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AN APPROACH TO THE CONSTRUCTION OF THE REGIONAL WATER RESOURCE MODEL

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Preface

There exist significant diversification of water demand/ supply models, each pursuing a different aim. The diversity of aim is the source of the diversity of models.

The particularities of the model elaborated arise from the requirements to its properties to be included as a submodel in the system of regional models, where the region is represented by many subregions, where demand is formed automatically and where on each iteration the data concerning the cost of water supply should be made more precise in conditions when the cost of the water supply in one subregion is the function of water supply of all the region.

This paper is an attempt to elaborate a special regional water resource model as a part of a more general system of regional models. The first test of this model was made for the Silistra region (Bulgaria), but the general idea is to make it broad enough for implementation in other cases.

Murat Albegov
November 1978

Abstract

The contemporary analysis of regional development is unthinkable without taking into account the water resource factor. Many specific properties of the regional water resource model developed for this purpose result from that information about water resources which is necessary for the regional planner. In actual fact, he would like to know not so much what the water supply or quality management system should be, as what the influence of water resource availability on regional development is. In the present approach such an influence is implemented through the mechanism of the total cost associated with the creation of regional water supply and treatment systems as well as the marginal water costs distributed geographically. The regional water resource model below consists of the two interconnected systems: water supply and water quality management. Many general statements are implemented for the Silistra Case Study and submitted in the conclusive section.

CONTENTS

Preface iii

Abstract v

I. INTRODUCTION 1

II. PLACE OF THE WATER RESOURCE MODEL IN THE GENERAL SCHEME 2

III. STRUCTURE AND PROBLEMS OF THE REGIONAL WATER RESOURCE MODEL 5

1. INITIAL DATA 6

2. LINKAGE OF THE WR AND RD MODELS 8

3. PRELIMINARY DESCRIPTION AND PROBLEMS OF THE WR MODEL 9

A. Water Supply System 11

B. Water Quality Management 19

IV. WATER SUPPLY MODEL FOR THE SILISTRA CASE STUDY 24

References 38

LIST OF FIGURES

1. General Scheme of the RD Models 3

2. Geographical Division of a Region into Districts ... 7

3. Structural Scheme of the WR Model 10

4. Numeration Scheme of Districts in the One-River-Basin Region 13

5. Structural Scheme of the WS System in the Silistra Region 27

6. Initial Data for the Silistra WS System 33

7. Basic Parameters of the Silistra WS System 34

8. Timetables of Fillings and Releases for Reservoirs . 35

9. Seasonal Unit Costs of Water for the Different Irrigated Areas 36

AN APPROACH TO THE CONSTRUCTION OF THE REGIONAL WATER RESOURCE MODEL

I. INTRODUCTION

There exist many kinds of water resource (WR) models that are distinguished mainly by the types of problem to be solved. Each model reflects the specific situation that needs to be analyzed (size of the territory; the number of rivers, lakes and possible reservoirs; static and dynamic consideration of the problem; the use of a deterministic or stochastic approach; and so on).

It is rather reasonable to include the water resource model into the system of regional models. Such a WR model must meet some specific requirements corresponding to the particularities of the regional problems, which need to be solved on a different level of analysis.

If we consider interregional problems, then the average data showing the cost of one cubic meter of water consumption can be used. Yet if the intraregional problems are under analysis then one needs to know not only the average data but also the marginal costs of consumption in each subregion (district) and at the same time, one needs to be sure that this water consumption is admissible while taking into account the withdrawal and discharge of water. The similar set of questions arises when analyzing the regional costs associated with the water quality management.

It is necessary to mention that regional analysis requires detailed data in terms of season (of consumption), space, time (years), etc. and that the WR model needs to be used only as a subsystem in the general scheme of regional models and at the same time should not be oversophisticated, but operational. Therefore the authors were forced to formulate their own version of a water resource model rather than to use one already developed. The elaboration of this model has not an object to compete with more detailed and sophisticated water supply or pollution models, but is oriented to making a sufficiently detailed description of water problems which need to be included in a more general system of regional models.

II. PLACE OF THE WATER RESOURCE MODEL IN THE GENERAL SCHEME

In the past year and a half, IIASA scientists have been working on the construction of a system of models for regional development (RD). This system is oriented towards a multi-stage approach to the solution of regional problems and includes many blocks which need to be combined into one system (see Figure 1). As can be seen, this system consists of a hierarchy of models. It begins with the model of regional specialization (on Level I) continues with the location of sectoral activities (on Level II) labor, capital, income and expenditure balance (on Level III), and ends with model of settlements and pollution (on Level IV).

