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Consistency Between Long-term Climate Target and Short-term Abatement Policy. Attainability Analysis Technique.

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Abstract

A crucial problem in climate policy decision making is to design mitigation strategies consistent with specified long–term stabilization targets. However due to a deep uncertainty in our understanding of climatic system functioning, it is often impossible to elaborate abatement actions over a long period of time. Therefore the usual policy way is to design actions for a short–term time period only, taking into account current possibilities and information, but keeping the possibility to meet a given long-term target. In this paper we propose a methodological framework based on control theory to tackle the consistency issue comprehensively. Our approach consists of two stages. On the first stage we identify a [consistency] set of all possible short–term states that keep the possibility to meet a given long-term target and on the second stage we assess the 'cost' of achieving a long–term target for every short–term state in the consistency set. To illustrate the approach we run the calculations for the DICE–94 model of the economics of global warming.

Key words: climate change, DICE model, abatement policy, short-term policy, climate target, consistency,
attainability domain, controllability domain, uncertainty analysis
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1 Introduction

The emissions of greenhouse gases are a side effect of human activities. Accumulation of these gases in the atmosphere results in global warming that is believed to have a significant impact on the environment. Elaborating of actions that should be undertaken to prevent negative impacts of global warming is the main subject of international negotiations devoted to climate change.

Our comprehension of potential damages of climate change defines a climate target. A number of climate targets have been proposed. For example, temperature targets can be defined as limiting warming to $2°C$, $2.5°C$, $3°C$ above the pre–industrial level. Taking into account the delay in the climate system, a climate target must be defined as *a long–term target*.

At the same time, a real implementation of abatement policy is based on *short–term actions* to be undertaken right now. Short–term actions are defined by international commitments devoted to the mitigation response to the climate change. These commitments define a volume of annual emissions reductions distributed among industrialized countries. Short–term policies are more realistic and politically conditioned.

An abatement policy should be designed in such a way that a specified long–term climate target will be reached. Consequently, a long–term target should be a major factor in designing short–term policy. International commitments prescribe abatement policies over a short–term time period and leaves long–term actions unconsidered. Therefore, there is a gap between a short–term policy and a long–term target that doesn't enable to assess the *consistency* between them directly. Consistency means that there exists an appropriate long–term policy that could be undertaken after a given short–term action in order to reach a given long–term target. A long–term policy has a supporting role in that definition of consistency and is used to verify the possibility of reaching a climate target.

The consistency issue brings up at least two important questions. The first question is how to estimate a set of possible short–term actions which are consistent with a specified long–term target. And the second question is which long–term actions are required to meet a long–term target depending on a chosen short–term policy and how much it costs.

The issue of linking long–term climate targets and short–term abatement policies has been considered by many authors. Corfee–Morlot and Höhne (2003) emphasize the importance of taking into account a long–term target when designing short–term policies, and consider how a long–term target should be defined to mirror dangerous consequences of climate changes. Rive et al. (2007) compute the probability of reaching a given long–term target for different short–term abatement policies using probability estimates of the climate sensitivity. O'Neill et al. (2005) use a notion of interim target to link a long–term target with short–term actions. That is a short list of papers devoted to the consistency issue.

This paper suggests an approach for the analysis of the consistency issue. We consider a model describing climate change due to human activities. An important part of the model we take for the investigation is *control parameters* which describe our abilities to affect climate system. These parameters reflect the concept of practical actions aimed at the mitigation of climate change impacts. For instance, a control parameter may represent the rate of emissions reductions. In such a way, the model is represented as a controlled dynamic system.

Our approach is based on control theory which provides a powerful apparatus for the investigation of controlled dynamic systems. *Controllability* and *optimality* are basic notions of control theory. The notion of controllability is related to the possibility of steering a dynamic system into a given target state using an appropriate control. Optimality means that we are looking for the best solution among a set of all possible solutions. We use the notion of controllability to investigate the consistency issue and the notion of optimality to indicate long–term actions which provide meeting a climate target at minimal cost.

2 Consistency analysis

2.1 Approach to consistency issue

Let us consider a control model describing climatic changes due to human activities over a given time period. The output (state) model's variables represent global climate parameters (such as the atmospheric temperature, CO_2 concentration, etc). The input (control) model's variables represent human actions aimed at the mitigation of climate change impacts (such as annual emissions reduction, etc). A sequence of actions is called to be a policy (control strategy). Every policy generates a model's trajectory that describes how climate parameters (the output variables) are changing due to the policy over a given time period. Desired range of values for climate parameters determines a climate target. Our aim is to construct such a policy that a corresponding model's trajectory meets a chosen climate target at a given time moment, that is, that a state of the model's trajectory at that time moment falls in the range of values prescribed by a climate target.

Consider model's trajectories over a time interval $[t_0, t^*]$, where t_0 is a initial time and t^* is a terminal time. It is assumed that the time interval is split into two periods: a short–term period $[t_0, t_*]$ and a long–term period $[t_*, t^*]$. Then every policy and trajectories corresponding to them are divided into two parts: a short–term part and a long–term part.

Let us specify a long–term climate target and a short–term policy.

Definition. A short–term policy and a long–term climate target is said to be consistent if there exists at least one long–term policy following the short–term policy that a corresponding trajectory meets the long–term climate target.

Figure 1 shows some long–term temperature target that defines an admissible range of the temperature at a terminal time t^*, and some short–term policy that generates a short–term trajectory, in particular a short–term temperature trajectory. We must check if there exists such long–term policy that a long–term temperature trajectory takes a value from a range prescribed by the temperature target at the terminal time t^*.

Let us consider steps of the consistency analysis. We take a prescribed short–term policy and construct a short–term trajectory corresponding to it. We call a trajectory's position at time t_* as a short–term state. Now we must check if there exist a long–term trajectory starting at the short–term state that meets a long–term target. To this end,

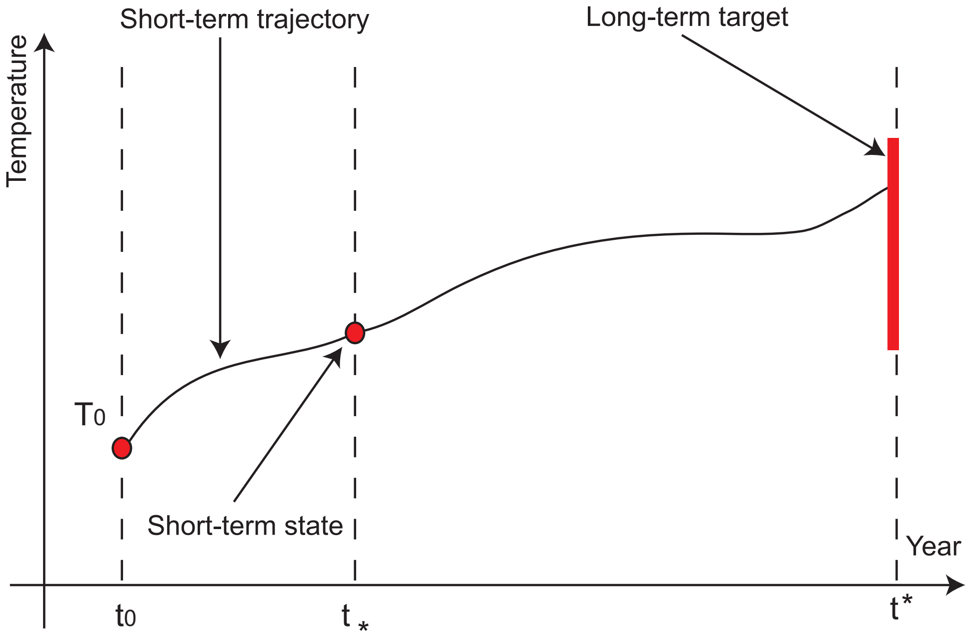

Figure 1: Consistency between long–term target and short–term policy.

we construct a set of all feasible states at time t_* from which the long–term target can be reached using some long–term policy. This step is based on the notion of controllability domain.

Fix an arbitrary short–term state (at time t_*) and look over all possible long–term policies. Each policy generates a trajectory starting at that state. If there exists at least one trajectory that meets the long–term target, then the state is included to the controllability domain. So, the controllability domain comprises the short–term states which are consistent with the long–term target (see Fig.2).

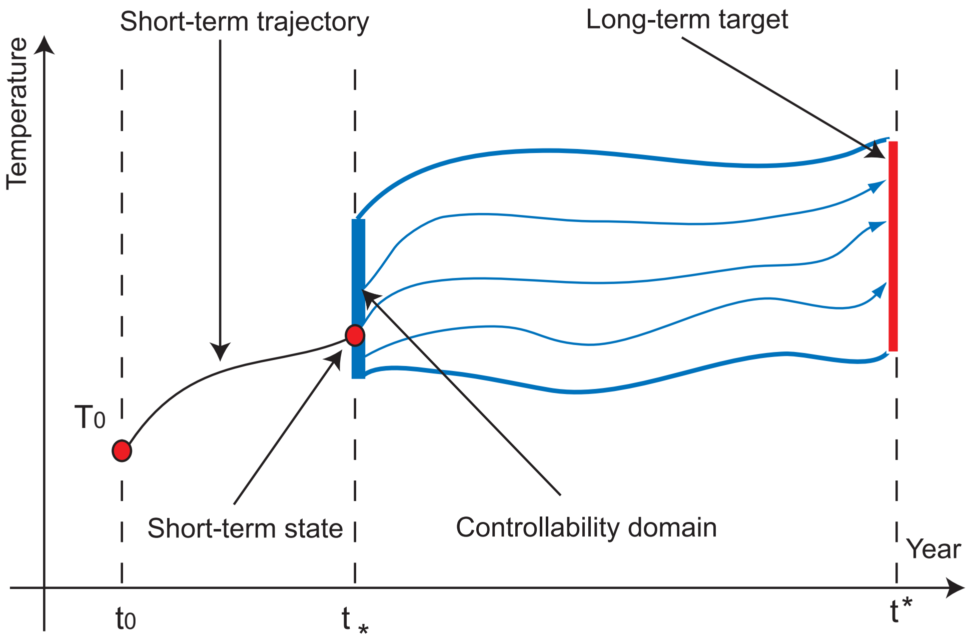

Figure 2: Controllability domain constructed for long–term target.

Using the notion of controllability domain, the consistency issue converts to the investigation of the following question: Does a short–term state lie inside a controllability domain constructed for a long–term climate target. If this is the case, the short–term policy and the long–term target are consistent.

At the next step, we estimate a set of all possible short–term policies which are consistent with the long–term target. For this purpose, we must construct a set of short–term states corresponding to all short–term policies. Let us introduce the notion of attainability domain for a short–term time period. Attainability domain is a set of all states which are reachable at time t_* from a given initial state.

