ENERGY POLICY IN A SMALL OPEN ECONOMY: THE CASE OF SWEDEN

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PREFACE

One of the tasks in the System and Decision Sciences Area is to develop quantitative models that can be used for analyzing applied policy problems. The following report develops a numerically formulated model of the Swedish economy, and uses it to evaluate a proposed energy strategy for Sweden.

The model used is a multisectoral economic growth model of the same type as Professor Leif Johanson's so-called MSG-model of the Norwegian economy. This approach has been adapted so as to be useful in analyzing the problems facing a small open economy that wishes to carry out an independent energy policy. Thus the model allows substitution between energy and other factors of production and has explicit export and import functions. While the model focuses on a specific problem, the same basic approach can be used for studying many other policy issues.

The basic policy question addressed is whether a constraint on the growth of energy consumption is compatible with conventionally measured economic growth in a small economy with a relatively large foreign trade sector. The results indicate that even if the substitutability of energy and other factors of production is low, it takes some 10-15 years before a zero energy growth strategy can have a significant impact on economic growth. However, the nature of the adjustment to the energy policy depends to a large extent on the elasticity of substitution between energy and other factors of production. The model shows that when that elasticity is high, the main element in the adjustment process is a reallocation of capital from the energy sector to the energy consuming sectors. When, however, the elasticity is assumed to be low, the main element is a significant change in the commodity composition of household consumption expenditures.

SUMMARY

In a small economy with a relatively large foreign trade sector, producers to a large extent must take as given prices on the world markets for goods and services. This means that the sectoral structure of production and employment is relatively sensitive to measures affecting domestic prices. For this reason some special problems are connected with economic policy in a small, open economy.

If such an economy plans to carry out an independent energy policy, aiming at a reduction in the growth of energy consumption, it faces at least two kinds of vexing trade-off problems. First, this energy strategy might have a negative impact on economic growth, that is, the energy policy might have a nonnegligible cost in terms of GNP or aggregate consumption growth. Second, a significant share of the reduction in energy consumption might be due to changes in the commodity composition of foreign trade, and thus in the sectoral structure of the production system. Thus the energy strategy might lead to a marked sectoral reallocation of the labor force, possibly combined with regional reallocation of the population. Such an outcome may not only cause difficult readjustment problems for industrial policy, but can also be in conflict with established goals related to regional development.

In this paper a multisectoral model of economic growth is developed and used for analysis of the economic impact of an energy strategy proposed by the Swedish government. According to the proposal, Sweden should aim at reducing energy consumption growth from a postwar average of 5% per annum to 2% per annum between 1973 and 1985 and to zero growth thereafter. The approach in this study is inspired by Professor Leif Johanson's so-called MSG-model of the Norwegian economy. Here the model has been adapted so as to be useful for analyzing the problems on which this study is focused. Thus the model allows substitution between energy and other factors of production, and it has explicit export and import functions.

The model is based on input-output data for Sweden. As far as possible the numerical values of various parameters in the model are based on econometric evidence. In many cases, however, such evidence is not available and the author had to rely on reasonable "guesstimates". The projections presented in the report should thus be regarded as tentative

rather than precise forecasts. However, the sensitivity of the results with respect to key assumptions has been investigated in detail, and therefore rather firm conclusions can be reached about the main results. The analysis was carried out for the period 1980 to 2000. The development of the economy in two cases was compared. In the first case there was no constraint on energy consumption growth. In the second, in line with Swedish policy, the growth of energy consumption was kept at 2% per annum between 1980 and 1985 and at zero growth thereafter.

The results indicate that for the studied 20-year period, the target energy consumption growth rate can be attained without significant costs in terms of GNP or aggregate household consumption losses. The loss in GNP due to the energy policy was only about 1% at the year 2000. In addition, the energy policy did not lead to significant changes in the sectoral allocation of the labor force. This is because it was primarily capital, available as a result of the reduced growth of the capital-intensive energy sector, that was used as a substitute for energy in the production sectors. However, the negative impact on economic growth increases over time. If the energy consumption is kept at the 1985 level for 5 or 10 more years, the reduction in the rate of economic growth tends to be substantial.

The model simulations were carried out under the assumption that the net savings ratio in the economy remains constant over the period in question. Since one effect of the simulated policy measures was that profits tended to decrease, this assumption might seem dubious. The tendency towards falling profits might lead to a reduction in the net savings ratio. In that case the proposed energy policy has an additional, indirect impact on economic growth.

In the model-economy the target energy consumption growth rate was attained by means of a tax on energy consumption. At the year 2000 the tax rate, which kept energy consumption at the target level, varied between 137% and 871%, depending on the assumption made about the elasticity of substitution between energy and composite capital-labor. Energy tax rates of this order of magnitude would obviously create economic incentives for the development of new energy sources and energy conservation methods. It is quite possible that a number of R & D investments in these fields would turn out to have a high rate of return. That is, by means of R & D investments the shape of the production functions would be changed so that the negative impact on economic growth of the energy policy would be mitigated and the tendency towards falling profits counteracted.

As expected, the proposed energy policy turned out to have a larger impact on economic growth, the lower was the elasticity of substitution between energy and composite capital-labor. This applied particularly on the sectoral level.

When the elasticity of substitution was assumed to be 0.50 in all sectors, neither the structure of the production system nor the commodity composition of household consumption was significantly affected. However, when the elasticity of substitution was assumed to be 0.10, attainment of the target energy consumption development was accompanied by significant changes in the commodity composition of household consumption. In addition the rate of reduction of industrial employment was increased by the energy policy measures.

Although reservations can be made, it seems that energy consumption in Sweden can be kept on the target development path proposed by the government at least during a period of 10-15 years without significant conflicts with other social and economic goals. Whether this is an "optimal", or justifiable, energy policy is another question, beyond the scope of this study.

Energy Policy in a Small Open Economy: The Case of Sweden

I. INTRODUCTION

In response to the oil crisis of 1973-1974 and increasing public concern about various side effects of energy consumption, a reorientation of Sweden's energy policy was initiated. In 1975, the general principles of a "new" energy policy were presented by the government and approved by parliament. Before the end of 1978 a major decision about the goals and means of future energy policy in Sweden is to be made by parliament.

According to the 1975 government proposal, Sweden's energy policy should aim to reduce energy consumption growth from a post-war average 5% per annum to 2% per annum between 1973 and 1985, and to zero growth from 1990. However, this is not a goal in itself. The basic idea is that the energy system should be transformed so as to reduce its environmental impacts as well as the country's dependence upon imported fuels. This transformation should, according to the government proposal, neither conflict with important social and economic goals nor lead to dramatic changes in the electricity supply conditions. The above mentioned growth figures were regarded as a reasonable compromise between these considerations.

This study is an attempt to quantify the impact of such an energy strategy for Sweden on the rate and pattern of economic growth. The study aims at identifying potential conflicts between energy policy goals expressed as target energy consumption growth rates, and goals related to aggregate economic growth as well as to the sectoral allocation of production and employment.

During the last few years a number of analyses of the macroeconomic impact of various national energy strategies have been carried out. See for instance Hudson and Jorganson (1974,1978), Manne (1977), Hogan and Manne (1977) and Ridker et al. (1977).

A common feature of these studies is that they deal with the USA, a large and relatively closed economy.

The Swedish economy, on the other hand, is small and has a relatively large foreign trade sector. In such an economy the producers are largely pricetakers on the world market for goods and services. Thus the demand for exports is elastic with respect to deviations between world market prices and domestic prices. The same applies to the demand for competitive imports, that is, imported goods that are domestically produced as well. When net export demand is elastic and the foreign trade sector relatively large, the sectoral structure of the economy is relatively sensitive to measures affecting domestic prices. This means that domestic energy taxation might bring about substantial changes in domestic energy consumption by changing the commodity composition of foreign trade. At given world market prices such structural changes in the economy do not necessarily lead to reductions in gross national product (GNP) or similar aggregate measures. Thus, at least for some time, there could be a rather weak relationship between aggregate economic growth and energy consumption growth. From this point of view a small, open economy has, *ceteris paribus*, a wider range of energy policy options than a large, relatively closed economy.

On the sectoral and regional level the trade-off problems connected with domestic energy policy might be more difficult in a small, open economy than in a large, relatively closed economy. When the sectoral allocation of production is sensitive to domestic energy policy measures, this might also apply to the sectoral allocation of the labor force and, possibly, the regional allocation of the population. Such an outcome of the energy policy may not only cause readjustment problems for industrial policy, but can also be in conflict with established goals related to regional development. Whether the above mentioned energy policy goals for Sweden are compatible with other economic policy goals depends on the quantitative importance of these effects together with the effects on aggregate economic growth resulting from the implementation of the energy policy.

Due to inertia in the economic system, short- and long-run effects of energy policy measures are likely to differ. This is especially true when a change in energy policy is anticipated by only a fraction of those affected by the measures. Short-run effects may include increased unemployment and capital losses. In the long run, however, a wide range of energy strategies are compatible with full utilization of the economy's resources. Instead the energy policy measures primarily affect the efficiency of resource allocation in the economy.

In this study, only long-run effects of energy policy measures are dealt with. That is, the estimated impact of energy policy measures refers to a situation where producers and consumers are completely adjusted to prevailing market prices. Energy policy measures are assumed to be gradually implemented and exogenous conditions are assumed to change smoothly over time.

The study is carried out by means of a numerically formulated multisectoral growth model of the Swedish economy. The model does not indicate "optimal" growth paths, but simulates the economy's development under certain assumptions about exogenous conditions. A number of "futures" of the Swedish economy are simulated. These "futures" are conditioned by two sets of assumptions. First, there are assumptions about exogenous conditions, such as world market trade and prices, domestic supply of capital and labor, as well as about the domestic energy policy that is adopted. Second, assumptions are made about various parameters in the model, such as the elasticity of substitution between energy and other factors of production, for which econometric estimates have not been available.