It is important to note here that the water supply problem is analyzed after the choice of regional specialization is made. This choice should be made on two different levels: for the region as a whole, and for each subregion (district). It means that during one iteration (which includes both Level I and Level II) for water supply models of different complexities (retional and subregional) the water demand is known and fixed. The particularities of intraregional water demand analysis consists in the interdependency of water costs in different subregions. If water consumption in one subregion is changed one cannot be sure that the system of water supply costs in all subregions would not be changed. And what is more, one can be sure that in a general case, they have to change.

The main feature of our approach is the simultaneous analysis of water supply costs in all subregions (districts) rather than the cost-benefit analysis of a particular subregion or a particular consumer. But for each subregion (and, respectively, for each consumer) the costs of water are assumed

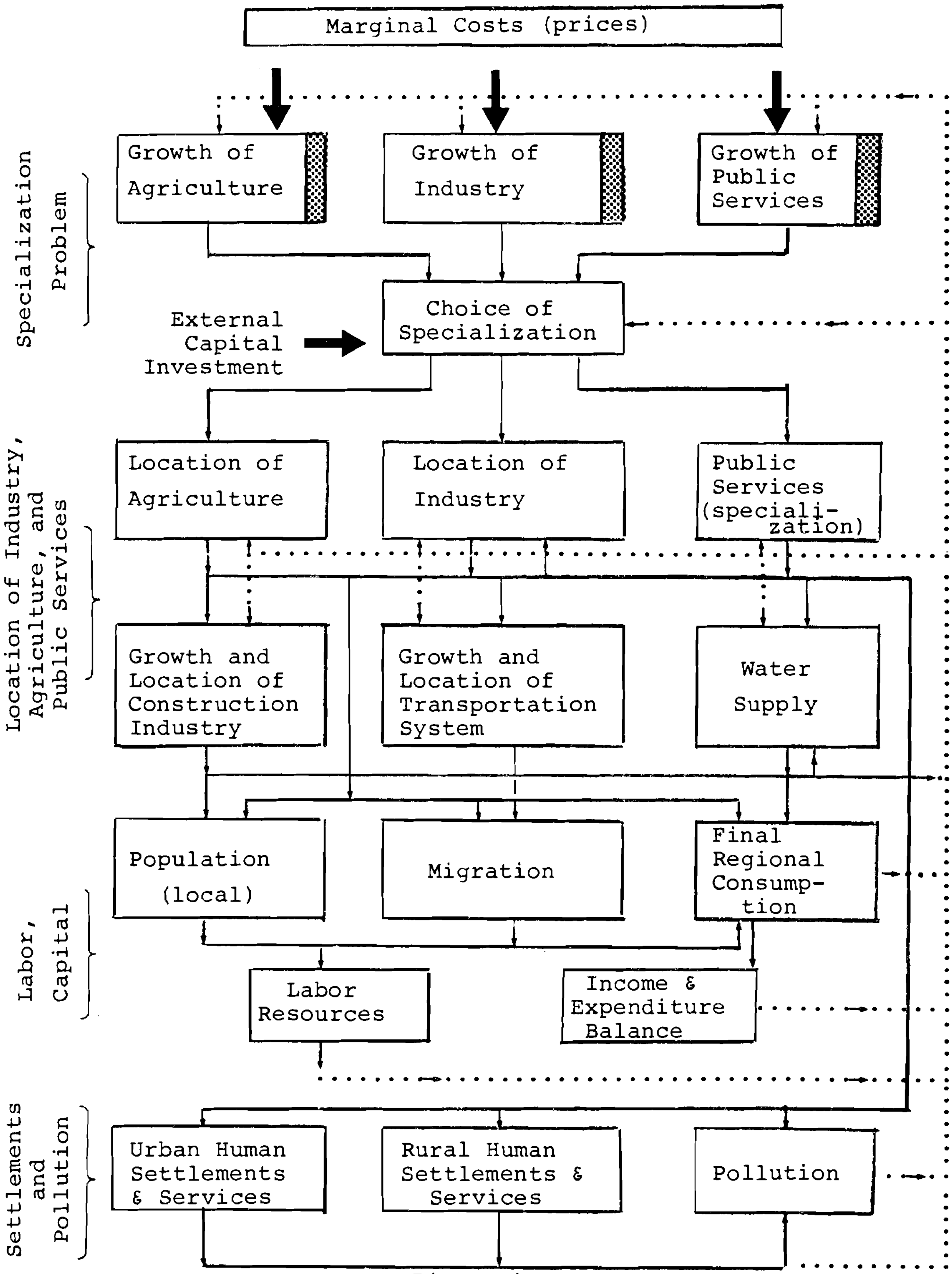


Figure 1

General Scheme of the RD Models

to be obtained and implemented for the choice of subregional specialization and economic growth.

