
2	.003219	.002868	.000541	.001077	.014089
3	.010028	.009433	.001209	.005212	.002132
4	.013270	.027934	.042965	.003702	.002746
5	.026561	.008918	.001696	.005826	.011363
6	.122000	.040354	.002683	.006461	.004080
7	.004672	.251657	.013901	.022087	.000915
8	.005217	.005353	.199585	.002954	.002556
9	.120296	.042234	.001313	.072650	.012534
10	.019705	.007840	.000505	.002785	.086213
11	.003878	.007108	.071394	.000425	.008180
12	.037362	.004833	.016813	.001154	.025101
13	.005805	.008311	.001023	.002814	.032485
14	.027900	.028544	.045916	.009739	.036772
15	.064794	.013109	.007693	.134764	.012660

Table 3 - continued

NR SEKTOROW	11	12	13	14
1	.000077	.000248	.055468	.006177
2	.000051	.000625	.013269	.014318
3	.000049	.000165	.016268	.011837
4	.005716	.002752	.024776	.004750
5	.000715	.001788	.070873	.009281
6	.001951	.182527	.023103	.013724
7	.039381	.000080	.006501	.008120
8	.391513	.000561	.040879	.029307
9	.000486	.002061	.027843	.005418
10	.000299	.003041	.121457	.010507
11	.384481	.000445	.003355	.008087
12	.018223	.044119	.088167	.003268
13	.003027	.000166	.039298	.007677
14	.013404	.000016	.087703	.007281
15	.040056	.000459	.048610	.004189

NR SEKTOROW	15	16	17	18
1	.001996	.266986	.097885	.013567
2	.001462	.101591	.060466	.092828
3	.002582	.151715	.077874	.127756
4	.001761	.090936	.012176	.100619
5	.002469	.242284	.027205	.027233
6	.000816	.185123	.020048	.060706
7	.000847	.128800	.010229	.082630
8	.002050	.062152	.019649	.115577
9	.004887	.241386	.020205	.084884
10	.001132	.288179	.047470	.011144
11	.001157	.299234	.011365	.005318
12	.011711	.287513	.005518	.001967
13	.004581	.363934	.026240	.040889
14	.005190	.244647	.002107	.007586
15	.019772	.224295	.013809	.009173

Table 4. Sector parameters

DANE SEKTOROW

NR	DELTA	BETA	K	TO	WSP.DYSKONTA
1	.0557	.3478	.4942E+00	3	.1000
2	.0344	.4704	.1163E+01	3	.1000
3	.0459	.3151	.1779E+01	1	.1000
4	.0348	.4031	.1105E+01	1	.1000
5	.0492	.6376	.6601E+00	1	.1000
6	.0544	.3582	.1592E+01	1	.1000
7	.0326	.1509	.2822E+01	1	.1000
8	.0582	.5477	.2816E+01	1	.1000
9	.0498	.3279	.1876E+01	1	.1000
10	,0673	.1737	.3423E+01	1	.1000
11	.0312	.8589	.6705E+00	1	.1000
12	.0524	.1188	.8543E+00	1	.1000
13	.0286	.8279	.2548E+00	1	.1000
14	.0202	.1071	.1778E+01	1	.1000
15	.0232	.4599	.3728E+00	3	.1000

Table 5. Initial values of the economy

WIELKOSCI POCZATKOWE

SEKTOR	PRODUKCJA	MAJATEK TRWALY	INWESTYCJE Z UWZGLEDNIENIEM OPOZNIEN		
1	.9493E+05	.25120E+06	.17790E+05	.19003E+05	.20714E+05
2	.5757E+05	.97895E+05	.77023E+04	.88808E+04	.10356E+05
3	.2107E+06	.15532E+06	.16862E+05		
4	.8969E+05	.95807E+05	.10533E+05		
5	.3796E+05	.62714E+05	.63677E+04		
6	.4640E+05	.36597E+05	.29365E+04		
7	.1131E+06	.58858E+05	.52505E+04		
8	.2102E+06	.98499E+05	.63343E+04		
9	.2488E+05	.15684E+05	.88630E+03		
10	.1827E+06	.60163E+05	.83548E+04		
11	.1986E+06	.51135E+06	.33689E+05		
12	.1377E+05	.23049E+05	.83200E+03		
13	.1016E+06	.37468E+06	.25412E+05		
14	.1094E+06	.66395E+05	.49692E+04		
15	.1937E+05	.61270E+05	.18810E+04	.20490E+04	.21500E+04

WARIANT OBLICZEN

WSPOLCZYNNIKI UDZIALU INWESTYCJI PRODUKCYJNYCH W DOCHODZIE (GAM) PROGNOZOWANE
MACIERZ WSPOLCZYNNIKOW PODZIALU INWESTYCJI WYZNACZANA PRZEZ OPTIM
STALE DO PROGNOZY GAM .179000 .007365

ROK	PROGNOZA LUDNOSCI (W MLN)
1971	32.8050
1972	33.0680
1973	33.3630
1974	33.6290
1975	33.8960
1976	34.1630
1977	34.4300
1978	34.6970
1979	34.9640
1980	35.2310

Table 6. Decision and exogeneous variables

ZMIENNE DECYZYJNE

ROK	POZYCZKI	WSP.UDZIALU INW.PROD.W DOCHODZIE
1971	-.17700E+05	.234461
1972	-.13200E+05	.239377
1973	.15000E+04	.247486
1974	.49700E+05	.256453
1975	.90300E+05	.266755
1976	0.	.277638
1977	0.	.286278
1978	0.	.297819
1979	0.	.310327
1980	0.	.323506

WSPOLCZYNNIKI PODZIALU INWESTYCJI

SEKTOR	1	2	3	4	5	6
1971	.02798	.02001	.07771	.03520	.03185	.01614
1972	.02421	.01728	.07056	.03101	.03035	.01452
1973	.02026	.01437	.06309	.02694	.02859	.01287
1974	.01616	.01131	.05544	.02303	.02658	.01122
1975	.01224	.00833	.04892	.01977	.02476	.00984
1976	.00880	.00563	.04609	.01807	.02413	.00920
1977	.00432	.00240	.04170	.01590	.02287	.00827
1978	.05924	.06261	.03615	.01335	.02018	.00712
1979	.12833	.16735	.03060	.01091	.01694	.00598
1980	.01173	.00904	.03206	.01520	.01732	.00680

SEKTOR	7	8	9	10	11	12
1971	.02239	.06190	.00103	.04038	.52864	.00259
1972	.02006	.05652	.00087	.03658	.55244	.00226
1973	.01769	.05101	.00072	.03264	.57731	.00194
1974	.01530	.04541	.00059	.02861	.60303	.00164
1975	.01330	.04060	.00049	.02522	.62460	.00140
1976	.01239	.03835	.00043	.02384	.63200	.00128
1977	.01106	.03508	.00037	.02160	.64650	.00112
1978	.00950	.03029	.00030	.01887	.55847	.00095
1979	.00802	.02520	.00024	.01619	.42082	.00079
1980	.00870	.02996	.00043	.01580	.72184	.00100

SEKTOR	13	14	15
1971	.11931	.01149	.00339
1972	.12998	.01032	.00305
1973	.14083	.00911	.00263
1974	.15163	.00788	.00215
1975	.16203	.00685	.00165
1976	.17225	.00639	.00116
1977	.18260	.00570	.00052
1978	.16789	.00490	.01019
1979	.13785	.00414	.02665
1980	.12418	.00441	.00152

Table 6 - continued

ZATRUDNIENIE W SEKTORACH

SEKTOR	1	2	3	4	5	6	7
1971	.4475E+06	.2337E+06	.1289E+07	.2967E+06	.2862E+06	.2938E+06	.8515E+06
1972	.4579E+06	.2407E+06	.1365E+07	.3111E+06	.2906E+06	.2991E+06	.8727E+06
1973	.4684E+06	.2476E+06	.1440E+07	.3256E+06	.2950E+06	.3043E+06	.8938E+06
1974	.4789E+06	.2545E+06	.1516E+07	.3400E+06	.2994E+06	.3096E+06	.9149E+06
1975	.4893E+06	.2615E+06	.1591E+07	.3546E+06	.3038E+06	.3148E+06	.9360E+06
1976	.4998E+06	.2684E+06	.1667E+07	.3690E+06	.3028E+06	.3200E+06	.9572E+06
1977	.5102E+06	.2754E+06	.1742E+07	.3835E+06	.3126E+06	.3253E+06	.9783E+06
1978	.5207E+06	.2823E+06	.1818E+07	.3980E+06	.3170E+06	.3306E+06	.9994E+06
1979	.5312E+06	.2892E+06	.1893E+07	.4125E+06	.3214E+06	.3358E+06	.1021E+07
1980	.5416E+06	.2962E+06	.1969E+07	.4270E+06	.3259E+06	.3411E+06	.1042E+07

SEKTOR	8	9	10	11	12	13	14
1971	.4668E+06	.1473E+06	.1495E+07	.5413E+07	.1797E+06	.1076E+07	.9676E+06
1972	.4816E+06	.1492E+06	.1564E+07	.5333E+07	.1758E+06	.1118E+07	.9811E+06
1973	.4965E+06	.1510E+06	.1634E+07	.5254E+07	.1720E+06	.1160E+07	.9945E+06
1974	.5113E+06	.1530E+06	.1703E+07	.5174E+07	.1681E+06	.1202E+07	.1008E+07
1975	.5262E+06	.1549E+06	.1772E+07	.5095E+07	.1642E+06	.1244E+07	.1021E+07
1976	.5410E+06	.1567E+06	.1842E+07	.5015E+07	.1604E+06	.1286E+07	.1035E+07
1977	.5558E+06	.1586E+06	.1911E+07	.4936E+07	.1565E+06	.1328E+07	.1048E+07
1978	.5707E+06	.1605E+06	.1980E+07	.4856E+07	.1527E+06	.1370E+07	.1062E+07
1979	.5855E+06	.1624E+06	.2050E+07	.4777E+07	.1488E+06	.1412E+07	.1075E+07
1980	.6004E+06	.1643E+06	.2119E+07	.4697E+07	.1449E+06	.1454E+07	.1089E+07

SEKTOR	15
1971	.4414E+06
1972	.4382E+06
1973	.4341E+06
1974	.4316E+06
1975	.4284E+06
1976	.4251E+06
1977	.4218E+06
1978	.4185E+06
1979	.4153E+06
1980	.4120E+06

Table 4 contains the sector's parameters where the following notation is used:

DELTA - δ_i BETA - β_i K - K_i TO - T_{0i}

WSP. DYSKONTA - ε_i

Table 5 contains initial values of the economy, where

SEKTOR - sector

PRODUKCJA - value of production

MAJATEK TRWALY - capital stock

INWESTYCJE Z UWZGLEDNIENIEM OPOZNIEN - investments according to investments lage

WARIANT OBLICZEN - variant of run

WSPOLCZYNNIKI UDZIALU INWESTYCJI PRODUKCYJNYCH W DOCHODZIE /GAM/ PROGNOZOWANE - the share of investments in national income is forecasted

MACIERZ WSPOLCZYNNIKOW PODZIALU INWESTYCJI WYZNACZANA PRZEZ OPTIM - allocation of investments among sectors derived by subroutine OPTIM

STALE DO PROGNOZY GAM .179000 .007365 - value of paramaters for forecasting of C_p and a_p (see point 9 in algorithm) respectively

ROK - year

PROGNOZA LUDNOSCI /W MLN/ - forecasting of population (in millions)

Table 6 contains decisions and exogeneous variables /ZMIENNE DECYZYJNE/ where

ROK - year

POZYCZKI - balance of international trade

WSP.UDZIALU INW.PROD.W DOCHODZIE - the investments in national income share

SEKTOR - sector number

WSPOLCZYNNIKI PODZIALU INWESTYCJI - rates of investments share for particular sector in each year

ZATRUDNIENIE W SEKTORACH - number of workers in each sector

Table 7 contains results of computation /WYNIKI Z MODELU/ where:

ROK - year

DOCHOD NAROD - national income

INWESTYCJE - investments

KONSUMPCJA - consumption

WSP.PODZ.DOCHODU - the share of investments in national income

STOPA WZROSTU - the rate of national income growth

DOCHOD DZIELONY - national income distributed

MAJATEK TRWALY W SEKTORACH - capital stock in each sector

Table 8 contains the volume indices of the economy (PODSTAWOWE WIELKOSCI EKONOMICZNE I WSKAZNIKI TECHN-EKON GOSPODARKI) where the following notation is used:

1 - gross product (in millions)
2 - capital stock (in millions)
3 - profit of the production subsystem
4 - employment in production subsystem (number of workers)
5 - wages fund in the production subsystem
6 - average monthly wage
7 - average increase of wages
8 - capital stock per workers
9 - productivity of capital stock
10 - value of capital stock on production value unit
11 - labour productivity
12 - net product
13 - gross product (depreciation excluded)
14 - net product (depreciation excluded)
15 - added value

Table 9 contains results of optimization run for one sector (No. 13 - communication and transportation). Following notation is used:

DELTA - δ_{13} K - K_{13} BETA - β_{13} TO - T_{013}

ROK - year

INWESTYCJE - investments

PRODUKCJA - gross production

DOCHOD - sector income

WSP INWEST PROD - sector investments share in total investment fund

PRZEPLYWY Z SEKTOROW - intersector flows

SEKTOR - sector

Table 10 contains the same indices as Table 8 but it is concerned with a particular sector (i.e. No. 13).

The historical run for national income and investments is shown in Figure 3.

The general organization of the programs and subroutines used for MR4 model is shown in Figure 4.

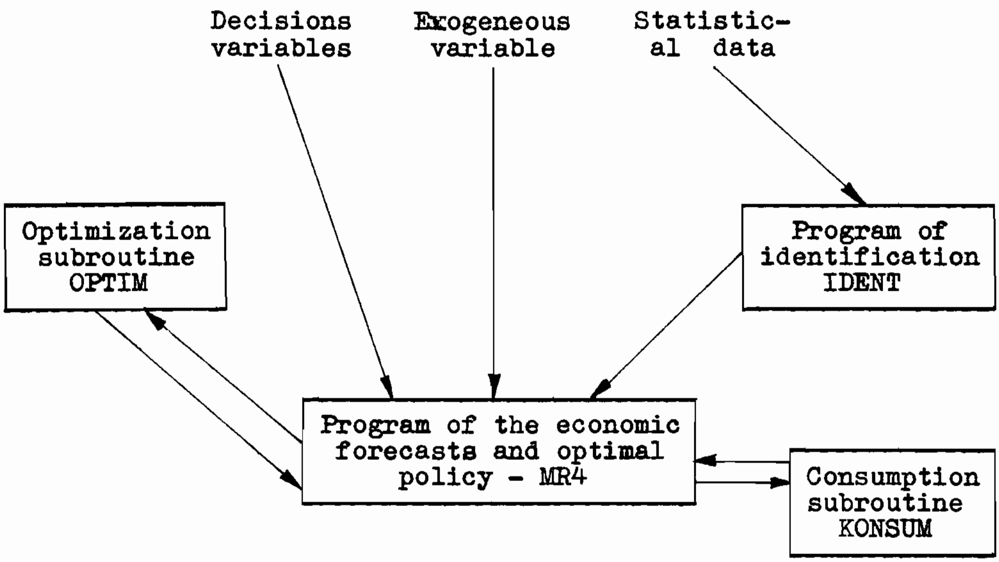

Figure 4. Packet of programs.

Table 7. Results of computation

WYNIKI Z MODELU

ROK	DOCHOD NAROD	INWESTYCJE	KONSUMPCJA	WSP.PODZ.DOCHODU	STOPA WZROSTU	DOCHOD DZIELONY
1971	.90416E+06	.20784E+06	.67862E+06	.23446E+00	.72810E+01	.88646E+06
1972	.99668E+06	.23542E+06	.74806E+06	.23938E+00	.10232E+02	.98348E+06
1973	.10979E+07	.27208E+06	.82731E+06	.24749E+00	.10156E+02	.10994E+07
1974	.12048E+07	.32172E+06	.93278E+06	.25645E+00	.97371E+01	.12545E+07
1975	.13262E+07	.37787E+06	.10387E+07	.26676E+00	.10080E+02	.14165E+07
1976	.14647E+07	.40665E+06	.10580E+07	.27764E+00	.10439E+02	.14647E+07
1977	.16147E+07	.46225E+06	.11524E+07	.28628E+00	.10241E+02	.16147E+07
1978	.17837E+07	.53122E+06	.12525E+07	.29782E+00	.10468E+02	.17837E+07
1979	.19582E+07	.60769E+06	.13505E+07	.31033E+00	.97851E+01	.19582E+07
1980	.21240E+07	.68714E+06	.14369E+07	.32351E+00	.84662E+01	.21240E+07

MAJATEK TRWALY W SEKTORACH

SEKTOR	ROK					
	1970	1971	1972	1973	1974	1975
1	.25120E+06	.26862E+06	.28582E+06	.30306E+06	.30768E+06	.31190E+06
2	.97895E+05	.10566E+06	.11392E+06	.12280E+06	.12693E+06	.13084E+06
3	.15532E+06	.16870E+06	.18121E+06	.19333E+06	.20512E+06	.21663E+06
4	.95807E+05	.10436E+06	.11100E+06	.11741E+06	.12362E+06	.12965E+06
5	.62714E+05	.66136E+05	.69555E+05	.73138E+05	.76936E+05	.81006E+05
6	.36597E+05	.38693E+05	.40875E+05	.42970E+05	.44991E+05	.46953E+05
7	.58858E+05	.62227E+05	.65393E+05	.68470E+05	.71462E+05	.74376E+05
8	.98499E+05	.10380E+06	.11395E+06	.12383E+06	.13353E+06	.14316E+06
9	.15684E+05	.15546E+05	.15183E+05	.14831E+05	.14491E+05	.14164E+05
10	.60163E+05	.65302E+05	.70113E+05	.74653E+05	.78945E+05	.83016E+05
11	.51135E+06	.53089E+06	.61189E+06	.70556E+06	.81616E+06	.94966E+06
12	.23094E+05	.24809E+05	.26290E+05	.27692E+05	.29020E+05	.30281E+05
13	.37468E+06	.39448E+06	.41311E+06	.43687E+06	.46721E+06	.50614E+06
14	.66395E+05	.69386E+05	.71992E+05	.74553E+05	.77071E+05	.79549E+05
15	.61270E+05	.64483E+05	.67808E+05	.71164E+05	.72473E+05	.73774E+05

Table 7 - continued

SEKTOR	ROK 1976	1977	1978	1979	1980
1	.31565E+06	.31878E+06	.32093E+06	.32131E+06	.31841E+06
2	.13446E+06	.13770E+06	.14031E+06	.14182E+06	.14148E+06
3	.22786E+06	.23868E+06	.24919E+06	.25922E+06	.26857E+06
4	.13552E+06	.14111E+06	.14652E+06	.15160E+06	.15623E+06
5	.85339E+05	.89718E+05	.94298E+05	.98737E+05	.10274E+06
6	.48856E+05	.50667E+05	.52418E+05	.54059E+05	.55550E+05
7	.77213E+05	.79961E+05	.82632E+05	.85207E+05	.87673E+05
8	.15271E+06	.16188E+06	.17092E+06	.17936E+06	.18685E+06
9	.13850E+05	.13544E+05	.13249E+05	.12961E+05	.12678E+05
10	.86877E+05	.90514E+05	.14666E+07	.16499E+07	.18003E+07
11	.11081E+07	.12760E+07	.14666E+07	.16499E+07	.18003E+07
12	.31477E+05	.32607E+05	.33678E+05	.34685E+05	.35625E+05
13	.55474E+06	.60939E+06	.67424E+06	.74108E+06	.80169E+06
14	.81986E+05	.84376E+05	.86724E+05	.89020E+05	.91256E+05
15	.75045E+05	.76240E+05	.77269E+05	.77941E+05	.77945E+05

Table 8. Volume indices of the economy

PODSTAWOWE WIELKOSCI EKONOMICZNE I WSKAZNIKI TECH-EKON GOSPODARKI

LEGENDA

1 - PRODUKCJA GLOBALNA (MLN ZL)
2 - MAJATEK TRWALY (MLN ZL)
3 - DOCHOD CALKOWITY PODSYSTEMU PRODUKCJI (MLN ZL)
4 - ZATRUDNIENIE W PODSYSTEMIE PRODUKCJI (L. OSOB)
5 - SUMA PLAC W PODSYSTEMIE PRODUKCJI (MLN ZL)
6 - SREDNIA PLACA (ZL)
7 - SREDNI WZROST PLAC (PROCENT)
8 - TECHNICZNE UZBROJENIE PRACY
9 - PRODUKTYWNOSC SRODKOW TRWALYCH
10 - MAJATKOCHLONNOSC
11 - WYDAJNOSC PRACY
12 - PRODUKCJA CZYSTA
13 - PRODUKCJA GLOBALNA NETTO
14 - PRODUKCJA CZYSTA NETTO
15 - PRODUKT DLA SPOLECZENSTWA

SYMBOL	ROK			
	1970	1971	1972	1973
1	.15110E+07	.21985E+07	.24321E+07	.26856E+07
2	.19695E+07	.20831E+07	.22581E+07	.24503E+07
3	.32957E+06	.46219E+06	.50437E+06	.54944E+06
4	.13693E+08	.13886E+08	.14079E+08	.14271E+08
5	.30689E+06	.44198E+06	.49231E+06	.54845E+06
6	.18677E+04	.26525E+04	.29140E+04	.32026E+04
7	0.	.42021E+02	.98600E+01	.99053E+01
8	.14383E+06	.15002E+06	.16039E+06	.17170E+06
9	.32315E+00	.43405E+00	.44137E+00	.44806E+00
10	.30945E+01	.23039E+01	.22656E+01	.22319E+01
11	.46480E+05	.65114E+05	.70793E+05	.76932E+05
12	.63646E+06	.90416E+06	.99668E+06	.10979E+07
13	.14350E+07	.21179E+07	.23449E+07	.25913E+07
14	.56045E+06	.82356E+06	.90947E+06	.10036E+07
15	.25356E+06	.38158E+06	.41717E+06	.45514E+06

SYMBOL	ROK			
	1974	1975	1976	1977
1	.29571E+07	.32495E+07	.35862E+07	.39476E+07
2	.26456E+07	.28711E+07	.31297E+07	.34011E+07
3	.59522E+06	.64562E+06	.70141E+06	.76091E+06
4	.14465E+08	.14658E+08	.14851E+08	.15044E+08
5	.60958E+06	.68063E+06	.76328E+06	.85376E+06
6	.35119E+04	.38695E+04	.42831E+04	.47293E+04
7	.96559E+01	.10185E+02	.10687E+02	.10419E+02
8	.18290E+06	.19588E+06	.21075E+06	.22608E+06
9	.45539E+00	.46193E+00	.46799E+00	.47475E+00
10	.21959E+01	.21648E+01	.21368E+01	.21064E+01
11	.83293E+05	.90481E+05	.98628E+05	.10733E+06
12	.12048E+07	.13262E+07	.14647E+07	.16147E+07
13	.28499E+07	.31406E+07	.34685E+07	.38208E+07
14	.11036E+07	.12173E+07	.13470E+07	.14880E+07
15	.49406E+06	.53670E+06	.58376E+06	.63419E+06

Table 8 - continued

SYMBOL	ROK 1978	1979	1980
1	.43508E+07	.47633E+07	.51513E+07
2	.37029E+07	.39941E+07	.42371E+07
3	.82648E+06	.89387E+06	.95844E+06
4	.15237E+08	.15430E+08	.15623E+08
5	.95722E+06	.10644E+07	.11656E+07
6	.52352E+04	.57485E+04	.62173E+04
7	.10697E+02	.98036E+01	.81553E+01
8	.24303E+06	.25886E+06	.27121E+06
9	.48170E+00	.49028E+00	.50129E+00
10	.20760E+01	.20396E+01	.19949E+01
11	.11707E+06	.12691E+06	.13596E+06
12	.17837E+07	.19582E+07	.21240E+07
13	.42141E+07	.46171E+07	.49971E+07
14	.16470E+07	.18120E+07	.19699E+07
15	.68980E+06	.74764E+06	.80430E+06

Table 9. Results of optimization run for sector No. 13

SEKTOR 13

DELTA = .0286 K = .2548E+00 BETA = .8279 TO = 1

ROK	INWESTYCJE
1970	.25412E+05

WYNIKI Z MODELU

ROK	PRODUKCJA	DOCHOD	INWESTYCJE	WSP INWEST PROD
1971	.11289E+06	.34622E+05	.24798E+05	.11931
1972	.12529E+06	.38426E+05	.30599E+05	.12998
1973	.14042E+06	.43067E+05	.38317E+05	.14083
1974	.15916E+06	.48813E+05	.48783E+05	.15163
1975	.18273E+06	.56043E+05	.61228E+05	.16203
1976	.21226E+06	.65100E+05	.70047E+05	.17225
1977	.24712E+06	.75791E+05	.84408E+05	.18260
1978	.28977E+06	.88871E+05	.89184E+05	.16789
1979	.33755E+06	.10352E+06	.83772E+05	.13785
1980	.38700E+06	.11869E+06	.85330E+05	.12418

Table 9 - continued

PRZEPŁYWY Z SEKTOROW

SEKTOR	ROK 1971	1972	1973	1974
1	.83727E+04	.92927E+04	.10415E+05	.11805E+05
2	.21347E+03	.23692E+03	.26554E+03	.30097E+03
3	.47228E+04	.52417E+04	.58748E+04	.66586E+04
4	.31286E+04	.34724E+04	.38918E+04	.44110E+04
5	.12914E+04	.14333E+04	.16064E+04	.18208E+04
6	.65330E+03	.72730E+03	.81515E+03	.92390E+03
7	.93819E+03	.10413E+04	.11670E+04	.13228E+04
8	.11548E+03	.12817E+03	.14365E+03	.16282E+03
9	.31766E+03	.35256E+03	.39515E+03	.44787E+03
10	.36671E+04	.40700E+04	.45616E+04	.51702E+04
11	.34171E+03	.37925E+03	.42506E+03	.48177E+03
12	.18739E+02	.20798E+02	.23310E+02	.26420E+02
13	.44362E+04	.49236E+04	.55183E+04	.62545E+04
14	.86662E+03	.96184E+03	.10780E+04	.12218E+04
15	.51713E+03	.57395E+03	.64327E+03	.72910E+03
16	.41083E+05	.45597E+05	.51104E+05	.57923E+05
17	.29621E+04	.32876E+04	.36847E+04	.41763E+04
18	.46158E+04	.51229E+04	.57417E+04	.65078E+04

SEKTOR	1975	1976	1977	1978
1	.13553E+05	.15743E+05	.18329E+05	.21492E+05
2	.34554E+03	.40138E+03	.46730E+03	.54795E+03
3	.76449E+04	.88803E+04	.10339E+05	.12123E+05
4	.50644E+04	.58828E+04	.68489E+04	.80310E+04
5	.20904E+04	.24283E+04	.28270E+04	.33150E+04
6	.10607E+04	.12322E+04	.14345E+04	.16821E+04
7	.15187E+04	.17641E+04	.20538E+04	.24083E+04
8	.18693E+03	.21714E+03	.25280E+03	.29643E+03
9	.51420E+03	.59730E+03	.69539E+03	.81541E+03
10	.59360E+04	.68953E+04	.80277E+04	.94132E+04
11	.55312E+03	.64251E+03	.74803E+03	.87713E+03
12	.30333E+02	.35235E+02	.41022E+02	.48102E+02
13	.71809E+04	.83414E+04	.97113E+04	.11387E+05
14	.14028E+04	.16295E+04	.18971E+04	.22246E+04
15	.83709E+03	.97236E+03	.11321E+04	.13274E+04
16	.66502E+05	.77249E+05	.89935E+05	.10546E+06
17	.47948E+04	.55697E+04	.64844E+04	.76036E+04
18	.74717E+04	.86791E+04	.10104E+05	.11848E+05

Table 9 - continued

SEKTOR	ROK 1979	1980
1	.25036E+05	.28704E+05
2	.63830E+03	.73182E+03
3	.14122E+05	.16191E+05
4	.93551E+04	.10726E+05
5	.38616E+04	.44273E+04
6	.19595E+04	.22466E+04
7	.28054E+04	.32164E+04
8	.34531E+03	.39590E+03
9	.94986E+03	.10890E+04
10	.10965E+05	.12572E+05
11	.10218E+04	.11715E+04
12	.56033E+02	.64242E+02
13	.13265E+05	.15208E+05
14	.25914E+04	.29710E+04
15	.15463E+04	.17729E+04
16	.12285E+06	.14084E+06
17	.88573E+04	.10155E+05
18	.13802E+05	.15824E+05

Table 10. Volume indices for sector No. 13

SEKTOR 13

LEGENDA

1 - PRODUKCJA GLOBALNA (MLN ZL)
2 - MAJATEK TRWALY (MLN ZL)
3 - DOCHOD CAŁKOWITY PODSYSTEMU PRODUKCJI (MLN ZL)
4 - ZATRUDNIENIE W PODSYSTEMIE PRODUKCJI (L.OSOB)
5 - SUMA PLAC W PODSYSTEMIE PRODUKCJI (MLN ZL)
6 - SREDNIA PLACA (ZL)
7 - SREDNI WZROST PLAC (PROCENT)
8 - TECHNICZNE UZBROJENIE PRACY
9 - PRODUKTYWNOSC SRODKOW TRWALYCH
10 - MAJATKOCHŁONNOSC
11 - WYDAJNOSC PRACY
12 - PRODUKCJA CZYSTA
13 - PRODUKCJA GLOBALNA NETTO
14 - PRODUKCJA CZYSTA NETTO
15 - PRODUKT DLA SPOLECZENSTWA

Table 10 - continued

SYMBOL	ROK 1970	1971	1972	1973	1974	1975
1	.10161E+06	.11289E+06	.12529E+06	.14042E+06	.15916E+06	.18273E+06
2	.37468E+06	.39448E+06	.41311E+06	.43687E+06	.46721E+06	.50614E+06
3	.31164E+05	.34622E+05	.38426E+05	.43067E+05	.48813E+05	.56043E+05
4	.10345E+07	.10765E+07	.11184E+07	.11604E+07	.12023E+07	.12443E+07
5	.36980E+05	.41083E+05	.45597E+05	.51104E+05	.57923E+05	.66502E+05
6	.29789E+04	.31804E+04	.33975E+04	.36702E+04	.40147E+04	.44539E+04
7	0.	.67665E+01	.68244E+01	.80260E+01	.93874E+01	.10941E+02
8	.36219E+06	.36646E+06	.36938E+06	.37650E+06	.38860E+06	.40678E+06
9	.18187E+00	.19191E+00	.20349E+00	.21556E+00	.22845E+00	.24212E+00
10	.54984E+01	.52108E+01	.49167E+01	.46391E+01	.43773E+01	.41302E+01
11	.65871E+05	.70328E+05	.75128E+05	.81157E+05	.88776E+05	.98489E+05
12	.68143E+05	.75705E+05	.84023E+05	.94171E+05	.10674E+06	.12254E+06
13	.90910E+05	.10162E+06	.11349E+06	.12794E+06	.14581E+06	.16827E+06
14	.57443E+05	.64438E+05	.72224E+05	.81694E+05	.93392E+05	.10809E+06
15	.20463E+05	.23355E+05	.26627E+05	.30590E+05	.35469E+05	.41588E+05

SYMBOL	1976	1977	1978	1979	1980
1	.21226E+06	.24712E+06	.28977E+06	.33755E+06	.38700E+06
2	.55474E+06	.60939E+06	.67424E+06	.74108E+06	.80169E+06
3	.65100E+05	.75791E+05	.88871E+05	.10352E+06	.11869E+06
4	.12862E+07	.13282E+07	.13701E+07	.14120E+07	.14540E+07
5	.77249E+05	.89935E+05	.10546E+06	.12285E+06	.14084E+06
6	.50050E+04	.56429E+04	.64142E+04	.72501E+04	.80722E+04
7	.12372E+02	.12745E+02	.13669E+02	.13032E+02	.11339E+02
8	.43131E+06	.45883E+06	.49211E+06	.52484E+06	.55137E+06
9	.25660E+00	.27195E+00	.28822E+00	.30546E+00	.32373E+00
10	.38971E+01	.36771E+01	.34696E+01	.32737E+01	.30889E+01
11	.11067E+06	.12478E+06	.14184E+06	.16032E+06	.17850E+06
12	.14235E+06	.16573E+06	.19433E+06	.22637E+06	.25954E+06
13	.19642E+06	.22971E+06	.27051E+06	.31638E+06	.36411E+06
14	.12650E+06	.14832E+06	.17507E+06	.20521E+06	.23664E+06
15	.49256E+05	.58386E+05	.69615E+05	.82360E+05	.95796E+05

B.I. ADAPTIVE MODEL FOR PRICES AND TECHNOLOGICAL CHANGE

K. Borowiecka, J. B. Krawczyk and R. Kulikowski

1. Introduction

As already stated in [A] the main problem in long-term, normative MRI-models is the optimum allocation of resources generated by the economy. From that point of view the MRI can be regarded as a long-term planning model. The planning of allocation of resources affects first of all the supplies of goods generated by the productive sectors. The demands depend on the GNP, i.e. the outcome of planning of supplies, and prices, which are a result of an equilibrium between supplies and demands. Besides the supplies depend on the change of technological coefficients (e.g. resulting from the change of prices) while demands depend on the consumption structure, which again changes along with prices. In other words, the development is a nonstationary process in which the complex interactions between the process and system coefficients (α_{ji}, λ_{ji}) take place. Since the statistical data available are given in monetary form we can identify these coefficients ex post only and indirectly (as shown by formula A.2.4(54)), by taking into account the price indices (which should be determined independently). In order to run the model ex ante we introduce the concept of an adaptive model for nonstationary development processes. According to that concept at each planning interval the technological and consumption-structure coefficients are fixed but may change when a sequence of planning intervals is used. In other words, the model of nonstationary development processes can be approximated by a sequence of stationary models with the coefficients which are constantly readjusted in such a way that the model is constantly adapting to the changing reality.

Such an approach enables us to derive the allocation strategies and price indices in an iterative form. We shall demonstrate that approach by using the MR4 model, described in [B].

2. Computation of Price Indices

As shown in [B], the sector outputs Y_i, $i = 1,\dots,n$, $n = 15$, can be written in the simple form

$$Y_i(t) \;=\; K_i \exp(\mu_i t)\, F_i(z_i) \tag{1}$$

where $F_i(z_i)$ is the productive capital, which depends on the investments intensity $z_i(t)$; K_i, μ_i = constant parameters identified ex post from historical data.

The formula (1) can be used also ex ante to derive the production forecasts within the given planning interval $[0,T]$. For that purpose the optimum strategy $z_i = \hat{z}_i$, determined also in Sec. B, should be set in (1).

The forecasts derived in that manner can be regarded as the supply of output production in monetary units. Since the future prices and technological coefficients are generally unknown at the planning stage ($t = 0$) the forecasts derived in that way do not tell us much about the production volume. By the arguments similar to these in Sec. A.2.3-3.2, we can show however that the term $K_i \exp(\mu_i t)$ can be regarded as a proxy for the neutral progress and

$$\pi_i = \prod_{j=0}^{n+2} (\alpha_{ji}/p_j)^{\alpha_{ji}/q_i}\, p_i^{1/q_i} \tag{2}$$

$$q_i = 1 - \sum_{j=0}^{n+2} \alpha_{ji} > 0, \quad i = 1,\dots,n$$

where: p_i = aggregated sector prices (p_0 = average wage); α_{ji} = technological coefficients; $n+1$, $n+2$ - correspond to the sector inputs from two foreign markets.

It should be observed that the production function (1) depends explicitely on the single production factor z_i only. However, the data regarding the rental prices of capital (ω_1) which was used in Sec. A2.3-3.2 are not available. For that reason we use a simplified expression (2) rather assuming that ω_1 and the rest of development factors (i.e. z_ν, $\nu = 3,\dots,m$) contribute to the output, i.e.

$$\ln \frac{\pi_i(t)}{\pi_i(t-1)} \triangleq \ln \pi_i^t = \bar{\mu}_i, \quad i = 1,\dots,n \tag{3}$$

where generally $\mu_i = \bar{\mu}_i + \tilde{\mu}_i$, and $\tilde{\mu}_i$ is the term responsible for the factors different from p_i, $j = 0,1,\dots,n+2$.

Since μ_i have been identified ex post (in Sec. B) the problem consists in the identification of $\tilde{\mu}_i$, $i = 1,\dots,n$. For that purpose the ex post data regarding price indices p_i^t, $i = 0,1, 2, \dots, n+2$, can be used. Using the data supplied by Central Statistical Office (GUS), and corresponding technological coefficients (α_{ji}) the parameters $\tilde{\mu}_i$, $i = 1,\dots,n$, have been derived. In Table 1 the derived μ_i, $\tilde{\mu}_i$, $i = 1,\dots,15$, parameters for MR4-model are given.

Table 1. The values of μ_i, $\tilde{\mu}_i$ coefficients

Sector No.	μ_i	$\tilde{\mu}_i$	Sector No.	μ_i	$\tilde{\mu}_i$
1	0.01798	0.0556	9	-.05166	.0098
2	.03334	0.01218	10	.01430	.07443
3	.02092	.14163	11	.03613	.19929
4	-.00398	.10273	12	-.02279	.02298
5	.03203	.08303	13	.05805	.06245
6	.00599	-.00020	14	.02864	.05791
7	.01799	.07485	15	.07840	.21817
8	.00930	.27304			

The negative values of μ_i observed in some sectors are mainly due to the decreasing price effects,which took place in Polish economy.

The model of price indices (for MR4) can be also written in the form

$$\ln p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ln p_j^t = q_i \bar{\mu}_i + \alpha_{oi} \ln p_o^t + \alpha_{n+1,i} \ln p_{n+1}^t + \alpha_{n+2,i} \ln p_{n+2}^t \qquad (4)$$

$$i = 1,\dots,n$$

where p_o^t = average wage index; p_{n+1}^t, p_{n+2}^t = prices for imported goods.

Using the terms of trade instead of p_{n+1}^t, p_{n+2}^t one gets (see Sec. A.3.2) the equivalent formulae:

$$(1 - \alpha_{n+1,i} - \alpha_{n+2,i}) \ln p_i^t - \sum_{j=1}^{n} \alpha_{ji} \ln p_j^t =$$

$$= q_i \bar{\mu}_i + \alpha_{oi} \ln p_o^t - \alpha_{n+1,i} \ln T_{n+1,i}^t - \alpha_{n+2,i} \ln T_{n+2,i}^t$$

$$i = 1,\ldots,n \qquad (5)$$

where $T_{n+j,i}$ = export to import price ratio or the so called "terms of trade", $j = 1, 2$.

It should be observed that price indices p_i^t, $i = 1,\ldots,n$, derived by (5) for given p_o^t, $T_{n+1,i}^t$, $T_{n+2,i}^t$ are positive. When p_o^t, $T_{n+1,i}^t$, $T_{n+2,i}^t$ are constant the derived p_i^t = const., $i = 1,\ldots,n$, $t \in [T,0]$.

The model described by (5) has been checked for accuracy in historical runs in the time period 1967-1973. The accuracy obtained for most of the sectors was arround 8%, which is rather good, in view of the fact that we deal with highly aggregated price, and terms of trade indices.

The next problems concerns the situation in which the price model (4),(5) is used ex ante. In that case the p_o^t, $T_{n+j,i}^t$, $i = 1,\ldots,n$, $j = 1, 2$, $t = 1,2,\ldots$, should be regarded as exogeneous parameters.

Since the average price p_o depends in MR4 on the sector employment X_{oi}, and output $\hat{Y}_i$ (see Sec. B), i.e.

$$p_o = \alpha_{oi} \hat{Y}_i / X_{oi}$$

the p_o^t depends mainly on the future employment rate, i. e. the demographic factors.

The econometric forecasts for future terms of trade can be also used. They are, however, not very reliable when long-term projections are needed. For that reason the method of investigation of different terms of trade-scenarios seems to be most appropriate.

The technological coefficients α_{ji}, which are assumed to be constant within the identification period, not necessary should keep the same values within the projection interval. On the opposite, in order to get an accurate description of real processes they should change according to the observed $\alpha_{ji}(t)$ trends. Using the adaptive approach, described in Sec. A.2.4 , (eq. (50)), we can use the values

$$\alpha_{ji} = \alpha_{ji}(0) \exp\left(-a_{ji} \frac{T}{2}\right), \quad j,i = 0,1,\dots,n+2 \qquad (6)$$

where $\alpha_{ji}(0)$ = the known estimates of α_{ji} at the initial moment of the planning interval $[0,T]$, as the expected model parameters within $[0,T]$. Obviously, for different T or - when the "moving planning technique" (with $[K,K+T]$, $K = 0,1,2,\dots$ sequences of intervals) is used - the α_{ji}, described by (6) are different and consequently the price indices derived by (4),(5) change.

As far as the values of μ_i are concerned it is necessary to observe that under the assumption of $\tilde{\mu}_i \approx$ const., $t \in [0,T]$, the changes of μ_i in formula (1) can be attributed to the price change, i.e. the change of $\bar{\mu}_i$.

On the other hand, using (1) in forecasting runs the μ_i should be regarded as expected price rates. There is no way to prove that the expectations regarding the future price rates were correct unless we compare the supply of production (expressed by (1)) with the demands for the outputs. To do that we can use the formulae (32), (33) of Sec. A.2.3:

$$\bar{Y}_i(t) = \sum_{\nu=1}^{m} \lambda_{\nu i} Z_\nu(t), \quad Z_\nu(t) = \gamma_\nu Z(t) \qquad (7)$$

which express the net sector productions $\bar{Y}_i$ by the expenditures $Z_\nu(t)$, i.e. - factors endowments.

Using the obvious relation

$$\underline{Y} = \left[I - A\right]^{-1} \bar{\underline{Y}} \triangleq Z \underline{1}$$

and (7) one can express the demands D_i for sectorial products in the form:

$$D_i(Y) = Y_i(t) = \sum_{\nu=1}^{m} l_{i\nu}\gamma_\nu Z(t) = Y(t-1)\sum_{\nu=1}^{m} l_{i\nu}\gamma_\nu \qquad (8)$$

$$i = 1,\dots,n$$

where $l_{i\nu}$ coefficients can be easily derived when the technological coefficients $\{\alpha_{ji}\},\{\lambda_{\nu i}\}$ are known.

In MR4 a relatively simple demand model has been used. There are m = 4 spheres of allocation of G.N.P.: Z_1 = investments, Z_2 = personal consumption, Z_3 = aggregated (social) consumption, Z_4 = stock of reserves (inventories).

It is assumed that γ_ν depend on G.N.P. per capita in constant prices, i.e.

$$\gamma_\nu = a_\nu (Y/pN)^{\varepsilon_\nu}, \quad \nu = 1,\dots,4 \qquad (9)$$

where N = population (exogeneous variable), p = aggregated price for the whole economy (supplied from the MRO model).

Table 2. Parameters of $\gamma_\nu = a_\nu(Y/pN)^{\varepsilon_\nu}$ econometrical model

	$\nu = 1$	$\nu = 2$	$\nu = 3$	$\nu = 4$
a_ν	0.09414	1.53428	0,06752	0.00554
ε_ν	0.35653	-0.31229	0.07845	0.70753
$\max \frac{\gamma_\nu - \tilde{\gamma}_\nu}{\gamma_\nu}$	3%	3%	5%	15%

Source: The C.S.O. Statistical Yearbook 1975. Data 1967-1974.

In Table 2 the a_ν, ε_ν parameters estimated from the statistical (GUS 1967-1973) data are given.

The technological coefficients ($\alpha_{ji},\lambda_{\nu i}$), which are needed for computation of $l_{i\nu}$ coefficients in (8), can be derived by the prediction formulae (6). Then, given an optimum investment strategy $z_i(t) = \hat{z}_i^o(t)$, $i = 1,\dots,n$, $t \in [0,T]$; the supplies

$Y_i(\hat{z}_i^o) = S_i(\hat{z}_i^o)$ and demands $D_i(Y^o)$, $Y^o = \sum_{i=1}^{n} Y_i(\hat{z}_i^o), i = 1,\dots,n,$

can be derived. Since, generally speaking, the demands may not coincide with supplies in Sec. A.2.3 an iterative approach for determination of optimum planning strategy has been proposed. According to that approach $\hat{z}_i^o$, $i = 1,\dots,n$, should be regarded as the first step, in the sequence $\hat{z}_i^{(k)}$, $k = 0,1,\dots,$of approximations of optimum planning strategies, which should converge to the strategy $\hat{z}_i$, $i = 1,\dots,n$, yielding the equilibrium between demands and supplies. The sequence of iterations enables us also to compute the corresponding $\pi_i^t(\hat{z}_i^k) = \pi_i^t(k)$, $k = 0,1,\dots,$ terms by the formulae (39) of Sec. A.2.3:

$$\pi_i^t(k) = \frac{D_i^t(Y^k)}{S_i^t(\hat{z}_i^k)} \pi_i^t(k-1), \quad k = 1,2,\dots \tag{10}$$

where $\pi_i^t(0) = \bar{\mu}_i$, $i = 1,\dots,n$.

At the second step one gets

$$\pi_i^t(1) = \frac{D_i^t\left[Y^o\right]}{S_i^t\left[\hat{z}_i^o\right]} \exp \bar{\mu}_i, \quad i = 1,\dots,n$$

Then, the supplies necessary to satisfy the equilibrium at the second step, become

$$Y_i^1(t) = K_i(1) \exp \left[(\ln \pi_i^t(1) + \tilde{\mu}_i)t\right] F_i(z_i), \; i = 1,\dots,n \tag{11}$$

where $K_i(1)$ are the corrected estimates of K_i.

Now we can solve again the optimum allocation of investment problem with the production functions (11). That yields the new set of $z_i = \hat{z}_i^1$, $i = 1,\dots,n$, strategies,which determine in turn the new set of supplies $S_i^t(\hat{z}_i^1)$, $i = 1,\dots,n$, etc. During that process the value of G.N.P. (Y^o) may change to the new value (Y^1) so the demans $D_i(Y^1)$, generally speaking, are different from $D_i(Y^o)$, $i = 1, \ldots, n$.

However, it can be shown that D_i do not change much and as a result one gets usually a fast convergence of the iterations (10). As a consequence the iterations $\hat{z}_i^{(k)}$, $i = 1,\ldots,n$, $k = 0, 1,2,\ldots$, converge fast to the strategy $\hat{z}_i$, which satisfies the equilibrium condition $D_i(Y) = S_i(\hat{z}_i)$, $Y = \lim_{k \to \infty} Y^k$, $\hat{z}_i = \lim_{k \to \infty} \hat{z}_i^k$, $i = 1,\ldots,n$.

It should be observed that the values $\bar{\mu}_i = \pi_i^t = \lim_{k \to \infty} \pi_i^t(k)$, $i = 1,\ldots,n$, enable us to derive the equilibrium price indices p_i^t, $i = 1,\ldots,n$, $t \in [0,T]$ by formulae (4), (5).

The iterative computations for the more general model, in which γ_ν, $\nu = 1,\ldots,m$, and D_i, $i = 1,\ldots,n$, depend on sectorial price indices p_i^t in an explicit form, are more complicated. The flow diagram, shown in Fig. 1 , illustrates the sequence of computations which can be used in that case.

Obviously, the quality of adaptive models under consideration depends much on the accuracy of estimation and prediction of technological coefficients. That problem will be discussed in the next section.

3. Estimation of Technological Coefficients

In order to estimate the technological coefficients for the purpose of construction of MR3 and MR4 models, the 15×15 input-output tables for Polish economy (GUS 1967-1973) have been used.

The data representing the flows $Y_{ji}(t)$, $i = 0,\ldots,17$, $j = 1, \ldots,15$, have been used twice. To derive the values of ϱ_i, a_{ji} and then b_{ji} (by formula (47) of Sec. A.2.4) in the identification interval (1967-1973) at the first stage. For that purpose the standard least squares method has been used. Then - starting with $T_1 = 1970$ and using the formula (48) of Sec. A.2.4 - the estimates for $\alpha_{ji}(1973)$ have been found. The values a_{ji} (in formula (50) of Sec. A.2.4) used for prediction purposes were

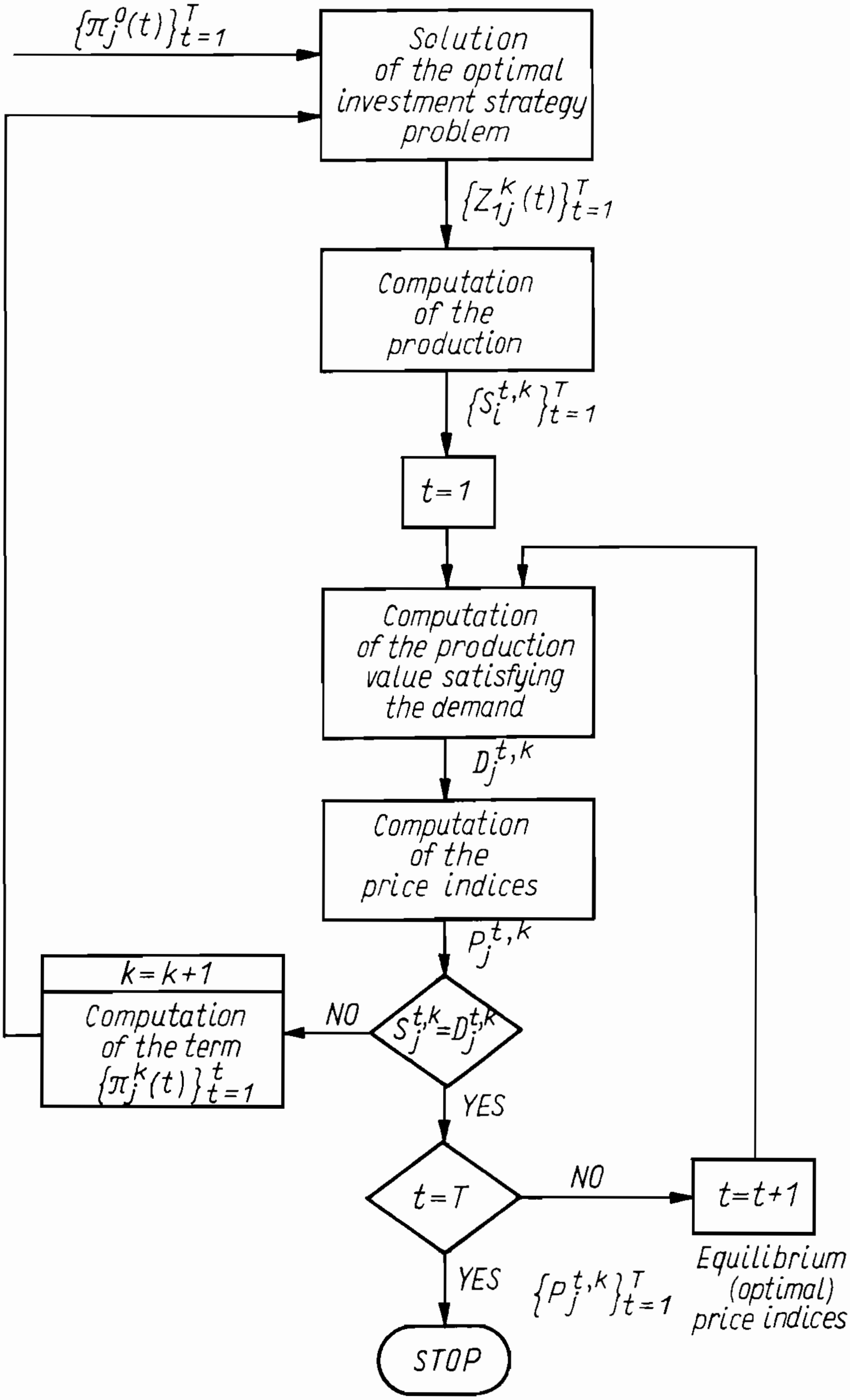
{πj0(t)} T t=1
Solution of the optimal investment strategy problem
{Z1jk(t)} T t=1
Computation of the production
{Sit,k} T t=1
t=1
Computation of the production value satisfying the demand
Djt,k
Computation of the price indices
Pjt,k
k=k+1
Computation of the term {πjk(t)} t t=1
NO
Sjt,k=Djt,k
YES
t=T
NO
t=t+1
YES
{Pjt,k} T t=1
Equilibrium (optimal) price indices
STOP

Figure 1.

also derived. The results of these computations are given in Tables 3, 4, 5.

In Table 6 the estimates of $\lambda_{\nu i}$ coefficients are given *).

At the second stage it was possible to compare the ex post accuracy of prediction of the $\hat{\alpha}_{ji}(t)$ with the data $\tilde{\alpha}_{ji}(t)$, t = 1971, 1972, 1973. The accuracy was generally satisfactory , For example: let us consider technological coefficients $\alpha_{11}(t)$ and $\alpha_{12}(t)$. The statistical data in the identification interval (1971-1973) are as follows:

$$\tilde{\alpha}_{11}(1971) = 0.189050 \qquad \tilde{\alpha}_{12}(1971) = 0.097624$$
$$\tilde{\alpha}_{11}(1972) = 0.162843 \qquad \tilde{\alpha}_{12}(1972) = 0.092624$$
$$\tilde{\alpha}_{11}(1973) = 0.160493 \qquad \tilde{\alpha}_{12}(1973) = 0.083322$$

while the values of technological coefficients, obtained by adaptive model become:

$$\hat{\alpha}_{11}(1971) = 0.181586 \qquad \hat{\alpha}_{12}(1971) = 0.098903$$
$$\hat{\alpha}_{11}(1972) = 0.174933 \qquad \hat{\alpha}_{12}(1972) = 0.092568$$
$$\hat{\alpha}_{11}(1973) = 0.167593 \qquad \hat{\alpha}_{12}(1973) = 0.086686$$

To compare the data ex post with the values received from adaptive model let us introduce:

a) relative error in one year

$$\Delta\alpha_{ji}(t) = \left[\hat{\alpha}_{ji}(t) - \tilde{\alpha}_{ji}(t)\right] / \tilde{\alpha}_{ji}(t) \cdot 100\%$$

and

b) relative mean square error

$$\Delta_S \alpha_{ji} = \frac{1}{(T_0 - T_1 + 1)\,\tilde{\alpha}_{ji}(T_1)} \sqrt{\sum_{t=T_1}^{T_0} \left[\hat{\alpha}_{ji}(t) - \tilde{\alpha}_{ji}(t)\right]^2}$$

The following values have been obtained:

*) The $\lambda_{j1} = 0$ for some of the sectors (i = 2,4,5,...) indicate that the goods necessary for capital investment enter the economy through the other sectors (e.g. trade and construction).

Table 3. The values of the coefficients $\hat{\alpha}_{ji}$ (1973) and ρ_i for j = 0,...,17, i = 1,...,15 (by formula 18))

j \ i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	.8336	.7192	.8071	.9426	.7442	.7898	.8682	1.2174	.5158	.8502	.9074	.6849	.8413	.9419	.8967
1	.5559	.3707	.7106	.4230	.9630	.8553	.8400	1.1156	1.3560	1.0421	1.8709	2.0476	1.1191	1.0217	1.9484
2	.5146	.9171	.7556	.6793	.1045	.4621	1.0000	1.0000	2.4281	.9915	2.0229	1.0000	1.0000	1.0000	2.1581
3	.9705	1.3085	.9967	1.4746	1.0784	1.0110	.7875	1.2106	1.9234	.9540	1.4717	.5865	.9915	.8223	1.9554
4	.4039	1.5049	.8273	1.1289	1.0309	.9641	.4118	.7902	1.1191	.8112	1.3391	.5195	1.2718	1.0656	1.6112
5	.2569	.8417	.4203	-.1108	1.0210	.6280	1.0000	1.0243	1.0000	.8141	1.0000	1.2713	1.5616	.9245	.9308
6	.5252	1.1266	.8211	.5247	.9922	.8998	.3038	.1068	1.8566	.5305	1.0000	1.9888	.4461	1.0455	-.2440
7	1.1420	1.2333	1.1623	.8211	.5719	1.3313	.7714	1.1676	.7596	1.0978	1.6771	1.1605	.7709	1.4893	1.3068
8	1.0000	1.0000	1.0000	.4651	1.0000	1.0000	2.9517	1.3861	1.0000	1.0000	1.0000	1.3399	1.0000	2.7807	1.0000
9	1.0000	1.0000	.9208	2.6019	1.8836	1.0000	2.3984	1.0000	2.1593	1.0000	2.0734	1.0000	1.0000	.5429	1.9524
10	1.1663	.6706	1.0000	1.0000	.3032	1.0000	1.0000	1.0000	2.6427	1.5886	2.2036	2.5487	3.0000	2.6309	3.0000
11	1.0000	1.0000	1.0000	.3769	1.0000	1.0000	1.1033	1.1880	1.0000	1.0000	.9384	1.7508	1.0000	-.4327	1.2616
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0490	1.0000	1.0000	1.0000	.8168	1.0000	1.7548	1.0000	1.0000	1.0000
13	.7858	1.1634	1.2620	1.3809	.9882	.9334	.8532	1.3164	.7594	.8684	1.0000	-.7000	1.4331	.3973	3.0000
14	1.0000	.8112	.6251	1.9259	1.1280	1.0000	1.0000	-.7000	-.3382	1.0000	.3861	1.0000	.7449	.5773	.5922
15	1.0000	1.0000	1.0000	1.0000	.9135	1.0000	1.0000	1.0000	1.2222	1.0000	1.0000	1.0000	.9435	1.2180	1.0000
16	1.0000	1.5870	2.0360	1.0857	3.0000	2.1601	2.2128	2.6774	2.0859	1.0000	1.6016	1.0000	2.6921	1.0000	.8426
17	1.7540	.5624	1.4371	.9182	2.1525	3.0000	.0071	-.3559	.5437	1.5849	1.6899	1.0000	-.0078	1.0000	.2438

Table 4. The values of the coefficients α_{ij} for j = 0,...,17, and i = 1,...,15 (by formula (20))

j \ i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	.0148	.0300	.0199	.0028	.0228	.0148	.0098	-.0174	.0593	.0199	.0048	.0241	.0176	.0082	.0135
1	.0429	.0657	.0295	.0540	.0014	.0115	.0098	-.0104	-.0442	.0015	-.0626	-.0737	-.0170	.0061	-.0818
2	.0272	.0095	.0248	.0324	.0806	.0418	.0247	-.0022	-.1000	.0008	-.0778	-.0002	.0774	.0205	-.0710
3	-.0011	-.0188	.0035	-.0439	-.0085	-.0000	.0171	-.0158	-.0163	.0065	-.0286	.0388	-.0029	.0212	-.0441
4	.0545	-.0606	.0204	-.0127	.0009	.0010	.0494	.0157	-.0212	.0257	-.0252	.0378	-.0365	.0005	-.0118
5	.0653	.0074	.0666	.1026	-.0050	.0265	-.0217	-.0094	-.0219	.0262	.0166	-.0183	-.0707	.0135	.0297
6	.0420	-.0215	.0152	.0431	-.0011	.0046	.0556	.0680	-.0528	.0551	-.0141	-.0654	.0654	.0041	.1234
7	-.0126	-.0377	-.0246	.0125	.0437	-.0279	.0184	-.0215	.0190	-.0104	-.0497	-.0131	.0235	-.0376	-.0203
8	.0120	-.0025	-.0212	.0463	-.0267	-.0060	-.1000	-.0313	-.0103	-.0444	.0242	-.0229	.0338	-.1000	.0438
9	.0007	-.0276	-.0191	-.1000	-.0524	.0261	-.1000	.0305	-.1000	.0079	-.0755	-.0237	-.0086	.0418	-.0687
10	-.0047	.0435	.0318	.0168	.0661	.0189	.0370	.0152	-.1000	-.0985	-.0680	-.0957	-.1000	-.1000	-.1000
11	.0091	-.0029	.0080	.0657	-.0025	.0137	-.0125	-.0194	-.0383	.0873	.0060	-.0461	-.0116	.1172	-.0075
12	-.0567	.0279	-.0366	.0216	.0032	-.0080	.0488	.0132	-.0892	.0492	-.0834	-.0591	.0408	.0598	-.0009
13	.0155	-.0185	-.0324	-.0309	.0006	.0054	.0141	-.0323	.0317	.0118	-.0237	.1235	-.0508	.0549	-.1000
14	-.0165	-.0087	.0334	-.1000	-.0093	.0381	-.0091	.1307	.1788	.0338	.0495	.0230	.0243	.0277	.0319
15	-.0091	-.0234	-.0164	-.0185	.0153	.0076	-.0008	-.0102	-.0268	-.0259	.0651	-.0420	.0133	-.0059	.0768
16	-.0308	-.0610	-.1000	-.0023	-.1000	-.0589	-.0963	-.0818	-.1000	-.0826	-.0580	-.0070	-.1000	-.0244	.0095
17	-.0695	.0474	-.0475	.0141	-.1000	-.1000	.0800	.1030	.0494	-.0735	-.0451	.0257	.1145	.0201	.0650

Table 5. The values of the coefficients b_{ji} for $i = 1,\dots,15$ and $j = 0,\dots,17$ (by formula (17))

j \ i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	.210172	.093921	.164192	.101467	.214721	.146220	.143820	.060891	.338575	.247707	.333747	.287475	.304172	.245626	.238902
1	.167593	.086686	.019153	.067612	.105107	.027752	.011822	.013754	.022289	.021216	.015908	.011763	.099914	.015845	.120271
2	.003753	.449083	.144955	.014760	.025157	.004963	.000922	.002151	.040572	.051149	.005938	.002455	.004388	.001798	.020118
3	.052097	.047510	.245244	.035158	.052306	.036031	.011659	.015842	.034818	.120781	.032990	.020342	.078978	.019602	.042298
4	.014259	.014462	.035099	.196597	.026202	.037878	.053490	.005119	.053355	.024061	.030603	.008510	.019094	.006291	.010182
5	.009194	.019383	.005260	.007751	.098806	.005427	.000816	.007900	.003915	.115800	.006101	.011525	.009263	.005572	.010775
6	.015386	.003093	.014109	.016178	.039121	.183857	.006843	.008262	.086414	.031325	.002531	.032145	.005833	.020537	.034791
7	.010223	.005018	.011228	.041049	.011606	.036071	.319711	.004850	.047240	.011501	.004064	.006564	.009938	.029517	.012198
8	.000507	.000518	.001014	.019221	.001295	.002219	.023205	.201434	.001233	.000373	.009668	.008812	.001453	.027718	.009544
9	.000769	.000915	.003966	.011186	.004048	.009043	.014390	.004864	.037945	.002960	.036361	.001468	.002647	.007450	.075525
10	.025260	.007350	.002815	.003287	.009045	.003475	.001564	.003069	.006546	.034257	.011157	.044679	.019917	.024684	.016674
11	.000083	.000048	.000051	.004300	.000689	.002119	.025942	.378268	.000383	.001067	.398012	.012756	.002755	.008829	.020015
12	.000160	.000811	.000130	.003584	.001835	.185265	.000124	.000695	.001160	.003835	.000218	.024145	.000258	.000045	.000459
13	.058596	.019522	.020481	.032467	.079721	.030158	.009727	.026399	.033507	.081347	.002485	.055035	.050110	.098187	.023376
14	.005856	.021913	.010342	.019760	.011573	.010025	.007284	.027666	.013563	.014325	.012085	.004009	.016386	.006641	.007677
15	.001799	.001204	.002300	.001491	.002720	.000893	.000834	.001812	.004144	.000939	.009148	.008566	.004283	.003544	.057728
16	.007971	.051602	.054885	.095593	.030710	.045961	.086225	.052373	.044336	.006734	.004959	.001980	.037767	.003574	.008453
17	.063183	.090064	.060553	.035265	.033761	.037104	.020298	.018372	.026853	.021965	.012261	.008479	.021080	.002570	.011827
ρ_i	.090971	.106485	.114854	.092569	.089753	.075298	.080632	.076149	.127999	.119312	.074090	.079455	.113739	.085360	.088972

Table 6. The values of $\lambda_{\nu i}$ coefficients

Sector	$\nu=1$ (capital investments)	$\nu=2$ (individual consumption)	$\nu=3$ (social and aggregated consumption)	$\nu=4$ (stock of reserves)
1	0.0	0.02874	0.11007	0.01185
2	0.0	0.00017	0.00506	0.07736
3	0.37458	0.05763	0.14595	0.43004
4	0.0	0.02688	0.12914	0.08589
5	0.0	0.00554	0.01753	0.02532
6	0.00184	0.02735	0.03452	0.03514
7	0.0	0.14430	0.04816	0.14650
8	0.0	0.35620	0.13859	0.04607
9	0.0	0.00829	0.02134	0.01366
10	0.61272	0.00375	0.03053	0.00394
11	0.00280	0.15731	0.02276	0.09732
12	0.0	0.00216	0.00126	0.00099
13	0.00163	0.03625	0.10876	0.00919
14	0.00644	0.13563	0.08560	0.00971
15	0.0	0.00982	0.04073	0.00703

$$\Delta\alpha_{11}(1971) = -3.9\% \qquad \Delta\alpha_{12}(1971) = 1.3\%$$
$$\Delta\alpha_{11}(1972) = 7.4\% \qquad \Delta\alpha_{12}(1972) = -0.06\%$$
$$\Delta\alpha_{11}(1973) = 3.9\% \qquad \Delta\alpha_{12}(1973) = 4.0\%$$

and

$$\Delta_S\alpha_{11} = 0.028 \qquad \Delta_S\alpha_{12} = 0.012$$

Than one can see that accuracy obtained is rather good. The values of the coefficients (by relations (47) and (50) of Sec. A.2.4) are as follows:

$$\bar{a}_{11} = 0.0404 \qquad \bar{a}_{12} = 0.0670$$
$$\rho_1 = 0.0910 \qquad \rho_2 = 0.1065$$
$$b_{11} = 0.5559 \qquad b_{12} = 0.3707$$
$$a_{11} = 0.0429 \qquad a_{12} = 0.0656$$

The predicted values which one obtains (by formula (50) of Sec. A.2.4) for period (1974, 1984) become:

$$\hat{\alpha}_{11}(1974) = 0.160560 \qquad \hat{\alpha}_{11}(1975) = 0.153823$$
$$\hat{\alpha}_{11}(1976) = 0.147368 \qquad \hat{\alpha}_{11}(1977) = 0.141184$$
$$\hat{\alpha}_{11}(1978) = 0.135260 \qquad \hat{\alpha}_{11}(1979) = 0.129584$$
$$\hat{\alpha}_{11}(1980) = 0.124147 \qquad \hat{\alpha}_{11}(1981) = 0.118937$$
$$\hat{\alpha}_{11}(1982) = 0.113946 \qquad \hat{\alpha}_{11}(1983) = 0.109165$$
$$\hat{\alpha}_{11}(1984) = 0.104584$$

and

$$\hat{\alpha}_{12}(1974) = 0.081178 \qquad \hat{\alpha}_{12}(1975) = 0.076020$$
$$\hat{\alpha}_{12}(1976) = 0.071189 \qquad \hat{\alpha}_{12}(1977) = 0.066665$$
$$\hat{\alpha}_{12}(1978) = 0.062429 \qquad \hat{\alpha}_{12}(1979) = 0.058462$$
$$\hat{\alpha}_{12}(1980) = 0.054747 \qquad \hat{\alpha}_{12}(1981) = 0.051269$$
$$\hat{\alpha}_{12}(1982) = 0.048011 \qquad \hat{\alpha}_{12}(1983) = 0.044960$$
$$\hat{\alpha}_{12}(1984) = 0.052103$$

B.II. OPTIMIZATION OF FOREIGN INVESTMENT CREDITS

Elzbieta Stapp

1. Introduction

The paper deals with foreign credits and international loans in the development model MR4 of planned economy.