The report is organized in the following way: in Section II the structural equations of the model are presented. Section III deals with some aspects of the solution procedure and Section IV with the empirical basis of the study. The results of the study are presented in Section V. Section VI contains a summary of the main results as well as some conclusions.

II. THE MODEL

The model used in this study is a so-called MSG model (Multi-Sectoral Growth). This kind of model is sometimes referred to as the Leif Johansen Model (see Blitzer et al., 1975, p. 100), since Leif Johansen (1959) introduced the special solution technique that makes numerically formulated general equilibrium models easy to handle. A somewhat refined version of Johansen's original model is used by the Norwegian Ministry of Finance for long-term forecasting purposes (Johansen 1974,1977), and recently Restad (1976) developed a MSG model to be used for similar purposes by the Swedish Ministry of Economic Affairs. In addition, Førsund (1977) has utilized a highly aggregated MSG model of the Norwegian economy for analysis of energy policy issues.

Except for complementary imports, foreign trade was exogenously determined in Johansen's model. Moreover the elasticity of substitution between energy and primary factors of production (capital and labor) was set equal to zero. Restad retained the latter assumption but made foreign trade an endogenous part of the model. However, the composition of aggregate exports was exogenously determined and so was the import share in the domestic supply of goods and services. A common feature of both models is that the change in the economy's aggregate capital stock and the labor force are exogenously determined, while the sectoral allocation of capital and labor is determined within the model.

In Førsund's model the elasticity of substitution between capital, labor and energy was unity. Foreign trade and aggregate capital formation were exogenously determined.

In the MSG model there is a nonzero elasticity of substitution between energy and primary factors of production, and that elasticity may differ between various sectors. There are also explicit import and export functions for each one of the trading sectors. However, as in the above mentioned models, both the total capital stock and the total labor force are determined outside the model, while the sectoral allocation of these factors of production are determined within the model.

II.1 Sectors and Variables

There are nine sectors in the model economy (see Table 1 below). The sector "basic processing industries" contains the mining industry, the paper and pulp industry, and the chemical industry. Sector 8, "capital goods", is a book-keeping sector where various produced goods are combined in fixed proportions. Thus the input-output coefficients of the capital goods sector define the composition of the economy's stock of real capital. There is only one kind of output from each sector, and each commodity is only produced in one sector. Thus the index "i" sometimes refers to "sector" and sometimes to "commodity i", the only output from sector i.

Table 1. Sectors of the model economy.

Sector	Code
Energy	0
Agriculture, forestry and fishing	1
Basic processing industries	2
Manufacturing industries	3
Transportation	4
Private services	5
Housing services	6
Public services	7
Capital goods	8
Households	C

Table 2 defines the variables and parameters of the model.

Table 2. Variables and parameters of the model.

A. <u>Exogenous variables</u>

G	public consumption
N	total labor force
K	total capital stock
I	total net investment
D	target surplus (deficit) on the current account
P_i^w	world market price of commodity i = 0,1,...,5, expressed in foreign currency
$\bar{P}_i$	world market price of complementary imports used in sector i = 0, expressed in foreign currency

B. <u>Endogenous variables</u>

X_i	gross output in sector i = 0,1,...,8
F_i	a composite capital-labor input used in sector i = 0,1,...,7
X_{ji}	input of commodity j = 0,1,...,5 in sector i = 0,1,...,8
K_i	capital stock in sector i = 0,1,...,7
N_i	employment in sector i = 0,1,...,7
$\bar{M}_i$	input of complementary import* in sector i = 0
C_i	household consumption of commodity i = 0,...,6
Z_i	export of commodity i = 1,2,...,5
M_i	import of commodity i = 0,1,...,5
P_i	price of commodity i = 0,1,...,8
W	index of the level of wages in the economy as a whole
W_i	wage rate in sector i = 0,1,...,7
R	index of the net return on capital in the economy as a whole
R_i	net return on capital in sector i = 0,...,7
Q_i	"user cost" of capital in sector i = 0,...,7
V	exchange rate (units of domestic currency per unit of foreign currency)
O	household consumption expenditure
Y	real gross national product
C	total real household consumption

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*Complementary imports is meant to imply the import of commodities that cannot (or at least are not) produced within the country.

Table 2. (cont'd)

C. Parameters*

a_{ji}	input of commodity j = 0,1,...,5 per unit of output in sector i = 0,1,...,8
$\bar{b}_i$	input of complementary imports per unit of output in sector i = 0
ρ_i	substitution parameter. The elasticity of substitution between energy and the composite capital-labor input in sector i = 0,1,...,7 is equal to $(1 - \rho_i)^{-1}$
α_i, γ_i	distribution parameters for sector i = 0,1,...,7
λ_i	rate of (neutral) technical change in sector i = 0,1,...,7
σ_i	rate of change of world market trade with commodity i = 1,2,...,5
δ_i	rate of depreciation of the capital stock in sector i = 0,1,...,7
ω_i	index of the relative wage rate in sector i = 0,1,...,7
β_i	index of the relative rate on capital in sector i = 0,1,...,7
η_i	elasticity of the household demand for commodity i with respect to total household consumption expenditures
η_{ij}	elasticity of the household demand for commodity i with respect to the price of commodity j
ε_i	price elasticity of export demand
μ_i	price elasticity of import demand
A_i, B_i	constants in the production and demand functions respectively

D. Energy policy parameters

τ	general value tax (or subsidy) on energy
ξ_i	value tax (or subsidy) on energy consumed in sector i = 1,2,...,7,C
T_i	$1 + \tau + \xi_i$

E. Notation conventions

If H is a variable in the model, then $\frac{dH}{dt} = H$ and $\frac{dH}{Hdt} = h$

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*Both parameters and exogenous variables are determined outside the model, the parameters being constants while the exogenous variables may change over time.

II.2 Technology

Gross output is a function of the input of a composite capital-labor input, energy and various intermediate goods. The elasticity of substitution between energy and the composite capital-labor input differ between the sectors, while the elasticity of substitution between energy and intermediate goods as well as between the composite input and intermediate goods is zero in all sectors.1/ The elasticity of substitution between capital and labor in the "production" of the composite input is unity in all sectors. Complementary imports (mainly crude oil) used in the energy sector only cannot be substituted for other factors of production. Finally, there are constant returns to scale in all sectors.

Using the symbols defined in Table 2 the technology can be described in the following way:

$$X_i = A_i \left\{ \gamma_i F_i^{\rho_i} + (1-\gamma_i) X_{0i}^{\rho_i} \right\}^{\frac{1}{\rho_i}} \quad ; \quad i = 0,1,\ldots,7 \quad . \tag{1}$$

The elasticity of substitution between energy and the composite input is equal to $(1 - \rho_i)^{-1}$.

Equations (2) - (4) make the description of the technology complete.

$$F_i = K_i^{\alpha_i} N_i^{1-\alpha_i} e^{\lambda_i t} \quad , \quad i = 0,1,\ldots,7 \quad . \tag{2}$$

$$X_{ji} = a_{ji} X_i \quad , \quad j = 1,2,\ldots,5; \quad i = 0,1,\ldots,8 \quad . \tag{3}$$

$$\bar{M}_0 = \bar{b}_0 \, X_0 \quad . \tag{4}$$

1/Of course, this relationship, as well as those presented in the following subsections, are "true" in the model only. The applicability of the model is discussed in Sections IV and VI.

II.3 Producer Behavior

The producers in the private sector of the economy are assumed to maximize their profits, while the public sector minimizes its cost for a given level of public consumption. The profit in sector i, Π_i, is defined by

$$\begin{aligned}\Pi_i = P_iX_i - T_iP_0X_{0i} - \sum_{j=1}^{5} P_jX_{ji} - W_iN_i - P_8\delta_iK_i \\ - R_iP_8K_i - V\bar{P}_i\bar{b}_iX_i \quad , \quad i = 0,1,\ldots,7 \quad .\end{aligned} \tag{5}$$

By using (3) we can define P_i^*, the sum of value added and energy costs in unit production costs, for commodity i as

$$P_i^* = P_i - \sum_{j=1}^{5} P_ja_{ji} - V\bar{P}_i\bar{b}_i \quad , \quad i = 0,1,\ldots,7 \tag{6}$$

where the last term on the right hand side is different from zero only for the energy sector. Moreover, for sector 8, the bookkeeping sector, P_i^*, must be zero, which means that

$$P_8 = \sum_{j=1}^{5} P_ja_{j8} \quad . \tag{7}$$

By defining "user cost of capital" in sector i, Q_i, by

$$Q_i = P_8(\delta_i + R_i) \qquad i = 0,1,\ldots,7 \tag{8}$$

the expression for Π_i becomes

$$\Pi_i = P_i^*X_i - T_iP_0X_{0i} - W_iN_i - Q_iK_i \quad ; \quad i = 0,1,\ldots,7 \quad . \tag{9}$$

Profit maximization implies that, in equilibrium, the value of the marginal product of each factor of production must be equal to its price. Moreover, when the level of output is fixed,[1] profit maximization is equivalent to cost minimization. Using the production functions (1), the composite input functions (2) and the definition of profit (9), the profit maximization conditions become

$$\gamma_i(1-\alpha_i)\left(\frac{A_iF_i}{X_i}\right)^{\rho_i} = \frac{W_iN_i}{P_i^*X_i} \quad , \qquad i = 0,1,\ldots,7 \quad ; \tag{10}$$

$$\gamma_i\,\alpha_i\left(\frac{A_iF_i}{X_i}\right)^{\rho_i} = \frac{Q_iK_i}{P_i^*X_i} \quad , \qquad i = 0,1,\ldots,7 \quad ; \tag{11}$$

$$(1-\gamma_i)\left(\frac{A_iX_{0i}}{X_i}\right)^{\rho_i} = \frac{T_iP_0X_{0i}}{P_i^*X_i}\,^{2/} \quad , \qquad i = 0,1,\ldots,7 \quad . \tag{12}$$

The formulation of the model implies that there is only one type of labor and that labor and capital can be moved between the sectors. This means that in equilibrium no intersectoral profit and wage differentials can exist. However, due to uncertainty, institutional factors, disequilibria, etc., such differentials are revealed by actual data. The sectoral profit and wage rates can be defined as functions of sectoral factors, β_i and ω_i, and the profit and wage rates, respectively, for the economy as a whole, so that

$$R_i = \beta_iR \quad , \qquad i = 0,1,\ldots,7 \quad ; \tag{13}$$

$$W_i = \omega_iW \quad , \qquad i = 0,1,\ldots,7 \quad . \tag{14}$$

Both Johansen and Restad regarded β_i and ω_i as institutionally determined constants. A better approach would perhaps be to simulate an adjustment process where intersectoral profit and

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[1] In the public sector, gross production is exogenously determined.

