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A Simple hybrid 6 sector I-O model

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Introduction

Most large dynamic economic models, including Input-Output models, are driven by a single-sector ‘driver’, based on the original economic growth model developed by Robert Solow (Solow 1956, 1957). That driver is usually a production function of the Cobb-Douglas, or CES form, with two factors of production, capital and labor, possibly augmented by quality multipliers. As the capital stock increases, depreciation accounts for an increasing share of investment, so in the long run growth per capita converges to the rate of population (labor force) growth, times the exogenous technical progress (or multi-factor productivity). In recent years a “human capital” approach may be used to get around the saturation built into the standard model. In practice, exogenous “technical progress” (or “total factor productivity”) account for a large fraction of historical growth, whereas energy is not included on the grounds that its cost-share is too small to matter. As a consequence, neoclassical theory effectively predicts that the economy can grow without limit (driven by technical progress) even without energy.

Following the Arab boycott and the resulting energy-crisis of 1973-74 there was a short period of interest among economists in the possibility of resource exhaustion and the implications for consumption. Using the single sector model, with a CES production function Dasgupta and Heal showed that, if the elasticity of substitution of man-made capital for resources is less than unity, consumption must eventually fall to zero (Dasgupta and Heal 1974). Solow showed that if the elasticity of substitution happens to be unity, which is the Cobb-Douglas case, then consumption can remain positive indefinitely provided the output elasticity of capital exceeds the output elasticity of the exhaustible resource (Solow 1974). Stiglitz showed that per capita consumption can be non-zero if resource-augmenting technological change exceeds the rate of population growth (Stiglitz 1974).Other scholarship along these lines was incorporated in the book *Scarcity and Growth Reconsidered* by Resources for the Future, Inc.(Smith and Krutilla 1979).

We do not doubt that the work cited above yielded some important insights. But the world has changed. The possibility of resource exhaustion seemed remote in 1974; it seems much less remote today. The assumption that capital and resources can be regarded as perfect substitutes for one another was regarded as an acceptable assumption for purposes of analysis, back in 1974. However, on closer scrutiny, this assumption cannot be taken seriously; capital investment can reduce resource requirements to some extent, at the margin, but they are not perfect substitutes and for that reason, if for no other, the simple Cobb-Douglas or CES production functions cannot be used to represent situations significantly different from the optimal combination.

Back in the 1970s there were attempts to introduce energy as a third factor of production, both within the Cobb-Douglas (or CES) frameworks(Allen and et al 1976; Allen 1979) and in the trans-log framework (Jorgenson 1983, 1984; Jorgenson and Lau 1984). These initiatives were not accepted by the economics profession generally, after the notion was “debunked” by the growth accountant Edward Denison of Brookings on the grounds that energy accounted for such a small “cost share” (fraction of GDP) that the output elasticity of energy must be negligible (Denison 1984, 1985). In other words, Denison argued that energy cannot have a significant impact on growth, even though the effects of oil price spikes suggested otherwise. Theory trumped observation.

In fact it is now known that the “cost-share” theorem cited by Denison is valid only in the special case of a single sector model if there are no physical constraints on the allowed combinations of the factors (Kuemmel, Ayres, and Lindenberger 2008). In the real world where such constraints do exist, the theorem is not valid and the output elasticity of energy can be much larger than its cost share. Indeed, recent econometric analysis of the impact of oil price spikes on growth seems to confirm this conclusion (Hamilton 2005, 2009).

In any case, we think the time has come to graduate from the oversimplified single sector model to a more complex and, hopefully, more realistic model from which interesting implications may be drawn regarding resource exhaustion and long-term growth and consumption, as well as other questions of interest in the present world.

Description of the model

Most I-O models are formulated in terms of money flows only. Hybrid versions of large models have been introduced in recent years, primarily to more accurately reflect emissions associated with process technologies . However we argue that important insights can be gained from a different formulations. The simplest version that reflects the main elements involves six sectors, four stocks, and three types of flows, as follows. The six sectors are

1. Extraction and harvesting of exergy inputs from the environment. These exergy inputs include agricultural products, fishery products, forest products, minerals and fossil fuels (coal, oil and gas).

2. Conversion of primary exergy inputs to energy carriers (especially electric power; we would like to include other energy carriers, especially liquid fuels, in this sector, although to do so makes the model significantly harder to quantify.).

3. Manufacturing, and construction of material goods, both durable and consumable. Durables include metal components, machines, vehicles, infrastructure, permanent structures (including housing) plus furniture, furnishings, household equipment, and other long-lived products such as books, clothing, etc. Consumables include lubricants, fertilizers, pesticides, chemical intermediates, food products, paper products, cleaning agents, cosmetics etc.

4. Services of all kinds, except government and banks,including transport, wholesale and retail trade, communications, health services, schools and universities, R&D and entertainment.

5. Banks and financial institutions providing credit (and holding debt)

6.Government, including law enforcement, courts, military, regulatory authorities, tax collection, .

Final demand consists of households, which consume both goods and services from other sectors and receive income from selling labor.

The four stocks are (i) labor (ii) capital equipment, durable goods, structures and infrastructure in use, (iii) knowledge (both as R&D stock and education) and (iv) money (debt). The labor stock is exogenous, but the labor flow is an output of households. The capital stock is an accumulation of outputs of the manufacturing sector. The knowledge stock is an accumulation of outputs of the R&D component of the service sector (as defined), and the money stock is an accumulation of debt, which is defined as the difference between revenues and expenditures of each sector. For simplicity we allow households and government to borrow, while the financial sector (part of services) provides the credit. Interest rates are set exogenously.