In that way an important, stimulating role of international cooperation is taken into account. The credits are used for financing new investments, for buying the best technology and licences, know-how, etc. In the present paper it is assumed that the volume of investments is limited.

The optimum development strategy consists in allocation of investments which maximize the total G.N.P. Investment credits are regarded as economicly effective if expected benefits enable to repay these credits with interest.

It is assumed that there exists a potential possibility to borrow the necessary sum of money or to buy (by a credit) the capital investments, technology, etc., on convenient terms.

That depends in turn on the rate of interest resulting from the money demand and supply on the international monetary market.

It is assumed that an increment of the G.N.P., generated by increased productivity due to the credits, enables to repay the loans.

Credits should be repaid by export of investment goods or by export of commodities produced in the factories created by using the credits (selfrepaid).

When economic benefits are greater than expenditures, a part of the G.N.P. generated by using credits enables to repay the credits. It enables also an increase of consumption out of production supplied by new factories.

It is also possible to derive the minimum amount of credits necessary for the repayment.

2. Statement of the Problem

The MRI model under consideration consists of n sectors which exchange the final products $Y_i(t)$, $i = 1,\ldots,n$. The sector production functions are of the generalized Cobb-Douglas form:

$$Y_i(t) = \left[\int_{-\infty}^{t} K_i \exp(-\delta_i(t-\tau))\, z_i^{\beta_i}(\tau - T_{0i})d\tau\right]^{q_i} \prod_{j=1}^{n} Y_{ji}^{\alpha_{ji}}(t) \quad (1)$$

where for any i, i = 1,...,n,

1) K_i = constant
2) δ_i = quantity of depreciation of capital in the i-th sector
3) z_i = investment intensity in the i-th sector
4) β_i = given constant, $0 < \beta_i < 1$
5) T_{0i} = construction delay in the i-th sector
6) Y_{ji} = intersectorial flow (from the j-th sector to the i-th)
7) α_{ji} = coefficient of elasticity
8) $q_i = 1 - \sum_{j=1}^{n} \alpha_{ji}$.

The decentralized system of decisions, typical for planned economies, has been adopted in the model structure. The i-th sector decision centre maximizes the new profits

$$\max_{Y_{ji}} y_i(t) = \max_{Y_{ji}} \left\{ Y_i(t) - \sum_{j=1}^{n} Y_{ji}(t) \right\}$$

As shown in Ref. [3] there exists optimum strategies

$$\hat{Y}_{ji}(t) = \alpha_{ji} \cdot \hat{Y}_i(t) \quad (2)$$

which maximize the net sector profits.

Then

$$\hat{y}_i(t) = \hat{Y}_i(t) \cdot q_i \quad (3)$$

where

$$\hat{Y}_i(t) = \int_{-\infty}^{t} K_i \exp(-\delta_{ii}(t-\tau)) \times z_i^{\beta_i}(\tau - T_{0i})d\tau \prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i} \quad (4)$$

The G.N.P. generated in the planning interval $[0,T]$ can be written as:

$$Y = \int_0^T y(t)\, dt \tag{5}$$

when $y(t) = \sum_{i=1}^{n} y_i(t)\, w_i(t)$, where $w_i(t)$ = nonnegative discount function

$$w_i(t) = (1 + \varepsilon_i)^{-t}, \quad 0 < \varepsilon_i < 1$$

Let us assume that the investments are limited by constant quantity Z, i.e.

$$\sum_{i=1}^{n} \int_0^T z_i(t)\, dt \leq Z$$

Let us consider the problem of optimum distribution of investments between n sectors, in such a way that the generated G.N.P. in the time period $[0,T]$ attains its maximum.

Taking into account (3)-(5) one obtains

$$Y = \int_0^T \sum_{i=1}^{n} q_i \prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i} K_i\, w_i(t) \times$$

$$\times \int_{-\infty}^{t} \exp\left(-\delta_i(t-\tau)\right) z_i^{\beta_i}(\tau - T_{0i})\, d\tau\, dt =$$

$$= Y_0 + \sum_{i=1}^{n} \int_0^T z_i^{\beta_i}(\tau) \int_{\tau + T_{0i}}^{T} \exp\left(-\delta_i(t - \tau - T_{0i}\right) \times$$

$$\times \prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i}\, q_i\, w_i(t)\, dt\, d\tau$$

where

$$Y_0 = \int_0^T \sum_{i=1}^{n} \int_{-\infty}^{0} K_i \exp(-\delta_i(t-\tau))\, z_i^{\beta_i}(\tau - T_{0i})\, d\tau \times$$

$$\times \prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i}\, q_i\, w_i(t)\, dt$$

Y_0 is the part of G.N.P. generated by the factories which started to produce before the planning interval $[0,T]$.

Let $f_i(t)$ be the following function:

$$f_i(t) = \begin{cases} \displaystyle\int_{t+T_{0i}}^{T} K_i \exp(-\delta_i(\tau - t - T_{0i}) \times \\ \qquad \times \displaystyle\prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i}\, q_i\, w_i(\tau)\, d\tau \\ 0 \end{cases}$$

Then the problem of maximalization of G.N.P. in the planning interval $[0,T]$, with limited investments, is equivalent to

$$\max_{z_i \in \Omega} \sum_{i=1}^{n} \int_0^T z_i^{\beta_i}(\tau)\, f_i(\tau)\, d\tau \qquad (6)$$

where

$$\Omega = \left\{ z_i(t): \sum_{i=1}^{n} \int_0^T z_i(t)\, dt \leqslant Z,\ z_i(t) \geqslant 0, \quad t \in [0,T],\ Z - \text{const.} \right\}$$

3. <u>Solution</u>

The optimum strategy which maximizes (6) becomes (see Appendix)

$$\tilde{z}_i(\tau) = \left(\frac{\lambda}{\beta_i\, f_i(\tau)} \right)^{1/(\beta_i - 1)}, \quad i = 1, \ldots, n$$

where λ is the solution of the equation

$$\sum_{i=1}^{n}\int_{0}^{T}\left(\frac{\lambda}{\beta_i f_i(\tau)}\right)^{1/(\beta_i-1)} d\tau - Z = 0$$

The optimum global investment in time τ can be written as $\tilde{z}(\tau) = \sum_{i=1}^{n} \tilde{z}_i(\tau)$.

Let Y - G.N.P. generated in the time interval $[0,T]$ using the optimum strategy.

The global investment determined by domestic resources, can be written as $\hat{z}_1(\tau) = \gamma(\tau)\sum_{i=1}^{n} y_i(\tau - 1)$ for $\tau \in (0,T)$, where $\gamma(\tau)$ = an exogeneous function describing relation between the consumption rate and the investments rate.

The realization of investment strategy $\tilde{z}$ requires usually a high concentration of investment intensity at the beginning of the time interval $[0,T]$.

On the other hand the following condition should take place:

$$(\exists t') \in [0,T] \quad (\forall t) \quad (t \leqslant t') \quad \tilde{z}(t) \geqslant \hat{z}(t)$$

Let us consider the time interval $[0,t']$.

The economic system under consideration is able to provide for investments $\hat{z}_1(t)$ only, so the realization of the optimum strategy $\tilde{z}(t)$ is possible in two alternative ways:

1) by the reduction of consumption rate,

2) by covering the difference $\tilde{z} - \hat{z}_1$ for any $t \leqslant t'$ with credits and international loans.

The first way can not be accepted, because the consument's demand has to be satisfied.

So the only way to solve our problem is the second one.

Let

$$\tilde{y}_i(t) = \int_0^{t-1} K_i \exp(-\delta_i(t-\tau-1))\, \tilde{z}_i^{\beta_i}(\tau)d\tau \times$$
$$\times \prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i}\, q_i$$

Let $\bar{z}(t) = \sum_{i=1}^{n} \bar{z}_i(t)$, where $\bar{z}_i(t) = \gamma(t)\, \tilde{y}_i(t-1)$, be the strategy, which uses two kind of resources: domestic and credits.

Let $T_1 \in (0,T)$ be the solution of the equation $\tilde{z}(T_1) = \bar{z}(T_1)$. Then the net credit value C which is necessary for optimum development within $[0,T]$ can be written as:

$$C = \int_0^{T_1} (\tilde{z}(t) - \bar{z}(t))dt$$

The potential possibility of credit repayment can be written as

$$OS = \int_{T_1}^{T} (\bar{z}(t) - \tilde{z}(t))dt$$

Assume that the repayment function can be described using the formula:

$$S(T - t) = k(t)\ (1 + \varepsilon)^{T-t} \quad \text{for} \quad t \in [0,T_1]$$

where: $k(t)$ = quantity of credit in the moment t, $k(t) = \tilde{z}(t) + - \bar{z}(t)$; ε = rate of interest.

The G.N.P. generated in time interval $[0,T]$ using the strategy $\tilde{z}$, after credits repayment can be described as:

$$Y_s = \tilde{Y} - \int_0^{T_1} S(T - t)dt$$

It is easy to show that if $\bar{Y}$ is the G.N.P. generated in time interval $[0,T]$ using the strategy $\hat{z}$ then $\tilde{Y} > \bar{Y}$.

It means that there exists a quantity Y_1 such that $\tilde{Y}-Y_1 = \bar{Y}$.

If $Y_1 > \int_0^{T_1} S(T - t)dt$ a surplus of G.N.P. can be achieved. It is done by an optimum credits distribution and a suitable rate of interest.

Otherwise repayment of credits is greater than the part of G.N.P. generated by using credits. In that case the use of credits is without purpose.

4. Discussion of Result

The solution of the foreign credit problem (see Sec. 3) was programmed on CDC CYBER 72 for time interval 1970-1980.

The algorithm was based on the formulae presented in Sec. 3 while data used are the same as in the Sec. [B]5.1 and the given interest rate.

It was assumed that the total investment for the period 1970 -1980 and integral constraint (credits available) is equal to corresponding investment value for the amplitude constraint (without foreign credits).

This assumption guarantees that all the investment are utilized.

The G.N.P. generated within the time interval under consideration enables the credit repayment. Besides, a wide margin of resources is created at the end of 1970-1980 interval. It enables an increase of consumption if necessary or - to start the new investments at the end of planning interval.

The results derived are given in Table 1.

As shown in Sec. 3 the G.N.P. generated using the strategy $\tilde{z}_i$ (with credits) can be described as

$$y(t) = \int_{-\infty}^{t} K_i \exp(-\delta_i(t-\tau))\, \tilde{z}_i^{\beta_i}(\tau - T_{0i})d\tau \prod_{j=1}^{n} \left|\frac{\alpha_{ji}}{\alpha_{ji}}\right|^{\alpha_{ji}/q_i} q_i$$

and the G.N.P. generated using the strategy $\bar{z}_i$(without credits) can be written as:

$$\bar{y}(t) = \int_{-\infty}^{t} \kappa_i \exp(-\delta_i(t-\tau))\hat{z}_i^{\beta_i}(\tau - T_{0i})d\tau \prod_{j=1}^{n} \alpha_{ji}^{\alpha_{ji}/q_i} \; q_i$$

Table 1

Year	G.N.P. (millions zl) without credits	with credits
1971	$.90416 \times 10^6$	$.90416 \times 10^6$
1972	$.99668 \times 10^6$	$.10456 \times 10^7$
1973	$.10979 \times 10^7$	$.12029 \times 10^7$
1974	$.12048 \times 10^7$	$.13126 \times 10^7$
1975	$.13262 \times 10^7$	$.14307 \times 10^7$
1976	$.14647 \times 10^7$	$.16053 \times 10^7$
1977	$.16147 \times 10^7$	$.17846 \times 10^7$
1978	$.17837 \times 10^7$	$.19976 \times 10^7$
1979	$.19582 \times 10^7$	$.22353 \times 10^7$
1980	$.21240 \times 10^7$	$.24992 \times 10^7$

The evaluated time interval for which the credit is lent becomes equal 3 years.

Table 2

Year	Value of credit (millions zl)	Repayment (millions zl)
1971	$.56679 \times 10^5$	
1972	$.82820 \times 10^5$	
1979	-	$.14771 \times 10^6$
1980	-	$.10867 \times 10^6$

The credits finance that part of investments (under integral constraint) which the system is unable to cover out of the domestic resources.

The value of credits taken repaid is given in Table 2.

The quantity of repayment is evaluated with respect to the interest rate 7.5%.

It can be observed that concentration of investments at the initial part of $[0,T]$ guaranteed a high rate of production growth. It gives also a high rate of the G.N.P. growth.

Table 3

Year	Rate of G.N.P. growth	
	without credits (%)	with credits (%)
1971	7.28	7.28
1972	10.23	15.64
1973	10.15	15.04
1974	9.74	9.11
1975	10.08	9.03
1976	10.44	12.20
1977	10.24	11.16
1978	10.47	11.93
1979	9.79	11.9
1980	8.47	11.8

The consumption for the optimum strategy of investments with credits exceeds the consumption without credits (Table 4).

Table 4

Year	Consumption (millions zl)	
	without credits	with credits
1971	$.67862 \times 10^6$	$.69217 \times 10^6$
1972	$.74806 \times 10^6$	$.79531 \times 10^6$
1973	$.82731 \times 10^6$	$.90519 \times 10^6$
1974	$.92278 \times 10^6$	$.97595 \times 10^6$
1975	$.10387 \times 10^7$	$.10491 \times 10^7$
1976	$.10580 \times 10^7$	$.11596 \times 10^7$
1977	$.11524 \times 10^7$	$.12737 \times 10^7$
1978	$.12525 \times 10^7$	$.14027 \times 10^7$
1979	$.13505 \times 10^7$	$.15417 \times 10^7$
1980	$.14369 \times 10^7$	$.16907 \times 10^7$

Therefore, we see that credits should be used for financing investments.

Appendix

Let E be the Hilbert space and let φ , f be functionals, φ : $E \to R$, f: $E \to R$.

Definition 1. We say that point $x_0 \in E$ is the regular point of functionals φ if $\varphi(x_0)=0$ and there exists a continuous linear operator A, $A:E \to R$, $Ag = \varphi'(x_0)g$ and A maps E onto R.

Definition 2. We say that point $x_0 \in E$ is the conditional maximum (minimum) point of the functional f(x) with respect to $\varphi(x_0) = 0$ iff for any point x, belonging to a neighbourhood of the point x_0, and satisfying condition $\varphi(x) = 0$, the condition $f(x_0) \geqslant f(x)$ $(f(x_0) \leqslant f(x))$ holds.

As shown in Ref. [2] the following theorem holds:

Theorem 1. Let x_0 be a conditional maximum point of functional f(x) with respect to $\varphi(x) = 0$ and let x_0 be a regular point of the variety $\varphi(x) = 0$, than there exists a linear functional k, $k: R \to R$, such that for functional $F(x) = f(x) - k\varphi(x)$ the condition $F'(x_0) = 0$ holds, i.e. $dF(x_0,h) = 0$ for any h, $h \in E$. Let $z(t) = (z_1(t),\dots,z_n(t))$ for $t \in [0,T]$ be a vector valued function, which is continuous and differentiable, i.e. $z \in C_n^1[0,T]$.

Let F be the functional, $F: C_n^1[0,T] \to R$,

$$F(z) = \sum_{i=1}^{n} \int_0^T z_i^{\beta_i}(t)\, f_i(t)\, dt \tag{A.1}$$

where:

I. $(\forall i)\ \ 0 < \beta_i < 1$

II. $(\forall i)\ \ f_i(t) = \begin{cases} \int_{t+T_{0i}}^{T} \varphi_i(\tau) w_i(\tau) \exp(-\delta_i)(\tau - t - T_{0i})\, d\tau & \text{for } t \in \langle 0, T-T_{0i} \rangle \\ 0 & \text{for } t \in \langle T-T_{0i}, T \rangle \end{cases}$

where:

II.1 $(\forall i)\ \ \varphi_i(\tau),\ w_i(\tau) \in C$

II.2 $(\forall i)\ \ T_{0i}, \delta_i$ = constants.

Let F_1: $C_n^1[0,T] \to R$ be the following functional

$$F_1(z) = \sum_{i=1}^{n} \int_0^T z_i(t)\,dt - Z \qquad (A.2)$$

where Z = constant.

We are looking for the extremum point of the functional F(z) with respect to the condition $F_1(z) = 0$.

Let z_0 be the extremum point, $z_0 \in C_n^1[0,T]$. Then if z_0 is a regular point of the variety $F_1(z) = 0$ and z_0 is a maximum point of functional F(z) then (by virtue of the Theorem 1) there exists a linear functional $k(F_1)$, k: $R \to R$, such that for the functional G(z), $G(z) = F(z) - k\,F_1(z)$ for condition $dG(z_0,h) = 0$ holds.for any h, $h \in C_n^1[0,T]$.

<u>Lemma 1</u>. Point z_0 is a regular point of the variety $F_1(z)=0$.

<u>Proof</u>. Let us define a functional A, A: $C_n^1[0,T] \to R$, in the following way:

$$Ag = F_1'(z_0)\,g, \quad g \in C_n^1[0,T]$$

We shall show that A maps $C_n^1[0,T]$ onto R.

In fact, let p be an arbitrary element of R. We shall prove that there exists an element $\bar{g} \in C_n^1[0,T]$ such that $A\bar{g} = p$.

Hence $z_0 \overset{df}{=} (z_{01},\dots,z_{0n})$

$$A\bar{g} = F_1'(z_0)\bar{g} = \frac{d}{dt} F_1(z_0 + t\bar{g})\Big|_{t=0} =$$

$$= \frac{d}{dt}\left[\sum_{i=1}^{n} \int_0^T (z_{0i}(\tau) + t\,\bar{g}_i(\tau))d\tau\right]\Bigg|_{t=0} =$$

$$= \sum_{i=1}^{n} \int_0^T \bar{g}_i(\tau)d\tau$$

Let $\bar{g} = (0,\dots,\frac{p}{T},\dots,0)$.

It is obvious that $\bar{g} \in C_n^1[0,T]$ and $A\bar{g} = p$. Therefore, z_0 is a regular point of the variety $F_1(z) = 0$. Q.E.D.

Taking into account Theorem 1 and Lemma 2 one can see that a linear functional k, k: $R \to R$ exists, such that for the functional $G(z) = F(z) - kF_1(z)$ the following condition $dG(z_0,h) = 0$ holds.

Let us remember that k: R ⟶ R. As an obvious consequence we obtain that there exists a constant λ, $\lambda \in R$, such that $kF_1 = \lambda F_1$.

Therefore $G(z) = F(z) - \lambda F_1(z)$.

Let us calculate $dG(z,h)$ for $h \in C_n^1[0,T]$:

$$dG(z,h) = \frac{d}{dt}\left[\sum_{i=1}^{n} \int_0^T (z_i(\tau) + t\, h_i(\tau))^{\beta_i} f_i(\tau)\, d\tau + \right.$$

$$\left. - \left(\lambda \sum_{i=1}^{n} \int_0^T (z_i(\tau) + t\, h_i(\tau))d\tau - Z\right)\right]\Bigg|_{t=0} =$$

$$= \sum_{i=1}^{n} \int_0^T \beta_i\, z_i(\tau)^{\beta_i - 1} f_i(\tau)\, h_i(\tau) d\tau - \lambda \sum_{i=1}^{n} \int_0^T h_i(\tau) d\tau =$$

$$= \sum_{i=1}^{n} \int_0^T h_i(\tau)(\beta_i\, z_i^{\beta_i - 1}(\tau)\, f_i(\tau) - \lambda) d\tau$$

Hence, at the extremal point the condition $dG(z,h) = 0$ holds for any $h \in C_n^1[0,T]$.

From $\beta_i z_i^{\beta_i - 1}(\tau)\, f_i(\tau) - \lambda = 0$ for $i = 1,\dots,n$ one gets

$$\tilde{z}_i(\tau) = \left(\frac{\lambda}{\beta_i\, f_i(\tau)}\right)^{1/(\beta_i - 1)} \quad \text{for } i = 1,\dots,n \qquad (A.3)$$

Taking into account the condition $F_1(z) = 0$ we obtain the following equation (with the unknown λ):

$$\sum_{i=1}^{n} \int_0^T \left(\frac{\lambda}{\beta_i\, f_i(\tau)}\right)^{1/(\beta_i - 1)} d\tau - Z = 0 \qquad (A.4)$$

This equation is equivalent to:

$$\lambda^{1/(\beta_i - 1)} \sum_{i=1}^{n} \int_0^T \left(\frac{1}{\beta_i\, f_i(\tau)}\right)^{1/(\beta_i - 1)} d\tau - Z = 0$$

Denote

$$f(\lambda) = \lambda^{1/(\beta_i-1)} \sum_{i=1}^{n} \int_0^T \left(\frac{1}{\beta_i f_i(\tau)} \right)^{1/(\beta_i-1)} d\tau - Z$$

and calculate $f'(\lambda)$ and $f''(\lambda)$

$$f'(\lambda) = \sum_{i=1}^{n} \frac{1}{\beta_i - 1} \lambda^{[1/(\beta_i-1)]-1} \int_0^T \left(\frac{1}{\beta_i f_i(\tau)} \right)^{1/(\beta_i-1)} d\tau$$

Hence $(\forall i)\beta_i - 1 < 0$ so $f'(\lambda) < 0$.

$$f''(\lambda) = \sum_{i=1}^{n} \frac{1}{\beta_i-1} \left(\frac{1}{\beta_i-1} - 1\right) \lambda^{1/(\beta_i-1)-2} \int_0^T \left(\frac{1}{\beta_i f_i(\tau)}\right)^{1/(\beta_i-1)} d\tau$$

It is possible to verify that $f''(\lambda) > 0$.

The equation $f(\lambda) = 0$ is solved by the Newton method (secant method):

Let us define a sequence $\{\lambda_n\}$ by induction:

1) λ_0 is an arbitrary element of R, such that $f(\lambda_0) > 0$,

2) $\lambda_{n+1} = \lambda_n - \dfrac{f(\lambda_n)}{f'(\lambda_n)}$, $n = 0,1,\ldots$.

Condition $f(\lambda_0)$ 0 implies that sequence $\{\lambda_n\}$ is converging to the solution of the equation (4) (see Ref. [1]).

References

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[3] R. Kulikowski: Optimization of Cooperation in n-Sector Development Model. Bull. Acad. Pol. Sci. Ser. tech. 1972 No. 12.

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C. PERSONAL AND AGGREGATE CONSUMPTION SUBSYSTEMS

Henryk Mierzejewski

1. Introduction

The National Income (N.I.), denoted here by $Z(t)$, generated in the Production Subsystem (see Part B) in the year t, is subdivided into Accumulation and Consumption,and afterwards - into different, hierarchically ordered, categories (see Part A Figs. 8 and 12). The share of particular components $Z_\nu(t), \nu = 1,\dots m$, (i.e. productive investments, personal consumption, aggregate consumption) in the N.I. distributed can be expressed by means of the allocation coefficients:

$$\gamma_\nu(t) = Z_\nu(t)/Z(t), \quad \nu = 1,\dots,m \tag{1}$$

The exemplary data on the distribution of N.I. in Poland are presented in Tables 1 and 2.

Assuming that the distribution takes place as a result of the maximization of the national utility function (see Part A. Introduction) in the form

$$U(Z) = \bar{U} \prod_{\nu=1}^{m} \left[Z_\nu(t) \right]^{\gamma_\nu(t)}$$

$$\sum_{\nu=1}^{m} \bar{\gamma}_\nu(t) = 1, \quad \bar{\gamma}_\nu(t) \geqslant 0, \quad \nu = 1,\dots,m \tag{2}$$

subject to the constraints

$$\sum_{\nu=1}^{m} Z_\nu(t) \leqslant Z(t)$$

one obtains that

$$\bar{\gamma}_\nu(t) = \gamma_\nu(t), \quad \nu = 1,\dots,m \tag{3}$$

According to relation (3), the 'ex post' data for the allocation of N.I. can be used for the construction of the utility function (2) (with the accuracy to the multiplier $\bar{U}$). That function can be used 'ex ante' to derive the future strategies for resources allocation.

Table 1. National income distributed (nett) (at current prices).

Specification	1970	1971	1972	1973	1974	1970	1974
	in 1000 mill. zl					percentage	
TOTAL	731.5	841.8	950.9	1114.1	1299.6	100.0	100.0
CONSUMPTION	548.0	594.7	649.7	726.8	832.3	74.9	64.0
Personal consumption	465.0	503.4	548.3	613.6	698.9	63.6	53.8
Aggregate consumption	83.0	91.3	101.4	113.2	133.4	11.3	10.2
ACCUMULATION	183.5	247.1	301.3	387.3	467.3	25.1	36.0
Productive investments	86.1	121.5	163.2	213.7	249.3	11.8	19.2
Nonproductive investments	53.4	62.8	90.2	86.6	104.3	7.3	8.0
Inventories (increment of working assets and reserves)	44.0	62.8	65.8	87.0	113.7	6.0	8.8

Source: Rocznik Statystyczny 1975, GUS Warszawa (Statistical Yearbook 1975, Printed by the Central Statistical Office of the Polish People's Republic).

Table 2. National income distributed (gross) (at current prices)

Specification	1970	1971	1972	1973	1974	1970	1974
	in 1000 mill. zl					percentage	
TOTAL	825.1	939.5	1057.6	1225.3	1416.9	100.0	100.0
CONSUMPTION	537.2	583.0	637.2	713.9	818.9	65.1	57.8
Personal consumption	457.5	495.6	540.2	605.1	690.1	55.4	48.7
Aggregate consumption	79.7	87.4	97.0	108.8	128.8	9.7	9.1
ACCUMULATION	287.9	356.5	420.4	511.4	598.0	34.9	42.2
Productive investments	179.7	219.3	268.4	324.5	366.6	21.8	25.9
Nonproductive investments	64.2	74.4	86.2	99.9	117.7	7.8	8.3
Inventories (increment of working assets and reserves)	44.4	62.8	65.8	87.0	113.7	5.3	8.0

Source: Rocznik Statystyczny 1975, GUS Warszawa.

Table 3

$\tilde{\gamma}(t)$	$\frac{\text{Productive investments } (t)}{\text{National income } (t)}$	$\frac{\text{Consumption } (t)}{\text{National income } (t)}$
Parameters		
$C = c$	$c = 0.179$ (T=56.1)	$c = 0.764$ (T=27.8)
$A(t) = a \cdot t$	$a = 0.007365$ (T=8.5)	$a = -0.028$ (T=7.9)
$B(t) = b \cdot t$	$b = 0$	$b = 0.195$ (T=3.4)
Statistical Analysis		
F	72.9	98.7
R	0.955	0.977
$\max \lvert \tilde{\gamma}(t) - \gamma(t) \rvert$	0.005	0.006
α	0.95	0.95
N	9	14
K	1	3
T_{KR}	2.447	2.228
F_{KR}	5.14	3.71

Data: 1960–1973; T – Student's test; F – Fisher's test; R – correlation coefficient; α – confidence level

In the research on MRI models the allocation coefficients(1) were approximated by the relation

$$\tilde{\gamma}_{\nu}(t) = C \left[\frac{Z(t-1)}{L(t)} \right]^{A(t)} \left[p_t \right]^{B(t)} \qquad (4)$$

where: $L(t)$ = population number in the year t; $p_t = p(t)/p(0)$ = price change index; C, $A(t)$, $B(t)$ = coefficients estimated by the linear regression method.

The model runs for (4) have shown a great compatibility with real data (1). The parameters of function (4) approximating the data (1) for productive investments and consumption [1] are given in Table 3, as an example. Models of coefficients of the N. I. distribution are used in the model MRI, especially in development forecasts programs (see Part B).

2. Modelling of the Personal Consumption Subsystem

The Personal Consumption Subsystem plays an important part in mutual relations between the Production Subsystem and the Aggregate Consumption Subsystem. A part of material goods and nonmaterial services is consumed by the population, while the two spheres of activity are provided with labour. Moreover, the subsystem influences market phenomena. Data on the employment structure: historical and forecasts are given in Tables 4–8 [2].

Table 4. Total employment in national economy (in thousands)

Year	Total employment	Material production sphere	Nonmaterial production sphere
1967	14402	13072	1330
1968	14730	13347	1383
1969	15036	13555	1481
1970	15175	13631	1544
1971	15482	13946	1536
1972	15947	13965	1982
1973	16399	14331	2068

Source: Total employment-The CSO Statistical Yearbook, 1974(Data do not comprise employees in budgetary units of the National Defence Ministry and the Ministry of Internal Affaire).Total employment in material production sphere-Tables of inter-sector flows.Total employment in nonmaterial production sphere-computed by the author.

Data on the age structure and education level of the population are given in Part C.III.

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[1] Numerical research on the distribution of N.I. was carried out by Hanna BURY and Maria SOSZYNSKA.

[2] The tables were prepared by Agnieszka GORECKA.

Table 5. Forecast of population in working age (in thousands)

Year	Men aged 18-64	Women aged 18-59	Total
1970	9294.5	9029.2	18323.7
1971	9463.1	9173.3	18635.4
1972	9630.4	9309.8	18940.2
1973	9815.2	9468.2	19283.4
1974	9988.2	9614.9	19603.1
1975	10156.6	9801.8	19958.4
1976	10330.7	9995.3	20326.0
1977	10479.9	10175.4	20655.3
1978	10620.1	10339.6	20959.7
1979	10738.9	10438.1	21177.0
1980	10867.5	10505.3	21372.8
1981	11011.0	10579.2	21590.2
1982	11150.3	10614.8	21765.1
1983	11276.3	10636.2	21912.5
1984	11366.3	10652.7	22019.0
1985	11430.0	10649.8	22079.8

Source: The C.S.O. Demographical Yearbook 1974.

Table 6. Total employment in material production sphere (in thousands)

Sector name	1967	1968	1969	1970	1971	1972	1973
1. Fuels and energy	413	415	421	424	455	465	466
2. Metallurgy	207	212	222	221	235	244	246
3. Machinery,electric equipment	1005	1073	1121	1152	1330	1383	1433
4. Chemical industry	241	256	267	270	300	315	326
5. Minerals	265	276	281	281	184	291	295
6. Wood and paper	275	278	285	288	290	297	308
7. Light industry	790	782	804	805	844	879	909
8. Food industry	416	424	431	432	474	488	498
9. Other industrial branches	139	144	144	144	146	148	153
10. Building industry	1251	1316	1343	1360	1457	1547	1706
11. Agriculture	5590	5628	5679	5687	5460	5183	5218
12. Forestry	192	191	188	188	184	173	169
13. Transport and communication	910	967	977	993	1117	1131	1146
14. Trade	933	935	938	932	931	977	1033
15. Rest of production	447	450	454	454	439	444	425
Total material production sphere	13072	13347	13555	13631	13946	13965	14331

Number of professionally active population - average number estimated by converting part-time employees into full-time employees.
Source: Tables of Inter-Sector Flows, 1967-1973.

Table 7. Forecast of total employment in material production sphere (in thousands)

Sector name	1974	1975	1976	1977	1978	1979	1980
1. Fuels and energy	479	489	500	510	520	531	541
2. Metallurgy	254	261	268	275	282	289	296
3. Machinery, electric equipment	1516	1591	1667	1742	1818	1893	1969
4. Chemical industry	340	354	369	383	398	412	427
5. Minerals	299	303	308	312	317	321	326
6. Wood and paper	310	315	320	325	331	336	341
7. Light industry	915	936	957	978	999	1021	1042
8. Food industry	511	526	541	556	571	585	600
9. Other industrial branches	153	155	157	159	161	162	164
10. Building industry	1703	1772	1842	1911	1980	2050	2119
11. Agriculture	5174	5095	5015	4936	4856	4777	4697
12. Forestry	168	164	160	156	153	149	145
13. Transport and communication	1202	1244	1286	1328	1370	1412	1454
14. Trade	1008	1021	1035	1048	1062	1075	1089
15. Rest of production	432	428	425	422	418	415	412
Total material production sphere	14464	14654	14850	15041	15236	15428	15622

Source: Estimated by the author according to the C.S.O. population forecast, and the Forecast of employment and education development in Poland. Committee of Research and Forecast "Poland 2000", Polish Academy of Sciences 1/1972.

Table 8. Employment in material production sphere (gross wages+insurances) (in mll.zl)

Sector name	1967	1968	1969	1970	1971	1972	1973
1. Fuels and energy	21302	22644	25799	26987	28864	31175	33651
2. Metallurgy	9481	10145	11020	11509	12364	13636	15311
3. Machinery,electric equipment	30067	42107	46009	48511	54271	60517	67198
4. Chemical industry	8521	9328	10027	10543	11757	13217	14462
5. Minerals	8340	9059	9489	9709	10303	11555	12869
6. Wood and paper	7465	8092	8637	8995	8793	10215	11067
7. Light industry	20683	22717	24190	24919	26859	29370	32349
8. Food industry	12491	13489	14052	14602	16628	18890	21025
9. Other industrial branches	4500	4770	4992	4925	5255	5794	6136
10. Building industry	40894	45044	48569	50150	55003	64046	73968
11. Agriculture	106889	110978	109458	110924	128417	146072	158036
12. Forestry	4386	4573	4598	4745	4639	5648	6313
13. Transport and communication	26137	29633	31350	32920	37648	41883	47692
14. Trade	24109	26358	25822	27966	31159	34487	40091
15. Rest of production	5154	5351	5278	5349	6187	6393	9141
Total material production sphere	338419	364288	379290	392754	438147	492898	555309

Source: Tables of Inter-Sector Flows, 1967-1973.

2.1. An Aggregated Version of the Subsystem

The personal consumption subsystem in the form of single sector (S_0) has been assumed. In order to standardize the method of approach, it is assumed according to 2 and 5 , that the sector S_0 can be described by production function (in the same way as in the case of material production sectors) in the form (see Part 2 Section 2)

$$Y_0(t) = \left[F_0(t)\right]^{q_0} \prod_{j=1}^{n} \left[Y_{j0}(t)\right]^{\alpha_{j0}} \left[Y_{U0}(t)\right]^{\alpha_{U0}} \tag{5}$$

where: α_{U0}, α_{j0} = positive coefficients given, $j = 1,2,\ldots,n$; n = number of production subsystem sectors;

$$q_0 = 1 - \left[\sum_{j=1}^{n} \alpha_{j0} + \alpha_{U0}\right] > 0 \tag{6}$$

$Y_{j0}(t)$ = flow of material goods from the j-th production sector to sector S_0 [3], $j = 1,\ldots,n$; $Y_{U0}(t)$ = flow of nonmaterial goods from aggregate consumption system U to sector S_0; $Y_0(t)$ = net incomes of the population in the year t (total value of employment in monetary units); $Y_0(t) = Y_{0P}(t) + Y_{0U}(t) + Y_{0R}(t)$; Y_{0P} = labour incomes in production subsystem; $Y_{0U}(t)$ = labour incomes in aggregate consumption subsystem; $Y_{0R}(t)$ = other sources of income (social benefits); $F_0(t)$ = positive function concerning the effects of an "investment" type consumption (fixed assets, influence of education, health care, etc.).

The connection of sector S_0 with the production and aggregate consumption subsystem is shown in Fig. 1, where the "investment" influence of a part of consumed goods is marked with a dotted line; $Y_{0j}(t)$ represent the labour incomes in particular production sectors, $j = 1,\ldots,n$.

At the present stage of research on MRI models the following form of the function $F_0(t)$ is used:

---

[3] All the flows are expressed in monetary units.

$$F_0(t) = \int_{-\infty}^{t} k_0(t-\tau)\, Y_{P0}^{\beta_0}(\tau)\, d\tau \qquad (7)$$

where $Y_{P0}(t) = \sum_{j=1}^{n} Y_{j0}(t)$, $0 < \beta_0 < 1$, and function $k_0(\cdot)$ in (7) is assumed in the form:

$$k_0(t) = \begin{cases} K_0 \exp(-\delta_0(t-T_{00})) & \text{for } t \geqslant T_{00} \\ 0 & \text{for } t < T_{00} \end{cases} \qquad (8)$$

The assumption (8) means that the effect caused by the impulse $Y_{P0}^{\beta 0}$ in the moment τ, appears in the moment $\tau + T_{00}$ (T_{00} = delay), and it is decreasing in time to the extent described by a constant depreciation $\delta_0 > 0$. The intensity of this process is described by the constant $K_0 > 0$. The function (8) is an approximation of a typical real process (Fig. 2) including aging (wear and tear) of fixed assets, acquired skills, etc.

In the further part of the paper, with regard to numerical computations, the relation (7) will be written in a discrete form, i.e. by virtue of (8) as

$$F_0(t) = \int_{-\infty}^{t} K_0 \exp(-\delta_0(t-\tau))\, Y_{P0}^{\beta 0}(\tau - T_{00}) \qquad (9)$$

It is possible to observe that relation (9) does not contain now the nonmaterial goods. It will be taken into account in the future research on MRI models.

Treating the sector S_0 as a productive sector, we assume also that it maximizes the value added (i.e. the savings):

$$D_0(t) = Y_0(t) - \left[\sum_{j=1}^{n} Y_{j0}(t) + Y_{U0}(t) \right] \qquad (10)$$

where $Y_{j0}(t) \geqslant 0$, $Y_{U0}(t) \geqslant 0$ are the decision variables, and $Y_0(t)$ given by equations (5) and (9).

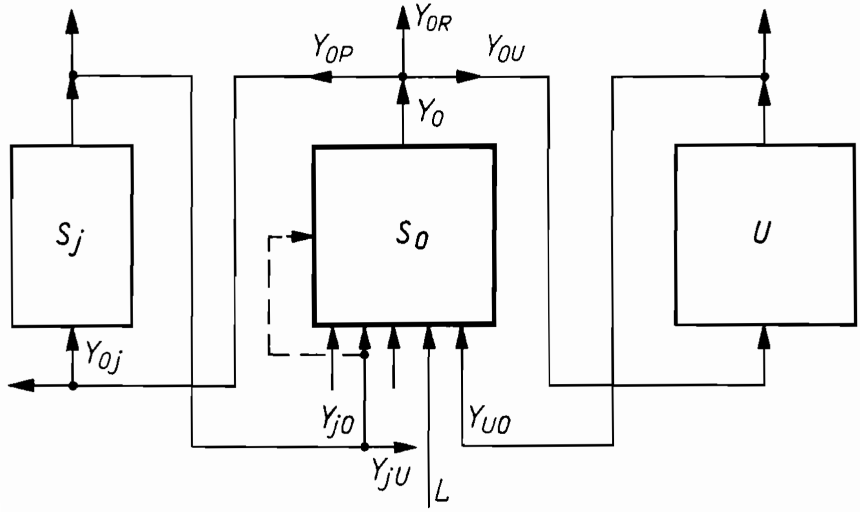

Figure 1.

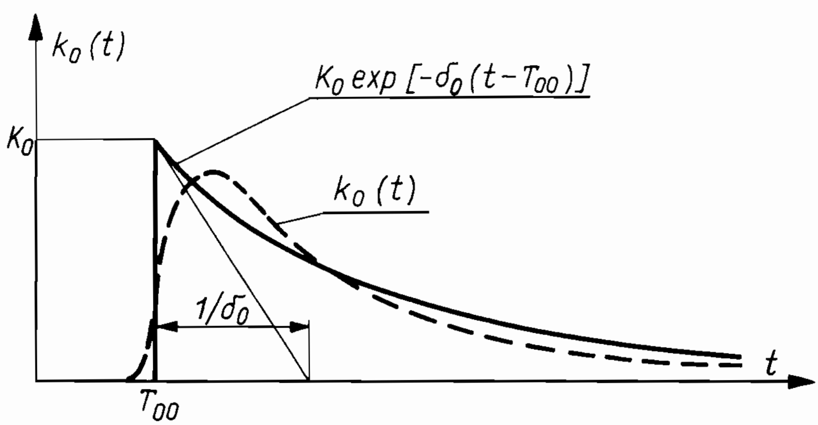

Figure 2.

As it was shown in [1] the following equations satisfy the optimal strategies (see Part A):

$$\begin{aligned} \hat{Y}_{j0}(t) &= \alpha_{j0}\, \hat{Y}_0(t), \quad j = 1,\dots,n \\ \hat{Y}_{U0}(t) &= \alpha_{U0}\, \hat{Y}_0(t) \end{aligned} \tag{11}$$

$$\hat{Y}_0(t) = F_0(t) \prod_{j=1}^{n} \alpha_{j0}^{\alpha_{j0}/q_0}\; \alpha_{U0}^{\alpha_{U0}/q_0} \tag{12}$$

$$D_0(t) = q_0\, \hat{Y}_0(t) \tag{13}$$

where q_0 is given by the relation (6), and $F_0(t)$ by the relation (9).

2.1.1. Identification of the sector S_0 production function

As a basis for identification of parameters of the sector S_0 production function we use the relations (11)-(13), i.e. we assume that the past decisions were optimal, and use the statistical data on $\tilde{Y}_0(t)$, $\tilde{Y}_{j0}(t)$, $\tilde{Y}_{U0}(t)$ for $t \in [t_p, t_k]$; t_p = initial year, t_k = final year [4].

Elasticity coefficients α_{j0}, α_{U0} can be derived from relation (11) as the average value [5]

$$\begin{aligned} \alpha_{j0} &= \frac{1}{t_k - t_p + 1} \sum_{t=t_p}^{t_k} \frac{\tilde{Y}_{j0}(t)}{\tilde{Y}_0(t)}, \quad j = 1,\dots,n \\ \alpha_{U0} &= \frac{1}{t_k - t_p + 1} \sum_{t=t_p}^{t_k} \frac{\tilde{Y}_{U0}(t)}{\tilde{Y}_0(t)} \end{aligned} \tag{14}$$

---

[4] In the Subsection 2.1.1 the values from statistical data are marked by a tilde, in contradistinction to the values computed on a model (without a tilde).

[5] In the research carried on MRI models the elasticity coefficients α_{j0}, α_{U0} were also assumed as variable in time and changes of their trends were forecasted.

On the basis of relation (6) we have

$$q_0 = 1 - \left(\sum_{j=1}^{n} \alpha_{j0} + \alpha_{U0}\right) \qquad (15)$$

Replacing (9) to (12) and dividing the obtained expression into two components we get

$$Y_0(t) = Y_0(t_p + T_{00} - 1)\ \exp\left(-\delta_0(t - t_p - T_{00} + 1)\right) +$$
$$+ q_{10} \sum_{\tau = t_p + T_{00}}^{t} K_0\ \exp\left(-\delta_0(t - \tau)\right)\ Y_{P0}^{\beta_0}(\tau - T_{00}) \qquad (16)$$

where

$$q_{10} = \prod_{j=1}^{n} \alpha_{j0}^{\alpha_{j0}/q_0}\ \alpha_{U0}^{\alpha_{U0}/q_0} \qquad (17)$$

The K_0 coefficient is identified by the least squares method, i.e. from the condition

$$\min_{K_0} \sum_{t=t_p}^{t_k} \left[\tilde{Y}_0(t) - Y_0(t)\right]^2 \qquad (18)$$

where $Y_0(t)$ given by the relations (16) and (17).

Equalizing the derivative of (18) to zero we obtain (with assumed parameters T_{00}, δ_0, β_0)

$$K_0 = \frac{\sum_{t=t_p}^{t_k} \left[\tilde{Y}_0(t) - \varphi_1(t)\right] \varphi_2(t)}{q_{10} \sum_{t=t_p}^{t_k} \left[\varphi_2(t)\right]^2} \qquad (19)$$

where

$$\varphi_1(t) = \tilde{Y}_0(t_p + T_{00} - 1)\ \exp\Big(-\delta_0(t - t_p - T_{00} + 1)\Big)$$
$$\varphi_2(t) = \sum_{\tau=t_p+T_{00}}^{t} \exp\left(-\delta_0(t-\tau)\right)\ \tilde{Y}_{P0}^{\beta 0}(\tau - T_{00}) \qquad (20)$$

and q_{10} is given in formula (17).

Numerical computations were based on data from Central Statistical Office in Poland. The statistical data covered the 4--year period (t_p = 1970, t_k = 1973). The division of material production sphere into n = 15 sectors was used. Computations were made according to the formulae (14)-(17), (19), (20). We used (19) many times to derive the K_0 coefficient, taking various sets of parameter values T_{00}, δ_0, β_0. The variant with the least square error (18) was selected. The results are presented in Table 9.

Table 9. Parameters of the sector S_0 production function
n = 15, identification interval 1970-1973

α_{10}	α_{20}	α_{30}	α_{40}	α_{50}	α_{60}	α_{70}
0.021428	0.000144	0.037215	0.019157	0.004652	0.019559	0.098042
α_{80}	α_{90}	$\alpha_{10;0}$	$\alpha_{11;0}$	$\alpha_{12;0}$	$\alpha_{13;0}$	$\alpha_{14;0}$
0.255829	0.003568	0.002828	0.106877	0.001265	0.027362	0.107914
$\alpha_{15;0}$	α_{U0}	q_0	T_{00}	δ_0	β_0	K_0
0.006409	0.156362	0.131789	1	0.1	0.95	9.6176×10^5

Based on the identified model parameters, model values $Y_{j0}(t)$ $Y_{U0}(t)$, $Y_0(t)$ (formulae (11), (16) and (17) were computed and compared with statistical data $\tilde{Y}_{j0}(t)$, $\tilde{Y}_{U0}(t)$, $\tilde{Y}_0(t)$ for t = = 1970,...,1973. The runs $Y_0(t)$ and $\tilde{Y}_0(t)$ are presented in Fig. 3, to illustrate how the model is fitting to the reality.

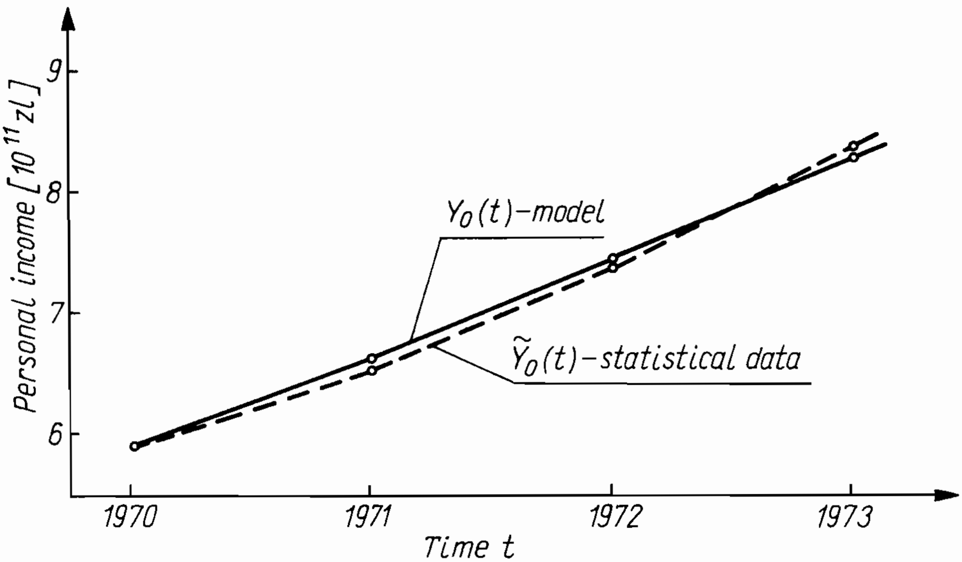

Figure 3.

2.1.2. Sector Development Forecast

Since the delay T_{00} appears in (16), the relations (9)-(11) and (16) allow to compute in an iterative way - from year to year - the future values $Y_{j0}(t)$, $Y_{U0}(t)$, $Y_0(t)$, i.e. for $t > t_k$. On this basis a forecast is derived [6] up to the year 1980, the results are presented in Table 10. Computations were performed on CYBER 72 computer (computing algorithm for the development forecast in economy is given in Part B).

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[6] The values $Y_{P0}(t)$ were obtained recurrently as

$$Y_{P0}(t) = Y_0(t) \sum_{j=1}^{15} \alpha_{j0}$$

Table 10

SECTOR S_0 PERSONAL CONSUMPTION

DATA

δ_0 = .1000 K_0 = .9618E+06 β_0 = .9500 T_{00} = 1

YEAR	Y_{P0}
1970	.42213E+06

MODEL RESULTS FORECAST 1971-1980

YEAR	POPULATION INCOME	Y_{P0}
1971	.64914E+06	.46209E+06
1972	.71004E+06	.50544E+06
1973	.77605E+06	.55243E+06
1974	.84755E+06	.60332E+06
1975	.92493E+06	.65841E+06
1976	.10086E+07	.71800E+06
1977	.10991E+07	.78240E+06
1978	.11968E+07	.85196E+06
1979	.13023E+07	.92703E+06
1980	.14160E+07	.10080E+07

FLOWS FROM SECTORS

SECTOR	YEAR			
	1971	1972	1973	1974
1	.13910E+05	.15215E+05	.16629E+05	.18161E+05
2	.93476E+02	.10225E+03	.11175E+03	.12205E+03
3	.24158E+05	.26424E+05	.28881E+05	.31541E+05
4	.12436E+05	.13602E+05	.14867E+05	.16236E+05
5	.30198E+04	.33031E+04	.36102E+04	.39428E+04
6	.12437E+05	.13604E+05	.14868E+05	.16238E+05
7	.63643E+05	.69613E+05	.76085E+05	.83095E+05
8	.16607E+06	.18165E+06	.19854E+06	.21683E+06
9	.23161E+04	.25334E+04	.27689E+04	.30240E+04
10	.18358E+04	.20080E+04	.21947E+04	.23969E+04
11	.69378E+05	.75887E+05	.82942E+05	.90583E+05
12	.82116E+03	.89820E+03	.98170E+03	.10721E+04
13	.17762E+05	.19428E+05	.21234E+05	.23191E+05
14	.70051E+05	.76623E+05	.83746E+05	.91462E+05
15	.41603E+04	.45506E+04	.49737E+04	.54319E+04

Table 10 - continued

SECTOR	YEAR 1975	1976	1977	1978	1979	1980
1	.19820E+05	.21613E+05	.23552E+05	.25645E+05	.27905E+05	.30343E+05
2	.13319E+03	.14524E+03	.15827E+03	.17234E+03	.18753E+03	.20391E+03
3	.34421E+05	.37536E+05	.40903E+05	.44540E+05	.48464E+05	.52698E+05
4	.17719E+05	.19322E+05	.21056E+05	.22927E+05	.24948E+05	.27127E+05
5	.43028E+04	.46922E+04	.51130E+04	.55676E+04	.60582E+04	.65874E+04
6	.17721E+05	.19325E+05	.21058E+05	.22930E+05	.24950E+05	.27130E+05
7	.90682E+05	.93889E+05	.10776E+06	.11734E+06	.12768E+06	.13882E+06
8	.23663E+06	.25804E+06	.28118E+06	.30618E+06	.33316E+06	.36226E+06
9	.32002E+04	.35988E+04	.39216E+04	.42703E+04	.46465E+04	.50524E+04
10	.26157E+04	.28524E+04	.31083E+04	.32846E+04	.36929E+04	.40045E+04
11	.98854E+05	.10780E+06	.11747E+06	.12791E+06	.13918E+06	.15134E+06
12	.11700E+04	.12750E+04	.13904E+04	.15140E+04	.16474E+04	.17913E+04
13	.25308E+05	.27598E+05	.30074E+05	.32747E+05	.35633E+05	.38746E+05
14	.99813E+05	.10885E+06	.11861E+06	.12915E+06	.14053E+06	.15281E+06
15	.59279E+04	.64644E+04	.70442E+04	.76104E+04	.83463E+04	.90754E+04

2.2. A Disaggregated Model

Notation:

V e c t o r s:

$\underline{G}(t) = \{G_i(t)\}_1^N$ = demand for material and nonmaterial goods

$\underline{p}(t) = \{p_i(t)\}_1^N$ = price of consumer goods

$\underline{w}(t) = \{w_i(t)\}_1^M$ = wages and social money benefits

$\underline{z}(t) = \{z_i(t)\}_1^N$ = consumption realized (purchases)

$\underline{X}_O(t) = \{X_{Oi}(t)\}_1^M$ = employment

$\underline{X}_{PO}(t) = \{X_{POi}(t)\}_1^{N_1}$ = material goods for individual consumption

$\underline{X}_{UO}(t) = \{X_{UOi}(t)\}_{N_1}^N$ = nonmaterial services for individual consumption

$\underline{X}(t) = \underline{X}_{PO}(t) + \underline{X}_{UO}(t)$

S c a l a r s :

k(t) = new credits and loans

s(t) = new savings

K(t) = population debts (credits and loans)

S(t) = savings account (banks, savings-and-loan cooperatives)

$Y_O(t)$ = net personal income of the population

$\overline{Y}_O(t)$ = income at the disposal

t = time

N = number of consumer goods taken into account

M = number of categories of wages and benefits taken into account

M a t h e m a t i c a l   r e l a t i o n s:

The relations among variables in the model are the following

$\underline{X}_O(t) = \underline{\alpha} L_O(t)$, ($\underline{\alpha} = \{\alpha_i\}_1^M$) = vector of employment factors)

$Y_O(t) = \langle \underline{w}(t), \underline{X}_O(t) \rangle + k(t)$

$\overline{Y}_O(t) = Y_O(t) - \delta K(t)$ (δ - rate of credit repayment)

$$K(t) = K(0) \exp(-\delta t) + \int_0^t \exp(-\delta(t-\tau))\, k(\tau) d\tau$$

$$G_i(t) = C_{gi}\left[\frac{\overline{Y}_0(t)}{L_0(t)}\right]^{\alpha_{Yi}}\left[p_i(t)\right]^{\alpha_{Pi}}\left[\frac{k(t)}{L_0(t)}\right]^{\alpha_{Ki}}\left[\frac{S(t)}{L_0(t)}\right]^{\alpha_{Si}} \quad 7)$$

$$i = 1,\ldots,N$$

$$\underline{z}(t) = \min\left[\underline{G}(t)\, L_0(t),\ \underline{X}_{P0}(t) + \underline{X}_{U0}(t)\right] \tag{21}$$

$$s(t) = \overline{Y}_0(t) - \langle \underline{z}(t), \underline{p}(t) \rangle$$

$$S(t) = S(0)\exp(\varepsilon t) + \int_0^t \exp(\varepsilon(t-\tau))\, s(\tau)\, d\tau$$

(ε = interest rate in savings)

where: vector parameters $\underline{\alpha}, \underline{\alpha}_Y, \underline{\alpha}_P, \underline{\alpha}_S, \underline{C}_g$ are derived 'ex post' from statistical data. Decision variables in the model are: wages $\underline{w}(t)$, prices $\underline{p}(t)$, credits $k(t)$ 8) and δ and ε .

A relation defining the demand for credit can be included additionally to the formula (21) as

$$k(t) = C_K\left[\frac{\overline{Y}_0(t)}{L_0(t)}\right]^{\beta_Y}\left[\frac{K(t)}{L_0(t)}\right]^{\beta_K}\left[\frac{\underline{X}(t) - \underline{z}(t), \underline{P}(t)}{L_0(t)}\right]^{\beta_z}\delta^{\beta_\varepsilon} \tag{22}$$

where $C_K, \beta_Y, \beta_K, \beta_z, \beta_\varepsilon$ are the factors derived 'ex post' from statistical data.

Purpose of the model

The model (21), (22) apart from its main purpose - the individual consumption subsystem in models MRI - can be also used independently, in, e.g.

- simulation research of prices and wages influence on the demand,
- simulation analysis of credits and savings politics,
- deriving prices (wages) with the assumed wages (prices)

---

7) In [10] the demand model is taken as a Stone's stochastic model.

8) In the future, prices and wages will be derived for the price and employment model. Credits K(t) will be linked with savings S(t).

and $\underline{X}(t)$ would assure the min $\| \underline{G}(t)\ \underline{L}_0(t) - \underline{X}(t) \|$,

- research on personal consumption share in national income.

A more detailed description of the model and results obtained are included in Part CI.

3. Modelling of the Aggregate Consumption Subsystem

An aggregate consumption subsystem (of the subsystem U) covers the sphere of nonmaterial services production and general consumption.

Similarly as in the sphere of material production we use a sector model of the subsystem U. In the present version of the model the subsystem U is derived into 4 sectors: education, research and development, health (including also social welfare and recreation services), and the "rest" sector. In the future new sectors will be separated from the rest sector. A diagram of

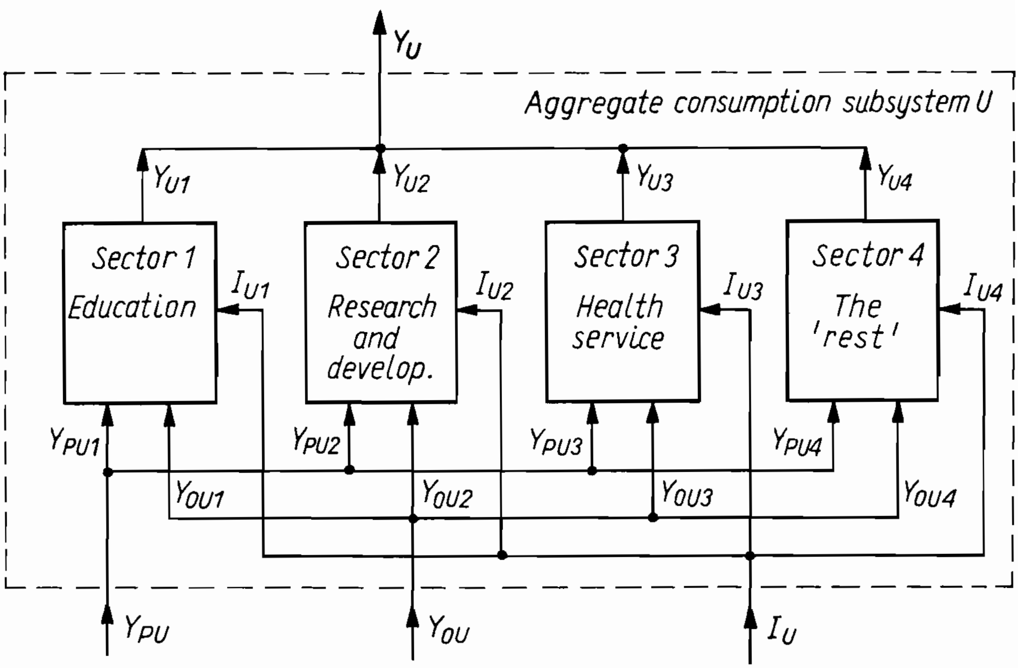

Figure 4.

the subsystem is shown in Fig. 4, where Y_{PU} - flow of material goods from a production subsystem, Y_{OU} - employment, T_U - investment outlays, Y_U - activity ("production") of the subsystem U.

3.1. Optimization of the Outlays Distribution

Since the statistical data published by C.S.O. do not contain the data on flows between the spheres of material produc - tion and nonmaterial services production, as well as the data on inter-sector flows in the subsystem U, we were compelled to apply slightly different methodology for construction of the subsystem U of our model.

Assuming that the influence of the aggregate consumption on the production sectors is similar to the productive investments in these sectors (inertia, delay), the sector production functions can be modified and taken in the form (see Pt. A Sec. 2.2)

$$Y_i(t) = \exp(\mu_i t) \sum_{\tau=-\infty}^{t-T_i} K_i \exp(-\delta_i(t-\tau-T_i)) \prod_{\nu=0}^{N} Z_{\nu i}^{\beta_{\nu i}}(\tau) \quad (23)$$

$$\sum_{\nu=1}^{N} \beta_{\nu i} = \beta < 1, \ \beta_{\nu i} \geqslant 0 \ \text{ for } \ \nu = 0,\dots,N, \ i = 1,\dots,n$$

where $Z_{0i}(\tau)$ = productive investments; $Z_{\nu i}(\tau)$ = outlays for the ν-th aggregate consumption sector,allocated for the activities in favour of the i-th production sector; μ_i, K_i, δ_i, $\beta_{\nu i}$, T_i = parameters derived from statistical data; n = number of sectors in the production subsystem; N = number of aggregate consumption sectors.

For the decision-makers an important problem is to find the optimal distribution of the limited assets $Z_U(t)$,which are used for aggregate consumption in particular consumption sectors.

The above problem can be formulated as the maximization problem of the income

$$Y = \sum_{t=0}^{T} g(t) \sum_{i=1}^{n} Y_i(t) \quad (24)$$

where: $g(t) = (1+\varrho)^{-t}$; ϱ = given discount rate; T = given planning period (e.g. 5-year); with constraints

$$\sum_{\nu=1}^{N} Z_{\nu}(t) \leqslant Z_U(t), \quad Z_{\nu}(t) = \sum_{i=1}^{n} Z_{\nu i}(t) \tag{25}$$

We can show that the optimal solution of the problem (24), (25) can be written in an explicit form:

$$\hat{Z}_{\nu}(t) = \frac{\sum_{i=1}^{n} \beta_{\nu i} f_i(t)}{\beta f(t)} Z_U(t), \quad \nu = 1,\dots,N, \; i = 1,\dots,n$$

$$\hat{Z}_{\ i}(t) = \frac{\beta_{\nu i} f_i(t)}{\sum_{j=1}^{n} \beta_{\nu j} f_j(t)} \hat{Z}_{\nu}(t) \tag{26}$$

where $f(t) = \sum_{i=1}^{n} f_i(t)$ and

$$f_i(t) = \left\{ \sum_{\tau=t+T_i}^{T} K_i \exp(-\delta_i(\tau-t-T_i))\, g(\tau) \times \prod_{\nu=1}^{N} (\beta_{\nu i}/\beta)^{\beta_{\nu i}} \left[Z_{0i}(\tau)\right]^{\beta_{0i}} \right\}^{\frac{1}{1-\beta}} \tag{27}$$

The problem of deriving the outlays Z_U for an aggregate consumption is connected with the problem of national income division. It has been already discussed in Section 1.