[2] By definition $T_0 = 1$.

wage differentials are gradually reduced and then take the final solution as a point of departure for the analysis of the impact of energy policy measures. However, that has not been done in this study. Instead β_i and ω_i are regarded as constants, reflecting institutional factors which remain unchanged during the simulation period.

II.4 Prices and Household Expenditures

By an appropriate choice of unit of measurement, domestic prices in the model economy become unity at the initial point in time.[1] The prices in the model economy are normalized so that the general level of prices is kept constant over time. Of course, relative prices may change. The normalization of the price level is carried out by means of the following equation:

$$\sum_{i=0}^{7} P_i X_i + \sum_{i=0}^{5} VP_i^w M_i + V\bar{P}_0 \bar{M}_0 = \sum_{i=0}^{7} X_i + \sum_{i=0}^{5} M_i + \bar{M}_0 \quad . \qquad (15)$$

The total real[2] household consumption is defined by

$$C = \sum_{i=0}^{6} C_i \quad . \qquad (16)$$

When prices are normalized by (15), it follows that there might be deviations between total real household consumption, C, and total household consumption expenditure, O. The quotient O/C defines the "implicit consumer price index" of the model economy. The demand for each kind of consumer goods and services is determined by the market prices of all goods and services and

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[1] All flows of commodities are expressed as values, using the prices prevailing at the initial point in time.

[2] That is, the value of household expenditure measured by the prices prevailing at the initial point in time.

and total household consumption expenditure, [1/]

$$C_i = B_i 0^{\eta_i} (T_C P_0)^{\eta_{0i}} P_1^{\eta_{1i}} \dots P_6^{\eta_{6i}} \quad , \quad i = 0,1,\dots,6 \ . \tag{17}$$

II.5 Foreign Trade

The demand for exports from sector i is basically determined by the world market trade with commodity i. However, the share of world market exports supplied by domestic producers is a function of the relation between the domestic price, expressed in foreign currency, of the commodity in question and the world market price of that commodity. Thus, the export demand functions can be written as

$$Z_i = Z_i^0 \left(\frac{P_i}{VP_i^w} \right)^{\varepsilon_i} e^{\sigma_i t} \quad , \quad i = 1,2,\dots,5 \ . \tag{18}$$

Since the model is fairly aggregated, the "commodities" of the model economy should not be regarded as individual products. Rather, they are commodity groups consisting of several different products which are either substitutes or complements to each other. This means that imported and domestically produced units of a certain "commodity" may not be perfect substitutes, and thus export and import of a certain "commodity" can take place simultaneously. Moreover, the share of imports in the domestic supply of a certain "commodity" is not completely elastic with respect to price differentials. Thus, the import functions can be written as

$$M_i = \frac{M_i^0}{X_i^0} \left(\frac{P_i}{VP_i^w} \right)^{\mu_i} X_i \quad , \quad i = 0,1,\dots,5 \ . \tag{19}$$

[1/] It should be noted that demand functions of this type, i.e., with constant elasticities with respect to expenditure and all prices, do not satisfy the budget constraint identically. However, the quantitative effect of this discrepancy is not likely to be important.

II.6 Capital Formation

In the model economy, the growth of the aggregate stock of real capital is an exogenously determined magnitude. The net investments in the economy as a whole are exogenously determined as well. Obviously the assumption about the change in the capital stock cannot be made independent of the assumption about the level of net investments. The link between these two assumptions is discussed in Section III.

II.7 Equilibrium Conditions for Goods and Factor Markets

In equilibrium, there must be equality between demand and supply on the markets for commodities, savings, labor and foreign currency. Thus, the following conditions have to be satisfied:

$$X_0 = \sum_{j=0}^{7} X_{0j} + C_0 - M_0 \quad , \tag{20}$$

$$X_i = \sum_{j=0}^{8} a_{ij}X_j + C_i + Z_i - M_i \quad , \quad i = 1,2,\ldots,5 \quad , \tag{21}$$

$$X_6 = C_6 \quad , \tag{22}$$

$$X_7 = G \quad , \tag{23}$$

$$X_8 = I + \sum_{j=0}^{7} \delta_j K_j \quad , \tag{24}$$

$$\sum_{j=0}^{7} K_j = K \quad , \tag{25}$$

$$\sum_{j=0}^{7} N_j = N \quad , \tag{26}$$

$$\sum_{i=1}^{5} \frac{P_i}{V} Z_i - \sum_{i=0}^{5} P_i^w M_i - \bar{P}_0 \bar{M}_0 = D \quad . \tag{27}$$

II.8 Definitions

GNP is defined by

$$Y = C + X_8 + G + \sum_{i=1}^{5} Z_i - \sum_{i=0}^{5} M_i - \bar{M}_o ; \tag{28}$$

and e_i, the energy input coefficients, by

$$e_i = \frac{X_{0i}}{X_i} . \tag{29}$$

II.9 Energy Policy

In the model economy, energy policy is carried out by means of an energy policy parameter, T_i, defined by

$$T_i = 1 + \tau + \xi_i \quad , \qquad i = 1,2,\ldots 7,c \tag{30}$$

where τ is a general value tax (or subsidy) on energy and ξ_i is a value tax (or subsidy) on energy consumption in sector i. The total domestic consumption of energy, E, is defined by

$$E = X_0 + M_0 . \tag{31}$$

In some applications E is an endogenous variable. Then τ is exogenous. In others E is exogenous, which means that τ is endogenous.

Obviously there are a number of additional energy policy measures available in the real world. For instance, the authorities can impose restrictions on the use of certain energy production technologies, regulate the emission of various pollutants, prescribe certain insulation standards for new houses, etc. Energy policy measures of this kind either change the shape of the production functions or make the range of feasible factor combinations more narrow than the range of technically feasible factor combinations.

As the model is formulated, it is easy to analyze the sensitivity of the solutions with respect to changes of the production functions. On the other hand, it is not very easy to know how a particular energy policy measure will affect the production functions. For this reason, the analysis in this study is confined to energy tax policy.

III. THE SOLUTION OF THE MODEL

III.1 General Remarks

All the variables of the model can be regarded as functions of time. By solving equations (1) - (30) for a number of points in time, the evolution of the model economy can be described. However, since many of the equations (1) - (30) are non-linear, the solution of the model is not a trivial problem.

What Johansen did was to differentiate all the relations with respect to time, and express the model in terms of relative rates of growth at the initial point in time. Due to the functional form of the model's structural equations, a linear equation system was then obtained. This linear equation system can be written

$$A\psi = B\phi \quad ,$$

where Ψ is the vector of relative rates of change of the endogenous variables and ϕ the vector of relative rates of change of the exogenous variables. If the number of endogenous variables is n and the number of exogenous variables m, A is a $n \times n$-matrix and B a $n \times m$ matrix. Thus the equation system has a unique solution.

In the solution matrix $A^{-1}B$ the element on the i^{th} row and the j^{th} column shows the impact of a given rate of change of the j^{th} exogenous variable on the rate of change of the i^{th} endogenous variable. Thus for a given set of assumptions about the exogenous variables, expressed as a vector ϕ^k, the rates of change of the endogenous variables, the vector ψ^k, are determined.

However, in order to get the model in such a simple form, one has to treat the values of the model's variables at the initial point in time as constants, while their relative rates of change are treated as variables. This is, obviously, valid only at that particular point in time.

Accordingly Johansen confined his analysis to the "growth tendencies" of the Norwegian economy at the initial point in time. Restad (1976, pp. 103-108) used the same method to approximate the model economy's development over a number of years. Since Restad's approach was adopted in this study as well, it should be described in some detail.

Given a data base for the point in time t, compatible with all the equations in the non-linear version of the model, the matrices A_t and B_t can be calculated. Using the solution matrix A_t^{-1} B_t and a set of assumptions about the exogenous variables, the development of the model economy between t and $t+\Delta$ is determined. If H_t denote the numerical value of the (exogenous or endogenous) variable H at t and h is the relative rate of change of H at t, the value of H at $t+\Delta$ is then calculated by means by the formula

$$H_{t+\Delta} = (1 + \Delta h)\ H_t \quad .$$

Using the resulting values of the model's variables at $t+\Delta$, the matrices $A_{t+\Delta}$ and $B_{t+\Delta}$ can be calculated. Then the solution matrix $A_{t+\Delta}^{-1}$ $B_{t+\Delta}$ together with assumptions about the exogenous variables determine the development between $t+\Delta$ and $t+2\Delta$. In this way it is possible to trace the whole development process over an arbitrary number of periods with the length Δ.

The problem is, of course, that the values obtained in this way for $t+\Delta$ may not be compatible with the non-linear version of the model. Moreover, the bias can be expected to increase for each step in the solution procedure. On the other hand, the bias appearing in each step can be expected to be smaller when Δ is smaller. Intuitively it seems reasonable to expect the bias emerging in a projection over a time period of given length to be smaller when Δ is smaller. However, no systematic analysis of this problem has been carried out within the frame of this study.