The three types of flows are (i) material (exergy) flows, from the environment, between sectors (as embodied in material goods) and to the environment as wastes, (ii) service flows – including labor and capital services – that have monetary value (hence prices) and (iii) money itself. The exergy flows are subject to physical balance conditions, which follow from the laws of thermodynamics. Similarly the labor supply is subject to constraints (for instance it cannot grow or decline by more than a small amount from year toyear.)The money flows, similarly, must satisfy accounting balances during any single period, though the money supply (the sum total of circulating currency plus credit) can grow as banks create credit. Banks, in turn, are subject to limits based on the value of assets.

The role of mass-exergy

Exergy is embodied in all mass flows, including waste flows from each sector. All input mass-exergy flows *X(t)* are converted to waste flows *W*(t) (neglecting time delays) except for mass-exergy embodied in *net* new (hard) capital formation *ΔK* in the period at time *t*. It is important to emphasize that we distinguish hereafter between the mass-exergy embodied in capital stock and the monetary cost of that capital, its price and it's economic productivity. It is easy to prove (and the proof is quite general for any number of sectors) that the mass-exergy flows satisfy the equation:

$$\Delta K(t) = X(t) - W(t) = X(t)E(t) \qquad (1)$$

where *X(t)* is the raw unprocessed mass-exergy input from the environment to the economy, representing agricultural harvests, fish, forest products, and minerals (including coal, oil and natural gas) and E is the exergy efficiency of the economy as a whole, defined in (1) or

$E(t) = \Delta K(t)/X(t)$

The aggregate waste *W(t)* is the sum of exergy wastes from each sector (including households), viz.

Exergy embodied in consumables is immediately converted into wastes (in the same

$$W(t) = \sum_{i=1}^{6} w_i \qquad (2)$$

time period), so intermediate flows simply cancel out. The only accumulation of exergy that occurs in the system is long-lived physical capital, which is one output of Sector 3 (by definition of the sector). Depreciation of the existing capital stock accounts for part of the capital output (more as the economy matures). depreciation can be introduced explicitly (contributing to intermediate flows) but the key result (1) does not change.

The sum of net new capital formation $\Delta K(t)$ over all time periods constitutes a

$$K(t) = \sum_{i=1}^{6} k_i \qquad (3)$$

material capital stock $K(t)$. The sectoral capital stocks k_i, provide capital services to the sectors, including households. These capital services are a form of (non-monetary) input to production and consumption. There are monetary costs attached to capital services, of course. The costs correspond to the costs of mobile capital goods shipped from sector #3 to the other sectors to replace depreciated capital. (n reality there are leasing agents in the service sector #4 that purchase the goods (e.g. aircraft) and lease them to users, such as airlines. But the main consequence of leasing is to increase the effective cost of capital, and the only contribution of the service sector is value added. The situation for immobile capital (buildings and infrastructure) is comparable, except that houses and small buildings are financed by mortgages, while most big projects such as highways, dams and bridges are built with money from bonds issued specifically for the purpose.

The capital stock per worker is an economic variable that normally increases over time: $\lambda(t) = K(t)/L(t)$ where $\lambda(t)$ is usually given in monetary units ($). The cost of capital, or the price of capital services, includes amortization (interest paid on debt) and other returns in the form of rents, dividends and royalties. Let the rate of return on capital (or the price of capital services) be designated as η, which we can also reasonably assume is the same for every sector. Then the sum of all payments for capital services is the total return on capital for the whole economy, is equal to the cost share for capital in the GDP which we have designated by the letter Y. In the US (circa 1950) the share of capital costs in the economy was about 30 percent:

$$\eta K = 0.3\, Y \qquad (4)$$

(In China, today, the capital share is apparently closer to 60 percent).

The capital stock k_6 belonging to the household (HH) sector includes houses, private cars, long-lived appliances, furniture, books and so forth, but not working farm animals (such as horses) or farm equipment, which belong to the agriculture subsector. Household capital (in monetary units) is a major part of the personal wealth of the households, the rest being money or money-equivalent assets such as stocks and shares (see below). However mortgage and other debt must also be taken into account for a wealth calculation. National wealth is also closely related to the magnitude of total K (including k_6) plus money and money-equivalent assets less debts.

The other variable that is normally regarded as a factor of production is aggregate labor, L, usually measured in man-years. Labor (as a flow) is produced only by the household sector, and is the only output of that sector:

$$L(t) = \sum_{i=1}^{5} l_i \qquad (5)$$

It is convenient to define the price of labor as θ (the wage rate), which we assume to be the same for unskilled labor in every sector. The product θL is the sum of all labor income, i.e. wages and salaries. In production function models, the monetary value of capital stock ηK is considered to be a factor of production, along with the value of labor services θL

In the US (circa 1950) returns to labor were about 70 percent of total GDP, viz.

$$\Theta L = 0.7\, Y \qquad (6)$$

Evidently the sum total of returns to capital and returns to labor constitutes the total GDP,

$$Y = \eta K + \Theta L \qquad (7)$$

which is also the sum total of value-added in the economy.

It is noteworthy that there are payments for the capital and labor needed to extract exergy from the environment, but there is no payment to the environment, *even though the economy could not exist without exergy inputs*. (Payments to farmers and mining companies or oil companies should be considered to be payments for the capital and labor inputs, not for the resources themselves.) Hence, there are strong reasons to think of exergy as a factor of production, even though it has no share of GDP. Equations (4) and (6) may be regarded as constraints on possible values of the various flows.

