THE ECONOMICS OF RISKS TO LIFE

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November 1978 WP-78-49

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PREFACE

IIASA has many reasons to be interested in the economics of risks to human life: the potential costs of nuclear hazards, the benefits of improved safety in industry, the possible payoffs of medical research, the pros and cons of a longevity that is gradually increasing. This paper develops a consistent method to evaluate change in mortality risk and loss or saving of human life.

This analysis, developed in the System and Decision Sciences Area, complements other work done in the Energy Program's Joint IIASA/IAEA Risk Project and in Management and Technology's Safety Standards Task.

The author would like to thank Neil Ericsson, Joanne Linnerooth, and Nathan Keyfitz for valuable discussions on the ideas of this paper.

ABSTRACT

This study asks two questions:

(1) What is the net value to the representative individual over his life-time of activities that alter age-specific mortality risks?

(2) What is the cost to the representative individual of activities that take a life at random at a given age?

Results, derived from an economic-demographic model with full age-specific accounting have a strong actuarial flavor: alterations in the mortality schedule, caused say by a medical breakthrough, should be assessed on the utility of expected additional life-years, production, and reproduction, less expected social costs of support. Loss of life at a specific age, due to an accident say, should be assessed on the opportunity costs of expected lost years of living, lost production and reproduction, less expected social support costs.

The results show that current methods, in general, leave out an important social transfer term, that "value of a life lost" is highly age-dependent, and that the degree of diminishing returns to consumption is crucial in calculations of the economic cost of risks.

1. INTRODUCTION

One of the more difficult questions the economist faces is how to assess activities -- engineering projects, safety procedures, medical advances -- that raise or lower risks to human life. It is clear that in most situations proper safety should be a matter of degree: engineering constructions should neither be infinitely solid nor excessively flimsy. But how safe is safe enough? What meaning can we give to phrases such as "the value of life" or "the cost of hazards to life"? And what are the economic consequences of the fact that mortality risks are gradually falling -- that life is lenghtening?

Most writing on the evaluation of risks to life falls into the large project evaluation literature. (For surveys see Linnerooth (1975) and (1978)). This literature concerns itself both with the social cost of projects that might increase risk of death: siting of nuclear power stations, large dams, and engineering constructions; and with the worth of activities that might prolong life: design of safer highways, changes in air safety regulations, and research to conquer cancer and heart diseases. Whichever type of activity is examined -- prolonging life or shortening it -- the arguments are symmetrical. Most of this literature bypasses consideration of individual lives lost or saved; instead it focuses on changes of overall risk of death be they positive or negative.

Two points of view dominate this evaluation-of-mortality-risk literature: the socalled "human-capital method", and the "willingness-to-pay" approach. Let us look at them in turn.

The human capital approach, designed to measure social loss due to increased accident or disease, has been proposed in various forms by Weisbrod (1961), Fromm (1962), Rottenberg (1967), Ridker (1970), and others. The basic method sums earnings foregone by individuals lost through death or incapicitation, and sets these against the net economic benefits of the project. Whether these earnings should be net of the individual's consumption or not has been the subject of some contention. Because the method is both conceptually simple and easily quantified, it is in widespread use by engineers and government agencies.

As a measure of social loss from projects that put life under risk, the human capital method is not without economic appeal. What it measures is the loss in national product attributable to mortality increase. Yet in recent years it has been roundly criticized by economists: from a welfare theory viewpoint it is founded on thin logic. As Schelling (1968) has argued, by concentrating only on GNP loss it ignores the individual's own desire to live. Under the human capital approach, a medical breakthrough that prolonged life from 70 to 80 years, for example, would have no social justification -- it would not raise GNP.

The willingness-to-pay method, proposed by Mishan (1971), does recognize the natural desire to live longer. Under this method a scheme that increased life from 70 to 80 would be socially justified if those who benefited were willing, in theory at least, to pay more for their extra years than the cost of the project. Wider social benefits, to close relatives for example, would be included by assessing willingness to pay for increased life of loved ones. This method, however, based on welfare utilities and not on dollar earnings, has obvious difficulties of quantification. Recently Conley (1973), Usher (1971) and Jones-Lee (1974), have proposed separate methods to put the criterion on a quantitative footing, by modeling the rational person's willingness to buy extra years and valuing it in dollar terms.

Both methods in current use, whether based on welfare theory or not, suffer a major deficiency. They are fundamentally partial-equilibrium approaches. They ignore the chain of wider economic transfers set up through society when life is lengthened. To return to the example, willingness-to-pay would approve an advance in life from 70 to 80 years if those affected and their kin were willing to pay the cost of the increase. Forgotten however, is that prolongation of life is not costless to wider society: those who live longer, consume longer, and this extra consumption must be financed by transfers from people at younger productive ages. Proper accounting would include, therefore, intergenerational transfer costs, felt in this case as a heavier social security burden on the young.

Ideally, in evaluating an activity that tends to prolong life, one would start with complete general-equilibrium knowledge of the economy and assess net benefits to each individual by tracing through the complex chain of social transfers resulting from the activity. The project could then be judged on some standard welfare basis, such as the compensated welfare criterion. In practice of course such knowledge is not available; this study instead constructs a model, which although stylized, does capture the important social transfers.