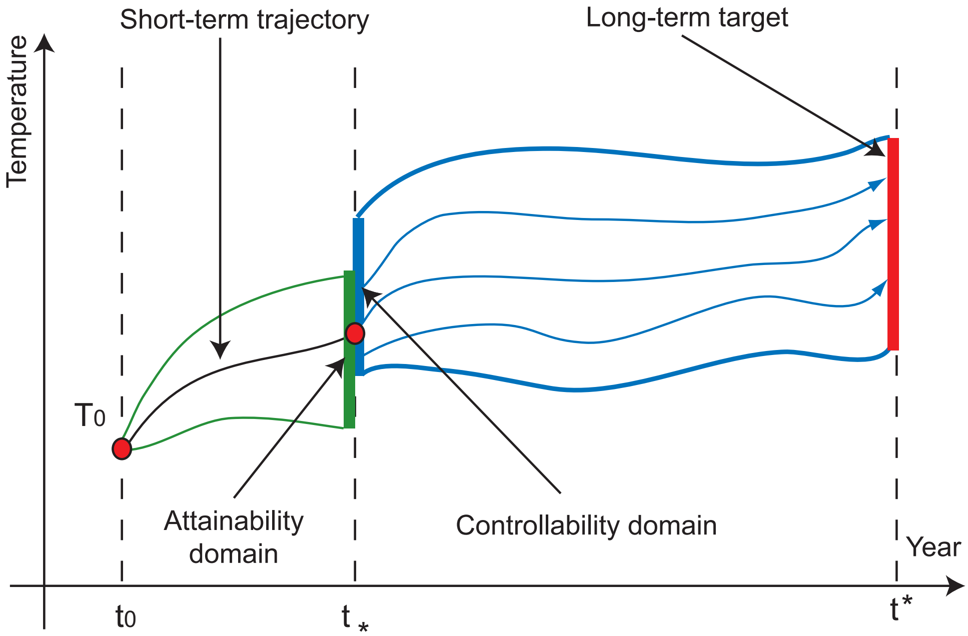

Figure 3: Intersection of controllability and attainability domains

The main question for the investigation is how the attainability and controllability domains overlap. The part of the attainability domain lying inside the controllability domain corresponds to short–term policies that are consistent with the long–term target.

We call the intersection of attainability and controllability domains as consistency domain (see Fig.3). Consistency domain comprises all short–term states which are reachable from a given initial state using some short–term policy and, at the same time, from which a given long–term target can be reached using some long–term policy.

Consistency domain is a powerful auxiliary notion that allows not only to reveal the consistency between a long–term target and a short–term policy but to compute important characteristics relating to the consistency issue as well. For example, let a model describe relationships between the economic development, GHG–emissions and the atmospheric temperature rise, then we can compute the maximum level of the emissions over a short–time period which is consistent with a given long–term temperature target and, moreover, to compute the optimal long–term policy that meets a given long–term target at minimal cost.

There are a number of effective methods which don't require scanning all policies to construct controllability and attainability domains. Further we describe a method based on the Pontryagin's maximum principle for constructing controllability and attainability domains.

2.2 Illustration of approach

This section aims to clarifying the notions introduced above. To this end, we consider the DICE–94 model and compute a consistency domain and a number of consistency characteristics. We don't make it our aim here to make comprehensive investigation of the DICE model, it will be done later.

The DICE–94 model describes relations between the world capital, K, the mass of GHG in the atmosphere M, the atmospheric temperature, T^U, and the temperature of the deep ocean, T^L. It includes two control variables: the saving rate of capital, s, and the rate of emissions reductions, μ.

At the start, we should specify input data. Let us take the following values

Short–term time period $[t_0,\ t_*]$	–	[2005, 2020];
Long–term time period $[t_*,\ t^*]$	–	[2020, 2100];
Long–term temperature target	–	$2°C$ (above pre-industrial level);
Initial concentration of CO_2 (2005)	–	808.9 (GtCO2eq);
Initial world capital (2005)	–	137 (trillions of 1990 dollars);
Initial atmospheric temperature (2005)	–	$0.7307°C$ (above pre-industrial level);
Initial ocean temperature (2005)	–	$0.0068°C$ (above pre-industrial level).

In accordance with the approach proposed, we construct attainability and controllability domains for the DICE model. A simplified version of the DICE model is considered here in order to decrease the computation complexity.

The attainability domain represents a set of all possible states which can be attained in 2020 from the given initial state. A state of the simplified DICE model comprises a couple of number at every time moment: the value of atmospheric concentration of CO_2 (M) and the value of the world capital (K). The attainability domain gives us a range of all possible atmospheric concentrations of CO_2 in 2020. The boundary of the attainability domain is highlighted with the red color in Fig. 4.

The controllability domain represents states in 2020 from which the long–term target can be reached. Using the controllability domain, we obtain a range of atmospheric concentrations in 2020 which are consistent with the given target. The boundary of the controllability domain is highlighted with the blue color.

The consistency domain is the intersection of the controllability and the attainability domains. It is highlighted with green color. You can see on the graph, that some states of the attainability domain lie outside of the controllability domain. It means that not all possible levels of CO_2 concentration in 2020 are consistent with the target. Figure 4 indicates that $2.0°C$ target can be reached if the CO_2 concentration in 2020 will be in the range from 780 GtCO2eq to 860 GtCO2eq.

An essential factor determining the possibility to reach a temperature target is the climate sensitivity which characterizes the increase in mean temperature due to carbon concentration doubling. Consequently, the value of climate sensitivity must essential affect the form of controllability domain as well as the form of consistency domain. Figure 5 shows how the form of consistency domain changes in relation to the value of climate sensitivity.

If the value of climate sensitivity is equal to $3.0°C$ then the attainability domain lies inside the controllability domain. It means that all possible levels of CO_2 concentration in 2020 are consistent with the target. The graph of the consistency domain indicates that for $3.8°C$ climate sensitivity a short–term policy must provide level of concentration in 2020 that is less than 795 GtCO2eq in order to keep the possibility of reaching $2.0°C$ target. Taking into account that the initial value of concentration is equal to 808.9 GtCO2eq, a short–term policy should provide reducing the level of atmospheric concentration by 2020

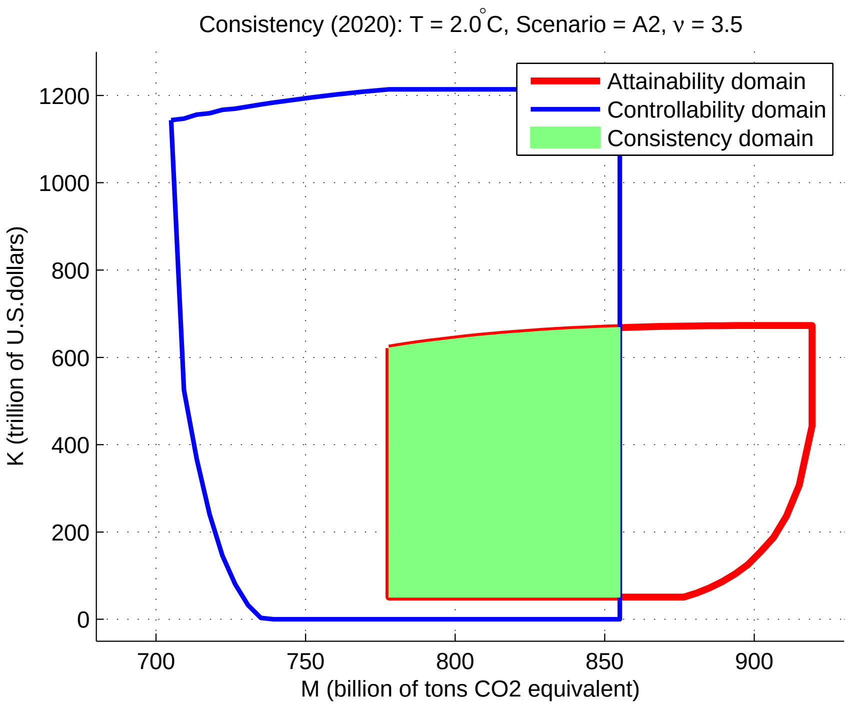

Figure 4: Consistency domain.

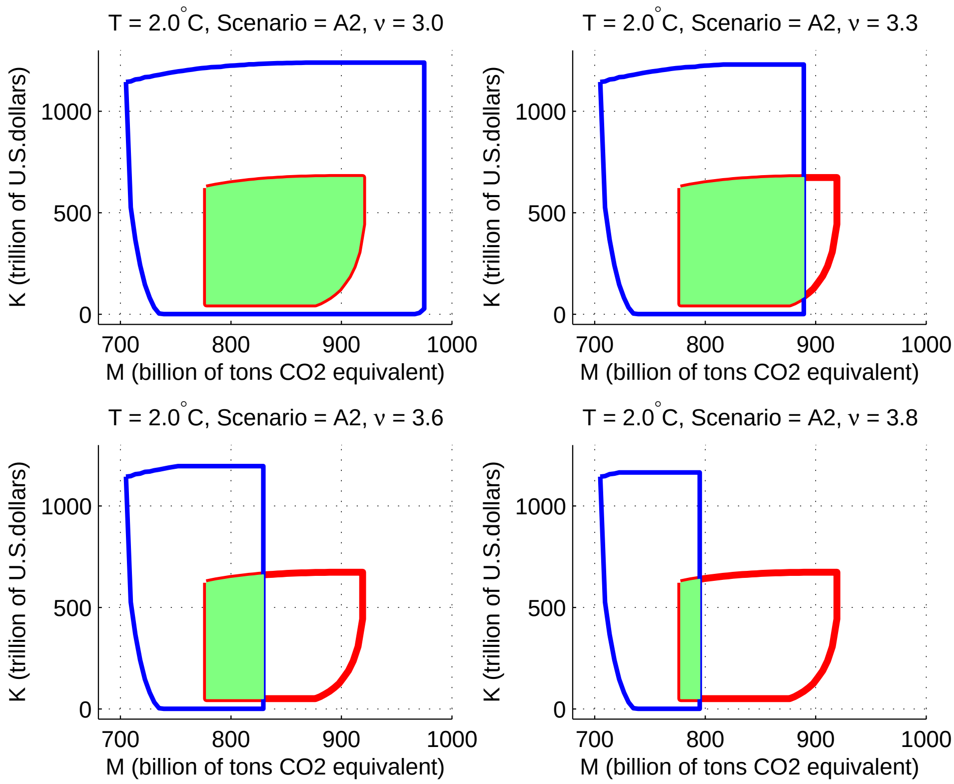

Figure 5: Dependence of consistency domain at 2020 on climate sensitivity. Climate target: $2°C$; Scenario: $A2$.

in relation to the initial level in 2005.

Let us calculate some consistency characteristics. The consistency domain indicates the levels of CO_2 concentration in 2020 which are allowable to reach $2.0°C$ target by 2100. We estimate the set of short–term policies which provide such level of concentration. To this end, we calculate the maximal allowable level of annual emissions for the short–term time period.

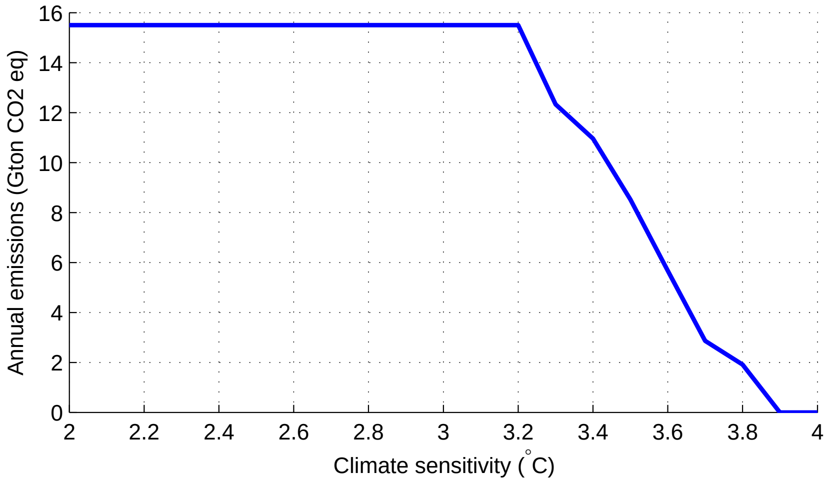

Figure 6: Maximal annual allowable level of emissions till 2020. (Climate target: $2°C$; Scenario: $A2$.)