Prices, credit and debt

Both η and Θ are essentially prices for capital services and labor services, respectively. However, physical flows of mass-exergy, as well as capital, labor and other services, can also be converted into money flows by means of prices. Money flows and balances can now be introduced, for each sector and for the economy as a whole. There is also a money stock for each sector and a total stock of money analogous to the capital stock.

Apart from physical currency, all money is really credit. Banks have the ability to create new money (as credit) by leveraging deposits. In principle, all debt/credit is created by sector #4, although it is convenient to attribute debts and changes in debt directly to the sectors.[3] There are other money-related services to the sectors (deposit, credit and transfer) performed by banks, which are also part of the service sector #4.

The money supply is much more than currency in circulation. In order of decreasing liquidity it starts with cash, demand deposits in banks, followed by savings deposits, commercial paper, certificates of deposit, bonds and mortgages. Every debt is also a monetary

3. The Federal Reserve Bank is included in sector #3 as part of the banking system, even though it is a government institution.

asset for the lender.[4] Hence the total money stock of the economy – excluding the value of equity shares in enterprises– can thus be equated to aggregate net debt *D(t)*, which is the sum of net sectoral debts $d_i(t)$.[5]

$$D(t) = \sum_{i=1}^{6} d_i \qquad (8)$$

and, of course

$$\Delta D(t) = \Delta \sum_{i=1}^{6} d_i \qquad (9)$$

The cost of debt depends on inflation, liquidity and perceived risk. In the real world interest rates vary from time to time, from sector to sector, and even from one individual to another, depending on those variables. The cost of debt can be equated roughly to an average interest rate ϕ. times the quantity of money owed *D*. The product ϕD is a part of the return on capital, but not the whole of it, because profits are also a form of return on capital. Debt service, as such, is distinct from other financial services such as brokerage, insurance and investment advice, all of which have their own prices.[6]

In exchange transactions between sectors money flows in the direction opposite to product or service flows. Costs of inputs from sector *i* to sector *j* are equal to the unit price p_i of the products of sector *i* multiplied by the quantity (either of mass-exergy or of services) flowing from *i* to *j*. The prices of other services and commodities are designated by p_i. In cases where the output of a sector consists of products with embodied mass-exergy, the value of the output is the product of mass flow times a price per unit mass. This applies to sectors #1, #2 and #3, which produce and sell material products.

For the service sector #4 and government #5 the outputs are mass-less and the value of the output can be expressed as a product of a price per unit service times a service flow. Sector #4 includes banks and financial services, but it also includes transportation, wholesale and

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4. It is tempting to think that assets and debts are always in balance, but it is not true, as the economy has recently demonstrated. Banks have created a lot of cfredit and lent a lot of money based on a capital base that turned out to be illiquid and possibly valueless, or very uncertain in value. But even though house prices have fallen considerably and many borrowers are “under water” the banks still expect to get their money back, even though millions of people have lost all their assets. In brief, a lot of money has literally disappeared, and the main losers are the borrowers, not the banks.

5. In principle debts are balanced by assets, but assets are not necessarily liquid (depending on markets) and security as regards debt is therefore a slippery concept. Since the gold standard was abandoned, Government debt is only secured by “faith and credit”. But history records many instances where government debts have been repudiated or inflated it away. Corporate bonds lose value if the company is forceed into bankruptcy (as exemplified by the recent exapmples of Chrysler and GM) . Mortgage debt loses value if the price of houses falls. Credit card debt is secured ultimately only by the card-holder’s reputation.

6. Strictly speaking, the deposit, credit and transfer services of banks should be treated as a separate sector, but in reality banks also perform other financial services while credit, in particular, is also provided by non-banks. Hence, it is more convenient to conceptually separate the functions of money from the institutions that manipulate it.

retail trade, commercial real estate, legal services, higher education, health care, publishing, communications and entertainment. Sector #5 is the government, at all levels, including public schools, police, public health, law enforcement, public roads, parks, water and sewage, airports, harbors and defense, plus regulatory authorities, tax collection and social security. All of these must be paid for by means of taxes and fees or loans. Thus tax revenues can be thought of as the price of government services, and a single tax rate on profits might be assumed for simplicity. However, there is no straightforward way to measure services except in terms of manpower, whence the value of services is generally equated to their cost in terms of wages and salaries. (This convention makes it difficult to measure productivity gains in the service sectors – a problem known as the "Baumol disease.")

The *value added* by each sector is the value of the output minus the cost of the purchased inputs, except for labor and the cost of capital services. *Profits* are the difference between output value and total costs. where "costs" must include taxes, wages and salaries, payments for capital services and debt service. Net profits at the sectoral level are either re-invested within the sector (to replace depreciated capital or to create new capital) or returned to owners as dividends or interest on bonds or loans. The fraction returned to owners (households) is denoted by α which we assume to be the same for all sectors. Of course the government makes no profit (normally), but collects taxes on gross profits of industry as well as salaries and wages. In the case of households, savings (income less expenditures) follow the same route, but the capital stock in question consists of consumer durables such as cars, clothes, furnishings and houses. Household savings can be negative (as has been the case in the US from 1990 through 2007).

We calculate total profit (surplus) Π for the economy as a whole by aggregating the sectoral income flows and outgo. All inter-sectoral flows cancel out exactly– because transactional income to any sector is also an expenditure to another sector. The only terms that do not cancel out are capital formation and debt changes, so we obtain:

$$\Pi = (p_2 + \mu)\Delta K - \mu X - \phi \Delta D \qquad (10)$$

This can be rewritten using equation (1), yielding

$$\Pi = p_2 E X - \mu X(1 - E) - \phi \Delta D \qquad (11)$$

The first term in (11) is the value of net capital formation at time t, in monetary terms, which is proportional to the price of capital goods at time t and the net resource (exergy) inputs to the economy at time t. The latter is the product of total resource (exergy) extraction *X(t)* times the net exergy efficiency *E(t)*. *E(t)* reflects changing technology, which depends on increases in the stock of knowledge. The stock of knowledge, in turn, is an output of the service sector (#4) by assumption. There is no fixed relationship between the total output of services and investment in the stock of knowledge, so this also must be regarded as exogenous (or as a control variable).