This study asks two questions:

(1) What is the net value to the representative individual over his life-time of activities that alter his life-risk -- activities that change the shape of his mortality age-schedule?

(2) What is the cost to the representative individual of activities that take a life at random at a given age?

Results are summarized in formal expressions for the net social worth of mortality variations and of age-specific lives lost. They show that, first, under the chosen life-cycle criterion, when risks are altered -- lowered say -- the utility value of expected additional life-years, production, and reproduction must be set against expected additional social support costs. Second, when an activity causes loss of life, age-incidence is all important: loss of life at age 30 is in general socially much more costly than loss of life at age 80. Third, the results show that (a) the willingness-to-pay criterion overstates the value of prolonging life and (b) under certain restricted circumstances the popular, but maligned human-capital method is in fact correct. Finally, if we are willing to put quantitative bounds on the tradeoff between additional lifeyears versus additional consumption, we can derive a numerical bracket in which the cost of lives lost must fall.

2. THE ECONOMICS OF CHANGES IN MORTALITY RISK

To set the context for the analysis I first set up a neoclassical, age-specific model of the economy and population. Within this model, the effect of a change in the mortality pattern on lifecycle wellbeing is then derived. Both population and economy are assumed in steady state growth, individuals are alike in tastes and behavior, and production shows constant returns. Later, I shall discuss whether the results hold up when these assumptions are relaxed.

Neoclassical Model

Begin with the economy. Output is produced by combining capital K and labor L in a constant-returns production function F. The economy stores no consumption goods. Output is split into consumption and investment in capital growth. Thus

$$(1) \qquad F(K(t),L(t)) = C(t) + \overset{0}{K}(t) \quad , \qquad C(t) \geqq 0 \quad .$$ [1]

For the population we need a fair degree of age-specific detail. The population grows according to the Lotka dynamics

$$(2) \qquad B(t) = \int_0^{\omega} B(t - x)p(t,x)m(t,x)\,dx$$

where B is births per unit time, p is the proportion who survive to age x at time t, and m is the proportion reproducing at age x, time t; ω is an upper bound on the length of life and the initial birth sequence is assumed given. This year's flow of births, in other words, is produced by those who were born x years ago and have survived to reproduce.

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[1] F is assumed concave, first-degree homogeneous and continuously differentiable; for simplicity capital depreciation is ignored.

Assume the population is *stable*,[2] and is growing exponentially at rate g. In this case equation (2) has the solution

$$B(t) = B(0)e^{gt} \tag{3}$$

where the growth rate g is connected to mortality p and fertility m by substituting (3) in (2) and cancelling B to yield

$$1 = \int_0^\omega e^{-gx}\, p(x)m(x)\,dx \quad . \tag{4}$$

If $\lambda(x)$ is the age schedule of labor participation, labor force L and total population N are given by

$$L(t) = \int_0^\omega B(t-x)p(x)\lambda(x)\,dx = B(t)\int_0^\omega e^{-gx}\, p(x)\lambda(x)\,dx \tag{5}$$

$$N(t) = \int_0^\omega B(t-x)p(x)\,dx = B(t)\int_0^\omega e^{-gx}\, p(x)\,dx \quad . \tag{6}$$

The labor/population ratio L/N and the birth rate B/N will be denoted by h(g) and b(g) respectively.

Individual consumption varies with age, as do the mortality, fertility, and labor participation rates above. (How it varies is determined below.) Putting population and economic variables together, we can express total consumption C as the sum of individual age-related consumption c(t,x) by

$$C(t) = \int_0^\omega B(t-x)p(x)c(t,x)\,dx \quad . \tag{7}$$

Later, we shall need three parameters: the average ages of producing A_L, consuming A_C, and reproducing A_M, in the population, defined by

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[2] That is, its age-specific rates of fertility and mortality and its normalized age-distribution are all constant over time; g is assumed positive.

$$A_L = \int_0^{\omega} x\, e^{-gx}\, p(x)\,\lambda(x)\,dx \Big/ \int_0^{\omega} e^{-gx}\, p(x)\,\lambda(x)\,dx$$

$$A_c = \int_0^{\omega} x\, e^{-gx}\, p(x)\,c(t,x)\,dx \Big/ \int_0^{\omega} e^{-gx}\, p(x)\,c(t,x)\,dx$$

$$A_m = \int_0^{\omega} x\, e^{-gx}\, p(x)\,m(x)\,dx \Big/ \int_0^{\omega} e^{-gx}\, p(x)\,m(x)\,dx \quad .$$

Assuming the economy has reached a Solow-type steady state, where the growth rate of the economy equals that of population and per capita variables are constant, and assuming investment is chosen to maximize total consumption, then

(8) $$\overset{0}{K}/K = g \quad ; \quad c(t,x) = c(x) \quad ; \quad \text{and} \quad F_K = g \quad .$$

One central fact in society is that consumption, which takes place at all ages, must be supported by production, which takes place only at labor-participative ages. The economy in other words functions at all times under the budget identity