Figure 6 presents the annual allowable level of emissions for various values of climate sensitivities. Value of allowable emissions is greater than 0 GtCO2eq if the vlaue of climate sensitivity is less than or equal to $3.8°C$. These values define a set of short–term policies which are consistent with $2°C$ target. The horizontal segment of the graph (the value of climate sensitivity lying in the range from $2.0°C$ to $3.2°C$) corresponds to the case where the attainability domain lies inside the controllability domain. In this case, the maximal level of emissions in 2020 is equal to 15.5 GtCO2eq. For values of climate sensitivity from $3.2°C$ to $3.8°C$, the curve of allowable emissions goes down. If the value of climate sensitivity is greater than $3.8°C$ then the level of allowable emissions is equal to 0, consequently in this case it is impossible to keep temperature less $2°C$ by 2100 using any abatement policy.

The next step of the analysis is an assessment of long–term actions needed to reach a long–term target. Consistency domain comprises all states in 2020 from which the long–term target can be reached. For each state from the controllability domain we calculate the minimal value of the accumulated abatement cost that must be payed in order to reach the target. Figure 7 presents results of the calculations.

Note that a value of abatement cost doesn't contain essential information for us by itself and must be considered in comparison to GDP, because the evolution of GDP might be essential differ for various scenarios and the same values of abatement cost for different scenarios mean different fraction of GDP to be payed for emissions reductions. Therefore, we compute the following two characteristics providing the minimal cost: the average cost of reduction as percent of GDP and average rate of annual emissions reductions (see Fig. 8). In fact, these two characteristics could be used as long–term actions needed to meet the long–term target at minimal cost.

Figure 8 can be read in the following way. A short–term policy determines a state (CO_2 concentration and capital) in 2020. If that state is consistent with the long–term

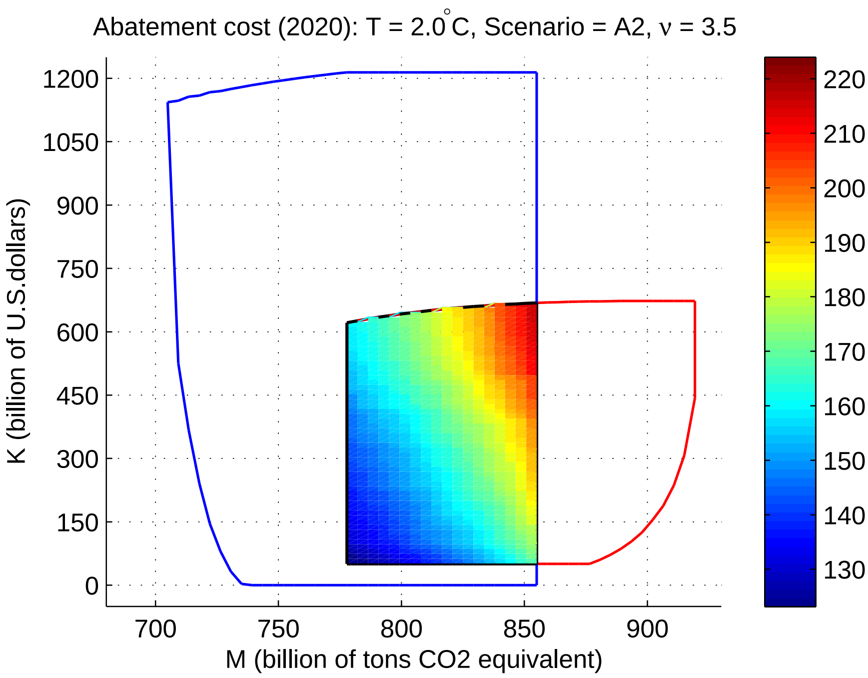

Figure 7: Accumulated abatement cost. (Climate target: $2°C$; Scenario: $A2$.)

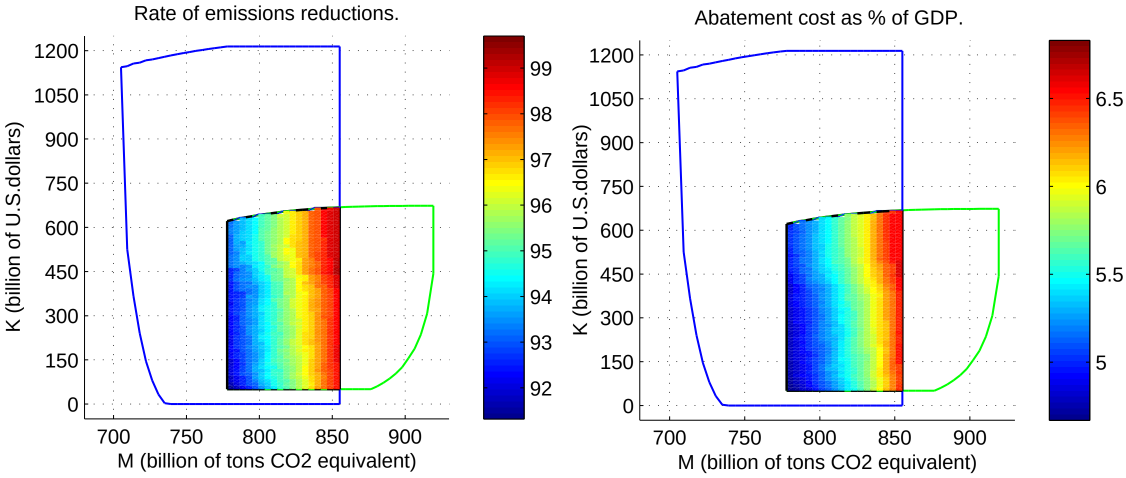

Figure 8: Average rate of annual emissions reductions and average abatement cost as percent of GDP. (Climate target: $2°C$; Scenario: $A2$; Climate sensitivity: $3.5°C$.)

target, then it lies inside the consistency domain and values corresponding to them in the graphs indicate the average rate of annual emissions reductions and the average cost of reduction as percent of GDP needed to reach long–term target. For example, if we provide 830 GtCO2eq concentration by 2020, we must cut 95% of emissions and it will require 5.5% of GDP to reach $2°C$ target.

3 Mathematical background

In this section, we describe the mathematical background that was used to construct consistency domains and calculate the minimal value of the abatement cost.

3.1 Attainability, controllability and consistency domains

Let us consider a controlled dynamical system described by differential equations

$$\dot{x} = f(t, x, u), \tag{1}$$

where $x \in \mathbf{R}^n$ is the state and $u \in \mathbf{R}^r$ is the control. The evolution of the system is considered over a given time interval $[t_0,\ t^*]$. The control $u = u(t)$ is any function restricted by the condition

$$u(t) \in U, \quad t \in [t_0, t^*], \tag{2}$$

where U is a given compact set.

Every initial state $x(t_0) = x_0$ of the system and every admissible control $u(t)$ satisfying the condition (2) determine a trajectory $x(t|t_0, x_0, u(\cdot))$ that describes the evolution of the system in time.

Definition. A state x is said to be attainable at time t from the initial state x_0, if there exists an admissible control $u(\tau)$, $\tau \in [t_0, t]$, such that $x(t|t_0, x_0, u(\cdot)) = x$.

Definition. The set $X(t|t_0, x_0)$ of all states x that are attainable at time t from the initial state x_0 is said to be attainable domain.

Thus

$$X(t|t_0, x_0) = \{x = x(t|t_0, x_0, u(\cdot)) : \ u(\tau) \in U,\ \tau \in [t_0, t]\}. \tag{3}$$

Consider some target set, $M \in \mathbf{R}^n$, which should be attained at a terminal time t^*. Let us define a set of states from which this target set can be attained.

Definition. The set $Y(t|t^*, M)$ of all states x at time t for which there exists an admissible control $u(\tau)$, $\tau \in [t_0, t]$, such that $x(t^*|t, x, u(\cdot)) \in M$ is said to be controllability domain.

Thus

$$Y(t|t^*, M) = \{x : \ x(t^*|t, x, u(\cdot)) \in M,\ u(\tau) \in U,\ \tau \in [t, t^*]\}. \tag{4}$$

Let us take two time periods $[t_0, t_*]$ and $[t_*, t^*]$, an initial state x_0 at time t_0 and a target set M at time t^*.

Definition. The set $C(t_*|t_0, x_0, t^*, M)$ of all states x at time t such that there exists an admissible control $u_*(\tau)$, $\tau \in [t_0, t_*]$, that $x(t_*|t_0, x_0, u_*(\cdot)) = x$ and there exists an admissible control $u^*(\tau)$, $\tau \in [t_*, t^*]$, that $x(t^*|t_*, x, u^*(\cdot)) \in M$ is said to be consistency domain.

Thus

$$C(t_*|t_0, x_0, t^*, M) = X(t_*|t_0, x_0) \bigcap Y(t_*|t^*, M). \tag{5}$$

3.2 Constructing attainability and controllability domains

The direct method for constructing attainability and controllability domains based on the definition requires the computation of model's trajectories for all admissible controls. The Pontryagin's maximum principle suggests an efficient way to construct theses domains indirectly. It allows to find the boundary points of attainability and controllability domains without scanning all possible controls.

The Pontryagin's maximum principle suggests the following scheme to construct the boundary of attainability domain for the system (1):

(i) Compose function defined by

$$H(t, x, u, \psi) = \langle \psi, f(t, x, u) \rangle\,, \tag{6}$$

where ψ is called as an adjoint variable and $\langle a, b \rangle$ denotes the inner product of the vector a with the vector b.

(ii) Find a control function $u^*(t, x, \psi)$ satisfying the following condition for every t, x, ψ:

$$H(t, x, u^*(t, x, \psi), \psi) = \max_{v \in U} H(t, x, v, \psi). \tag{7}$$

(iii) Compose the system of differential equations defined by

$$\begin{cases} \dfrac{dx}{dt} & = & \dfrac{\partial H(t, x, u^*(x, \psi), \psi)}{\partial \psi}, \\ \dfrac{d\psi}{dt} & = & -\dfrac{\partial H(t, x, u^*(x, \psi), \psi)}{\partial x}, \\ x(t_0) & = & x_0, \\ \psi(t_0) & = & \psi_0. \end{cases} \tag{8}$$

(iv) Find the solution $(x(t), \psi(t))$ of (8) for each vector ψ_0 having unit length.

The set of vectors $x(t^*)$ obtained from the step (iv) comprises all boundary points of attainability domain at time t^* and possibly some inner points. The inner points don't contain additional information about attainability domain and must be separated from the boundary ones.

Let us remark that all trajectories bringing the system on the boundary of attainability domain are generated by control function defined by (7).

It is more convenient for us to use another form of the Pontryagin's maximum principle. Consider the vectogram for the dynamic system (1)

$$F(t, x) = \{f(t, x, u) : \ u \in U\},$$

and the support function of set $F(t, x)$

$$c(F(t, x), \psi) = \max_{f \in F(t,x)} \langle f, \psi \rangle .$$

If the function $c(F(t, x), \psi)$ is differentiable in x and ψ, then (8) can be rewritten as follows

$$\begin{cases} \dfrac{dx}{dt} & = & \dfrac{\partial c(F(t, x), \psi)}{\partial \psi}, \\ \dfrac{d\psi}{dt} & = & -\dfrac{\partial c(F(t, x), \psi)}{\partial x}, \\ x(t_0) & = & x_0, \\ \psi(t_0) & = & \psi_0. \end{cases} \tag{9}$$

Then we take various values for the vector ψ_0, solve the system of ordinary differential equations (9) and finally get the boundary points of attainability domain.