The second term in (11) is the cost of waste disposal, also in monetary terms, which also declines as *E* increases (other factors remaining equal). The third term reflects the impact of increasing aggregate debt during the period. *As X and D increase over time aggregate profit* *(surplus) can become negative unless the efficiency E increases rapidly enough to compensate*. The current global recession may be an illustration of this phenomenon.

Equation (11) is a fundamental relationship between the physical world and the economic system. The derivation is extremely simple and involves very few economic assumptions. It is interesting to consider how the fundamental equations would change if there was an institution representing “the environment” and capable of charging for its exergy services and re-investing the revenues by purchasing other services such as reforestation, restocking of fisheries and so forth.

If exergy inputs were not free, we could imagine a tax rate ξ applied to inputs X, resulting in an increase in the cost (and price) of outputs x_{ij} and other downstream services. However the funds would have to be recycled somehow, probably as a revenue stream to the government sector #5. In this case the “returns” to exergy inputs ξX would constitute a significant component of the GDP, reducing the cost shares for labor and capital.

The classic “model” (really a caricature) of an economy consists of a production sector and a consumption sector in balance. Households produce labor, while household expenditures for products and services pay the wages of workers who provide the household income. Profits add to household income, permitting savings and investment. But this model economy produces only non-material goods and consumes no energy (exergy). Our hybrid economy therefore reduces to the classic case if we set $X = 0$, meaning no inputs of mass-exergy. In this case there can be no physical capital, so $K = 0$, hence no production. Hence the classic case is both unrealistic and uninteresting.

If we assume constant E and no population growth, then ηK will be bounded from above, which allows for little or no economic growth. For a growing economy $K(t)$ must grow geometrically, if not exponentially, which implies a comparable growth rate for the product $E(t)X(t)$. But resource constraints will limit the growth of $X(t)$, which implies that efficiency $E(t)$ must carry most of the burden if economic growth is to continue. However, at some future time, E will reach a maximum (the theoretical maximum is $E = 1$) and economic growth will necessarily come to a stop. This would be the ultimate limit to growth.

Prices.

To convert physical flows and service flows into monetary flows, prices are needed. The price of a unit of mass-exergy at time $t = 0$ must be equal to the cost of extraction plus a profit margin. The cost of extraction is the quantity of labor required, times the unit price plus the quantity of capital service required, also multiplied by the unit price .of purchased capital services. The price of the product of each sector is based on both the cost of the services provided by the capital equipment belonging to the sector and the cost of the direct labor, plus the cost of other goods or services purchased by the sector, plus taxes, plus a profit margin. The value-added is, of course the difference between the cost and the value of the product of each sector.

The money units are arbitrary, so we can begin by specifying the price (wage) of a unit (man-hour or man-year) of unspecialized labor as θ, Then other wages (e.g. for skilled professionals) will be multiples of θ. The price of capital goods (actually, the use of capital) must be a multiple of its labor and power equivalent. The price of a horse-equivalent (at time $t = 0$) should thus be equal to the yearly wage of about 5 unskilled human workers, or 5θ.

For services by highly trained professionals, such as pilots, lawyers, surgeons, accountants, actuaries and so on, the number of years needed for training beyond elementary school (literacy, numeracy) might be the most appropriate multiple of the simple number of people in those professions. However, lacking a model of educational achievement over time, we suggest that the price of quantity of service at time $t = 0$ should be equal to the manpower required at twice or possibly three times the wage rate θ for unskilled manpower plus the cost of purchased goods or services times a profit margin.

The growth mechanism

Each sector is regarded as a decision-maker. The major levers on the economic system in our model are Δd_i, $\Delta k_{i.}$ and Δe_i. However, the decision-maker must begin by assessing its current situation, resulting from its own decisions made in the previous periods, as well as outcomes of previous decisions made in other sectors (suppliers, customers, regulators). Outcomes are probabilistic, not deterministic, so the first step in the decision process at time t is to calculate the available stocks of capital, labor and money and the total monetary value of all assets of the sector, resulting from the outcomes of previous transactions and investment decisions. Deviations between actual and previously expected outcomes – including, but not only, profits (or losses) from the previous period – must also be noted and taken into account. Savings or borrowing decisions for the current period need to be made, and these decisions may depend on cumulative results over a number of previous periods. However, the details of an hypothetical evaluation algorithm need not be considered here.

In principle, each sectoral decision-maker would like to respond only to actual current demand from downstream customers, including households, before deciding on labor needs and input requirements from upstream suppliers. However, in reality (and in a model) such decisions have to be made on the basis of expectations, informed by previous experience. So each sector is likely to assume demand from customers will be the same as the previous period, as adjusted to reflect probable growth (or shrinkage) of the economy based, in turn, on changes in the overall money supply. Here a trend calculation covering several prior periods will be needed.

Then, given the assumed demand, the necessary labor costs and interest payments on invested capital are determined, and the expected profit, along with expected tax payments, are determined. The decision-maker then decides how much to invest in new capital, how much to pay in dividends and how much (if any) to borrow. These decisions are all made before the actual demand from other sectors or households is known. Thus, there is always a gap between expected results and actual results, which must be carried forward to inform the decisions in the next period.