Within the Norwegian Ministry of Finance a method for computation of exact solutions to the MSG-model has been developed.[1/] It is based on the approach described above, but, by means of an iterative procedure, the solution obtained after each step is made compatible with the non-linear version of the model. For $\Delta=3$ years, the value used by the Norwegian Ministry of Finance, the bias turned out to be relatively unimportant, and the iterative procedure converged after a small number of iterations.

However, this study has been carried out without access to a program for exact solution of the model. Instead Restad's approach was adopted. The length of the sub-period was set equal to five years, that is $\Delta=5$. After the first step, there was a difference between output, as determined by the production functions, and demand in the size order 1-1.5% in each of the production sectors. Although disturbing, this bias was regarded as acceptable.

Another point is that both the aggregate stock of capital and aggregate net investments are exogenous variables in the model. Obviously there is a relation between changes in the stock of capital and net investments; the numerical values of the exogenous variables k and i cannot be chosen independently. In order to make the values of k and i consistent with each other the following approximate but computationally simple procedure is adopted. Thus it is noted that

$$\frac{K(t+\Delta) - K(t)}{\Delta} \approx I(t) \left(1 + \frac{i}{2}\Delta\right) .$$

Division by K(t) yields

$$\frac{K(t+\Delta) - K(t)}{K(t)\Delta} \approx \frac{I(t)}{K(t)} \left(1 + \frac{i}{2}\Delta\right)$$

[1/] See Johansen (1974, chapter 10) and Spurkland (1970).

or

$$k \approx \frac{I}{K}\left(1 + \frac{i}{2}\Delta\right) .$$

In the matrix $A^{-1}B$ mentioned above, two elements represent the sensitivity of the growth of GNP (the variable y) with respect to the growth of net aggregate capital formation (the variables k and i). Using the information contained in these multipliers, k and i can be chosen so that the net savings ratio, I/Y, remains constant over time.

III.2 The Linearized Version of the Model

Table 3 contains all the equations of the linearized version of the model, and in the Appendix the derivation of each individual equation is briefly described. Throughout Table 3, endogenous variables are written on the left hand side and exogenous variables on the right hand side. Capital letters denote the value of the variable in question at the initial point in time. The coefficients A_{ij} in equations M4 and M7 are defined by

$$A_{ij} = e_{ij} - a_{ij} \text{ where } e_{ij} = \begin{matrix} 1 \text{ when } i = j \\ 0 \text{ when } i \neq j \end{matrix} .$$

It should be noted that the formulation of the model can be changed by means of the parameter θ.[1] When $\theta = 0$, total energy consumption is endogenously determined while the general energy tax rate, τ, is an exogenous variable. When $\theta = 1$, on the other hand, total energy consumption is exogenously determined, while the tax rate which is sufficient to induce that level of total energy consumption, τ, is determined within the model.

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[1] See equation M7:i, M10:i and M18.

Table 3. The equations of the linearized version of the model.

Number	Equation
M1:i	$x_i - \frac{\alpha_i}{1-\alpha_i} \frac{\omega_i WN_i}{P_i^* X_i} k_i - \frac{\omega_i WN_i}{P_i^* X_i} n_i - \frac{T_i P_0 X_{0i}}{P_i^* X_i} x_{0i} = \frac{1}{1-\alpha_i} \frac{\omega_i WN_i}{P_i^* X_i} \lambda_i \quad ;$ $\qquad i = 0,1,\dots,7 \quad .$
M2	$Yy - Cc - X_8 x_8 - \sum_{i=1}^{5} Z_i z_i + \sum_{i=0}^{5} M_i m_i + \bar{M}_0 \bar{m}_0 = Gg$
M3	$\frac{P_0}{P_0^*} p_0 - \sum_{j=1}^{7} \frac{P_j a_{j0}}{P_0^*} p_j - \frac{V\bar{P}_0 \bar{b}_0}{P_0^*} v + (1-\rho_0) x_0 + \rho_0 \alpha_0 k_0 + (\rho_0 - \alpha_0 \rho_0 - 1) n_0 - w = -p_0 \lambda_0 + \frac{V\bar{P}_0 \bar{b}_0}{P_0^*} \bar{p}_0$
M4:i	$\sum_{j=1}^{7} \frac{P_j A_{ji}}{P_i^*} p_j + (1-\rho_i) x_i + \rho_i \alpha_i k_i + (\rho_i - \alpha_i \rho_i - 1) n_i - w = -p_i \lambda_i$ $\qquad i = 1,2,\dots,7 \quad .$
M5:i	$\frac{P_8(\delta_i + \beta_i R)}{Q_i} p_8 + \frac{P_8 \beta_i R}{Q_i} r + k_i - w - n_i = 0 \quad ,$ $\qquad i = 0,1,\dots,7 \quad .$

Table 3. (cont'd)

M6	$\frac{P_0 - P_0^*}{P_0^*} p_0 - \sum_{j=1}^{7} \frac{P_j a_{j0}}{P_0^*} - \frac{V\bar{P}_0 \bar{b}_0}{P_0^*} v + (\rho_0 - 1) x_{00} - (\rho_0 - 1) x_0 = \frac{V\bar{P}_0 \bar{b}_0}{P_0^*} \bar{p}_0 \quad ;$	
M7:i	$\sum_{j=1}^{7} \frac{P_j A_{ji}}{P_i^*} p_j + (\rho_i - 1) x_{0i} - (\rho_i - 1) x_i - p_0 - \theta \frac{\dot{\tau}}{T_i} = (1-\theta) \frac{\dot{\tau}}{T_i} + \frac{\dot{\xi}_i}{T_i}$	$i = 1, 2, \ldots, 7$.
M8	$P_8 p_8 - \sum_{j=1}^{5} P_j a_{j8} p_j = 0$	
M9	$\sum_{i=0}^{7} (P_i - 1) X_i x_i + \sum_{i=0}^{5} (VP_i^W - 1) M_i m_i + (V\bar{P}_0 - 1) \bar{M}_0 \bar{m}_0 + \sum_{i=0}^{7} P_i X_i p_i + [V\bar{P}_0 \bar{M}_0 + \sum_{i=0}^{5} VP_i^W M_i] v =$ $= - \sum_{i=0}^{5} VP_i^W M_i p_i^W - v\bar{P}_0 \bar{M}_0 \bar{p}_0$	
M10:i	$c_i - \eta_i 0 - \sum_{j=0}^{6} \eta_{ji} p_j - \theta \frac{\dot{\tau}}{T_c} \eta_{0i} = (1-\theta \; \frac{\dot{\tau}}{T_c} \eta_{0i} + \frac{\dot{\xi}_c}{T_c} \eta_{0i}$	$i = 0, 1, \ldots, 6$.
M11:i	$z_i - \epsilon_i p_i + \epsilon_i v = \sigma_i - \epsilon_i p_i^W$	$i = 1, 2, \ldots, 5$.

Table 3. (cont'd)

M12:i	$m_i - \mu_i p_i + \mu_i v - x_i = -\mu_i P_i^W$,	$i = 0,1,\ldots,5$.
M13	$\overline{m}_0 - x_0 = 0$,	
M14:i	$x_{0i} - x_i - \frac{\dot{e}_i}{e_i} = 0$	$i = 0,1,\ldots,7$.
M15	$\sum_{j=0}^{7} \frac{K_j}{K} k_j = k$,	
M16	$c - \sum_{i=0}^{6} \frac{C_i}{C} c_i = 0$,	$i = 1,2,\ldots,5$.
M17	$X_0 x_0 - \sum_{j=0}^{7} X_{0j} x_{0j} - C_0 c_0 + M_0 m_0 = 0$	
M18	$(1-\theta)Ee - X_0 x_0 - M_0 m_0 = -\theta Ee$	

Table 3. (cont'd)

M19	$\sum_{j=0}^{7} A_{ij}X_jx_j - C_ic_i - Z_iz_i + M_im_i = 0$
M20	$x_6 - c_6 = 0$
M21	$x_7 = g$
M22	$x_8 - \sum_{j=0}^{7} \frac{\delta_jK_j}{X_8} k_j = \frac{I}{X_8} i$
M23	$\sum_{i=1}^{5} \frac{P_i}{V} Z_iz_i + \sum_{i=1}^{5} \frac{P_i}{V} Z_ip_i - \sum_{i=1}^{5} \frac{P_i}{V} Z_iv - \sum_{i=0}^{5} P_i^wM_im_i - \bar{P}_0\bar{M}_0\bar{m}_0 = \bar{P}_0\bar{M}_0\bar{p}_0 + P_0^wM_0p_0^w -$ $- \sum_{i=1}^{5} P_i^w(Z_i-M_i)p_i^w + \dot{D}$
M24	$\sum_{j=0}^{7} \frac{N_j}{N} n_j = n$

IV. THE EMPIRICAL BASIS OF THE STUDY

Two kinds of data are needed in this study. The first is a complete description of the state of the economy in terms of intersectoral and final deliveries of goods and services, capital stocks, prices, etc. at a particular point in time. The second is estimates of the parameters of the production, household demand, export and import functions.

The data used in this study are primarily those prepared by the Ministry of Economic Affairs for the above mentioned study by Restad. The estimates of the intersectoral flows and other variables describing the state of the economy were obtained by means of an econometric model, used for forecasting the development of the Swedish economy between 1975 and 1980. Thus, the "initial year" in this study is 1980. In Table 4, some key figures from the data base are presented, while the complete data base can be obtained from the author upon request.