Constructing a controllability domain for a given target set uses the same algorithm based on the Pontyagin's maximum principle. There is only difference in the initial conditions for the system (9). Let us take some convex target set M and composed the following system of differential equations

$$\begin{cases} \dfrac{dx}{dt} & = & \dfrac{\partial c(F(t, x), \psi)}{\partial \psi}, \\ \dfrac{d\psi}{dt} & = & -\dfrac{\partial c(F(t, x), \psi)}{\partial x}, \\ x(t^*) & = & x^*, \\ \psi(t^*) & = & -\psi^*. \end{cases} \tag{10}$$

Note that the initial conditions of the latter system of differential equations is assigned for a final time moment.

We will use the notion of normal vector of a set to describe the procedure for constructing controllability domain.

Definition. A vector ψ is said to be a normal vector of a convex set M at a boundary point x if the following inequality is fulfilled for every $x \in M$

$$\langle \psi, x - x_0 \rangle \leq 0.$$

Let us find the solutions $(x(t), \psi(t))$ of the system (10) for each vector x^* lying on the boundary of the set M and each vector ψ^* that is a normal vector of the set M at the boundary point x^*. The set of obtained vectors $x(t_0)$ comprises all boundary points of the controllability domain at time t_0 and possibly some inner points which should be excluded.

Using the algorithm described above, we get the boundary points of attainability and controllability domains. Consistency domain is the intersection of these two domains.

3.3 Calculating abatement cost

Steering the dynamic system into a target set could be implemented by various control strategies. Let us introduce an objective function to be minimized by choosing a control strategy. That means that we are looking for such control that brings the dynamic system from a given initial state to a target set and has the minimal value of the objective function. In such a way, we obtain the following optimal control problem

$$\begin{array}{rcl} \text{minimize } J & = & \int_{t_*}^{t^*} f^0(t, x(t), u(t)), \\ \dot{x}(t) & = & f(t, x(t), u(t)), \\ x(t_*) & = & x_*, \\ x(t^*) & \in & M, \\ u(t) & \in & U, \\ t & \in & [t_*,\ t^*]. \end{array} \tag{11}$$

Calculating the minimal value of abatement cost is an optimal control problem (11). We use an appropriate version of the dynamic programming method to find a solution for the problem.

Let us introduce a time grid

$$t_* = t_1 < t_2 < \ldots < t_N = t^*,$$

and assume that control is a piecewise constant function on the grid.

The dynamic programming method suggests the following scheme to compute the minimal value of the objective function $J^0(x_*)$ depending on a given initial state x_* and find a corresponding optimal control.

Let us consider a sequence of functions defined recurrently

(i) For every $x \in G_N = M$,

$$V_N(x) = 0;$$

(ii) For every $i = 1, \ldots, N-1$ and every $x \in G_i$,

$$V_i(x) = \min_{v \in U_i(x)} \left\{ V_{i+1}(x(t_{i+1}|t_i, x, v)) + \int_{t_i}^{t_{i+1}} f^0(t, x(t), v)dt \right\},$$

where

$$G_i = \{x : \text{ exists } u \in U \text{ that } x(t_{i+1}|t_i, x, u) \in G_{i+1}\},$$

$$U_i(x) = \{u \in U : \ x(t_{i+1}|t_i, x, u) \in G_{i+1}\}.$$

Then the following equality holds

$$J^0(x_*) = V_1(x_*).$$

Calculating an optimal control $(u_1^*, u_2^*, \dots, u_{N-1}^*)$, where u_i corresponds to a time interval $[t_i, t_{i+1}]$, $i = 1, \dots, N-1$, consists of the follows steps. We put the initial state $x_1 = x_*$ and find a control $(u_1^*, u_2^*, \dots, u_{N-1}^*)$ from the conditions

$$\begin{cases} V_i(x(t_{i+1}|t_i, x_i, u_i^*)) + \int_{t_i}^{t_{i+1}} f^0(t, x(t), u_i^*)dt \\ \qquad = \min_{v \in U_i(x_i)} \left\{ V_i(x(t_{i+1}|t_i, x_i, v)) + \int_{t_i}^{t_{i+1}} f^0(t, x(t), v)dt \right\}, \\ x_{i+1} = x(t_{i+1}|t_i, x_i, u_i^*), \\ i = 1, \dots, N-1. \end{cases}$$

The described scheme can be applied to the model to calculate the minimal value of the abatement cost needed to reach a given long–term target. Optimal control providing the minimal cost is used to calculate the characteristics of long–term actions such as the average rate of emissions reductions and the average abatement cost as percent of GDP.

4 DICE-94 model

4.1 Model

The DICE–94 model ([4]) is the most popular model describing relations between climate change and economic development. We use this model to investigate the consistency issue and compute the consistency characteristics.

The DICE model describes the evolution of the following global parameters

$K(t)$	–	the world capital,
$M(t)$	–	the mass of GHG in the atmosphere,
$T^U(t)$	–	the atmospheric temperature,
$T^L(t)$	–	the temperature of the deep ocean.

We are in a position to control the model's trajectories by choosing values for two control variables

$s(t)$	–	the saving rate of capital,
$\mu(t)$	–	the rate of emissions reductions.

The evolution of the world capital is given by

$$\frac{dK(t)}{dt} = s(t)Q(t) - \delta_K K(t),$$

where $Q(t)$ is the production output and δ_K is the rate of depreciation of the capital stock. The production output is given by the Cobb-Douglas production function

$$Q(t) = \Omega(t)A(t)L(t)^{1-\gamma}K(t)^{\gamma},$$

where γ is the elastic of output, $\Omega(t)$ describes the impact of climate change on output, $A(t)$ is the size of the world technology stock, and $L(t)$ is the size of the world population. The functions $A(t)$ and $L(t)$ are given exogenously.

The next equation is the definition of the function $\Omega(t)$ suggested by Nordhaus

$$\Omega(t) = \frac{1 - b_1\mu(t)^{b_2}}{1 + \theta_1 T^U(t)^{\theta_2}}.$$

The accumulation of the GHGs in the atmosphere is given by

$$\frac{dM(t)}{dt} = \alpha E(t) - \delta_M (M(t) - \tilde{M}),$$

where $E(t)$ is the emission of GHGs, α is the marginal atmospheric retention ratio, δ_M is the rate of the deep ocean's uptake of atmospheric carbon, and $\tilde{M}$ is the preindustrial amount of atmospheric carbon.

The emission of GHGs is given by the following equations

$$E(t) = (1 - \mu(t))E_r(t),$$
$$E_r(t) = \sigma(t)A(t)L(t)^{1-\gamma}K(t)^{\gamma},$$

where $E_r(t)$ is the emissions in the absence of controls, $\sigma(t)$ is the base-case ratio of industrial emissions to output ($\sigma(t)$ is given exogenously).

The relationship between amount of GHGs in the atmosphere and radiative forcing is given by

$$F(t) = \eta \log_2 \left(\frac{M(t)}{\tilde{M}} \right) + O(t),$$

where η is the increase in radiative forcing due to doubling of CO_2 concentrations from preindustrial levels, and $O(t)$ represents the forcing of other GHGs.

The next equations describe the change of atmospheric and deep ocean temperatures due to radiative forcing

$$\begin{aligned}
\frac{dT^U(t)}{dt} &= \frac{1}{R_1} \left(F(t) - \lambda T^U(t) - \frac{R_2}{\tau_{12}} (T^U(t) - T^L(t)) \right), \\
\frac{dT^L(t)}{dt} &= \frac{1}{\tau_{12}} (T^U(t) - T^L(t)),
\end{aligned}$$

where R_1 is the thermal capacity of the upper layer of the ocean, R_2 is the thermal capacity of the deep ocean, τ_{12} is the rate of the top-down transfer of carbon in the ocean, and λ is a feedback parameter.

The parameters η and λ determine the value of climate sensitivity ν as follows

$$\nu = \eta / \lambda.$$

Climate sensitivity is the equilibrium increase in mean temperature due to carbon concentration doubling.

4.2 Simplified model

The simplified DICE-94 model is more convenient to make numerical calculation. The simplified model comprises two differential equations

$$\begin{aligned}
\frac{dK(t)}{dt} &= \frac{s(t)(1 - b_1 \mu(t)^{b_2}) A(t) L(t)^{1-\gamma} K(t)^{\gamma}}{1 + \theta_1 \left(M(t)/\tilde{M} \right)^{\theta_2}} - \delta_K K(t), \\
\frac{dM(t)}{dt} &= \alpha (1 - \mu(t)) \sigma(t) A(t) L(t)^{1-\gamma} K(t)^{\gamma} - \delta_M (M(t) - \tilde{M}).
\end{aligned} \tag{12}$$

The equations of simplified model don't contain temperature as a state variable. The parameters δ_M and δ_K of the model were calibrated so that trajectories of the simplified model approximate trajectories of the original model ([6]).

We use both the original and simplified models. The original model is used to treat a climate target given in terms of temperature and the simplified model is used for the numerical calculations.

4.3 Emissions and abatement cost

Let us consider some control strategy $(\mu(t), s(t))$ over a time interval $[t_*,\ T]$. This control strategy determines a model's trajectory $(M(t), K(t))$. In accordance with the DICE model, the volume of cut of GHG–emissions at time t is defined as

$$\mu(t)E_r(t),$$

and the cost of reducing emissions at time t is defined as

$$b_1\mu^{b_2}(t)Q(t).$$

We introduce three notions based on the last expressions:
(i) Discounted accumulated abatement cost over a time interval $[t_*,\ T]$

$$\int_{t_*}^{T} e^{-\rho t} b_1\mu^{b_2}(t)Q(t)dt; \tag{13}$$

(ii) Average rate of emissions reductions

$$\frac{\int_{t_*}^{T}\mu(t)E_r(t)dt}{\int_{t_*}^{T}E_r(t)dt}; \tag{14}$$

(iii) Average abatement cost as percent of GDP

$$\frac{\int_{t_*}^{T} e^{-\rho t} b_1\mu(t)^{b_2}Q(t)dt}{\int_{t_*}^{T} e^{-\rho t}Q(t)dt}. \tag{15}$$

We are looking for such control strategy that a corresponding trajectory meets a given long–term target. However, various strategies could satisfy this requirement. Therefore, there is an opportunity to choose the most appropriate strategy in some prescribe sense. We design a long–term abatement policy to minimize abatement cost. Having gotten the optimal control strategy in term of the DICE's control parameters, we use the average rate of emissions reductions and the average abatement cost as percent of GDP as characteristics of practical long–term abatement actions.

4.4 Scenarios

There is an uncertainty in behavior of some parameters used in the model. For instance, there is no exact information about the size of the world population in the future or the the size of the world technology stock. We are able to predict behavior of these parameters using historical data, but we are not able to take into account all future factors influencing the climate and economic structures. An approach addressing this issue is to consider a number of different *scenarios* which describe behavior of uncertain parameters.

The DICE model contains a number of functions to be defined exogenously. Namely, we have to specify the size of the world population, $L(t)$, the world technology stock, $A(t)$, and the base-case ratio of industrial emissions to output, $\sigma(t)$. We choose these exogenous functions in accordance with some specified baseline scenario which describe evolution of global parameters (such as the world population, GDP and GHGs emissions) in the absence of controls. We will consider two different baseline scenarios GGI–$A2$ and GGI–$B1$ depicted in fig. 9 ([10]) and calibrate the DICE model according to each of them.