(9) $$C \equiv F(K,L) - gK \quad .$$

that is,

$$\int_0^{\omega} B(t-x)\,p(x)\,c(x)\,dx \equiv (F/L - g\,K/L) \int_0^{\omega} B(t-x)\,p(x)\,\lambda(x)\,dx \quad .$$

Using (3) and dividing through by $B(t)$, with usual per-unit labor notation this societal budget constraint becomes

(10) $$\int_0^{\omega} e^{-gx}\, p(x)\,c(x)\,dx \equiv (f(k)-gk) \int_0^{\omega} e^{-gx}\, p(x)\,\lambda(x)\,dx \quad .$$

Thus intergenerational transfers are introduced by the inescapable requirement that, when growth, labor-participation rates, and the capital-output ratio remain unchanged, any increase in consumption for one age-group must be matched by decreases for other age-groups.

To complete the model, it remains to determine the lifecycle pattern of consumption. It is assumed that people individually allocate their consumption to maximize their expected lifetime welfare W, where

$$(11) \qquad W = \int_0^{\omega} U[c(x),x]p(x)dx \quad .$$

In aggregate, of course, they must do this in such a way that the societal budget constraint continues to hold at all times. The standard consumption-loan mechanism arranges this (Samuelson (1958)): a market interest rate appears which encourages people to distribute their consumption over their lifecycle in such a way that the social budget constraint is always met. The exact mechanisms of this need not concern us; it is sufficient to say that the individual spreads his consumption so that lifecycle welfare is maximized subject to (10) being met. Finding the lifecycle consumption pattern is thus a simple constrained variational problem, the solution of which yields

$$(12) \qquad \partial U/\partial c(x) = \partial U/\partial c(0)e^{-gx} \quad .$$

Thus lifecycle consumption is patterned according to age-related need so that its marginal usefulness is the same at all ages, modified only by the ability to invest at an interest rage g, which equals the rate of population growth. Condition (12) therefore is the continuous-age generalization of Samuelson's "biological interest rate" condition.

All preliminaries are now completed. Population and economic growth are well-defined ((3),(4) and (8)), as is the pattern of lifecycle consumption (12). And the social budget identity (10) connects the demography of consumption with that of production.

Change in Age-Specific Risks

We now introduce a particular, but small, age-specific change in age-specific risks, so that the mortality schedule $p(x)$ becomes $p(x) + \delta p(x)$, and derive the implications for our chosen criterion -- the representative person's lifetime utility, W.[1]

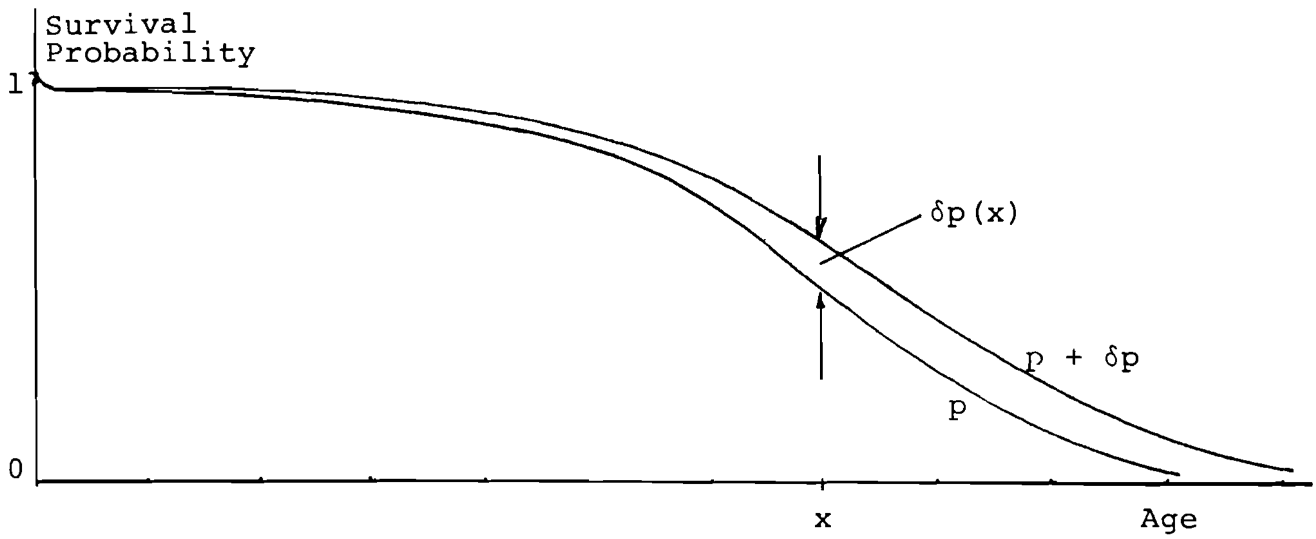

Figure 1.

Figure 1 illustrates $\delta p(x)$ for a decrease in the incidence of cancer (scale of $\delta p(x)$ exaggerated slightly). For convenience, I shall assume in this section that the mortality variation lengthens life; for shortened life the argument is symmetrical.