In Table 5, the parameters of the household demand functions used in this study can be seen. With one exception, housing services, the figures are obtained from Restad (1976, p. 110) where the demand for housing services was treated as an exogenously determined magnitude. However, the price of energy is a relatively important determinant of the price of housing services (see Table 4), and changes in the consumption of housing services have a relatively large impact on the total consumption of energy. Thus, given the purpose of this study, it is not satisfactory to treat housing expenditures as an exogenously determined datum. Instead, it is, somewhat arbitrarily, assumed that the demand for housing services is unitary price and expenditure elastic.

In Restad's model there are no explicit export and import functions. Consequently the numerical values of the parameters in the trade functions of the model used in this study could not be obtained in the same easy way as the parameters of the household expenditure functions. Unfortunately there was no other suitable study available. The "solution" to this problem was simply to assume a set of, seemingly, reasonable parameters

Table 4. Selected data about the Swedish economy, estimates for 1980.

Sector	Input of energy per unit of output[1]	Share of total energy consumption	Share of GNP originating in the sector	Share of exports	Share of employment	Share of capital stock
Energy	0.0418	0.0373	0.043	-	0.008	0.064
Agriculture, forestry and fishing	0.0418	0.0396	0.039	0.009	0.052	0.042
Basic processing industries	0.0509	0.1579	0.088	0.257	0.062	0.064
Manufacturing industries	0.0151	0.1678	0.353	0.630	0.289	0.135
Transportation	0.0348	0.0426	0.057	0.054	0.069	0.078
Private services	0.0254	0.0959	0.182	0.050	0.255	0.108
Housing services	0.1461	0.1902	0.067	-	0.006	0.359
Public services	0.0272	0.0829	0.172	-	0.260	0.150
Household consumption	0.0285[2]	0.1859	-	-	-	-

[1] Both energy and output is measured in terms of Skr in 1968 prices.

[2] Share of expenditures on energy in total consumption expenditures.

Source: Restad (1976, pp. ,132-133)

Table 5. Estimated price and expenditure elasticities of the household for consumer goods and services.

	Demand for goods from sector	Elasticity with respect to....							
		P_0	P_1	P_2	P_3	P_4	P_5	P_6	C
0	Energy	-0.3373	-0.0671	-0.0768	-0.7444	-0.0613	-0.2050	0	1.4919
1	Agriculture, forestry and fishing	-0.0120	-0.1193	-0.0247	-0.1300	-0.0193	-0.0658	0	0.3711
2	Basic processing industries	-0.0310	-0.0557	-0.3125	-0.4919	-0.0280	-0.0648	0	0.9269
3	Manufacturing industries	-0.0228	-0.0541	-0.0469	-0.6778	-0.0375	-0.1253	0	0.9244
4	Transportation	-0.0299	-0.0537	-0.0614	-0.5955	-0.2291	-0.1640	0	1.1936
5	Private services	-0.0349	-0.0627	-0.0716	-0.6947	-0.0573	-0.4714	0	1.3926
6	Housing services	0	0	0	0	0	0	-1.0000	1.0000

Source: Restad (1976, p. 110)

and investigate to what extent the results were sensitive to the assumptions on this particular point. The adopted numerical values of the price elasticity parameters in the export and import functions are discussed in Section V in connection with the description of the so-called "base" case.

Except for the substitution parameters, ρ_i in eq. (1), the parameters of the production functions are obtained by using Eqs. (10)-(12) and income distribution data. The determination of the numerical values of the substitution parameters is, however, a little bit more complicated.

During the last few years a number of studies of the substitutability of energy and other factors of production or between various kinds of energy have been carried out. Although these studies differ from the present one in terms of the specification of the production functions as well as the level of aggregation, some results can be used as a basis for assumptions about the substitution parameters in the model used in this study.

In a study by Berndt and Wood (1975),[1/] based on aggregated time-series data for the American industry, capital and labor were found to be complements (that is, the estimated elasticity of substitution had a negative value), while energy and labor turned out to be substitutes. Similar results, but quite different values, were obtained in a study by Denny and Pinto (1975),[2/] based on aggregated time-series data for the Canadian industry. To the extent that these results are valid, the specification of the production functions in the model used in this study is rather dubious.

[1/] A homothetic translog production function, where output was a function of the input of capital, labor, energy and material, was used. The elasticity of substitution between each pair of inputs was estimated.

[2/] A generalized non-homothetic Leontief production function with capital, labor, energy and materials as inputs was used, and the elasticity of substitution between each pair of inputs was estimated.

However, in a study by Gregory and Griffin (1976),[1] based on a cross-section of data from nine different countries, both capital and energy as well as labor and energy were found to be substitutes. The estimated elasticity of substitution between capital and energy was close to 1.0 for all countries, while the corresponding figure for labor/energy was 0.8. Thus, to the extent that these findings are valid, production functions of the type used in this study can be justified. Moreover, the elasticity of substitution between energy and the composite capital-labor input can be assumed to be positive and not much less than unity.

This is not a place for a detailed discussion of the merits and drawbacks of various studies in this field of econometrics. However, it seems more appropriate to base a long-run study like the present one on results obtained on the basis of cross-sectional rather than time series data.

Yet it is not reasonable to assume that the elasticity of substitution between energy and capital-labor is close to unity. This is because Gregory's and Griffin's results apply to the industry as a whole rather than to individual sectors. Thus, part of the estimated substitutability is the result of structural change within the industry.[2] If these results are directly applied to individual sectors in a multisectoral model, the effect of structural change on energy consumption will be counted twice. Thus, even if Gregory's and Griffin's results are accepted, the elasticity of substitution should be a bit less than unity on the sectoral level.

Apart from these considerations the results obtained in Gregory's and Griffin's study seem, intuitively, a little bit too "optimistic" in terms of the substitutability of energy and other factors of production. This statement is, of course,

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[1] The same approach as in Berndt's and Wood's study was used, but only three inputs, capital, labor and energy, were distinguished.

[2] An attempt to estimate the impact of structural change on the change in energy consumption during a 10-year period is made in Bergman (1977).

difficult to defend, but reference to Manne (1977, p. 10), who considers an elasticity of substitution between energy and capital-labor equal to 0.25 for the economy as a whole to be the "best" estimate, could perhaps be made.

Obviously there is not a very solid ground for assumptions about the substitutability of energy and other factors of production. In this study the elasticity of substitution between energy and capital-labor is assumed to be 0.25 in the "base case". In the so-called "rigid" case, the corresponding figure is 0.1, while it is 0.5 in the so-called "flexible" case.

V. RESULTS

In the first step of the analysis, a "base case" is calculated. To a large extent this case is based on assumptions made in a recent long-term economic forecast published by the Ministry of Economic Affairs (1975). However, since neither the functional form of the structural equations nor the numerical values of various parameters in the model presented above are tested against actual data, the base case should not be regarded as a forecast. Instead it can be said to represent a plausible, but not necessarily the most probable, development of the Swedish economy.[1/] The basic issue in this step of the analysis is whether or not the growth of energy consumption is likely to be higher than the target growth rate put forward in the 1975 government proposal.

In the next step it is assumed that domestic energy policy is directed towards reducing the growth of energy consumption to 2% per annum between 1980 and 1985 and to zero growth thereafter. The impact of this strategy on GNP and other economic variables is calculated not only for the "base" case, but also for two polar cases: one where the technology is "rigid" in terms of energy input coefficients and one where it is "flexible".

[1/] In the terminology of Johansen (1977), the base case can be regarded as a "projection".

V.1 The Base Case

In the base case it is assumed that the net savings ratio remains approximately constant between 1980 and 2000. This means that the economy's aggregate capital stock grows by approximately 2.0% per annum. [1/] In accordance with the projections made by the Ministry of Economic Affairs, the labor force, measured in man-hours, decreases by 0.2% per annum between 1980 and 1990 and by 0.6% per annum between 1990 and 2000. On the same basis the growth of public consumption is assumed to be 2.5% per annum between 1980 and 2000.

The trade on international markets where Swedish producers compete is assumed to grow by 4% per annum during the entire period. Except for oil prices, world market prices, expressed in foreign currency, are assumed to remain constant in real terms. World market prices of crude oil as well as refined petroleum products are assumed to increase by 2% per annum in real terms between 1980 and 1990. For the period of 1990-2000 the corresponding figure is 5%.

No model of international trade flows has been available. Thus, it has not been possible to test whether or not the assumptions made about world market conditions are consistent with each other.

As was mentioned in the preceding section, no estimates of the price-elasticity parameters in the trade functions have been available. It seems reasonable, however, to assume that the demand for imports is less price elastic than the demand for Sweden's exports. This is because a substantial part of Sweden's imports are complementary rather than substitutes to domestically produced goods and services. In accordance with the discussion in the introductory section, the demand for Sweden's exports should be quite price elastic. In particular this holds for standardized products like the output from "basic processing industries".

[1/] In the forecasts made by the Ministry of Economic Affairs, this figure was assumed to be 3%. Thus a gradual increase in the net savings ratio was assumed.

The specific assumptions made are the following: the absolute value of the price elasticity of the export demand for output from "basic processing industries" is assumed to be 3.0. The corresponding figure for "manufacturing industries" is 1.5, and 1.0 for the other exporting sectors.[1/] In all import functions the absolute value of the price elasticity parameter is set equal to 0.5.

The last set of assumptions concerns the productivity of the combined capital-labor input.[2/] Here the assumptions are based on the above mentioned forecast by the Ministry of Economic Affairs. Thus, in "basic processing industries" the productivity of composite capital-labor is assumed to grow by 3% per annum. For "agriculture, forestry and fishing" and "manufacturing industries" the corresponding figure is 2.5%, while it is 2% in transportation and 1% in the remaining sectors.

The main results obtained in the base case are presented in Table 6, together with results from a projection denoted "rapid growth". This case differs from the "base" case with regard to the assumptions made about the productivity of the

Table 6. Calculated annual change of selected macroeconomic variables, 1980-2000, percentage points.