In the present versions of model MRI it is assumed that for each productive sector S_i "the factor coordination principle" holds (see Part A eq. 24 of Sec. 1.2). Then, according to (26) of Section 1.2 Part A, eq. (23) can be presented as:

$$Y_i(t) = \bar{K}_i \exp(\mu_i t) \int_{\tau=-\infty}^{t} \exp(-\delta_i(t-\tau-T_i))\, Z_{0i}^{\beta_i}(\tau) \tag{28}$$

The influence of $Z_{\nu i}$, $\nu = 1,\dots,N$, on the production of sector S_i, i = 1,...,n, is considered in parameters $\bar{K}_i$, $\bar{\mu}_i$ and β_i estimated on the basis of statistical data.

The production functions in the form of (28) were the basis for numerical computations of models MRI (see Part B).

3.2. The Sector Models

The disaggregated models of particular sectors of the subsystem U should cooperate in the future versions of the model with an aggregate consumption subsystem.The aim of the education sector model is to adjust the supply of trained labour in the education system, to the demand for a labour force (according to the skills) resulting from the model MRI. A model of the functioning of education system in Poland is presented in Pt.C.III. Methodological foundation of modelling of the research and development are included in [7] while mutual relations between these sectors and economy are presented in [6].

In [8] the model for the number of research workers of different qualifications in Poland has been investigated.

The goal of modelling of the health service system in Poland is to define the relations between government expenditures on health protection (maintenance and investment expenditures) and the effects of health service. A detailed description of the model is given in Part C.II.

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C.I. PERSONAL CONSUMPTION

Hanna Bury

1. Introduction

In the social and economic development models the personal consumption sphere plays an important part in connecting the flow of material goods and nonmaterial services with the labour supply.

In the MR4 model the personal consumption subsystem receives a part of goods and services produced by the production and aggregate consumption subsystems, and provides these subsystems with labour.

The personal consumption subsystem is regarded as the single sector S_0, connected with the production subsystem by the goods flow and with the aggregate consumption subsystem by the services flow. It is connected with both subsystem by labour flows, expressed in monetary units.

These interconnections are shown in Fig. 1.

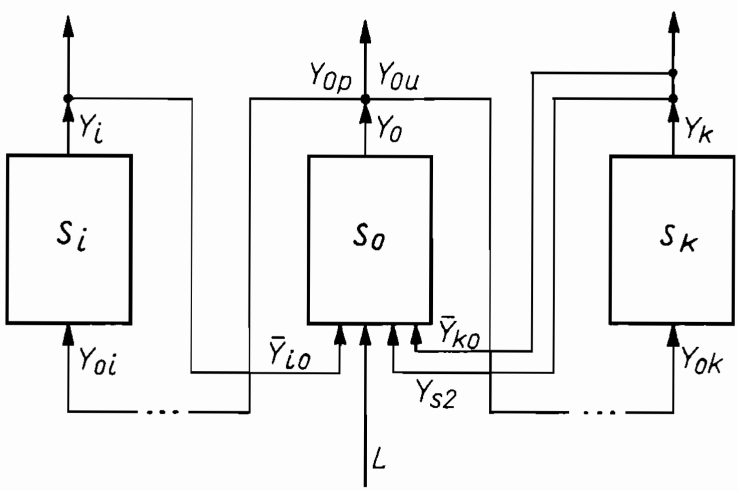

Figure 1.

Y_i = production of the i-th production sector, i = 1,...,n

$\bar{Y}_{io}$ = supply of the i-th material commodity (purchased from population personal income)

Y_k = production of the k-th sector of U subsystem, k = n+1,...,N

$\bar{Y}_{ko}$ = supply of the k-th nonmaterial service (purchased from population personal income)

L = demographic vector

To describe the personal consumption subsystem the following notation has been used:

$NI(t)$ = national income distributed

$Y_O(t)$ = net income of the population

$Y_O^*(t)$ = net personal income of the population

$\bar{Y}_O(t)$ = income at the disposal

$Y_{Op}(t)$ = wages and salaries in production subsystem

$Y_{Ou}(t)$ = wages and salaries in aggregate consumption subsystem

$Y_s(t)$ = social benefits

$Y_{s1}(t)$ = social money benefits

$Y_{s2}(t)$ = social benefits in kind

$Y_{Ox}(t)$ = other incomes, i.e. incomes of peasant population from agricultural production, incomes of owners of private enterprises and the so called free occupations, incomes of the population from other sources

$K(t)$ = population debts (credits and loans)

$k(t)$ = credits granted for population

$S(t)$ = savings deposits

$s(t)$ = new savings

$Y_{pO}(t)$ = material goods for private consumption

$Y_{uO}(t)$ = nonmaterial services for private consumption

$L_O(t)$ = population number

$w(t)$ = average net wage per annum

$\underline{G}(t) = \left\{G_i(t)\right\}_{i=1}^{N}$ = demand for material goods and nomaterial services

t = time

n = number of consumer goods taken into account

$N-n$ = number of consumer services taken into account

In the personal consumption model the relations between material goods and nonmaterial services supply, the demand submitted by consumers, the consumers income distribution as well as the goal of the subsystem are described.

The model consists of submodels containing demography, prices, incomes, market, savings and credits.

2. Utility Function

The long term goal of the personal consumption subsystem is to maximize the utility resulting from the consumption of material goods, especially durables, and nonmaterial services, which can be regarded as personal investments (e.g. education, public health and social welfare, etc.).

The following form of the utility function is assumed [1]:

$$U(Y) = U_0 \prod_{i=1}^{N} Y_{i0}(t)^{\gamma_i(t)} \qquad (1)$$

where: $\gamma_i(t)$ = given (identified) functions, $\gamma_i(t) > 0$, $i = 1, \ldots, N$; $\sum_{i=1}^{N} \gamma_i(t) < 1$; $Y_{i0}(t)$ = consumption of i-th material commodity for $i = 1, \ldots, n$.

In the case of the nonmaterial services consumption out of population income, or from social benefits in kind, the $Y_{i0}(t)$ depends on the government expenditures on the i-th nonmaterial (aggregate consumption) sphere:

$$Y_{i0}(t) = c_i(t) \sum_{\tau=0}^{t} k_i(t,\tau)\, Z_i^{\beta_i}(\tau - T_i), \quad i = 1, \ldots, N \qquad (2)$$

where: $k_i(t,\tau)$ = given, nonnegative functions; $k_i(t,\tau) = 0$ for $\tau < t$; $k_i(t,\tau) = k_i(t-\tau)$ in the case of the stationary model.

The typical example of $k_i(t,\tau)$ function is

$$k_i(t,\tau) = \begin{cases} K_i \exp(-\delta_i(t - T_i)), & t \geqslant T_i \\ 0, & t < T_i \end{cases} \qquad (3)$$

where: K_i, δ_i, T_i = given (identified) parameters; $c_i(t)$ represents the current state outlays; $\beta_i(t)$ = given (identified) functions; $Z_i(t)$ = the government investment expenditures in the i-th sector of the aggregate consumption.

It is assumed that $\sum_{i=n+1}^{N} Y_{i0}(t) = Y_{u0}(t)$.

The optimum consumption strategy can be derived by the maximization of the utility functional

$$\int_0^T v_u(t)\ U(Y)\ dt \tag{4}$$

where U(Y) is given by (1); $v_u(t)$ = given weight function.

Subject to the constraints

$$\sum_{i=1}^{N} Y_{i0}(t) \leqslant Y_0(t) \quad \text{for} \quad t = 0,\dots,T \tag{5}$$

$$\sum_{i=n+1}^{N} Z_i(t) \leqslant Z(t) \quad \text{for} \quad t = 0,\dots,T \tag{6}$$

or

$$\sum_{i=n+1}^{N} \int_0^T v_i(t)\ Z_i(t)\ dt \leqslant Z_N,\ Z_N = \int_0^T v_N(t)\,\gamma_N(t) NI(t-1) dt$$

where: Z(t) = the government investment expenditure in the year t; Z_N - the total government investment expenditures in the aggregated consumption sphere in (0,T); $\gamma_N(t)$ = the nonmaterial investment share in national income distributed; $v_i(t)$, $v_N(t)$ = given weight functions, i = n+1,...,N.

The solution of (4) gives (see [1]) the optimal consumption strategies for material goods and nonmaterial services.

3. Structure of the Personal Consumption Subsystem

The structure of the personal consumption subsystem is shown in Fig. 2.It is assumed that the subsystem consists of the following submodels: demography, wages, prices, demand, market and savings credits.

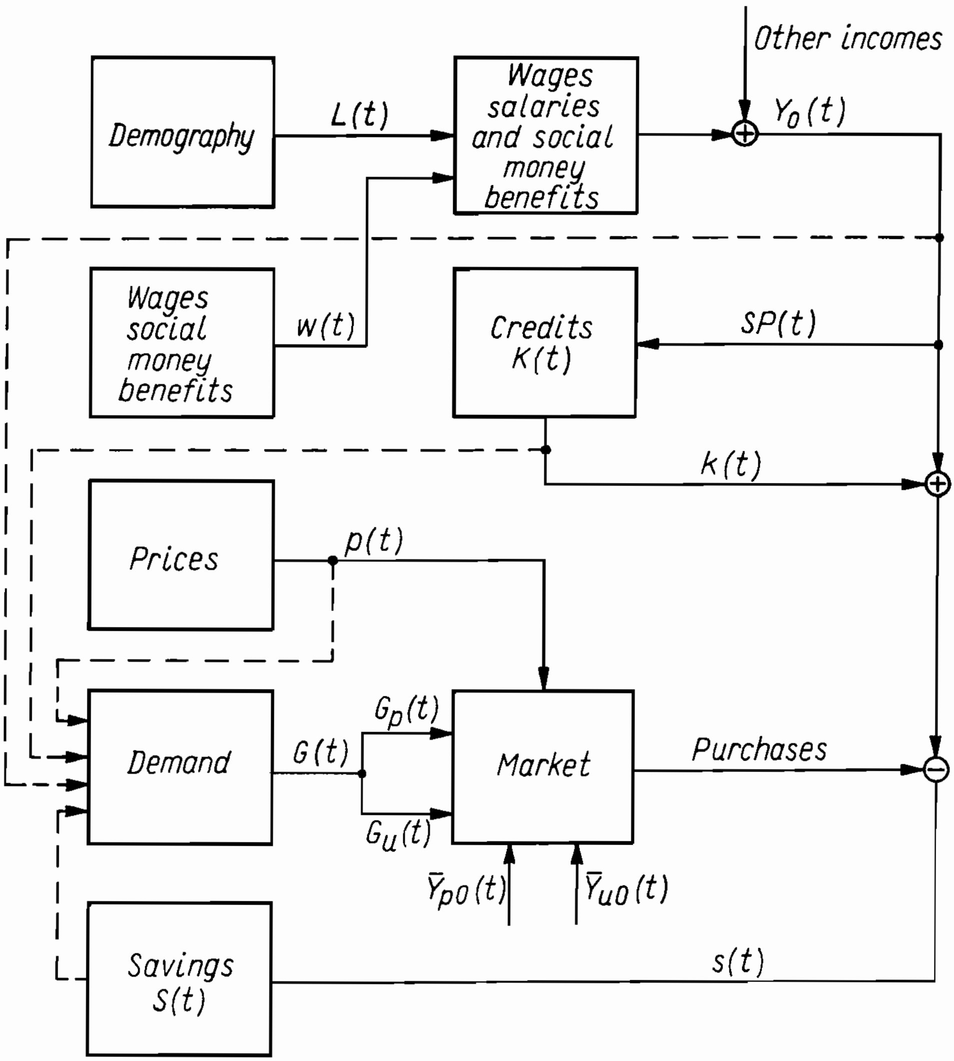

Figure 2.

To maintain the unity of the MR-models methodology we assume that the output of the personal consumption subsystem is the population income $Y_O(t)$:

$$Y_O(t) = Y_{Op}(t) + Y_{Ou}(t) + Y_s(t) + Y_{Ox}(t) \tag{7}$$

where

$Y_{Op}(t) = \sum_{i=1}^{n} Y_{Oi}(t)$, $Y_{Oi}(t)$ = wages and salaries in the i-th production sector

$Y_{Ou}(t) = \sum_{j=n+1}^{N} Y_{Oj}(t)$, $Y_{Oj}(t)$ = wages and salaries in the i-th sector of aggregate consumption subsystem.

The net personal income $Y_O^*(t) = Y_O(t) - Y_{s2}(t)$.

The population income is increased by the balance of credits

$$SK(t) = k(t) - SP(t),\ SP(t) = \text{credits repayment.}$$

The income at the disposal $\bar{Y}_O(t) = Y_O(t) + SK(t)$.

The distribution of population income becomes

$$\bar{Y}_O(t) = Y_{PO}(t) + Y_{UO}(t) + Y_{s2}(t) + Y_R(t) + s(t)$$

where

$Y_{PO}(t) = \sum_{i=1}^{n} Y_{iO}(t)$, $Y_{iO}(t)$ = consumption of the i-th material commodity

$Y_{UO}(t) = \sum_{j=n+1}^{N} Y_{jO}(t)$, $Y_{jO}(t)$ = consumption of the j-th nonmaterial service

$Y_R(t)$ = other expenditures.

We are now able to describe the submodels of the personal consumption subsystem.

A demographic submodel provides information about the population number, its age and professional structure and about the employment in the economy. The demographic vector L(t) has the following components:

$$L(t) = \begin{bmatrix} L_O(t) \\ X_{O1}(t) \\ \vdots \\ X_{ON}(t) \\ X_{Os}(t) \end{bmatrix}$$

where:

$X_{Oi}(t)$ = employment in the i-th sector of economy

$\sum_{i=1}^{n} X_{Oi}(t) = X_{Op}(t)$ = employment in production subsystem expressed in natural units

$X_{Os}(t)$ = number of persons who get the various kind of social benefits (in monetary units).

In the submodel of wages the average wage in sectors and in the whole economy, as well as the value of social benefits are given.

Social (money) benefits are given to the people who already do not work professionally (pensioners) and those who still do not work (students and pupils), but get scholarhips and grants, as well as for employed in the economy, who get statutory and workers benefits.

The sum of wages, salaries and of social money benefits becomes

$$Y_L(t) = \sum_{i=1}^{N} w_i(t)\ X_{Oi}(t) + \sum_{i=1}^{r} u_i(t)\ X_{Osi}(t) \tag{8}$$

where: $w_i(t)$ = the average net wage per annum in the i-th sector; $u_i(t)$ = the average value per capita of the i-th kind of social (money) benefits; $\sum_{i=1}^{r} X_{Osi}(t) = X_{Os}(t)$.

The prices-submodel gives information about the retail prices, their structure and dynamics.

The demand submodel which defines consumer demand for commodities is an important element of the personal consumption subsystem. The demand function is assumed in the following general form (see Ref. [3]):

$$\tilde{G}(t) = c\left[\mu(t)\right]^a \left[p(t)\right]^b \left[\tilde{k}(t)\right]^e \left[\tilde{S}(t)\right]^d \tag{9}$$

where: $\tilde{G}(t)$ = demand per capita; $\mu(t)$ = measure of population incomes, e.g. average wage per annum or value of national income distributed per capita; $p(t)$ = retail price of the commodity (this price is expressed as an index); $\tilde{k}(t)$ = value of credits per capita granted for consumers; $\widetilde{S}(t)$ = savings per capita; c,a,b,d,e = given (identified) coefficients; c = const., $c > 0$; a = demand elasticity with respect to income; b = demand elasticity with respect to price; d = demand elasticity with respect to savings; e = demand elasticity with respect to credits.

$G(t) = \widetilde{G}(t)\, L_0(t)$ - the global demand for a given commodity submitted by consumers may be expressed as follows:

$$G(t) = c_0\left[\mu(t)\right]^a \left[p(t)\right]^b \left[k(t)\right]^e \left[S(t)\right]^d \tag{10}$$

In the first version of personal consumption submodel the following relations were assumed

$$G_p(t) = c_p(t)\left[w(t)\right]^{a_p} \left[p_p(t)\right]^{b_p} = \text{for material goods} \tag{11}$$

(regarded as single commodity)

$$G_u(t) = c_u(t)\left[w(t)\right]^{a_u} \quad \text{for nonmaterial services} \tag{12}$$

(regarded as single commodity)

where $p_p(t)$ = retail price index number of goods bought by population.

The consumers demand is realized on the market with material goods flows supplied by the production subsystem.In the market submodel the consumers demand is equalized to the supply of goods $\overline{Y}_{p0}(t) = \sum_{i=1}^{n} \overline{Y}_{i0}(t)$.

The demand-supply balance is executed only for commodities purchased from population personal income and does not concern the consumption of social benefits in kind and nonmaterial services.

In the market submodel the following problem has been solved:

(a) the global demand-supply balance

$$\min_{w(t),\,p_p(t)} \left\{ c_p\left[w(t)\right]^{a_p} \left[p_p(t)\right]^{b_p} - \overline{Y}_{p0}(t)\right\}^2 \tag{13}$$

with constraints

$$w(t) \quad (1 + z)\, w(t - 1),\; p_p(t) \leqslant (1 + r)p_p(t - 1)$$

where $z > 0$, $r > 0$, z and r are decision variables;

(b) after solution of (a) in the case of total demand-supply balance, i.e. when the average wage $w(t)$ and total consumption price index are fixed, the partial balance is to be realized

$$\min_{p_i(t)} \left\{ c_i(t) \left[w(t) \right]^{a_i} \left[p_i(t) \right]^{b_i} - \overline{Y}_{0i}(t) \right\}^2 \qquad (14)$$

with constraints

$$p_i(t) \leqslant (1 + r_i)\, p_i(t - 1)$$

where $r_i > 0$ a decision variable, $p_i(t)$ = the i-th material commodity retail price.

The value of demand for a given commodity may be also influenced by credits; value of credits granted may be also a decision variable by determining the population income.

The credits model is described in the following form:

$$K(t) = K(t_0)(1 - \zeta)^{t-t_0} + \sum_{\tau=t_0+1}^{t} k(\tau)(1 - \zeta)^{t-\tau} \qquad (15)$$

where: ζ = rate of credit repayment; $K(t)$ = population debt.

It is assumed that the repayments ($SP(t)$) are proportional to debt, i.e.

$$SP(t) = \zeta\, K(t - 1)$$

In the credits model two kinds of credits are specified: credits for instalment purchases of goods and services,and credits for residential building. They are both described by the relation (15).

As a result of market purchases a part of population income is saved. We assume the following description of the savings model:

$$S(t) = S(t_0)(1 + \varepsilon)^{t-t_0} + \sum_{\tau=t_0+1}^{t} s(\tau)(1 + \varepsilon)^{t-\tau} \qquad (16)$$

where ε = interest rate in savings.

The global consumption of material goods (purchased from consumers personal income)

$$Y_{PO}(t) = \sum_{i=1}^{n} Y_{iO}(t) = \gamma_2(t)\ NI(t) \tag{17}$$

is one of the categories of national income distribution. The γ_2 function which determines the rate of personal consumption of material goods in national income is defined in the following form (see Part C. Introduction):

$$\gamma_2(t) = c\left[\frac{NI(t-1)}{L_O(t)}\right]^E \left[p(t)\right]^e \tag{18}$$

where c, E, e = given (estimated) coefficients, and may be modified by taking into account the last year consumption

$$\gamma'_2(t) = c\left[\frac{NI(t-1)}{L_O(t)}\right]^E \left[p(t)\right]^e \left[Y_{PO}(t-1)\right]^f \tag{19}$$

According to [2] it is assumed that the consumers demand for i-th commodity may be defined as follows:

$$\begin{aligned} \tilde{Y}_{i\nu}(t) &= \lambda_{i\nu}(t)\ Z_\nu(t), \quad \nu = 2 \\ Z_\nu(t) &= \gamma_\nu(t)\ NI(t) \end{aligned} \tag{20}$$

Comparing (20) and (11) one gets the following relation:

$$\lambda_{i2}(t) = \frac{c_i(t)\left[w(t)\right]^{a_i}\left[p_i(t)\right]^{b_i}}{Z_2(t)}$$

which may be applied for forecasting the λ-coefficients for the personal consumption sphere.

4. Empirical Results

The model described above was simulated in CYBER 72 computer, based on Central Statistical Office data for time interval 1965-1974. Parameters of the relations (11), (12) and (15) have been identified by the least squares method assuming that the past demand was equal to the consumption (i.e. we assume that the past demand has been realized).The results of identification are given in Table. The computation accuracy varies from 5 to 8%.

(a)

Demand	c	a	b
for material goods	5.87	1.314	.3394
for nonmaterial services	.3492	1.653	

(b)

Credits	
for individual residential building	0.055
for instalment purchases of goods and services	0.9

References

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[2] K. Borowiecka, J.B. Krawczyk, R. Kulikowski: Adaptive Model for Prices and Technological Change

[3] H. Wold: Demand Analysis. Uppsala 1952.

C.II. HEALTH SERVICE

Michał Bojańczyk

1. Introduction

The growing interest of different scientific centres in the construction of models of socio-economic growth by means of the system analysis methods can be observed recently. At present the system of models describing economic and social phenomena, as well as their interactions is being developed in Polish Academy of Sciences 1 .The consecutive modifications of the normative, macroeconomic, multisectorial model MR which describes the most important features of economic growth, and constitutes the so--called core-model, are the fundamental element of the system mentioned above (Part A, Fig. 13). Several other models as regional and sectorial ones (health care system model included in the latter) are cooperating with the core-model.

The health care system activities belong to the most important ones of the so-called non-material services production sphere (Part C). The goal of the health care sector - to cure, to protect against the diseases and generally speaking to prolong the longitude of a human life in the best possible physical and psychical state - is rather easy to be presented in a colloquial formulation. However, it is much more difficult to formalize this problem in a form of mathematical description of the quantitative relationships between main variables characterizing this system (see [8]).

The state defined as "health" is identified often with the negation of "illness" conception but the above definition does not satisfy anybody. Nevertheless, it is much easier to formulate the formalized description of the states of temporal or permanent disability, and after having it negated to get a description of the health level of the society.

Then such indices as morbidity, prevalence, incidence or mortality, fundamental in practice of the statistical offices of health sector, will be used for the construction of the relationships describing the activity of Health Care System (HCS).

The schema given below (Fig. 1) shows the main interactions of the core-model with the HCS model.

The health status of the society can be defined mainly by

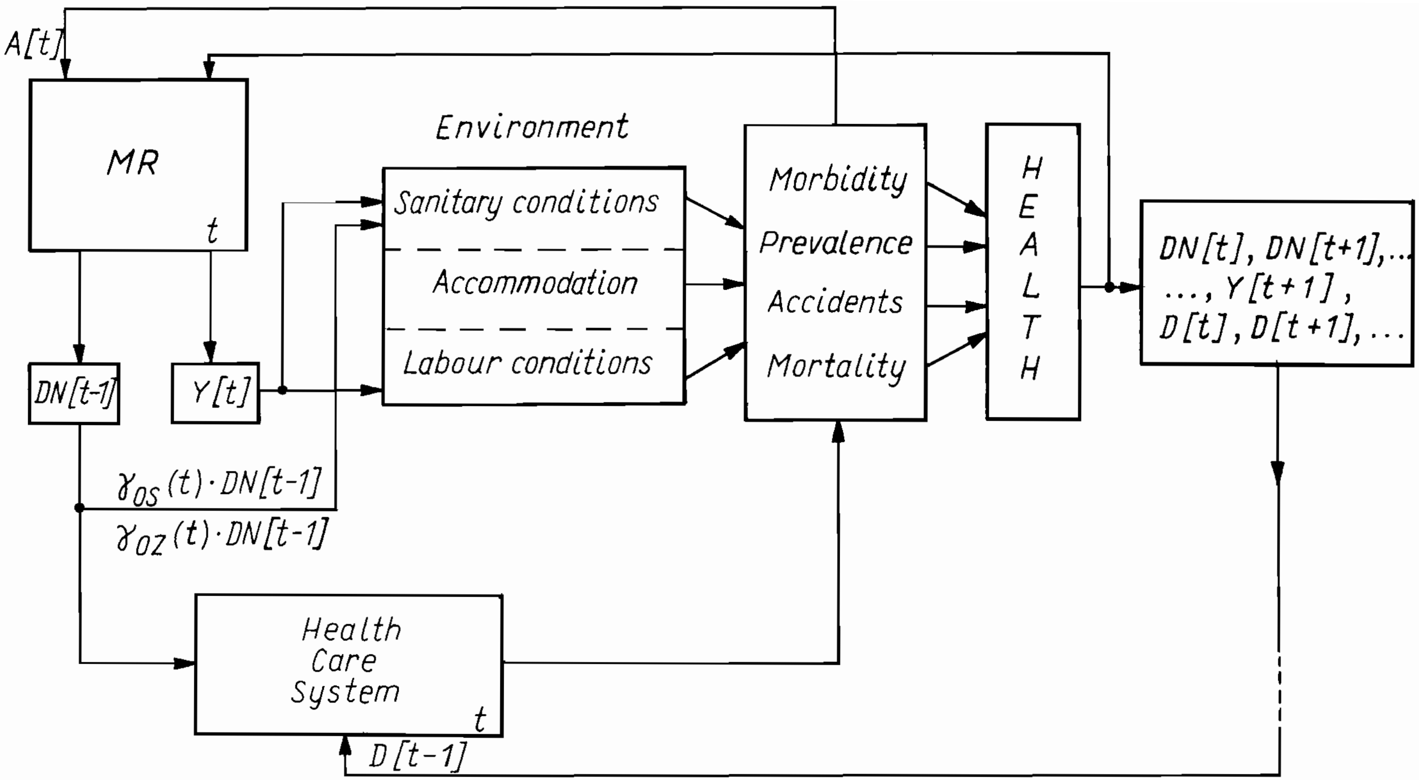

Figure 1. Health care system's interactions with other subsystems of the complex model
DN [t] = the national income in the year t, Y [t] = G.N.P. in the year t,
D [t] = population in the year t, $\gamma_{OS}(t)$ = the coefficient denoting the expenditures for the environmental protection to the national income ratio,
$\gamma_{OZ}(t)$ = the same as above but for the health care system.

the environmental conditions (sanitation,social environment,accomodation, nutrition, etc.) the biological characteristics of the population and - last but not least - the activity of HCS - see [9] and [10].

We can define the goal of HCS as follows: to satisfy (perhaps to modificate as well) the social demand for health services in the best possible way in the meaning of the given performance index and subject to the existing constraints. One assumes that the quantities such as the policy of economic growth, the regional and branch structure and levels of production and employment, the gross national product and the national income, the expenditures for health services and environmental protection, the demographic forecast, etc. are given exogeneously.

The HCS receives a certain part of the national income and realizes a social goal described above - to improve the health status of the whole society.

The government spends some amount of money on the HCS to cover the growing cost of health services. More services are supplied bigger expenditures are to be spent on them (see Fig. 2 - curve CD).

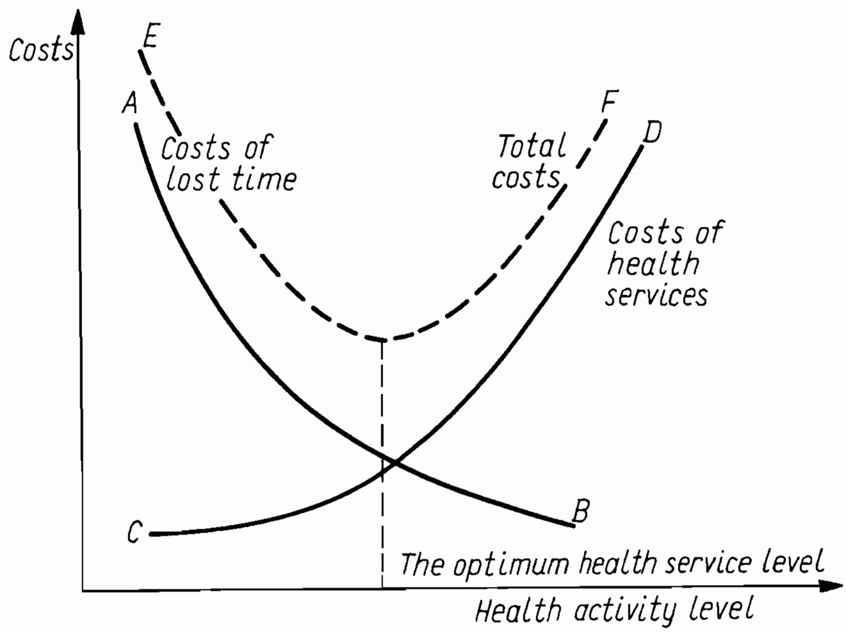

Figure 2. Costs of health care strategy.

Chosen statistical data on government expenditures and existing health services level

Year	Expenditures (million zl)			Investments (million zl)	Employment per 10000		Material base		Activity	
	total	OPC	IPC		physicians	nurses	beds per 10000	dispensaries	hospital stays/ 10000	consultations per citizen
1965	18009	2856	5463	x	12.6	24.4	59.3	5177	930.5	3.7
1966	20017	3745	6309	1596	13.0	25.7	60.7	5238	x	x
1967	22076	4200	6905	1580	13.4	26.9	61.1	5318	x	x
1968	23730	4500	7426	1731	13.8	28.4	61.4	5354	x	x
1969	25179	4788	7834	1831	14.5	30.1	61.9	5423	x	x
1970	26881	5004	8167	2236	15.1	31.5	62.9	5566	994.5	4.5
1971	27437	5521	8883	2773	15.6	32.7	63.6	5689	1042.3	5.0
1972	31712	6421	9869	3653	16.0	34.0	64.0	5819	1054.5	5.3
1973	x	x	x	x	16.4	35.1	64.9	x	1082.1	5.7

Some chosen statistical data of government expenditures and the existing health services level are shown in Table.

On the other hand for given demand for health services and certain health services level obtained there exists a corresponding cost of the lost time due to diseases and deaths. These costs could be calculated by taking into account such quantities as labour productivity, professional activity, labour force structure, socially lost time, salaries level, etc. It is to note that costs of the lost time decrease with the growth of health services level (understood as number of personnel, number of beds and consulting rooms - generally speaking: material base and employment level in the HCS) - see curve AB in Fig. 2.

The above mentioned tendencies are shown on a diagram (see Fig. 2).

For the existing health care system and socio-economic environment one can define the total costs connected with chosen health protection strategy as a sum of the cost of health services and the costs of the time lost - see curve EF in Fig. 2. There appears a problem to be discussed in the future: to choose an optimal health services level that minimizes this total cost. By now one considers the allocation of resources problem for given expenditures level.

The HCS model given in this paper is a multisectorial dynamic model with a hierarchical structure of management. The following problems will be presented - the description of therapeutical activities (in-patient care - IPC and out-patient care - OPC):

- the demand definition for health services;

- the optimal allocation of resources (financial and material ones).

2. The Therapeutical Activities (IPC and OPC) Modelling

The main problem in the modelling of system activities consists in the choice of outputs (describing the production level of a given system) and inputs (representing the production factors),and also in the definition of the functional relationships between inputs and outputs.

The Cobb-Douglas type production function has been chosen to describe both of the activities - in-patient care (IPC) and out-patient care (OPC) - because of its well known substitution of the production factors feature - see [6]. It is also possible to take linear form equations (Leontieff-type [2]) or even mixed type equations (Leontieff-Cobb-Douglas [2]). All these formulations should have a relationship describing the technology of the defined therapeutical processes. In the future it will be necessary to investigate the important problems of the organizational-technological progress (the change of therapeutical methods, the introduction of new drugs, etc.).

The main principle of the description of activities were to divide the whole population into the age (index i,(i = 1,...,I) and illness (index k, k = 1,...,K) groups.In such a way we may avoid such typical problems as:

- assignment the arbitrary weights to the age and illness groups (necessary to formulate optimization problems of aggregated products), and also
- averaging the production processes with significantly different parameters.

All input and output variables are understood as statistical one-year data values.

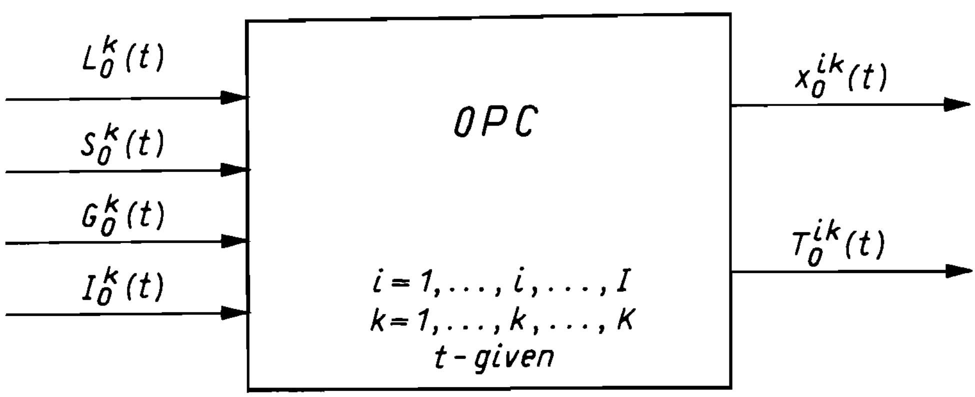

Figure 3. Out-patient care activity (OPC).

In Fig. 3. the scheme of out-patient care activity is shown. $x_0^{ik}(t)$ denotes the number of medical examinations in OPC-units (for people in the i-th age and k-th illness groups) in the year t, $T_0^{ik}(t)$ denotes the average time of the therapy prescribed by a physician, appropriate for the i-th age and k-th illness group in the year t.

The following functional relationships describe the OPC activity:

$$x_0^{ik}(t) = k_1^{ik}\left[L_0^k(t)\right]^{\psi_L^{ik}}\left[S_0^k(t)\right]^{\psi_S^{ik}}\left[G_0^k(t)\right]^{\psi_G^{ik}}\left[I_0^k(t)\right]^{\psi_I^{ik}} \qquad (1)$$

$$T_0^{ik}(t) = k_2^{ik}\left[\frac{L_0^k(t)}{D(t)}\right]^{\eta_L^{ik}}\left[\frac{S_0(t)}{D(t)}\right]^{\eta_S^{ik}}\left[I_0^k(t)\right]^{\eta_I^{ik}} \qquad (2)$$

where

k_1^{ik}, k_2^{ik} = constants

$D(t)$ = total number of people in the year t

$L_0^k(t)$ = number of physicians working in OPC-units and treating the patients belonging to the k-th illness group in the yeat t

$S_0^k(t)$ = the same as above but for an auxiliary personnel (nurses, etc.)

$G_0^k(t)$ = number of consulting rooms used in the k-th illness category in OPC-units in the year t

$I_0^k(t)$ = intensity of therapy in the k-th illness group (expenditures for drugs, injections - generally speaking other expenditures than for salaries and maintenance costs for existing buildings)

$\psi_L^{ik}, \psi_S^{ik}, \psi_G^{ik}, \psi_I^{ik}$ = corresponding elasticities of the Cobb--Douglas function describing the production of health services in OPC subsystem

$\eta_L^{ik}, \eta_S^{ik}, \eta_I^{ik}$ = corresponding elasticities of the Cobb-Douglas function describing the therapy time.

In analogous way the IPC activity has been described-Fig. 4:

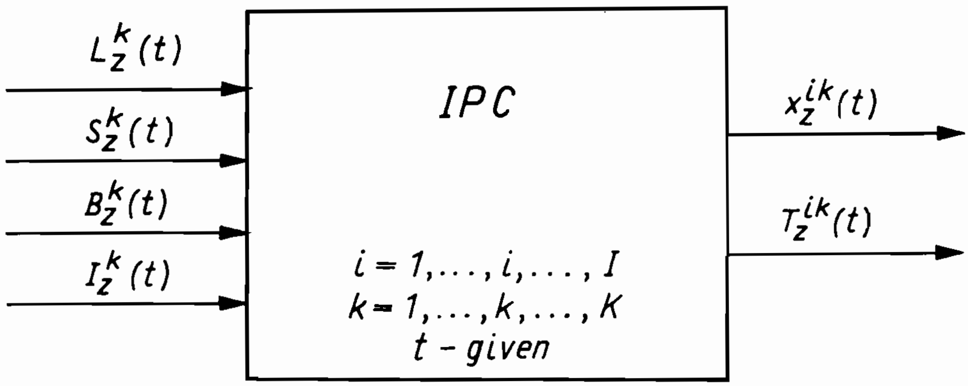

Figure 4. In-patient care activity (IPC).

$x_z^{ik}(t)$ = number of people belonging to the i-th age group who are treated in IPC units in the k-th illness category in the year t

$T_z^{ik}(t)$ = average duration of the patient stay in IPC units characteristic for the i-th age and k-th illness groups in the year t.

The relationships given below describe the IPC activity:

$$x_z^{ik}(t) = k_4^{ik}\left[L_z^k(t)\right]^{\varkappa_L^{ik}} \left[S_z^k(t)\right]^{\varkappa_S^{ik}} \left[B_z^k(t)\right]^{\varkappa_B^{ik}} \left[I_z^k(t)\right]^{\varkappa_I^{ik}} \tag{3}$$

$$T_z^{ik}(t) = k_5^{ik}\left[\frac{L_z^k(t)}{B_z^k(t)}\right]^{\varrho_L^{ik}} \left[\frac{S_z^k(t)}{B_z^k(t)}\right]^{\varrho_S^{ik}} \left[I_z^k(t)\right]^{\varrho_I^{ik}} \tag{4}$$

where

k_4^{ik}, k_5^{ik} = constants

$L_z^k(t)$ = number of physicians working in IPC-units and treating the patients belonging to the k-th illness group in the year t

$S_z^k(t)$ = the same as above but for an auxiliary personnel

$B_z^k(t)$ = number of beds occupied by the patients from the k-th illness group in the year t

$I_z^k(t)$ = intensity of therapy in the k-th illness group (see for OPC-description)

$\varkappa_L^{ik}, \varkappa_S^{ik}, \varkappa_B^{ik}, \varkappa_I^{ik}$ = corresponding elasticities of the Cobb-Douglas function describing the production of health services in IPC subsystem

$\varrho_L^{ik}, \varrho_S^{ik}, \varrho_I^{ik}$ = corresponding elasticities of the Cobb-Douglas function describing the mean duration of patient's stay.

The given above equations (1)-(4) describe the IPC and OPC activities from the point of view of the health services production capacity.

3. Demand of Health Services

Let the morbidity trends in the age and illness groups be assumed to be given in the model. They constitute the foundations for defining the demand for health services in both chosen therapeutical activities IPC and OPC (rehabilitation could be considered additionally).

So, one assumes that the sequences

$$\alpha_o^{ik}(t),\ \alpha_z^{ik}(t),\quad i = 1,\ldots,I,\ k = 1,\ldots,K \qquad (5)$$

are known, where $\alpha_{(\cdot)}^{ik}(t)$ denotes the relative demand for health services in the (.) activity in the i-th age and k-th illness group referred to the total number of the people belonging to this i-th age group (for example counted for 10000 people in this age group in the year t).

As it was mentioned above we propose to find these values resulting from the analysis of trends observed in the past, say, M years (or for the so-called civilization diseases -could be defined taking into account the data from highly developed countries as Sweden for example). They describe - in the sense of average - the health status of the society characteristic for existing sanitary conditions, working conditions and the level and quality of the present state of health services (diagnosis and therapy effectiveness, extension of prophylaxis and protection) - see [5]. It is not earlier, that these factors have been considered and included to (5) in the form of causal-consequential relationships that one could define more precisely the demand for health services - see [3] and [7].

Using the forecasts of (5) type and knowing the numbers of people in the age groups $D^i(t)$ one can describe the demand for health services in the k-th illness group as follows:

$$\bar{x}_o^k(t) = \sum_{i=1}^{I} \alpha_o^{ik}(t)\, D^i(t) \tag{6}$$

$$\bar{x}_z^k(t) = \sum_{i=1}^{I} \alpha_z^{ik}(t)\, D^i(t) \tag{7}$$

and - introducing the auxiliary variables -

$$l^i(t) = D^i(t) \Big/ \sum_{i=1}^{I} D^i(t) = D^i(t)/D(t) \tag{8}$$

$$\bar{x}_o^k(t) = D(t) \sum_{i=1}^{I} l^i(t) \quad {}_o^{ik}(t) \tag{9}$$

$$\bar{x}_z^k(t) = D(t) \sum_{i=1}^{I} l^i(t) \quad {}_z^{ik}(t) \tag{10}$$

4. Formulation of the Optimization Problems

4.1. Introduction

The expenditures and resources allocation among the illness groups and different fields of health activities in a chosen planning period is the main problem under consideration. It is of a hierarchical and complex structure.

At the beginning we may introduce the following definition of the socially lost time (denoted in the text by A) as a res-

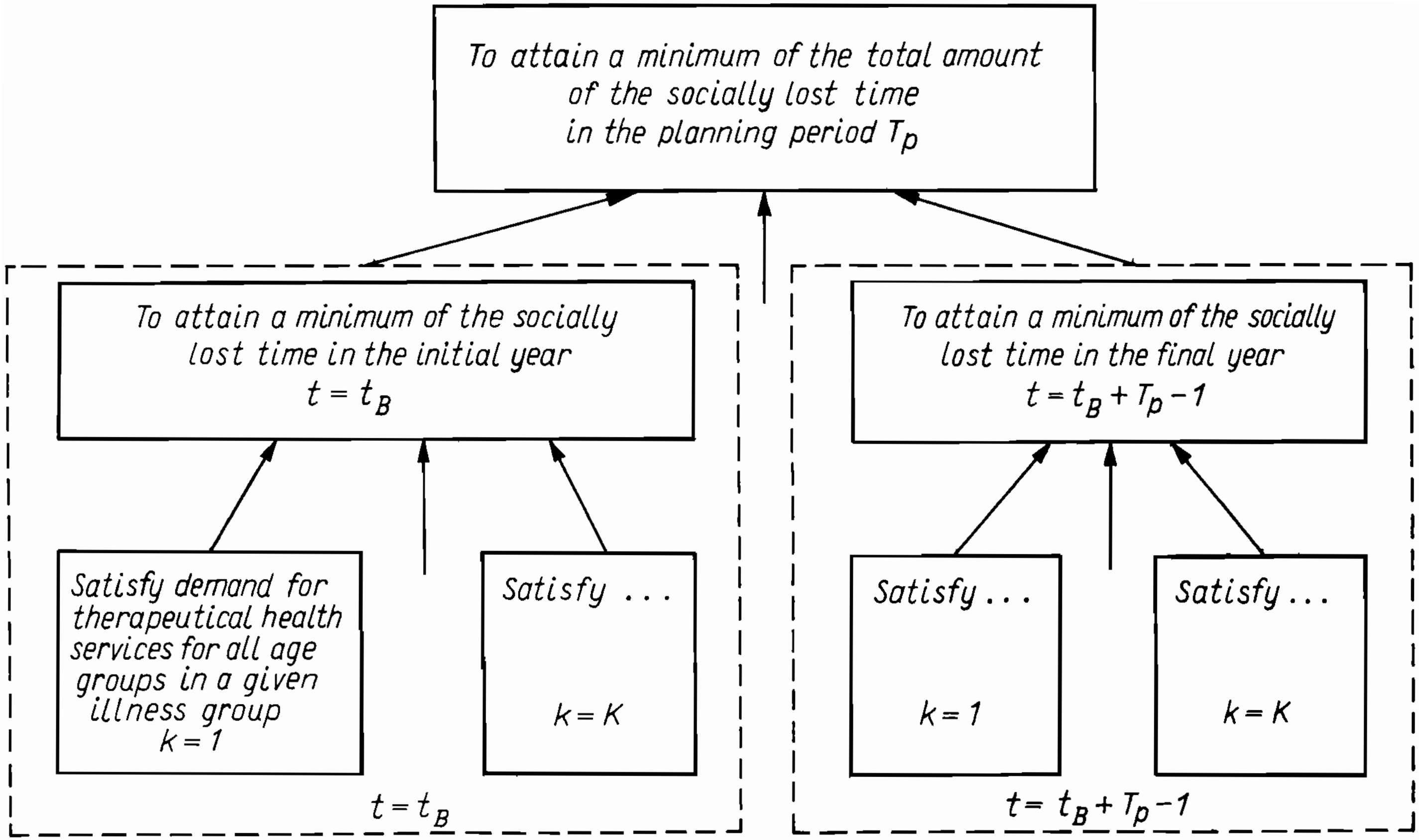

Figure 5. Optimization problems structure.

ult of the diseases and deaths (i.e. the total amount of time lost by all the numbers of the society - to loose the time means to be excluded from the social activities, e.g. from work for employed people, education for pupils and students, leisure time, etc.). With the help of the introduced above definition one can formulate in a general way the optimization considered in details a little further. It has been shown in a diagram in Fig. 5.

So the main management principle for HCS is to allocate the expenditures and resources (current expenditures and investments) in such a way that the socially lost time in the planning period T_p/$A(T_p)$/ obtains its minimum, and at the same time the demand for health services is satisfied, as well as possible (other social and economic factors given exogeneously).

The management structure is a multilevel one. The main informations for HCS taken from the core-model MR [4] are:

- the total expenditures (as the part of the national income) for the HCS activities (global - for one-year periods and for the planning period T_p or divided into investments and current expenditures),
- the total industrial production (G.N.P.) and its sector structure,
- the current flows from individual sectors (for example - pharmaceutical industry of the chemical sector),
- the employment level and its sectorial structure.

As soon as the optimization problems defining the goal and the quality of the HCS activity are solved - we are informed about the scale of a decrease in the disponible fund of work (i.e. of the total amount of work hours). This in turn results in the losses in G.N.P. and the national income due to partial use of the disponsible human productive potential.

The upper level problem - to minimize the socially lost time in the planning period - is a dynamic one.

The possibility to satisfy the actual demand for health services depends on the allocation strategy of the expenditures and material resources for the therapy (and other health activities -as prophylaxis for example-if such are to be included in the model in the future) - see Fig. 8 - in one-year periods and in the whole planning period. It is also influenced by the investment strategy for the enlargement or modification of existing health services production capacities (building facilities and experts).

An obvious dependence of the demand for health services and the production capacities on the activity of HCS in the past is tried to be represented (though it is not the only dependence - the influence of the other socio-economic factors for the sake of the present paper is to be considered treated as exogenous).

4.2. Optimization of Therapeutical Activities in One-year Period (Levels I and II)

Consider the k-th illness group and the initial period time $t = t_B$. One considers at first the following static local problems of the lowest management level (denoted as the optimization level I): to minimize the deviation between the manifested demand for health services in different fields of therapeutical activity (for given k-th illness group and chosen t time-period) and the potential supply defined by the actual state of material and human resources and also by the level of current expenditures - see Fig. 6).

At the level I one gets the following informations from the upper level II:

$\Gamma^k(t)$ = current expenditures for the k-th illness group in the year t

$L^k(t)$ = number of physicians working in IPC and OPC, and treating the illness belonging to the k-th illness group in the year t

$S^k(t)$ = the same as above but for the auxiliary personnel

$G_0^k(t)$ = number of the consulting rooms needed for the treatment of the illnesses from the k-th group in the year t

$B_z^k(t)$ = number of beds occupied by the ill people of the k-th illness group in the year t.

Before the control problem for this optimization problem has

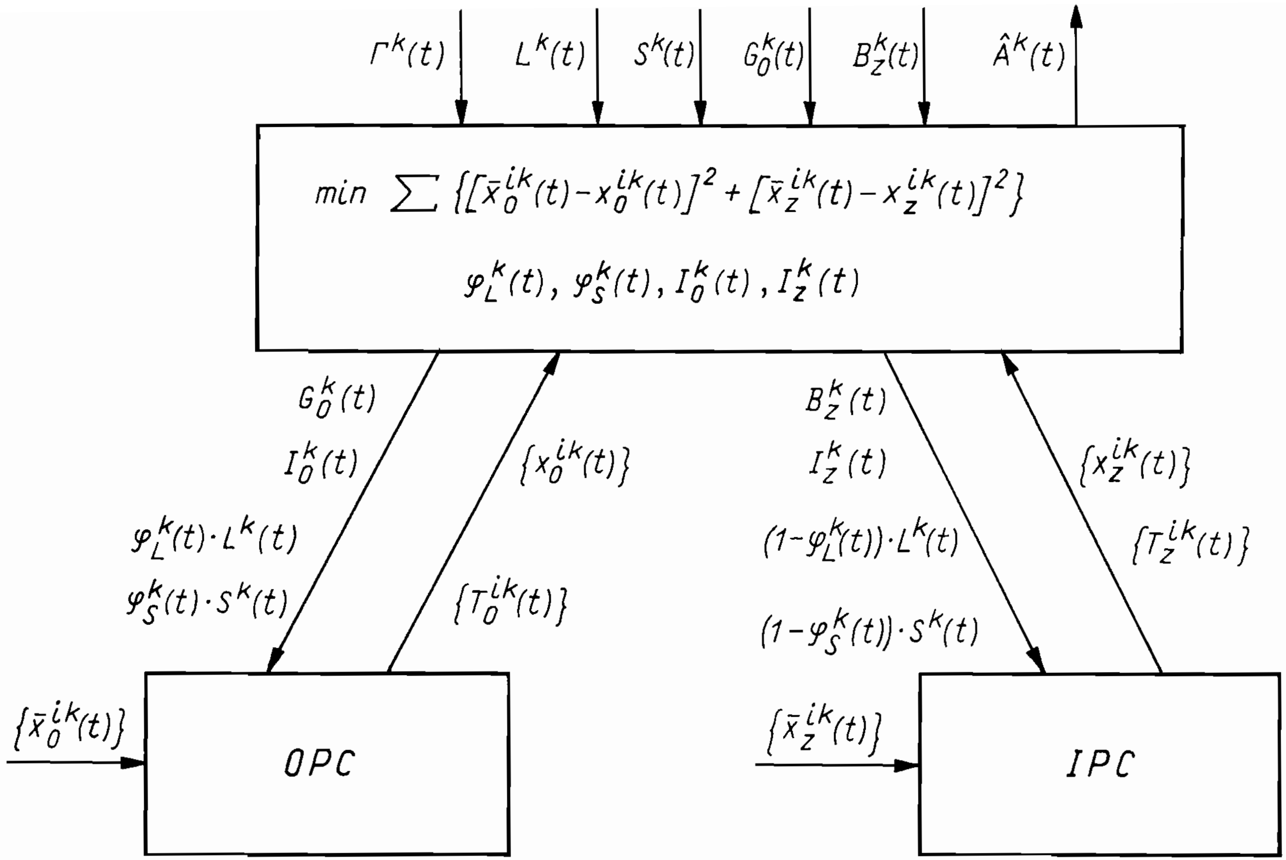

Figure 6. Optimization level I (demand satisfaction for the k-th illness group and time t).

been formulated it is necessary to introduce the following variables:

$\varphi_L^k(t)$ = the coefficient of distributing the physicians in OPC activity for the k-th illness group in the year t

$\varphi_S^k(t)$ = the same as above but for the auxiliary personnel.

So the quantities - characterized by the equations given below

$$L_0^k(t) = \varphi_L^k(t)\, L^k(t) \tag{11}$$

$$S_0^k(t) = \varphi_S^k(t)\, S^k(t) \tag{12}$$

denote respectively the numbers of physicians and auxiliary personnel who work in OPC and treat the illnesses of the k-th illness group while the quantities:

$$L_z^k(t) = (1 - \varphi_L^k(t))L^k(t) \tag{13}$$

$$S_z^k(t) = (1 - \varphi_S^k(t))S^k(t) \tag{14}$$

denote the use of the production factors such as employment for IPC activity.

If we assume the complete use of the physicians, the auxiliary personnel and the material base (consulting rooms, beds) the equations (1)-(2) and (3)-(4) can be rewritten in a form:

$$x_0^{ik}(t) = \bar{k}_1^{ik}(\ {}_L^k(t))^{\psi_L^{ik}}\ (\varphi_S^k(t))^{\psi_S^{ik}}\ (I_0^k(t))^{\psi_I^{ik}} \tag{15}$$

$$T_0^{ik}(t) = \bar{k}_2^{ik}(\ {}_L^k(t))^{\eta_L^{ik}}\ (\varphi_S^k(t))^{\eta_S^{ik}}\ (I_0^k(t))^{\eta_I^{ik}} \tag{16}$$

$$x_z^{ik}(t) = \bar{k}_4^{ik}(1 - \varphi_L^k(t))^{\varkappa_L^{ik}}\ (1 - \varphi_S^k(t))^{\varkappa_S^{ik}}\ (I_z^k(t))^{\varkappa_I^{ik}} \tag{17}$$

$$T_z^{ik}(t) = \bar{k}_5^{ik}(1 - \varphi_L^k(t))^{\varrho_L^{ik}}\ (1 - \varphi_S^k(t))^{\varrho_S^{ik}}\ (I_z^k(t))^{\varrho_I^{ik}} \tag{18}$$

where

$$\bar{k}_1^{ik} = k_1^{ik}(L^k(t))^{\psi_L^{ik}}\ (S^k(t))^{\psi_S^{ik}}\ (G_0^k(t))^{\psi_G^{ik}} \tag{19}$$

$$\bar{k}_2^{ik} = k_2^{ik}(L^k(t))^{\eta_L^{ik}} (S^k(t))^{\eta_S^{ik}} (D(t))^{-(\eta_L^{ik} + \eta_S^{ik})} \tag{20}$$

$$\bar{k}_4^{ik} = k_4^{ik}(L^k(t))^{\varkappa_L^{ik}} (S^k(t))^{\varkappa_S^{ik}} (B_z^k(t)^{\varkappa_B^{ik}} \tag{21}$$

$$\bar{k}_5^{ik} = k_5^{ik}(L^k(t))^{\varrho_L^{ik}} (S^k(t))^{\varrho_S^{ik}} (B_z^k(t))^{-(\varrho_L^{ik} + \varrho_S^{ik})} \tag{22}$$

At level I one solves the following local problem:

$$\min_{\varphi_L^k(t), \varphi_S^k(t), I_0^k(t), I_z^k(t)} \sum_{i=1}^{I} \left[(x_0^{ik}(t)-\bar{x}_0^{ik}(t))^2+(x_z^{ik}(t)-\bar{x}_z^{ik}(t))\right]^2 \tag{23}$$

subject to the local inequalities

$$0 \leqslant \varphi_L^k(t) \leqslant 1 \tag{24}$$

$$0 \leqslant \varphi_S^k(t) \leqslant 1 \tag{25}$$

$$x_0^{ik}(t) \leqslant \bar{x}^{ik}(t), \quad i = 1,\dots,I \; ^{1)} \tag{26}$$

$$x^{ik}(t) \leqslant \bar{x}^{ik}(t), \quad i = 1,\dots,I \; ^{1)} \tag{27}$$

$$\sum_{i=1}^{I} x_z^{ik}(t) \; T_z^{ik}(t) \leqslant 365 B_z^k(t) \tag{28}$$

$$I_0^k(t) + I_z^k(t) \leqslant \bar{\Gamma}^k(t) \tag{29}$$

where

$$\bar{\Gamma}^k(t) = \Gamma^k(t) - c_1^k(t)L^k(t) - c_2^k(t)S^k(t) + - c_3^k(t)G^k(t) - c_4^k(t)B^k(t) \tag{30}$$

---

[1] These manifested demands for health services given by formulae (9)-(10), (53)-(54) could be multiplied by "safety" factors.

where: $c_1^k(t)$, $c_2^k(t)$ = the average one-year salaries of the physicians and auxiliary personnel in the k-th illness group; $c_3^k(t)$ = the necessary current unit expenditures for maintenance of the existing facilities in OPC; $c_4^k(t)$ = the same as above but for IPC.

From the solution of this problem for the quantities

$$\Gamma^k(t),\ G_0^k(t),\ B_z^k(t),\ S^k(t),\ L^k(t) \tag{31}$$

given parametrically we get the quantities

$$\hat{A}_0^k(t) \quad \text{and} \quad \hat{A}_z^k(t) \tag{32}$$

which characterize the therapeutical process for the k-th illness group under conditions of the optimal allocation (in a sense of the performance index (23)) of the expenditures and the material resources among the two therapeutical activities under consideration: IPC and OPC. It is necessary to discuss the meaning of the quantities $A_0^k(t)$ and $A_z^k(t)$ - their sum represents the total time socially lost due to the illness from the k-th illness group in the year t.

So

$$A^k(t) = AI^k(t) + AII^k(t) + AIII^k(t) \tag{33}$$

$$AI_0^k(t) = \sum_{i=1}^{I} x_0^{ik}(t)(1 - a_0^{ik}(t))T_0^{ik}(t) \tag{34}$$

is the total time of illness for the persons treated in OPC ($a_0^{ik}(t)$ = the mortality in the i-th age and k-th illness group for the illness stadium appropriate for treatment in OPC),

$$AII_0^k(t) = \sum_{i=1}^{I} x_0^{ik}(t)a_0^{ik}(t)\left[T^{i,\infty}(t) - T^i\right] \tag{35}$$

represents the time socially lost due to deaths of persons treated in OPC where

$T^{i,\infty}(t)$ = average life expectancy for the i-th age group
T^{i} = average age in the i-th age group

and finally

$$AIII_0^k(t) = f\left[\Delta \bar{x}_0^{ik}(t)\right] \tag{36}$$

is the time socially lost due to the deaths of persons who stayed beyond the OPC and the HCS as a whole (as the result of the unsatisfying of the demand) where

$$\bar{x}_0^{ik}(t) = \bar{x}_0^{ik}(t) - x_0^{ik}(t) \geqslant 0 \tag{37}$$

denotes the extent of the unsatisfaction because of (6)-(7), $\bar{x}_0^{ik}(t)$ = the demand for health services in OPC in the i-th age and k-th illness group, $x_0^{ik}(t)$ = services supplied, while $f\left[\Delta \bar{x}_0^{ik}(t)\right]$ is a certain function (see (60)).

In analogous way one defines the quantity $A_z^k(t)$.

Let the quantity

$$\hat{A}^k(t) = \hat{A}_0^k(t) + \hat{A}_z^k(t) \tag{38}$$

be defined as the total time socially lost due to the illnesses (and deaths) belonging to the k-th illness group.

Next we can introduce the following variables:

$LN^{*}(t)$ = increase in the total number of physicians in the year t

$L^kS(t)$ = number of physicians working in the k-th illness group at the beginning of the year t

$\xi^k(t)$ = coefficient denoting how many of new physicians (from the total number LN(t)) will work in the k-th illness group in the year t

$L(t)$ = total average number of physicians in the year t

$S^{*}(t)$ = total number of auxiliary personnel in the year t

$G_0^{*}(t)$ = total number of consulting rooms in the OPC activity in the year t

$B_z^{*}(t)$ = total number of beds in the IPC activity in the year t

Obviously

$$L^k(t) = LS^k(t) + \xi^k(t)LN^{*}(t) \tag{39}$$

At the upper level (II) one considers the allocation problem for the one-year expenditures $\Gamma(t)$ and the disponible resources $LN^{*}(t)$, $S^{*}(t)$, $G_o^{*}(t)$, $B_z^{*}(t)$ among the individual illness groups $k \in \{1,\dots,K\}$ i.e. the minimization problem (see Fig. 7).

$$\min_{\Gamma^k(t),\xi^k(t),S^k(t),G_o^k(t),B_z^k(t)} \sum_{k=1}^{K} \hat{A}^k(t) \tag{40}$$

subject to the following equality and inequality constraints:

$$0 \leqslant \sum_{k=1}^{K} \Gamma^k(t) \leqslant \Gamma(t) \tag{41}$$

$$\sum_{k=1}^{K} \xi^k(t) = 1 \tag{42}$$

$$\sum_{k=1}^{K} G_o^k(t) = G_o^{*}(t) \tag{43}$$

$$\sum_{k=1}^{K} B_z^k(t) = B_z^{*}(t) \tag{44}$$

$$\sum_{k=1}^{K} S^k(t) = S^{*}(t) \tag{45}$$

$$c_1^k(t)L^k(t) + c_2^k(t)S^k(t) + c_3^k(t)G^k(t) + c_4^k(t)B_z^k(t) < \Gamma^k(t) \tag{46}$$

$$G_o^k(t) \geqslant 0, \quad k = 1,\dots,K \tag{47}$$

$$B_z^k(t) \geqslant 0, \quad k = 1,\dots,K \tag{48}$$

$$S^k(t) \geqslant 0, \quad k = 1,\dots,K \tag{49}$$

$$\xi^k(t) \geqslant 0, \quad k = 1,\dots,K \tag{50}$$

As a result of the solution of problem belonging to these two levels (I and II) one gets (for the system with the manage-

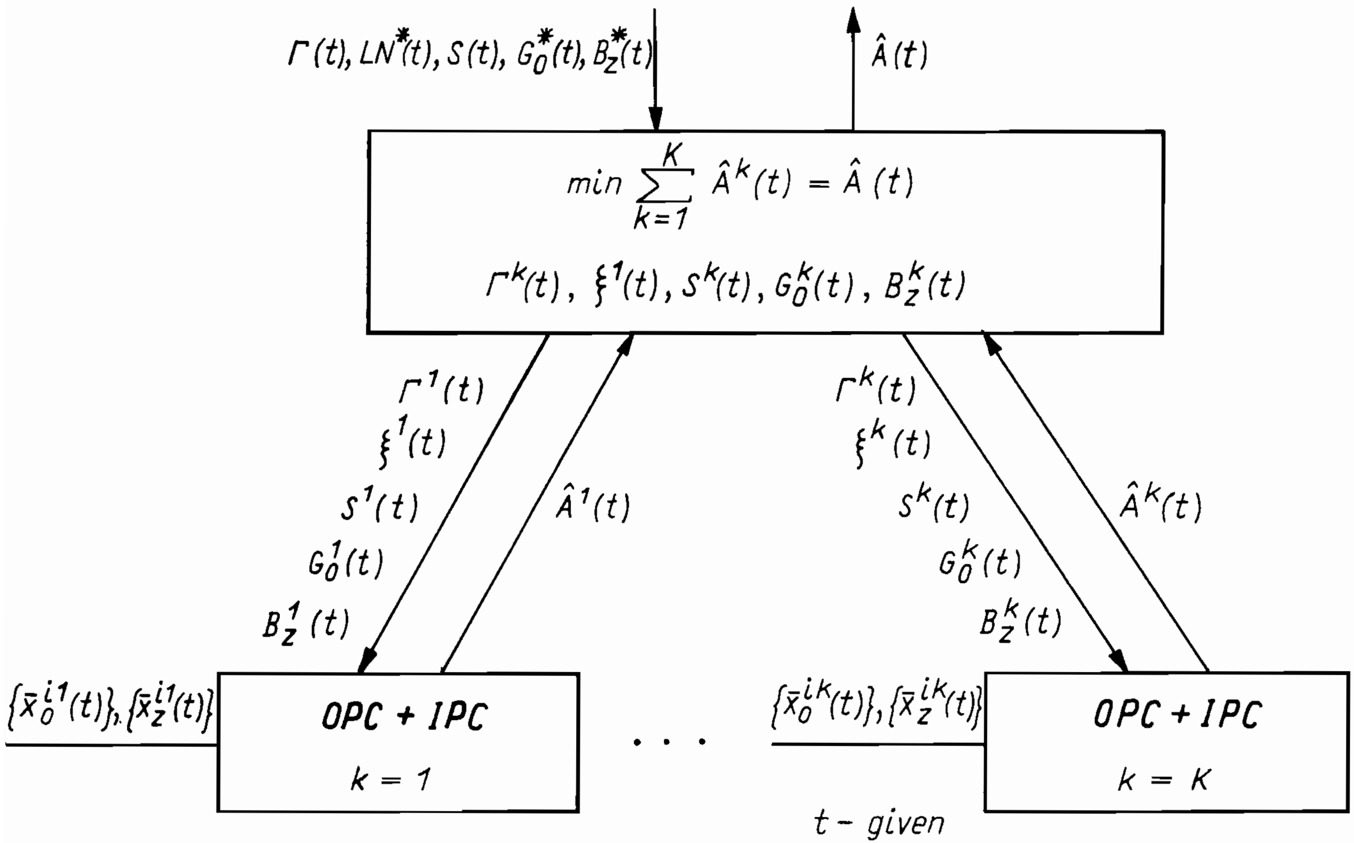

Figure 7. Optimization level II (minimization of the socially lost time in year t).

ment structure given by the performance indices (23) and (40)) the minimal time socially lost due to the illnesses and deaths for a given:

- level of current expenditures for HCS in the year t, and
- investment strategy in the part (by now given parametrically).

4.3. Demand for Health Services Confronted with the Production Capacity

Up to now it was assumed that the forecasts of the demand for health services in the activity (.) in a form of a set of the coefficients $\alpha^{ik}_{(\cdot)}(t)$ (eq. (5)) are given. Now the demand for health services in the activity (.) in the year t_B+1 should be found knowing the forecasts $\alpha^{ik}_{(\cdot)}(t_B)$, $\alpha^{ik}_{(\cdot)}(t_B+1)$ and the possibilities of the demand realization.