When the mortality schedule changes, several variables are forced to change with it: the growth rate g, the consumption

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1) A word on the choice of expected lifetime utility as the social criterion. It is quite legitimate to ask what are the consequences of risk-change for any arbitrary criterion. Suitability of a particular criterion depends on how well it "represents" social interests and on the "reasonableness" of the implications, both judgmental matters. Assuming risk-neutral individuals with identical tastes who fulfill the von Neumann-Morgenstern choice axioms, W is arguably representative. Reasonableness of implications will be judged later.

pattern $c(x)$, lifecycle utility W, and others. I shall write $\delta g[\delta p(x)]$ as the differential change[1] in growth due to the particular age-specific mortality variation $\delta p(x)$. Where the variation $\delta p(x)$ is understood, I shall simply write δg. Similar practice will be followed with other variables.

At this point some new notation will be useful. Let

$$
\begin{aligned}
U_{ex} &= \int_0^\omega U[c(x),x]\,\delta p(x)\,dx \quad , \\
c_{ex} &= \int_0^\omega e^{-gx}\,c(x)\,\delta p(x)\,dx \quad , \\
L_{ex} &= \int_0^\omega e^{-gx}\,\lambda(x)\,\delta p(x)\,dx \quad , \qquad (13) \\
v_{ex} &= \int_0^\omega e^{-gx}\,m(x)\,\delta p(x)\,dx \quad .
\end{aligned}
$$

The first three can be viewed as the expectations of extra utility from lengthened life, of extra lifetime consumption, and of extra man-years of production resulting from the particular variation $\delta p(x)$. The fourth, v_{ex}, is the change in reproductive value -- loosely speaking expected additional children per person due to the mortality variation. (The last three are discounted because future consumption utilities are later valued to date of birth.)

To derive $\delta g[\delta p(x)]$, the change in the intrinsic growth rate due to the mortality variation, recall equation (4):

$$1 = \int_0^\omega e^{-gx}\,m(x)\,p(x)\,dx \quad .$$

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[1] Technically $\delta g[\delta p(x)]$ is a Fréchet differential -- a differential whose argument is a function and not a single valued variable.

Using the appropriate chain rule

$$0 = \int_0^{\omega} e^{-gx}\, m(x)\, \delta p(x)\, dx \; - \; \delta g \int_0^{\omega} x\, e^{-gx}\, m(x)\, p(x)\, dx \quad ,$$

whence

$$(14) \qquad \delta g[\delta p(x)] = \frac{\int_0^{\omega} e^{-gx} m(x)\, \delta p(x)\, dx}{\int_0^{\omega} x\, e^{-gx}\, m(x)\, p(x)\, dx} = v_{ex}/A_m \quad .$$

The altered mortality pattern affects the growth rate by the change in reproductive value divided by the average age of motherhood (average length between generations). Note that if the mortality variation affects only post-reproductive ages, v_{ex} is zero, so that no change in the growth rate occurs.

We can now derive the change in expected lifetime welfare, $\delta W[\delta p(x)]$. From (11)

$$(15) \qquad \begin{aligned} \delta W &= \int_0^{\omega} U[c(x),x]\, \delta p(x)\, dx + \int_0^{\omega} \partial U/\partial c(x)\, \delta c(x)\, p(x)\, dx \\ &= \int_0^{\omega} U[c(x),x]\, \delta p(x)\, dx + \partial U/\partial c(0) \int_0^{\omega} e^{-gx}\, \delta c(x)\, p(x)\, dx \quad . \end{aligned}$$

Lifecycle welfare is changed directly by extra years and indirectly by the alteration in consumption pattern needed to accommodate these extra years. The latter can be evaluated by taking differentials across the societal budget identity (10). This yields, on collecting terms,

$$(16) \qquad \begin{aligned} 0 &= \int_0^{\omega} e^{-gx}\, c(x)\, \delta p(x)\, dx + \int_0^{\omega} e^{-gx}\, \delta c(x)\, p(x)\, dx - (f(k)-gk) \int_0^{\omega} e^{-gx}\, \lambda(x)\, \delta p(x)\, dx \\ &\quad + \delta k(f'-g) \int_0^{\omega} e^{-gx}\, \lambda(x)\, p(x)\, dx - \beta\, \delta g \quad , \end{aligned}$$

where

$$\beta = \int_0^{\omega} x e^{-gx} c(x) p(x) dx - (f(k)-gk) \int_0^{\omega} x e^{-gx} \lambda(x) p(x) dx - k \int_0^{\omega} e^{-gx} \lambda(x) p(x) dx \quad .$$

From the savings rule $f' = g$ the fourth term in (16) disappears. Where $\bar{c}$ is per capita consumption, β, the lifecycle value of a marginal increase in the growth rate, can be expressed as

$$(17) \qquad \beta = 1/b[\bar{c}(A_C - A_L) - k\,h] \quad .$$

Finally, using (16) to substitute for the second term in (15), and noting that for constant returns $f - gk$ is F_L, we obtain

$$\delta W = \int_0^{\omega} U[c(x), x]\, \delta p(x) dx + \frac{\partial U}{\delta c(0)} \left\{ F_L \int_0^{\omega} e^{-gx} \lambda(x)\, \delta p(x) dx - \int_0^{\omega} e^{-gx} c(x)\, \delta p(x) dx + \beta \delta g \right\}.$$