	Base Case	Rapid Growth Case
GNP	2.0	3.6
Real household consumption	2.8	4.4
Energy consumption	2.3	4.2
Industrial employment*	-2.6	-1.3

*That is, employment in "basic processing" and "manufacturing" industries.

[1/] That is "agriculture, forestry and fishing", "transportation" and "private services".

[2/] That is, the parameter λ_i in eq. (2).

combined capital-labor input and capital formation. Thus, annual rates of change are one percentage point higher in the "rapid growth" case than in the base case. This means that the productivity assumptions in the "rapid growth" case is close to the actual productivity growth experienced in Sweden during the 1950s and the 1960s.

In comparison with the experience during the period 1950-1972, the base case represents a reduction of the growth of GNP and, in particular, energy consumption.[1/] Yet the level of energy consumption in the year 2000 is 43% higher than the level compatible with the target growth rate for energy consumption mentioned in the introductory section.

The growth of real private consumption is only slightly below the "normal" postwar figure. The declining industrial employment is a continuation of a postwar trend; labor productivity increases faster than production in industry and consequently the demand for labor decreases in that sector.

The increasing share of consumption in GNP is the result of a gradual improvement of Sweden's terms of trade in the base case. This outcome to a large extent depends on the assumptions made about the world market prices of industrial goods as well as the assumptions about the price elasticity parameters in the foreign trade functions. If, for instance, the world market price of the output from "basic processing industries" is assumed to decrease by 1% per annum in real terms rather than remain constant, the growth of real private consumption is reduced by 0.6 percentage points per annum.

In the "rapid growth" case the growth of GNP is "normal" according to postwar standards. However, as in the base case the increase in the "energy" intensity of GNP is considerably slower than during the period 1950-1972. In order to discuss this result, it is appropriate to decompose the total base case change in energy consumption between 1980 (t=0) and 2000 (t-T) into a

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[1/] During this period the average annual growth rates for GNP and energy consumption were 3.6% and 5% respectively. Thus the "energy intensity" of GNP grew by approximately 1.4% per annum.

number of components. The following identity is then utilized:

$$\underbrace{E(T) - E(0)}_{\text{TOT}} = \underbrace{\sum_{i=0}^{7} e_i(0)[\bar{X}_i - X_i(0)]}_{\text{VOL}} + \underbrace{\sum_{i=0}^{7} e_i(0)[X_i(T) - \bar{X}_i]}_{\text{COMP}} +$$

$$+ \underbrace{\sum_{i=0}^{7} [e_i(T)-e_i(0)]X_i(T)}_{\text{INP}} + \underbrace{[C_0(T)-C_0(0)]}_{\text{DIR}}$$

where the variable $\bar{X}_i$ represents the hypothetical production in sector i if aggregate production is equal to aggregate production at t+T and the composition of aggregate production is equal to the composition of aggregate production at t=0[1/], and

TOT: the total change in energy consumption;

VOL: the change in energy consumption due to change in aggregate production, provided aggregate production is composed in the same way as at the initial point in time;

COMP: the change in energy consumption due to change composition of aggregate production;

INP: the change in energy consumption due to changed energy input coefficients;

DIR: the change in energy consumption due to changed direct consumption of energy in the household sector.

[1/] Thus $\bar{X}_i = \frac{X_i(0)}{X(0)} X(T)$ where $X(t) = \sum_{i=0}^{7} X_i(t)$.

This formula was used in conjunction with the results obtained in the base case simulation. The results are presented in Table 7.

Table 7. Decomposition of the total change in energy consumption, 1980-2000, in the base case.*

TOT	=	VOL	+	COMP	+	INP	+	DIR
9.2		5.1		1.4		-.5		3.2

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*Expressed in 10^9 SKr at 1968 prices.

Behind the positive figure denoted COMP in Table 7 are primarily two counteracting trends. One is that the production in "basic processing industries" grows slower than aggregate production, which tends to reduce the energy intensity of GNP (see Table 4). This development is due to an absolute decline by 1.2% per annum of exports from this sector. In turn, this depends on an unfavourable development of domestic production costs in this sector in relation to world market prices. The other trend is the relatively rapid growth of the production of "housing services", which tends to increase the energy intensity of GNP.

Both these trends seem reasonable; but still there is reason to believe that the COMP figure in Table 7 is somewhat to low. This is because the structure of intersectoral deliveries, except for deliveries from the energy sector, is kept constant during the period 1980-2000. During the first postwar decades there was a trend towards more input of industrial goods per unit of output in the service sectors. A continuation of such a trend would increase the growth of production in "manufacturing industries" and, as a result "basic processing industries", thus increasing energy intensity of GNP. However, a *ceteris paribus* 10% increase of production in the latter sector in the year 2000 would only increase the COMP figure in Table 7 from 1.4 to 1.7.

The negative figure denoted INP in Table 7 reflects a reduction in energy input coefficients by less than 0.5 percentage points per annum. In comparison to the postwar experiences these figures seem fairly low. During the period 1950-1972 the industry's average energy input coefficient declined by 2.1% per annum in spite of an annual decrease of energy prices by 2.9% in real terms.[1/] The figure denoted DIR reflects an annual growth at 3.6% of direct consumption in the household sector. Behind this figure are the relatively rapid growth of real private consumption and a comparatively high income elasticity for energy.

On balance the base case consumption of energy per unit of GNP might represent an underestimation, but the opposite is also possible. If the base case figures are accepted, energy consumption in Sweden can be expected to grow more slowly for the rest of this century than during the first three postwar decades. That also holds in the case with "rapid growth" assumptions.

However, the "optimistic" GNP growth assumption in conjunction with such assumptions about technical change in the energy sector that the price of energy continues to decrease by 2.9% per annum leads to an annual GNP growth rate at 3.7% and an annual energy consumption growth rate at 4.7% in the model simulation. These figures are quite close to postwar averages. On the basis of these results it seems that the relatively small difference between the target energy consumption growth rate proposed by the government and the expected growth rate at "unchanged energy policy" and base case assumption primarily depends on the reduction in the growth of GNP together with slightly increasing energy prices. In any case, the difference between the target energy consumption growth rate proposed by the government and the growth rate obtained in the base case model simulation is only 1.8 percentage points per annum between 1980 and 2000, which is considerably less than expected when the 1975 government proposal was presented.

[1/] In the base case simulation there was a slight increase in the price of energy.

V.2 The Impact of a Constraint on Energy Consumption

In this section it is assumed that a constraint is imposed on energy consumption. Thus, energy consumption is allowed to grow by 2% per annum between 1980 and 1985 and then remain constant. This policy is implemented by a value tax on all energy purchases. The tax revenues are assumed to be immediately distributed to the private sector. Thus, the energy tax only affects the relative market price of energy, while the size of the public sector is unaffected by the energy policy measures.[1/]

In Table 8 the main results, obtained in the "base" and the "rapid growth" cases with a constraint on energy consumption, are summarized.

Table 8. Calculated values of selected macroeconomic variables in the year 2000 under various assumptions about productivity growth and energy policy, 1980 = 100.

	The "base" case		The "rapid growth" case	
	No constraint on energy consumption	Constraint on energy consumption	No constraint on energy consumption	Constraint on energy consumption
GNP	148	147	202	196
Real household consumption	174	174	243	238
Energy consumption	163	110	231	110
Industrial employment	60	58	77	58

[1/] This implies that there are no direct costs for the implementation of the energy policy measures. In the real world a number of additional civil servants would probably have to be employed.

On the basis of the results presented in Table 8, energy policy of the kind discussed here seems to have a minor impact on the rate of economic growth. In the "base" case the effect corresponds to less than 1% of GNP at the year 2000, while the corresponding figure is 3% in the "rapid growth" case.

The impact of the energy policy can also be expressed in terms of the additions to the average working week which are necessary in order to fully compensate for the impact of the energy policy measures on GNP. In the "base" case, this figure is 3/4 hour per week and in the "rapid growth" case, 1 hour per week at the year 2000.

Obviously the economic impact of the energy policy measures to a large extent depends on the substitutability of energy and other factors of production. For this reason, the analysis of the base case is carried out for two additional sets of assumptions about the substitutability of energy and composite capital-labor. In one case the technology is said to be "rigid" in terms of energy input coefficients. Thus, the elasticity of substitution between energy and the composite capital-labor input is assumed to be 0.1 in all sectors. In the other case, where the technology is said to be "flexible", the corresponding figure is 0.5.

Given the other base case assumptions, including the assumption about no constraint on energy consumption, the rate and pattern of economic growth is practically the same in the "rigid" and the "flexible" case as in the base case. That also applies to energy consumption.[1/] Thus, the impact of the energy policy measured in the two cases can easily be compared.

In the "rigid" case the constraint on energy consumption reduces the rate of GNP growth by 0.1 percentage point per annum. This means that, as compared with a case without a constraint on energy consumption, the level of GNP by the year 2000 is about 2%

[1/] The rate of economic growth is slightly more rapid in the "flexible" case than in the "rigid" case.

lower in this case. In the "flexible" case the corresponding figure is lower than that of the "base" case.

These results are somewhat surprising. Even more surprising is perhaps that the energy consumption constraint has practically no impact on aggregate real consumption, either in the "rigid" or in the "flexible" case. This is because the slower growth of oil imports, resulting from the slower growth of energy consumption, leads to an improvement in the terms of trade.1/ Thus, the impact on the level of consumption by the reduction in GNP is almost entirely offset by an increase of the share of consumption in GNP.2/

Although the energy policy tends to reduce employment in industry, and in particular "basic processing industries", the results indicate that a constraint on energy consumption has a very small macroeconomic impact over a period of 20 years. However, the energy strategy has an impact on the economy, and that impact differs considerably between the "rigid" and the "flexible" cases. In Table 9 the difference in energy consumption is decomposed using the formula presented on p. 32. The results indicate that the nature of the adjustment mechanism to a large extent depends on the substitutability of energy and the composite capital-labor input.