Let the k-th illness group in the moment t_B be considered. The demand for health services in both fields of activity (OPC and IPC) will be given by the following formulae:

$$\bar{x}_o(t_B) = \left[\sum_{i=1}^{I} l^i(t_B)\alpha^{ik}_o(t_B)\right] D(t_B) \tag{51}$$

$$\bar{x}^k_z(t_B) = \left[\sum_{i=1}^{I} l^i(t_B)\alpha^{ik}_z(t_B)\right] D(t_B) \tag{52}$$

where

$D(t)$ = the total number of people in the year t

$l^i(t)$ = the proportion of the i-th age group in D(t).

The similar equations can be written for the age and illness groups:

$$\bar{x}^{ik}_o(t_B) = \alpha^{ik}_o(t_B) l^i(t_B) D(t_B) \tag{53}$$

$$\bar{x}^{ik}_z(t_B) = \alpha^{ik}_z(t_B) l^i(t_B) D(t_B) \tag{54}$$

One assumes further that the realization level of health services will satisfy the following conditions:

$$x_o^{ik}(t_B) \leqslant \bar{x}_o^{ik}(t_B), \quad i = 1,\ldots,K \tag{55}$$

$$x_z^{ik}(t_B) \leqslant \bar{x}_z^{ik}(t_B), \quad i = 1,\ldots,K \tag{56}$$

So the total time socially lost due to the illnesses from the k-th illness group in the year t_B for the given state of the medical knowledge and the actual environmental conditions can be defined by:

- disability time of persons treated in OPC and IPC - the components of the type

$$(1 - a_{(\cdot)}^{ik}(t_B))x_{(\cdot)}^{ik}(t_B)T_{(\cdot)}^{ik}(t_B) \tag{57}$$

- morbidity in the illness group - the components of the type

$$x_{(\cdot)}^{ik}(t_B)a_{(\cdot)}^{ik}(t_B)\left[T^{i,\infty}(t_B) - T^i\right] \tag{58}$$

for persons treated (the notation is the same as in (4.2)) and

$$f_{(\cdot)}\left[\Delta\, \bar{x}_{(\cdot)}^{ik}(t_B)\right] \tag{59}$$

for persons beyond treatment in the activity field (.).

Finally

$$A^k(t_B) = \sum_{i=1}^{I}\left[1 - a_o^{ik}(t_B)\right]x_o^{ik}(t_B)T_o^{ik}(t_B) +$$

$$= \sum_{i=1}^{I}\left[1 - a_z^{ik}(t_B)\right]x_z^{ik}(t_B)T_z^{ik}(t\ ) +$$

$$= \sum_{i=1}^{I} a_o^{ik}(t_B)x_o^{ik}(t_B)\left[T^{i,\infty}(t_B) - T^i\right] +$$

$$= \sum_{i=1}^{I} a_z^{ik}(t_B)x_z^{ik}(t_B)\left[T^{i,\infty}(t_B) - T^i\right] +$$

$$+ \sum_{i=1}^{I} \gamma_{o}^{ik}(t_B)\left[\bar{x}_o^{ik}(t_B) - x_o^{ik}(t_B)\right]\left[T^{i,\infty}(t_B) - T^i\right] +$$

$$+ \sum_{i=1}^{I} \gamma_{z}^{ik}(t_B)\left[\bar{x}_z^{ik}(t_B) - x_z^{ik}(t_B)\right]\left[T^{i,\infty}(t_B) - T^i\right] \qquad (60)$$

where: $\gamma_o^{ik}(t_B), \gamma_z^{ik}(t_B)$ = corresponding morbidities for the i-th age and k-th illness group for people beyond treatment in OPC and IPC (but manifesting tne demand for these types of health services according to the forecasts (53)-(54)).

The demand for health services in the year t_B+1 will be given by the formulae presented below:

$$\bar{x}_o^{ik}(t_B+1) = \alpha_o^{ik}(t_B+1)l^i(t_B+1)D(t_B+1) + $$
$$+ \beta_o^{ik}(t_B)\left[\bar{x}_o^{ik}(t_B) - x_o^{ik}(t_B)\right] \qquad (61)$$

$$\bar{x}_z^{ik}(t_B+1) = \alpha_z^{ik}(t_B+1)l^i(t_B+1)D(t_B+1) + $$
$$+ \beta_{z1}^{ik}(t_B)\left[\bar{x}_z^{ik}(t_B) - x_z^{ik}(t_B)\right] + $$
$$+ \beta_{z2}^{ik}(t_B)\left[\bar{x}_z^{ik}(t_B) - x_z^{ik}(5\ )\right] \qquad (62)$$

where

$$D(t_B+1) = \sum_{i=1}^{I} D^i(t_B+1) \qquad (63)$$

$$D^i(t_B+1) = \frac{1}{\delta}\Big\{D^{i-1}(t_B) - \sum_{k=1}^{K} x_o^{i-1,k}(t_B)a_o^{i-1,k}(t_B) + $$
$$- \sum_{k=1}^{K} x_z^{i-1,k}(t_B)a_z^{i-1,k}(t_B) - \sum_{k=1}^{K}\gamma_o^{i-1,k}(t_B)\cdot$$
$$\cdot\left[\bar{x}_o^{i-1,k}(t_B) - x_o^{i-1,k}(t_B)\right] + $$
$$- \sum_{k=1}^{K}\gamma_z^{i-1,k}(t_B)\left[\bar{x}_z^{i-1,k}(t_B) - x_z^{i-1,k}(t_B)\right]\Big\} +$$

$$+ \frac{\delta - 1}{\delta}\Bigg\{D^{i}(t_B) - \sum_{k=1}^{K} x_o^{ik}(t_B)a_o^{ik}(t_B) - \sum_{k=1}^{K} x_z^{ik}(t_B)a_z^{ik}(t_B) +$$

$$- \sum_{k=1}^{K} \gamma_o^{ik}(t_B)\left[\bar{x}_o^{ik}(t_B) - x_o^{ik}(t_B)\right] +$$

$$- \sum_{k=1}^{K} \gamma_z^{ik}(t_B)\left[\bar{x}_z^{ik}(t_B) - x_z^{ik}(t_B)\right]\Bigg\} \qquad (64)$$

with assumption made that the age distribution is uniform and the age groups are δ-years long and the coefficients $\beta_o^{ik}(t_B)$, $\beta_{z1,2}^{ik}(t_B)$ are the "transition" coefficients (they show the way how the unrealized demand from year t_B will pass to the year t_B+1). The unrealized demand in any year (for example $t=t_B$) results in two effects:

- the growing total mortality effect (usually $\gamma_{(\cdot)}^{ik}(t) > a_{(\cdot)}^{ik}(t)$,
- the demand growth effect in the coming year (the "transition" of potential patients between the activity fields - from OPC to IPC).

The equations (61)-(64) define the total demand for health services, the development of the HCS taken into account (i. e. its production capacities).

4.4. Some Remarks on the Investment Allocation Problem

As it has been mentioned in 4.1 and 4.2 the investment activity carried on for the needs of the HCS allows for the enlargement of its production possibilities.

They are defined by:

- "natural" constraints (e.g. the number of beds and consulting rooms, the number of physicians and auxiliary personnel,the production capacity of the pharmaceutical industry, etc.), and
- financial constraints (total current expenditures for:salaries, drugs, food, materials, etc.).

The set of the "natural" constraints can be modified by the investment process. These constraints can be divided into two groups:

- constraints of the material base,
- employment level.

To the first group we include, for example, the number of beds and consulting rooms, whereas to the second - the inequalities set describing the number and structure of the employment in HCS (e.g. specialization structure, the number of physicians to the number of auxiliary personnel ratio, etc.).

We can transform the material base by:

- construction of new hospitals and dispensaries (new investments),
- modification of the existing health units (equipment modernization, changes of the speciality),
- repairs of the old health units.

Here, only the problem of new investments influence on the change of beds number $B_z^{k*}(t)$ (the material base of the IPC subsystem) and consulting rooms number - $G_o^{k*}(t)$ (the material base of the OPC subsystem) is considered.

The description of these relationships by the following simplified formulae:

$$B_z^{k*}(t) = B_z^{k*}(t_B) + K_z^k \sum_{\tau = t_B - TI_z}^{t - TI_z} IN_z^k(\tau) \qquad (65)$$

$$G_o^{k*}(t) = G_o^{k*}(t_B) + K_o^k \sum_{\tau = t_B - TI_o}^{t - TI_o} IN_o^k(\tau) \qquad (66)$$

where: $IN_{(.)}^k(t)$ = investment expenditure in the year t in the (.) activity in the k-th illness group and $TI_{(.)}$ = investment delay time - is proposed.

The medical personnel employment and its structure depend - in the planned economy - on the enrollment and education system in medical schools. They should be adjusted to the demand for health services. Planning in the educational system (money and resource allocation among medical groups and specialities) is a

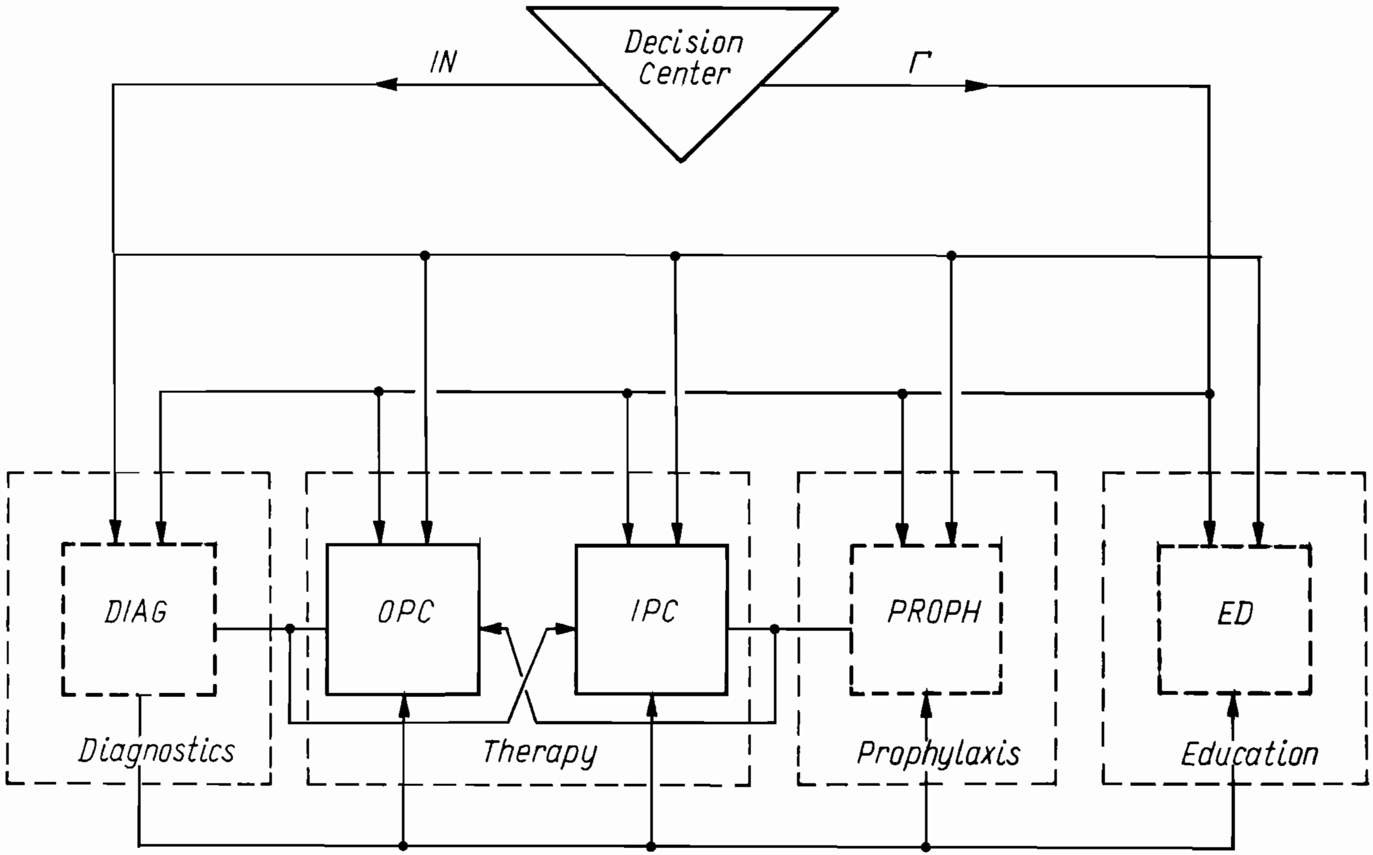

Figure 8. Interactions in health care activities.

very important problem - the effects of earlier decisions are to be seen only after a relatively long time (e.g., education duration time, the period of specialization, etc.).This problem has not been considered in the present paper.It should be mentioned, however, that a possible modification of the management structure (including the educational system - belonging to the main activities of the Ministry of Health and Social Welfare - to the HCS model being constructed - see Fig. 8) has to be discussed in the future. It could be described in the form analogous to (65)-(66) - as in the first approach.

For the present version of the HCS model the quantities describing the employment characteristics (and other denoted by the asterisk in formulae (39),(43)-(44)) are considered as given exogeneously.

As it was discussed above the constraints sets $\Omega^{k}_{(\cdot)}(t)$ can be transformed thanks to the investments

$$\left(\Omega^{k}_{(\cdot)}(t) = \Omega^{k}_{(\cdot)}\left[t,\left\{IN^{k}_{(\cdot)}(\ )\right\}^{t-TI_{(\cdot)}}_{t_B-TI_{(\cdot)}}\right]\right)$$

The investment strategy has to be chosen in such a way that it satisfies the manifested social demand for health services in the planning period,as well as possible.The quality measures (performance indices) which allow to evaluate the satisfaction degree have been previously proposed (see (23), (40)). The investment allocation strategy will be chosen taking into account the expenditure limits (the above mentioned group of the financial constraints).

4.5. Optimization Problem for the Planning Period T_p

The consideration in points 4.2 and 4.3 (the one-year optimization problem) and in point 4.4 is the starting point for the formulation of the dynamic optimization problem of the resources and money allocation for the whole planning period T_p. The static optimization problem has been solved for given parametrically:

- investment expenditures for activity fields (indices "o" for OPC and "z" for IPC) and illness groups (index "k") in consecutive years

$$\left\{IN_o^k(t)\right\}_{t_B}^{t_B+T_p-TI_o-1} \qquad (67)$$

$$\left\{IN^k(t)\right\}_{t_B}^{t_B+T_p-TI_z-1} \qquad (68)$$

- current expenditures

$$\left\{\Gamma(t)\right\}_{t_B}^{T_p+t_B-1} \qquad (69)$$

So, if the total budget expenditures for the HCS in the planning period T_p (from t_B until t_B+T_p-1)

$$W\ SOZ(T_p) \qquad (70)$$

are given (from MR models [4]) - then the goal of management center of the HCS can be formulated as follows) - see Fig. 9.

Find

$$\min_{\left\{IN_o^k(t)\right\},\left\{IN_z^k(t)\right\},\left\{\Gamma(t)\right\}} \sum_{t=t_B}^{t_B+T_p-1} \hat{A}(t) = \hat{A}\left[T_p,\ W\ SOZ(T_p)\right] \qquad (71)$$

subject to the budget constraint

$$\left\{ \sum_{t=t_B}^{T_p+t_B-1} \Gamma(t) + \sum_{t=t_B}^{t_B+T_p-1-TI_o} \sum_{k=1}^{K} IN^k(t) + \right.$$

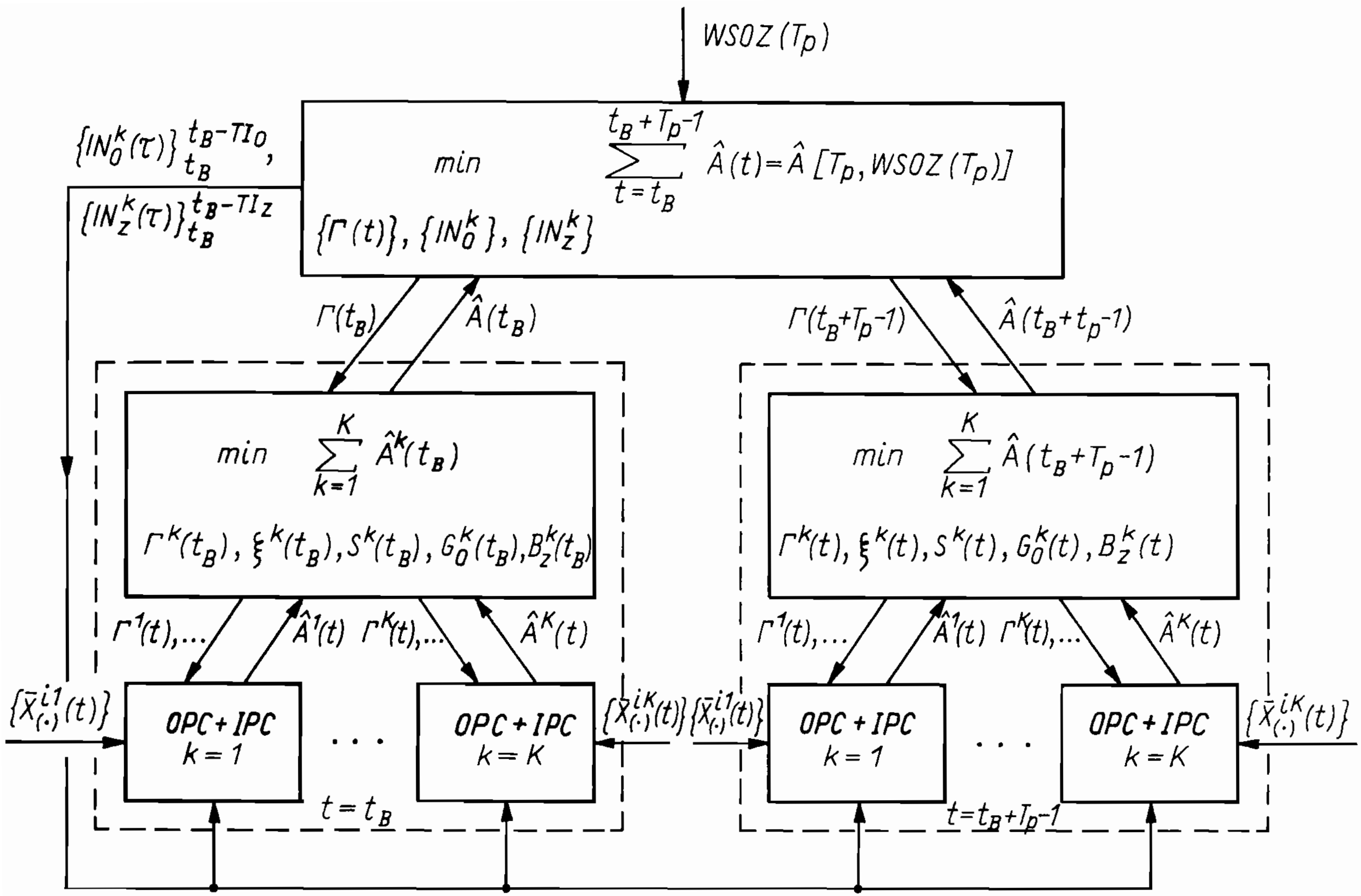

Figure 9. Optimization level III (dynamic problem for the planning period T_p).

$$+ \sum_{t=t_B}^{t_B+T_p-1-TI_z} \sum_{k=1}^{K} IN_z^k(t) \Bigg\} \leqslant W\ SOZ(T_p) \qquad (72)$$

In the present paper the problem of the resources allocation in the planning period has been discussed.The model formulated, the chosen performance indices and management structure enable us to calculate the minimum of the socially lost time $\hat{A}\left[T_p,\ W\ SOZ(T_p)\right]$ subject to the existing government expenditures constraint. The HCS gets money from budget (the expenditures level results from the division of national income strategy adopted in MRI series - see part C).

The decision center of the Ministry of Health and Social Welfare is to divide sparse resources among the activities and disease groups in such a way to get the best results. The model is assumed to be used as an advisory tool for a sectorial planning center.

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C.III. EDUCATION

Wojciech Rokicki

1. Introduction

The main objective of the educational subsystem activity is to meet the socio-economic demands and needs.

Therefore educational subsystem should:

a) supply the economy with the qualified labour;

b) ensure (for all persons) attainment at least a given level of education (in Poland at the present time the primary education is compulsory - see Fig. 1);

c) satisfy the educational aspirations of the whole society.

In Poland education on all levels is free of charge. The funds necessary for education constitute part of national income, within the aggregated consumption. A simple way of determination of qualified labour demand for the economy is outlined in the first chapter of the paper.

Since the educational subsystem "produces" educated labour the following pecularities of the process of education must be taken into account:

1) in the process of education the resources obtained from the other sectors of economy are used; the amount of these resources is limited;

2) the number of people who undergo educational process cannot be abruptly changed by means of additional increase in maintenance expenditures, that needs time and new investments;

3) five to ten years elapse from the moment of starting the education up to the moment of graduation;

4) the number of children for education depends on demographic factors.

Since the human activity in the economic system lasts for 30 -40 years the high quality of educational process is of great importance and therefore the number of graduates in any period of time, cannot be increased at the expence of the quality. Bearing in mind the mentioned above it is quite obvious that the usage of models while making the decisions on education, is very helpful. Analysing the structure of educational system and the statistical data available the five types of schools, denoted by indices $s \in S$, $S = \{1,\dots,5\}$ (see Fig. 1), will be used in the

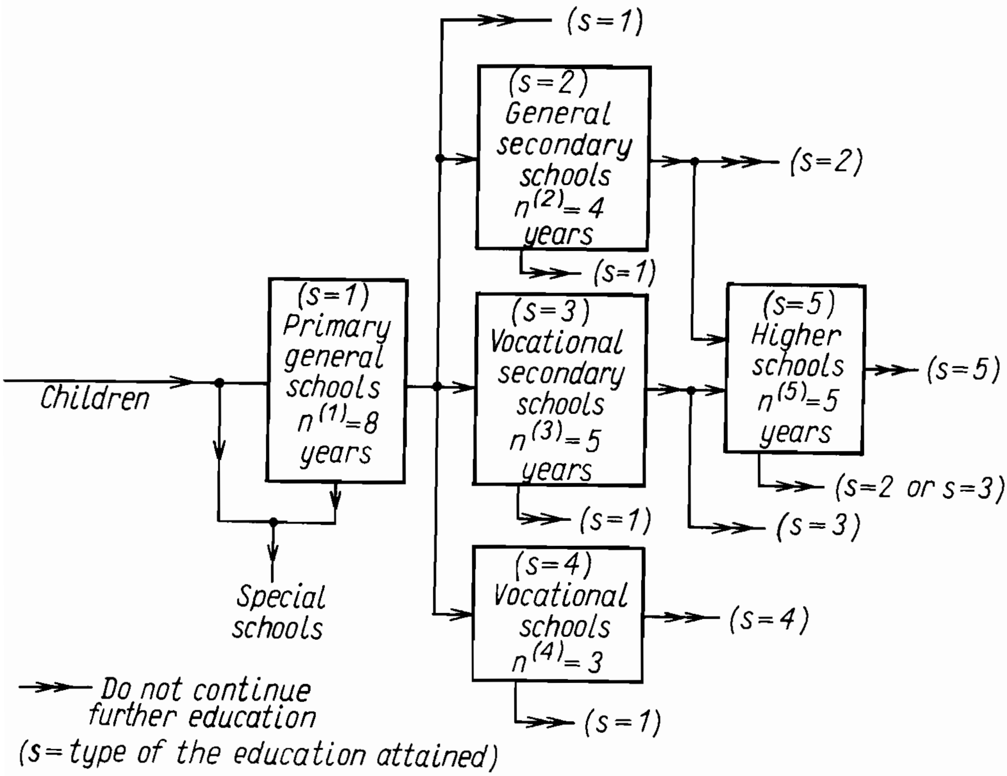

Figure 1. Basic structure of the present educational system in Poland. x)

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x) The preparatory studies for the reform of educational system in Poland are under way. The compulsory teaching will be extended to 10 year period. Further education will be continued at 2 year vocational schools and in schools with a specialized curricula. Educational system at higher schools will not be changed.

model considered.The problem of allocation of graduates (starting to work or continuing the education at schools of higher level) is being considered.

Possible supply of labour to the economy (according to the type of education received) and necessary inputs and increase in employment in the educational subsystem is calculated for each strategy of allocation of graduates.The problem of coordination of educational subsystem with the economy, so that the supply of labour (graduates from various schools) is being adjusted to the demand is of great importance.It is also possible to estimate the demand expressed in terms of money on the basis of forecasts obtained from the MRI model.

The MRI sector production functions include explicit dependence on the amount of labour (see Parts A, B) considered in terms of money ($Y_{oi}(t)$ = funds of wages and salaries of the employees in sector i in the year t). The demand for labour calculated on the basis of MRI model is connected by means of wages and salaries with the forecasts of all labour resources (measured by number of persons).

The type of education is not taken into consideration here and direct association of educational subsystem with the economy is difficult. Therefore the values $Y_{oi}(t)$ should be divided into various types of labour education($Y_{oi}^{(s)}(t)$ = fund of payment for employees with s-type of education). In sector production functions (see Parts A, B) that corresponds to the replacement of factor

$$\left[Y_{oi}(t)\right]^{\alpha_{oi}} \quad \text{by} \quad \prod_{s \in S} \left[Y_{oi}^{(s)}(t)\right]^{\alpha_{oi}^{(s)}}$$

where $\sum_{s \in S} \alpha_{oi}^{(s)} = \alpha_{oi}, \quad \alpha_{oi}^{(s)} \quad 0.$

It is easy to show (in the same way as in Parts A,B) that under optimum strategy

$$\hat{Y}_{oi}^{(s)}(t) = \alpha_{oi}^{(s)} \hat{Y}_i(t)$$

where $Y_i(t)$ = value of production of the sector i.

The values $\alpha_{oi}^{(s)}$ can be estimated on the basis of statistical data in the same way as α_{oi} by means of adaptive procedure described in Part B.

Assuming the dependence of wages and salaries on time, as follows:

$$w^{(s)}(t) = w^{(s)}(T_p)(1 + \varepsilon^{(s)})^{t-T_p} \qquad (1)$$

where:

$t = T_p, T_p+1, \ldots, T_k$

$w^{(s)}(T_p)$ = average annual wage or salary of an employee with s-type of education in a year t

$\varepsilon^{(s)}$ = average growth rate of the corresponding wages and salaries,

one can obtain the number of the employees demanded by sector i (according to the education):

$$Z_i^{(s)}(t) = \frac{Y_{oi}^{(s)}(t)}{w_i^{(s)}(t)} \qquad (2)$$

Next equation express influence of new employment $\bar{z}_i^{(s)}(t)$ on $Z_i^{(s)}(t)$:

$$Z_i^{(s)}(t) = Z_i^{(s)}(t-1)(1 - \lambda^{(s)}) + \bar{z}_i^{(s)}(t) \qquad (3)$$

where $0 < \lambda^{(s)} < 1$, and $\lambda^{(s)}$ should be estimated by statistical data.

The total demand $\bar{z}^{(s)}(t)$ for new employees with s-type of education outside the educational sector becomes

$$\bar{z}^{(s)}(t) = \sum_{i=1}^{n} \bar{z}_i^{(s)}(t) \qquad (4)$$

Denoting by $z^{(s)}(t)$ the supply of labour for the economy, after taking into consideration the demands the problem of adjusting of supply to the demand may be formulated in a following way:

$$\min \sum_{s \in S} \sum_{t=T_p}^{t=T_k} \omega^{(s)}(t)\left[z^{(s)}(t) - \bar{z}^{(s)}(t)\right]^2 \tag{5}$$

where $\omega^{(s)}(t)$ = given weight functions.

The present version of the model is charaэterized by great flexibility (it enables to analyse any strategy of allocation of graduates, adding new types of education, etc.).

The algorithm for the computer will be supplemented by appropriate optimization procedures, enabling us to improve the strategy of the allocation of graduates subject to the given goal (e.g. functional (5)).

2. Process of Education - Pupils' Flows,Use of Resources

Denote by $i = 1,\dots,n^{(s)}$ the indices describing classes, where $n^{(s)}$ = period of education in schools s (e.g. in primary schools, $s = 1$, $n^{(1)} = 8$ years - see Fig. 1).

In order to simplify the notation,while considering the same type of schools, index s will be dropped.

All the values which depend on time t are measured at the end of the corresponding calendar year.

Numbers of pupils $x_i(t)$ (specified at the end of a year t) which undergo the educational process in a given class i, within a year (t+1), are divided at the end of this year into two parts:

$$x_i(t) = p_i(t + 1) + n_i(t + 1) \tag{6}$$

where $p_i(t+1)$ = numbers of pupils who can continue their education and go to the higher class (i+1),or - numbers of graduates if $i=n$ $\Omega(p_n(t) = g(t))$; $n_i(t+1)$ denotes those pupils which have to repeat the same class and those which drop out.

The pupils which start their education in a given class i at the end of year t - $x_i(t)$ consists of: those who finished the former class (the first component of (6a)) and those who repeat the same class (the second component of (6a)):

$$x_i(t) = a_{1i}p_{i-1}(t) + a_{2i}n_i(t) \tag{6a}$$

where a_{1i}, a_{2i} = positive numbers such that

$0 < a_{1i} \leqslant 1$, $a_{1i} \cong 1$, $0 < a_{2i} < 1$

if $i = 1$ and $s = 1$ then $p_0^{(1)}(t)$ will denote new pupils who have started their education (in Poland - 7 years old).

This numbers are determined by demographic factors. For schools different from primary ones $(s > 1)$ $p_0^{(s)}(t)$ will denote pupils which graduated from schools of lower level and which continue their education in schools of higher level. It is assumed further on, that the number of pupils promoted from lower to higher class for given s depends on the number of pupils who started their education in a given class $x_i(t)$ and on conditions in which the whole process takes place (determined by existing and provided resources) [1)]

$$p_i(t+1) = k\left[x_i(t)\right]^{c_{p_i}} \prod_{j=0}^{m} \left[E_{ji}(t)\right]^{c_j} \tag{7}$$

where: k, c_{p_i}, c_j = positive numbers such that: $0 < c_j < 1$, $\sum_{j=1}^{m} c_j < 1$,

$j = 0,1,\ldots,m$ = index of the type of resources, $E_{ji}(t)$ = amount of resources of the type j destined for educational process on the i-th class level within a year (t+1) (specified at the end of a year t).

The form of (7) shows that the substitution between various types of resources and the number of pupils $x_i(t)$ is possible. This phenomenon, to some extent, takes also place in reality. Nonlinearity of production function is of such a type that the greater is the value of the resources the smaller are effects of their increase.

The values of parameters, estimated for general secondary

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1) Function (7) can be regarded as local approximation of the real situation. Values $E_{ji}(t)$, $x_i(t)$ $(i = 1,\ldots,n,\ j = 0,\ldots,m)$ should satisfy the condition $p_i(t+1) \leqslant x_i(t)$, $i = 1,\ldots,n$.

Table 1. Population of 15 years of age and over, by educational attainment

Level of education	Years	Percentages of total population of 15 years and over
Higher	1960	2.1
	1970	2.7
s = 5	1974	3.2
Secondary vocational	1960	10.3
and general	1970	13.4
s = 2 and s = 3	1974	15.7
Vocational	1960	3.1
	1970	10.6
s = 4	1974	12.6
Primary	1960	39.3
	1970	48.8
s = 5	1974	48.5

Source: Statistical Yearbook of Education 1974/1975. Central Statistical Office, Warsaw 1976.

schools in Poland, for two types of resources $j = 0$ and $j = 1$ are follows: $c_0 = 0.05$, $c_1 = 0.2$, $c_{p_i} = 0.7$ (it is assumed that c_{p_i} are equal). $E_0(t)$ denotes the number of classrooms and $E_1(t)$ = number of teachers in the years 1971-1974.

Hence it can be seen that the number of classrooms bears its influence on the number of promoted pupils. This influence is however much weaker then that, which depends on the number of teachers or the number of pupils who start the education (because $c_0 \leqslant c_1 < c_{p_i}$).

It should be stressed that better teaching conditions when greater quantity of resources is provided are reflected in increased pupils and graduates knowledge (qualitative aspect) and in larger percentage of promoted pupils among those who start their education (quantitative aspect).

The estimation of parameters of function (7) takes into account the quantitative aspect only. The calculation of pupils promoted should be preceded by allocation of resources provided to particular classes in a given type of schools. It is assumed that resources are allocated in such a way that after taking into account the constraint

$$\sum_{i=1}^{n} E_{ji}(t) \leqslant E_j(t), \quad j = 0,1,\ldots,m, \quad i = 1,2,\ldots,n \tag{8}$$

the number of all pupils promoted in each t year

$$P(t + 1) = \sum_{i=1}^{n} p_i(t + 1) \tag{9}$$

is maximum.

The solution of the above problem is formulated by Theorem 1 (proved in 6).

<u>Theorem 1</u>. There exist a unique solution $\hat{E}_{ji}(t), j = 1,\ldots,m$, $i = 1,\ldots,n$, of the optimization problem (7)-(9):

$$\hat{E}_{ji}(t) = \frac{f_i(t)}{F(t)} E_j(t) \tag{10}$$

$$f_i(t) = \left[x_i(t)\right]^{(c_{p_i}/q)} \tag{11}$$

$$F(t) = \sum_{i=1}^{n} f_i(t) \tag{12}$$

$$q = 1 - \sum_{j=0}^{m} c_j \tag{13}$$

Moreover, the following relations take place:

$$\hat{P}(t+1) = k\left[F(t)\right]^{q} \prod_{j=0}^{m} \left[E_j(t)\right]^{c_j} \tag{14}$$

$$\hat{p}_i(t+1) = k \frac{f_i(t)}{\left[F(t)\right]^{1-q}} \prod_{j=0}^{m} \left[E_j(t)\right]^{c_j} \tag{15}$$

In case if certain groups of resources $E_j(t)$ are limited as follows:

$$\sum_{j \in G} E_j(t) \leqslant E^{(G)}(t), \text{ where set } G \subset \left\{0,1,\ldots,m\right\} \tag{16}$$

The value $E^{(G)}(t)$ can be divided into parts $\hat{\hat{E}}_j(t), j \in G$, such that $\hat{P}(t+1)$ and $\hat{p}_i(t+1)$ are maximum. Hence

$$\hat{\hat{E}}_j(t) = \frac{c_j}{\sum_{j \in G} c_j} E^{(G)}(t) \quad \text{for} \quad j \in G \tag{17}$$

and

$$\max \hat{P}(t+1) = \hat{\hat{P}}(t+1) = k\left[F(t)\right]^{q} \varphi(E) \tag{18}$$

$$\max \hat{p}_i(t+1) = \hat{\hat{p}}_i(t+1) = k \frac{f_i(t)}{\left[F(t)\right]^{1-q}} \varphi(E) \tag{19}$$

where:

$$\varphi(E) = \left(\prod_{j \in G} [c_j]^{c_j}\right)\left(\sum_{j \in G} c_j\right)^{-\sum_{j \in G} c_j} \times$$

$$\times \; (E^{(G)}(t))^{-\sum_{j \in G} c_j} \prod_{j \notin G} (E_j(t))^{c_j} \qquad (20)$$

Theorem 1 permits to deal with the allocation of resources even when the values of $E_j(t)$ are small.

According to the (14), (15) and (7) the percentage of promoted $p_i(t + 1)$ out of $x_i(t)$ would get diminished then and the number of drop - outs would increase. But the considerations cannot be restricted to examining the quantitative aspect only. Reducing values of the resources (e. g. number of teachers) would lower the level of teaching, and thus the level of knowledge of graduates. Therefore additional restrictions should be introduced, the implementation of which would enable to improve or at least to assure certain stability in the quality of education. This is expressed in the following form:

$$X(t) \leqslant E_j(t)\, \mu_j(t) \quad \text{for} \quad j \in J,\; J \subset \{0,1,\ldots,n\} \qquad (21)$$

$$X(t) = \sum_{i=1}^{n} x_i(t) \qquad (22)$$

and $\mu_j(t)$ = nonincreasing positive functions of time.

The values of functions $\mu_j(t)$ represent the average numbers of pupils for one unit of j-type of resources in t year (e.g. the number of pupils for one teacher).

So the improvement of teaching conditions is associated with the decreasing of $\mu_j(t)$ values.

Those may be exogeneously given functions (e.g. the implementation of programme aimed at improvement of teaching conditions) or they can depend on increase of prices, when j = the type of sources considered in terms of money.

3. Allocation of Graduates to Various Types of Schools

Let $A^{(s_i)}$, $s_i \in S$, denote set of indices of schools $s \in S$ in which a student can start his education on condition that he is a graduate of school s_i, e.g. $A^{(1)} = \{2,3,4\}$ (see Fig. 1).

Let $B^{(s_j)}$, $s_j \in S$, denote set of indices of schools $s \in S$ the graduates of which can continue their education in schools s_j, e.g. $B^{(5)} = \{2, 3\}$.

$d^{(s_i,s_j)}(t)$ denotes relation

$$d^{(s_i,s_j)}(t) = \frac{x_1^{(s_i,s_j)}(t)}{g^{(s_i)}(t)}, \quad s_j \in A^{(s_i)} \tag{23}$$

where $x_1^{(s_i,s_j)}(t)$ = the number of graduates of s_i-type schools admitted to the first class of s_j-type schools at the end of year t.

Hence $0 \leqslant d^{(s_i,s_j)}(t) \leqslant 1$ and $\sum_{s \in A^{(s_i)}} d^{(s_i,s_j)}(t) \leqslant 1.$

Graduates of schools s_i who do not continue their education (so they can start to work in a year t) can be counted as follows:

$$l^{(s_i)}(t) = g^{(s_i)}(t)\left(1 - \sum_{s \in A^{(s_i)}} d^{(s_i,s)}(t)\right) \tag{24}$$

Moreover, in the same year those who graduated of schools s_i earlier $m^{(s_i)}(t)$ and continued their education in schools $s \in A^{(s_i)}$ but gave up within a year t can start their work

$$m^{(s_i)}(t) = \sum_{s \in A^{(s_i)}} \gamma^{(s_i,s)}(t) \sum_{i=1}^{n^{(s)}} \left[p_{i-1}^{(s)}(t)(1 - a_{1i}^{(s)}) + \right.$$
$$\left. + n_i^{(s)}(t)(1 - a_{2i}^{(s)}) \right] \qquad (25)$$

where

$$\gamma^{(s_i,s)}(t) = \frac{\sum_{\tau=t-n^{(s)}}^{t-1} g^{(s_i)}(\tau)\, d^{(s_i,s)}(\tau)}{\sum_{\tau=t-n^{(s)}}^{t-1} \sum_{s' \in B^{(s)}} g^{(s')}(\tau)\, d^{(s',s)}(\tau)} \qquad (26)$$

where $\gamma^{(s_i,s)}(t)$ = there are coefficients of the structure of admission, so that

$$0 \leqslant \gamma^{(s_i,s)}(t) \leqslant 1, \quad \sum_{s_i \in B^{(s)}} \gamma^{(s_i,s)}(t) = 1$$

Relation (26) helps to specify in a simplified way (because of the lack of statistical data) the number of graduates of schools s_i out of a total number of drop-outs of $A^{(s_i)}$ schools. The simplification is based on the assumption that the structure of graduates and drop-outs in a year t (according to schools of which they graduated earlier) is assumed to be the same as the structure of admission to classes $1,2,\dots,n^{(s)}$ in the years $t-n^{(s)}, t-n^{(s)}+1,\dots,t-1$. The simplification deals with the graduates of general secondary (s_i = 2) and vocational secondary (s_i = 3) schools admitted to higher schools (s = 5). Since $g^{(2)}(t) \gg x_1^{(5)}(t)$ and $g^{(3)}(t) \gg x_1^{(5)}(t)$ the errors resulting from the simplification mentioned can be neglected.

Since the annual statistical data for Polish educational system deal with a few types of resources, the index j will take the values j = 0, 1, only (m = 1):

$E_1^{(s)}(t)$ for j = 1 - number of teachers

$E_0^{(s)}(t)$ for j = 0 - capital stock (general equipment at schools accomodations possibilities, etc.)

The constraints (21) should be fulfilled for j = 0, 1.

The remaining types of resources considered, belong to the group which is not taken into account in the "production function" (7). The values of these resources will be determined by the relations given in Sec. 6.

4. Investments and Capital Stock

Resources of capital stock $E_0^{(s)}(t)$ (a state at the end of the year t) utilized in the process of education in s-type of schools within a year (t+1) are directly connected (by constraints (21)) to the number of pupils starting their education in all classes together (see (22)) at the end of a year t.

The number of pupils in $2 \leqslant i \leqslant n^{(s)}$ classes during the year (t+1) is determined by the number of pupils in the given classes and by resources provided for $1 \leqslant i \leqslant n^{(s)}$ classes a year before (i.e. at the end of a year t, compare (6), (7)). Therefore constraints (21) influence admission to the first classes of those schools. Since all 7 years old children must be admited in the corresponding years in the case of primary schools (s = = 1), in which the education is compulsory,the constraints (21) may be difficult to satisfy. In Poland, where the structure of population by age is very differentiated (Table 2b) this phenomenon could be easily observed.

The influence of demographic factor is not so strong as far as the education at the higher level is concerned.

The values of $E_0^{(s)}(t)$ resources depend on earlier investment outlays.

The following relations renders a current $E_0^{(s)}(t)$ value (index s is dropped):

$$E_0(t) = \sum_{\tau=-\infty}^{t} k_0(t-\tau)\left[I(\tau)\right]^{\beta} \qquad (27)$$

The relation (27) is a discrete nonlinear version of continous Volterra operator:

Table 2. Population projections

2a. Estimated population

Year	Population (in thousands)
1970	32 658
1971	32 909
1972	33 202
1973	33 512
1974	33 846
1975	34 024.4
1976	34 301.0
1977	34 568.6
1978	34 839.9
1979	35 093.2
1980	35 334.5
1985	36 522.0
1990	37 422.8

2b. Population projection in five year age groups (in thousands)

Age \ Year	1975	1980	1985
0-4	2784.7	2898.2	2805.7
5-9	2538.7	2760.3	2884.8
10-14	2717.6	2524.1	2756.6
15-19	3370.4	2703.0	2518.1
20-24	3453.6	3356.5	2691.8
25-29	2928.5	3415.6	3337.8
30-34	1863.4	2891.4	3393.5
35-39	2123.6	1831.5	2866.1
40-44	2216.7	2085.4	1810.1
45-49	2206.6	2170.0	2048.3
50-54	1926.8	2146.6	2113.9
55-59	1224.0	1847.3	2061.5
60-64	1395.0	1143.7	1731.7
65-69	1331.5	1243.5	1028.4
70-74	955.5	1101.6	1032.1
75-79	587.4	698.4	809.2
80 and over	400.4	517.4	632.5

Sources:
Table 2a, years 1970-1974: Statistical Yearbook of Poland. Central Statistical Office, Warsaw 1975.
Table 2a, years 1975-1990 and Table 2b: Population projections: Statistical Yearbook of Poland (Demography). Central Statistical Office, Warsaw 1973.

$$e_0(t) = \int_{-\infty}^{t} k_0(t-\tau)\, I(\tau)\, d\tau$$

where:

$\beta > 0$ is the estimated parameter

$k_0(t - \tau)$ = nonnegative function, the parameters of which are to be estimated from the past statistical data, such that: $k_0(t - \tau) = 0$ for $t - \tau < T_0$

T_0 = minimum of time elapsing from starting the investment to the moment when new capital stock is obtained.

If $k_0(t - \tau)$ is given by

$$k_0(\xi) = \begin{cases} 0 & \text{for } \xi < T_0 \\ K_o \exp(-\delta_0(\xi - T_0)) & \text{for } \xi \geqslant T_0 \end{cases} \tag{28}$$

where: $\xi = t - \tau$; K_0, δ_0 = nonnegative constants (to be estimated) and δ_0 = parameter describing depreciation of the capital stock, then (27) can be expressed in the form

$$E_0(t) = E_0(t - 1) \exp(-\delta_0) + K_0\left[I(t - T_0)\right]^{\beta} \tag{29}$$

Taking into consideration that $I(t) \geqslant 0$ and denoting

$$B(t) = \frac{E_0(t) - E_0(t - 1) \exp(-\delta_0)}{K_0} \tag{30}$$

by virtue of (29) one gets:

$$I(t - T_0) = \begin{cases} \left[B(t)\right]^{1/\beta} & \text{for } B(t) \geqslant 0 \\ 0 & \text{for } B(t) < 0 \end{cases} \tag{31}$$

The minimum values of $E_0(t)$, $t \in [T_p, T_k-1]$, necessary for imolementation of the strategy of allocation of graduates $d^{(s_i,s)}(t)$, where $t \in [T_p, T_k-1]$, $s_i \in S$, $s \in A^{(s_i)}$, during the projection period T_p, T_k can be calculated by (21). Then one gets from (31) the corresponding values of the investment outlays in the particular years $t \in [T_p, T_k-T_0-1]$.

Cases in which $I^{(s)}(t_0 - T_0) = 0$ and defined by (30) $B^{(s)}(t_0)$ has a negative value prove certain surplus of capital stock values in the year t_0 and s-type of schools. This surplus makes it possible to improve conditions of teaching and result in a better quality and larger quantity of promoted and graduates.

If there are some total restrictions on investments imposed other variants of $d^{(s_i,s)}(t)$ strategy, ensuring better utilization of capital stock in particular years, should be chosen.

Relations based on capital stock values in the year t for s-type of schools, namely $E_{OA}^{(s)}(t)$ - i.e. the actual value and the minimum value $E_{OC}^{(s)}(t)$ obtained by (21) can be used for formulating a criterion for the above mentioned selection, such as

$$\eta_0^{(s)}(t) = \frac{E_{OC}^{(s)}(t)}{E_{OA}^{(s)}(t)} \qquad (32)$$

Since by (21) one gets $E_{OA}^{(s)}(t) \geqslant E_{OC}^{(s)}(t)$ then $\eta_0^{(s)}(t) \in \in (0, 1)$.

The utilization of capital stock is the better, the higher is $\eta_0^{(s)}(t)$ value.

Total investment outlays needed in the particular years can be derived by the formula

$$I(t) = \sum_{s \in S} I^{(s)}(t), \quad t = T_p, T_p+1, \ldots, T_k-T_0-1 \qquad (33)$$

5. Teachers' Labour Resources

Number of teachers (assuming the same time of work and equal qualifications for all teachers) can be taken as a measure of teachers labour resources. A better measure here, however, would be teachers payment, but teachers salaries in Poland have been undergoing certain increases recently, not necessarily connected with any additional raise of qualifications level. And this is the reason for which quantity of teachers was assumed as a measure for $E_1^{(s)}(t)$ resources.

It is assumed also that condition (21) must be fulfilled in this case (i.e. j = 1) and new teaching staff can constitute only of graduates with the higher type of education (s = 5).

Employment of teachers in various types of schools can be expressed by following equation:

$$E_1(t) = E_1(t-1)(1-\lambda_1) + e_1(t) \tag{34}$$

in which: $\lambda_1 > 0$ denotes parameter accounting for decreasing of the number of teachers (changing the profession, deaths, retirements), $e_1(t)$ = number of newly employed teachers in t year.

The $E_1(t)$ values are calculated for a given $d^{(s_i,s)}(t)$ strategy, in the same way as in the case of investment outlays (using constraints (21) for j = 1).

Assuming $e_1(t) \geqslant 0$ one can get by (34), for the given type of schools (s dropped)

$$e_1(t) = \begin{cases} E_1(t) - E_1(t-1)(1-\lambda_1) = E(t) & \text{for } E(t) \geqslant 0 \\ 0 & \text{for } E(t) < 0 \end{cases} \tag{35}$$

Negative $e_1(t)$ values, obtained from relation (34) without taking into account constraint $e_1(t) \geqslant 0$ can be interpreted similarly to the case of investment outlays (see Part 5). They would show certain surplus of number of teachers (in s-type schools and t year), going beyond a minimum level specified by condition (21).

The change in employment of teachers or change of their qualifications would reveal certain shortcomings, namely:

1) changing of qualification (even if it is connected with teaching in another type of school) would require long time of adaptation;

2) the lack of feeling of stability in performing the job would deprive the teaching personnel of supply of the most valuable candidates.

It does not change the fact, however, that in case of some disciplines and schools those activities may do not have these shortcoming. Such detailed problems, however, are not analysed in a presented model. Besides, it is felt that the danger connected with the surplus of teachers is rather theoretical than real and may occur only at the initial runs of seeking $d^{(s_i,s)}(t)$ strategy. It is assumed therefore, that needs for teachers are balanced separately for various types of schools with taking into account the $e_1^{(s)}(t) \geqslant 0$ constraint.

Relation (36) can be helpful for assessment of the degree of utilization of $E_1^{(s)}(t)$ resources

$$\eta_1^{(s)}(t) = \frac{E_{1C}^{(s)}(t)}{E_{1A}^{(s)}(t)} \tag{36}$$

where: $E_{1C}^{(s)}(t)$ = the minimum value computed from (21), $E_{1A}^{(s)}(t)$ = the actual number of teachers in t year and s-type of schools.

In case of the higher schools (s = 5) two groups of teachers $E_1^{(5)}(t)$ and $R_2(t)$ are introduced and

$$E_1^{(5)}(t) = R_0(t) + R_1(t) \tag{37}$$

where: $R_0(t)$ = number of assistant lecturers and senior lecturers, $R_1(t)$ = number of lecturers (adjuncts) employed in higher schools in t year.

The $R_2(t)$ denotes the number of professors. They carry on also the research work and are responsible for the scientific level of lecturers (and thus indirectly - for the level of knowledge of students). Therefore the following constraint is assumed:

$$X^{(5)}(t) \leqslant R_2(t)\, \mu_2(t) \tag{38}$$

where $\mu_2(t)$ is the nonincreasing function of time.

Since the group of lecturers (37) have direct and the greatest influence on the education of students, in the "production function" (7) only $E_1^{(5)}$ was used.

When one uses the model, specification of quantity of persons in those groups is needed. The problem of the quantitative development of scientific staff has already been considered [9] and the same methodology can be used there.

Let $r_i(t)$ denotes the number of promoted from (i - 1) group to i group in the year t (i = 1, 2).

The following relations show connections between $r_i(t)$ numbers:

$$r_i(t) = \sum_{\tau=-\infty}^{t} k_i(t-\tau) r_{i-1}(\tau)$$

where: $r_0(t)$ = number of newly employed assistants,graduates of higher schools, in the year t; $k_i(t - \ )$ = nonnegative function such that

$$k_i(t-\tau) = 0 \quad \text{for} \quad (t-\tau) < T_i$$

In the simplest case

$$k_i(\xi) = \begin{cases} 0 & \text{for } \xi < T_i \text{ or } \xi > T_{im} \\ K_i \exp\left(-\frac{1}{\tau_i}(\xi - T_i)\right) & \text{for } T_i \leqslant \xi \leqslant T_{im} \end{cases} \quad (39)$$

where: $\xi = t - \tau$; T_i = minimum period, needed by a member of (i-1) group to be promoted to i group; T_{im} = maximum period, limited by regulations or old age;, τ_i, K_i = nonnegative estimated numbers.

The parameters of the model described were estimated by the least squares method for whole Poland using the CSO statistical data of groups to be promoted. The following values of parameters for i = 1 and i = 2 have been obtained:

$$T_1 = 5 \text{ years}, \ \tau_1 = 8.3 \text{ and } T_2 = 4 \text{ years}, \ \tau_2 = 0.32.$$

The numbers of people in $R_i(t)$ groups (i = 0,1,2) can be computed from the following equations:

$$R_i(t) = R_i(t-1)(1-\lambda_{R_i}) + r_i(t) - r_{i+1}(t) + r_{in}(t) \quad (40)$$

i = 0,1,2; $r_3(t) = 0$; $\lambda_{R_i} > 0$, where λ_{R_i} is the parameter expressing number of people falling off the group (change of work, retirement, death); $r_{in}(t)$ = the number of newly employed people in higher schools in the year t and entering i group; and $r_{0n}(t) = 0$.

According to (37) and (40) for $\lambda_{R_0} = \lambda_{R_1}$ the number of new teaching assistants

$$e_1^{(5)}(t) = \begin{cases} E_1^{(5)}(t) - E_1^{(5)}(t-1)(1-\lambda_{R_0}) + r_2(t) = D(t) & \text{for } D(t) \geq 0 \\ 0 & \text{for } D(t) < 0 \end{cases} \quad (41)$$

can be computed taking into account the additional constraint $r_0(t) + r_{1n}(t) = e_1^{(5)}(t) \geq 0$.

If it is additionally assumed that persons coming to $E_1^{(5)}(t)$ group are graduates from higher schools,employed as assistants, then $e_1^{(5)}(t) = r_0(t)$.

Equation (40)for i = 2 allows to check if the process of promoting among teaching staff in higher schools, is intensive, enough to ensure the fulfilment of constraint (38). If it is not, assuming $r_{2n}(t) \geq 0$ one obtain from (40)

$$r_{2n}(t) = \begin{cases} R_2(t) - R_2(t-1)(1-\lambda_{R_2}) - r_2(t) = R(t) & \text{for } R(t) \geq 0 \\ 0 & \text{for } R(t) < 0 \end{cases} \quad (42)$$

The total demand for new teachers for the educational subsystem is:

$$e_1(t) = \sum_{s \in S} e_1^{(s)}(t) \quad (43)$$

6. Maintenance Expenditures on Education

The maintenance expenditures are not taken into account in "production function" (7) in the explicit form. However,the expenditures connected with employing teachers and other personnel as well as the rest of expenditures, will be derived separately. Expenditures of the first type can be specified if the employment of the persons in various educational groups with an average salary are known.

The numbers of employed teachers are computed from the relation given in Part 5. The work of the remaining part of employees, is connected with services rendered in that field. Therefore the numbers of the employees, depending on numbers of pupils X(t) and teachers $E_1(t)$, can be calculated.

Let $Z^{(s_i)}_{(s_j)}(t)$ be the number of school employes, where s_i = the type of school, s_j = the type of education, $s_i \neq 5$, $s_j \neq 5$, which can be computed as follows:

$$Z^{(s_i)}_{(s_j)}(t) = b^{(s_i)}_{1(s_j)} \left[X^{(s_i)}(t)\right]^{b^{(s_i)}_{2(s_j)}} \left[E_1^{(s_i)}(t)\right]^{b^{(s_i)}_{3(s_j)}} \qquad (44)$$

where:

$b^{(s_i)}_{1(s_j)}$, $b^{(s_i)}_{2(s_j)}$, $b^{(s_i)}_{3(s_j)}$ = nonnegative numbers estimated parameters.

For $s_j = 5$ the number of employees with university degree, outside teachers, in $s_i \neq 5$ types of schools becomes:

$$Z^{(s_i)}_{(5)}(t) = b^{(s_i)}_{1(5)} \left[X^{(s_i)}(t)\right]^{b^{(s_i)}_{2(5)}} \left[E_1^{(s_i)}(t)\right]^{b^{(s_i)}_{3(5)}} - E_1^{(s_i)}(t) \qquad (45)$$

The number of employees with university degree employed in higher schools except $\sum_{i=0}^{2} R_i(t)$ is equal

$$Z^{(5)}_{(5)}(t) = b^{(5)}_{1(5)} \left[X^{(5)}(t)\right]^{b^{(5)}_{2(5)}} \left[\sum_{i=0}^{2} R_i(t)\right]^{b^{(5)}_{3(5)}} - \sum_{i=0}^{2} R_i(t) \qquad (46)$$

It is assumed, that numbers of employees except teachers, are balanced within the whole subsystem of education irrespective of the type of school and according to the type of education. The number of employees in all types of schools having s_j education should be

$$Z_{(s_j)}(t) = \sum_{s_i \in S} Z^{(s_i)}_{(s_j)}(t) \tag{47}$$

The changes connected with employing new people $z_{(s_j)}(t)$ are expressed by (48)

$$Z_{(s_j)}(t) = Z_{(s_j)}(t-1)(1-\lambda_{s_j}) + z_{(s_j)}(t) \tag{48}$$

In order to calculate personnel expenditures, the average income which depends on time, is introduced in respective groups in the form of (1).

Within the whole subsystem personnel expenditures in the t year are:

$$C_p(t) = \sum_{s_i \in S} \sum_{s_j \in S} \left[w^{(s_i)}(t)\, E_1^{(s_i)}(t) + Z^{(s_i)}_{(s_j)}(t) w^{(s_i)}_{(s_j)}(t) \right] + R_2(t)\, w_R(t) \tag{49}$$

where: $w^{(s_i)}(t)$ = average annual teachers income in s_i schools, $w^{(s_i)}_{(s_j)}(t)$ = income of persons employed in s_i schools having s_j type of education, $w_R(t)$ = average annual income of professors.

The remaining expenditures can be calculated from the econometric relations

$$C_r^{(s)}(t) = b_4^{(s)} \left[X^{(s)}(t)\right]^{b_5^{(s)}} \left[E_1^{(s)}(t)\right]^{b_6^{(s)}} e^{b_7^{(s)} t} \tag{50}$$

where: $b_i^{(s)}$, $i = 4,5,6,7$ = estimated nonnegative parameters, the factor $e^{b_7^{(s)} t}$ taken into account the increase of expenditures on education, caused by the increase of prices and raising of the education level in all types of schools.

For all types of schools in the year t the following relation can be obtained:

$$C_r(t) = \sum_{s \in S} C_r^{(s)}(t) \qquad (51)$$

The total maintenance expenditures on the educational subsystem, connected with a selected $d^{(s_i,s_j)}(t)$ strategy of allocation of graduates in the year t become

$$C(t) = C_p(t) + C_r(t) \qquad (52)$$

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DISCUSSION

Peschel pointed out that, even if domestic prices and production are fixed, a stochastic component should be added to other variables, such as foreign trade. Kulikowski replied that this is foreseen in their future work. Under the time horizon given, however, this is of little importance even for investments that are either firmly planned or determined as a result of an optimization within the model.

Richardson raised the general question of to what extent this model reaches the real decision maker. Kulikowski replied that the modeller must at least have contact with the policy maker in order to understand his problems and his ways of thinking. Second, one must make the model known to policy makers' immediate advisors who may have more time than the policy makers themselves.

Herrera asked whether maximizing per caput GNP will actually give the best possible result for the people. Kulikowski replied that the utility function can be freely chosen. What an optimal strategy is will depend on the utility function chosen.

Roberts asked whether the model also yields an optimal population growth rate. Kulikowski replied that this question will be studied in the future. M'Pherson asked whether the model remains stable, given the rather long feedback delays involved. Kulikowski agreed that the system certainly is unstable. When the delays are greater the alpha values should be smaller. The alphas were kept constant and equal for computational convenience; but that could easily be changed.

Osterrieth wondered whether a constant parallel shift of the time horizon in dynamic programming amounts to an infinite horizon, or to no dynamic view at all? Kulikowski replied that a moving five-year time horizon maps reality in most centrally governed states. It may not be optimal, but it is the methodology used.

M'Pherson wondered whether it is sufficient to consider labor as an exogenously given variable and to optimize only the capital input. Kulikowski replied that one model allocates capital; while labor is determined by other (partial) models, including dynamic input-output coefficients.

Keyzer asked whether the input-output coefficients are technical coefficients or value coefficients, i.e. to what extent they reflect price variations. Kulikowski replied that they are in monetary terms. Keyzer wondered in what way the well-known difficulties arising from this fact are being overcome in the model. Kulikowski admitted that he had no patent solution. Keyzer stressed furthermore that the nonlinear estimation procedures employed certainly result in a very good fit;

but how about the economic significance? Kulikowski replied that, in order to overcome this difficulty, they have tested various production functions, finally limiting themselves to relatively simple ones, in order to minimize the identification problems.

Burke wondered to what extent quality changes could be reflected by the model. Kulikowski replied that MRI tries to approach this problem in connection with a general treatment of technological change.

Panov pointed out that, in a socialist economy, prices are not determined by equilibria of supply and demand but by social policy. Kulikowski replied that price subsidies can well be reflected by his model. Prices serve mainly as an instrument for an efficient allocation of scarce resources, e.g. of investments.

Kaya asked how reliable the input-output coefficients used in the model might be, given the fact that input-output tables are scarce and often based on rough estimates. Kulikowski replied that the Polish Central Statistical Office had furnished them with reliable input-output coefficients for seven years.

Mierzejewski replied to a question from M'Pherson that the model inputs are measured in monetary units; no general statement can be made about the outputs.

Richardson asked whether the training of health care personnel is treated within the health care system, or whether a coupling with the education system has been considered. Bojanczyk replied that the health care system so far, has been limited by the data available. The explicit recognition of health care education remains yet to be added, as well as other aspects, for example, to optimize with respect to total cost involved, i.e. the cost of the health care system itself plus the cost of time lost due to illness.

Fleissner asked whether, under this definition, time of children or of retired people lost due to illness would also be considered "cost". Bojanczyk agreed that the goal function should include this "cost", too.

Questioned by Richardson, Bojanczyk replied that the number of age categories and of types of diseases chosen depends on the availability of statistical data. In Poland, they were able to identify 10 age groups and about 15 groups of diseases.

Kaya asked about the kind of relationship between supply and demand used in the model. Bojanczyk replied that this relationship is not easily visible in the model.

Peschel asked whether preventive medicine is explicitly recognized in the model, and what happens if the planning procedure used in the model requires a fluctuating amount of resources, e.g. doctors. Bojanczyk replied that the inclusion of preventive medicine will be implemented soon and that the number of doctors is determined by the input of new doctors and the retirement of old.

Karlqvist stressed the difficulty of defining demand for health care in an appropriate way; it is well-known that an increase in health care may result in an increased demand for it. Questioned by McLean, Bojanczyk replied that with regard to the incidence of particular diseases, the population was not subdivided by sex.

Herrera pointed to the danger of a strictly economic goal function. Carried to the extreme, such a goal function might mean that if the cost of maintaining somebody alive were higher than the cost of someone else coming into the world, one should let him die. The social objectives of any model should be explicitly stated before the formulation of the goal function. He wondered about the objectives of the health care model and the education model. If, as is being stated, quality should not be sacrificed to quantity, this might be to the detriment of the right of any human for education, unless education were considered only an input to the productive system. Rokicki replied that the goal "quality goes before quantity" does not apply to primary school, which is compulsory. But the formulation of the objective function is quite open.

Mottek clarified that both the consumption function and the investment function of education must be considered. If, as in Poland, ten years of education are compulsory, this corresponds to the "right for education".

Lord Kennet asked to what extent the system of norms and values underlying the model had been formalized or, at least, made explicit. Kulikowski replied that the model was started four years ago trying to map the relationships in Poland to the best of the modellers' knowledge. He agreed that it would be important to investigate more thoroughly the correspondence between the model and the system of values in socialist countries as stated by expert authors.

Richardson asked for which parts of the model programs are actually running and which results the authors would judge to be of greatest importance. Kulikowski replied that for all parts of the model displayed during the Conference working computer programs exist, with different degrees of accuracy. The second question is not as easy to answer as all results are being given to the sponsors; it is up to them to decide what use to make of them. But the work has certainly contributed (and the continuation of the work should further contribute) to a better understanding of the underlying relationships. Global or national modelling is a long process of approximations to a constantly changing reality; this work is only a step in this development.

Gelovani wondered to what extent it is justified to use Cobb-Douglas functions in education or health systems that are more oriented toward social than toward economic goals. M'Pherson added that MRI seems to reflect the same economic principles as SARUM. Panov stressed that it is necessary to distinguish between classical methods developed by econometrics and mathematical programming, and by different economic systems. If one uses these classical methods within a socialist economy, one must be aware that they are not fully capable of describing the system. Gelovani added that such methods should be used only within the framework given by the system. Keyzer said that it is interesting to see that in the MRI production takes place without externalities so that a competitive market could realize the same allocations as the government does. The problem is of course that competitive markets do not exist. But in a modelling sense the simplifying assumption of perfect knowledge in planning comes very close to the assumption of perfect competition as many economists have shown. Kulikowski replied that the main concern of MRI was an efficient allocation of resources within a planned economy.

Mottek underlined again that maximization of GNP does not correctly reflect the goals of a socialist economy, goals that include job safety, social relations, and other aspects that cannot be accounted for in GNP. But this could, in principle, be handled by poly-optimization.

Lord Kennet stated that SARUM had been characterized to be demand driven; what might be considered the dominant factor in MRI? Kulikowski replied that they had been trying to incorporate the factors determining demand to the best of present-day knowledge, in awareness of the shortcomings of such an approach.

McLean came back to the question of applying optimization criteria in an education model--if properly disaggregated, the model might yield the result that all students of humanities should take up control engineering instead. Kulikowski admitted that supply and demand, in the education model, do not fit. The main problem in "optimizing" an educational model, however, lies elsewhere: because of the long time spans involved to train any specialist, the demand for different professions in different years should be known well ahead of time.