Reexpressed in more convenient notation this becomes our first main result. The net lifecycle utility value of an age-specific change in mortality risk is given by

$$(18) \quad \delta W = U_{ex} + \frac{\partial U}{\partial c(0)} \left\{ F_L \cdot L_{ex} - c_{ex} + v_{ex} \beta / A_m \right\} .$$

Lifecycle welfare increase	=	Utility of extra life-years	+	Value of extra labor years	-	Social cost of consumption upkeep	+	Value of additional children

The net individual lifecycle welfare increase thus consists of four components. When mortality is improved, the individual is blessed with extra years of life, extra years of productive work if pre-retirement years are affected, and extra children if reproductive years are affected. On the other hand extra years must somehow be supported. The third term shows the total amount of consumption support needed -- a burden on social security, or a burden on private savings earlier in life, or a burden on one's children, depending on the particular social arrangement that ensures support for the elderly.

These welfare changes occur at different periods in the lifecycle. Those at younger age-groups carry the consumption cost; only in later life do they reap the utility of extra years, the costs now turned over to a new generation. To the extent that population is growing, younger age-groups are larger than older ones and transfers toward later ages are easier on the individual; this is why the analysis discounts costs at rate g over the lifecycle in the above terms.

3. DISCUSSION

Any risk evaluation method must unavoidably compare two very different things: the enjoyment of additional living (U_{ex} in (18)), and the enjoyment of additional consumption (the terms within the bracket). It would be of great advantage to express all terms in consistent units. Assume that U is the same at all ages, with constant elasticity of consumption ε, given in the usual way by

$$\varepsilon = \frac{dU}{dc}\frac{c}{U(c)} \quad .$$

It can then be shown that (18) reduces to

$$(19) \qquad \delta W = \frac{dU}{dc}\left[\left(\frac{1}{\varepsilon} - 1\right) c_{ex} + w \cdot L_{ex} + \beta/A_m \cdot v_{ex}\right]$$

where $w(=F_L)$ is the wage rate. Utility of additional years now becomes c_{ex}/ε when translated into consumption terms. When ε = 0.5 for example, additional years are valued at twice the consumption needed to support them. Taking the terms within the bracket in (19), the mortality pattern change is worth to the individual marginal consumption equivalent to

$$(20) \qquad \text{C.E.}[\delta p(x)] = \left(\frac{1}{\varepsilon} - 1\right) c_{ex} + w \cdot L_{ex} + \beta/A_m \cdot v_{ex} \quad .$$

Example 3.1. To illustrate (20), let us assess the worth to the individual of elimination of cardiovascular diseases in the U.S. Using the cause-deleted lifetables of Preston, Keyfitz, and Schoen (1972), Table 1 in the Appendix shows the age-specific mortality variation that would result. Under 1975 U.S. data (again see Appendix) and definitions (13), complete elimination of cardiovascular diseases yields the differentials

Extra Years	c_{ex}(\$)	L_{ex}(years)	v_{ex}
10.33	56,870	0.6918	.00135

Cardiovascular diseases attack for the most part post-productive and -reproductive ages. Hence, though longevity increases considerably, expected working-life and expected number of children increase only a little.

Where ε = 1.0, 0.6, and 0.3

$$\text{C.E.} = \left.\begin{matrix}(1.0 \;\;\; - 1)\ 56{,}870 \\ (1.667 - 1)\ 56{,}870 \\ (3.333 - 1)\ 56{,}870\end{matrix}\right\} + (13{,}749)\ 0.692 + (-68{,}125)\ 0.00135 = \left\{\begin{matrix}\$\ \ 9{,}400 \\ \$\ 47{,}300 \\ \$142{,}100\end{matrix}\right.$$

This of course does not imply the U.S. should spend corresponding amounts per person on cardiovascular elimination. A flood of research dollars would by no means guarantee such a breakthrough. The illustration however gives an idea of the potential returns to the individual.

The example shows the crucial, but arbitrary element in the evaluation of mortality change is the degree of diminishing returns to consumption -- the degree to which pure enjoyment of additional years is offset by its consumption cost. In our well-off society we could expect additions to longevity to outweigh consumption considerations (ε is low), but in poorer societies (ε is high) utility of additional living might be offset by the additional burden of support; in certain nomadic tribes for example, older members, if no longer productive, are expected to separate themselves and die.[1]

[1)] Even in Western society, life could not be extended much beyond 100 years unless retirement age were also increased. See Boulding (1965) for an entertaining essay on the economic menace of extreme longevity.

One often hears two different ethical arguments where activities that put life at risk are under discussion: "life is infinitely valuable" versus "social product is what counts". In our schema these follow from different positions on returns to consumption. When ε tends to zero, (20) shows that additional life-years outweigh any consumption considerations: activities should be judged only on whether they preserve and prolong life. When ε is one, "utility is consumption", and extensions to life are perfectly offset by their consumption cost: only social product considerations remain. Normally, where returns to consumption are in the usual range, ε between zero and one, (20) retains elements of both ethical positions.