1/World market oil prices are assumed to increase by 2% per annum between 1980 and 1990 and by 5% per annum between 1990 and 2000. The world market prices of other traded commodities are assumed to remain constant in real terms.

2/The same mechanism is at work in the "rapid growth" case. That can be shown in the following way. Real private consumption is about 50% of GNP. The development of aggregate net investment and public consumption are exogenously determined in the model. Thus, provided that share of net exports in GNP is constant, the impact of the energy policy on private consumption should be about twice as big as the impact on GNP. As can be seen in Table 8, that is not the case. Consequently the energy policy tends to improve the terms of trade and, thus, increase the share of private consumption in GNP.

Table 9. Percentage shares of the reduction in energy consumption by the year 2000, resulting from a constraint on energy consumption that can be assigned to various components under various assumptions about the substitutability of energy and composite capital-labor.

Elasticity of substitution	VOL	COMP	INP	DIR
0.10	24	6	32	39
0.25	13	6	57	23
0.50	2	5	79	13

In the "rigid" case the reduction of direct energy consumption in the household sector is the quantitatively most important part of the total change in energy consumption. Due to gradually increasing energy taxation the market price of energy grows by 10% per annum in this case. As a result, direct consumption of energy in the household sector grows only by 0.6% per annum as compared to 3.9% in the case without a constraint on total energy consumption. The energy input coefficients in the production sectors are not very much affected by the increasing market price of energy. They decline by less than 1% per annum in all sectors. As a result, reductions in energy input coefficients represent a fairly limited share of the total adjustment. Changes in the structure of the production system represent an even smaller share of the change in energy consumption. Nevertheless the energy policy leads to a more rapid reduction of industrial employment: -3.0% per annum as compared to -2.6% per annum in the case without constraint on total energy consumption.

In the "flexible" case energy input coefficients decline by 2.2 - 3.2% per annum. As a result, almost 80% of the change in energy consumption can be assigned to substitutions of the composite capital-labor input for energy in the production sectors.

However, the decrease in energy input coefficients is accomplished primarily by means of input of more capital. This capital is available as a result of the reduced growth of the energy sector. One can say that capital is used for "energy conservation" rather than for energy production purposes.

In this case the growth of direct consumption of energy in the household sector is not affected by the energy policy to the same extent as in the "rigid" case. The annual growth rate is 2.8%, that is, the reduction due to the energy policy is slightly more than one percentage point per annum. Consequently only 13% of the total change in energy consumption can be assigned to changes in direct consumption of energy in the household sector.

In both the "rigid" and the "flexible" cases higher energy taxes tend to reduce exports from "basic processing industries" and increase exports from "manufacturing industries". However, the resulting impact on the sectoral allocation of employment is not significant. The reason for this is that the reduced growth of the capital intensive energy sector leaves a larger share of net capital formation to be used as a substitute for energy in the production sectors. If the price elasticity of export demand is higher than assumed in the base case, domestic energy taxation tends to have a significant impact both on the structure of the production system and the sectoral allocation of the labor force. The results from a few experiments can be mentioned.

If the price elasticity of the demand for exports from the industrial sectors is assumed to be -5 rather than -3 and -1.5 respectively, an annual increase of the energy tax rate with 10 percentage points would reduce the growth of production in "basic processing industries" by 0.4 percentage points. The corresponding figures for employment and exports would be 0.3 and 0.9 respectively. If the price elasticity figures are assumed to be -10, the corresponding values become 1.9, 1.8 and 3.2.

According to the 1975 government proposal the reason for imposing a constraint on energy consumption growth is the side effects associated with conventional fuels and electricity generation technologies. In the model simulations the target energy consumption growth rate was attained by means of a general value tax on energy purchases. The tax rate, which is endogenously determined in the model, indicates the marginal value in excess of production costs of one unit of energy. Thus the tax rate can be interpreted as a shadow price of "clean and safe energy", that is, the marginal willingness to pay for one unit of energy from a source without the side effects associated with conventional energy sources.

At the year 2000 the endogenously determined energy tax rate was 137% in the "flexible" case, 398% in the "base" case, and 871% in the "rigid" case. These results indicate the importance of the substitutability of energy and other factors of production. They also show that over a period of 20 years, a constraint on energy consumption growth is likely to create substantial economic incentives for the development of energy sources without negative environmental and safety side effects.

So far the model results seem to indicate that attainment of the target growth rate for energy consumption proposed by the Swedish government would not have significant negative effects on conventionally measured economic growth. However, the picture becomes a little bit different if the inpact is studied year by year rather than for the entire period 1980-2000. It then turns out that under "base" case assumption the energy policy measures have practically no impact on GNP until the last five-year period. A similar pattern can be seen in the development of factor prices (Table 10).

Table 10. Reduction in the annual rates of change of wages and profits in the "base" case due to the constraint on total energy consumption percentage points.

	1980	1990	2000
Wage-index*	-0.1	-0.4	-0.5
Profit-index**	-0.3	-0.8	-1.2

* The variable W in the model.
** The variable R in the model.

Thus, for some time energy consumption can be kept constant without significant reductions in the rate of economic growth. As time goes by, however, such a policy leads to a change in the economy's aggregate factor proportions; more capital is accumulated but it has to be combined with a constant amount of energy and, under the base case conditions, a slowly decreasing labor force. Accordingly the "law of diminishing returns" comes into operation. Wages and, in particular, profits are negatively affected and the rate of economic growth is reduced. Over time those effects become increasingly important, and more so the less flexible the technology is. However, over a 20-year period, the constraint on energy consumption does not seem to have significant effects on economic growth.

This conclusion is, however, subject to at least one important qualification. The reduction in the rate of profit due to the energy policy measures may not be compatible with the assumption about a constant net savings ratio. At least some additional policy measures may be needed in order to prevent a drop in total net investments. If it is assumed that such measures are not implemented and that the tendency towards reduced profits is offset by a drop in net investments, then the constraint on energy consumption leads to an additional reduction in economic growth. Under base case conditions such investment behavior leads to an additional reduction in GNP growth by, on the average, 0.3 percentage points per annum 1980-2000. This means that the level of GNP at the year 2000 should be reduced by another 6 percentage points.

VI. SUMMARY AND CONCLUSIONS

The purpose of this study has been to investigate to what extent there is a conflict in a small open economy between economic policy goals related to the growth of GNP or similar measures, and an energy policy aimed at zero growth in energy consumption. The analysis has focused on Sweden, a small economy with a relatively large foreign trade sector. In Sweden energy policy presently aims at reducing the growth of energy consumption to 2% per annum to 1985 and to zero growth thereafter provided such an energy policy is not in conflict with other social and economic goals. The analysis has been carried out by means of a numerically formulated model of the Swedish economy, and it has been focused on the period 1980-2000.

The results indicate that for the studied 20-year period, the target energy consumption growth rate can be attained without significant costs in terms of GNP or aggregate household consumption losses. In addition the energy policy did not lead to significant changes in the sectoral allocation of the labor force. This is because it was primarily capital available as a result of the reduced growth of the capital intensive energy sector that was used as a substitute for energy in the production sectors. However, the negative impact on economic growth increases over time. If the energy consumption is kept at the 1985 level during 5 or 10 additional years, the reduction in the rate of economic growth tends to be substantial.

The model simulations were carried out under the assumption that the net savings ratio in the economy remains constant over the period in question. Since one effect of the simulated policy measures was that profits tended to decrease, this assumption might seem dubious. The tendency towards falling profits might lead to a reduction in the net savings ratio, so that the proposed energy policy has an additional, indirect impact on economic growth. If, as an extreme example, the tendency of falling profits is completely balanced by reductions in total net investments, the previous conclusions have to be somewhat modified. Under base case assumptions, at the year 2000 the level of GNP is 7% lower in the case with a constraint on energy consumption. When capital formation was treated as an exogenous variable, the corresponding figure was 1%.

This case is extreme for two reasons. First, the energy policy measures can be combined with other measures for preventing the fall in profits. The existing tax system has a number of parameters which could be used for such purposes. Second, an important class of investment opportunities does not exist in the model economy: investments in R & D activities. This point, perhaps, needs some clarification.

In the model economy the target energy consumption growth rate was attained by means of a tax on energy consumption. At the year 2000 the tax rate, which kept energy consumption at the target level, varied between 137% and 871%, depending on the assumption made about the elasticity of substitution between energy and composite capital-labor. Energy tax rates of this order of magnitude obviously would create economic incentives for the development of new energy sources and energy conservation methods. It is quite possible that a number of R & D investments in these fields would turn out to have a high rate of return. That is, by means of R & D investments the shape of the production functions would be changed so that the negative impact on economic growth of the energy policy would be mitigated and the tendency towards falling profits counteracted.

As expected, the proposed energy policy turned out to have a larger impact on economic growth where the elasticity of substitution between energy and composite capital-labor was low. In particular this applied on the sectoral level.

When the elasticity of substitution was assumed to be 0.50 in all sectors, neither the structure of the production system nor the commodity composition of household consumption was significantly affected. Thus, from a welfare point of view, GNP and aggregate household consumption has roughly the same meaning in the case with the energy policy measures as in the case without such measures.

However, when the elasticity of substitution was assumed to be 0.1, attainment of the target energy consumption development was accompanied by significant changes in the commodity composition of household consumption. In addition the rate of reduction of industrial employment was increased by the energy policy measures. This means that changes in aggregate measures such as GNP or aggregate household consumption become more difficult than usual to evaluate from a welfare point of view.

Obviously, the assumption about the substitutability of energy and other factors of production is an important one. On the basis of the econometric literature in this field it is difficult to say what would be the most realistic assumption in a study like this. However, the econometric results indicate that 0.10 is a rather "pessimistic" assumption, while 0.50 does not seem to be overly "optimistic".