Decision-making by households is essentially the same, except that spending is based on expected income from labor and dividends, adjusted up or down on the basis of deviations of actual results in the previous period from expectations in that period. If actual income was greater than expected income, anticipated expenditures in this period may be a little greater than last time, and conversely.

In the case of government, expected income from tax revenues and fees, based on the previous period, will be the basis of expected current expenditures (fiscal policy), current tax rates and current interest rates (monetary policy). Again, there will be a comparison between the actual and expected results from the previous period, and this will also influence current policy. If tax revenues were higher than expected, current rates may be cut, or expenditures may be increased. If money supply growth has accelerated, interest rates may be increased, and so on.

Note that prices, or price changes, do not appear explicitly anywhere in this scheme. Only money flows, which are the product of material flows multiplied by prices, are considered. However, none of the assumed stocks (physical capital, labor or money) are infinite, nor is the exergy input flow. If the demand for labor exceeds the number of available workers (or man-hours) wages must rise to balance supply and demand. If the demand for capital exceeds the available stock, capital prices must rise, and if the demand for money (debt) exceeds the willingness of creditors to lend, interest rates must rise.

Up to this point, we (in our role as hypothetical decision-makers) have not explicitly considered wastes or energy efficiency. However, suppose the output of sector #1 (extraction) is geologically limited, so the sector cannot increase output simply by deploying more labor or capital. In this case prices would have to rise to balance the demand for exergy with supply. Or, as an alternative, we can suppose that government imposes a disposal charge on waste exergy. This adds a cost to every sector in proportion to its consumption of exergy embodied in products or services (such as electric power). This disposal charge – analogous to a carbon tax or "cap and trade" permit – could be a form of income for the government, or it could be returned directly to households as "owners" of the environment.

So far there is no mechanism for producing new knowledge or technological change. We could add a new sector to include all activities related to education, training and R&D, or these activities could be treated as a subsector of services (#4). Inputs to this sector would be payments from other sectors, including government and households. However the output would be "knowledge", with an economic role similar to capital, except that knowledge increases the productivity of the capital stock, the labor stock and the exergy input, as measured by the efficiency E.

Payments from each of the other sectors would be of two types. The first type would cover the maintenance cost of the existing knowledge stock, i.e. education and training, and it would be paid partly by government and partly by households. Costs would be relatively fixed (like taxes). The second type of payment, to purchase new knowledge (R&D), would be a variable for decision-makers, although perhaps only incremental changes would be allowed. The benefits would be applied exclusively to to labor quality. The purchaser of R&D is either the government or industry, but the economic productivity gains would be mostly from the industry component. These gains would be allocated between capital goods quality and exergy efficiency. The allocation may be made exogenously or endogenously.

There is another possible mechanism for driving growth, namely to introduce an explicit price elasticity of demand. Costs will fall (by assumption) thanks to economies of scale in production and/or increasing experience. However, for any given product, there is a declining marginal utility for customers, which creates a saturation effect on total demand. The same is true on the cost side: as costs fall, whether thanks to economies of scale or experience, the rate of cost reduction also declines.

The only way to overcome the saturation problem, as noted long ago by Joseph Schumpeter, is to innovate. When a new product or process is introduced, assuming it is competitive in cost and performance at the time of introduction, the market for the old one rapidly disappears, because as the new product draws customers away from the old one, the economies of scale can be lost (as seems to be the case right now for GM) because fixed costs are much harder to eliminate than variable costs. The substitution process as a whole has been called "creative destruction" for obvious reasons. .

Unfortunately, the innovation process *per se* is difficult to model. This topic will be considered in a later paper.

Appendix: Peter Fleissner

To end up with a hybrid model, its structure is developed step by step. The idea is to develop a generic structure of a dynamic input output model with n+1 sectors (the last sector is the banking sector) where physical capital investment increases capital stock. At the same time the financial basis for capital investment is determined. Savings of households are derived from the difference between wages and consumption, money savings of firms are fed from the difference between profits and capital investment. There is a special sector with different behavior, namely banks. Their turnover is the difference between interest paid back by firms and households for credits and the interest paid to depositors and investors who provide money capital to the banking sector.

In this version of the model there is no technical change. Growth is just related to the extension of capital stock in equilibrium. The composition of capital investment corresponds to the composition of capital stock in place. For the moment there is no depreciation of capital stocks included. The accounting of the economy can be done in two different ways:

(a) In terms of values (physical quantity times unit price)

(b) In standardized coefficients (unit prices, or inputs of sector i per unit of output of sector j like the Leontief matrix **A**) or in mass-exergy units.

Both versions are available for computer implementation on a laptop. In the following equations output **x** is measured in value terms, the matrices **A**, **C** and **S** represent therefore also values (e.g. in million EUROs). There are four basic types of equations:

(1) Flow equations (static balance equations on an annual basis) in physical units
(2) Flow equations in money terms
(3) Dynamic stock equations (dynamic balance equations) for physical units
(4) Dynamic stock equations in money terms

Matrices are written in fat caps (e.g. **A**).
Units of measurement = price times volumes (or turnover)
The unit price of waste = 0
The unit price of capital stocks = unit price of capital investments (flow)

1. Static closed equilibrium model

Starting point is a **static closed equilibrium model** with *n* sectors, with workers (in this version = households) and with branches of production (to represent a cluster of firms), intermediary goods, consumption and capital investment (gross for net), no foreign sector, no state sector yet. Workers earn wages to buy consumer goods, capitalists make profits to buy capital investment goods. Money does not explicitly show up. There could be a difference between individual profits and capital investment, and also between wages and consumption, but the total amount of profits equals the total value of capital investment.