We can use (20) to comment on the two methods in present use. *Willingness-to-pay*, as presently used, ignores the negative social burden term. In the usual case where the reproductive term is negligible it will therefore overstate the value of mortality reduction and unduly bias against risky projects. *Human capital* fares slightly better. In the special case where (a) altered risks do not affect childbearing ages, (b) population growth is vanishingly small, and (c) utility shows constant returns to consumption ($\varepsilon = 1$), additional life-years would be exactly offset by their consumption cost, so that (20) would reduce to

$$C.E. = w \cdot L_{ex} \quad .$$

In this case, and this case only, the (gross) human capital method would be justifiable and correct.

Extensions. Two extensions of the analysis of the previous section are worth looking at. First, where life of loved ones is valued, person i's utility rate U^i might include the extra enjoyment α^i that loved ones j (with age differences a_j) are alive:

$$U^{i*} = U^i + \sum_j \alpha_j^i p(x + a_j) \quad ,$$

whence lifecycle welfare becomes for person i

$$W^i = \left[\int_0^{\omega} U^i p(x)\,dx + \sum_j \int_0^{\omega} \alpha_j^i p(x+a_j)\,p(x)\,dx\right] .$$

Under this criterion the social value expression (18) would contain an extra kith-and-kin term:

$$\sum_j \alpha_j^i \int_0^{\omega} (\delta p(x+a_j)\,p(x) + p(x+a_j)\,\delta p(x))\,dx .$$

Lessened mortality risk, in other words, is twice valuable -- it increases both the chances parents and grandparents will survive to be enjoyed, and that we will survive to enjoy our children and grandchildren.

Second, a change in length of life may be accompanied by a change in the age of retirement or in the age-specific labor participation schedule. For this case the L_{ex} term should then reflect extra labor years due to increased participation, as well as increased survival.

Robustness. How robust is the analysis of Section 2 when the assumptions of the model are replaced by more realistic ones? Recall that we assumed economic and demographic steady-state growth, constant returns in production, perfect life-cycle financial markets, and similar individuals who face similar mortality schedules.

Note first that the most important factors are scarcely changed under increased realism. When risks to life fall for the population (a) the individual does enjoy extra years, extra working life, and perhaps extra reproduction, and (b) whatever the support mechanism for old age, be it gifts to tribal elders, Robinson Crusoe stockpiling, or a government social-security system, consumption must still be set aside for lengthened life (although the amount may now depend on the transfer mechanism). With non-constant returns in production and imperfect lifecycle markets, the valuation of these factors would change however. The marginal value of consumption may well vary more widely than in (12), labor would not necessarily be paid its marginal product, and the value of growth, β, would be altered. With non-optimal investment, an extra capital-labor ratio adjustment term would enter. These changes are relatively minor. More important is the case where altered mortality risks strike the population unevenly, or the mortality change comes suddenly, or demographic and economic growth varies widely from steady-state. In this case, some people may reap the benefits of increased life and production, while others bear the consumption costs. For example, a sudden mortality improvement can be a windfall to the elderly -- they enjoy extra years while escaping the corresponding extra support of the generation that went before.

4. VALUE OF LIFE

Until now I have viewed activities that put life under hazard in rather inconvenient terms as causing variations in the mortality age-profile. Is it possible to proceed more directly and value actual lives lost or saved? In the literature, most writers prefer to deal with marginal changes in risk rather than with direct loss of life, feeling possibly that increase of risk is more approachable somehow, less awesome, than loss of life. From an actuarial viewpoint, however, risk and death cannot be separated. For any sizable population, an increase in age-specific risk means, in lifetable terms, an increase in numbers of deaths at specific ages. We might therefore expect valuation of risk and valuation of lives lost to be closely connected.

Let us ask a specific question. Suppose in the community an unspecified activity were to take on average one life at random per year, at age a, how much consumption would the community as a whole be prepared to give up to rid itself of the increased risk? The result will be called the Social Consumption Equivalent (S.C.E.) of Life, at age a.

To answer this question go back to the lifetable -- to how $p(x)$ is constructed. A lifetable is calculated by taking a base number of births, $\bar{B}$, (usually 10,000) and observing the year-by-year decrements in survivorship. Assume that every $\bar{B}$ people born undergo one additional death at age a. Until age a there is no difference in survivorship; at age a there are $\bar{B}\,p(a) - 1$ survivors instead of $\bar{B}\,p(a)$; at age $x > a$ there are $(\bar{B}\,p(a) - 1)\,\frac{p(x)}{p(a)}$ survivors instead of $\bar{B}\,p(x)$. The additional death causes a variation in the mortality schedule equal to the difference in numbers surviving divided by the base:

$$\delta p(x) = \begin{cases} 0 & 0 \leq x \leq a \\ p(x)/p(a)\bar{B} & a < x \leq \omega \end{cases} .$$