Panov stressed that the optimization criteria used are useful also in an additional way: they allow one to assess how much it costs not to stick to them, but to follow a different social policy instead. Kulikowski agreed and added that the purpose of the model was to help the planner, not to replace him--if economic efficiency were the only goal, no planners or decision makers would be needed.

Parikh outlined that the main difference between SARUM and MRI seemed to be the way in which savings decisions are taken. For the rest, both models aim at maximizing consumption, within the frame given by prices and by income.

EVALUATION OF THE SARU WORLD MODELS

J. A. Clark, J. M. McLean and P. Shepherd

INTRODUCTION

Since January 1976 the Science Policy Research Unit has been engaged in an evaluation of the world models being developed by SARU. The project has been split into two phases, each of six months duration. The model as presented to us was a simplified version of the full three-stratum model constructed by the Systems Analysis Research Unit (SARU), involving a single economic stratum interacting with a "world shop" treated in a very simple manner. The data provided were to a large extent hypothetical.

Unfortunately, there are no means by which the validity or usefulness of a model of this type can be established in any absolute sense. "Laws" of economic and social behaviour are not only more difficult to establish than those of natural science (where, for example, the possibility of verification through experimentation exists), nor can they necessarily be considered "immutable", i.e. expected to hold in all societies and at all times in the foreseeable future.

The model is essentially "descriptive" rather than "prescriptive"; a number of mechanisms, based on economic theory and empirical data, have been chosen which interact dynamically to provide a description of the temporal evolution of the world system.

Perhaps the only "objective" assessment of the model one can make concerns its internal consistency. The adequacy of the philosophical and theoretical basis of the model, and the choice of representations, inevitably have a high subjective component. Even in sensitivity testing, i.e. the determination of the effect on output of changes in input, conclusions are dependent on how much uncertainty in input values is regarded as "reasonable", and on how much variation in output is thought to be "tolerable". Even high sensitivity is not necessarily a bad thing; while it reduces the confidence one may have in implementing certain policy choices suggested by the model, it may indicate that the world is in reality highly sensitive to such choices.

In addition to investigating the general philosophical and theoretical basis of the model, and the particular representations used, we have attempted to ascertain the structure of the model in terms of the strength of the relationships operating between pairs of variables and the feedback processes operating within it. These properties, while rarely considered explicitly when a model is under construction, are critical to its operation and form a very useful alternative perspective from which it can be viewed. We have used the results obtained from this "structural analysis" work first to derive some further insight into the reasons why the model functions as it does, and secondly to suggest some experimental computer runs that could usefully be performed to gain further information about it.

In this paper we present only some general remarks on the model and some results obtained from the structural analysis work. In the course of our work, we have also carried out a number of runs designed to test the sensitivity of the model to changes in structure and parameters; however, many of the results concern very specific points not relevant to the version of the model as presented at this conference, so we largely confine ourselves here to general observations.

GENERAL REMARKS

The position adopted by the SARU team is that of neoclassical economic theory; no account is taken of differences in sociopolitical organization between different regions, it being argued that all economies operate according to the same rules whatever their ostensible philosophical or ideological differences may be. Only in trade between regions is a distortion of the free-market mechanism explicitly introduced.

The important question is whether the present structure is "adequate" for a basic core model. In particular:

(a) Are real-world deviations from the mechanisms described sufficiently small to be described as "perturbations", with reference to the kind of answers required from the model?

(b) Is the model structure flexible enough to incorporate deviations that cannot be so described, or does the structure assumed inherently constrain the output characteristics to an unacceptable degree?

(c) Even if the mechanisms described do not reflect real-world behaviour, does their use still provide a useful aid? For example, if the way stocks and prices are represented does not accurately describe the instrument for equating demand and supply, are they still useful constructs for simulating the link?

With reference to (a), there is nothing smooth about the historical development path traced by individual economies; wars, depressions, cartel action by producers, and withdrawal of labour by trade unions are examples of "disruptions" arising from a breakdown of some sort in the operation of a perfect free market. It can be argued that many such occurrences are short-term in nature and need not be included in a model designed to examine long-term trends. However, some distortions from a smooth neoclassical growth path cannot be considered in this way. Indeed, it is really the deviations from this picture that one is interested in; if the world *did* proceed in this way, and was expected to continue to do so, there would be little reason for concern or interest in the future. The immediate question, posed in (b) above, is whether the model provides a useful basis for investigation of the effects of these deviations. The SARU group have anticipated one such problem in the concept of stock- and flow-limited sectors, with the possibility of supply constraints on raw materials. Others can be introduced by simulating real-world events by exogenous amendment of certain relationships.

Applicability of the Model

The main focus intended for the model is food and agriculture. This is a good choice, first because of its importance to mankind and also because depletion effects are more readily modelled for this sector than for any other. Comparatively reliable data exist for potential arable land and costs of land development; far less is known about raw material or fossil fuel reserves. The situation for the latter examples is also complicated by the large number of potential substitutes available, which would each require a separate sector and about which few data exist. The model is constructed in a way that ensures a fair degree of "flexibility" in that, for example, the level of aggregation can be changed and extra subsectors added for particular purposes. Nevertheless, the neoclassical structure assumed does appear to prohibit experimentation with certain actual or potential real-world phenomena, a situation we would regard as inevitable given that no theory has an infinitely broad domain of applicability. Direct interference with the price mechanism, for example, leads to apparently unreasonable results, and price changes can only be invoked indirectly by changing, for example, the output or demand functions. As we have implied under (c) in our list of assessment criteria, this need not be a problem in itself but does, in our view, restrict somewhat the range of scenarios that could be generated with the model in its present form.

Similarly, it seems unlikely that the present structure would permit testing of a price policy such as the common agricultural policy (CAP). It is worthy of note that governments in nearly all developed countries appear to consider an agricultural policy of some kind as a necessity, so provision for such a policy to be applied exogenously would be of value.

Treatment of Uncertainty

Uncertainty about the future can be considered to be of two main types: first, that deriving from lack of knowledge of cause-effect relationships and from imprecise data; secondly, that caused by decisions taken by actors in the system which seem inherently impossible to forecast. An example of the former is the quantity of fossil fuels economically recoverable from the earth. An example of the latter is the possible change in consumer spending patterns as values change.

If the uncertainty in either of these categories is considered to be too great to be realistically related to other specific model variables it can be specified exogenously to give one particular "scenario" for the future. In our version of the SARU model, population growth and technology can be treated in this way. Utility curves can also be easily amended if required, to represent changes in spending patterns from those suggested by historical data. Of course, the form of any of the other relationships *can* be changed, but by treating a variable as endogenous the assumption is made that it can be meaningfully related dynamically to other variables.

Theoretical or empirical support is provided for all the relationships used in the model, and this represents a substantial advance over the early work of Forrester and Meadows. Whether this support is "sufficient" to justify endogenous treatment depends on the nature and accuracy of the output required and the sensitivity of the model to changes in the assumed structure. Related to this is the fact that it may be useful to treat certain variables exogenously so that policy options can be simulated. Some, such as foreign aid, have been mentioned by SARU but are not included in our version of the model; if required, specification of some others may require modifications to be made to model mechanisms. For example, the fact that the main inputs to the agricultural system are being modelled optimally precludes exogenous choice between investment in irrigation or fertilizer (this mechanism is not incorporated in the version of the model we have tested).

In the model, population growth is treated exogenously. Income and income distribution are widely thought to affect fertility. While we tend to agree that causal models representing population variation are unsatisfactory, we would hope that the work SARU have performed on the relationship between food intake and mortality could be introduced to provide feedback between the food sector and death rate, we also suggest that the population variable be disaggregated in order that the strong dependence of food requirements on age could be incorporated. The influence of population growth on economic evolution, and the reverse influence, constitutes a major area of debate--perhaps the single greatest area of differing philosophy between futures researchers--and development of the model to allow useful experimentation on this question would be very valuable.

Dominant Variables

Results of runs we have performed suggest that the overall behaviour of the model is largely determined by the overall growth parameters (population and technology, including the exponents of the production function and any depletion mechanism); the utility curves, and overall growth, determine output from each sector. Many of the other parameters, such as stocks, serve the function of ensuring that behaviour is stable and "realistic".

Overall Growth Parameters

It is clear that population and technology are extremely important to model evolution. With no depletion effects, and assuming equilibrium growth, the Cobb-Douglas function gives $q = m(\ell-\lambda) + \ell$, where q, m and ℓ are growth rates in output, technology, and labour, respectively, and λ is the exponent of capital. For a "Malthusian" outcome, with population growing more rapidly than output, this gives $m(\ell-\lambda) < 0$, or m negative. Now the depletion functions used in the model have the effect of increasing the capital and labour input per unit of output, i.e. they represent an exact equivalent to a negative rate of technological growth. Thus a Malthusian outcome occurs if the depletion functions become more significant than those representing technological improvement.

Utility Curves

The demand side is highly dependent on the shape of the utility curves, giving the distribution of consumption between sectors and hence influencing the relative investment allocated to them. In our version of the model, which did not include irrigation sectors and hence could not incorporate depletion effects in agriculture, these curves were completely dominant in determining consumption patterns. The adjustment in these patterns incurred by the mechanism incorporating changes in relative prices had a very small effect. Given the difficulties involved in deriving price elasticities and other problems arising through their use, the inclusion of the price effect mechanism may not be justified for some purposes.

Input-Output Coefficients

In the model, input-output coefficients are taken as constant except in the case of fertilizers and in depleting sectors. For the latter, it is assumed that the coefficient will increase as depletion effects--negative technology--become significant. While it is extremely difficult to relate possible future reduction in the coefficients in sectors subject to technical improvements this does seem to represent a pessimistic bias. Runs we have performed illustrate the sensitivity of results to assumed reductions in the value of the coefficients, a sensitivity compounded by "multiplier" effects.

For the basic input-output data, representative countries or regions in each stratum have been chosen. We appreciate that the scarcity of data is often severe, particularly in developing countries, but it would be useful if one of two other countries or regions in the strata 2 and 3 could be selected at random and compared with that chosen to give some measure of confidence that the selected subregion is indeed "typical".

Overall, we consider that the model represents as coherent and consistant a statement of the neoclassical world view as we have seen; the structure and calibration of the model have been carried out with care and obvious expertise. We think that it will provide a useful and stimulating input to the field of futures research and think the project is eminently worth while. The problems that SARU are analysing are profoundly important and the fact that the task is difficult should not deter the effort, which we fully expect to yield valuable insights, particularly into the effects of physical resource depletion and substitution between alternatives.

STRUCTURAL ANALYSIS

We feel that one of the most satisfactory features of the SARUM project has been the extent to which the constructors of the model have themselves experimented with the model structure. The SARU team have performed and reported the results of a wide range of sensitivity analyses, and devoted a great deal of time to the testing and evaluation of their model. In order to complement the analysis already performed we decided to apply some techniques for the analysis of system structure developed at the Science Policy Research Unit (SPRU) to the SARU model as a major part of our overall evaluation.

The motivation for analysing the structural features of a model is to help provide an interpretation of behaviour from a viewpoint distinct from that obtained by the study of individual relationships. In particular, it is designed to indicate the major "driving" mechanisms of a model, to assess its feedback loop structure, and to explain its behaviour under the influence of exogenously applied changes of various kinds. It is hoped that this will provide insights that will enable the modellers better to understand model operation and, if appropriate, to change the structure.

Technical Discussion

The first stage in our systematic analysis of SARUM was to derive automatically an interaction matrix providing a linear representation of the state of the model at a particular time period during a run. The computer program designed for this purpose (AUTO) was implemented and this task was satisfactorily completed. The procedure is described below:

Consider a typical equation of a dynamic simulation model (DSM) that contains statements about the effect that the variables B,C,D, and E have upon variable A:

$$A = B * C + D * E$$

This can be represented in the following notation:

$$B \rightarrow A \quad C \rightarrow A \quad D \rightarrow A \quad \text{and} \quad E \rightarrow A \ ,$$

where $B \rightarrow A$ signifies that B affects A or alternatively that the value of A is partly determined by B.

For a complete set of equations the relations implied will define a graph (Figure 1). For non-trivial models this graph will have structural features that can form the basis for analysis and comment. It is possible to quantify the "effect" that one variable is having or could have upon another. We define this "effect" as the strength of the relation.

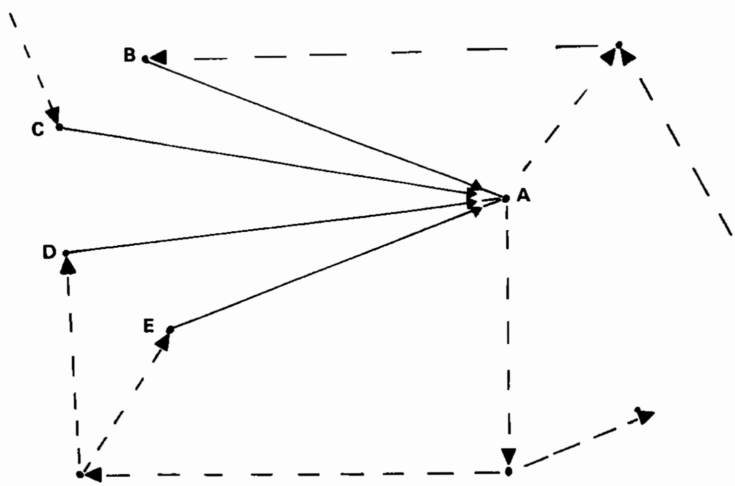


Figure 1.

The strength of the relation B → A is given by:

$$S_{B \to A} = \frac{\partial A}{\partial B} \quad .$$

The physical interpretation is that, if B were to increase by ΔB, then the resultant change in A would be

$$\Delta A = \Delta B \; S_{B \to A} \quad .$$

The partial derivative can be determined by a numerical approximation. If

$$y = f(x)$$

$$y + \Delta y = f(x + \Delta x) \quad ,$$

$$\text{then} \quad \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - y}{\Delta x} \quad ; \qquad (1)$$

$$\text{if} \quad \Delta x \to 0 \quad , \text{ then } \frac{\Delta y}{\Delta x} \to \frac{\partial y}{\partial x} \quad .$$

This procedure can be implemented on the computer by manipulating the original function into the form of (1) and evaluating this expression at a particular point in model time. In general the model under study will be non-linear and an estimate of the range of validity of this numerical approximation could be made by evaluation of the second derivative in a similar manner. A more satisfactory test of the numerical accuracy of the results can be obtained by using the linear system as a dynamic model, as described later, and comparing the results obtained with those of the original model.

The next stage in our analysis was to identify the complete set of feedback loops contained in the matrix and thus determine the dominant feedback structure of the model. Our first attempts to do so revealed that the interaction matrix, and thus the model it represents, contains a surprisingly large number of feedback loops. Indeed the model contains so many loops that a one-hour run of our algorithm, on the ICL 1906A of the SRC ATLAS Laboratory, detected 200,000 loops; the program indicated that the search had not been completed, and on inspection the first 12 variables of each of the 200,000 loops were found to be identical.

At this point we realized that a full feedback analysis of SARUM was not feasible unless the speed of our algorithm was increased. We were a little deterred by the fact that the method we were using at the time, due to Syslo, was claimed to be, in theory, the most efficient for the purpose. Nevertheless, we rewrote the algorithm starting from first principles and discovered that our new version was approximately three times faster than that we had been using previously. We applied our new algorithm to the 10-sector interaction matrix and a one-hour run similar to that described above detected approximately 600,000 feedback loops again indicating that the search had not been completed. Unfortunately, even in this case, the first 10 variables on each of the loops were found to be identical which suggested that even this rather extensive search had not progressed far through the structure of the model.

Thus, it appeared that a full search for all the feedback loops contained in the interaction matrix of the 10-sector model was not feasible within the time-scale of the first phase of this research project. We accordingly transferred our attention to the 2-sector version of the model. We felt that this considerably simplified version of the model would embody most of the significant structural features to be found in the more sophisticated version. In particular the feedback structure of the two versions would show a close correspondence. This viewpoint was supported by the fact that the feedback loops discussed in the SARU model description as presented to us relate exclusively to feedback processes contained within a single sector.

It was a relatively simple process to repeat the automatic interaction-matrix-generating procedure (AUTO) in conjunction with the 2-sector data set. The resulting interaction matrix contains 309 relationships linking 152 variables (here vectors and arrays with different subscripts are treated as separate variables).

Again we ran our improved feedback algorithm for one hour in an attempt to list all the feedback loops contained in the model. This time the program found 542,528 loops but again had not been able to complete the search in the time available. This time, however, only the first three variables on each of the loops remained the same. We were somewhat encouraged by this result as it suggested that an exhaustive feedback loop search was at least feasible, if time consuming. We accordingly modified our algorithm so that it could be set to search through the model not from the beginning but from a path containing a specified set of variables. In this way we were able to continue the search through the model in a series of one-hour computing runs. (One hour is the maximum practical length of run on the ATLAS Laboratory computer.) This process has not yet been completed at the time of writing this report. We were able to estimate, however, from the runs already completed that the 2-sector model structure contains approximately 20 million distinct feedback loops. However, this number can be lowered considerably by reducing and simplifying a linearized form of the model, as described below. We have been able to show that such a simplified linear model is capable of reproducing the output characteristics of the original model to a good approximation. Further, it appears to react to perturbations in a very similar way.

By performing the linearization procedure at a set of distinct time points, a number of linear models can be obtained on which a sequence of further operations aimed at improving understanding of model structure can be performed. This sequence is summarized in Figure 2 and described below.

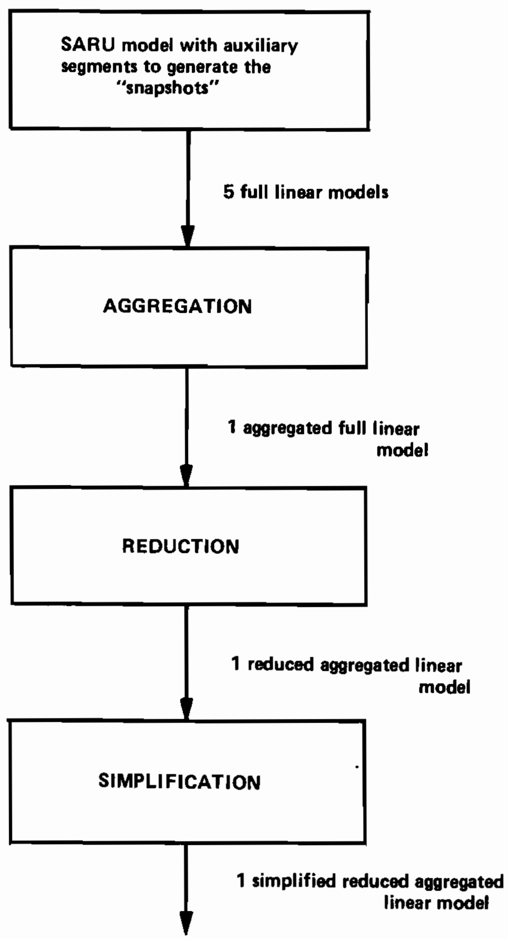


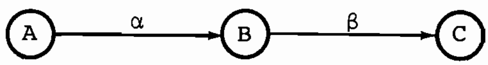
Figure 2. Flow chart of production of the linear model.

Reduction of the Model

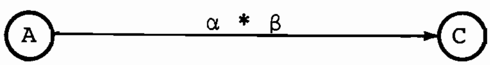
When a structural model is derived from the DSM in the above manner all the variables (auxiliary, rates, and levels) will be present. In the introduction it was stated that the behaviour of the model can be retained by reducing the model to a set of levels for a discrete system.

In graphical form this corresponds to the following operations:

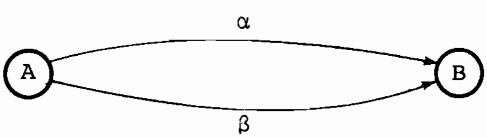
(1) replacing 2 paths in series:



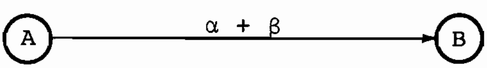
by one and therefore losing the intermediate variable:



(2) replacing 2 paths in parallel:



by one:



This process enables all intermediate variables present in a model structure to be systematically deleted. Removal of *any* variable will change the structure to a certain degree, the change being dependent on the relationships that involve the removed variables and the features of the structure under study. By removing variables (or nodes) from the graph fine detail is lost in explicit form, but retained implicitly; this may be acceptable and, for large models, even desirable.

By using the FORTRAN version of SARUM with the base run of the 2 sector (1 stratum) data set, five structural "snapshots" were produced between the years 10 and 50 at 10 year intervals. These five models, each containing the full set of about 170 variables, were then reduced onto each of three distinct variable sets, containing respectively the following (as designated in the FORTRAN listing):

(1)	*POPO	TOTLAB	EMPI	E	EPC
	*GR	PCI	AVR	*DEBT	*CUMD
	*CALS	*AL(1)	*AL(2)	*CO(1)	*CO(2)
	*CSTP(1)	*CSTP(2)	*ETA(1)	*ETA(2)	*EXR
	*GROW(1)	*GROW(2)	*SO(1)	*SO(2)	*WSM(1)
	*WSM(2)				

where an asterisk denotes a state variable.

(2) State variables and Q(1), Q(2) E VIN.

(3) State variables only.

Aggregation

Having obtained the linear models by using the three alternative variable sets at five different time points, we decided to produce a single model from a combination of the "snapshots" with the hope that this "aggregate" model would reproduce the behaviour of the original non-linear model more closely. If the system interaction matrices for the "snapshots" are

$$A_1, A_2, A_3, \ldots, A_k \quad ,$$

the elements of the aggregate system interaction matrix are defined as:

$$A_{ij} = \sqrt[k]{\prod_{\ell=1}^{k} (a_\ell)_{ij}} \quad ,$$

which is the geometric mean of the corresponding elements of the matrices.

At this point we can compare the behaviour of the aggregate reduced linear model (ARL) with that of the full non-linear FORTRAN model. The equation used to run the ARL model is:

$$x(t + DT) = x(t) + \hat{A}(x(t) - x(t - DT))$$

where the x(t) are set to the values of the variables given by the FORTRAN model at time 10.0 + DT and the x(t - DT) are equated to the values of the variables at time 10.0 (DT = 1/6 year).

Table 1 contains the values (which have been scaled for ease of comparison) of the linear model containing the set of variables (1) above compared to the original FORTRAN model. These values are shown at time = 10 (the start of the run) time = 30 and time = 50 (the end of the run). The derivation of the values appearing in the last two columns is discussed later.

Table 1.

	Original Model			Aggregate Reduced Model		Simplified Aggregate Reduced Model	
	T=10	30	50	30	50	30	50
E	83	163	306	176	301	170	293
AL(1)	95	133	183	136	176	132	168
AL(2)	649	781	939	768	911	739	828
CO(1)	117	288	629	330	648	317	609
CO(2)	25	54	105	58	109	54	103
CSTP(1)	881	821	837	814	822	806	806
CSTP(2)	101	101	101	101	98	100	95
ETA(1)	107	135	154	137	154	139	164
ETA(2)	893	865	846	863	846	865	851
GROW(1)	113	145	186	145	186	145	186
GROW(2)	113	145	186	145	186	145	186
POPO	200	244	298	244	298	244	298
SO(1)	243	464	864	507	865	459	784
SO(2)	68	133	250	145	256	137	238
WSM(1)	220	449	855	500	846	441	752
WSM(2)	66	128	241	138	244	129	220
TOTLAB	743	912	1120	903	1090	870	994
EPC	42	67	103	68	104	65	101
PCI	213	218	233	223	213	229	217
CUMD	41	76	131	96	169	90	150

Table 2 shows similar data for the linear model containing variable set (2); as can be seen, the values obtained for variables common to both linear models are very close indeed, and show good agreement with those obtained from the original non-linear model. Table 2 also shows the values at year T = 50 given by the "snapshot" models from which the aggregate reduced model was derived.

Table 2.

	Original Model (T=50)	Aggregate Reduced Model (ARL) (T=50)	"Snapshot" Taken at Time				
			10	20	30	40	50
E	306	300	237	265	300	342	395
AL(1)	183	175	150	166	177	187	197
AL(2)	939	913	921	916	912	908	901
CO(1)	629	641	446	529	638	791	1010
CO(2)	105	109	85	96	109	125	147
CSTP(1)	837	824	836	830	826	822	818
CSTP(2)	101	98	96	97	98	99	99
ETA(1)	154	154	142	151	155	157	158
ETA(2)	846	847	858	849	845	843	841
GROW(1)	186	186	186	186	186	186	186
GROW(2)	186	186	186	186	186	186	186
POPO	298	298	298	298	298	298	298
SO(1)	864	863	675	756	857	989	1160
SO(2)	250	255	204	228	256	290	333
WSM(1)	855	843	622	726	843	995	1190
WSM(2)	241	243	197	218	243	274	313
VIN	685	654	462	547	653	791	974
Q(1)	977	923	672	790	926	1100	1310
Q(2)	274	268	214	238	268	304	349
CUMD	131	168	129	147	169	194	226

Very similar results were also obtained from the linear model comprising state variables only. In Figure 3 (A and B) the derivations of the values obtained in this model from those in the non-linear model are plotted as a function of time. In Figure 4 the evolution of variables CO(1) and CO(2) are each compared in the two models, to show that agreement is in fact close even for those variables that a superficial inspection of Figure 3 would suggest do not match well.

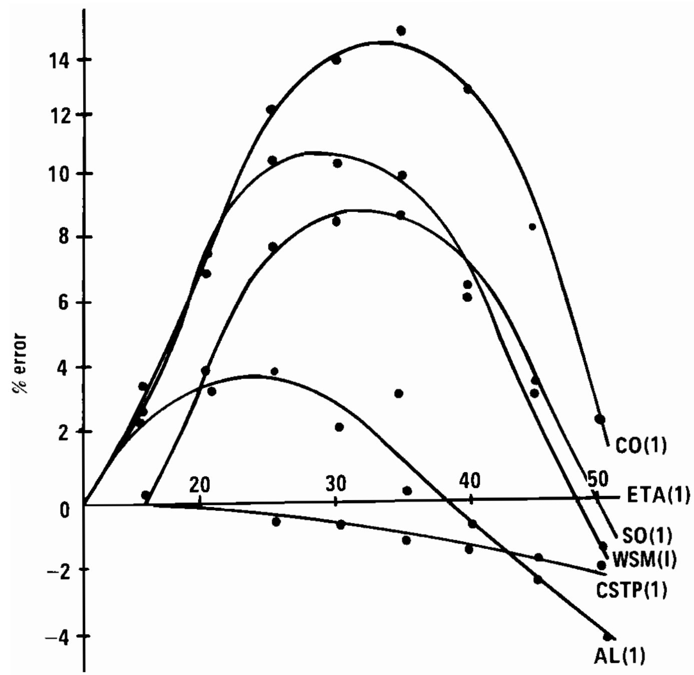


Figure 3A. Percentage error of the key variables in the 1st sector of the linear model over time.

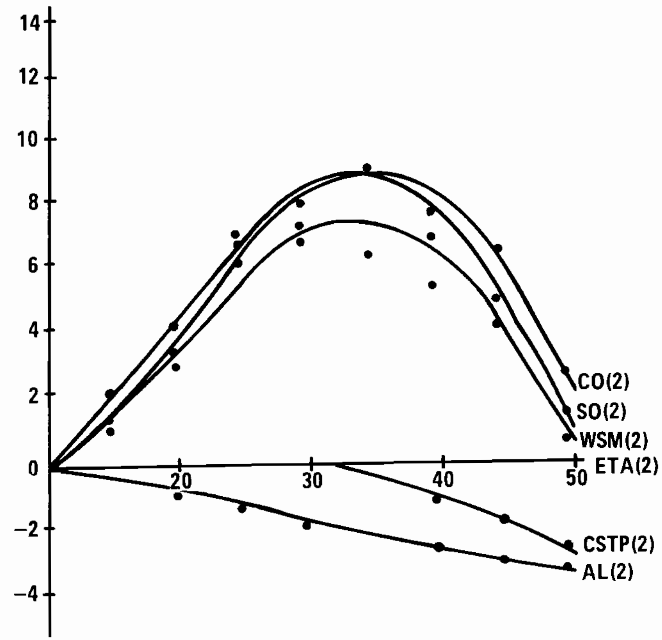


Figure 3B. Percentage error of the key variables in the 1st sector of the linear model over time.

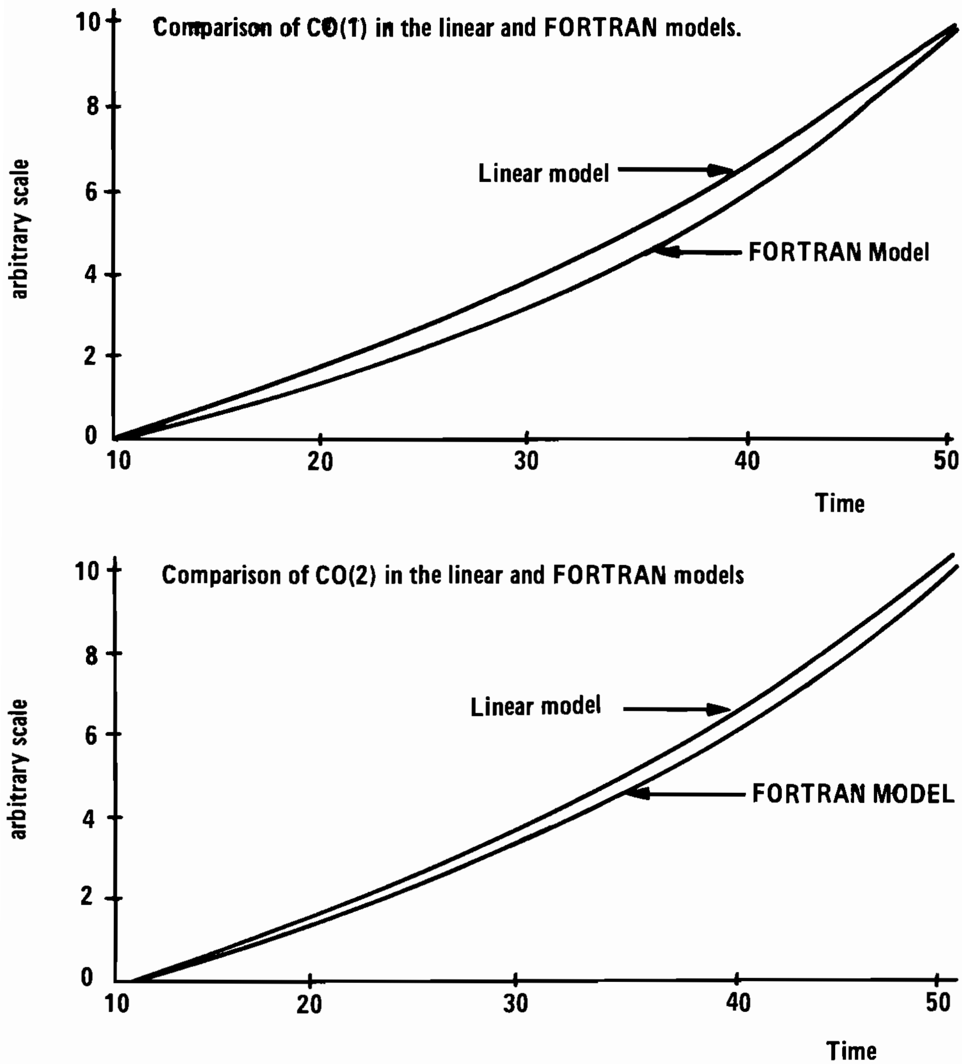

Comparison of CO(1) in the linear and FORTRAN models.
arbitrary scale
10
8
6
4
2
0
10
20
30
40
50
Time
Linear model
FORTRAN Model
Comparison of CO(2) in the linear and FORTRAN models
arbitrary scale
10
8
6
4
2
0
10
20
30
40
50
Time
Linear model
FORTRAN MODEL

Figure 4.

Simplification

The system interaction matrix $\hat{A}$ determines the behaviour or trajectory of the state variables. Many state variables are determined by groups of other state variables; for the ith variable, the equation above can be written:

$$x_i(t + DT) = x_i(t) + \sum_{j=1}^{n} \hat{a}_{ij}(x_j(t) - x_j(t - DT) \quad .$$

Now, if one element in the summation is small compared with the others, the $\hat{a}_{ij}$ may be set to zero and the trajectory of x_i only marginally changed. Physically we are deleting relations that have little importance in determining model behaviour. This may allow, for example, the system to be decomposed into subsystems or a hierarchy more easily. The validity of the simplification may be checked by running the simplified model as before and noting the differences introduced.

Table 3 indicates the quantities $\hat{a}_{ij}(x_j(t) - x_j(t - DT))$ where CO(1) has been selected as the variable j and the rows i are labelled by the variables that directly affect it. The matrix as used for the different time columns is the reduced linear model for that time point. As can be seen, the main determinant of CO(1) is itself (this is because of the "self loop" term). There are numerous criteria that could be adopted for deleting relations with no significant effect on CO(1) (two obvious candidates are CSTP(2) and EXR) or for deleting two relations whose effects on CO(1) balance or nearly balance each other out (GROW(1) and GROW(2)). The general rule adopted in this case has been to delete a relation if all the entries in the row (for that relation) are less than 0.1% of the largest entry in the corresponding column; this is equivalent to deleting a relation if, for the whole time period, at least one other variable has more than 1000 times the effect of the variable under consideration.

This procedure requires modification in some cases. Consider the hypothetical column:

A	1000
B	-1000.01
C	0.99

Clearly C is a candidate for deletion on the 0.1% criterion.

Table 3. $\hat{a}_{ij}(x_j(t) - x_j(t - DT))$ for the five "snapshots" - showing all variables affecting CO(1).

	T = 10	20	30	40	50
E	11.45	14.62	20.18	29.17	43.41
*EPC	1.662	2.497	3.999	6.663	11.35
*AVR	-1.125	-2.134	-2.619	-3.496	-5.266
*AL(1)	6.147	7.820	11.34	17.19	26.74
*AL(2)	-2.592	-3.243	-4.815	-7.291	-11.12
CO(1)	1152.	1364.	1870.	2737.	4130.
*CO(2)	-1.870	-1.901	-1.938	-2.013	-1.921
*CSTP(1)	-5.842	-3.046	-3.646	2.336	5.565
*CSTP(2)	-.1584	-.1438	.1956	.5401	.9632
*EXR	0	0	0	0	0
*GROW(1)	4.953	7.150	10.50	15.71	23.88
*GROW(2)	-5.203	-7.266	-10.68	-16.08	-24.62
SO(1)	-24.66	-28.82	-40.96	-60.94	-92.09
SO(2)	29.19	45.37	62.14	87.83	128.3
WSM(1)	27.38	31.71	43.22	62.69	93.64
WSM(2)	-27.96	-43.99	-61.15	-87.15	-127.8

EXR is constant for the run

However, if this rule were followed, instead of a change* of

$$1000 - 1000.01 + 0.99 = 0.98$$

the change would be

$$1000 - 1000.01 = 0.01$$

which is very different. It would be better to delete the other two on the basis that they balance each other.

The relations marked with an asterisk in Table 3 are those that may be deleted without significantly altering the trajectory

*ignoring the units and the time period during which that change occurs of CO(1). When the same simplification rule was applied to the other 25 variables, a total of 77 relations were deleted. These deletions gave a simplified aggregate reduced linear (SARL) model, which was run in the same manner as the ARL model, with results as shown in Table 1. At first it may seem strange that certain variables track with the original FORTRAN model variables better than the ARL model (CO(1) and CO(2) for example). In fact this is to be expected; the ARL model had slight errors that may have been reduced or increased by the simplification process. If the problem of "balancing" relations is ignored then the simplification procedure can be made fully automatic. A computer program was written that simply deleted any relation according to the 0.1% criterion outlined above. This program was used to create the simplified version of the linear model containing variable set (2) which formed the basis for discussion in our final report to SARU.

SUMMARY

A set of reduced aggregate linear models have been produced that closely follow the behaviour of the original non-linear SARU model. Thus non-linearities appear not to be important over the range considered. The SARL models were derived from the model structure, not from the data or output. (The ability of the linear models to duplicate other behaviour modes is discussed elsewhere.) By using the same technique of generating an interaction matrix containing partial derivatives, a non-linear model could have been obtained.

The linearization procedure, while aiding considerably the analysis and interpretation of the feedback loop structure of the model, can also very readily provide direct information on model dynamics. For example, both Professor Rademaker and ourselves have shown that the reason for the exponential growth pattern produced in the early period of the "standard run" of Forrester's world model was not, as is commonly assumed, due to the structure of the population subsystem. The latter is in fact intrinsically stable, as can be seen from the corresponding diagonal element and the interaction matrix; the only unstable subsystem is capital investment, which is thus the driving force of growth in the model.

As shown above, no such "unexpected" mechanisms are present in the SARU model; all state variables (except population and technology, the "exogenous" driving variables) are intrinsically stable. The values of the interaction matrix coefficients, however, yield information on the response of the system to externally applied perturbations, and allow estimates to be made of the relative significance of feedback loops in the model.

Space, unfortunately, does not permit us to present here the details of our comprehensive analysis of the linear model whose production was described above. Readers are referred to our final project report to SARU.

DISCUSSION

Richardson claimed that the policy maker might ask two questions: one, as to a simplified and more comprehensible underlying structure of the model, and a second one concerning the validity, whether the model structure is a reasonable representation of reality. Has the Sussex group made an analysis of the validity of SARUM? McLean replied that a "structural analysis" cannot be claimed to test the adequacy of the model too well. The main emphasis of their work was a better understanding of the model as such.

Burke, seconded by M'Pherson, pointed out that linearization of a model is not unique, so it is dubious to what extent one may draw conclusions from a linearized model to the general nonlinear form. McLean replied that, for certain investigations, local linearization will reproduce the model sufficiently well.

Osterrieth wondered whether the methodology applied by SPRU can be applied in case of discontinuity caused by changes in constraints. McLean replied that, due to limited resources, such discontinuities have not been dealt with so far.

Hopkins raised the general question of how useful the SPRU technique might be for another model developed by some other group, or whether the SPRU technique is only applicable for models of a certain size. McLean replied that the purpose of the technique is to simplify the complicated structure of a model; hence, the technique is independent of the model's size.

COMPARISON OF SARUM AND MRI

M. Keyzer

It is very difficult to compare SARUM and MRI which are such big models without going to the computer and playing around with them for a while. So here I will only say something about the models from a theoretical point of view, in the way an economist looks at this kind of work.

Yesterday, Professor M'Pherson referred to the most essential common theoretical feature of the two models, namely, that both try to allocate resources in such a way that the consumer is satisfied. In the MRI model, the government can limit this with what may be called "social restrictions", but if one looks only at the mathematical structure, they are comparable. As soon as one looks deeper, the models are not comparable at all. I have a general criticism about both papers which is that, while the assumptions are stated very explicitly and with great modesty, the difference between practical assumptions and descriptive assumptions is not always clear. All assumptions are presented as being descriptive while after the discussions and after having read the whole paper, one finds that many of them are really of a practical nature. An example in the SARU model is the heaps with the stock mutations.

I realize that to do Monte Carlo simulations with the model and to run it often, if you have such a big model you do not want a simultaneous system, but a simple recursive form; this is quite legitimate. But it should not be brought forward under the cover of disequilibrium economics as there are much more acceptable ways of introducing disequilibrium features in models. If the stocks, to take this as an example, are just adjusted for the differences between the supply and demand, there is no net need for change in prices under "perfect" competition. Under "imperfect" competition there can be a change in price but then prices are not given for the producers. There are many such examples. The trade bias matrix in the SARU model is also not as complete as the economist would like to see. In the MRI model the relation of the Cobb-Douglas form is not based on estimation of any elasticity of substitution; it is just very practical. Again, this is quite legitimate, but it should be said at the beginning that certain assumptions are necessary to make the approach feasible. Kulikowski has promised us that he will introduce constant elasticity of substitution functions. I can understand that if one keeps it that way in all sectors, one can realize something, but if one tried to do it more the way the econometrician would like one would get in more trouble—perhaps one would not get any result at all. To summarize this first point, it would have been easier if the practical assumptions had been separated better from the really descriptive assumptions.

This brings me to a description of the methodologies of both projects. Mr. Roberts has said the SARUM project started from criticism of the Meadows and Forester work--especially the criticism on the part of the economists that the resource allocation mechanisms in the Meadows model were not good, so that the system was insufficiently adaptive. Another criticism was with the level of aggregation both in regions and in commodities, and the SARUM group have tried to overcome these shortcomings, and, I think, succeeded. A second line of criticism from economists was hardly expressed because it was expressed by silence. The "establishment" in the economic profession (Nordhaus is an exception) has hardly given any open reaction to what has been done by the system dynamics people. I can only guess the reasons for this because they were not expressed. But from discussions I have had with economists I conclude that there is a combination of profound contempt and jealousy. There have been all kinds of scientists, engineers, mathematicians, people not trained in the social sciences, who have just entered the field and have not bothered about discussions, controversies, and problems that have been discussed for years in the economic profession. They just followed their intuition and formulated their models. They disregarded problems with all kinds of things like aggregation levels, insignificant coefficients, etc., and just built the model they wanted to build or that the policy makers wanted them to build. I felt this tension when I read the SARUM paper but I think it had already been felt with the Mesarovic model. This problem should be expressed because if not you get all kinds of antagonistic feelings developing. When I read the paper on the SARUM model I had the feeling that the authors have really succeeded in building the model they wanted to build, with certain aggregation levels, incorporating all sectors, etc. And it makes the econometrician who is bothering with solutions of simultaneous equations, optimization problems, etc., feel jealous. The others just say that they are not considering simultaneity and the like. They concentrate fully on their flow charts and discuss them with everybody; after a while everybody agrees with those flow charts. The most serious disadvantage when building a model in an econometric fashion is that one spends one sixth of the time on theory and model development, and the rest with technicalities. One has little time to think of the relative problem so one feels jealous. On the other hand when one looks at the final result the jealousy is over and the old trail becomes appealing again. Let us consider a model where a good historical fit has been reached at the expense of many degrees of freedom and where there has been no test of significance. A very complex sensitivity analysis performed afterwards does not mean very much when neither the empirical relevance nor the theoretical consistency of the model have been checked. The economic profession will not be able to build a world model for many many years, if at all, according to its own criteria.

In the MRI model the situation is substantially different because here we have a strict, basically very simple, but powerful structure that achieves full consistency between all sectors in the economy. In the MRI approach one can satisfy all the balance equations, but again I have problems with the empirical relevance of the model. I had the impression that it is somewhat too idealized--just like if one were to assume "perfect" competition in market economy. I know that Kulikowski and his colleagues are aware of this. I am just saying that uncertainty and imperfect knowledge were not introduced into the model so that hardly any social restrictions on policy were represented. The model by itself has potential but it should try to introduce more imperfections. It has succeeded in achieving a balance between centrally planned producer decisions and the consumer. Although not proved in the paper that the equilibrium will always exist, I think it is easy to show that there is an equilibrium inherent in this kind of a structure. I doubt whether the equilibrium will always exist especially if one introduces the kinds of problems mentioned.

Why is modelling not done by economists? Why is it done by control scientists? The optimization part of the work should be done at least with the help of control scientists but, certainly in the Netherlands, the behavioral description of producer, consumer, and government is traditionally the field of the economist. I think that if an economist tried to describe the production structure of his country, he would start with more general production functions and only slightly restrict them in order to satisfy certain conditions for good behavior. He would not decide on a complete specification before having really reached the empirical level and he would then estimate general functions lest the significance of the parameters meant introducing all kinds of lags. Some coefficients he would want to be significant would not be nicely structured and the model would not work. That is why it is good if some fresh contributions from outside are given here to show that if you do not care about empirical tests, you can formulate a structure that leads to interesting results.

To give another example of the difference in approach: an economist would never analyze optimal population policy before having analyzed whether there are externalities in the economy and whether there are congestional or depletional effects. One can get nothing more than an optimum population structure, and not for a very long run analysis, because one cannot just for the sake of an optimum population structure go on with population growth forever. The economist would not have an answer for these questions. What economist could really say what the externalities are? So I am not saying that the economist would have a better answer. If the policy makers asked him, he would say "if you have very strong depletion effects you should not stimulate population growth; otherwise you should." And this is why I say that as an economist I do not know exactly how to react to this kind of work. I am taking the position of the economist because I know very well that sociologists and biologists take exactly the same kind of position towards economists as economists do towards engineers. The psychologists find our production functions far too simple; labor productivity is not a technical coefficient; it is a behavioral relation and one should perform all kinds of studies on motivation, etc. to understand it. I think that the first stage to contributing to the global modelling field should be from the economist. I do not say that it will be the most relevant one. It is just that in the global modelling field and at IIASA in general, the way systems analysis is discussed leaves the economist with the kind of problem I have described.

DISCUSSION

Pestel asked where, in the distinction between the economist's and the control engineer's view, the systems analyst finds his place. Roberts commented that his group, in their work, at least subjectively was trying to apply as much economic thinking as seemed appropriate. But economic thinking alone does not suffice; they have tried to have as much dialogue between disciplines as possible. M'Pherson pointed out that the real difference lies in the attitude toward the problematique; whereas economists, aware of all the shortcomings of their theories, are hesitant to build general models, control engineers tend to say that the problematique must be solved, albeit with crude tools.

Bruckmann warned against too much generalization; what is being said about "economists" should rather be said about "a substantial part of traditionally trained economists".

Herrera stressed the difference between interdisciplinary collaboration and real interdisciplinary research, which requires one to spend an entire first phase on learning to understand each other before any fruitful work can begin.

Parikh added a word of caution about the control engineer's approach. If, as it is being claimed, the man with a bicycle gets ahead faster than the driver of a car caught in a traffic-jam, it is because the bicyclist has a pre-fixed vision of his goal. The control engineer may be in danger of picking just those tools from other disciplines' toolchests that he needs in order to arrive at goals he may have decided upon before starting his work. There is a further difference: in engineering work, there is an immediate feedback--a poorly designed machine will not run. But how can the control engineer who has built a long-term global model be judged? Hence, some of the economist's caution ought to be applied.

Kulikowski added that his group had found it most fruitful to have engineers and economists work together.

COMMENTS ON THE SARU GLOBAL MODEL

M. Slesser

On behalf of the group working on energy and food production within the Energy Studies Unit at Strathclyde University, I should like to make a brief comment on the agricultural sector of the SARU model. Roberts et al. note, in our opinion quite rightly, that "the constraint on agricultural output is related to the flux of energy over the input area". The same sentence terminates with the clause "and efficiency improvement is the crucial aspect (rather than capital/labour enhancement)".

If efficiency improvement is taken to imply more yield per unit input (other than solar energy), then it is logical enough to propose a logistic curve for output per unit area. However, after three and a half years of study in this field, during which we have collected data from over two hundred randomly selected systems of food production throughout the world, we have detected no evidence to support a logistic curve, at least in terms of resource input. Our finding has been that the effect of genetic improvements has generally been to permit the plant to make better use of higher inputs, rather than yield more per unit input. Figure 1 shows that over ten orders of magnitude of input intensity, output intensity has followed a law of diminishing return: the double logarithmic plot a straight line was obtained.

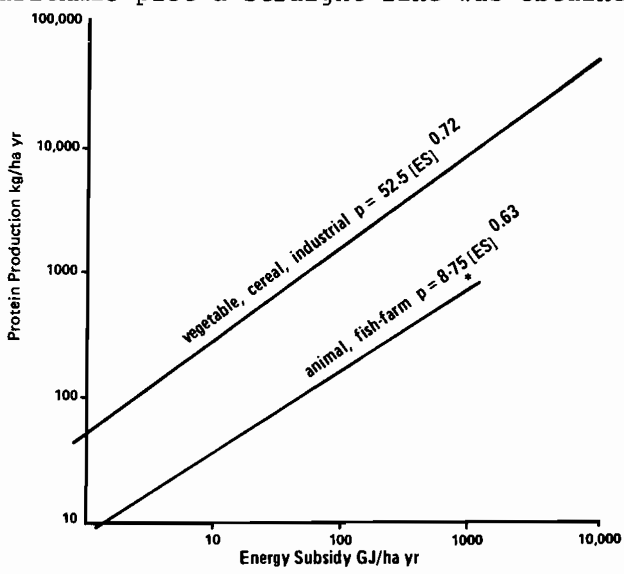


Figure 1. Intensity of protein production plotted against GER of network inputs to food production process.

Vegetable cereal and industrial food production followed the equation $P = 52.5(ES)^{0.72}$, while animal and fish-farm production followed $P = 8.75(ES)^{0.63}$, where P is the protein yield in kg/ha year, and ES is the energy subsidy expressed as GJ/ha year.

A word is necessary to explain energy subsidy. We follow the rules for energy analysis proposed by International Federation of Institutes for Advanced Study (IFIAS), whereby an energy analysis is made in terms of the prime energy resource consumed in order to make available a given good or service at some point in the economic system. The point we have chosen is the farm gate. Normally this is called the GER (gross energy requirement) and is the sum of the prime energy requirement of all the network inputs to the process of production. We call it energy subsidy, simply because we regard this energy resource input as a subsidy to that provided by sunlight.

The area term is obtained by summing all the land required to produce the final output. For example, a fish farm may yield 20 tons of fish per hectare of pond per year, but the food fed to the fish may be grown on 200 ha. Together with land requirement for industrial inputs, the true area, and the one used in the calculation will be slightly above 220 ha.

Several points emerge from Figure 1. Firstly, we do not dispute that in money terms the value of output may often rise per unit of input. But money also embraces labour costs, and labour may be used more efficiently in an intensive agricultural system. Secondly, the import of our plot is that in energy resource terms, each doubling of output intensity requires an approximate tripling of input. Thirdly, it appears virtually impossible to intensify animal and fish farm production beyond a level of around 800 kg/ha year (protein), because at that level of intensity the bulk of the animal foodstuff is coming from vegetable and cereal producing systems that use large areas for production. Finally, the convergence of the animal and vegetable plots occurs at some production intensity lower than that found in natural primeval forest. However, our interest is the intensive region.

Finally, even when it is possible to show that genetic improvement can give a true increase in output per unit input, such is only true for a specific subsystem within the whole system. Our view of the modelling process is that one should look at this whole system, not the parts, and that rather than synthesize from the elements upwards in the model's hierarchy, it is better to devolve downwards. Thus an overall picture such as Figure 1 reveals is the one to incorporate into a global model.

One of the duties that falls upon me now is to throw a spanner in the works--to use an English metaphor. If the following comments seem to apply to SARUM rather than to MRI, I think it is because SARUM is a global model and MRI is a national model. The first one is: has a model constructed on this type of basis really any validity at all beyond the short term? And secondly: are they using the right unit of account in looking at the future? The basis of SARUM is the nine conditions expressed in their full technical report. And every one of the nine conditions are made to apply to future events, which are gauged by study of past events calibrated in money terms and extrapolated to the future as functions of money-based parameters like GNP per capita. I find it quite impossible to see how one can judge the money price of an act far into the future. I am not alone in this. If you travel by air, as most of you do, you will notice that airline companies no longer publish their fares along with their schedules.

Money is a value judgement, one which we exercise extremely skillfully in our everyday transactions. But how can we really attribute a value judgement to something that is fairly far into the future? This process is handled by the science of economics which is essentially a behavioural science, for it deals with how people make choices. A money based allocation mechanism is an extremely effective way of judging what I shall do with my money today, but I do not know that one can really make models about how people will make future choices. Because this is what these models seem to be trying to do. I seriously question whether a model using money as a unit of account can have validity for more than a very short time ahead.

My purpose is to argue, and this fits in remarkably well with the WELMM approach of the energy group at IIASA, that there is a case for abandoning money as a unit of account in future modelling and replacing it with a set of prime resources. They fall into three quite distinct categories. First of all those that are finite--stocks if you like. It has been said by several people that there cannot be any shortage of minerals, because after all when we make a motor car or a washing machine, we still have the substances that go to make it. They are not lost. The quantities of resources in the earth are so vast that the only question at issue is how dilute the supply will eventually become. And we have some very interesting flux resources, coming in at virtually a standard rate, like solar energy and rainfall.

Secondly, we have renewable resources of which it seems to me we only have one important one--people. There are so many variables associated with people that we need to take this very seriously. And we do. Their ideas produce new technology. We can train people. We can detrain people. We can even condition people, I am afraid.

Thirdly, and finally, we have only one non-renewable resource that really should be expressed in negentropy terms, but that we usually express as energy (fossil, fission, and possibly fusion energy). There are obviously enormous numbers of feedback loops. We get biomass by using land but we also need inputs. These inputs come by the use of negentropy and finite stock. I am not going to spend time discussing this because it must be very obvious to all of you here, but I suggest that we have to formulate our production functions and our allocation mechanisms around these basic facts of the world's resources. Simply as an illustration, I present to you the outcome of looking at the food sector of the economy in these terms. If I were giving you a full picture of the food sector I would not simply be restricting myself to energy, I would be looking at people and I would be looking at the quality of the people involved. The intriguing thing about this plot--which is purely an empirical plot obtained by examining, on more or less a random basis, a very large number of food producing systems from the extreme of free range cows in the Peruvian Altoplano to the other extreme of fish protein concentrate and synthesis--is that a correlation holds well over eight orders of magnitude. And we have taken the results of Pimental and Leach and others and we find that their data fall nicely into this plot, (Figure 1) with a correlation coefficient of about 0.91. These two plots emerge, the top one for industrial proteins, for cereals and for vegetables, and the bottom one for animal and fish-farm protein. The star at the end of the animal and fish-farm protein line is the point beyond which it appears that you cannot intensify the system without beginning to use industrially produced inputs to the animal and fish-farm world. Though there is nothing conclusive about this, it is part of the way that we can proceed in trying to look at the future though using a different unit of account. Using such plots as Figure 1, one can assess, knowing the amount of land that is available in different parts of the world, what the prime energy consequences are of a growing population on a given area of land. One can also establish what the prime energy consequences are of a given level of diet. For example, the prime energy consequences for Japan to provide her entire animal and vegetable protein diet from within her own resources is really quite a frighteningly large quantity of energy.

In conclusion, the best we can hope to do in modelling the future is to assess the primary resource consequences of perceived choices, expressing resources here not as mineral resources but as energy resources, people, and the fluxes that come with us. Some of you may feel that this is a somewhat defeatist attitude. I would like to suggest that it is quite the opposite. Life is extremely boring if we know every single move that lies ahead of us for the next few months. Some uncertainty is absolutely essential to have any sort of vitality. On the other hand, we do want to know where our next meal is coming from and the one tomorrow and the one the day after and certainly maybe ten years from now. I suggest to you that global modelling should concern itself with the resource consequences of perceived choices of life-style. This is something I think it has the potential to do with a fair degree of precision. Even the model that has been presented by the SARU people, sophisticated and very good though it is, still allows us all to dispute the results because it has to make quite a number of assumptions, one of which is the price of energy in the future.

DISCUSSION

McLean asked to what extent the relation shown in Figure 1 between food production and energy subsidy supports the view that future analyses should not be based on money terms. Slesser replied that Figure 1 should only serve as an example to show better than any economic analysis what the future needs of food production are. Burke agreed that money is a poor measurement because it is not conservative; things do not always have the same value. The use of money terms fallaciously insinuates virtually instantaneous substitution between conservative entities like materials, energy, and people. What is commonly referred to as a "crisis" is, in fact, the result of difficulties in making this kind of substitution.

Parikh warned that the use of physical units is essentially the same as using constant prices. Furthermore, the explicit recognition of prices, albeit in the form of shadow prices, has the great advantage of permitting optimization procedures.

Batteke supported Slesser's view; environmental analyses trying to express in dollar terms lives of people, lives of fish, and the mercury content in tissue are more than unsatisfactory.

Roberts, summarizing, said that, originally, the SARU group had tried hard to express everything almost entirely in energy terms. But if one pursues this path, one must go it to the end and give an entire picture of the future in energy terms--try to communicate this to the people and see how they receive it! Hence, the avoidance of pitfalls associated with money as a unit of measurement should not be carried to the extreme where more might be lost than gained.

OTHER MODELLING WORK

GLOBAL/NATIONAL MODELLING WORK AT THE INTERNATIONAL LABOUR ORGANISATION

M. J. D. Hopkins

INTRODUCTION

This note describes some of the long-term modelling work being performed by a small team in the World Employment Programme of the International Labour Organisation (ILO). Most of the resources devoted to modelling in this program have been directed to the building of long-term models at the national level, particularly of countries in the Third World. However, some work has been undertaken at the global level to see how global models can help us understand the type of development strategies best suited to eliminating poverty in the Third World. The ILO has not built a global model itself but has used an adaptation of an existing one--the Fundacion Bariloche Latin American world model--to provide illustrative projections. In this paper a short description of the global modelling work will be given followed by a description of the national models that have been and are being built under the generic name of BACHUE.

GLOBAL MODELLING WORK

Early in 1975 I was asked to provide a quantitative picture of alternative patterns of development to the year 2000 as background to the World Employment Conference held in Geneva in June 1976 [1, 2]. To do this a goal was defined in terms of a "defined" set of basic needs (food, housing, health, education) for each of 15 subregions of the world. Next the desired GNP growth rates, with and without various degrees of internal income redistribution needed to enable the poorest 20 per cent of the population to meet these basic needs targets by the year 2000 were estimated. A comparison of these desired growth rates with the actual growth rates of 1960-1970 lead to our general conclusion that the defined set of basic needs could not be met for most of the developing subregions without greatly increased GNP growth rates or, alternatively, without internal income redistribution as well as fairly high rates of growth.

The methodology to reach these conclusions is described in the Background Papers to the World Employment Conference [3]. Briefly the methodology was as follows:

An income was estimated which an individual would need in order to be able to consume a bundle of goods associated with the defined set of basic needs.* To estimate this income the Bariloche model was used. It should be emphasized that the Bariloche model was not used as originally intended by its authors, i.e. to provide a riposte to the *Limits of Growth* work, but was revised to suit our different purposes. In this I had the help of Hugo Scolnik of the Bariloche Foundation and Mick McLean of Sussex University.

The Bariloche model maximizes a life expectancy function which is a function of the basic needs included in the model. Capital and labour are allocated optimally to each of the basic needs producing sectors subject to a number of constraints. At the point at which the basic needs targets are met the GNP per capita can be taken to be the minimum income** required to satisfy the defined set of basic needs.

The major revisions to the Bariloche model to enable us to do this were as follows.

The model was calibrated (albeit rather crudely) to 15 subregions from the original four Bariloche subregions in order that the income target eventually arrived at could be inputted into yet another world model--the Leontief World Economy Model.*** This was done at our request by the Leontief group at Harvard in order to understand the implications of our income targets and different income distribution patterns for such things as balance of payments, investment, natural resources, aid, employment, and environment. Their results are summarized elsewhere [3] but in general they support the general conclusions of our own global work noted above.

The Bariloche basic needs targets for food were substantially reduced and varied between regions following the Food and Agriculture Organization's (FAO) minimum requirements to allow for climatic differences. The implications of a 25 per cent reduction in the Bariloche targets for housing and education needs were also assessed (to between 5 and 7.5 m^2 of housing per person and to 9 years of education per person).

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*A defined set since not all basic needs were included in the Bariloche model nor can all basic needs be quantified--the need to participate in the decisions affecting oneself, or the need to live in peace, for example.

**This is of course subject to a number of reservations made more explicit in [2].

***This could be done since the Leontief model is composed of an underdetermined set of linear equations, so for example inputting income targets and the consequent GNP growth rates into this model fully determines the system of equations and thus allows the implications of our income targets to be known.

For each of the subregions at the point at which the basic needs targets were reached the GNP per capita was read off. This figure was taken to be the income required to satisfy the defined set of basic needs as explained above.

Other quantifiable basic needs were implicitly allowed for in the income target by forcing the proportion of income allocated to consumption to be constant over time and the same as initial conditions. In fact relaxing this constraint, as a basic needs oriented society may try and do, could reduce the desired GNP growth rates eventually arrived at, i.e. the GNP growth rates that we stipulate as being necessary to meet basic needs would not have to be so high if the consumption constraint is relaxed.

Parameterizations were performed on a number of constraints to investigate the effect on the income target and in consequence the desired growth rates. Not much variation was found here largely because the constraint above dominated.

The GNP per capita income taken from runs of the Bariloche model for each subregion is the income needed to satisfy the defined set of basic needs. In order for the poorest 20 per cent of the population to receive this target income, the average income would obviously need to be much higher. The average income for each subregion needed was computed using income distribution data compiled from World Bank sources. If we choose the year 2000 for the time at which the average income is to be achieved (and by implication, the target income for the poorest 20 per cent) we can compute the income per capita growth rates needed from now until then in order that the poorest satisfy their basic needs. We can also compute overall GNP growth rates by taking UN population projections.*

The main conclusion from this work was that with existing patterns of income distribution, all of the regions of the developing world would need to attain growth rates of between 8 and 12 per cent per annum between now and the end of the century in order to meet the defined set of basic needs. This is nearly double the growth rates attained by developing countries in the 1960s and would not appear to be feasible except for a few countries, thus implying the necessity of substantial income redistribution within these subregions in addition to fairly high rates of growth.**

These illustrative results were a substantial force behind the declaration of principles agreed upon at the World Employment Conference by all ILO member States and organizations [4] that

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*UN "low" population projections were used in order to provide estimates on the conservative side.

**But much less than 8 to 12 per cent depending on how much income is redistributed.

"past development strategies in most developing countries have not led to the eradication of poverty and unemployment..." and that "strategies and national development plans and policies should include explicitly as a priority objective the promotion of employment and the satisfaction of the basic needs of each country's population". This view was reiterated at the meeting of the heads of state of the non-aligned countries held in Sri Lanka in August 1976. However whether each country will, in fact, move away from growth oriented development strategies of the past towards strategies aimed at both growth and redistribution in order to satisfy basic needs remains to be seen.*

Work is to continue next year on global modelling to investigate these preliminary results more fully than hitherto following the general approach of the Bariloche model but without using it. In parallel with these activities work is to proceed to define basic needs more precisely by taking account of what people want, to design and aid national governments to execute household surveys to measure the existing deficiencies and, in consequence, hopefully to stimulate the necessary changes.

NATIONAL LEVEL MODELLING WORK

Introduction

At the national level, dynamic simulation models with the generic name of BACHUE are being developed to examine the interrelationships, over the long term (10 to 30 years), between economic change and population size and structure. Most resources have been allocated to the national level because this, in the end, is the level at which the necessary changes needed to improve people's well-being are to occur. It should be emphasized that BACHUE is not a world model as such, even though it is being applied to many countries of the globe.

BACHUE is not intended to produce a single statement or prediction of how any particular country will develop. It is designed with the basic belief in mind that changes thought of and implemented today shape the future as much as and perhaps more than observed economic and social structures--whether such changes are policy measures of an existing government, or whether they are radical or revolutionary changes of government or society. It thus aims to evaluate policies or strategies of development over the medium to long term. Our particular view is that the evaluation should be framed with respect to how well the chosen policies perform in terms of reaching adequate employment

*Unless, of course, some subregions in the developing world can reach the very high growth rates needed in order that enough income trickles down to the poorest. There has not been much evidence of this phenomenon in the developing regions to date.

levels, eliminating poverty, and achieving a more equitable income distribution. While the model can give significant indications on a number of subjects in the short term, it is aimed at studying the effects of policies on population, employment, incomes, and income distribution in the longer term, and this is reflected in model structure. Use of BACHUE should not preclude different types of models examining different questions, but where possible such models should not be used independently of each other.

To serve as an effective and realistic tool to help in policy making, specific models of the BACHUE type have to be developed for each country where the methodology is to be applied. Thus the model developed and recently finished* for the Philippines cannot be applied to Kenya, Brazil, or Yugoslavia, where other BACHUE models are being developed,** merely by changing the data in the model. Clearly there are many structural, accounting, and conceptual relationships that are generalizable--for instance the population accounting and major concepts developed for the Philippine model are being used and improved upon for the Kenyan version of BACHUE. On the other hand, the economic structures of Brazil and Kenya are completely different, the labour markets of the Philippines and Yugoslavia operate on entirely different principles, the importance of regional differences varies, and so does the cultural context of household decision making. The issues of interest for policy also vary widely. Thus any attempt to develop a model in the BACHUE series for a country other than the Philippines involves major differences in structure from BACHUE-Philippines. Nevertheless some ideas on the types of issues that BACHUE can explore can be gained from the use of the Philippine model. In the remainder of this article, therefore, we focus on selected aspects of the use of BACHUE-Philippines.

Overall Structure of BACHUE-Philippines

In order to adequately understand results from the model, some knowledge of its content and structure is required. Here we give a very brief outline of the elements that make up the BACHUE-Philippines model; a detailed specification, together with policy evaluations and sensitivity analysis, is reported elsewhere [5].

BACHUE-Philippines contains three main sub-systems - economic, labour market and income distribution, and demographic. The major modelling effort was therefore concentrated on the labour market and income distribution and demographic subsystems and especially on their interrelationships. By contrast, some major components of economic behaviour are omitted or determined

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*Insofar as a model is ever finished.