The simplest version of a static input output scheme with 3 quadratic matrices and one output vector in equilibrium (all markets are cleared, no leaking to stocks) is described by the following parameters and variables (elements of arrays measured in bill. EURO) which could be taken empirically from national input-output tables.

A…intermediary goods matrix
C…consumption matrix
S…surplus matrix
x…output (column vector)

1…column of 1's (just for the summation of components of a matrix – row sum)
1'…row of 1's (column sum)

Primal problem: (read it horizontally: use of output by sector of production)
$(\mathbf{A} + \mathbf{C} + \mathbf{S})1 = \mathbf{x}$

Primal problem in detail:
Intermediary goods sold = $\mathbf{A}1$
Consumption $\mathbf{c} = \mathbf{C}1$
Capital investment $\mathbf{s} = \mathbf{S}1$
Output = $\mathbf{x}$

Dual problem: (read it vertically: input by sector needed for production)
$\mathbf{1}'(\mathbf{A} + \mathbf{C} + \mathbf{S}) = \mathbf{x}'$

Dual problem in detail
Cost of intermediary goods = $1'\mathbf{A}$
Wages $\mathbf{w} = 1'\mathbf{C}$
Profits $\pi = 1'\mathbf{S}$
Output = $\mathbf{x'}$

Clearing of markets
Wages equal consumption: $\mathbf{w}1 = 1'\mathbf{C}1 = 1'\mathbf{c}$
Profits π equal cap. investment: $\pi\ 1 = 1'\mathbf{S}1 = 1'\mathbf{s}$

2. Dynamic extension

In addition to model 1 capital accumulation is provided by a change of capital stock $\mathbf{K}_t$ by net investment $\mathbf{S}_n$

$$\mathbf{K}_{t+1} = \mathbf{K}_t + \mathbf{S}_n = \mathbf{K}_t + (\mathbf{S} - \mathbf{S}_d)$$

$\mathbf{S}_n = (\mathbf{S} - \mathbf{S}_d)$ net investment
$\mathbf{S}_d$…Depreciation matrix

3. Monetary Extension

Money savings/increase of debt of households, $\mathbf{s}_{hh}$
(r_l interest rate for lending money to banks, r_b interest rate for borrowing from banks, $r_l < r_b$)

$$\mathbf{s}_{hh} = \mathbf{w} - 1'\mathbf{C} + r_l\ \mathbf{m}_{hh} \quad \text{if } \mathbf{m}_{hh} > 0$$

$$\mathbf{s}_{hh} = \mathbf{w} - 1'\mathbf{C} + r_b\ \mathbf{m}_{hh} \quad \text{if } \mathbf{m}_{hh} < 0$$

Money savings/increase of debt of firms, $\mathbf{s}_f$

$$\mathbf{s}_f = \mathbf{p} - 1'\mathbf{S} + r_l\ \mathbf{m}_f \quad \text{if } \mathbf{m}_f > 0$$

$$\mathbf{s}_f = \mathbf{p} - 1'\mathbf{S} + r_b\ \mathbf{m}_f \quad \text{if } \mathbf{m}_f < 0$$

Money(+)/Debt(-) stocks of households, $\mathbf{m}_{hh,t}$, at time t

$\mathbf{m}_{hh,t+1} = \mathbf{m}_{hh,t} + \mathbf{s}_{hh}$

Money(+)/Debt(-) stocks of firms, $\mathbf{m}_{f,t}$, at time t

$\mathbf{m}_{f,t+1} = \mathbf{m}_{f,t} + \mathbf{s}_{f}$

Turnover of banks, (approximation)

$$\boldsymbol{x}_{banks} = \begin{array}{l} \sum_{j=1}^{n} -r_l\, m_{hh,j} \quad (if\ m_{hh,j} > 0) + \sum_{j=1}^{n} -r_l\, m_{f,j} \quad (if\ m_{f,j} > 0) \\ \sum_{j=1}^{n} -r_b\, m_{hh,j} \quad (if\ m_{hh,j} < 0) + \sum_{j=1}^{n} -r_b\, m_{f,j} \quad (if\ m_{f,j} < 0) \end{array}$$

Description of the model

The closed economy consists of six sectors and is quantitatively defined by input-output tables of intermediate consumption, final uses and value added. The six sectors are:

1. Extraction
2. Electricity
3. Manufacturing/construction
4. Services
5. Banks
6. Government.

The sectors *Banks* and *Government* need special treatment, because their total output depends on redistributions of direct income of other sectors to them.

According to the accounting standards of the European Union domestic production is shown is shown explicitly in the input-output tables, while the distribution side of value added is not elaborated at all. Primary distribution is given by consumption of fixed capital, gross wages (including social security contributions) and operating surplus (gross).This primary distribution is transformed into a generic type of secondary distribution where banks and the state have a specific role. In the primary distribution *Banks* earn only fees for their services, and the state receives only remuneration for its production activities. Interest paid by non-banks (firms and households) are part of the secondary distribution, as are all kinds of taxes. By creating the secondary distribution of value added its total sum remains constant, while the income distribution is changed.

A finer breakdown of value added is presented using the following categories:

1. consumption of fixed capital (depreciation)
2. indirect taxes
3. profit taxes
4. wage taxes

5. operating surplus (net)
6. net wage.

Taxes are the product of a tax rate times a tax base. Profit taxes are based on non-negative gross profits.

The financial assets/debts of firms, households and state are held by the *Banks* sector. Assets are rewarded by *Banks* with an interest rate $r_{\text{borrowing}}$, credits have to be paid for with an interest rate r_{lending} ($r_{\text{lending}} > r_{\text{borrowing}}$). The payments of interest are deducted from the surplus variables and added to the wage income. Of course, the redistribution does not change the amount of the GDP.