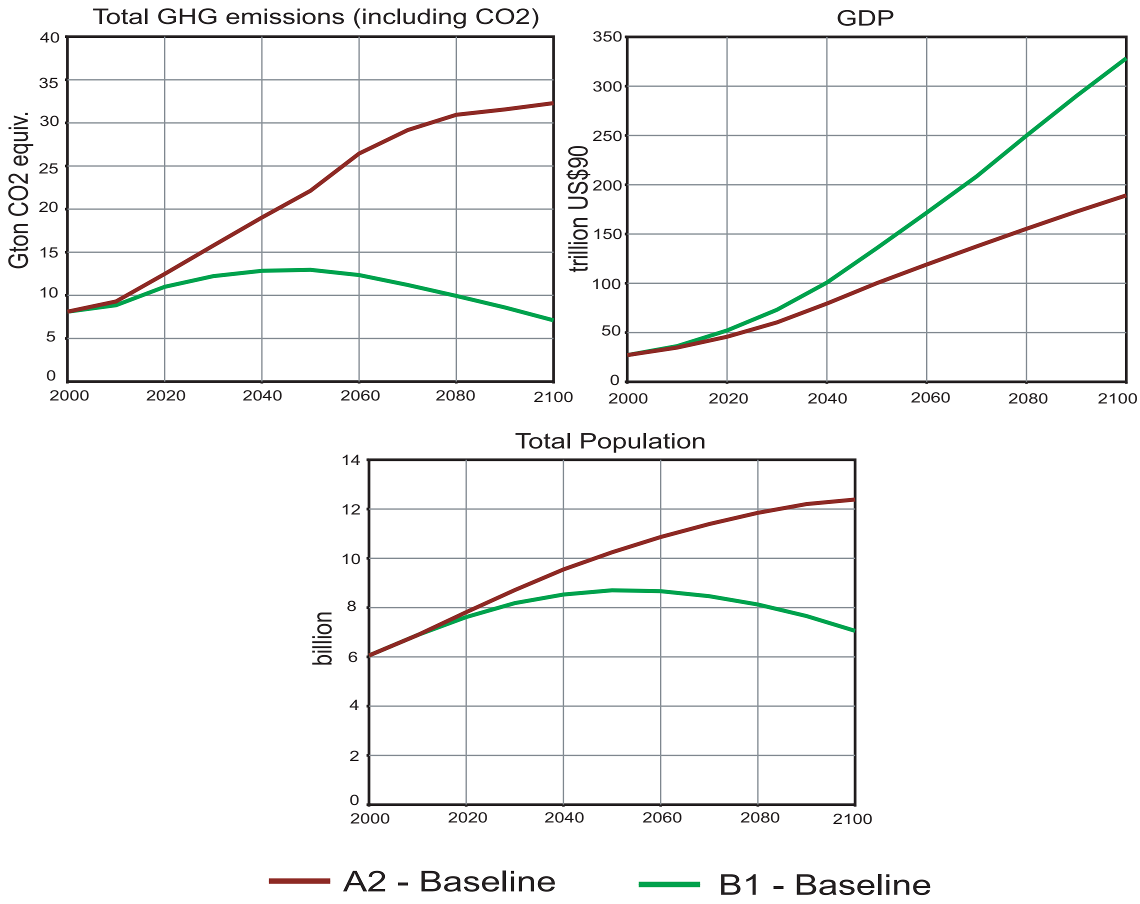

Figure 9: Scenarios.

4.5 Consistency analysis

In this section, we use the DICE–94 model to carry out the analysis of the consistency between long–term targets and short–term policies for various scenarios.

We take the following input data

Short–term time period $[t_0,\ t_*]$	–	$[2005,\ 2020]$;
Long–term time period $[t_*,\ t^*]$	–	$[2020,\ 2100]$;
Initial concentration of CO_2 (2005)	–	808.9 (GtCO2eq);
Initial world capital (2005)	–	137 (trillions of 1990 dollars);
Initial atmospheric temperature (2005)	–	$0.7307°C$ (above pre-industrial level);
Initial ocean temperature (2005)	–	$0.0068°C$ (above pre-industrial level).

Let us consider $2°C$ temperature target and $A2$ scenario. Figure 10 shows results of the consistency analysis if the value of climate sensitivity is equal to $3.0°C$. We can see that all possible short–term policies are consistent with the long–term target because the attainability domain lies inside the controllability domain, in other words, all possible couples of values (CO_2 concentration and world capital) in 2020 keep possibility to meet the long–term target. For each such state we compute the average rate of emissions reductions and the average abatement cost as percent of GDP needed to reach the long–term target if we start from that state in 2020. We obtain that depending on the CO_2 concentration in 2020 it will require to cut from 84% up to 94% of emissions and it will cost from 3.4% up to 5.2% of GDP. Therefore, the aim to reach $2°C$ target in the case $A2$ scenario and $3.0°C$ climate sensitivity seems to be difficult for the accomplishment.

Figures 11, 12, 13 show results of the consistency analysis for greater values of climate sensitivity. In this case not all values of CO_2 concentration in 2020 keep possibility to reach the long–term target and, consequently, not all short–term policies are consistent with the target. Moreover, the rate of emissions reductions and the cost to provide these reductions must be greater in comparison to the case where the value of climate sensitivity equals $3.0°C$.

Since consistency domain is computed, we can compute the maximal allowable level of the emissions over the short–term time period, which is consistent with the long–term target. Note that we are carrying out the analysis under a given scenario that defines changing CO_2 concentration in the absence any policy. Therefore, scenario implicitly determines the maximal *possible* level of the emissions. At the same time, we compute the maximal *allowable* level of emissions to keep possibility to meet a given long–term target.

Figure 6 shows the maximal annual allowable level of CO_2 emissions corresponding to various climate sensitivities under $A2$ scenario. We can see that if the climate sensitivity turns out to be less than or equal to $3.2°C$ then we must not exceed the level of about 16 GtCO2eq. If the climate sensitivity is greater than $3.2°C$ then the curve of allowable emissions goes down up to the climate sensitivity of $3.9°C$ where the long–term target becomes inconsistent with any short–term policy.

Figures 15, 16, 17, 18 show the results of the consistency analysis for $2°C$ temperature target under $B1$ scenario. Comparison of the results for these two scenarios indicates that the allowable ranges of CO_2 concentrations in 2020 are the same for both $A2$ and $B1$ scenarios. This means that scenarios don't affect the possibility of reaching long–term target. Indeed, let us consider a long–term abatement policy that provides cutting emissions by 100% (this is an admissible policy). In this case, there is not any emissions for any scenario. Therefore, the possibility to reach long–term target depends on initial concentration only. It provides an explanation that ranges of allowable concentrations are similar for different scenarios. Moreover, it emphasizes the importance to compute not only consistency domain but long–term actions needed to reach a long–term target as well.

We can see in the figures that scenarios essentially affect long–term actions. For $B2$ scenario and climate sensitivity of $3.0°C$, we obtain that depending on the CO_2 concentration in 2020 it will require to cut from 65% up to 85% of emissions and it will cost from 2% up to 4.5% of GDP. That is lesser requirements than for $A2$ scenario. However, the more value of the climate sensitivity the more requirement on the emissions reductions. If the value of climate sensitivity is equal to $3.8°C$ then we must cut from 92% up to 97% of emissions and it will cost from 5.5% up to 6.3% of GDP.

Figures 14 shows the maximal allowable level of the emissions under $B2$ scenario. These values are less than corresponding values for $A1$ scenario, because $B2$ scenario prescribes lesser level of CO_2 concentration than $A1$ scenario.

Figures 19 – 28 presents the results of the consistency analysis for $3.0°C$ climate target. We can see that in this case the consistency takes place if the value climate sensitivity takes values that are greater than $6.2°C$. This target requires lesser emissions reductions. By comparison, if the value of climate sensitivity is equal to $3.0°C$, it will require to cut from 65% up to 85% of emissions to meet the target under $A2$ scenario and cut from 2% up to 20% of emissions under $B1$ scenario.

Figures 29 gathers the result of the calculation of the maximal allowable level of the emissions reductions over the short–term time period for various climate targets and scenarios.

5 Conclusion

The approach we have presented allows to investigate the consistency issue between a given long–term target and short–term policies. This approach is applicable to various models. The main advantage of the approach is that it allows to assess all possible policies rather than a single policy and a single trajectory corresponding to it. In such a way every possible policy can be check if it is consistent with a target and, if that is the case, it can be compared to other policies consistent with the target. These results can be presented in a clear graphical way. This feature of the approach gives a powerful framework to decision makers for choosing an appropriate policy.

In the context of climate change, the approach is useful for analysis of the consistency between a long–term climate target and short–term abatement policies. In particular, if we consider the issue of limiting temperature rising due to CO_2 emissions, the method estimates which short–term policies for emissions reduction are consistent with a chosen climate target and computes which long–term policies for emissions reductions meet the target at minimal cost, depending on a preceding short–term policy. To illustrate the approach, we have carried out consistency analysis with the DICE-94 model for various long–term targets, values of climate sensitivities and socioeconomic development scenarios.

6 Computational results for DICE model

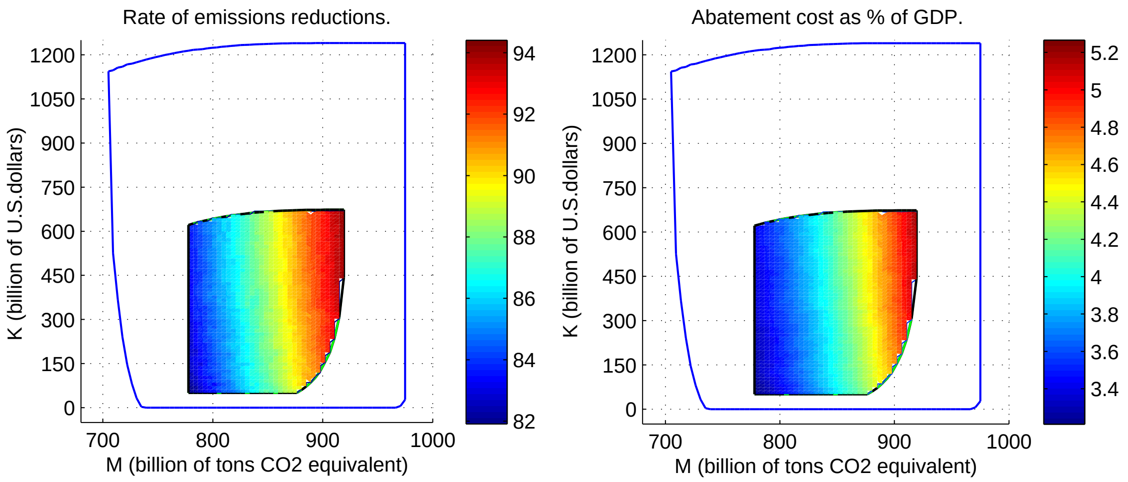

Figure 10: Climate target: $2°C$; Scenario: $A2$; Climate sensitivity: $3.0°C$.