**A full bibliography of papers on the BACHUE models is given at the end of this paper - see [5] to [12].

outside the model--this includes most prices, aggregate investment, and some aspects of the production process.

The economic subsystem is a demand-centred model generating sectorial outputs and value added. The emphasis on demand derives from the fact that this is a major link between population growth, and employment and income distribution. Supply constraints can be modelled by imposing an overall target for total output based on planning targets or estimates of possible growth rates. Alternatively, a balance of payments constraint or other types of constraint can be imposed. The sectorial components of output, however, are always endogenous. Final demand, by sector, is determined by household consumption (in turn a function of population size and structure), exports, government expenditure and private investment. By using an input-output production system, output, imports, and value added by sector are determined. If aggregate supply is constrained, the output growth demanded in any sector is revised so that the overall constraint is not exceeded. Agricultural output growth is specifically restricted, and changes in the balance of supply and demand for agricultural goods lead to changes in the agriculture-industry terms of trade.

In the determination of employment and wages, the employed are distinguished from the self-employed, and separate labour markets are specified for skilled and unskilled, urban and rural, and modern and traditional sectors. Dualism is represented by the tendency for excess labour supply to be absorbed by traditional sectors; unemployment is modelled, but is treated as more a social than an economic phenomenon. Both rural-urban and intersectorial migration of workers are considered explicitly, and have an equilibrating effect on wage differentials. Labour supply is determined by age-sex specific labour force participation rates, which are functions of jobs in modern sectors, education levels, household incomes, and occupation of household head.

The wages and employments computed by this system generate household incomes aggregated into rural and urban income distributions. The latter--in addition to being major outputs of the model--enter into the determination of household expenditure and savings.

The demographic system determines marriage rates, fertility, migration, education, and mortality. Marriage rates depend on education levels and female labour force status. Fertility is affected by female labour force participation, mortality, education, and the proportion of the population that is agricultural. It can also be modified (exogenously) by family planning programs. Rural-urban migration is based on education levels, rural-urban wage differences, rural-urban income distribution differentials, sex, age, and marital status. Education levels are determined by school graduation rates, in turn functions of government policy. Mortality is a function of the level and distribution of income. The system then tracks the disaggregated population as it is born, gets older, is educated, migrates, and dies.

There are a number of ways in which policy changes can be simulated in the model. Firstly, certain specified variables are or could be directly under government control--for example government expenditure, taxation, and exchange rates. These in themselves are policy instruments. Secondly, many variables can be indirectly modified more or less easily by policy intervention--for instance, modern sector wages, education graduation, or rural-urban migration. These variables can be changed by government action, either by influencing their determinants (increasing rural education would promote migration to urban areas) or by modifying them directly through detailed decisions, laws, and incentives that are not themselves incorporated in the model.

Some Model Outcomes

In this section, we summarize the contents of a recent article [6] that illustrated two features of the Philippine model. The first illustration linked the global modelling work to the national modelling work by using the income target for the Asia subregion as a target for development planning in the Philippine model. Since household incomes distribution is endogenous in BACHUE-Philippines, this income target could be compared against model outcomes for the income of the poorest 20 per cent. In this way, the effects of individual policies and combinations of them could be evaluated with regard to their success in maximizing the income of the poorest and--a complementary but not identical objective--in redistributing income as much as possible. The policy package chosen could be likened to measures that a progressive government would probably introduce. The main elements of the package in urban areas were nationalization of large-scale industry, policies to promote small-scale industry and policies to increase labour intensity through using labour intensive techniques; in rural areas, there was a public works program, increased agricultural productivity growth, a change in external trade patterns to increase demand for agricultural output and a minor land reform. Overall, migration from rural to urban areas was provoked, there was an increase in education growth and a large program of wage subsidies and government transfer payments through the income tax system. Despite all these changes, the package was nevertheless only one of a large number of possibilities and it should, therefore, be treated as an example rather than as a definite basic needs strategy. The results showed that the resulting income of the poorest in the year 2000 did not meet the income target despite the fact that by the year 2000 average incomes in the economy were over double the income level needed. This was mainly because the distribution of income could not be equalized enough. This, in turn, was due to the structural inequalities built into the system through urban dualism, rural-urban differentiation and, above all, returns to land and capital. The implication here is that more radical measures than those applied need to be taken (given that the income target is not set too high) should foreign help on a significant scale not be forthcoming.

One of the more interesting aspects of the package of policies, briefly described above, was the extent to which they had demographic effects. The policy package was associated with quite significant declines in fertility, largely because of increasing levels of socioeconomic development, e.g. increasing education levels and decreasing mortality. A second feature of the policy package was the importance of increased rural to urban migration in shifting the incidence of poverty from rural to urban areas but nevertheless somewhat reducing overall inequality. This reduction occurs mainly because people in urban areas have larger incomes on average than in the rural areas and the new migrants thus have a better chance of improving their income earning capacity in urban areas in comparison with the rural areas.

Conclusion

Future work will continue to focus on and improve the modelling work at the national level since it is here that most decisions are taken, and it is here that the socio-political obstacles to development are located--and these are perhaps the most important obstacles to development. Also it is at the national or subnational level that work at the micro-level (e.g. household surveys) needs to be undertaken in order to establish the links in the causal chain from which planning models are calibrated. Ideally, however, there should be a mix of micro and macro work at the national level coupled with modelling work at the global level. If this is not done the poor behavioural base that global models have used in the past will continue to draw such models into disrepute and thus the persuasiveness that they arguably have had in focusing world attention on critical issues will quickly disappear.

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DISCUSSION

Kulikowski asked whether fertility change and the impact of fertility on the labor supply is being accounted for, whether there is a feedback between fertility and the allocation of the consumption among different social groups. Hopkins replied that relations of this kind are included in the model.

McLean pointed out that in her South Korean econometric model, Irma Adelman found out that certain redistribution policies not only affect the migration from rural to urban areas, but also the distribution between agricultural income groups. Did Hopkins' model yield similar results? Hopkins said that, although his model is not as disaggregated, results point in a similar direction.

Roberts questioned the mechanism in the model according to which population increase will level off as soon as income rises above a "poverty line". In a fast-rising population, the pressure on land to grow the necessary food might push up prices to such an extent that the poverty line cannot be surpassed. Hopkins replied that in addition to the supply effects, also demand effects must be considered. Also technical progress will play a substantial role and this makes it difficult to arrive at "true" relationships.

FUTURE TRENDS OF AGRICULTURE IN DEVELOPING COUNTRIES

B. Choe, M. Osterrieth, and J. Waelbroeck*

INTRODUCTION

In this paper, we survey the agricultural performance of six developing regions**, and assess their prospects till 1985. Historical series for aggregate production, trade and domestic use of agricultural products have been constructed from commodity by commodity volume data from the US Department of Agriculture (USDA) and the Food and Agriculture Organization (FAO), using as weights the 1967/1969 average international prices, to make regional figures comparable. The figures so obtained cannot be compared to national accounts aggregates: goods sold domestically are often sold at prices that differ substantially from the levels that prevail in international trade, both because of transport and trading margins, and because of price controls, taxes, and subsidies imposed by governments.

The projections are summarized in Table 1. They rely on a complete study of the foodgrain sector in developing countries (Hadler, 1976).

What we do is to extend the analysis to cover the entire agricultural sector of developing countries. The main achievement of this paper is to integrate the analytical results obtained in various fields by the International Bank for Reconstruction and Development (IBRD) (Less Developed Countries (LDCs) growth projections, commodity market analysis, price forecasts) into a simple and consistent framework. At this stage, however, many relevant elements had to be left out of the explicit projection apparatus. This is true of domestic price movements and of demand for agricultural inputs, for which data is hard to collect and still unreliable. We did, of course, look into these questions for the past, and our findings help to qualify the discussion of the projections.

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*The authors work in the Economic Analysis and Projections Department of the World Bank. The views presented in this paper are their own, and not necessarily those of the Bank. The original paper presented at IIASA had to be considerably reduced in size for publication. The original version is available on request.

**The list of countries covered by the study is annexed.

Table 1. Summary of the projections (1967-69 prices).

	Production Consumption								Exports				Imports			
	10^9 US$		Growth rates		10^9 US$		Growth rates		10^9 US$		Growth rates		10^9 US$		Growth rates	
	1974	1985	1962 -74	1975 -85	1974	1985	1962 -74	1975 -85	1974	1985	1962 -74	1975 -85	1974	1985	1962 -74	1975 -85
Oil Exporting Regions	14.1	23.2	2.4	4.5	14.3	25.7	2.8	5.3	1.18	1.70	0.5	2.7	1.51	4.13	5.8	9.5
North Africa & Middle East*	5.2	7.9	2.7	3.8	5.5	8.9	3.2	4.5	0.48	0.71	0.2	1.8	0.85	1.71	3.7	6.3
Sub-Saharan Africa	13.2	18.4	3.1	3.1	11.2	15.6	3.4	3.2	2.55	3.74	2.2	2.9	0.53	0.96	4.2	4.4
South Asia	36.3	49.4	2.2	2.7	36.0	50.5	2.3	3.0	1.19	1.49	-1.4	1.2	0.90	2.68	-1.3	8.6
East Asia	10.7	15.9	3.2	3.4	10.2	16.0	3.7	4.0	2.21	3.60	3.2	4.2	1.68	3.69	5.9	7.1
Latin America & the Caribbean	33.3	48.0	2.8	3.6	29.6	43.1	3.0	3.8	5.02	7.02	2.1	2.3	1.2	2.09	5.4	3.8
All Developing Countries	112.9	162.8	2.6	3.4	106.9	159.8	2.8	3.8	12.65	18.24	1.7	2.7	6.68	15.27	4.0	7.1

*1963-1974 growth rates because 1961 was a very poor harvest year.

There are significant differences in the approach followed between foodgrains and other agricultural products. Grains use up some 70 percent of cultivated land, with quite low yields in dollars per hectare. Thus, it seems reasonable to assume that production is constrained by available resources in land and by existing technology. Other crops are more labor or capital intensive and use a much smaller amount of land, and most of them yield much higher returns per hectare than cereals. Therefore, it is logical to expect that if demand expands sufficiently, land use will be shifted from foodgrains to these other crops.

The structure of the projection model is then as follows: consumption is basically income determined, while exports are projected exogenously. In the case of foodgrains, production is considered as limited by resource availability, and also treated exogenously. Imports are then determined residually. In the case of other products, production is determined by demand, and imports are linked to consumption or income.

In the next sections, consumption, production, and trade developments are analyzed separately. We hope to contribute to a better understanding of the potential that policy planners in developing countries should try to exploit, and of the considerable problems that they may have to face.

THE CONSUMPTION OF AGRICULTURAL PRODUCTS

Experience

Consumption of agricultural products per head has not changed much during the period 1962-1974. Although consumption per head increased in other regions, in South Asia supplies have not matched the increase of population.

The figures of Table 2 show clearly why a successfully developing country cannot afford to neglect agriculture over the longer run. There are very large differences in the average per capita consumption of agricultural products across regions. Consumption in Latin America, the wealthiest of the regions, is more than twice that in the poorest region (South Asia). Yet the consumption of the average Latin American is in turn only a fraction of the level attained by developed countries. While in the medium run it may be possible to cope with lagging agricultural production by importing or by allowing prices to rise, only very small and highly specialized countries could resort to these policies permanently.

Figures on average consumption levels may of course underestimate the magnitude of the problem, since they conceal the variations in food consumption among classes of the population. Unequal distribution of the fruits of growth might explain why

Table 2. Factors influencing per capita consumption of agricultural products in developing countries.

	Per capita consumption in 1972	Increase in per capita consumption of agricultural products 1962-72	Increase in per capita incomes 1962-72	Increase in ratio agricultural to GNP prices* 1962-72	Changes in share of agricultural population in total population: difference between 1962 and 1972 shares**
	(1967-69 $)	----------------	% per annum	----------------	
Oil Exporting Countries	46	0.1	4.8	0.8	-0.10
North Africa & Middle East***	66	0.3	2.6	1.6	-0.07
Sub-Saharan Africa	48	0.8	1.9	-0.9	-0.06
South Asia	45	-0.6	1.1	1.2	-0.07
East Asia	56	1.0	4.9	1.2	-0.12
Latin America & Caribbean	101	0.1	2.7	0.8	-0.08

*This period is chosen rather than 1971-1974 because 1972 was a more normal year for prices than 1974.
**Extrapolated from FAO data.
***1963-1972, as 1961 was a very bad harvest year.

per capita consumption of agricultural products has been practically stagnant in the oil exporting economies. Quick urbanization may also have a perverse influence on the distribution of available supplies between population groups, the diet of the poor city dweller being for a given income level probably worse than that of his rural counterpart. In some dramatic cases, production itself affects the consumption of some population groups. There is no doubt that farmers reduce their consumption of their own products when the harvest is bad.

Finally, prices have a substantial impact on demand. The price elasticity of demand for food is generally thought to lie between -0.25 and -0.50, which means that a moderate price increase of 10 percent would suffice to reduce consumption by 2.5 to 5 percent. The figures of Table 2 suggest that agricultural prices have increased more quickly than GNP prices in all but one region, probably keeping consumption levels low. Data on domestic prices, unfortunately, are notably weak, and do not permit a rigorous analysis at this point.

Methodology and Assumptions

The projection model simply links per capita consumption to per capita income by means of constant elasticity coefficients*. This approach is quite straightforward in the case of food products, if one agrees that income distribution and location effects are small in comparison with population and income effects. It is more debatable in the case of agricultural raw materials, which should rather be linked directly to industrial production. The share of raw materials in total agricultural consumption, however, is too small (less than 6 percent at 1967-1969 prices for the developing world) to make the incorporation of industrial production in the model worthwhile. Both product groups are thus treated in a similar way.

The GNP growth rates used in the projections (Table 3) have been generated by the Bank's SIMLINK model (Hicks, 1976). They rest on a fairly optimistic view of the economic prospects of industrial countries: an annual real growth rate of approximately 5 percent as from 1976, inflation rates which fall from 8.4 percent in 1977 to a steady 7 percent in 1980-1985, and no major cyclical movements. The population growth rates are based on the 1974 country by country projections of the United Nations (medium variant).

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*The elasticities have been estimated commodity by commodity and are documented in the complete version of the paper. In the case of non-grain products, the regional observations were pooled in a single cross-section time-series regression to obtain an "average" elasticity coefficient, applied in all the regional projections.

clearer when the variations in demand are compared to the GNP

Table 3. Income and population growth by regions.

	GNP		Population		GNP per capita	
	1962-74	1975-85	1962-74	1975-85	1962-74	1975-85
Oil Exporting Countries	8.0	11.2	2.5	3.1	5.5	8.1
North Africa & Middle East	5.4	6.6	2.5	2.8	2.9	3.8
Sub-Saharan Africa	4.4	4.6	2.6	2.8	1.8	1.8
South Asia	3.3	3.8	2.4	2.6	0.9	1.8
East Asia	7.4	7.3	2.7	2.5	4.7	4.8
Latin America	5.9	6.1	2.8	2.8	3.1	3.3

Demand Pressure on the Agricultural Sector

Some implications of our projections are worked out in Table 4. It clearly appears that the projections for poor countries are a cause for serious concern. Increases in per capita consumption of agricultural products are expected in all of the six regions, but they will be moderate in the poor regions, Sub-Saharan Africa and South Asia. The effort involved in obtaining these gains, measured in terms of share of demand for agricultural goods in total increase of GNP, is much stronger in the poor regions. This simply reflects the truth that the pressure on the agricultural sector arising from demand effects of growing incomes is particularly strong at low levels of development.

To test the sensitivity of the projections to change in the GNP and population assumptions, we increased the GNP growth rate by 1 percent in one simulation (Case A) and reduced the population growth rate by 0.5 percent in another (Case B). The effect of the perturbations can be measured by comparing the increase in agricultural consumption obtained in the various simulations, expressed as a percentage of the 1972 level of agricultural consumption, and of the increase of GNP over the projection period.

The impact on agricultural demand (columns 5 to 7) strongly reflects the regional income elasticities embodied in the model (column 3). The meaning of these figures, in terms of the global effort involved to meet the demand requirements, is clearer when the variations in demand are compared to the GNP

Table 4. The demand pressure on the agricultural sector.

	Agricult. cons. (1972) (10^9 1972 $)	Share of agr. cons. in GNP (1972)	Income elasticity of agric. consump.	Rate of growth of agr. con. per capita (base case) (1972-1985)	Increase in agric. cons. (1972-1985), as a percentage of the 1972 level*			Share of agricultural consumption in the base case increase in GNP (1972-1985)*		
					Base Case	Case A (high GNP)	Case B (low pop.)	Base Case	Case A (high GNP)	Case B (low pop.)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Oil Exporting Countries	20.0	20.4	0.30	2.4	100.5	+6.9	-6.8	7.1	+0.5	-0.5
North Africa	5.8	25.7	0.48	1.8	79.3	+9.3	-4.7	15.9	+1.9	-0.8
Sub-Saharan Africa	8.3	28.4	0.30	0.5	53.0	+4.5	-5.5	19.6	+1.7	-2.1
South Asia	41.0	46.6	0.42	0.4	46.6	+7.1	-4.2	37.8	+5.8	-3.4
East Asia	14.2	25.0	0.25	1.2	62.0	+5.9	-5.6	10.3	+1.0	-0.9
Latin America	26.2	13.4	0.28	1.0	62.2	+5.9	-5.7	6.8	+0.6	-0.6

*The figures for case A and B are additive differentials from the base case.

increase over the period (column 8 to 10). As expected the impacts are now much stronger in South Asia than in the other regions.

A more refined analysis of the projections shows that the share of staple food and tropical foodstuffs in total consumption will decline in favor of raw materials and "other" products (vegetables, meat, and dairy products). This simply reflects the changing patterns of consumption associated with rising incomes.

THE PRODUCTION OF AGRICULTURAL PRODUCTS

Experience

Agriculture still accounts for a large fraction of GNP in developing countries, and made a substantial contribution to GNP growth over the period studied (Table 5). The highest growth of agricultural output was observed in East Asia, reflecting among other things the favorable price policies of Korea, Malaysia, and (for export crops other than rice) Thailand. South Asia registered the slowest increase, partly because of the poor harvests of the early 1970s, but also because of inadequate price incentives and the neglect of policy makers throughout the region. Population growth has offset most of the increase in production. Whereas in developed countries agricultural production per head increased by 1.2 percent per year, the increase achieved by developing countries was almost negligible. However, there were differences in performance among the various regions: output per head fell in South Asia, which dominates the developing countries' average and in oil exporting countries, but elsewhere per capita production rose slightly.

Various elements explain the poor performance of agricultural production in the past. In many countries, the incentives to expand agricultural output may have been inadequate: in their urge to industrialize their countries, planners favored policies that either discriminated against agriculture, or at least, like industrial protectionism, distorted the pattern of incentives to the detriment of that sector. Incomes per head in agriculture are everywhere very low: only 20 percent of incomes per head of the other sectors in the oil exporting countries and in Sub-Saharan Africa; 33 percent in North Africa, East Asia, and Latin America; 40 percent in South Asia (1961). The situation has improved, and though there remain some striking cases of pricing bias against agriculture, in many countries the terms of trade of agriculture have improved (see Table 2). This has resulted both from the response of market forces to lagging output, and from planner's more favorable attitude towards agriculture, evidenced by producer price increases in Korea, Argentina, Pakistan, and more recently in Burma, Thailand, and Chile. The more outward-looking trade policies adopted by countries such as Korea, Taiwan, Brazil, and Colombia have indirectly improved the incentives to farmers in those countries.

Table 5. Agricultural production in developing regions.

	Share of agricul-ture in GNP	Share of agriculture in increase in GNP	Growth of agricultural output		Growth of agricultural output per capita	
	1972 %	1962-74 %	1962-74	1975-85	1962-74	1975-85
				% per annum		
Oil Exporting Countries	20.7	8.4	2.4	4.5	-0.1	1.4
North Africa & Middle East	25.2	14.1*	2.7*	3.8	0.2*	1.0
Sub-Saharan Africa	34.8	31.8	3.1	3.1	0.5	0.3
South Asia	47.1	22.5	2.2	2.7	-0.2	-0.1
East Asia	26.0	13.1	3.2	3.4	0.5	0.9
Latin America	15.5	8.5	2.8	3.6	0.0	0.8

*The figures for North Africa & the Middle East refer to 1963-1974, because 1961 was a very bad harvest year.

Factors of Agricultural Growth

Improvements of yields made a substantial contribution to the growth of agricultural output between 1961 and 1974, mainly in South Asia, East Asia and the Middle East region, where they account respectively for 63, 54, and 50 percent of the growth of total agricultural output. They were small in Sub-Saharan Africa and Latin America, where extension in the cultivated area was largest.

A closer examination of the past performance suggests, unfortunately, that there has been some decrease in the returns to additional inputs into agriculture in the early 1970s as compared with the 1960s. Though harvest fluctuations make it difficult to judge trends for shorter periods, it is possible to discern differences when the rates of growth of agricultural output for 1962-1965, 1966-1970, and 1971-1974 are compared (Table 6). The figures for 1966-1970 reflect the introduction of new high yielding varieties of wheat and rice in South Asia, which led to remarkable increases in wheat production in India and Pakistan, and in rice production in the Philippines. The impact of this on the total output of all developing countries was only marginal, however. The general slowdown of agricultural growth in 1971-1974, which is suggested by Table 6 is a cause for concern.*

Table 6. Growth of agricultural production by subperiods. (Compound rates of growth, based on indices weighted by 1967/1969 average international prices)

	1961-62 to 1965-66	1965-66 to 1969-70	1969-70 to 1973-74
Oil Exporting Countries	1.73	3.11	2.39
North Africa & Middle East	3.63	3.27	3.15
Sub-Saharan Africa	3.56	3.96	1.75
South Asia	0.20	5.14	1.09
East Asia	4.22	3.35	2.79
Latin America & Caribbean	3.23	2.84	2.27
All Less Developed Countries	2.15	3.84	1.91

It is interesting to compare the growth profiles of agricultural output and of the total flow of inputs into the sector. For this purpose, we built an index which aggregates estimates of changes in cultivated area, irrigated area, fertilizer use and available labor force (Table 7). This index does not cover all inputs. For instance, extension services and investment in roads are not included, but it may serve as a rough measure of the volume of inputs going into agriculture.**

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*For South Asia, the fact that 1970 was a favorable year exaggerates both the 1966-1970 gain and the 1971-1974 slowdown. The 1975-1976 South Asian harvest has been so bountiful that it is proving difficult to find storage for it. But according to the latest estimate, the bumper harvest in India implies only a 1.5 percent annual rate of growth of output between 1970 and 1976.

**How to aggregate the various inputs in a single index is of course open to question. Here, we proceeded as follows: labor is valued at its 1961 marginal productivity (simply taken as half its average productivity). Fertilizers are valued on the base of their yields (10 tons of grains per nutrient ton). The contribution of these two inputs to the increase of output can then be assessed. Land is assumed to account for the remainder of agricultural growth. Its contribution grows in proportion to an available land quantity index in which irrigated land is weighted to reflect that it is three times as productive as unirrigated land. The method insures that the overall growth of input and output is the same, but allows for different time patterns.

The figures suggest that the slowdown in the rate of growth of agriculture which took place in the early 1970s was not generally due to a decrease in the flow of inputs into that sector. The notable exception is South Asia, where the figures are strongly influenced by the stagnation of fertilizer consumption in the early 1970s, which was partly due to physical shortages.

Table 7. Rate of growth of inputs into agriculture.

	1962-65	1966-70	1971-74
Oil Exporting Countries	1.6	1.5	2.0
North Africa & Middle East	2.6	1.7	4.0
Sub-Saharan Africa	3.2	3.2	3.4
South Asia	1.7	1.9	1.8
East Asia	2.5	2.3	2.4
Latin America & Caribbean	1.8	1.5	1.6

The fact that so far in the 1970s returns to agriculture have been less than in the preceding decade should not be taken to imply that agriculture has entered a phase of steadily decreasing returns. While it might very well be that the land brought into cultivation at the margin is less fertile than the land previously used, and that the best opportunities for irrigation have been exploited, two factors certainly contribute to alleviate the fear of steadily decreasing returns. First, the green revolution gave an exceptional spur to production of rice in Asia and Africa and of wheat in South Asia, pushing the growth rates of agricultural production in 1965-1966 to 1969-1970 to exceptionally high levels. Second, fertilizer use, traditionally oriented to the production of high yielding crops, is now more and more directed towards the production of grains, which have lower yields in dollar per hectare. The figures would then reflect a change in the mix, rather than diminishing returns in the production of individual crops*.

*Another element of reassurance is that yields per hectare for grains, cotton and tobacco in developing countries remain far below the levels attained by developed countries (40 percent on the average). To raise yields would of course require a large research effort, and a mobilization of the small scale traditional sector.

The Projections

Our projections for agricultural output (Table 5) are reasonably optimistic. The future growth of agriculture would be higher than in the past decade in five of the six regions, and stagnating in the last. We expect middle income regions to do significantly better than Sub-Saharan Africa and South Asia. In the latter region, output per head might, again, decrease.

The growth rates assumed for grains (Table 8) strongly influence the global results (Hadler, 1976). They are based on the regional performances between 1960 and 1974, together with commodity and region specific information. They assume some improvements over the historical, internal terms of trade facing agriculture, and that the rate of adoption of existing agricultural technology will proceed as in the past. They are, for the most part, close to the estimates for the observation period. The major exceptions are, in the case of wheat, South Asia and Latin America, and in the case of rice, South Asia. With respect to wheat production, the introduction of the high yielding varieties in South Asia in the late 1960s gave a spur to production which, owing to resource constraints, probably cannot be maintained in the future. In Latin America, the trend rate of growth of wheat production bears little relation to the production capacity of the region and, in the light of recent policy developments, a substantial increase in productivity has been assumed. Rice production in the developing countries is dominated by South Asia, which has the slowest rate of growth of production in any developing region. Irrigation and improved cultural practices contain the potential for substantially raising productivity, and production is thus expected to accelerate slightly.

As far as other products are concerned, we explained that production is seen as adapting to domestic demand and trade conditions. The major purpose of the model is to derive the implications for agricultural production of specified income, population, and trade prospects. The question we have to answer is thus whether these implications are realistic.

If income distribution grows more equal than in the past, there is little doubt that demand will exert a considerable pressure on the agriculture of the *oil producing countries*. Because of high oil revenues, investments will be able to reach much higher levels than in the past. Venezuela, Indonesia, Iran, and Nigeria have announced ambitious plans to expand agricultural production, so that high growth rates of output do not seem unrealistic in the future.

The increase in growth in the small *North African* and *Middle East* region seems less realistic. However, the exploitation of trade opportunities with the neighboring oil exporters could probably, if policies are adequate, lead to some increase in output of high yielding crops and livestock products. Progress will have to come mainly from increases in labor productivity, which will not be forthcoming without substantial improvement in infrastructure and supporting services.

Table 8. Growth rates* of grains production in developing countries.

	Wheat		Coarse Grains		Rice	
	1962-74	1975-85	1962-74	1975-85	1962-74	1975-85
Oil Exporting Countries	2.9	2.0	-0.2	0.0	5.0	4.5
North Africa and Middle East**	2.6	2.5	1.5	2.5	2.1	3.6
Sub-Saharan Africa	4.6	4.0	1.3	2.2	2.9	3.0
South Asia	6.9	2.0	0.5	1.0	1.8	2.0
East Asia	-3.9	3.0	5.5	4.0	2.1	2.8
Latin America & Caribbean	0.5	3.0	4.2	4.7	2.6	2.6
All Less-Developed Countries	4.4	3.4	2.2	2.8	2.3	2.6

*Trend growth rates for the past, compound growth rate for the future.
**1963-1974 because 1961 was a bad harvest year.

Indications are that in most of *Asia*, further increases in the cultivated area will be costly to achieve. In South Asia there is now little land left to be opened up. There is a vast potential for profitable irrigation using the Himalayan waters on which much of the region's agriculture is based, though the cost of this will be somewhat higher than irrigation costs in the 1960s, since some of the most attractive tubewell irrigation and river control possibilities have already been developed. In Thailand, maize has accounted for a significant fraction of the expansion of area, but further expansion is jeopardized by the exhaustion of usable land. Malaysia still has substantial land areas suitable for production of rubber, palm oil, and other tree crops. In South Asia also, the projected acceleration of the growth of agriculture will require a substantial effort, in terms of public investment and planning. The price incentive system has to be revised to encourage agricultural output.

In *Latin America*, by contrast, there is no scarcity of cultivable virgin land. Yet the areas recently opened up have either been less fertile and less accessible, or have cost more to develop, than those on which the continent's agriculture was initially based. Thus although it is physically feasible a large expansion of the area under cultivation cannot be expected without appropriate price policies and sufficient public investment in infrastructure to make it commercially attractive. Land reform and other institutional changes would probably increase the productivity of the land under cultivation.

In conclusion, the output projections do not seem unrealistic, but they imply a substantial effort in all the regions. The increasing realization by governments in many countries of the importance of agriculture in economic development is auspicious. This shift in attitude is in part the result of increasing concern for rural poverty, and of the publicity given to projections of growing foodgrains deficits. But many governments have also learned from the bitter experience of dwindling agricultural exports, rising imports, and domestic food shortages.

AGRICULTURAL TRADE

Exports

Perhaps somewhat surprising is the fact that in recent years agricultural exports accounted for only 15 percent of developing countries' total exports of goods and nonfactor services. They amount, however, to 30 percent of exports from the oil-importing developing countries; there are 47 developing countries that depend on agricultural exports for half or more of their total foreign exchange earnings.

The agricultural exports of developing countries (Table 1) rose by only 1.7 percent per year in 1962-1974, compared with a 7.3 percent rate of increase of their total exports, and a 4.0 percent increase of their agricultural imports. Developing countries have tended to lose market shares to developed countries (Table 9). If in 1974 the developing countries' shares in world trade had equalled those they attained in 1961, the volume of their exports would have grown by 2.3 percent annually rather than by 1.7 percent over those 13 years.

Table 9. Developing countries' share in world exports of some natural products.

	Meat	Grains	Citrus	Sugar	Tobacco	Wool	Cotton	Fats & Oils
1961	0.38	0.21	0.52	0.94	0.47	0.21	0.57	0.52
1974	0.35	0.17	0.42	0.91	0.49	0.12	0.62	0.43

This poor export performance is also due to the slow increase in world demand for tropical noncompeting foodstuffs, and to the emergence of synthetic substitutes for the agricultural raw materials exported by the developing world (Table 10). The market lost to the latter products between 1961 and 1974 may be worth as much as $\$4 \times 10^9$ in the latter year, of which developing countries would have captured roughly half had they maintained their share of world production of synthetic and natural products*.

Table 10. World production of natural and synthetic fibers, 1962-1974.

	1961 (million metric tons)	1974	Annual Growth Rate (%)
Rubber			
Natural	2.10	3.51	4.0
Synthetic	2.50	7.26	8.5
Clothing Fibers			
Natural	11.64	14.27	1.6
Synthetic	3.52	10.96	9.1
Jute and Hard Fibers			
Natural	1.87*	1.45	-1.7*
Synthetic	-	0.38	infinite

*1960 and 1961-1974 rate of increase

The growth projections of LDCs' exports (Table 1) are based on the commodity by commodity projection of the World Bank's experts. They are commented on briefly below.

For the many products exported both by developed and developing countries, developing countries should primarily seek to improve their market shares, since for many important products there is a good chance of reversing the past trend. The developing countries' share of the vegetable oils market is expected to improve. Exports of palm oil have been rising swiftly, as well as the soybean exports of Brazil; it appears that Argentina too may become a major exporter of soybeans.

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*This calculation assumes that synthetics replace natural products on an equal weight basis.

In the rubber market a significant part of the increase in the share of synthetics has reflected the inability of natural rubber producers to increase production in proportion to the increase of demand for elastomers. Natural rubber remains price-competitive with synthetic. If producers succeed in their efforts to prevent sharp price fluctuations and keep prices competitive, it is probable that their sales will not suffer from the expected slowdown of the growth of rubber consumption in the world.

Cane sugar exports face a major threat from the competition of corn-based sweeteners, but it does not seem that these new substitutes will take over a large fraction of the world sugar market, unless sugar prices are allowed to reach new heights. Sugar is one of the cheapest sources of energy, and it is only worth transforming foodgrains into sugar if sugar prices are high compared with production costs.

There are profitable export opportunities for meat and foodgrains also. Production of beef is rising rapidly in estates in the interior of Brazil and of some Andean countries; in Africa also there have recently been substantial investments in meat production. The production costs of this beef are low and it is very competitive on world markets. Argentina's improved pricing policies are expected to enable it to maintain or improve its share of the rapidly expanding world grains market; Thailand's exports will, however, grow rather slowly.

It would also be possible to increase cotton exports, which are a large source of foreign exchange for Pakistan and for a number of countries in Africa. The prospects are less favorable for other fibers. Jute and sisal are losing ground to the competition of new synthetic fibers.

Developing countries are the only exporters of four products: bananas, tea, coffee, and cocoa. As the demand for these products is quite inelastic, the main goal of exporters should be to avoid the chronic surpluses which have plagued several of these commodities in the past. The chief objective of commodity agreements concluded so far has, however, been the reduction of short-run price fluctuations. A cocoa agreement has just been signed. Tea investments have recently been reduced, partly because of concern about the impact of oversupply on prices, and this should lead to firmer prices after a few years. The serious damage inflicted by the recent frost in Brazil has considerably strengthened the position of coffee. But it is important that coffee producers should not react with excessive new planting as they did to a similar frost in the 1950s, since this would again lead to an excess of supply. Any improvement in banana prices, finally, will depend on the ability of producers to achieve accord about appropriate levels of export taxes and production control.

Imports

Imports of agricultural products by developing countries (see Table 1) amount to about half the value of their total exports. They increased rapidly between 1961 and 1974, particularly those of food. It is useful to distinguish between three regions that are largely self-sufficient and three that are not. The countries of Latin America and Sub-Saharan Africa continue to be largely self-sufficient in food* and in most of them agricultural imports do not pose a serious balance of payments problem. The South Asia countries only import a small proportion of their total supplies, but their export earnings are so small that these imports are a heavy burden on the balance of payments.

The three other regions depend on imports for 11-16 percent of the agricultural products they consume, and account for 60 percent of developing countries' agricultural imports. The main importing countries among them have substantial foreign exchange earnings, and their agricultural imports have not posed serious balance of payments problems.

Projected grain imports (see the last section of the paper) result directly from the demand and supply assumptions that we have discussed earlier. In the case of other products we had to face the difficult problem of forecasting imports directly. Imports are, of course, a very volatile item expanded to meet temporary domestic shortages, and then cut back to reduce balance of payments deficits. In a medium-term model such as ours, there is no other alternative than to link imports per capita to income, via elasticity coefficients. We estimated these by regression analysis, using either the relevant regional series only or the full set of all regional observations. Whenever it was impossible to detect any significant or plausible relationship with income, it was assumed that the ratio of imports to domestic consumption of the product considered would remain unchanged at its 1970-1972 level.

The model forecasts substantial increases in the growth rates of foodgrains and other agricultural imports, with the major exception of Latin America. The situation with respect to dependence on imports is totally different for the more or less self-sufficient regions (Sub-Saharan Africa, South Asia, and Latin America) and the other regions**. In the first group total imports will remain a small fraction of total consumption, and the increased dependency on imports is a cause for concern only in the case of grains in South Asia. In the second group, dependence on imports is of a more structural nature. In these countries, living standards are improving rapidly, and many of

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*They import respectively 4 and 5 percent of their total agricultural consumption.

**We project that in 1985 imports would amount to 16 percent of domestic consumption in the oil exporting countries, 19 percent in North Africa, 6 percent in Sub-Saharan Africa, 5 percent in South Asia, 23 percent in East Asia and 5 percent in Latin America.

them have little land that could be opened up readily for agriculture. Their comparative advantage is such that they will rely increasingly upon agricultural imports and industrial production, and they offer other developing countries attractive and expanding markets for agricultural exports.

The Foodgrains Gap Problem

The developing countries' increasing imports of foodgrains have attracted a great deal of notice in recent years. Attention has focused on the countries of South Asia and in particular Bangladesh, and on the emergency cereals imports which were required when Ethiopia and the Sahelian countries were hit by drought. However, the middle-income countries account for by far the greatest increase in food-grains imports of developing countries. East Asia's imports have grown fastest. Table 11 gives the net foodgrains imports for 1961 and 1974, together with the projections for 1985 under three sets of assumptions concerning GNP and population growth. The table illustrates how sensitive the projections for South Asia are to changes in assumed population and GNP growth rates. Under the standard assumptions of this paper, the net foodgrains imports of developing countries would reach a level of 87 million tons in 1985, of which about 19 percent would consist of rice. Only 24 percent of the total amount would go to South Asia, against 30 percent to oil-exporting countries and 22 percent to East Asia.

Table 11. Net foodgrain imports under alternative assumptions (million metric tons).

			1985		
	1961	1974	Base Case	Case A* (high GNP)	Case B** (low pop.)
Oil Exporters	3.4	10.6	25.5	27.4	22.7
North Africa & Middle East	3.9	6.2	14.3	16.1	13.3
Africa, South of Sahara	0.8	2.7	5.4	5.7	3.9
South Asia	3.9	7.3	20.6	29.1	15.4
East Asia	0.8	5.8	18.8	22.5	17.1
Latin America	-2.2	0.1	2.7	7.4	-0.3
Total LDCs	10.6	32.7	87.1	108.2	72.1

*GNP growth rate increased by 1 percent in all regions.
**Population growth rate reduced by 0.5 percent in all regions.

Table 12. Estimated cost of foodgrains deficits, and total net foreign exchange receipts from agriculture (1967-69 prices; 10^9 US$).

	Oil Exporting Countries	North Africa & Middle East	Sub-Saharan Africa	South Asia	East Asia & Pacific	Latin America	Total
Cost of Foodgrains							
Deficits							
1974	0.9	0.4	0.3	0.4	0.5	-	2.4
1985 (Base)	2.3	0.9	0.5	1.9	1.5	0.3	7.3
1985 (Case A)*	2.5	1.0	0.6	2.6	1.7	0.5	8.9
1985 (Case B)**	2.0	0.8	0.4	1.2	1.2	-	5.6
Total Net Receipts from Agricultural Trade							
1974	-0.3	-0.4	2.0	0.3	0.5	3.8	6.0
1985 (Base)	-2.4	-1.0	2.8	-1.2	-0.1	4.9	3.0
1985 (Case A)*	-2.9	-1.2	2.7	-1.9	-0.6	4.6	0.7
1985 (Case B)**	-2.1	-0.9	2.9	-0.5	0.2	5.2	4.7
Total Exports							
1985 (Base)	20.1	6.3	11.4	6.0	40.7	33.3	117.8

*GNP growth rate increased by 1% in all regions.
**Population growth rates reduced by 0.5% in all regions.

Table 12 evaluates the cost of foodgrains imports at 1967-1969 international prices and compares them to the total contribution of agriculture to the balance of payments. The figures clearly show that from the balance of payments point of view, grain imports are a major problem only in the case of South Asia, because of the very small levels of exports expected in that region. In the North Africa and the Middle East region (and in countries of the middle belt of Africa) food imports account for a smaller but still significant fraction of export receipts. Africa as a whole is, however, a net exporter of agricultural products, as is Latin America. In judging the figures for oil-producing countries, it should be remembered that they are expressed in 1967-1969 prices; unless oil prices drop food imports will account for a much smaller fraction of 1985 foreign exchange receipts than is suggested by the table.

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ANNEX: COUNTRIES COVERED BY THE STUDY

Oil Exporters

Nigeria, Gabon, Indonesia, Brunei, Ecuador, Venezuela, Trinidad/Tobago, Algeria, Iran, Bahrain, Oman, Iraq, Kuwait, Libya, Qatar, Saudi Arabia, United Arab Emirates.

North Africa and Middle East

Low-Income Countries

Yemen, People's Dem. Rep., Yemen, Arab Rep.

Middle-Income Countries

Egypt, Morocco, Syria, Tunisia, Jordan, Lebanon.

Africa, South of Sahara

Low-Income Countries

Ethiopia, Kenya, Malagasy Rep., Sudan, Tanzania, Zaire, Chad, Mali, Mauritania, Niger, Upper Volta, Burundi, Central African Rep., Dahomey, Gambia, Guinea, Lesotho, Malawi, Rwanda, Sierra Leone, Somalia, Togo, Comoro Islands, Uganda.

Middle-Income Countries

Cameroon, Ghana, Ivory Coast, Liberia, Senegal, Zambia, Botswana, Congo People's Rep., Equatorial Guinea, Mauritius, Swaziland, Angola, Cape Verde, Guinea-Bissau, Mozambique, Namibia, Reunion.

South Asia

Bangladesh, Burma, India, Pakistan, Sri Lanka, Afghanistan, Nepal, Bhutan, Maldives.

East Asia and Pacific

Low-Income Countries

Khmer Rep., Laos, Timor, Vietnam Rep.

Middle-Income Countries

Korea, Rep. Malaysia, Philippines, Thailand, Fiji, Papua/New Guinea, Singapore, West Samoa, Br. Solomons, Fr. Polynesia, Hong Kong, Macao, New Caledonia, China Rep.

Latin America and Caribbean

Argentina, Bolivia, Brazil, Chile, Colombia, Dominican Rep., Guatemala, Jamaica, Mexico, Peru, Bahamas, Barbados, Costa Rica, El Salvador, Guyana, Honduras, Nicaragua, Panama, Paraguay, Uruguay, Belize, Guadeloupe, Martinique, Netherlands Antilles, Surinam.

DISCUSSION

Pestel pointed out that it might be misleading to deduce from low imports a high degree of self-sufficiency; the particular country might just be too poor to pay for higher imports. Osterrieth agreed that this is an important point from the viewpoint of effective demand, or actual nutritional requirements.

Pestel stated that, for most commodities, the model is demand-driven, except for grain. For what time-horizon is a supply constraint for grain stipulated? Osterrieth replied that these constraints seem to be valid for the time-horizon chosen, i.e. 1985.

Keyzer asked for an explanation of the cost index that seems to reflect a tenfold increase of the fertilizer price. Osterrieth replied that this is the result of the assumption of a fixed relationship between the price of fertilizer and the price of grain.

Richardson stressed the importance of the model with regard to the fact that the World Bank might base its decisions on specific model outcomes. Osterrieth replied that the model was just completed; hence, it could not be used in the decision-making process of the World Bank as yet. One main result seems to be that, in developing countries, more emphasis should be put on price policies. Furthermore, at a lower level of development the agricultural sector should receive higher priorities.

A STEP TOWARDS ESTIMATING PARAMETERS OF IMPUT-OUTPUT TABLES

Y. Kaya, T. Imoto, and T. Yamaoka

INTRODUCTION

In recent years, efforts in global modelling have been made in various areas in the world. The LINK project by Klein [1], the Limits to Growth model originated by Forrester [2] and developed by Meadows [3], the Multiregion hierarchical model by Mesarovic and Pestel [4], and the Modelo Mundial Latinoamericano by Fundacion Bairloche [5] are among these. The authors have also developed a normative multiregion model [6] and a macroeconomic model[7].

Throughout the processes of model construction, however, model builders inescapably face the serious difficulty of the shortage of data especially in developing countries. The data of age structure, for example, are indispensable in the construction of a demographic model but only those in developed countries and some developing countries are obtainable. Since the global model covers all regions in the world the model builders have to complement the data not available from the existing data bank.

Input-output tables of a country's economy are considered to be the most useful data in construction of a multi-industry model, because they supply basic information of interindustry activities. Nevertheless construction of input-output tables requires tremendous efforts mainly by the government. The tables do not exist in some developing countries and even where they do the quality--accuracy of data, number of sectors, etc.--of the tables differs very much from country to country.

The world has become more and more closely related in a network of international policy and economy, and the necessity of adjustment of the present system of international division of labor is recognized by which each country may develop in good balance with others.

Multi-industry models are in this sense the most useful in investigating the various impacts of change in the pattern of the international division of labor in the world. The data of input-output tables of countries are then of primary importance but because of their nonavailability model builders often substitute parameters of the tables of one country for another. The tables of this kind may be inconsistent with existing data on the country's economy.

The objective of the authors' research is to estimate parameters of input-output tables of various countries in which the tables themselves are not available, with the use of what data does exist and the data of other countries. The tables thus obtained are consistent with the existing data and general characters of the tables, so that they may be said to be the best estimate in the present circumstances. This paper describes first the general data situation and secondly the computational results of estimation of investment and consumption converters.

DATA SITUATION

Input-Output Tables

Needless to say the more real input-output tables available the better. Figure 1 shows the distributions of available tables in the United Nations' library and in the authors'. It can be seen that the tables of about half the developing countries are not available from these libraries.

Bottomley's library [8] covers a little more than these.

These tables provide some data for global modelling, but its usefulness is very limited because of differences in table specifications. These are now briefly described.

Division of Industry into Sectors

Division of industry differs from country to country. For example the industry producing coal products is separated in ordinary tables into several tens of divisions except in the US tables. Some sector of one country may overlap two or more sectors of other countries. The number of sectors of a multi-industry model of the world will inevitably be limited to 5-10.

Price System--Producer's or Purchaser's Prices

Basically two price systems are adopted in input-output tables, namely, the producer's prices and the purchaser's prices. The difference between them is that the purchaser's prices include the transport costs, wholesale and retail trade mark-ups, insurance and warehouse costs, and net indirect taxes, while the producer's prices do not. Since the basic idea of input-output tables is the proportionality of material inputs to the gross output, the producer's price table may be more stable and thus more desirable. It is however easier to construct purchaser's price table because then every industry transaction can be measured at inputs to purchasers. Some countries prepare tables of both kinds (most developed countries are in this category) but others do not.

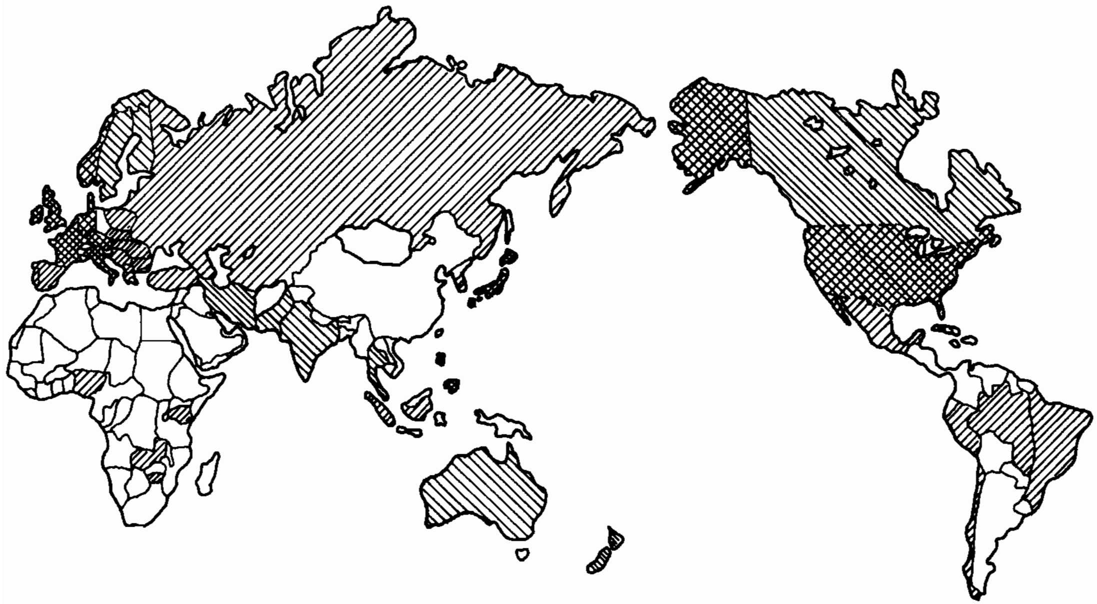

Figure 1. Distribution of available input-output tables.

Price System--Constant or Current Prices

Transactions in tables are measured either in current prices of the year concerned or in constant prices of a given year. If the standard year were the same throughout all countries, measurement by constant prices in the year would be the most welcome. The standard year, however, is not common although 1970 is now adopted in many advanced countries. In any case, only a limited number of tables of this kind are available.

The transformation of current price tables to constant price tables requires knowledge of price deflators of goods produced in each sector of an input-output table, but only deflators of very aggregated sectors such as of agriculture and manufacturing are internationally available.

Treatment of Imports

There are two kinds of treatments of imports, namely, competing imports and noncompeting imports. The main difference is that the imported goods are considered as competing perfectly with domestic ones in competing imports table while in noncompeting tables they are not.

Whether the imports are really competing with domestic ones depends on the kinds of goods and of the importing and exporting countries, but the trouble in global modelling is that in many countries only one of these two is available.

Efforts should be made to overcome these differences in specifications of input-output tables.

Availability of Other Data on National Economies

The following equation is a basic one for input-output tables of the competing imports type:

$$X = AX + C + I + Is + E - M \quad , \tag{1}$$

$$C = Cg + Cp, \tag{2}$$

$$= Kcg\tilde{C}g + Kcp\tilde{C}p, \tag{3}$$

$$I = KI\ \tilde{I} \tag{4}$$

$$KvX = V \tag{5}$$

where A, Kcg, Kcp, and KI are parameter matrices and other capital letters are n-dimensional vectors (n is the number of sectors)

X is the gross output,

V, the product of value added (or gross domestic product),

C, the final consumption expenditure by sector,

Cp, the final private consumption expenditure by sector,

Cg, the final government consumption expenditure by sector,

I, the fixed capital formation by type of capital goods,

Is, the increase in stocks by sector,

E, the exports,

M, the imports,

$\tilde{C}p$, the final private consumption expenditure according to purpose,

$\tilde{C}g$, the final government consumption expenditure according to purpose,

$\tilde{I}$, the fixed capital formation by kind of economic activity of owner,

Kcg, the government consumption converter matrix,

Kcp, the private consumption converter matrix,

$\tilde{K}I$, the investment converter matrix, and

Kv, the ratio of value added.

Some of the data on these vectors are available from various United Nations statistics even when input-output tables themselves are not. The *Yearbook of National Accounts* (NA) supplies the data on Cp, Cg, I, $\tilde{I}$, Is, and V. The *Yearbook of Trade Statistics* supplies E and M, and *The Growth of the World Economy* (GWI) supplies X, V, Ip, and Ig of manufacturing industries.

Table 1 shows the percentage of the population of the world for whom data exists.

$$\text{The percentage existence of data} = \frac{\text{sum of population of countries in which data exist}}{\text{sum of total population in the area}} \times 100\% \qquad (6)$$

It is seen that the data on consumption expenditure are fewer than other data while those on investment are comparatively more available. These data may be used to estimate parameters of input-output tables as will be described later in this paper, but it is to be noticed that these data do not constitute a consistent system. Classification of economic activities in NA is

Table 1. Data existence: percentage of population.

	$\overline{cg}$	$\overline{cp}$	$\overline{i}$	$\overline{is}$	$\overline{m}$	cp	cg	I	$\tilde{I}$	Is	V	X
1. Japan	100	100	100	100	100	100	100	100	100	100	100	100
2. Australia & New Zealand	100	100	100	100	100	81.7	100	81.7	81.7	59.2	59.2	59.2
3. Canada	100	100	100	100	100	100	0	100	100	100	100	100
4. United States	100	100	100	100	100	100	100	100	100	100	100	100
5. EC	100	100	100	100	100	100	79.4	100	100	48.9	100	100
6. Other Advanced Countries	100	100	100	100	100	61.9	54.2	87.9	61.1	75.5	91.3	89.5
7. East Asia	85.1	85.1	85.1	77.2	85.1	85.1	77.2	85.1	77.2	0	77.2	77.2
8. ASEAN	100	100	100	100	100	38.7	20.4	68.1	0	0	59.4	59.4
9. West Asia	86.2	86.2	86.2	86.2	86.2	39.8	47.3	39.8	8.5	0	47.9	47.9
10. Other Asia	82.5	82.5	82.5	82.5	82.5	10.5	1.4	10.5	1.4	0	1.4	1.4
11. Middle East	77.5	77.5	77.5	34.5	77.5	3.9	16.0	26.8	8.0	0	59.4	58.8
12. Africa												
North Africa	83.3	83.3	83.3	83.3	83.3	11.4	11.4	83.3	47.0	0	54.3	54.3
West Africa	77.7	77.7	77.7	77.7	77.7	7.8	6.6	70.9	0	0	63.0	58.7
Central Africa	61.2	61.2	61.2	61.2	61.2	0	7.7	33.5	32.9	0	27.7	27.7
East Africa	89.8	89.8	89.8	89.8	89.8	20.2	31.4	68.1	55.5	0	70.8	70.8
13. Caribbean & Latin America												
Central America	98.8	98.8	98.8	98.8	98.8	11.9	1.8	21.6	6.9	0	79.9	85.3
South America	100	100	100	100	100	2.7	8.1	89.4	8.1	0	51.5	58.2
14. USSR & Eastern Europe (including Cuba)	68.1	92.5	92.5	92.5	92.5	6.1	0	0	92.5	13.9	6.9	11.7
15. China & Other Asian Socialist Countries	2.2	2.2	2.2	2.2	2.2	2.2	0	2.2	0	0	0	0

$\overline{i}$, total of fixed capital formation
$\overline{is}$, total of increase in stocks
$\overline{m}$, total of imports

completely different from that in GWI, in which the manufacturing industries aggregated into one sector in NA are divided into almost 30 sectors. The statistics of GWI usually lack data on small factories so that its data should be modified with help of NA.

The price system is also a crucial point when using these data. Almost all vectors listed above are measured in NA in terms of purchaser's prices but in some countries data on fixed capital transformation in terms of producer's prices are available. The inconsistency of data described above should be taken into account when the data are used.

ESTIMATION OF PARAMETERS OF INPUT-OUTPUT TABLES

The final target of our research is to estimate the parameters of input-output tables of the 15 aggregated areas listed in Table 1, which correspond to the areas adopted in one of the world models the authors are constructing [8]. The methodology for the estimation will be different according to the degrees of availability of data, and establishment of the methodology is still under way. We now describe the general process of estimating parameters of a country's input-output table when the data of almost all items of UN National Accounts and the World Industry of that country are available, and the methodology and computational results of estimating consumption converters Kc (aggregation of Kcg and Kcp) and investment converters KI.

General Process of Parameter Estimation

As described above the tables in terms of producer's prices are preferable to those in terms of purchaser's price. The data in statistical yearbooks are however measured in purchaser's prices and the transformation from the one to the other requires data of transport and service costs and indirect taxes, which are virtually not available. Purchaser's price thus becomes an inescapable choice as a price basis. The division of industries into sectors depends on data availability and temporarily the division shown in Table 2 is adopted. The outline of estimating vectors and parameters of the input-output tables from the data concerning items listed in GWI is the following.

V (Gross Domestic Output)

From NA, gross domestic outputs of sector 1, 2, and 9 and the sum of 3-8 are available. Each gross domestic output of the sectors 3-8 is estimated as the same percentage of the products of value added listed in GWI.

Table 2. Division of industries into sectors.

1. Agriculture, Forestry, and Fishing
2. Mining
3. Light Industry
4. Metal and Chemical Industry
5. General Machinery
6. Electric Machinery
7. Transport Equipment
8. Other Manufacturing
9. Services

X (Gross Output) and Kv (Ratio of Value Added)

Data of gross outputs of manufacturing industries (3-8) are listed in GWI, but considering that GWI lacks data of small size industries the data should be modified with the help of NA. Our approach is to assume that the ratio of value added Kv calculated from GWI is valid. This Kv can be estimated by the value added divided by gross output. X's of sectors 3-8 are then estimated by dividing V by Kv.

The gross outputs of sectors 1, 2, and 9 are not obtainable from these sources and so should be estimated by introducing some assumption such that Kv's of these sectors are the same as those in some specified country.

Consumption Expenditure, Cp, $\tilde{C}p$, Cg, and $\tilde{C}g$

Data on $\tilde{C}p$ and $\tilde{C}g$ are in principle available from NA. Cp and Cg are neither available from NA nor from GWI. Our approach is first to estimate Kcg and Kcp of the countries for which input-output tables are available, by the method described later, and secondly to find characteristics of these converters and apply these to known $\tilde{C}p$ and $\tilde{C}g$, resulting in Cp and Cg.

Gross Fixed Capital Transformation I and $\tilde{I}$

I's and $\tilde{I}$'s of sector 1, 2, and 9 and the sum of sectors 3-8 are available from NA. The sum of sectors 3-8 are distributed into sectors in proportion to the percentage of $\tilde{I}$ in GWI. I's of sectors 3-8 are determined from the investment converter KI estimated as described later.

Increase in Stocks I's

Total of I's is available from NA. This is distributed to all sectors in proportion to the amount of fixed capital formation. This treatment is conventional but may not have much effect on the tables because of relative smallness of I's.

Exports E and Imports M

Detailed data of exports and imports are available from the UN *Trade Statistics*. The remaining problem is how to estimate the tariff for imports. Imports in *Trade Statistics* are usually measured by CIF which does not include any tariff. This is still left for further research.

Input Coefficient Matrix A

Given all the data, A is estimated in almost the same way as KI, Kcg, and Kcp.

Estimation of Private Consumption Converter Kcp

A process of estimating Kcp is now described. In this process the following assumptions are made:

- Cp and $\tilde{C}p$ are given. The former is obtained from input-output tables and the latter from NA.
- Kcp for Japan can be used as a basis for a standard converter, around which converters of other countries are distributed.

Let Kcp be the private consumption converter to be estimated.

Elements of Kcp should satisfy the following constraints:

$$Kcp.ij \geq 0 \quad ; \quad \text{for all } i,j \tag{7}$$

$$\sum_i Kcp.ij = 1 \quad ; \quad \text{for all } i,j \tag{8}$$

$$Cp = Kcp\,\tilde{C}p \quad , \tag{9}$$

where Kcp.ij indicates (i,j) element of Kcp. Then the problem is to find Kcp to minimize the distance from $\overset{o}{Kcp}$:

$$J = \sum_{i,j} wij(Kcp.ij - \overset{o}{Kcp}.ij)^2 \tag{10}$$

under the constraints of (7) to (9).

The problem is how to choose $K\overset{o}{c}p$, and the authors temporarily adopted the following method:

- Estimate the Kcp of various countries with the use of Kcp of Japan as $K\overset{o}{c}p$.

- Average the Kcp's obtained and replace it into former $K\overset{o}{c}p$.

- Repeat until $K\overset{o}{c}p$ converges.

In other words the average of Kcp's of various countries based on Japan's Kcp is used as $K\overset{o}{c}p$.

The classification of private consumption by object is shown in Table 3 and the sectors corresponding to Cp is in Table 4.

Table 3. Division of private consumption by object.

1. Food, Beverages, and Tobacco
2. Clothing and Footwear
3. Gross rent, Fuel, and Power
4. Furniture, Furnishings, and Household Equipment and Operations
5. Medical Care and Health Expenses
6. Transport and Communication
7. Recreation, Entertainment, Education, and Cultural Services
8. Miscellaneous Goods and Services

Table 4. Division of private consumption by use of goods.

1. Agriculture, Forrestry, and Fishing
2. Mining
3. Light Industry
4. Metal and Chemical Industry
5. Electric Machinery
6. Transport Equipment
7. General Machinery
8. Electricity, Gas, and Water
9. Construction
10. Merchandise
11. Transport and Communication
12. Financing and Services

The problem then becomes the iterative use of quadratic mathematical programming and some results are shown in Table 5. The target countries are those of advanced countries (Australia, Belgium, Finland, the Netherlands, Norway, and Denmark) and those of Asian countries (the Philippines, Thailand, and Korea). The distance in the parameter plane between the orginal Japanese converter and their converters are calculated and listed in Figure 2, where Asian countries are very different from the others. It is easily seen from Figure 2 that the converters thus estimated are in two groups, the western countries and the eastern countries. This result may be a reflection of validity of the converters.

Table 5. Examples of private consumption converter.*

Japan 1970

	1	2	3	4	5	6	7	8
1	0.14336	0.00000	0.00302	0.00513	0.00000	0.00000	0.00000	0.00000
2	0.00000	0.00000	0.00411	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.58289	0.86884	0.00029	0.10860	0.05976	0.00000	0.20977	0.00000
4	0.00000	0.00000	0.03628	0.07134	0.21469	0.00000	0.00106	0.00000
5	0.00000	0.00000	0.00000	0.24435	0.00085	0.00000	0.00315	0.00000
6	0.00000	0.00000	0.00000	0.16700	0.00058	0.00000	0.00215	0.00000
7	0.00000	0.00000	0.00000	0.12478	0.00043	0.00000	0.00161	0.00000
8	0.00000	0.00000	0.09082	0.00000	0.00000	0.00000	0.00000	0.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
10	0.23450	0.11236	0.01698	0.09137	0.10780	0.00000	0.08177	0.00000
11	0.03925	0.01881	0.01062	0.01279	0.02120	0.83160	0.01903	0.01498
12	0.00000	0.00000	0.83787	0.17466	0.59468	0.11840	0.68146	0.98502

The Netherlands 1970

	1	2	3	4	5	6	7	8
1	0.07983	0.00000	0.01249	0.00989	0.00000	0.00000	0.00000	0.00000
2	0.00000	0.00000	0.01033	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.43881	0.83136	0.00123	0.25057	0.12681	0.00000	0.46112	0.00000
4	0.00000	0.00000	0.07661	0.02022	0.16035	0.00000	0.00046	0.00000
5	0.00000	0.00000	0.00000	0.09906	0.00101	0.00000	0.00236	0.00000
6	0.00000	0.00000	0.00000	0.05485	0.00071	0.00000	0.00115	0.00000
7	0.00000	0.00000	0.00000	0.10370	0.00061	0.00000	0.00166	0.00000
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
10	0.48135	0.16864	0.07972	0.26395	0.26683	0.00000	0.20796	0.00000
11	0.00000	0.00000	0.00000	0.00000	0.00000	0.35377	0.00000	0.00000
12	0.00000	0.00000	0.81962	0.19775	0.44368	0.64623	0.32528	1.00000

Korea 1963

	1	2	3	4	5	6	7	8
1	0.72970	0.00000	0.00337	0.00857	0.00000	0.00000	0.00000	0.00000
2	0.00000	0.00000	0.03405	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.18506	0.87332	0.00000	0.14987	0.04294	0.00000	0.25406	0.00000
4	0.00000	0.00000	0.21237	0.24806	0.52674	0.00000	0.00322	0.00000
5	0.00000	0.00000	0.00000	0.07452	0.00000	0.00000	0.00138	0.00000
6	0.00000	0.00000	0.00000	0.02209	0.00000	0.00000	0.00053	0.00000
7	0.00000	0.00000	0.00000	0.17837	0.00045	0.00000	0.00287	0.00000
8	0.00000	0.00000	0.07053	0.00000	0.00000	0.00000	0.00000	0.00000
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
10	0.07390	0.11402	0.00504	0.12468	0.07780	0.00000	0.09695	0.00000
11	0.01134	0.01266	0.00000	0.01112	0.00793	0.86572	0.01448	0.00987
12	0.00000	0.00000	0.67457	0.18316	0.34408	0.13428	0.62725	0.99020

*The row numbers indicate the division of private consumption by use of goods of Table 4, and the column numbers, those of Table 3.