The following control variables are available, which allow for a change of the distribution of value added:

r_b	… interest rate for credits
r_l	… interest rate for assets at banks
t_ind	… tax rate of indirect taxes
t_profits	… tax rate of profits
t_wages	… tax rate on wages
deprec_rate	… depreciation rate (for the moment related to output, not to fixed capital)

In addition, one can control the fraction of public investment on total investment:

fraction_public_investment … fraction of public investment on total investment

and a leverage_factor which limits the maximum amount of credits given by banks with respect to the level of their own financial assets.

Dynamics

For each sector and for households a stock of fixed capital (physical capital expressed in currency units) and a stock of financial assets resp. debt is given. Fixed physical capital of firms resp. state is updated annually by net private resp. public investment, financial assets are updated by the difference of surplus (including depreciation, minus taxes and interests) minus investment. For the households of each sector assets are updated by the difference of net wages plus capital income minus consumption.

The updated stocks represent the basis for production of the following year. There are many ways how to define new output.

The simplest way would be just to take the capital productivity (= **x/K**) of the last year and multiply it by capital of the following year. Please take note that this capital productivity is related to sector output, not to value added. The new output allocated to the diagonal of a diagonal matrix gives the new values of demand by a right hand multiplication of the (column-standardized by output) input-output coefficients.

Even such simple measure will create a difference between the output produced and the output demanded. One could take just note of the difference and submit it to a stocks variable, or one could include relative prices and modify them according to the difference between supply and demand on the level of stocks accumulated. In this case one has to define a certain behavior how firms and households react to a change in prices. An iterative procedure could possibly create prices and volumes at equilibrium.

To match demand to supply a simple price mechanism is built in. In our case supply is given by capital stock times capital productivity. Unfortunately, demand $\boldsymbol{x_d}$ is not always equal to supply $\boldsymbol{x_s}$, so a change in prices could fix this difficulty to stimulate or to reduce demand such that it is equal supply.

$$\boldsymbol{X}_s = \boldsymbol{X}_d \times \boldsymbol{P}^{-\alpha}, \text{ and } \boldsymbol{P}^{\alpha} = \frac{\boldsymbol{X}_d}{\boldsymbol{X}_s} \text{ which implies that } \boldsymbol{P} = \left(\frac{\boldsymbol{X}_d}{\boldsymbol{X}_s}\right)^{1/\alpha}$$

In the context of a standardized input matrix demand is given by $\mathbf{x}_d$ **= (A + C + S)**$\mathbf{x}_s$, where $\mathbf{x}_s$ is the vector of new output. To bring up demand to the level of supply, we have to multiply the elements of $\mathbf{x}_d$ by $\boldsymbol{P}^{-\alpha}$.

To end up with new tables we have to multiply **A, C and S** from the left side with the diagonal matrix of the price vector $\boldsymbol{P}_j(1 - \alpha_j)$ from the right with the diagonal matrix of supply vector $\mathbf{x}_s$. These matrices should give the basis for determining the next cycle. Now the prices are no longer constant, but variable. All the essential variables are updated

x_update,
A_update,
C_update,
S_update,
K_update,
K_hh_update,
m_update,
m_hh_update,

and go as inputs into the next cycle.