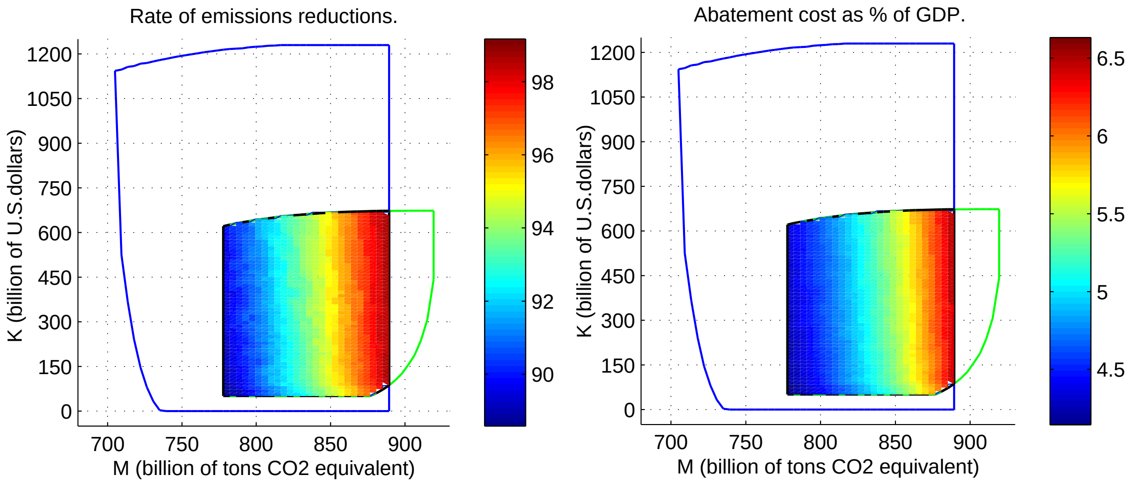

Figure 11: Climate target: $2°C$; Scenario: $A2$; Climate sensitivity: $3.3°C$.

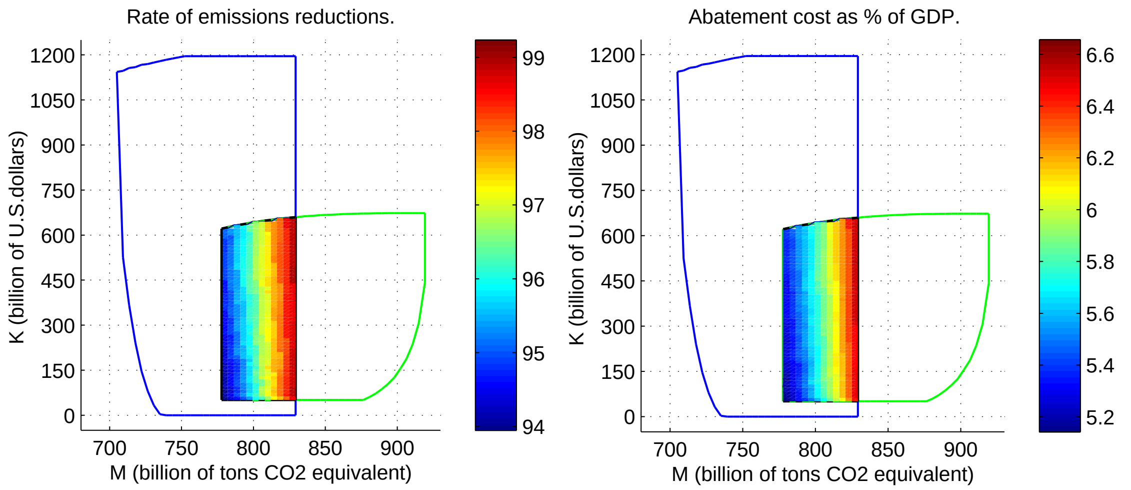

Figure 12: Climate target: $2°C$; Scenario: $A2$; Climate sensitivity: $3.6°C$.

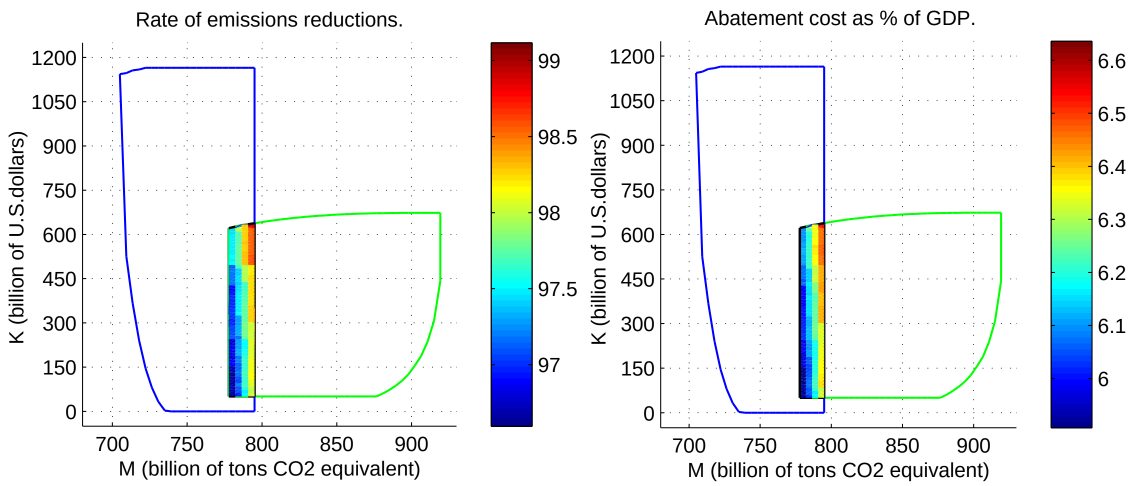

Figure 13: Climate target: $2°C$; Scenario: $A2$; Climate sensitivity: $3.8°C$.

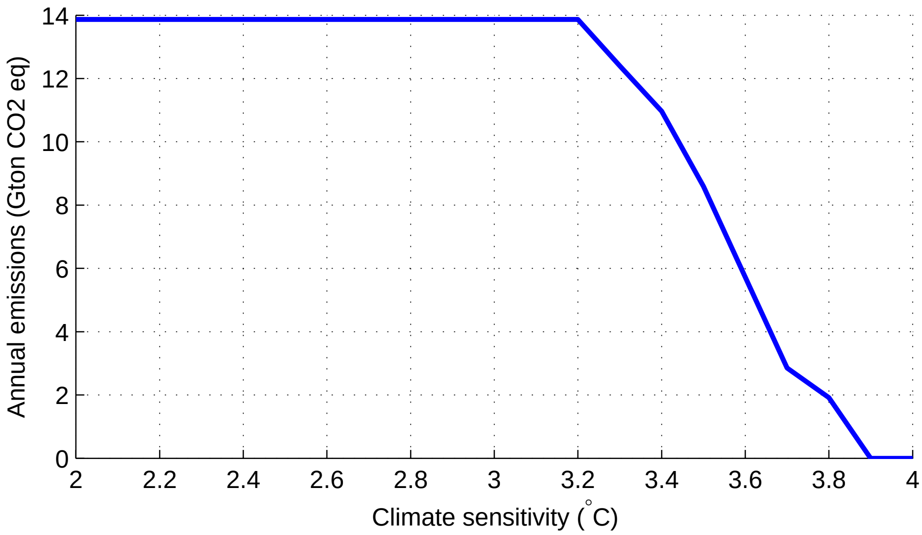

Figure 14: Maximal annual allowable level of emissions till 2020. (Climate target: $2°C$; Scenario: $B1$.)

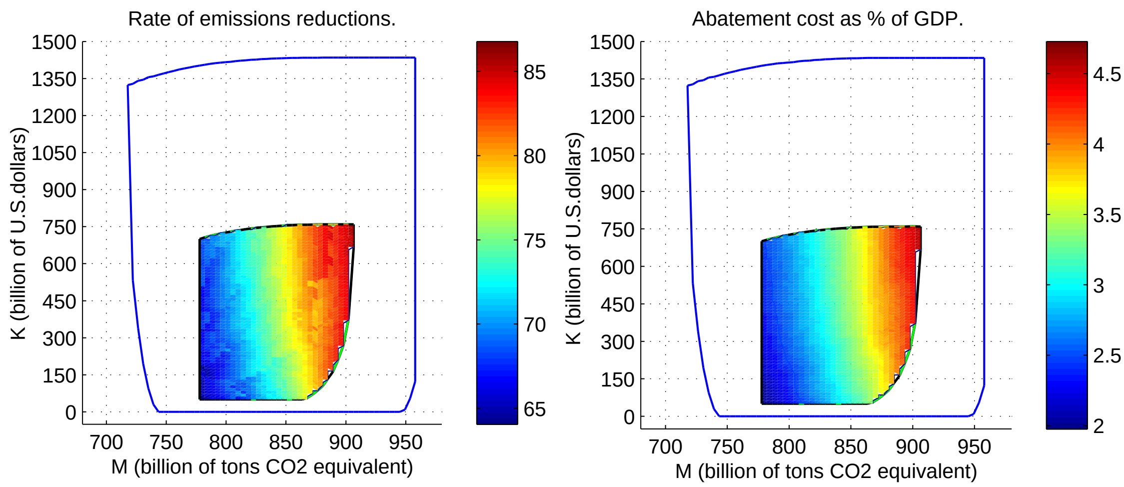

Figure 15: Climate target: $2°C$; Scenario: $B1$; Climate sensitivity: $3.0°C$.

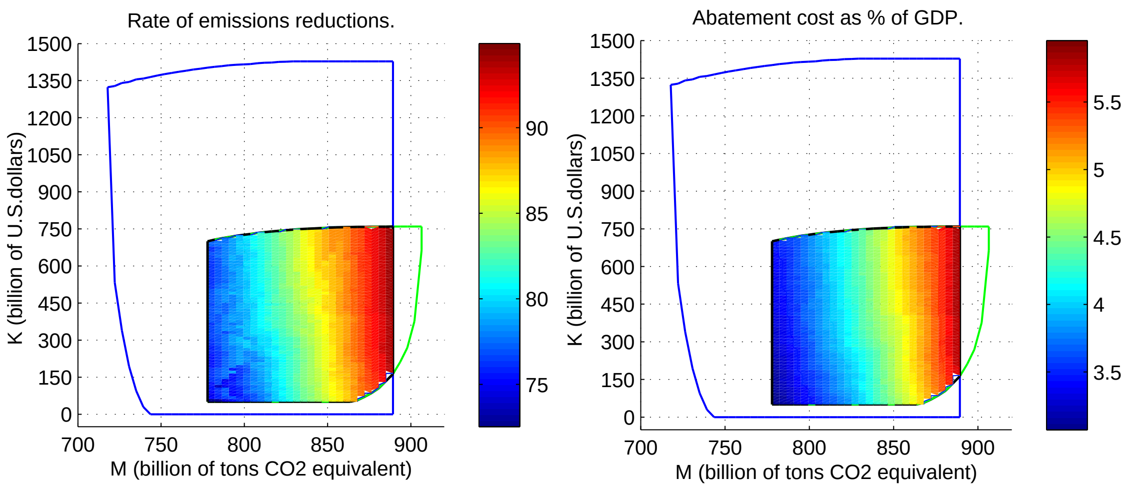

Figure 16: Climate target: $2°C$; Scenario: $B1$; Climate sensitivity: $3.3°C$.