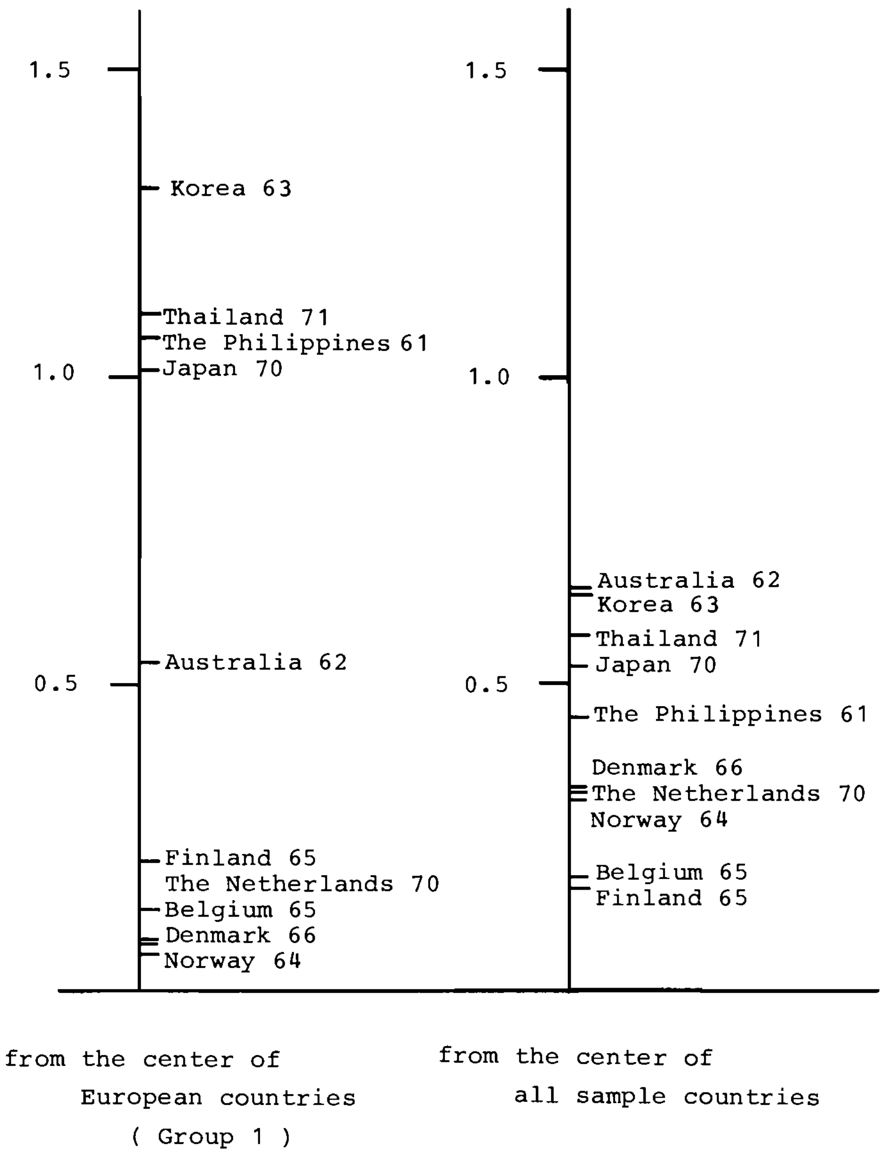

Figure 2. Geometric distances of consumption converters.

Estimation of Investment Converter KI

Almost the same process as used for the estimation of Kcp can be applied for the estimation of KI. In case of KI, however, both I and $\tilde{I}$ are available (in principle) from NA and GWI so that the process may be applied to many countries. The formulation of the problem is as follows:

$$\text{Constraints:} \quad KI.ij \geq 0 \quad : \quad \text{for all } i,j \quad , \qquad (11)$$

$$\sum_i KI.ij = 1 \quad : \quad \text{for all } i,j \quad , \qquad (12)$$

$$I = KI.\tilde{I} \quad . \qquad (13)$$

$$\text{Criterion:} \quad J = \sum_{i,j} Wij(KI.ij - \overset{o}{KI}.ij)^2 \rightarrow \min \quad . \qquad (14)$$

Since some components of I are unknown these components are also selected as independent variables. $\overset{o}{KI}.ij$ is again initially that of Japan and then the average of estimated KI.ij's. Nine advanced countries, four medium-advanced countries and six African developing countries were selected and the computational results shown in Table 6. Sectors corresponding to I are the same as those in Table 4, and the sectors corresponding to $\tilde{I}$ are those in Table 4 plus governmental sectors of agriculture, transport and communication, general administration, and other services. Figure 3 shows that the converter of Japan is closer to those of advanced countries than to developing countries, while its Kcp is less close. This seems reasonable considering that the characteristics of industrial structure in modern society is the same all over the world.

CONCLUSION

Our research has come to the point where consumption and investment converters of various countries can be estimated. The next stage will be to find a functional relation between input coefficient matrices and other variables specific to countries, and to estimate these matrices. The work is now being done in cooperation with our main project of global modelling. Results of this should come at the end of next year.

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Table 6. Examples of investment converter.*

Japan 1965

	Private inv.												Government Inv.			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.0982	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0899	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0291	0.0144	0.0223	0.0101	0.0156	0.0126	0.0193	0.0122	0.0442	0.0190	0.0063	0.0079	0.0175	0.0105	0.0096	0.0127
4	-0.0082	-0.0114	-0.0212	-0.0068	-0.0002	0.0040	0.0038	-0.0232	0.0066	-0.0114	-0.0170	-0.0047	0.0198	-0.0148	-0.0082	0.0170
5	0.0223	0.0605	0.0404	0.1115	0.2272	0.0714	0.1609	0.2005	0.0632	0.0562	0.0223	0.0411	0.0276	0.1921	0.0182	0.0736
6	0.1207	0.1495	0.0933	0.0986	0.0552	0.0644	0.0882	0.0109	0.1967	0.3762	0.5232	0.0725	0.0660	0.1872	0.0219	0.0430
7	0.2327	0.2101	0.4437	0.2908	0.3211	0.2559	0.4003	0.1142	0.4457	0.0491	0.0284	0.0301	0.1171	0.0809	0.0400	0.1773
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.4551	0.5043	0.3063	0.4135	0.2966	0.4956	0.2601	0.6259	0.1504	0.4379	0.3691	0.8366	0.6614	0.5262	0.8903	0.6496
10	0.0634	0.0529	0.1007	0.0739	0.0793	0.0900	0.0567	0.0410	0.0772	0.0470	0.0592	0.0149	0.0000	0.0143	0.0244	0.0213
11	0.0063	0.0193	0.0075	0.0083	0.0051	0.0061	0.0105	0.0185	0.0159	0.0059	0.0033	0.0013	0.0006	0.0031	0.0038	0.0032
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Italy 1971

	Private Inv.												Government Inv.			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0903	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0469	0.0153	0.0308	0.0154	0.0163	0.0131	0.0207	0.0116	0.0469	0.0250	0.0173	0.0085	0.0176	0.0113	0.0097	0.0143
4	0.0114	-0.0101	-0.0145	-0.0023	0.0003	0.0044	0.0049	-0.0220	0.0076	-0.0085	-0.0025	-0.0089	0.0198	-0.0143	-0.0091	0.0181
5	0.0508	0.0626	0.0610	0.1297	0.2296	0.0730	0.1653	0.2041	0.0772	0.0790	0.0496	0.0589	0.0278	0.1936	0.0263	0.0772
6	0.0780	0.1442	0.0501	0.0287	0.0514	0.0589	0.5701	0.0028	0.1528	0.3127	0.3941	0.0000	0.0646	0.1788	0.0000	0.0376
7	0.2587	0.2124	0.4647	0.3025	0.3230	0.2570	0.4031	0.1180	0.4528	0.0636	0.0515	0.0385	0.1172	0.0820	0.0432	0.1811
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.4508	0.5021	0.2830	0.4319	0.2933	0.4964	0.2656	0.6306	0.1605	0.4587	0.4065	0.8769	0.6619	0.5291	0.8997	0.6421
10	0.0801	0.0534	0.1102	0.0805	0.0803	0.0905	0.0584	0.0383	0.0828	0.0569	0.0666	0.0196	0.0000	0.0156	0.0260	0.0233
11	0.0231	0.0200	0.0147	0.0136	0.0058	0.0065	0.0119	0.0166	0.0195	0.0126	0.0169	0.0063	0.0007	0.0038	0.0043	0.0066
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Zaire 1970

	Private Inv.												Government Inv.			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0804	0.0266	0.0281	0.0199	0.0157	0.0132	0.0205	0,0071	0.0517	0.0274	0.0296	0.0074	0.0559	0.0127	0.0661	0.0727
4	0.0252	0.0020	-0.0145	0.0038	0.0000	0.0046	0.0049	-0.0253	0.0124	-0.0034	0.0053	-0.0149	0.0542	-0.0119	0.0327	0.0690
5	0.1422	0.0749	0.0512	0.1260	0.2273	0.0727	0.1631	0.1921	0.0767	0.0702	0.0632	0.0278	0.0768	0.1918	0.1025	0.1637
6	0.1707	0.1132	0.0638	0.0780	0.3533	0.0588	0.0823	0.0127	0.1889	0.3812	0.4982	0.0527	0.1075	0.1949	0.0310	0.1212
7	0.3837	0.2143	0.4491	0.2998	0.3207	0.2564	0.4020	0.0972	0.4570	0.0590	0.0621	0.0202	0.1759	0.0834	0.1372	0.2798
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.4731	0.3000	0.3722	0.2988	0.4975	0.2579	0.6702	0.1042	0.3956	0.2289	0.8918	0.4443	0.5058	0.4197	0.1343
10	0.1417	0.0665	0.1050	0.0833	0.0791	0.0904	0.0577	0.0340	0.0864	0.0559	0.0829	0.0104	0.0462	0.0180	0.0943	0.0952
11	0.0560	0.0294	0.0122	0.0168	0.0051	0.0065	0.0116	0.0120	0.0229	0.0141	0.0297	0.0045	0.0391	0.0053	0.0594	0.0641
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

*The row numbers and column numbers up to 12 indicate the division of Table 4. Column numbers over 12 indicate: 13, Agriculture, Forrestry and Fishing; 14, Transport and Communication; 15, Administration; and 16, Other Industrial Activities.

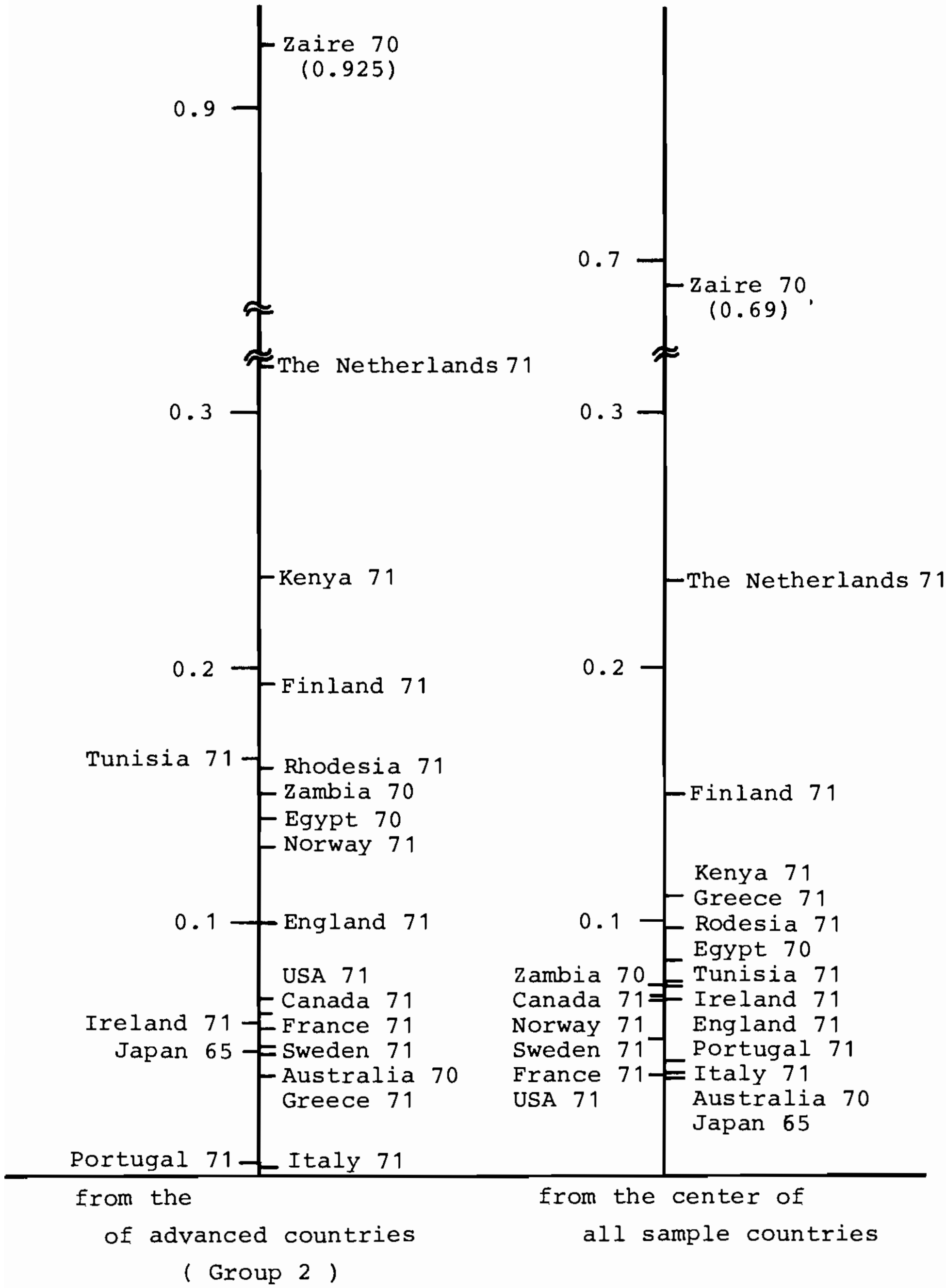

Figure 3. Geometric distances of investment converters.

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DISCUSSION

McPherson pointed out that, to his knowledge, input-output tables are only accounting procedures displaying economic data that fall within one particular period of time. In a dynamic approach, because of the time lags involved data from different time-periods are necessary. How is this difficulty overcome? In reply, Kaya referred to the extensive literature and practical experience gained in dynamic input-output theory, including Japan (National Economic Agency).

Bruckmann wished to clarify a possible misunderstanding between dynamic input-output theory, which attempts to combine fixed input-output coefficients with some dynamic economic theory, and dynamic input-output tables, which attempt to forecast the input-output coefficients of future input-output tables. Kaya agreed that for the latter, theoretical approaches are often not applicable owing to the lack of comparable data from the past; hence, his group had to adopt another kind of methodology.

Bruckmann asked whether environmental aspects were built into the input-output analysis, along the lines pursued by Leontief and by Allan & Kneese. Kaya replied that the problem certainly is an important one, in particular in a country like Japan, but that its specific inclusion would have little impact on economic growth. In his opinion, environmental data cannot and should not be measured in money terms. The specific recognition of environmental impacts is being pursued in his country by different groups with a different model.

A SIMULATION MODEL FOR EXPLORING FUTURE TRENDS IN WASTE LOADINGS TO THE UPPER GREAT LAKES

J. P. H. Batteke

INTRODUCTION

This is a brief report on the application of systems modelling for the purpose of management of the international waters of the Great Lakes. The work was originally undertaken for the Upper Lakes, Superior and Huron. That task will be completed by January 1977. Currently, preparations are also being made to apply the model to the Lower Lakes, Erie and Ontario. In this version, the new estimate will be both an update and an expansion of similar estimates prepared for the International Joint Commission in 1970. This work is to be completed by the spring of 1977.

There are significant differences between this model and the majority of the models that have been presented at the IIASA global modelling conferences:

- It was developed in response to a management need. The timing for the completion and the format of the results was specified in that context.

- It had a limited purpose and scope, and it might, therefore, suffer some lack of flair in sophistication.

- Its results had to gain acceptance by the International Joint Commission and by the representatives of State, Provincial, and Federal governments having jurisdiction over these waters and basins.

- The model may not be pristine in a strict disciplinary sense but, rather, it breathes a pragmatism necessary to achieve the desired goals with the limited tools available.

Purpose

The purpose of the modelling work is to determine--where possible--the nature of quantitative relationships between demographic, social, and economic trends in the basin, and their demands on the water resource. This can be formulated more precisely as follows: the purpose is to explore, in quantitative terms, the range of plausible future trends in waste loadings for a number of parameters, in order to spot critical developments and to provide an early warning--along with costs and impact estimates--for the formulation of policies and action plans by both nations.

Background

The cooperative approach regarding boundary problems between the USA and Canada is based on the Boundary Treaty of 1909. This Treaty resulted from the recognition, on both sides, of the international equality of rights in the use of boundary waters. The Treaty provided the legal principles and international institutional structures to guide the control and usage of the Great Lakes and the St. Lawrence River. In 1911, the International Joint Commission was created as a result of this Treaty. Its main responsibilities were to investigate boundary management issues referred to it by either, or both, the US and Canadian governments. The International Joint Commission was, in the words of J.C. Day [1], given quasi-judicial, investigative, and administrative powers to discharge its responsibilities. It frequently holds public hearings as a means of gathering information.

With the Canada-United States Great Lakes Water Quality Agreement of 1972, a set of comprehensive studies of the Lakes was initiated. Two International Joint Commission References originated from it:

- A Reference on the Pollution from Land Use Activities.
- A Reference on the Water Quality of the Upper Great Lakes.

The remainder of my discussion will deal with the latter of the two References.

The Reference questions are:

- Are the waters of Lake Superior and Lake Huron being polluted on either side of the boundary to an extent that is causing, or is likely to cause, injury to health or property on the other side of the boundary; or that is causing, or likely to cause, a degradation of existing levels of water quality in these two lakes or in downstream portions of the Great Lakes system?
- If the answers to the foregoing questions are yes, to what extent, by what causes, and in what localities is such pollution taking place?
- If the Commission should find that such pollution is taking place, what remedial measures would, in its judgement, be most practicable to restore and protect the quality of the waters, and what would be the probable cost?

- In the event that the Commission should find that little or no such pollution is taking place at the present time, what preventive measures would, in its judgement, be most practicable to ensure that such pollution does not occur in the future and what would be the probable cost?

The task was undertaken by the Upper Lakes Reference Group with representatives of Federal, State, and Provincial governments. The operational responsibilities were divided into four subtasks, each dealing with a broad set of disciplinary aspects. They are:

- Background studies on hydrology, climate, socioeconomic conditions, and demography. Future trends resulting from human activities. The simulation model was developed under this group.

- Main lake data collections and analysis; including the study of transboundary movements of pollutants.

- Present materials input measurements at tributary mouths and direct inputs to the lakes.

- Coastal and embayment pollution problems, and the study of nearshore/open water exchange mechanisms.

A committee of eight co-chairmen (each subtask has a US and Canadian co-chairman) was responsible for the production and co-ordination of the work, and for the integration of the results. The final reports to the International Joint Commission will be completed early in 1977.

MODEL DESIGN PRINCIPLES AND SPECIFICATIONS: A SUMMARY DESCRIPTION

The length of the Lakes' response time (Lake Superior has a flushing time of 113 years; Lake Huron, 17 years) required that the analysis time period be set at 50 years.

The critical importance of determining the extent of transboundary pollution precluded the use of separate national models. In separate models, the variation in output, due to differences in methodologies, would defeat the purpose of the model, i.e. in providing insight into present and possible future occurrences of transboundary pollution. The operation of the unified model was funded jointly by Canada's Department of the Environment and the USA's Environmental Protection Agency.

One would hope and expect that a model of this caliber could be linked closely with a Lake behavioural model. Unfortunately, the state of the art in large-lakes modelling has not yet reached the capability to handle the large number of chemicals monitored. In addition, problems of monitoring the segmentation of the lakes, and the related exchange mechanisms, have prevented the application of comprehensive systems modelling techniques to date. The Waste Loadings Trends Model, therefore, describes only events and trends between the outer basins' boundaries and the shoreline.

The regional aspects of the pollution studies were introduced by selecting regions on the following set of criteria:

- the major watershed as a management unit;
- the homogeneity of economic activity in a group of watersheds; and
- the inclusion of embayment units, which are of importance to the studies of the fourth subtask.

A set of 30 waste parameters was selected for analysis in the model although, in various localities, more than those 30 have been monitored. The list is shown in Table 1.

Table 1. Chemical waste parameters in model.

Common Metals	Nitrogenous Nutrients
Calcium	Nitrogen-Inorganic
Iron	Ammonia
Magnesium	
Potassium	Phosphorus Nutrients
	Total Dissolved
Trace Metals	Phosphorus
Aluminum	
Cadmium	Non-Specific Inputs
Chromium	Oil
Copper	Dissolved Solids
Lead	Suspended Solids
Manganese	
Mercury	
Nickel	Phenols
Titanium	
Zinc	Silica
Inorganic Anions	Boron
Fluoride	
Chloride	
Bromide	
Cyanide	
(Sulphuras) Sulphate	

METHODOLOGY

During the formative stages of the model, various research proposals, some containing quite interesting promises, were put before us. However, the conviction that, to date, we do not have the capabilities to map out social behavioural models with some degree of success led to the decision not to gamble on such promises. Although "concept-rich", such applications of the Forrester-type systems dynamics model are often frustrated by the lack of a proper data back-up. In addition, the concepts are often open to much criticism--which I do not have to repeat here--from the side of economists, political scientists, and sociologists. However, the other alternative, the use of econometric models, was not considered to suit the purpose either. The inherent rigidity, common to such models, which is, of course, very useful for describing the "economic-world", is not suitable for incorporating all social and specific technological aspects of the relationships between the wealth and waste of the Great Lakes' citizens.

An additional constraint existed. In the multi-government operating environment, forecasts and plans existed that could not be ignored (remember the requirement of the International Joint Commission's acceptance). Since the best of these plans and forecasts resulted from serious and excellent scientific work, there was neither the need nor the desire to ignore them. This meant that the model would have to have capability of acting as a "clearing house" for the merging of often incompatible input data.

The considerations discussed in the previous paragraphs have led to the adaptation of a hybrid-model structure, that incorporates the advantages of econometric and systems dynamics modelling, while largely circumventing their major shortcomings.

Acknowledging that econometric models are still the best tools available for describing the distributive aspects of society, we have used their inputs to give the Waste Loadings Model what we call an *economic spine*.

Impact multipliers were generated to deflect the *spine*, if non-productive investments in pollution abatement were to affect the national performance. The technical and operational abatement aspects and the anticipated change in social and legislative aspects were "woven" around these economic spines.

I should make clear now that the model is a highly disaggregated one; it deals with 11 regions, 30 waste parameters, 31 industry groups, 3 different levels of municipal waste treatment capabilities (including the relative removal capacities for each parameter), and a set of continuous-scale factors for the removal of industrial wastes.

Returning to the economic spines, they were formed from the results of previously conducted economic forecasting work. In the USA, that was provided by the OBERS projections. The acronym OBERS signifies a unified effort of the Office of Business Economics of the US Department of Commerce, and the Economic Research Service of the Department of Agriculture, in which an integrated set of projections was developed under a common set of procedures

--relative to Water Resources Regions and Subregions. These projections were extended to the year 2020. In Canada, an interdepartmental disaggregated econometric model, CANDIDE (with more than 2000 simultaneous equations), was utilized. However, for the purpose of this project, CANDIDE results had to be related to the watershed regions, and the model had to be extended over an additional time period from the existing 1985 limit to the year 2020. Both US and Canadian projections provided sets of different economic scenarios, all of which could be linked to the simulation model in a similar way. Needless to say, all scenarios are "non-revolutionary" and, consequently, the variations of the impacts on waste loadings do not offer any great surprises.

The generalized structure of the simulation model is shown in Figure 1. It is a structure in which "hard" deterministic data and relations are merged with "soft" data; for example, opinions, judgements, and expectations. The model provides the mechanical structure that allows a systematic and consistent accounting of most of the relevant factors. An attempt was made to fully cover the human-physical circuit from perception to action; this is shown in Figure 2.

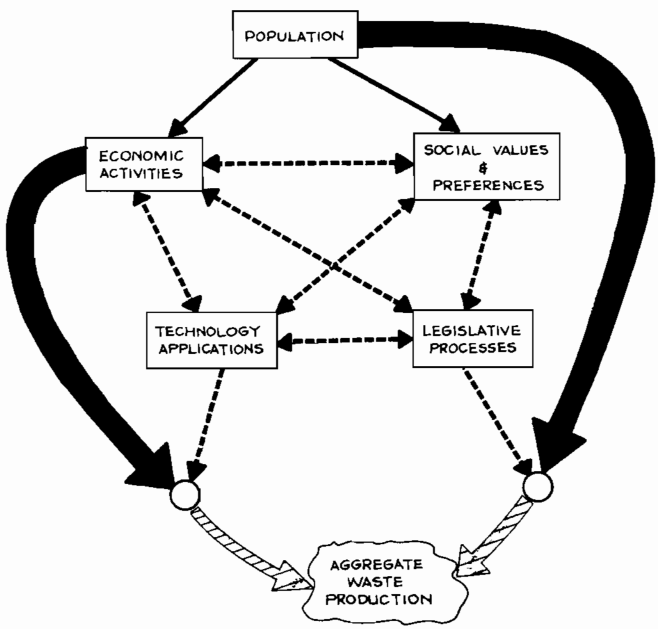

Figure 1. Generalized structure of the model.

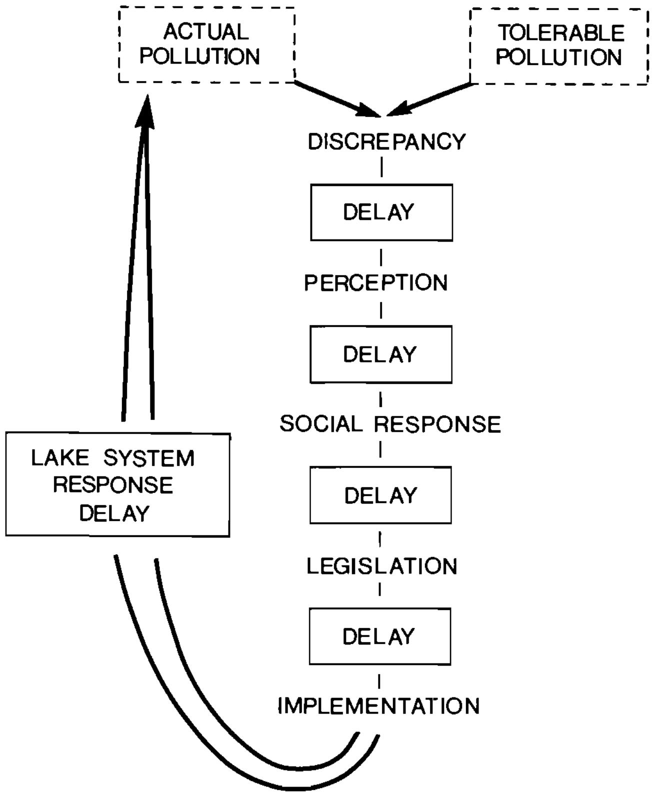

Figure 2. The pollution perception and response cycle.

GENERAL STRUCTURES OF ALGORITHMS

The algorithms are designed to calculate the production of raw waste on the basis of unit load relationships. That is:

- per capita loads generated by urban population and urban activities that have to be handled by the municipal treatment systems; and

- per dollar of value added loads for each parameter in any of the 31 industry groups.

Tables 2 and 3 show examples of selected sets of unit loads per capita, and per million dollars of value added. A considerable effort went into establishing these industrial unit load values. The statement made earlier that the model was perhaps

Table 2. Municipal waste production. (Examples of waste produced--before treatment--by domestic, commercial, service sector, and minor industrial activities in a municipality.)

Selected Waste Parameters	kg per capita, per year
Total Phosphorus	1.2
Total Nitrogen	6.8
Total Dissolved Solids	99.0
Chloride	13.5
Silicate	2.5
Sulphate	11.3
Ammonia	3.8
Cyanide	0

Table 3. Industrial waste production. (Examples of the by-products of a typical industry group (primary metals) per dollar of output (value added).)

Selected Waste Parameters	kg per million dollars output, per year
Total Phosphorus	9
Total Nitrogen	1,430
Total Dissolved Solids	222,000
Chloride	820
Silicate	450
Sulphate	1,700
Ammonia	2,820
Cyanide	130

not sophisticated should be qualified now. A good practical sophistication was demonstrated in the adaptation of the model algorithms to the available data that, as you know, are always short of the needs. As a general comment regarding this data availability aspect it follows that concept-sophistication in model building is often unwarranted; it might add to the promise, but it certainly does not add to realism of the model output.

Returning to the discussion of the algorithms. At the next step, the algorithms proceed to compare this raw production with existing treatment capacities and their removal efficiencies for each parameter. In most cases, the raw municipal and industrial wastes will undergo some form of treatment before being discharged to the open waters in the river basin. What is then discharged after such treatment, if any, could be called the net-waste production. This net-waste production for each region will be affected by the economic, social, demographic, and technological forces that determine the growth and public attitudes in those regions. The attitudes of the public and the available technology, together with the willingness to invest in abatement equipment, will reduce the net-waste production relative to the raw-waste production.

The sociological, technological, and institutional aspects, that in fact determine present and future efforts of society towards pollution abatement, had to be introduced into the model. For this purpose, a separate research project, using the Delphi technique, was established. The project focused on general trends and on "synergistic effects", i.e. the new and not immediately foreseeable developments resulting from a set of somewhat independent and discernable trend developments in different sectors of human activities. The results of these findings have been translated, through a quantitative point system, to fit the simulation model's input requirements.

The uncertainty about the future values of many variables and the requirement that the model has some "clearing house" capability has led to a selection of a set of variables that can be used as convenient entries for simulation purposes. With that set of simulation variables shown in Table 4, the model has become a vehicle for a quantitatively oriented dialogue, in which relatively precise data can be merged with far less precise data elements. Through this structure, the model can serve its purpose of being a tool for "exploration of possible critical developments" in the future. Note that each variable is either a vector or a multidimensional matrix with such dimensions as time periods, regions, waste parameters, industry groups, population, etc.

I spoke earlier about the regions' net-waste productions. These amounts will, in principle, *never* equate with what passes the shoreline into the lake. It is clear that the route from establishment-outfall to tributary mouth is one in which much can happen to the quantity of the loadings. The values thus obtained are, in fact, theoretical waste loadings values; what is now needed is to introduce the influences--be they positive or negative--of the streams and basins on these loads.

The empirical aspects of the loadings are brought to bear in the manner shown in Figure 3. Thus, the waste load trends resulting from the model are based on the observed loads (at the tributary mouths and direct discharge points) in 1973 and 1974, on which are superimposed the future trends as determined by social, economic, and technological factors.

Table 4. Simulation Variables

1. National Population Growth USA
2. Provincial Population Growth Ontario
3. Alternative Economic Scenario USA
4. Alternative National Economic Scenario Canada
5. Regional Populations Adjustments
6. Urban/Regional Population Ratios Adjustments
7. Adjustments Per Capita Municipal Waste Products
8. Adjustments Per Dollar Industrial Waste Products
9. Adjustments Capital Efficiency Ratio
10. Adjustments Per Capita Water Use
11. Adjustments Proportion Municipal Treatment Capacity in Primary, Secondary, and Tertiary Treatment
12. Industrial Treatment Efficiency in Base Year (1973/1974)
13. Desired Industrial Treatment Level
14. Desired Municipal Treatment Capacity

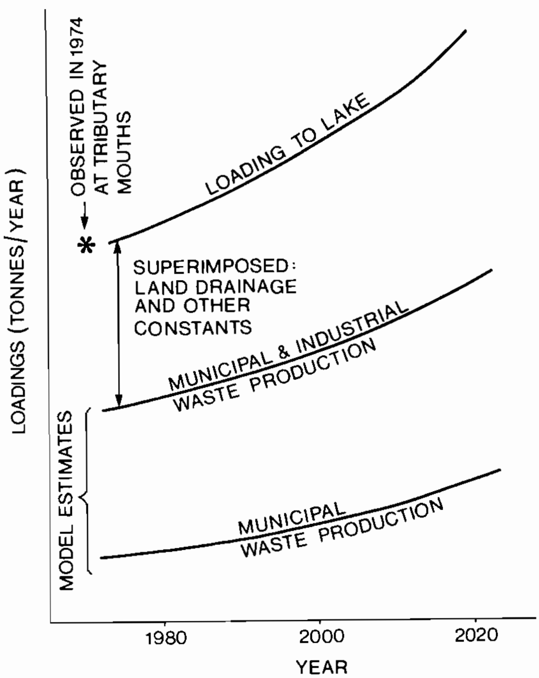

Figure 3. Loadings and waste production.

In this approach, municipal and industrial waste production are taken as the *most significant explanatory agents* of change of the levels of waste loadings. Lack of detailed understanding of what happens to the waste as it travels through the river basin on its way to the lake and a lack of data on the diffuse-source waste-load components has made such an approach necessary. The consequence of this is that land drainage, assimilation, and other physical factors such as climate, soils, and geomorphology are introduced as constants. This is not an entirely unreasonable assumption, since the rivers are relatively short in length.

FUTURE TRENDS IN THE BASIN

Let us now look at the subject of the model: the trends development in the basins and the nature of the results in terms of waste loads and abatement expenditures.

A comparison of the commonly accepted estimates of growth (Figure 4) of the various aspects of the human factor shows where our problem is. Even with the moderate population expansion anticipated in North America, important polluting activities--economic output and the generation and consumption of energy--show five- to tenfold increases over the period to 2020.

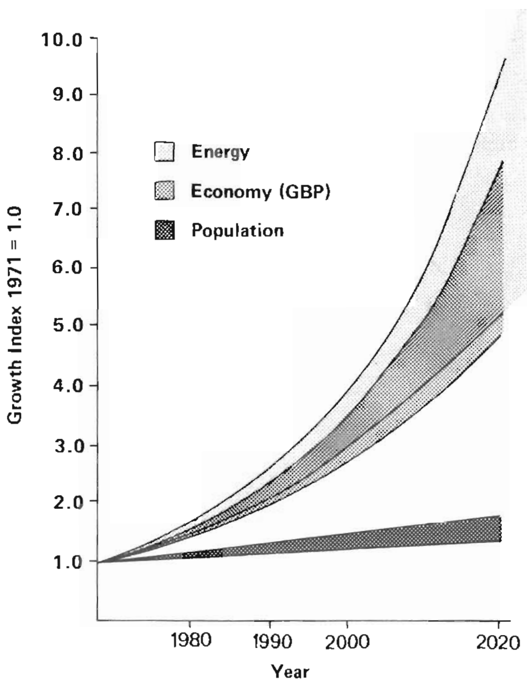

Figure 4. Growth rates.

The planners envisage that the Upper Lakes Basin will remain a low density area. It is the habitat of just over one percent of the total US and Canadian population. It also contributes approximately one percent to the combined GNPs. Future growth of urban areas is largely affected by employment opportunities, and by the pull of larger cities. Some growth might result from government policies aimed at a more equitable national distribution of employment and income. In the period to 2020, the Upper Lakes Basin's populations are expected to grow from 1.8 million to 2.4 million in the USA and from 1.0 million to 1.9 million in Canada. The population distributions over the subbasins are shown in Figures 5 and 6.

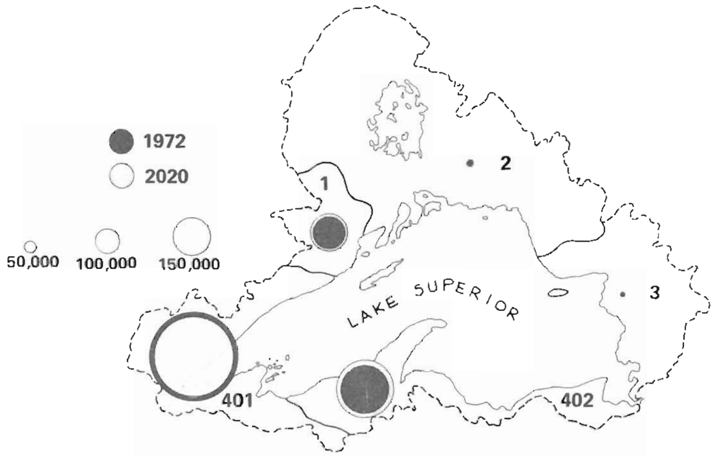

Figure 5. Lake Superior drainage basin and estimates of regional population growths.

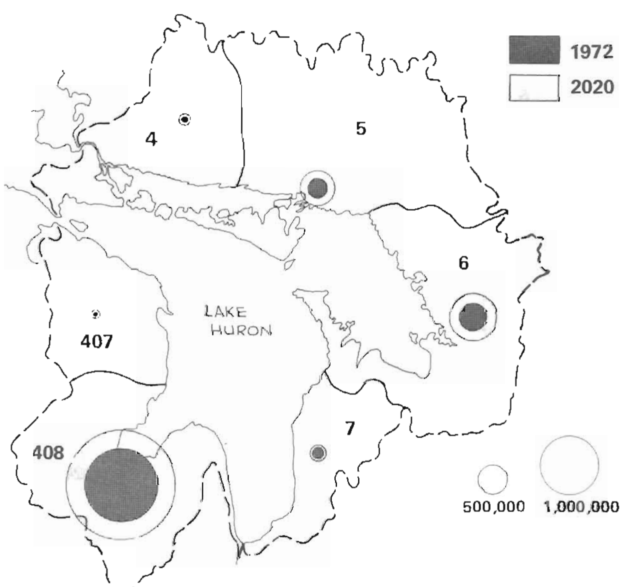

Figure 6. Lake Huron drainage basin and estimates of regional population growths.

Present economic activities and projected future growth of the Canadian Basin are determined by the development of resource industries: mining in the Superior Basin and the Sudbury Region; primary processing--pulp and paper, and wood--in Thunder Bay; and the primary metal industry at Sault Ste. Marie. Manufacturing is the growth sector in the Georgian Bay and Southern Lake Huron Basin. Note that "Forestry and Fisheries" in the bar-charts (Figures 7 and 8) are narrowly defined as: harvesting the trees and fish. The pulp and paper, the saw mills, and the fish processing industry are all classified under "Manufacturing".

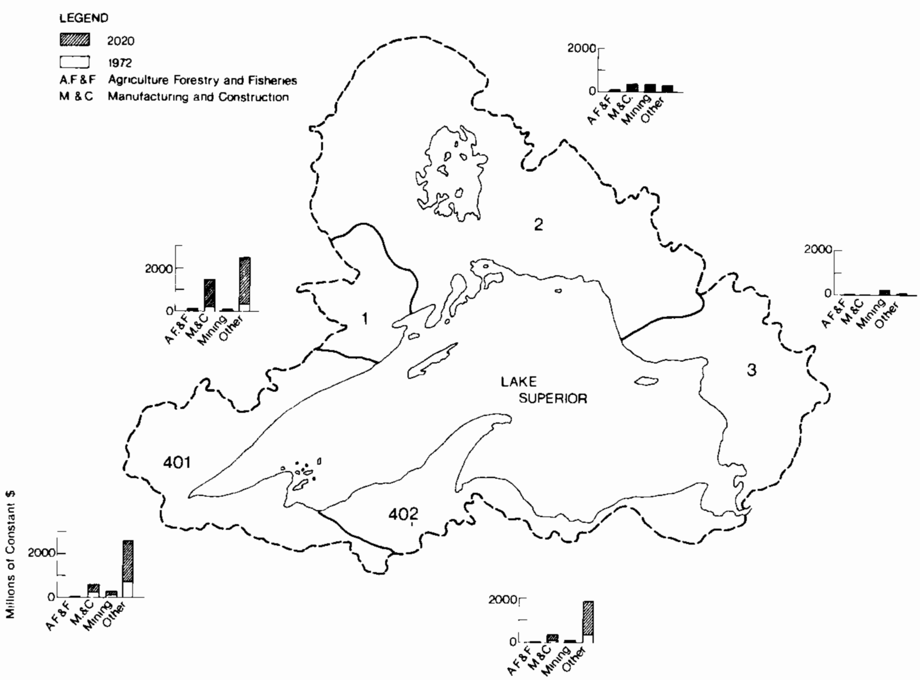

Figure 7. Lake Superior drainage basin and estimates of regional economic activity (economic activity measured as constant $ output.).

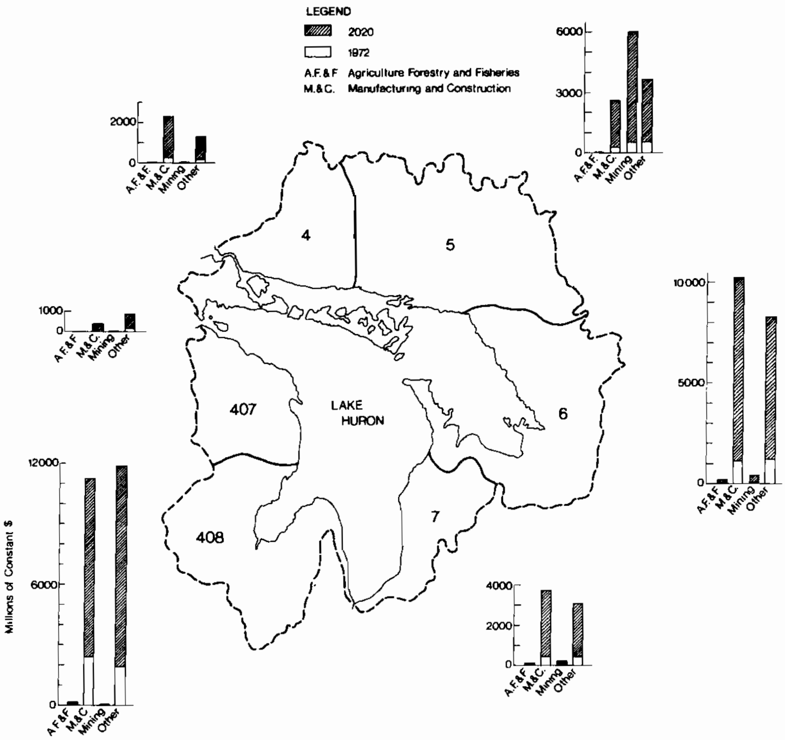

Figure 8. Lake Huron drainage basin and estimates of regional economic activity (economic activity measured as constant $ output).

In the US Superior Basin, the public and service sectors--classified under "Other" in the bar-chart--are the major components of growth. Manufacturing, together with the service sector, are key growth areas in the Huron Basin. US and Canadian real economic growth is expected to be five times and seven times, respectively, for the period to 2020.

Three scenarios were developed to explore trends in loadings from point-sources. The first, depicted by the full lines, lower graph of Figure 9, shows what might happen to the quantities of annual waste discharges if private and public capital expenditures on abatement remain at the current percentage--0.1 percent of Gross Basin Product (GBP)*--throughout the simulation period.

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*This term is an indicator of economic output for the region. It is defined in a similar way to gross national product.

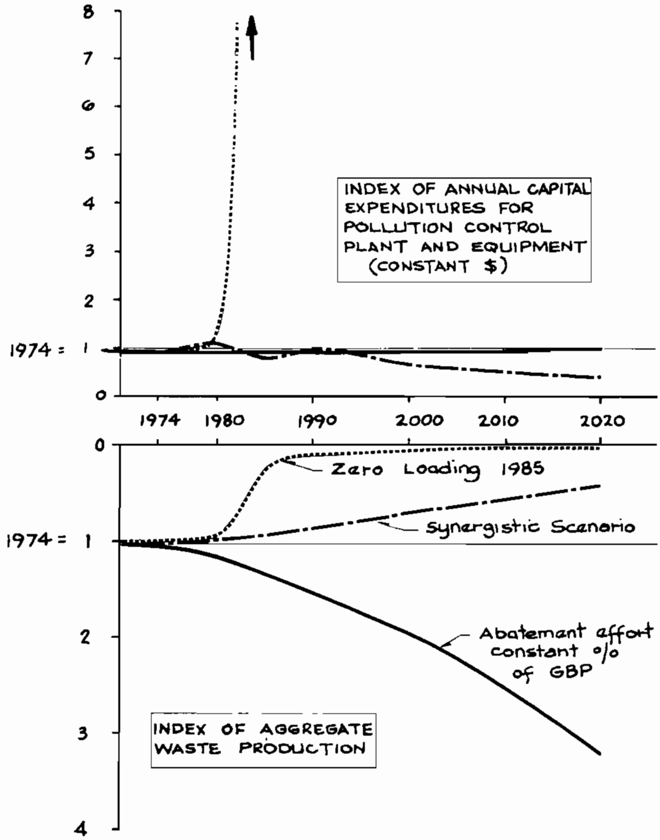

Figure 9. Alternative policy options (30 parameters).

This seems to be the upper limit of pollution that we may expect. Note that this line represented the average trend behaviour (for the purpose of this picture) where, in full, there are 30 different lines--one for each parameter. Note also that the values shown here are subject to considerable revision, since the analytical work with the model has not been finalized yet.

The point-source loadings are only part of the problem. Figure 10 shows how the total of phosphorus loadings are derived from contributions from land-drainage and atmospheric sources, in addition to loadings from point-sources. Atmospheric loadings result from long-range transport patterns leading from continental metropolitan areas to the Great Lakes.

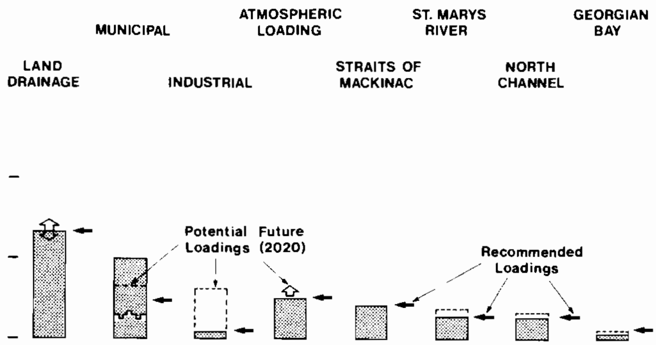

Figure 10. Main body of Lake Huron-phosphorus loading.

Source: [2]

A second scenario, shown as the dotted lines in Figure 9, studied the lowest limit, which aims at zero-discharge by 1985 for all waste parameters. The resulting investment requirements--in the upper graph--are excessive. This is due to the progressively increasing costs of removing the last 5 or 10 percent of the pollutants. Zero loadings as a goal for all nutrient and toxic substances is, therefore, impractical as costs are disruptive to the national economies. Although no across-the-board removal is possible, selective prohibition of the discharge of certain toxicants and selective removal of a proportion of the nutrients is a practicable and necessary goal.

A third scenario, the synergistic scenario, shown as the broken lines in Figure 9, was developed to demonstrate that the combined effects on net-waste production of increased public, governmental, and commercial attention to environmental problems will be larger than the sum of these effects taken individually.

Thus, some changes in production technology (e.g. more recirculation) will pre-empt the need for treatment. Further the development of a competitive and aggressive pollution control industry could drastically reduce per unit treatment costs. A schematic explanation of this phenomenon is given in Figure 11. There are indications that this industry has already realized a strong growth in recent years. Although the potential for the realization of such a favourable scenario exists, it will not actually happen without an increased commitment to this goal, on the part of the public, governments, and industry. In additior to pollution control measures, this commitment would include programs to encourage research, development, production, and application of innovative non-polluting industrial processes and non-polluting residential water use processes. Moreover, the need for the conservation of energy will also automatically induce application of less polluting technology.

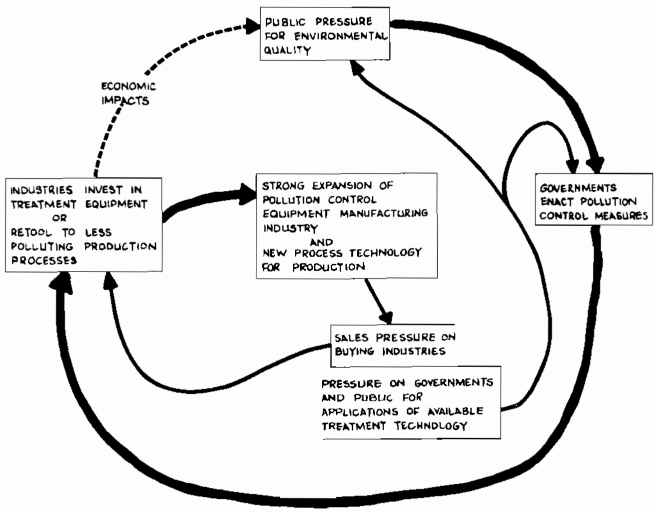

Figure 11. The role of a growing pollution control equipment manufacturing industry.

There is a growing body of evidence that a gradual transition from a polluting society to a non-polluting society--as far as point-sources is concerned--is possible without disruptive economic sacrifices [3, 4, 5]. Intensive public participation in this process and the development of commercial interests in new opportunities will both lead and support meaningful government steps to this end. The warning signal that comes from these scenarios will hopefully provide the stimulus for concerted action, which might lead to actual developments along the lines of our more favourable scenario.

CONCLUSION

The modelling work related to the Great Lakes water quality is not a one-shot affair. The subject matter is too complex and too dynamic to allow such a simplistic round-up. As I mentioned earlier, work is now in progress on another International Joint Commission's set of reference questions. These deal with all the nutrients and toxicants that are discharged to the lakes and have their origin in diffuse sources. They constitute, as shown in Figure 10, a significant portion of the total loads to the lakes. It is obvious that not all, or not even a large part, of the diffuse sources loads will allow human control in the future.

In conclusion, the model did not attempt to map out a future. However, by using available data, it demonstrated the boundaries of environmental consequences of the populace's and politicians' choices. The usefulness of a systematic analysis of such choices has been demonstrated in this work for the International Joint Commission.

Where no comparable institutions exist for other important inland seas, I would like to suggest that IIASA consider carefully the possibility of applying similar research techniques to the Mediterranean and the Baltic Sea, which are both subjected to environmental stresses of the same nature as those affecting the Great Lakes. Such research would provide IIASA with a unique opportunity for conducting applied systems analysis in critical areas of international interest.

I would like, at the end of this presentation, to acknowledge the staff of the modelling team. It was only through their individual skills and persistence that the model moved successfully from the planning to the implementation stage. Their names are listed in Appendix I. The staff publications, working papers, and main data references have been listed in Appendix II.

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APPENDIX I. PROJECT STAFF

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APPENDIX II. LIST OF WORKING PAPERS AND DATA REFERENCES

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THE STATISTICS CANADA LONG-TERM SIMULATION MODEL

W. Page

INTRODUCTION

The Statistics Canada Long-Term Simulation Model (SCLTSM) provides a framework within which the long-term trends in major economic variables can be analysed in terms of feasibility and internal consistency, and by means of simulation, these growth paths of the Canadian economy can be explored. Simulation (as opposed to forecasting) implies a simple and understandable representation of structure, a high degree of disaggregation, and computability; long term implies a preoccupation with physical (as opposed to financial) phenomena and an abstraction from short-term cyclical activity; and feasibility implies that in the long term, the economic system faces a set of constraints--imposed by the supply of labour, the rate at which the system can accumulate capital, the technology available for transforming raw materials into usable products, and the availability of raw materials.

The model should not be thought of as a device that delivers information or forecasts to the user, but rather an educational process through which the user learns what is feasible and, more importantly, what is not. The model user, wishing to examine the implications of a possible scenario or forecast of interest to him, describes a scenario to the model in terms of a relatively small number of major exogenous variables and parameters, or if he wishes, in terms of many more detailed variables and parameters. The model generates a solution and issues a report which draws the user's attention to results that may not be feasible or consistent; the model itself does not react to apparent disequilibria or inconsistencies. The user must supply and try out various combinations of possible reactions to inconsistencies:

- by altering the model by choosing optional specifications,
- by resetting major exogenous variables,
- by resetting major parameters,
- by setting or resetting minor or more detailed exogenous variables, and
- by altering minor or detailed structural parameters.

The economic system delivers goods and services to people; the physical system is the stock or reservoir of materials, some of which are renewable by means of physical processes and some of which are not. The economic system takes materials from the physical system and transforms them by processes that may use capital and labour into products ready for final consumption, and returns materials to the physical system in the form of industrial by-products and spent consumer or capital goods (Figure 1). At present we are concerned only with measuring and accumulating selected flows of non-renewable resources as they are drawn into the economic system.

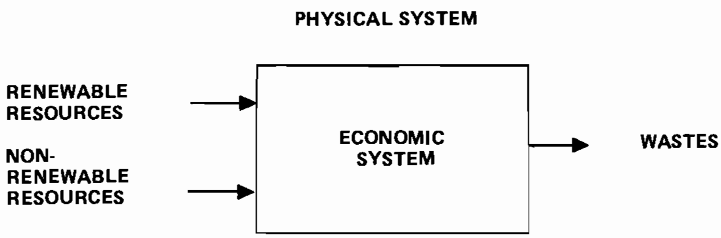

Figure 1. Economic and physical systems.

THE MODEL

The economic system consists of two major submodels, the demographic submodel and the economic submodel (Figure 2). The demographic submodel includes population, household formation, and labour force participation, and the economic submodel includes final demand formation, output determination, and the labour, capital, and resource requirements. Demographic variables from Model A are used to set levels of demand in Model B.

There is no feedback from B to A in the model structure itself. If, for example, the labour supply as calculated in A is not consistent with labour requirements as calculated in B, the user supplies the reaction in a subsequent simulation. Model C brings together results for different parts of the model and generates a report to assist the user in assessing the feasibility and consistency of the simulation he has defined. The user can react to this as outlined earlier.

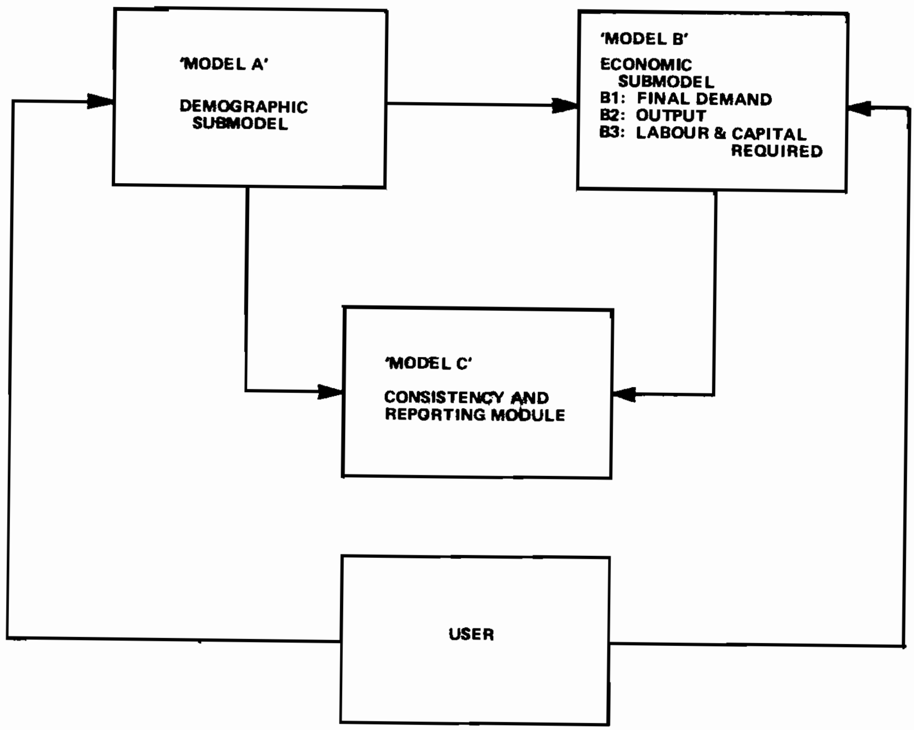

Figure 2. Structure of the long-term simulation model.

Recursive Structure

The model system is recursive in two senses, intratemporal and intertemporal. Within each time period each block need be calculated only once; the sequence goes from Model A to B1 to B2 to B3 (see Figure 3). The system moves stepwise through time; i.e., from t = 1,...,n. Because there are no diagonal arrows, it is feasible to move columnwise from left to right instead of row-wise--e.g., all of the Model A calculations can precede Model B calculations. The downward arrows generally designate the use of a stock variable where the stock in t+1 depends at least in part upon the stocks in t. It is feasible for each submodel to have its own time pattern as long as the time pattern of the blocks on the right is a subset of the time pattern of the blocks to the left. For example, Model A has a single year time interval and Model B has a variable time interval which, in our estimation, should be five years. This recursive structure leads to computational efficiency.

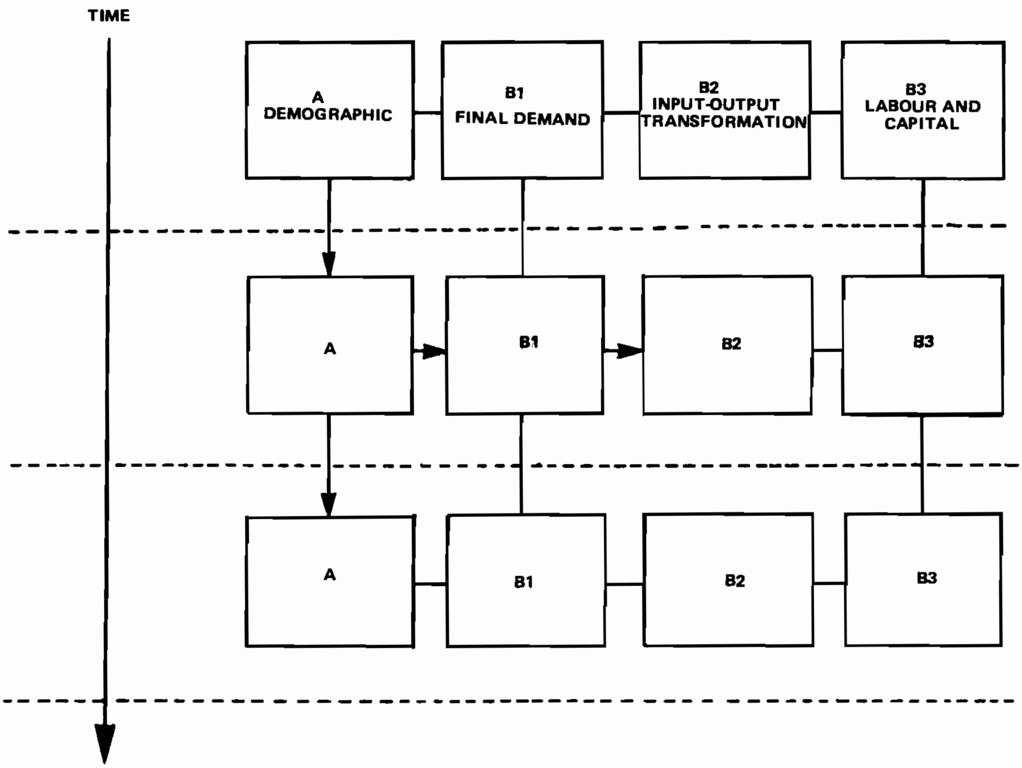

Figure 3. Recursive structure.

Demographic Submodel A

The demographic submodel A (Figure 4) calculates population and household variables available to Model B and labour supply, which along with labour requirements from Model B form the labour consistency checks.

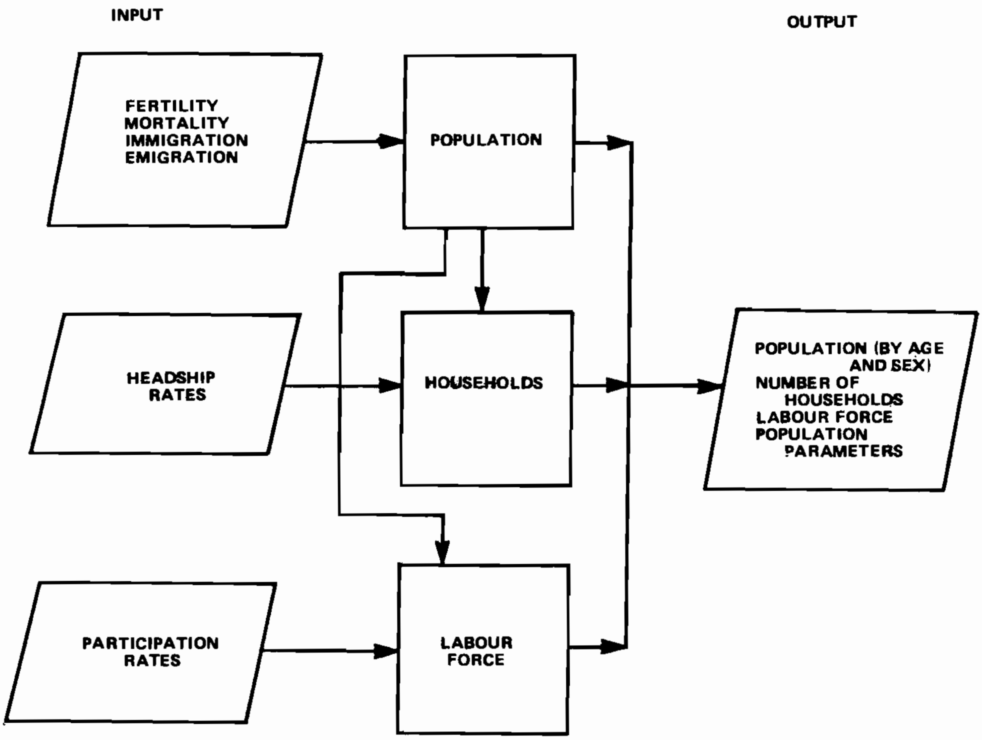

Figure 4. Submodel A.

Population

Population in time t is calculated by taking population in t-1, ageing it by one year, adding births and immigrants and substracting deaths and emigrants (all calculations are performed each year by single year age-sex groups). The major exogenous variables are immigration and emigration; the major parameter, fertility. The user may give the value for the long-term equilibrium total fertility rate (i.e. value at 1985) and the total immigration and emigration. Minor parameters are age-sex specific mortality rates, immigration and emigration, and parameters determining age-sex distribution of fertility rate.

Households

The number of households is calculated by applying age-sex specific "headship rates" to 12 population groups. Headship rates for age-sex specific groups are projected into the future as a linear function of time (for a chosen number of years and then held constant).

Labour Force

Participation rates for specific age-sex groups are applied to a source population (total population less armed forces, etc.) Of less importance are participation rates for specific age-sex groups whose default values have been estimated by means of econometric analysis of historical data.

Submodel B1

Model B1 calculates final demand in terms of 105 categories (see Figure 5). The commodity composition of final demand is obtained by the application of fixed converter coefficients to these final demand categories. Special investment projects may be simulated by adding the commodity requirements to other final demands by commodity.

Consumer Expenditures

It is convenient to consider the calculation of total consumer expenditures as an operation distinct from the distribution of the total over the 40 component categories.

Total Consumer Expenditures

The user may specify either the total consumer expenditures or growth rate of total, or the per capita consumer expenditures or growth rate of per capita consumer expenditures. The major exogenous variable is the total consumer expenditure (or its growth rate) and the major parameters are per capita consumer expenditures (or growth rate). If the first option is followed, total consumer expenditures per capita are calculated for use elsewhere in Model B1 by dividing total consumer expenditures by the population from Model A. If the second option is followed, total consumer expenditure is calculated by multiplying per capita consumer expenditures by the population from Model A.

Distribution of Total Consumer Expenditures

The user may chose either the relative share method of distributing total consumer expenditures among the 40 categories, or the method of distributing total consumer expenditures according to an estimated relation between expenditures on categories and persons per household (no further options for consumer expenditures are available if this option is chosen).

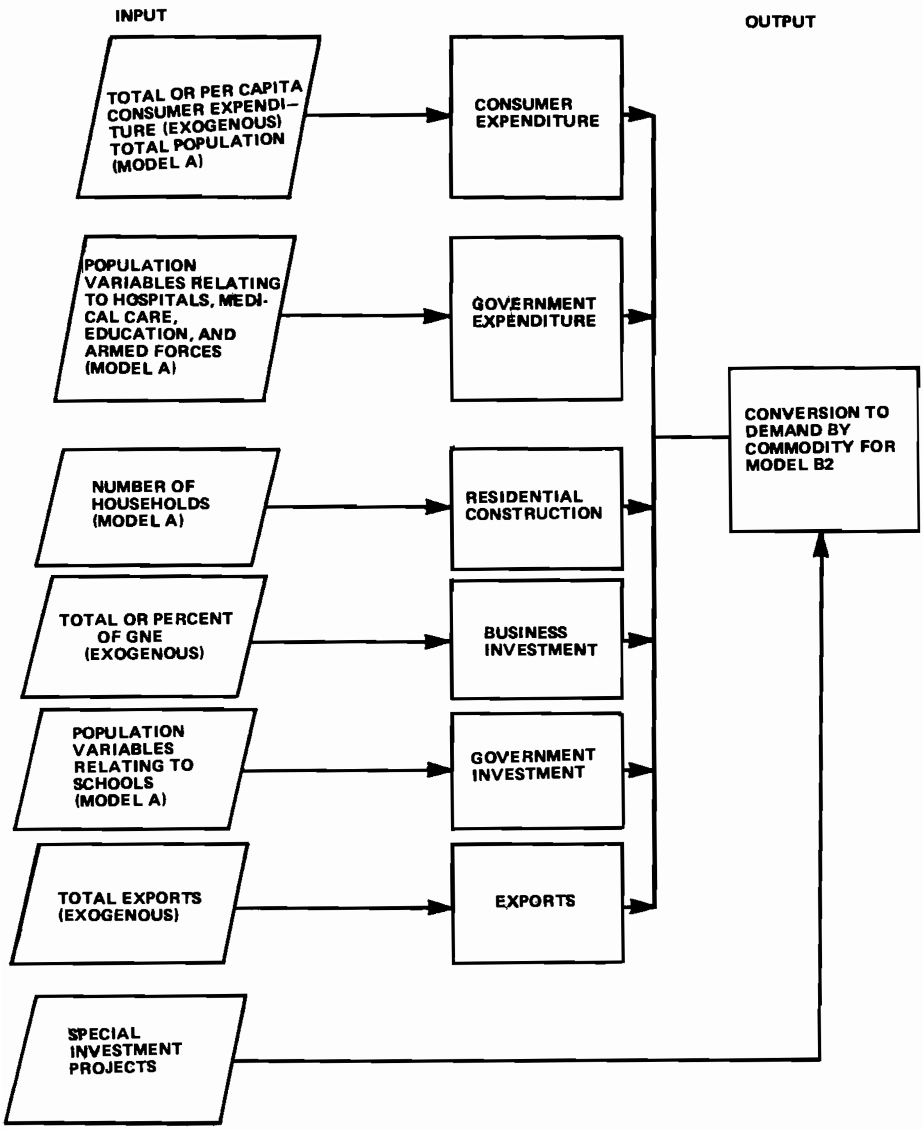

Figure 5. Submodel B1 final demand determination.