Although reservations can be made it seems that energy consumption in Sweden can be kept at the target development path proposed by the government at least during a period of 10-15 years without significant conflicts with other social and economic goals. Whether this is an "optimal", or justifiable, energy policy is another question, beyond the scope of this study.

It does not seem worthwhile to extend the analysis to the period after the year 2000. If the development of the model economy is simulated over a number of additional decades, with given technology and with the level of energy consumption kept at the 1985 level, it eventually collapses. But the technology cannot be regarded as given and constant over time. This is especially the case in a period where relative prices change substantially. R & D activities are likely to contribute to the development of new energy sources, new energy conservation methods and more flexible production techniques. In addition, they might lead to better methods of handling the side effects of existing energy sources, thereby removing the motive for an energy policy of the kind discussed in this study. This does not, of course, mean that everything is fine a few decades into the next century. It only means that no conclusions about that period can be made on the basis of this study.

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Appendix: The derivation of the equations of the linearized version of the model

Equations M1:0 - M1:7

The relative rate of change of production can be written

$$(D1)\quad x_i = \frac{\partial X_i}{\partial F_i} \cdot \frac{F_i}{X_i} f_i + \frac{\partial X_i}{\partial X_{0i}} \cdot \frac{X_{0i}}{X_i} x_{0i} \quad ; \qquad i = 0,1,\dots,7 \quad .$$

Differentiation of (1) with respect F_i and X_{0i}, respectively, yields

$$(D2)\begin{cases} \dfrac{\partial X_i}{\partial F_i} \cdot \dfrac{F_i}{X_i} = \gamma_i \left(\dfrac{A_iF_i}{X_i}\right)^{\rho_i} \\ \\ \dfrac{\partial X_i}{\partial X_{0i}} \cdot \dfrac{X_{0i}}{X_i} = (1 - \gamma_i) \left(\dfrac{A_iX_{0i}}{X_i}\right)^{\rho_i} \end{cases} \qquad i = 0,1,\dots,7 \quad .$$

Taking logs of (2) and differentiation with respect to time yields

$$(D3)\quad f_i = \alpha_i k_i + (1 - \alpha_i) n_i + \lambda_i \quad ; \qquad i = 0,1,\dots,7 \quad .$$

Using (10) - (12) and substitution of (D2) and (D3) in (D1) yields

$$(D4)\quad x_i = \frac{Q_iK_i}{P_i^*X_i} k_i + \frac{W_iN_i}{P_i^*X_i} n_i + \frac{W_iN_i + Q_iK_i}{P_i^*X_i} \lambda_i + \frac{T_iP_0X_{0i}}{P_i^*X_i} x_{0i}$$

$$i = 0,1,\dots,7 \quad .$$

(D4) can then be written

$$(\text{M1}:i) \qquad x_i - \frac{\alpha_i}{1-\alpha_i} \frac{\omega_i W N_i}{P^*_i X_i} k_i - \frac{\omega_i W N_i}{P^*_i X_i} n_i - \frac{T_i P_0 X_{0i}}{P^*_i X_i} x_{0i} =$$

$$= \frac{1}{1-\alpha_i} \frac{\omega_i W N_i}{P^*_i X_i} \lambda_i \quad ; \quad i = 0,1,\dots,7 \quad .$$

Equation M2

This equation is obtained directly from(28) by differentiation with respect to time.

Equations M3 and M4:i

Taking logs and differentiation of (10) with respect to time yields

$$(\text{D5}) \qquad p^*_i + (1 - \rho_i) x_i + \rho_i f_i = w_i + n_i \quad ; \quad i = 0,1,\dots,7 \quad .$$

Differentiation of (6) with respect to time yields

$$(\text{D6}) \begin{cases} p^*_0 = \dfrac{1}{P^*_0} \left[P_0 p_0 - \sum\limits_{j=1}^{5} P_j a_{j0} p_j - V \bar{P}_0 \bar{b}_0 (v + \bar{p}_0) \right] \quad ; \\[2em] p^*_i = \dfrac{1}{P^*_i} \left[P_i p_i - \sum\limits_{j=1}^{7} P_j a_{ji} p_j \right] \quad . \end{cases}$$

Next we define $A_{ji} = e_{ji} - a_{ji}$ where $e_{ji} = \begin{cases} 1 \text{ when } i=j \; ; \; j = 1,2,\dots,7 \\ 0 \text{ when } i \neq j \; ; \; i = 0,1,\dots,7 \; . \end{cases}$

Eq. (D6) can then be written

$$\text{(D6)}\quad \begin{cases} P_0^* = \dfrac{P_0}{P_0^*}\, p_0 - \sum\limits_{j=1}^{5} \dfrac{P_j a_{j0}}{P_0^*}\, p_j - \dfrac{V\bar{P}_0\bar{b}_0}{P_0^*}\, v - \dfrac{V\bar{P}_0\bar{b}_0}{P_0^*}\, \bar{p}_0 \quad ; \\[2em] p_i^* = \sum\limits_{j=1}^{7} \dfrac{P_j A_{ji}}{P_i^*}\, p_j \quad . \end{cases}$$

Eq. (14) yields

$$\text{(D8)}\qquad w_i = w \quad ; \qquad i = 0,1,\ldots,7 \quad .$$

Substitution of (D3), (D7) and (D8) in (D5) then yields

$$\text{(M3)}\quad \frac{P_0}{P_0^*}\, p_0 - \sum_{j=1}^{7} \frac{P_j a_{j0}}{P_0^*}\, p_j - \frac{V\bar{P}_0\bar{b}_0}{P_0^*}\, v + (1 - \rho_0)x_0 + \rho_0\alpha_0 k_0 +$$

$$+ (\rho_0 - \alpha_0\rho_0 - 1)n_0 - w = - \rho_0\lambda_0 + \frac{V\bar{P}_0\bar{b}_0}{P_0^*}\, \bar{p}_0 \quad ;$$

$$\text{(M4:i)}\quad \sum_{j=1}^{7} \frac{P_i A_{ji}}{P_i^*}\, p_j + (1-\rho_i)x_i + \rho_i\alpha_i k_i + (\rho_i - \alpha_i\rho_i - 1)n_i - w = - \rho_i\lambda_i \quad ;$$

$$i = 1,2,\ldots,7 \quad .$$

Equation M5:i

Taking logs of (11) and differentiating with respect to time yields

$$\text{(D9)}\qquad p_i^* + (1-\rho_i)x_i + \rho_i f_i = q_i + k_i \quad ; \qquad i = 0,1,\ldots,7 \quad .$$

Substitution of (13) in (8) and differentiation with respect to time yields

$$\text{(D10)} \qquad q_i = \frac{P_8(\delta_i + \beta_i R)}{Q_i}\, p_8 + \frac{P_8 \beta_i R}{Q_i}\, r \quad ; \qquad i = 0,1,\ldots,7 \quad .$$

Substitution of (D5), (D8) and (D10) in (D9) yields

$$\text{(M5:i)} \qquad \frac{P_8(\delta_i + \beta_i R)}{Q_i}\, p_8 + \frac{P_i \beta_i R}{Q_i}\, r + k_i - w - n_i = 0 \quad ; \ i = 0,1,\ldots,7 \quad .$$

Equations M6 and M7:i

Taking logs of (12) and differentiating with respect to time yields

$$\text{(D11)} \begin{cases} p_0^* + (\rho_0 - 1)x_{00} - (\rho_0 - 1)x_0 = p_0 \quad ; \\ \\ p_i^* + (\rho_i - 1)x_{0i} - (\rho_i - 1)x_i = p_0 + t_i \quad ; \quad i = 1,2,\ldots,7 \quad . \end{cases}$$

Differentiation of (30) with respect to time

$$\text{(D12)} \qquad t_i = \frac{1}{T_i}\,(\dot{\tau} + \dot{\xi}_i) \quad ; \qquad i = 1,2,\ldots,7 \quad .$$

Substitution of (D7) and (D12) in (D11) and rearrangement of terms yields

$$\text{(M6)} \quad \frac{P_0 - P_0^*}{P_0^*}\, p_0 - \sum_{j=1}^{7} \frac{P_j a_{j0}}{P_0^*}\, p_j - \frac{V\bar{P}_0 \bar{b}_0}{P_0^*}\, v + (\rho_0 - 1)x_{00} - (\rho_0 - 1)x_0$$

$$= \frac{V\bar{P}_0 \bar{b}_0}{P_0^*}\, \bar{p}_0 \quad ;$$

$$(\text{M7:i}) \quad \sum_{j=1}^{7} \frac{P_j A_{ji}}{P_i^*} p_j + (\rho_i - 1) x_{0i} - (\rho_i - 1) x_i - p_0 - \theta \frac{\dot{\tau}}{T_i} = (1-\theta) \frac{\dot{\tau}}{T_i} + \frac{\dot{\xi}_i}{T_i} \quad ;$$

$$i = 1,2,\ldots,7 \quad ;$$

where θ is either 1 or 0 .

Equations M8 - M17

These equations are obtained directly from (7), (15), (17), (18), (19), (4), (29), (25), (16) and (20) respectively by differentiation with respect to time.

Equation M18

Differentiation of (31) with respect to time yields (18)

$$(\text{M18}) \quad (1 - \theta)\, Ee - X_0 x_0 - M_0 m_0 = -\theta Ee$$

where θ is either 1 or 0.

Equation M19:i

Differentiation of (21) and using the definition of A_{ij} yields

$$(\text{M19}) \quad \sum_{j=0}^{7} A_{ij}\, X_j\, x_j - C_i c_i - Z_i z_i + M_i m_i = 0 \quad ; \quad i = 1,2,\ldots,5 \quad .$$

Equations M20 - M24

These equations are obtained from (22), (23), (24), (27) and (26) respectively by differentiation with respect to time.