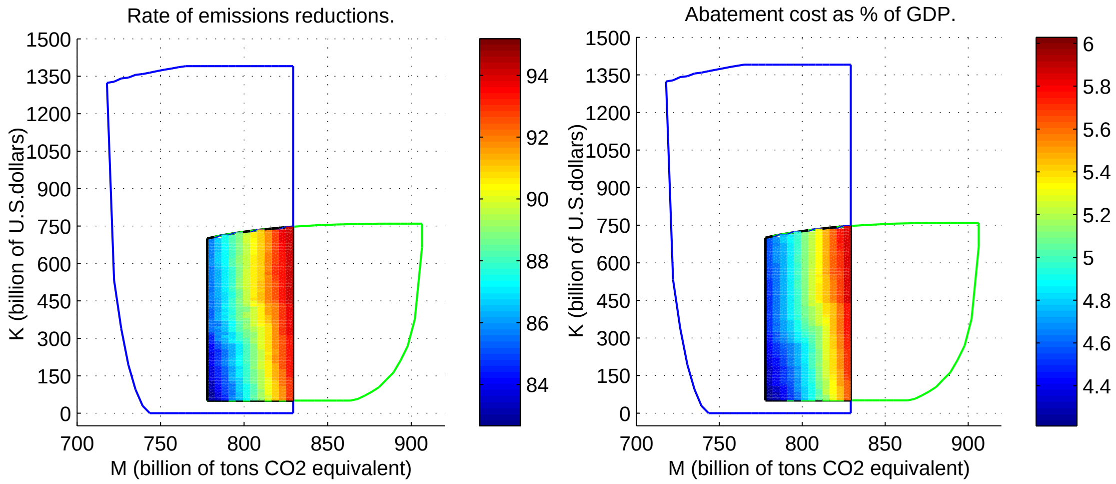

Figure 17: Climate target: $2°C$; Scenario: $B1$; Climate sensitivity: $3.6°C$.

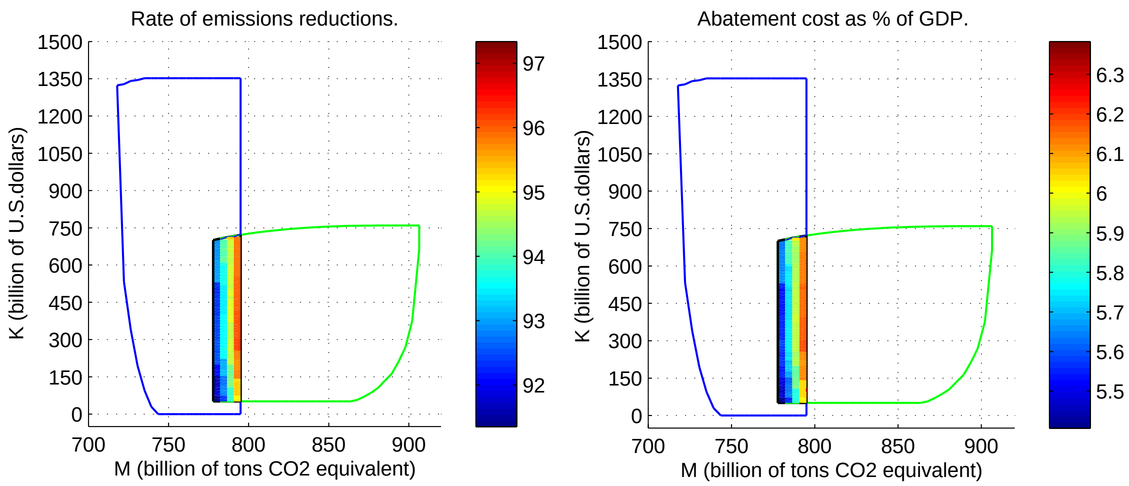

Figure 18: Climate target: $2°C$; Scenario: $B1$; Climate sensitivity: $3.8°C$.

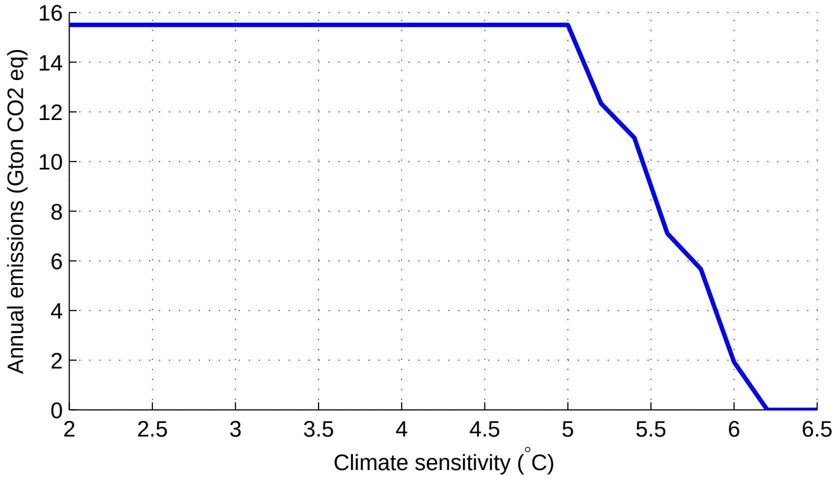

Figure 19: Maximal annual allowable level of emissions till 2020. (Climate target: $3°C$; Scenario: $A2$.)

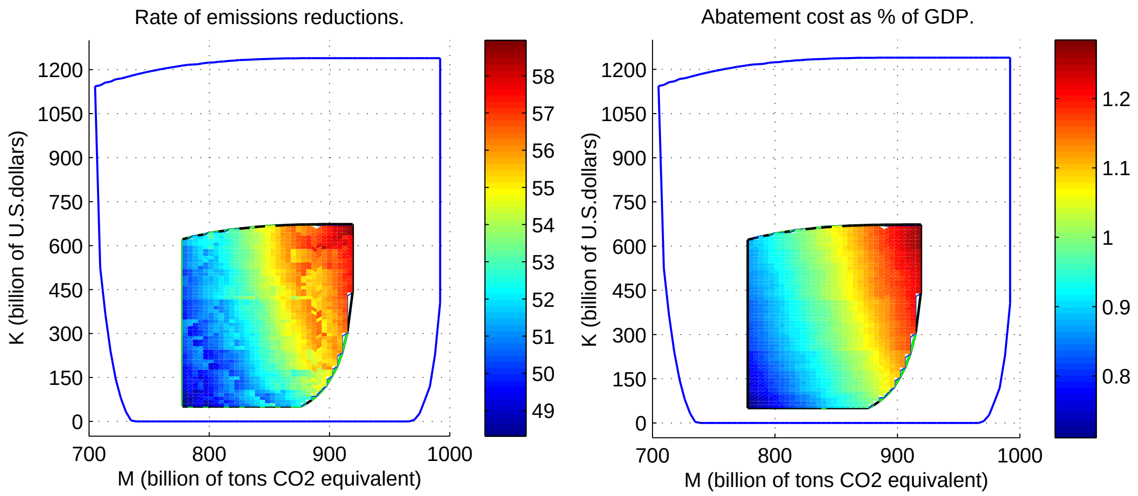

Figure 20: Climate target: $3°C$; Scenario: $A2$; Climate sensitivity: $3.0°C$.

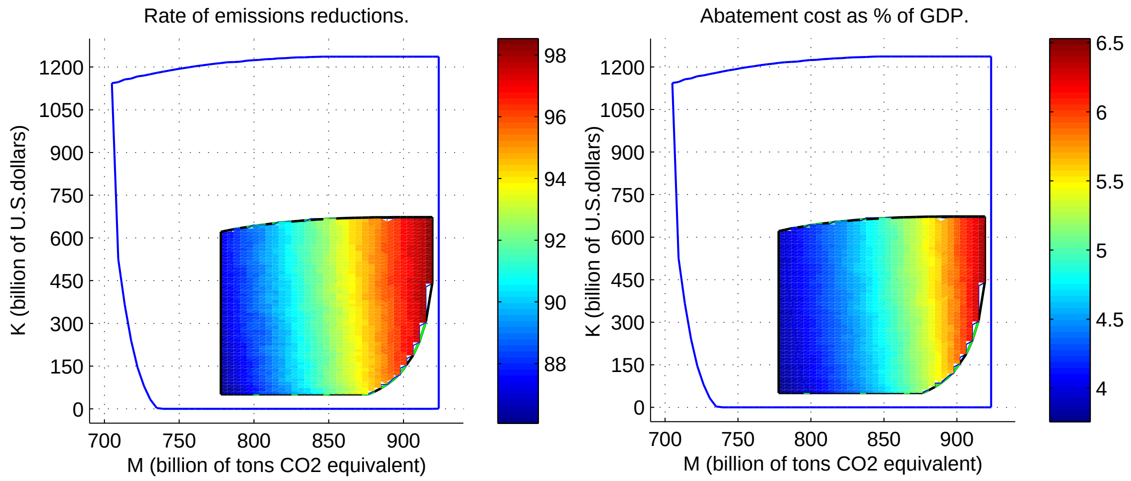

Figure 21: Climate target: 3°C; Scenario: $A2$; Climate sensitivity: 5.0°C.

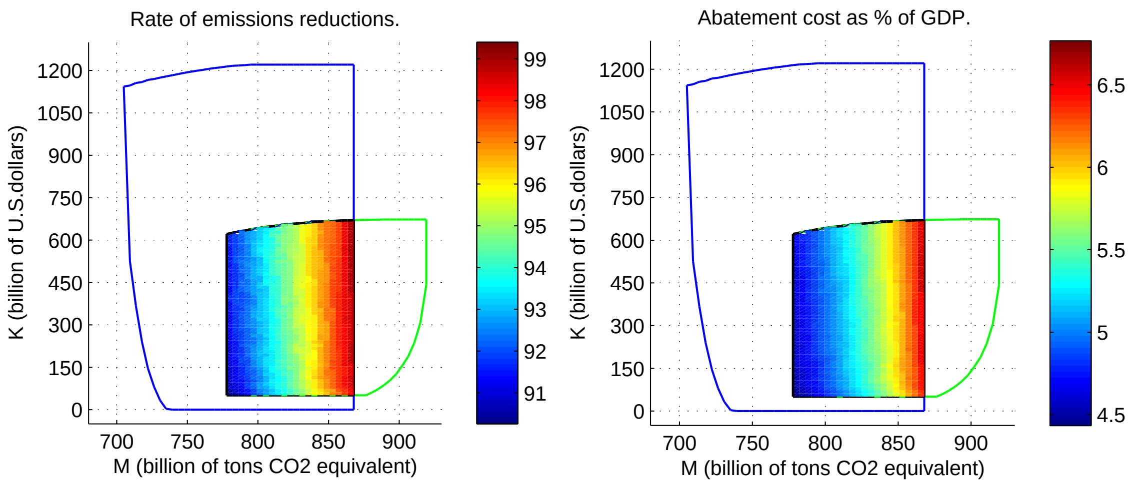

Figure 22: Climate target: 3°C; Scenario: $A2$; Climate sensitivity: 5.5°C.

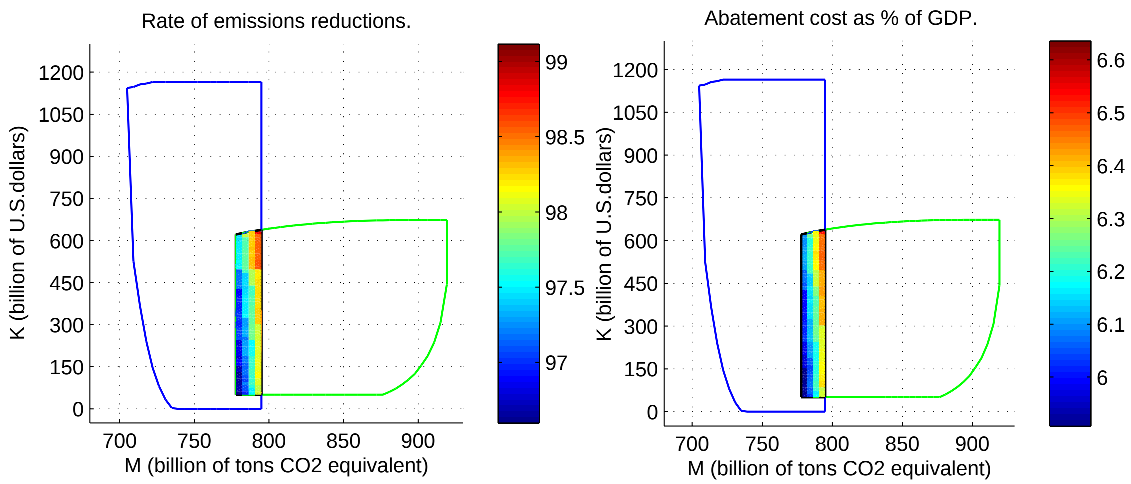

Figure 23: Climate target: 3°C; Scenario: $A2$; Climate sensitivity: 6.0°C.