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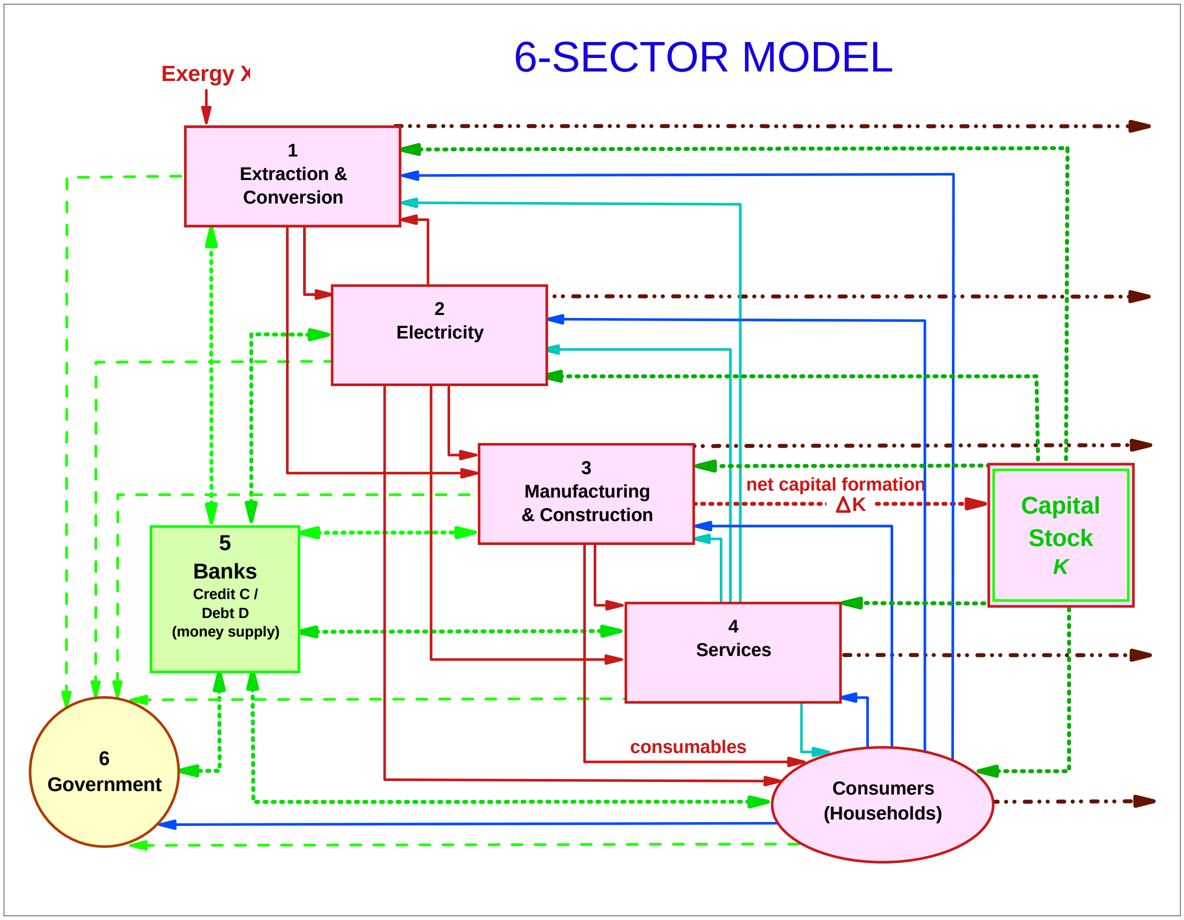
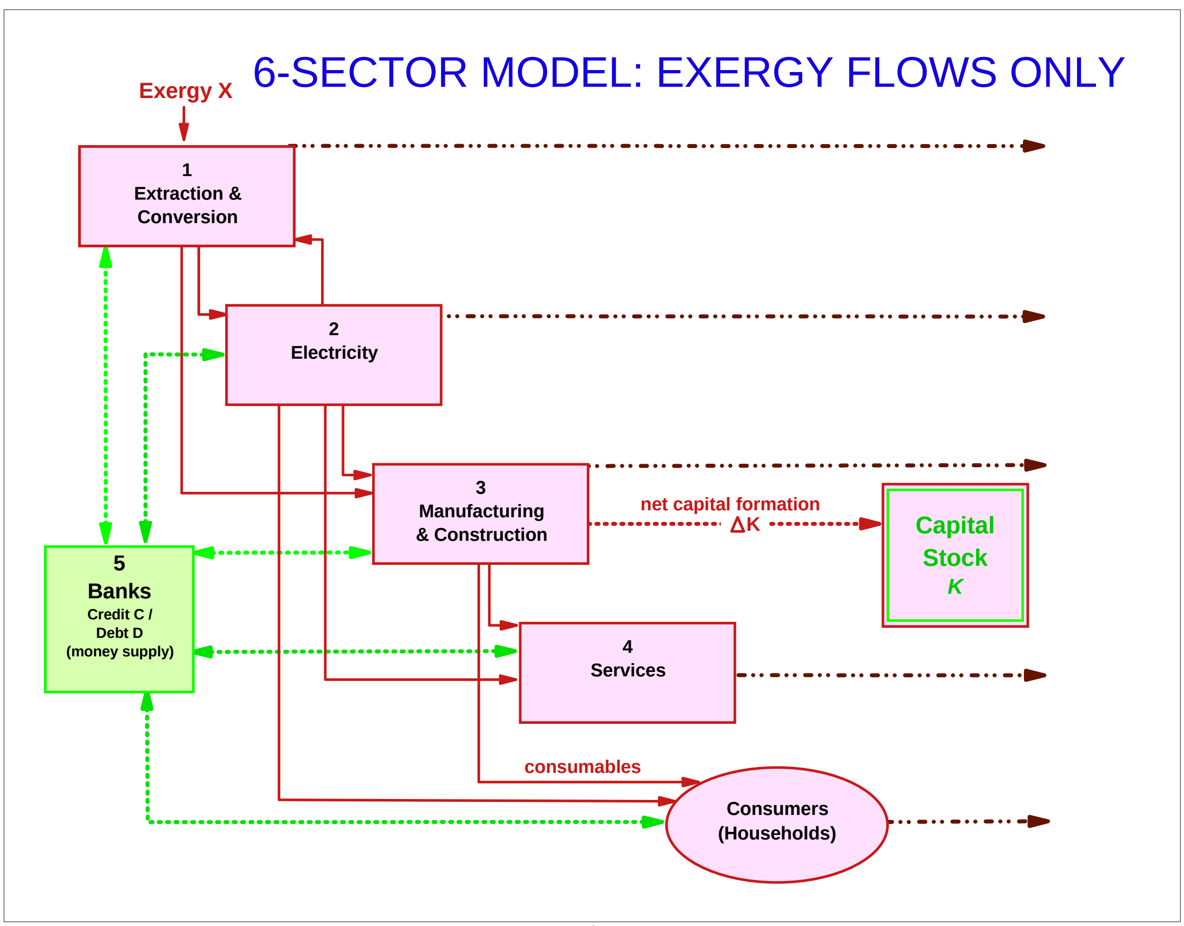


6-SECTOR MODEL
Exergy X
1
Extraction &
Conversion
2
Electricity
3
Manufacturing
& Construction
net capital formation
ΔK
Capital
Stock
K
4
Services
5
Banks
Credit C /
Debt D
(money supply)
6
Government
consumables
Consumers
(Households)



6-SECTOR MODEL: EXERGY FLOWS ONLY
Exergy X
1
Extraction &
Conversion
2
Electricity
3
Manufacturing
& Construction
net capital formation
ΔK
Capital
Stock
K
4
Services
5
Banks
Credit C /
Debt D
(money supply)
consumables
Consumers
(Households)