I shall write $p(x)/p(a)$ as $p(x|a)$, the probability of survival to age x given survival already to age a. For each representative person, from (20), expected lifecycle welfare is lowered in consumption terms an amount

$$\text{C.E.} = \left(\frac{1}{\varepsilon} - 1\right)\int_a^\omega e^{-gx} c(x) \frac{p(x|a)}{B(t)}\,dx + w\int_a^\omega e^{-gx}\lambda(x)\frac{p(x|a)}{B(t)}\,dx + \frac{\beta}{A_m}\int_a^\omega e^{-gx} m(x)\frac{p(x|a)}{B(t)}\,dx . \tag{21}$$

In the spirit of project evaluation, we may sum this amount over the individuals affected -- hence multiply by $\bar{B}$ -- to arrive at the social consumption equivalent, S.C.E., that would compensate for increased risk corresponding to loss of one life at age a. This yields our second main result -- a result that has an obvious actuarial interpretation

$$\text{S.C.E.}(a) = \left(\frac{1}{\varepsilon} - 1\right)\int_a^\omega e^{-gx} c(x)\,p(x|a)\,dx + w\int_a^\omega e^{-gx}\lambda(x)\,p(x|a)\,dx + \frac{\beta}{A_m}\int_a^\omega e^{-gx} m(x)\,p(x|a)\,dx . \tag{22}$$

Consumption Equivalent of Life, Age a	=	Value of Remaining Consumption years at Age a	+	Value of Remaining Earning years at Age a	+	Value of Remaining Reproduction at Age a

Where consumption is roughly constant at level $c(a)$ over the remaining years, this can be written as

$$(23) \qquad S.C.E.(a) = c(a)\left(\frac{1}{\varepsilon} - 1\right)\tilde{e}_a + w.\tilde{e}_{\ell a} + \frac{\beta}{A_m}\tilde{e}_{ma} \quad ,$$

where $\tilde{e}_x$, $\tilde{e}_{\ell x}$, $\tilde{e}_{mx}$ are the (discounted) expected values of remaining survival-years, labor-years, and net fertility at age x. A marginal life lost, in other words, is valued in terms of opportunity lost -- opportunity to enjoy further life, to produce further output, to have additional children, less of course, consumption support costs no longer necessary.

To give an idea of magnitudes, Table 4.1 illustrates $\tilde{e}_x$, $\tilde{e}_{\ell x}$, $\tilde{e}_{mx}$ and the S.C.E. for U.S. 1975 data (see Appendix), under different returns to consumption.[1]

Age		0	10	20	30	40	50	60	70	80
$\tilde{e}_x$		70.3	62.5	52.9	43.5	34.3	25.6	18.0	11.7	6.7
$\tilde{e}_{\ell x}$		31.6	32.5	31.4	24.7	17.6	10.8	4.4	0.3	-
$\tilde{e}_{mx}$		0.921	0.949	0.882	0.339	0.038	-	-	-	-
SCE $1000	ε=1.0	371	382	371	316	239	148	61	4	-
	ε=0.6	668	664	619	520	399	265	139	54	31
	ε=0.3	1,412	1,371	1,240	1,027	796	557	336	179	110

Table 4.1

Thus far we have assessed the cost of a single life lost at a given age. How might we extend the analysis to numbers of lives lost at various ages? Consider an activity R (airline accidents say) that costs $D\,e^{gt}$ lives in year t, numbers of deaths small relative to total deaths and growing at the same

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1) S.C.E. at age 0 would not be a suitable way to measure the desirability of introducing an additional birth: the analysis calculates how much those *already* born would give up to avoid certain types of risk. Also, this being steady-state analysis, it assumes all cohorts face the same experience; for reasons given in the robustness subsection, (22) would not be perfectly correct as a valuation of a life lost on a once-only basis. Note that the SCE figures in this illustration contains no cost of loss to relatives or friends.

rate as the population. Assume these deaths are distributed as $d(a)e^{gt}$ at age a, so that the probability that a life lost to this activity is aged a is $\phi_R(a) = d(a)/D$. In our analysis cost of lives lost is imputed to this year's cohort which stands to lose $d(a)e^{g(t+a)}$ lives at age a in year t + a. The cost-of-life argument above is additive over lives lost, therefore for this activity in year t, total (consumption-equivalent) losses are

$$\text{Total S.C.E.} = \sum_a d(a)\, e^{gt} \cdot e^{ga} \cdot \text{S.C.E.}(a) \quad .$$

Finally, multiplying above and below by D, gives the needed result

$$\text{(24)} \qquad \text{Total S.C.E.} = D\, e^{gt} \cdot \sum_a \phi_R(a) \cdot e^{ga} \cdot \text{S.C.E.}(a) \quad .^1$$

Cost of lives lost, in other words, is the number of deaths times the expected cost of a death in the activity in question.