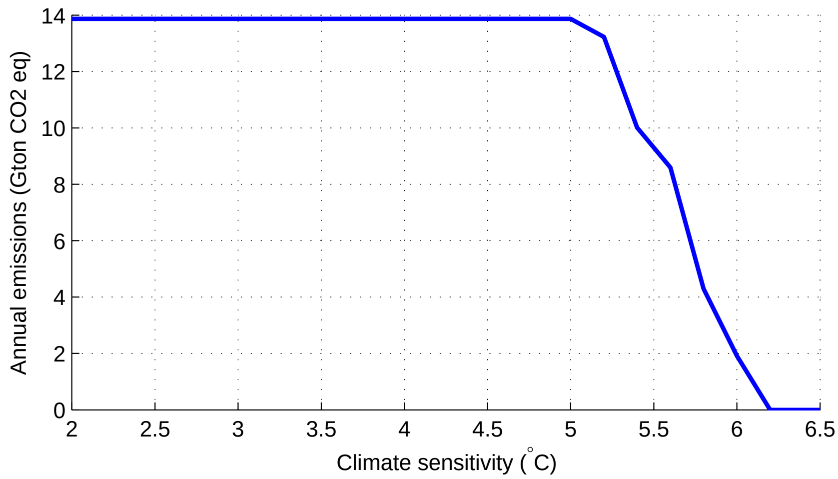

Figure 24: Maximal annual allowable level of emissions till 2020. (Climate target: $3°C$; Scenario: $B1$.)

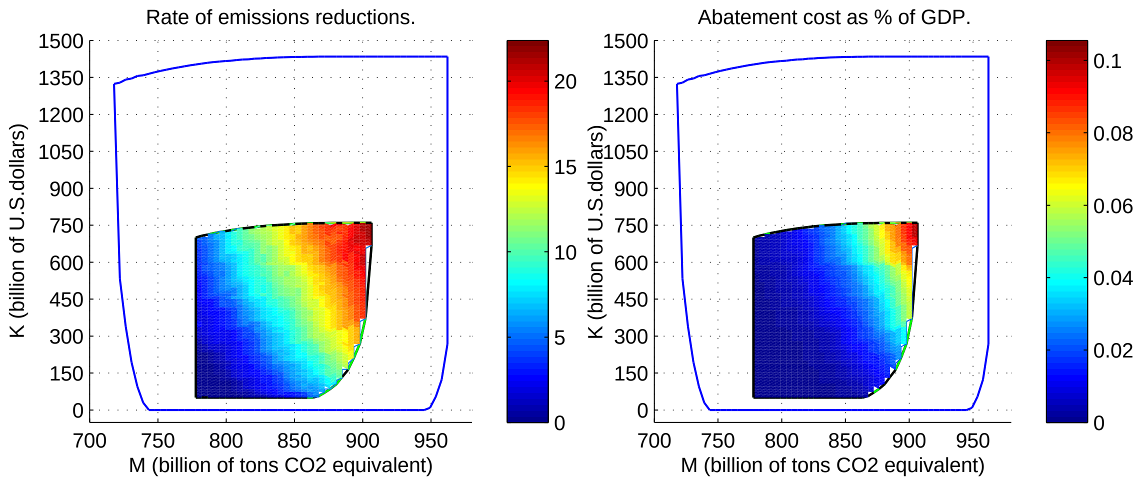

Figure 25: Climate target: $3°C$; Scenario: $B1$; Climate sensitivity: $3.0°C$.

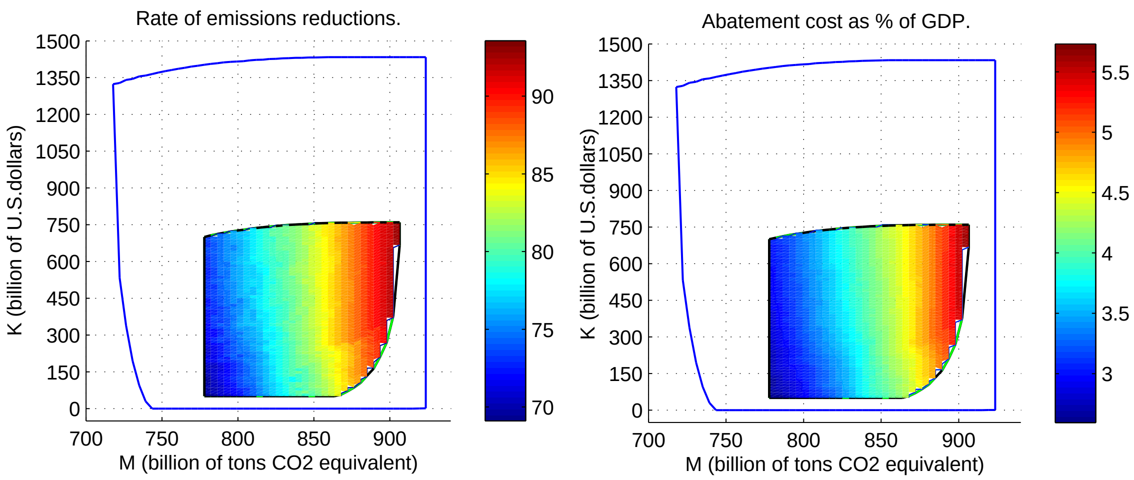

Figure 26: Climate target: $3°C$; Scenario: $B1$; Climate sensitivity: $5.0°C$.

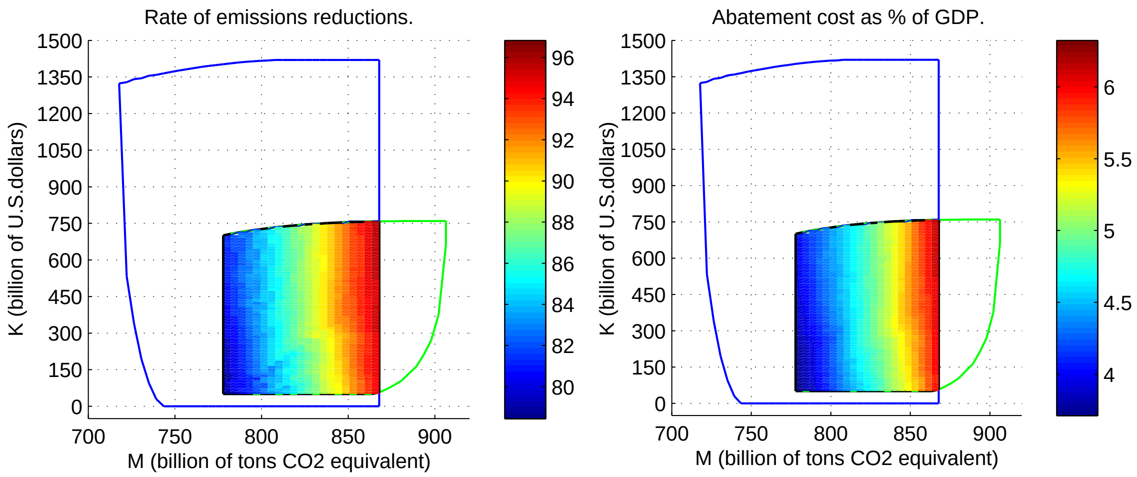

Figure 27: Climate target: $3°C$; Scenario: $B1$; Climate sensitivity: $5.5°C$.

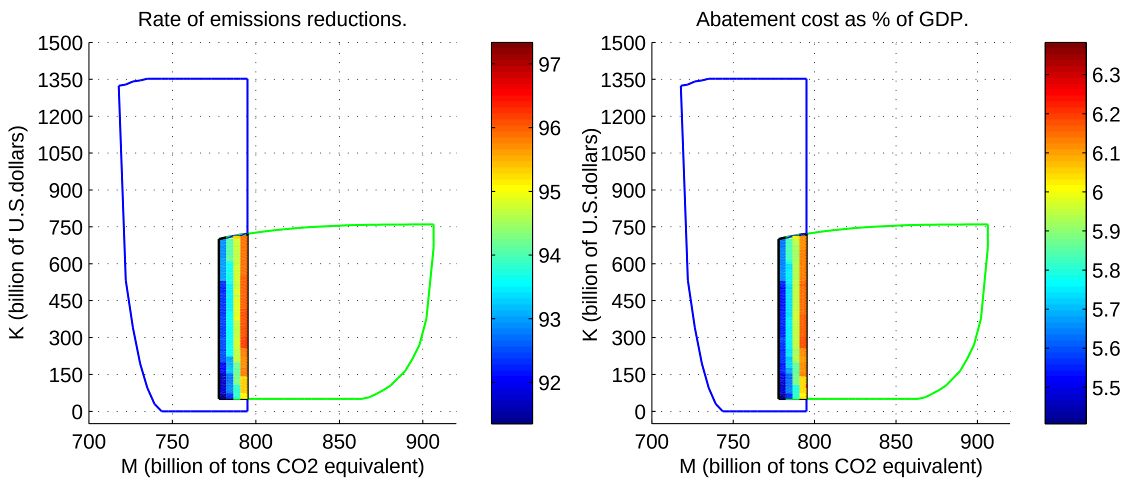

Figure 28: Climate target: $3°C$; Scenario: $B1$; Climate sensitivity: $6.0°C$.

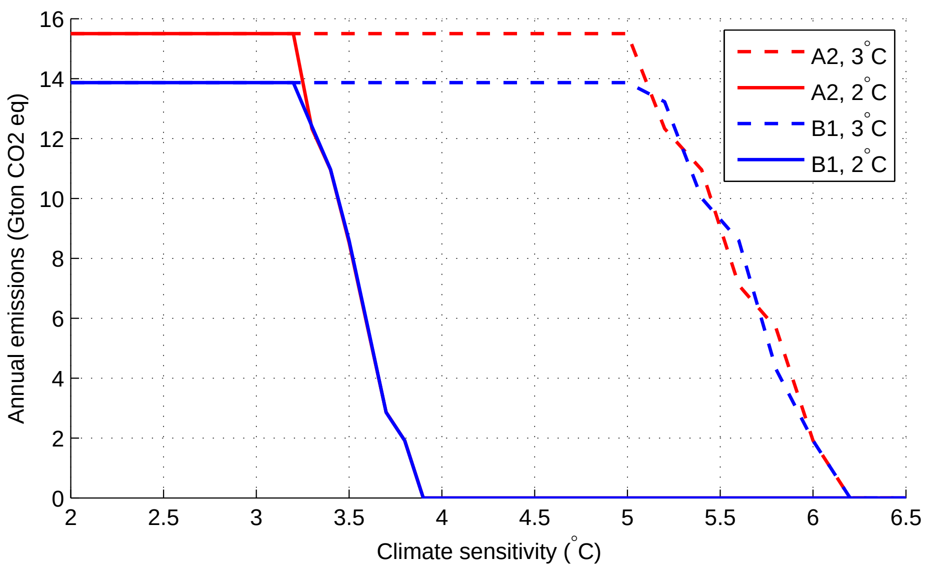

Figure 29: Maximal allowable level of emissions till 2020.

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