If the first option is chosen, the total is distributed among 40 component categories by means of a set of share coefficients that change through time. If the second option is chosen, consumer expenditures by category are determined as a linear function of total consumer expenditures and persons per household estimated over the past. The values of the categories are constrained to add to total consumer expenditures. Users may intervene in the share mechanism by setting values, growth rates, or overriding shares for specific consumer expenditure categories or major groups; then the remaining share coefficients are adjusted to assure additivity to the total.

Minor exogenous variables are consumer expenditures for specified categories or major groups, and minor parameters are the time varying share coefficients whose default values have been estimated by econometric analysis of historical data, or shares for major groups of consumer expenditures.

Government Expenditures

Government expenditure on hospitals, education, defence, medical expenses, and other current expenditures are determined separately; the first four types are related to the appropriately weighted segments of population; the others are related to consumer expenditures per capita. The value added share of each category is estimated separately.

Residential Construction

The number (stocks) of housing units is related to the total number of households determined in Model A; a single multiple distinction is made by using an estimated ratio of single housing units to the total stock of housing; and the additional units are then calculated as the difference between the stocks in t and the stocks in t-1 after allowance is made for retirements from the stock; finally the number of additions is valued in terms of 1961 dollars.

Business Investment

The user may supply either the absolute value of the average annual rate of increase of non-residential construction and of machinery and equipment investment, in which case the major exogenous variables are the value of non-residential construction and of investment in machinery and equipment, or the business investment share of GNE and the ratio of machinery and equipment investment to business construction investment. Here the major parameters are business investment share of GNE and the ratio of machinery and equipment to business construction investment.

Government Investment

Government investment in schools and other buildings, highways, other engineering structures, and machinery and equipment are determined separately.

Exports

The user supplies total exports (or their growth rate). These are distributed among 43 categories of exports by means of a set of time varying coefficients. He may set values or growth rates for specified categories or major groups and the rest of the share coefficients will be adjusted to maintain additivity to the total. The major exogenous variables are total exports (or growth rate of), the minor exogenous variables exports by category or major groups (or growth rate of), and the minor parameters the time varying share coefficients econometrically estimated from historical data or shares for major groups. The user may also intervene in the composition of exports at the level of 680 commodities.

Submodel B2

Model B2 is an input-output transformation of final demands by commodity into industry and commodity outputs that also calculates import requirements. The transformation takes place at the most disaggregated level available--211 industries and 680 commodities (see Figure 6). The user is offered a choice of input-output transformation of either, a fixed import share version,

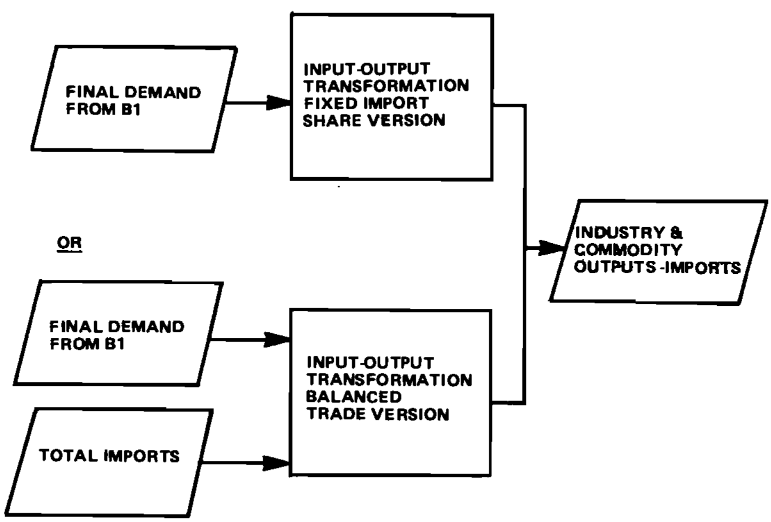

Figure 6. Submodel B2 output determination.

or a variable import share (balance trade) version that takes total imports as an exogenous variable and adjusts the share coefficients in such a way that imports sum to the prespecified total. The major exogenous variable in this case are total imports, and the parameters are market share, import share, and import coefficients estimated from 1961 and 1966 input-output tables (import share coefficient weights if the second option is used).

Submodel B3

Model B3 takes industry and commodity outputs from B2 and calculates labour and capital requirements. It also accumulates the stream of investments into capital stocks and the use of nonrenewable resources (see Figure 7).

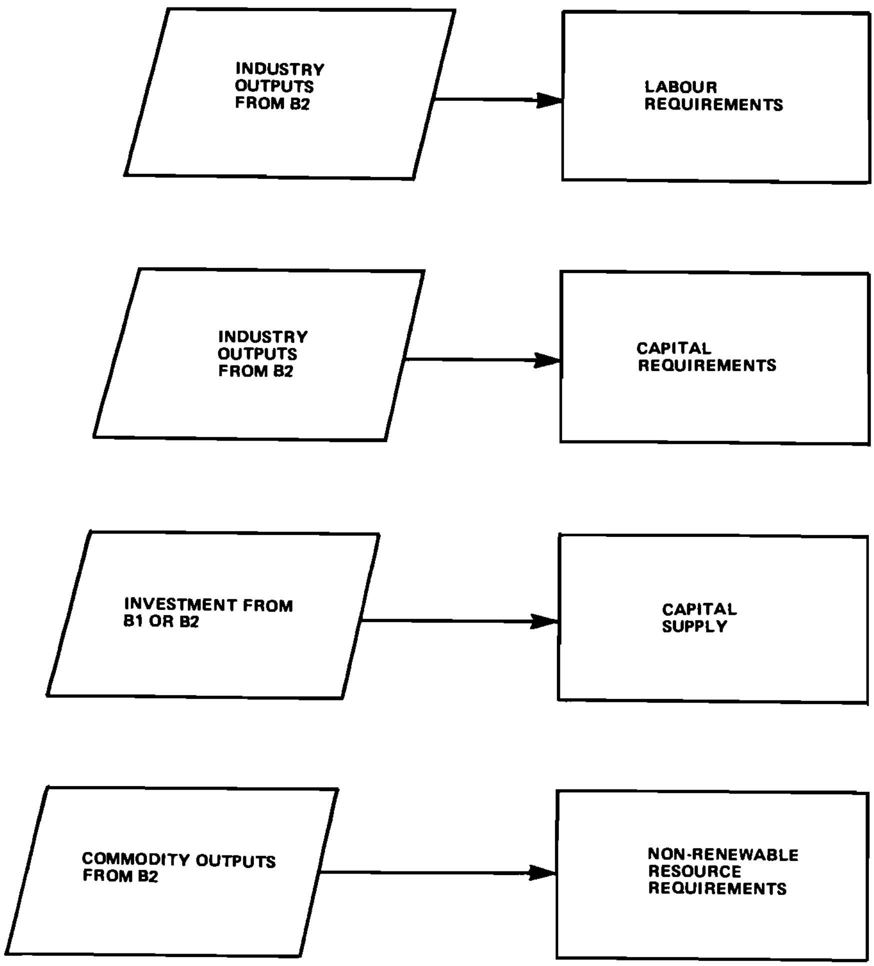

Figure 7. Submodel B3 labour, capital, and resource requirements.

Labour Requirements

These are calculated by multiplying industry outputs from B2 by industry specific time varying labour output ratios. The minor parameters are industry specific time varying labour output ratios and default values estimated by analysis of historical data.

Capital Requirements

These are calculated by multiplying industry outputs from B2 by industry specific time varying capital output ratios. The minor parameters are industry specific time varying capital output ratios and default values estimated by analysis of historical data.

Capital Supply

Capital stocks of structures and machinery and equipment are calculated by updating the stocks from one period to the next with the stream of business investment and allowing for retirements from the stocks according to the economic lives of assets. The minor exogenous variables are the economic lives of assets.

Non-Renewable Resources

Flows of selected non-renewable resources are accumulated. It is necessary to interpolate between simulation years.

Model Software

Figure 8 shows the model software. Execution can be interrupted after Model A and B1 and reports obtained before incurring the expense of the B2 calculations.

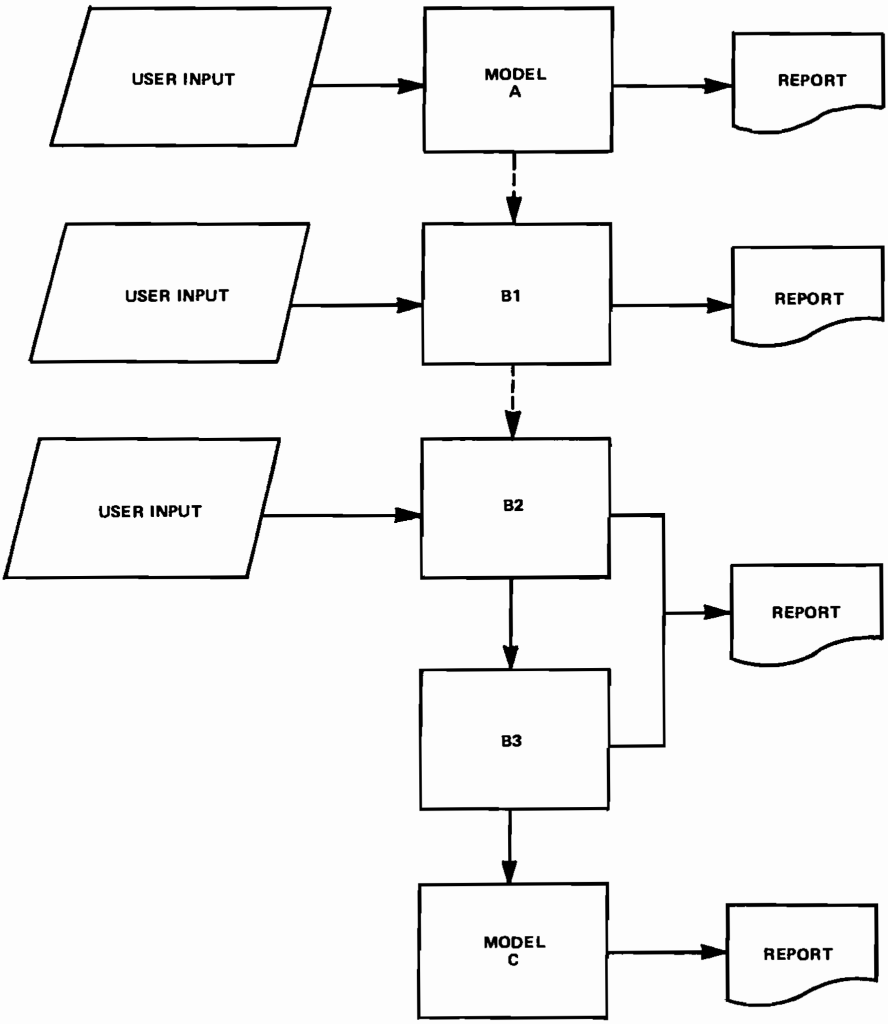

Figure 8. Model software.

THE MODEL-BASE-SYSTEM (MBS) PROJECT— INTENTIONS AND ACTIVITIES

S. Dickhoven

The intention of the MBS project, briefly described in the following pages, is to develop a software system that will support the flexible setup and handling of computer-based planning models of different methodological types. The result of these efforts will be a special "operating system for models" [1].

The project derives from a survey [2] on computer-based planning in the FRG, done by our institute in 1974. The survey showed that there exist many technical problems with models developed by external institutions using various modelling philosophies and different computer and operating system environments. Insofar as many of these technical problems would seem to be avoidable, given modern software and OS-capabilities, we undertook a second survey [3] on existing modelling software, which produced the following conclusions:

- The scene in this software sector is very diverse. This of course is a natural phenomenon that always occurs in any relatively new and rapidly developing area of data processing.

- Most of the analyzed software instruments in this sector make use of what now are somewhat antiquated software techniques--in comparison with those that, using modern operating systems, should currently be possible.

- The development of software instruments in this area normally proceeds in two steps. At the beginning most efforts are spent on increasing methodological support to satisfy the needs of the model builders, while interfaces to facilitate communication and understanding for nontechnical users are often neglected. But after the first versions are in use, and more users who did not participate initially in model development become involved, problems of handling it become more and more important for further development. This second step however is often never fully carried out, because the original software concept does not allow it or because the software developers are the only users of their system, or because they have inadequate resources available to work on these problems.

One of the few exceptions in this field of software systems is the TROLL-System for econometric applications that has been developed at MIT and is now held by the National Bureau of Economic Research (NBER), Cambridge, Mass. Besides rather good methodological facilities in the field of econometrics it supports its user by a wide range of operating system functions (for example: data management, special edit-functions, macro- and default-facilities, monitoring), using rather modern computing techniques.

The MBS, in a comparable manner to the econometric system connected with two or three existing construction-oriented systems such as DYNAMO and MEBA (a German econometric system) and offering interfaces to some data base systems and data analysis packages, will support as far as possible the following groups of modelling activities:

- specification of formal model structure and of structural parameters,
- generation and loading initial data,
- call and processing of models of different types,
- model-linkage,
- adaptation of external models,
- data analysis and report generation, and
- management and documentation of data and models.

In developing such an operating system for simulation models we hope to enable (as far as possible) nonexperts with computers to work with different models and model types in a rather unusual manner.

Assisted by the pilot users of our system, we intend to promote standardization efforts in modelling, e.g., by setting formal model interface standards that are common to a wide range of different model types, and that help facilitate the linkage of different models or at least of all model types, developed for or granted by the MBS pilot users.

Although only a few steps in this direction seem to be possible, we hope to make a little progress; for standardization efforts are a necessary supposition for transfer of model know-how and increase of cooperation between model-builders and users towards better modelling. To support these efforts, we have initiated a discussion circle through a workshop on modelling software [4], in which developers and users of socioeconomic models as well as developers and users of modelling software are engaged in clarifying the picture in this software sector and in promoting cooperation in further software development.

The project that started at the beginning of the current year is now in the software design phase, at the end of which there will be a *User's Guide* issued. This project phase, which should end in the first half of 1977, will address problems of the integration of existing model construction systems, data base systems, and analyzing systems into the MBS or the development of language concepts for both an extended (FORTRAN- or TROLL-similar) system language, to include both a modelling language and statements for system control. After this conceptual phase, a revision phase will take place in which real modelling tasks of different types are simulated to discover weaknesses of the system. The project--including programming, testing and implementation of the first version--is scheduled for completion by the end of 1978.

Further Project Data

- Financial support: German Federal Ministry for Research and Technology.
- Pilot users: German Federal Ministry for Education and Science.
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TOWARDS AN INDICATOR SYSTEM FOR THE RELATIONSHIP BETWEEN POPULATION AND BASIC HUMAN NEEDS

J. M. Richardson, Jr.

This brief paper is a progress report on a project oriented towards the development of an aggregate set of indicators for the relationship between population and the fulfillment of basic human needs. After a review of relevant literature, a prototypical index, focusing on population, employment, urban development, health, and food has been developed. Eight countries have been selected for the purpose of making a preliminary assessment and providing a basis for refinement and modification of the index.

RATIONALE

In 1974, the closely intertwined issues of population and food were addressed by the nations of the world at international conferences in Bucharest and Rome. A third issue, habitation, was examined at the Vancouver Conference in May-June 1976. In June the International Labour Organisation held a special intergovernmental conference on unemployment. Underlying all of these gatherings of our diverse and fractious global community is a common theme: recognition that fulfillment of certain fundamental human needs must assume much greater importance as a goal of national and international policy.

It is apparent that issues related to population, food, employment, health, and habitat will receive increasing attention in international forums during the next quarter century. Perhaps most influential in presenting this view have been several studies sponsored by the Club of Rome that have forcefully argued that continued growth in population, profligate attitudes toward resources, and narrowly focused national policies will lead to fundamental and disastrous changes in the human condition.

If fulfillment of basic human needs is to become a more important and explicit goal of national and international policy, the development of comparable, recognized, and widely accessible measures of performance relevant to this goal is essential. There is not now in existence even an adequate data base which would support such a development. While segments of the total issue are under active study, there exists no ongoing effort to develop an aggregate indicator, supported by an appropriate data base that could focus national and international attention on the total problem of fulfilling basic human needs. To address this critically important area of national and international policy, the project to develop a composite index of population and the satisfaction of basic human needs has been established as a joint undertaking of the World Population Society and The American University's Center for Technology and Administration. The basic conception of the project, as well as its underlying rationale, were first presented at the International Population Conference "Since Bucharest--and the Future" of the World Population Society, held in Washington, D.C. in November 1975.

DEVELOPMENT OF A PROTOTYPICAL INDEX

Development of a prototypical index has been the project's first objective. The basic development strategy is depicted in Table 1.

Table 1. Developmental strategy.

Literature search and review

- History of major indexes in wide use
- Theory of index number construction
- General Studies and surveys of index number construction

Identification of significant factors

Formulation of prototypical composite index

Selection of a spectrum of nations for analysis

Preliminary data base development

Calculation of preliminary results using prototypical composite index

Modification of composite index based on preliminary results

Review of the relevant literature was focused on the history of major indexes in use, the theory of index number construction, and general studies of index number construction. A study by the National Academy of Sciences *Planning for Environmental Indices*, February 1975, has proved to be particularly helpful. Seven major functions of indexes were identified in this report (Table 2). This enumeration of functions emphasizes the symbiosis of science, art, and public relations which characterizes a good index. Such a symbiosis is much easier to describe than to achieve.

Table 2. Functions of a good index.

Enhance communication

Highlight significant conditions

Illustrate major trends

Inform both professionals and laymen about the status of a particular issue

Assist in formulating and measuring progress towards goals

Facilitate the judgment and decision-making process

Source: *Planning for Environmental Indices*, National Academy of Sciences, Washington, 1975

Structuring a complex phenomenon in terms of a manageable number of well-defined issue areas is an extremely difficult task. Many hours of discussion were required to achieve a reasonable, workable compromise between desirable and achievable objectives.

We have chosen to focus on the relationship between basic human needs and the capacity to meet basic human needs in the following four areas:

- health,
- housing,
- medical services, and
- employment.

A somewhat more complex factor, urban complexity and the capacity to manage urban complexity is incorporated to recognize the fact that meeting human needs is often vastly more difficult in urban settings. Our choices reflect a decision to focus only on the most basic material human needs. In particular, the perplexing areas of environmental and psychic needs have been excluded. It is hoped that this focus will simplify measurement problems and lead to a greater degree of consensus regarding the factual basis for index values. Our preliminary investigations demonstrated that reasonably adequate, comparable data will be difficult enough to gather even given this very simple conception of basic human needs.

The basic structure of the prototypical index is illustrated in Figure 1. Prototypical subindexes measure the relation between human needs and the capacity to meet those needs. Agregate performance is weighted by a prototypical index of a nation's capacity to manage existing levels of urban complexity. A more mathematical description has not been included, because of space limitations, but may be obtained by contacting the author.

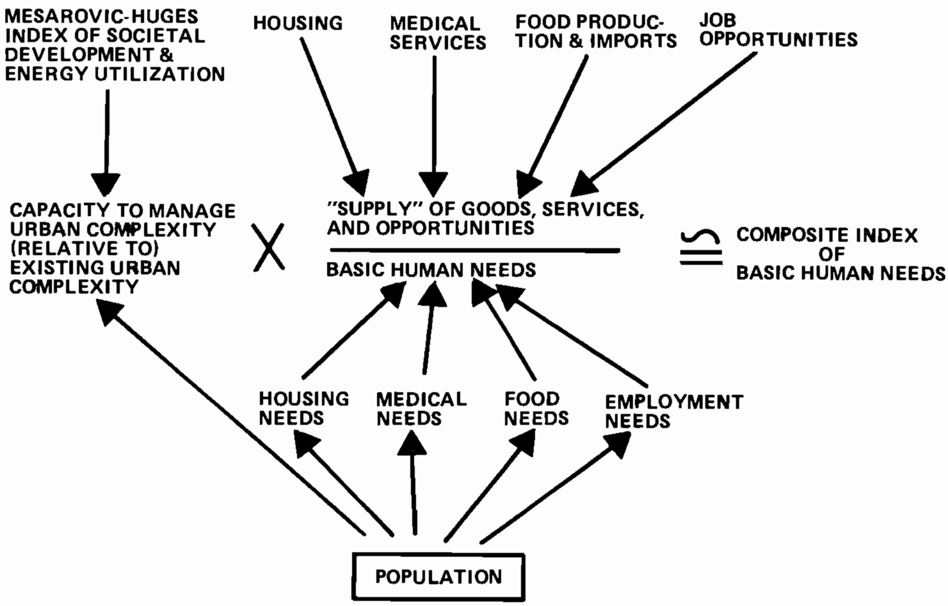

Figure 1. The composite index.

Seven countries have been selected for the purpose of developing a data base for preliminary investigation and refinement of the index. The countries are Argentina, Bangladesh, Brazil, The Netherlands, Ghana, Japan, and the USA. Puerto Rico, although not an independent nation, will also be examined. Data collection has proved to be somewhat more difficult than anticipated (even after extensive experience with global data). However preliminary results, for at least some nations should be forthcoming in the near future.

EUROPE PLUS THIRTY—A FACTUAL NOTE

Lord Kennet

The Europe Plus Thirty project took place between June 1974 and September 1975. In September 1975, it submitted a report in answer to the questions which had been asked of it by the Council of Ministers of the European Community. These questions were: should the EC have a long-term multi-sectoral forecasting capacity? If so, how should it work and who should it report to? Should there be a European Office of Technology Assessment?

The answers were: Yes, there should be a forecasting capacity; it should be between 30 and 50 people, and should use existing institutions where convenient. Modelling was regarded as one implement in the forecaster's battery. There should not be a European Office of Technology Assessment, but the continuing forecasting unit should do some technology assessment. It should report equally to the European Commission and the European Parliament.

The next move must be a proposal for action from the Commission to the Council of Ministers. The present Commission retires in its entirety at the end of this year, and it is expected that the proposal will be made by the new Commission which comes in on 1 January, 1977.

The report will be published by the Cambridge University Press for the European Commission, under the title of *The Futures of Europe*, on 4 November, 1976. This edition will be in English. Editions in other Community languages are to follow.

MECHANISMS OF SOCIOPSYCHOLOGICAL DEVELOPMENT

J. Millendorfer

INTRODUCTION AND ACKNOWLEDGMENTS

The so-called soft variables have been gaining importance in discussions on the future. The importance of the question of values, motivations, and ways of acting has been emphasized by all--even by those who develop mathematical models on the basis of hard variables and who dispense with soft variables because they are difficult to quantify. This paper is to be understood as an attempt to carry out hard observations of soft variables, using a comparison by countries of variables that are understood to be indicators for these soft variables. In addition to my own experience, which I collected with Dr. C. Gaspari in a comparison by countries of hard variables, essential prerequisites for this paper were the cooperation with psychologists of the Study Group of International Analyses, in particular (Mrs.) Dr. A. Fuchs, and the discussion with experts in psychology and social psychology, in particular with Professor V. Frankl, Professor E. Ringl and Professor H. Strotzka. May I herewith express my gratitude for the cooperation, for the valuable comments and for the fertile critique.

EARLY HISTORY OF THE WORK AND STATEMENT OF THE PROBLEM

While investigating long-term societal development processes [1], in particular those of economic development, a residual factor was enunciated in connection with the developed, so-called general production function [2]; this residual factor describes the efficiency of the use of the production factors and it has five different values in the five different greater zones of the world. Culturally determined values, motivations, and pattern of behavior were assumed to be the explanation for this residual factor. It was shown during a further investigation of this problem that the effects of these soft variables were not confined to the production region, but that they also touch on the life-region: the emphasis moved then to the latter one. I have chosen out of many observations of relationships in this region to illustrate the problem (Figure 1). One can see that the suicides in the countries of origin and the suicides of US immigrants coming from those countries show a highly positive rank-correlation. So the following questions can be raised: Firstly, are there mentality differences in the various countries, that determine--at least in the first generation--certain patterns of behavior of the immigrants originating from those countries;

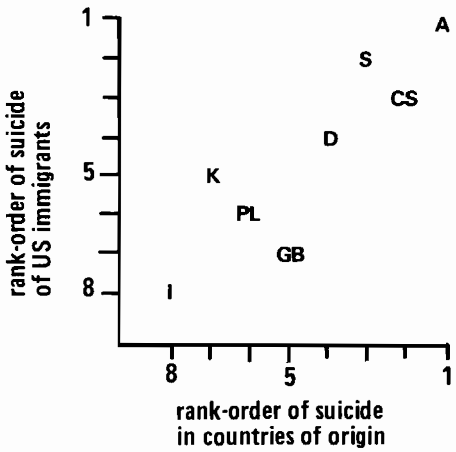

Figure 1. Comparison of the number of suicides of US immigrants from different countries and the number of suicides in the countries of origin.

secondly, are there indicators for such pattern of behavior; and thirdly, are there sociopsychological mechanisms that can be described by these indicators.

During the empirical investigation of these questions on the basis of heuristic hypotheses stimulated by already existing theories (e.g. "Praesuicidales Syndrom" of E. Ringel [3]) the relations shown in Figure 2a between suicides and divorces emerged showing a very stable correlation: it appears in all cross sections all the way up to the beginning of the 20th century. Observations on the correlation between psychoses and car accidents (Figure 2b) agree with the opinion of traffic psychologists that psychologically linked human failure is a main cause of accidents. A cross-check, namely the empirical examination of the hypothesis that traffic accidents depend on traffic density, did not show any significant correlation. Starting from these single observations, stimulated and interpretable by the theory, one can raise the question of whether these observations are to be viewed within a greater framework of relations.

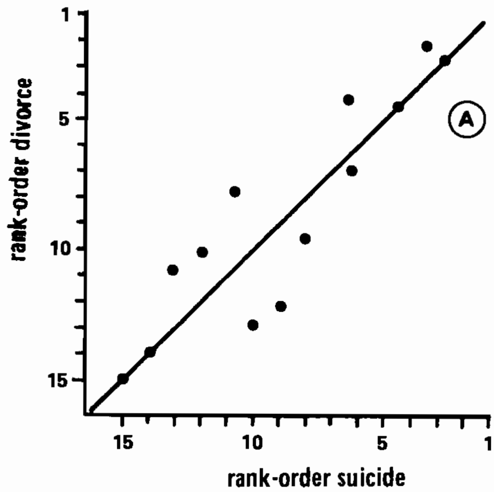

Figure 2a: Correlation between suicides and divorces in European countries.

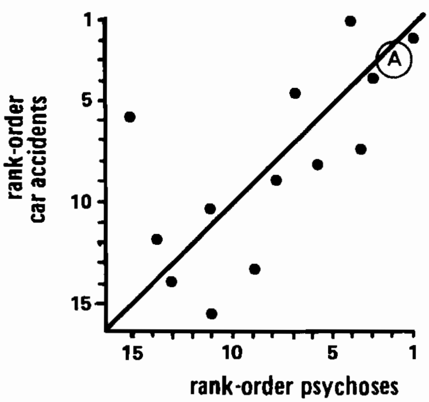

Figure 2b: Correlation between psychoses and car accidents.

THE METHOD OF INVESTIGATION

The important question of the method of investigation can be dealt with only briefly: It is based on a process of model development, described in [4]. Starting from a first observation or hypothesis on the basis of a "pragmatic decision" [5], a preliminary model is developed. The predictions of this first model, are checked empirically; this examination yields additional information and leads to improvement of the model, which is followed by a third such step, etc. until a comparison of the model with reality shows no necessity or possibility of a correction or improvement of the model. A peculiarity of the method used in this work is the understanding of certain variables as indicators for more global variables hidden behind them, as shown in Figure 3.

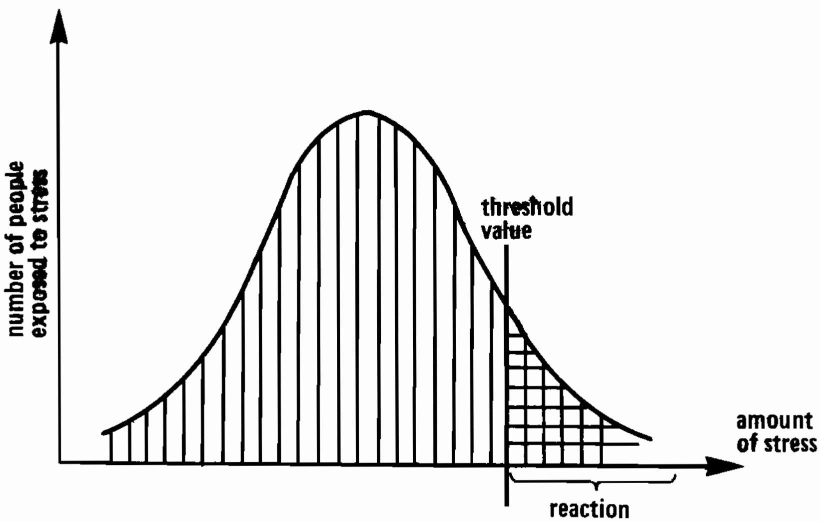

Figure 3. The extreme case of a variable as an indicator for the total extent. The intensity of a certain variable (e.g. the intensity of a certain sociopsychological stress) is plotted against the number of persons subject to this variable.

From a certain threshold value of stress on, extreme behavior can be observed. Keeping the threshold value constant, the extent of extreme behavior can be interpreted as a measure of the global variable. The method is sensitive to disturbances of the threshold value (e.g. to the change of provisions concerning divorces, if one takes divorces as a measure of the extent of troubled marriages), but it yields stable results, if the threshold value is kept constant.

ADDITIONAL OBSERVATIONS

If one correlates the four variables of Figures 2a and 2b in a different way, the pattern shown in Figure 4 results.

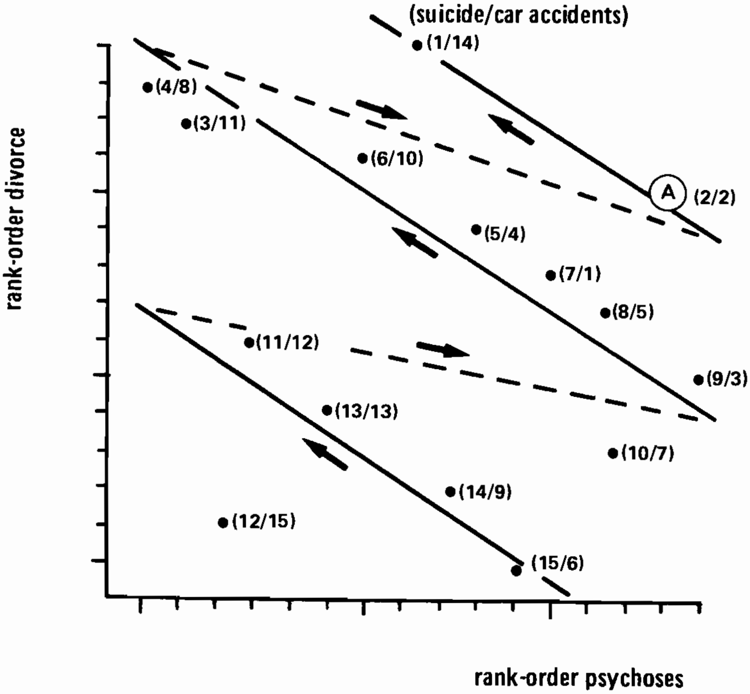

Figure 4. Comparison of divorces and psychoses, and suicides and car accidents. The numbers in parentheses behind the country symbols: first number, rank-order of suicides; second number, rank-order of car accidents. High frequencies correspond to rank-number 1, low frequencies to rank number 15.

At first sight, no correlation can be recognized; but if we follow the numbers assigned to the rank-order of suicides, we can see that, with few exceptions, they decrease along the lines in the direction of the arrows, i.e., that the suicides increase in this direction. If we look at the traffic accidents, we can recognize principally the opposite trend. The sum of suicides and car accidents results in each case along a full line in a certain level--highest along the line in the right upper corner and lowest along the line in the left lower corner (Figure 5). We can interpret this line of constant sum of suicides and car accidents rates as a stress-level. At the highest stress level the number of divorce rates and suicide rates on one hand and of psychoses and traffic accidents on the other hand are highest. On lower levels the data are lower. There is--formally identical with the constant sum of suicides and car accidents and the increase of suicide in direction of the arrows--a substitution of car accidents by suicides along the full lines in the direction of the arrows and vice versa. Going from one level to another, there is--if one follows the rank orders of suicides--in each case a jump from the left end of one level to the right end of the other.

Before interpreting this correlation, let me represent the four variables in a different way (Figures 5 and 6).

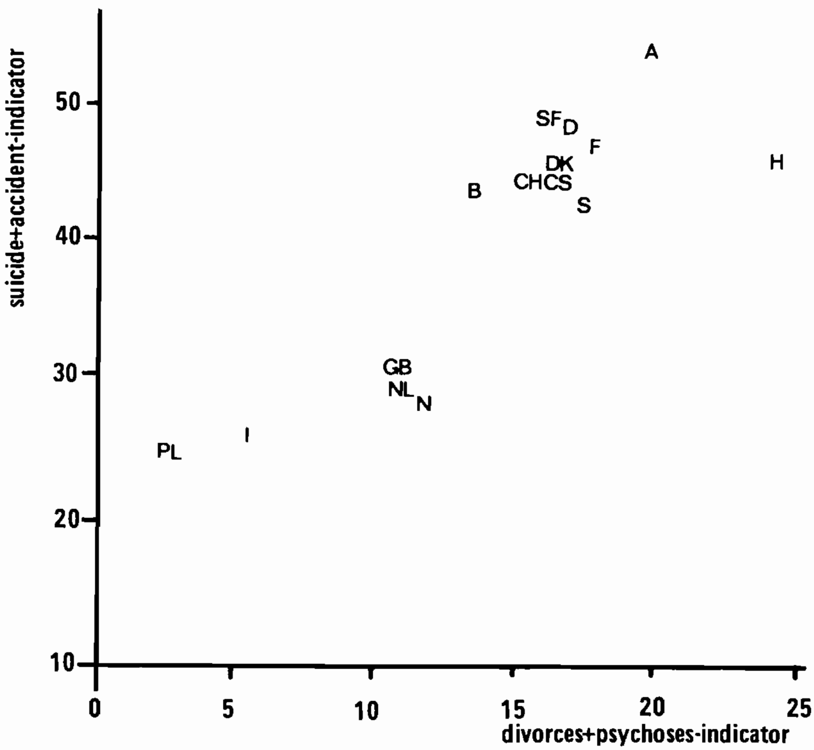

Figure 5. Comparison of a weighted sum of suicides and an index of accidents, and a weighted sum of divorces and an index of psychoses.

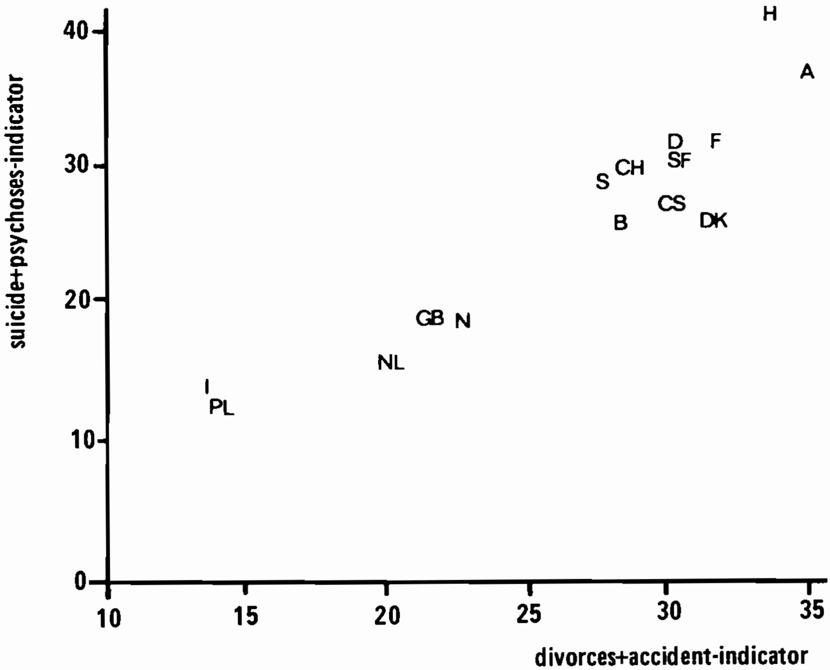

Figure 6. Comparison of a wieghted sum of suicides and an index of psychoses, and a weighted sum of divorces and an index of accidents.

The weighting factors of the sums were determined, starting with the parameters that result from the simple regressions represented in Figure 2 based on the idea of a projection of the observed points onto a plane perpendicular to the regression line, such that the sum of the squares of the distances between the points was a minimum. The method leads in both cases to the same clusters of countries. These clusters again correspond to the stress lines of Figure 4 (excepting Italy and Poland, for which according to the cluster method a fourth, very low stress level resulted). Figures 4 to 6 show the same result in that there are clearly distinguishable groups of countries that emerge during the observation of the correlations of our four variables for the year 1966: there are empty spaces between these groups of countries. One can raise the question of whether the relation of the variables in these groups of countries describes something like a stable system status and that the spaces between the variables something like an unstable system status, correspondingly not occupied.

A FIRST CONTENT ORIENTED INTERPRETATION

The probability for suicides to occur rises with every stress level: this is accompanied by an increase in divorces and psychoses. If we interpret divorces as an indicator for disturbed human relationship and psychoses as an indicator for loss of realitis, a connection can be made between our results and Ringel's findings, namely that the probability of suicide depends on the amount of self-aggression in an individual, caused by disturbed human relationships and loss of realitis. We can interpret the substitutions between suicides and car accidents, if we consider traffic accidents as an indicator of external aggression (against others) and suicide as an indicator of internal or self-aggression, as a substitution between external and internal aggression at every level.

Observing one stress level, we can see that the indicator of external aggression decreases as soon as the indicator of internal aggression increases. This process develops up to a certain point, whereupon the one system becomes unstable and changes over to another more stable state. The instability occurs in subsystems, namely in the small groups based on close face-to-face human relations. These subsystems become stabilized by diminishing the communication to outside namely by worsening the loose human relations and by suddenly increasing external aggression. This change shows up in the diagram as a jump from one level to another.

The same process starts again at the next level. The loose human relations measured by traffic accidents become better, the close relations of the partners measured by divorce rate become worse. Since the level as such is higher, the entire situation is worse. If we propose that the human being needs close human relations and that he needs small groups in a dynamic sense, we can postulate that when the group relations, for instance in a family, become worse, the entire system becomes unstable. This system gains stability by isolation against external influences through worsening the loose human relations and the loss of sense of reality, shown by the rise of psychoses. The disturbance of loose relations shows up as a rise in traffic accidents.

The process begins in Europe by a slow breaking down of human relations in the system of marriage. This system becomes more and more unstable, and the resulting jump leading to a higher level only worsens the situation. Although it seems that the jump stabilizes the close human relations by isolating the individual, both the close and the loose human relations become worse. Thus an iterative process of breaking down of close and loose human relations emerges; it leads the individual to a situation that makes a suicide possible and ever more probable.

Many more relations fit the pattern of our observations and forms the self-consistent theoretical framework that serves as the sole background for the interpretation of our hypotheses.

A check of this interpretation using the path analysis from the model depicted in Figure 7 yielded a positive result.

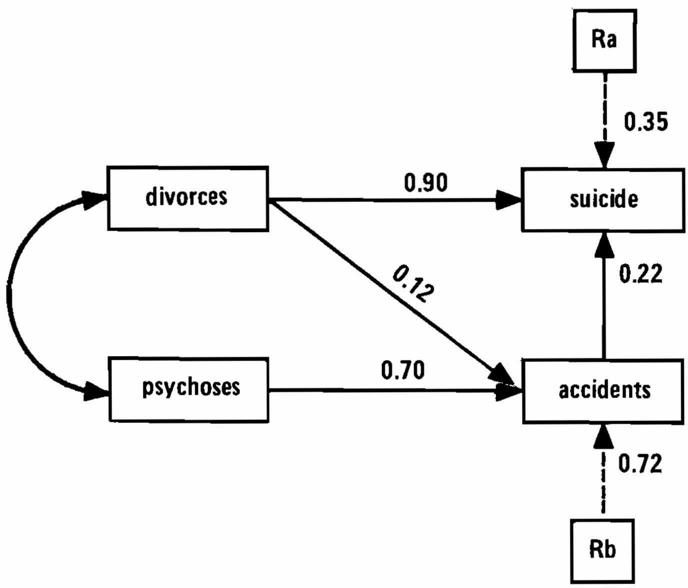

Figure 7. Model of causal relationships tested by path analysis.

A Formal Completion of the First Interpretation

If we consider "close human relations" as relations within a group, then we can conceive of their improvement as an intensification of the bonds in the group. If we look at the intensification of the bonds in a subsystem as an "organization" and at the relation of this subsystem to other subsystems and higher-placed levels as an "integration", then improved "loose human relations" mean an improved integration. The decay of a system through disorganization of subsystems and their disintegration in the global system goes in the opposite direction. Looked at this way, the described mechanism is an interaction between disorganization and disintegration. The disorganization leads to a status unstable for the subsystem, and this status is transformed through a disintegration of the global system into a status more stable for the subsystem. A further disorganization leads again to the unstable status, which in its turn leads to a jump into a further disintegration. Vice versa, the bonds in the system would have to be strengthened by an interaction of organization and integration. A formal task resulting from these considerations would be an examination of the correlations between partial stabilities (in the sense of stability of subsystems) and total stabilities of complex systems.

ADDITIONAL OBSERVATIONS

The variable psychoses and psychoneuroses shows a high negative correlation with age-corrected mortality for cervical cancer in a comparison by countries (Figure 8). This observation is in no way surprising: On the one hand, based on the observation that cervical cancer has never been observed in

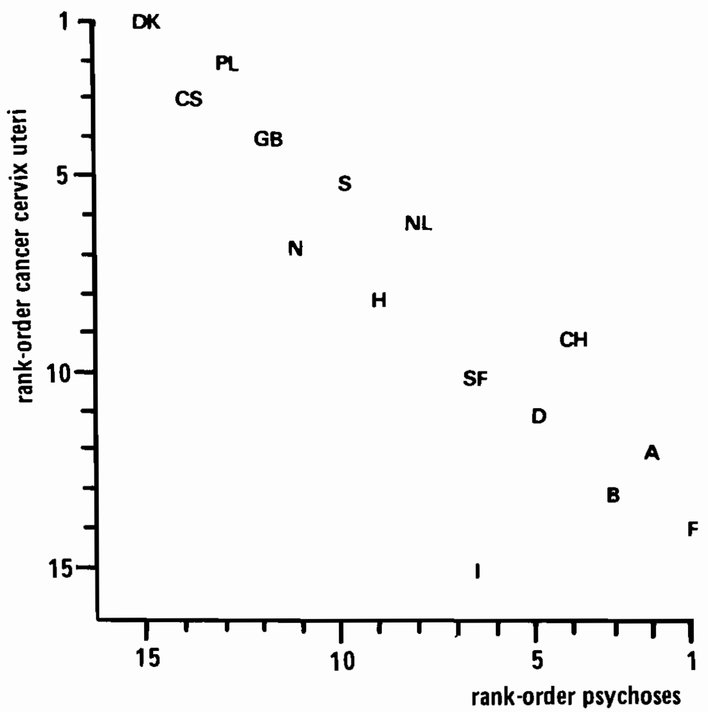

Figure 8. Comparison of rank-orders of psychoses and psychoneuroses (age-corrected mortality rate for cases of death in the international classification A67 and A68) and cancer cervix uteri (age-corrected mortality rate).

nuns, whereas it has often been observed in prostitutes (e.g. in the town of Laredo, situated at the Mexican border and renowned for that), there has for a long time been the conjecture that cervical cancer has to do with sexuality and promiscuity. These conjectures are supported by new observations to be described later on. On the other hand, S. Freud has claimed that sexuality that has not been lived out could be the cause of psychoses. Thus Figure 8 would correspond-- assuming that the above mentioned conjecture applies-- to Freud's theory.

Investigations of E.B. Lucky and G.D. Nass [6] and of H.C. Christenson [7] show that there are great differences in attitude towards and behavior relating to sexuality in the countries Denmark, Sweden, Belgium, the UK, Germany, Norway, Canada, and the USA. A rank-order of these countries corresponding to higher promiscuity, according to polls, corresponds exactly to the rank-order of European countries corresponding to cervical cancer (rank-order correlation 1). Moreover, recent results of cancer research give strong support to the conjecture that cervical cancer has to do with sexuality.

> Papers given by various epidemiologists at the recent Florida conference on the epidemiology of cervical carcinoma emphasized that some relationships are based on solid facts, whereas others are essentially legends and myths. There was a clear relationship between the risk of cervical carcinoma and the age of the first intercourse, and also the number of sexual partners. There was no significant correlation with the number of pregnancies or the circumcision status of the male partner. An important piece of information concerned the contraceptive device: obstructive contraception was clearly of preventive value. The conclusion was drawn that a carcinogenic agent is transmitted from the male to the female during adolescent intercourse that contributes to the risk of cervical carcinoma [8].

In a discussion the psychosomatists and cancer researchers have shown an identical point of view. Cervical cancer can thus be used as an indicator for sexuality in comparisons of countries.

A SECOND INTERPRETATION

Inserting the sexual indicator into Figure 4, instead of psychoses, results in the same pattern, only with opposite signs (Figure 9). The number of suicides increases with increasing sexuality until it reaches an unstable point, from which (with a further increasing number of suicides) the jump

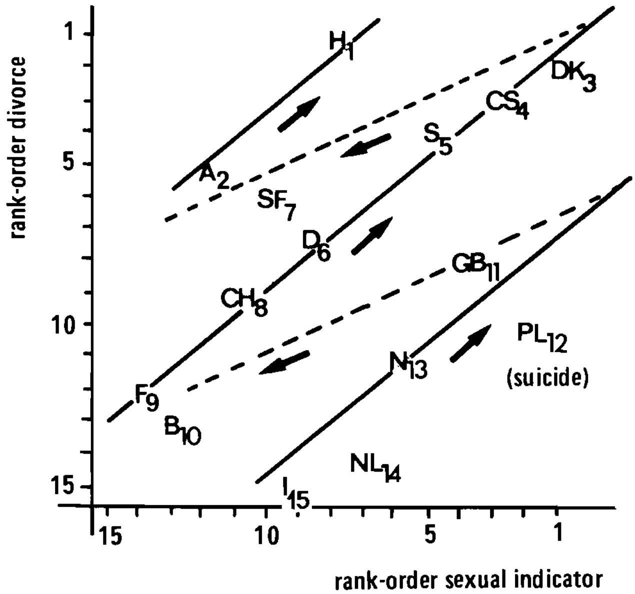

Figure 9. Modified Figure 4, abscissa: rank-order of the sexual indicator (instead of psychoses); ordinate: rank-order of divorces. The number after each country symbol is the rank-order of suicide.

to the next-higher stress level takes place; the jump is accompanied by a decrease in sexuality and an increase of psychoses (Figure 8). After a new increase of sexuality the number of suicides increases again, until it reaches the next unstable point, etc.

It is of interest that family quality and sexuality show a negative correlation along the level lines. Particular consideration should be given to the fact that the correlations between the variables along the level lines and those from a level line to another one are of opposite sign. This means that opposite points of view must be adopted if one reasons along the level lines or if one considers the relations between the level lines. Perhaps this could explain the contrary nature of discussions about sexuality.

A path analysis of the second interpretation failed at first due to purely formal difficulties caused by an extremely high correlation ("multi-colinearity") of both variables, psychosis and cervical cancer (as sexual indicator), which were looked at as causal variables. Fortunately, it appears that the causal interpretation of the correlation between these variables has been confirmed by a connection of the latest results of cancer research and sexology as well as by an old psychoanalytical theory based on Sigmund Freud's findings.

EXAMPLE OF APPLICATION, SUMMARY, AND PROSPECTS

An indicator for family quality was developed during a series of family surveys on the basis of comparison by countries. If one inserts the rank-orders of the countries according to this indicator into Figure 4, a worsening of family quality can be observed in the direction of the arrows, along the full lines, whereas an improvement takes place in the direction of the arrows along the broken lines. This can be considered as a supporting argument for the interpretation elucidated earlier. Additional information from the results of sexology research--inserted into Figure 9--shows that the so-called single-concept (as opposed to the double-concept) becomes more pronounced in the direction of the arrows. Thus the mechanism described could be conceived of as a process leading towards a single concept through alternating disorganization and disintegration. At present investigations are being carried out to confirm and modify the correlations described into a larger framework of relations. The iterative process of derivation of improved models for hard observations on soft variables goes on.

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DISCUSSION

M'Pherson agreed that it is very important to have the production functions express explicitly also the quality of management, the quality of labor, the efficiency of a society in turning its resources into production, etc. He wondered, though, to what extent statistics on suicides, divorces, car accidents, etc., can express quality of life--a society with no divorces at all may not be a happy society, it may just be operating under a different culture. McLean pointed out that low efficiency coefficients of a society may simply reflect a conscious societal choice to have a better life rather than economic efficiency.

Burke stressed that, in pursuing this approach further, one should increasingly be preoccupied with nonevents as a yardstick for quality of life than with events--the nonbirth of babies, nonstrikes, etc. In reply, Millendorfer pointed out that many more variables were used than those he had focused on. Furthermore, he wished to distinguish carefully between variables one is really interested in and variables used as indicators for them. Thirdly, he wished to stress that by no means all of STUDIA's work is dealing with soft variables; it seemed important to him, however, in a conference of this kind to lay particular emphasis on these variables, as in practical work, they are often badly neglected.

GENERAL EVALUATION

SUMMARY

K. S. Parikh

After nearly four days of discussion of various technical points, I feel that in a short summary one should forget the various technicalities. I shall therefore only give a broad summary of what I felt was the major thing I discovered in the models presented here.

First SARUM: I liked the fact that, in SARUM, an attempt is made to estimate the various relationships econometrically rather than to describe them as if one were sitting under a tree with divine wisdom. Maybe this was necessary in earlier efforts, but as we grow more experienced in global modelling it is better to really try to come to grips with reality as far as possible. I found the fact that this is not a conclusion in search of a model to be something quite admirable. In some of the earlier efforts I felt that they were models of conclusions in search of models. After saying this I would like to point out one thing to Mr. Roberts. I am not so optimistic about the use of such models or of the effectiveness of specialists in affecting policies as he seemed to be in his concluding observations.

In India we have a custom of consulting astrologers. When some match has been proposed, one consults an astrologer who compares the horoscopes of the potential bride and the bridegroom. Normally, what one does is to look around till one finds an astrologer who says "go ahead, this is a good match". On the other hand if you do not want to accept the match, you find someone who says, "Oh, this is terrible". This is the kind of behavior politicians also have. They shop around till they find a specialist who tells them what they really want to hear. Although I think it is worthwhile to develop models, one should be a bit more realistic about the potential usefulness of models for policymaking.

Now to the normative model of planning. I am really impressed by it, particularly when I compare it with the models of planning used in India. For example, let me just describe briefly the characteristic of the planning model used for deciding the fifth five-year plan in India. It was a static target year resource allocation model in which the levels of trade were prescribed externally. Also there was no internal income generation in the model and there was no adjustment of consumption patterns to prices. In all these senses the MRI is an improvement. It is a dynamic model and in it consumption is internally determined. It responds to relative price changes and also the subsequent version endogenizes the decisions on fertility and population as well. I am impressed. I think it will be very useful when fully developed.

AN APPRAISAL OF GLOBAL MODELLING ACTIVITIES

J. M. Richardson, Jr.

In my assessment of the Conference, I wish to return to a theme that I have emphasized throughout the four days we have been together. The theme is the usefulness of models to decision makers. It is significant to note that this theme has received a similar emphasis in discussions of the future of IIASA by Professor Gvishiani, the retiring director, Dr. Raiffa, and the new director, Dr. Levien. These statements are all found in IIASA's 1975 *Annual Report*.

In his "preface" to the report: Professor Gvishiani emphasizes that

> ...in stressing the applied aspect of our methodologies we must look still harder for productive links with the decision makers in gaining and justifying their confidence. IIASA's role in doing such a service to science will be profoundly appreciated by researchers and decision makers and will bring us new allies and supporters.

In his speech at the November council meeting, 1975, Professor Raiffa emphasized a similar theme:

> IIASA will in the long run, have to justify its existence by making *substantial contributions to real problems*. [emphasis added] We will judge our leadership on how well it seizes the opportunities that arise. Our future leaders must dare to take on those risky problems where the greatest potential benefits lie.
>
> Let us be courageous, let us believe in ourselves, and let us create an institution that is relevant for the precarious world we live in.

It is clear from the "director's message" for 1976 that substantial contributions to real problems will be a continued area of major, even increased, emphasis at IIASA. Dr. Levien has said that

> ...it has been desirable to interlink the efforts we have begun and *focus on those topics that are likely to have a strong impact on matters of importance*. [emphasis added]

The field of global modelling is in its very early stages. To borrow a phrase from Thomas Kuhn, it does not yet have a well defined paradigm. As scientists then, it is proper for us to be concerned with those issues of theory and methodology that will, as they evolve, define the boundaries of this new field. But our concern with issues pertaining to science as science must not direct us away from the major objective of global modelling activities: to significantly contribute to an understanding of critical problems now facing mankind. Accordingly, it is essential in my judgment that we pose to all global modelling efforts, the three questions that I have posed on several occasions over the past four days:

- What critical problems of substance have been identified by this modelling activity?

- What, if anything, do the models have to offer in the way of solutions to these problems?

- What degree of confidence can be placed in these solutions?

As scientists, we are properly aware of the severe limitations of our methods. This is nowhere more true than in the field of global modelling. Yet we should be reminded of the observation of Professor Pestel, expressed two years ago at the first conference on global modelling.

> We strongly resent the suggestion, frequently made, that scientists should not publish statements and recommendations regarding situations in which there are elements of judgement or uncertainty and for which "scientific tests" cannot be conducted. This would effectively eliminate scientists from any consideration and discussion of mankind's long-term future, where such a degree of certainty is just not attainable. Scientists could thus be unduly penalized and forced to leave the arena of public discussion to those who have even less information about the world's future course.

Before closing, it seems appropriate to ask whether IIASA should sponsor a fifth global modelling conference. If so, one possibility, would be to use a similar form to the one used successfully in this conference and its predecessors. However in urging that IIASA and its member organizations do sponsor a fifth global modelling conference, I should like to urge a somewhat different focus. By September of 1977, four years will have elapsed since the publication of *Limits to Growth*. Three years will have passed since the first results of the Mesarovic-Pestel work were presented at IIASA. The global modelling effort headed by Herrera and Lindemann will also have been on-going for a substantial period of time as will those which we have had presented to us today. I believe it would be appropriate, at a fifth global conference to assess the future of this field by devoting some attention to its brief, but already rich past. The conference would address itself to the issues of outcome, process, and implementation.

The first part of the conference would be devoted to a careful and critical review of previously initiated global modelling activities, hopefully by the leaders of the major groups. Each group would be asked to respond to the following questions.

- What is the present state of the modelling activities of this group? What refinements of the model have been made since it was first presented? What steps have been taken, through validation and sensitivity analysis, to improve the confidence of decision makers in the results of the model?

- What is the state of the data base and documentation supporting the model? What steps have been taken to resolve the problems of data availability noted when the model was first presented? Has there been an attempt to focus on those data problems that careful sensitivity analysis suggested to be of particularly critical importance?

- What has been the impact of the modelling efforts of this group? In particular, what changes in policies or in the policy making process can either directly or indirectly be linked to this modelling effort? Second, what, if any, changes in the world have occurred as a result of this activity?

The second section of the conference would examine the process of organizing interdisciplinary global modelling efforts, presenting results, and interfacing with decision makers. Each of the major global modelling groups would be asked to discuss the way in which their research was organized, and the way in which resources were allocated to developing the model, collecting data, negotiating with decision makers and the scientific community for acceptance of the results and publicizing the results. In every global model effort, difficult and critically important judgments have had to be made in each of these areas. "Success" of the modelling effort has been determined as much by these decisions as by the content of the model. Obviously, some of these issues are extremely sensitive. However they are also critically important. It is to be hoped that IIASA could provide a forum in which they could be discussed candidly and openly. Such discussions could perform an invaluable service for the international community of scientists as it becomes increasingly involved in the process of policy making.

The third section of the conference would be devoted to a discussion of other models that have actually been implemented and had an impact on the policy-making process. A number of such models exist, both in government and industry. Emphasis would be on concrete, observable results and the way in which these results had been achieved.

I hope that the organization of a fifth global modelling conference will receive serious consideration. There are few activities, in my judgment, that are more central to the ideas that motivated the creation of IIASA. I share and warmly endorse Professor Raiffa's validictory remarks which are as appropriate now as when he offered them:

> I believe IIASA has a mission. It must keep its doors open so that scientists who have a broad vision of tomorrow's world can communicate with each other. ...We must learn today how we can jointly grapple with some real problems so that we shall be better able to grapple with the strain of equally devastating problems that will surely shape our societies in the not distant future.

CONCLUDING REMARKS

G. Bruckmann

After the flattering remarks by John Richardson, I feel obliged not to let you all leave before having had to listen to a final word from my side, too. Let me thank you, both in an orthodox and in an unorthodox fashion.

First to the orthodox fashion. My first word of thanks goes to the major modelling groups; first of all for having come and presented the models at all, which is not so self-explanatory because all of them are involved very greatly in their work, and to tear a whole fine group away from their work for an extended period of time causes troubles well-known to us all. Second, for having really exceeded all former modelling efforts presented at IIASA by the extremely fine preparation: This was the first time that all papers of all the major models were available several weeks ahead of time. Everybody who knows the terrible impact of deadline, will again know what that means. Third, for the great pain to which they had gone in presenting the papers here. The fact that they repeated most of contents of the papers is not to be blamed on them but rather on many of us who, in spite of having received the papers well ahead of time, had not studied them sufficiently.

This brings me to a suggestion made by several participants in the Conference--that maybe in the next conference we should go one step further. If there is a fifth global modelling Conference, it could be envisaged to cut out the presentation of the papers altogether (or to reduce them to short summaries of 5-10 minutes), and to have instead discussants who give a 20 minute discussion of each of the papers, before the floor is opened to general discussion. If this procedure is announced along with the papers ahead of time, it will exert some pressure on those who might otherwise not find the time to study the papers sufficiently.

My second word of thanks goes to all other gentlemen who have agreed to give a paper today, following our old tradition to have a fourth day set aside to inform the audience on any new modelling efforts going on.

My third word of thanks goes to all discussants who have participated in the discussion. Again I do not think it is only my feeling that the discussion of this Conference has been of a much higher quality than at any former conference we have had.

And this brings me now to my unorthodox winding-up. I have the definite feeling that something has emerged that could be called a "community spirit" of the global modelling group in the whole world--a community feeling that, in my opinion, seems to be stronger than in many other comparable disciplines.

I have given some thought to what might be the sociological roots for this. I found two--there may be others or I may be wrong--one: global modelling, in my opinion, is so vast a field, there remains so much to be done, that nobody needs to be envious of results that a colleague may have found instead of himself. But that alone does not suffice; there is a second aspect, in my opinion, namely: that global modelling is so muddy a field, that one cannot really earn great merits in a short space of time. So in my opinion it attracts a special brand of people who are humble and modest of spirit, more devoted to the task than to visible earnings, be it in money or reputation terms. And given these two factors it should not be surprising that our discussions have been very frank, very open minded, very direct, without any mental reservation or hidden traps. All the contributions have been in a very friendly, constructive, helpful, and genial tone. And it was this combination of frankness and friendliness that perhaps gave me the feeling of an emerging community spirit.

R. E. LEVIEN

I shall keep my closing comments very short. I want to say only that we were very happy to have you with us. I am pleased that the meeting has been successful and that the proceedings, interaction, and discussion have continued the traditions developed in the first three conferences. I hope, although we do not have formal plans established yet, that there will be a fifth conference in September of 1977 and I look forward to seeing you all then. There are other models to talk about and I think the proposals that Dr. Richardson made are challenging. IIASA is here and working all year round. I think by now most of you consider yourself as part of the extended IIASA family and we will be delighted to have you visit and participate in many ways in the ongoing program. So with these brief remarks let me bring the meeting to an end and thank you all again for your participation.

APPENDIXES

APPENDIX 1: AGENDA

Monday, 20 September 1976

10:00-10:15	Welcome	R.E. Levien
10:15-10:30	Introduction to the Conference	G. Bruckmann
10:30-12:30	SARUM	P.C. Roberts et al.
14:30-16:00	SARUM (cont.)	P.C. Roberts et al.
16:30-18:00	SARUM (cont.)	P.C. Roberts et al.

Tuesday, 21 September

09:00-10:30	SARUM (cont.)	P.C. Roberts et al.
11:00-12:30	SARUM (cont.)	P.C. Roberts et al.
14:30-16:00	MRI	R. Kulikowski et al.
16:30-18:00	MRI (cont.)	R. Kulikowski et al.

Wednesday, 22 September

09:00-10:30	MRI (cont.)	R. Kulikowski et al.
11:00-12:30	MRI (cont.)	R. Kulikowski et al.
14:30-16:30	MRI (cont.)	R. Kulikowski et al.
17:00-18:00	MRI (cont.)	R. Kulikowski et al.

Thursday, 23 September

09:00-09:30	Continuation of the Bariloche Model	A. Herrera
09:30-10:00	Evaluation of the SARU World Models	J.M. McLean
10:30-11:00	Comparison of SARUM and MRI	M. Keyzer
11:30-12:00	Global/National Modelling Work at the ILO	M. Hopkins

Thursday, 23 September (cont.)

12:00-12:30	Future Trends of Agriculture in Developing Countries	M. Osterrieth
13:30-14:20	A Step to Estimating Parameters of Input-Output Tables	Y. Kaya
14:20-15:10	Upper Great Lakes Waste Loadings Trend Simulation Model	J.P.H. Batteke
15:20-15:40	Summary of Modelling Activities at Statistics Canada	W. Page
15:40-16:00	The GMD-Project "Model Base System"	S. Dickhoven
16:00-16:30	The Relation Between Food Energy, and Space	M. Slesser
16:30-16:50	An Indicator System for the Relationship Between Population and the Satisfaction of Basic Human Needs	J.M. Richardson, Jr.
16:50-17:30	Mechanisms of Sociopsychological Development	J. Millendorfer
17:30-18:00	General Evaluation	K.S. Parikh G. Bruckmann R.E. Levien J.M. Richardson, Jr.

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SARUM AND MRI: DESCRIPTION AND COMPARISON OF A WORLD MODEL AND A NATIONAL MODEL

Proceedings of the Fourth IIASA Symposium on Global Modelling, September 20-23, 1976

Gerhart Bruckmann, Editor

Since 1974, the International Institute for Applied Systems Analysis has been monitoring developments in the field of global modelling. IIASA conferences on global modelling are intended to serve as a forum for exchanging information among modelling groups. Five such conferences have been held, each focusing on one or two major global models approaching completion. This fourth conference was mainly devoted to a comparison of two models: one by the Department of the Environment of the United Kingdom (the Systems Analysis Research Unit Model—SARUM) and one by the Polish Academy of Sciences (the Long-Term Normative Model of Development—MRI).

SARU was established in the UK in 1972 following a world conference at Stockholm that was the first to be concerned with the environment, resource use, and population growth. The UK government recognized that the problem areas were closely enough connected to warrant investigation of their interactive effects, and commissioned SARU to advise on the value of global models and the extent to which quantitative methods could illuminate world problems. As the full version of SARUM has been published elsewhere, these Proceedings just contain a summary, which, however, should give sufficient insight to show the functioning of the model and its main results.

MRI, however, is published in full, as this is the only publication of the model in a Western language. This part of the book describes the methodology used for the construction of 10 long-term normative models of development of Poland constructed by a team of scientists from the Polish Academy of Sciences. The MRI system of models has a decentralized structure with a core model and a number of sectorial submodels such as: industrial production, agriculture, personal and aggregate consumption, regions of the country, ete. It takes into account the complex interactions between production, consumption, demography, environment, technological change, foreign trade, etc. The existing decision and management structure as well as regional and international linkages are included. Ten-year forecasts of national development under optimum allocation of resources (maximizing GNP and other criteria) and alternative strategies have been investigated with the MRI.

The final section is devoted to short presentations and discussions of other recent modelling efforts from around the world.

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