Example 4.1. In the U.S., introduction of the 55 m.p.h. speed limit shows clearly in the motor-vehicle death statistics as saving roughly 9000 lives per year.[2] What is the social gain? Assuming lives are saved in the same proportions as road accidents normally strike the population, the probability distribution of a life saved is:

Age	0	10	20	30	40	50	60	70	80
$\phi(a)$	.027	.045	.224	.152	.119	.117	.119	.116	.081

Expected social cost of a single motor vehicle death is then

$$\sum_a \phi_R(a)\, e^{ga}\ \text{S.C.E.}(a) = \begin{cases} \$763{,}000 & \varepsilon = 0.3 \\ \$369{,}000 & \varepsilon = 0.6 \\ \$212{,}000 & \varepsilon = 1.0 \end{cases} \quad .$$

---

1) The e^{ga} factor enters to preserve consistency: the cost-of-life argument was developed on a cohort (lifecycle) basis whereas deaths are introduced on a period (current year) basis.

2) Total motor-vehicle deaths are given in the U.S. 1977 Statistical Abstract as 54.6, 54.7, 56.3, 55.5, 46.4, 46.0, 47.1 (thousand) from 1970-1976. Nathan Keyfitz called my attention to these figures. Preston et al. (1972), for U.S. 1964, was used as source for $\phi(a)$.

From (24), 9000 deaths averted are worth \$6.9, \$3.3, or \$1.9 billion per year in consumption terms, depending on whether $\varepsilon = 0.3$, 0.6, or 1.0.

5. CONCLUSION

This paper derived expressions for the value of activities that alter the mortality schedule and for the cost of premature loss of life, under specific assumptions and a life-cycle criterion. Full age-specific accounting, where labor participation, consumption, fertility, mortality, and utility are dependent on age, brings an actuarial precision to the results: alterations in the mortality schedule should be assessed by the difference they make to expected length of life, production, reproduction and consumption support; loss of life should be assessed by the expected opportunity costs of lost years, production and reproduction, less support costs.

It should be emphasized that these results depend highly on our choice of life-cycle criterion. A life lost at age 80 has less opportunity to contribute to this criterion than one lost at age 30, hence the implied "value of life" decreases with age. This sits comfortably, for the most part, with our intuitive feelings; if we felt on the other hand that "a life is a life whatever the age" our criterion would be no longer appropriate.

The degree of returns to consumption figures large in the economics of mortality risk. While individual terms in an evaluation can be assessed quite accurately, overall valuation of mortality change requires comparison between increased longevity and its consumption cost: it requires taking a stance on returns to consumption. One possible way to deal with this is to set reasonable bounds for returns to consumption; the value of mortality change can then be bracketed quite accurately.

6. APPENDIX

The illustrations use U.S. data chosen to correspond to year 1975.[a]

For the cardiovascular example, the following survival schedules are used:[b] $p(x)$ is the usual survival table; $p_E(x)$ would obtain if cardiovascular diseases were eliminated; $\delta p(x)$, the variation caused by eliminating these diseases, is the difference.

Age	0	10	20	30	40	50	60	70	80
$p_E(x)$	1.000	97029	96402	95260	93747	.91056	.85807	76547	.62615
$p(x)$	1.000	.97000	.96343	.95091	93149	.88841	.79228	.61135	.34853
$\delta p(x)$	0.000	.00029	.00059	.00169	.00598	.02215	.06579	.15412	.27762

Table 1

Labor participation and fertility schedules are:

Age	-16	16-19	20-24	25-29	30-44	45-49	50-54	55-59	60-64	65+
$\lambda(x)$[c]	-	.549	.751	.755	.744	.738	.704	.646	.479	.131

Age	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
$m(x)$[d]	.0006	.0290	.0595	.0566	.0272	.0101	.0024	.0001

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a) All data are for male and female combined.

b) From latest available cause-of-death lifetables: Preston et al. (1972), for U.S. 1964. The 1964 survival probabilities are used throughout the illustrations; mortality in the U.S. has changed but little in the last 15 years. For conciseness, only 10 year intervals are shown above; all calculations however were based on 5-year intervals. Preston (1976) contains further details on cause of death.

c) Source: ILO Year Book 1976; data for U.S. 1975.

d) Source: Statistical Abstract of the U.S. 1977; data for U.S. 1974.

Other data: $L = 94{,}793{,}000$ [c] $N = 213{,}137{,}000$ [d]

$\bar{c} = \$6{,}142$ [e] $K = \$4{,}303$ billion.[e]

In the absence of a usable consumption age-schedule, it is assumed that those 15 and under consume one-half of an adult's standard consumption; those 65 and above, three quarters. This yields for consistency with average consumption $\bar{c}$

Age	0-15	15-65	65+
c(x)	3582	7164	5373

Computations on the above data, smoothed where necessary, yield

$g = 0.00$; $b = 0.148$; $A_c = 38.07$; $A_L = 38.92$; $A_m = 25.21$

$\beta = -\$1{,}716$ million;[f] $\beta/A_m = -\$68{,}125$; $w = \$13{,}749$ [g].

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e) 1977 U.S. Stat.Abst.; data for 1975 in current dollars. In the consumption figure, government expenditures were taken as part of consumption. K represents total reproducible assets.

f) For discussion on β, the value of a marginal increase in g, and why it is negative, see Arthur and McNicoll (1978).

g) For consistency with the lifecycle model here ($g = 0$), w, the wage rate was computed from (12): $\int c(x)p(x)dx \equiv w\int\lambda(x)p(x)dx$.

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