The chosen sequence of analysis calls for different types of data concerning water supply. On the first level, when the choice of regional specialization is considered, one needs data for the region as a whole and the average cost for water consumption (per cubic meter) can be used.

On the second level, when the problem of intraregional location is considered, one needs to know not only the costs of water consumption in different subregions, but also what maximum quantities of water withdrawn are available and how stable are the costs of water supply in each subergion (district).

From the above-mentioned the following requirements to the water supply (WS) model can be formulated:

For the region as a whole:

a) average cost per one cubic meter of water consumption (which is mostly needed for some preliminary and additional calculations, as for example, interregional comparisons);
b) maximum available water for consumption;
c) maximum available water for withdrawal (if this is necessary for a particular region); and
d) the total expenses for the water supply.

For each subregion:

a) cost per one cubic meter of water consumption;
b) maximum available water for consumption for each subregion or several subregions; and
c) maximum available water for withdrawal for each subregion or several subregions (if necessary).

The analogous information should be obtained by solving the problem of the regional water quality management. Mainly, this is the costs associated with the maintenance of the proper water quality in a region and their dependence on the geographical location of wastewater discharges.

It is clear that developing such a system of models is a long and time-consuming process. Thus, in a first stage of analysis, in accordance with the nature of the Notec (Poland) and Silistra (Bulgaria) regional problems, it is planned to organize the coordination of the following four models: (1) regional industry; (2) regional agriculture; (3) regional water supply; and (4) a model to estimate the future regional labor resources.

Though this scheme of models is much simpler than the above, the role of the WS model is in principle the same. Therefore, the efforts to resolve the practical problems of Notec and Silistra will serve at the same time as a basis for the construction of a more comprehensive system of models.

III. STRUCTURE AND PROBLEMS OF THE REGIONAL WATER RESOURCE MODEL

We do not here pretend to encompass all of the problems arising in water use modeling in a regional context. The main consideration is to assess the water resource factor in regional development rather than to develop a calculation technique for a complex water economy system.

It should be noted that there exist many well-developed methods for the calculation of water economy system, in particular in the USA, the USSR, and in Canada [1-8]. In essence, these methods are rather complicated in order to be used in the iterative process of search for the best RD alternative. At the same time, they do not cover all the questions arising in modeling the regional water use such as taking into account the dynamics of water supply system during the planning period, determining the specific water costs distributed over space and time, etc. Specifically, these methods could be applied to the only RD alternative that is best.

The major part of this paper is restricted to a development of structure and a statement of problems for the regional WR model and does not contain the algorithms for their solution. Many statements are implemented in the concluding section concerning the development of a water supply system for the Silistra case study.

The main goal of RD modeling is to determine the location and development level for regional production units. Undoubtedly, water resources can be of essential importance in regional planning. At this point, the regional planner is interested not so much in what the regional water supply or wastewater treatment system should be, as in what the influence of water resource availability is on regional development. Of course, regional development is determined not only by water resources. Hence, the main idea of our approach follows, namely, the WR model is presented as a submodel in the regional system of models, working interactingly. In other words, the WR model is not independent and is subordinated to the specific requirements of regional planning.

The search for the best RD alternative is supposed to be realized by means of a directed reset of those. That is why the basis of constructing the WR model is the exact water economy analysis of some RD alternative. On the other hand, the WR model, below, should provide the recommendations for a choice of the next RD alternative.

1. INITIAL DATA

Let some region be set up with the source of its water resources being one or more river systems. While constructing the WR model, let us proceed from the following assumptions:

1. The geographical division of the region into a number of districts ($i = 1,\dots,n$) is set up (see Figure 2). In essence it is a priori aggregating of a large dimension system such as the whole region. Each district can include different kinds of production units but is characterized by indivisible water economy characteristics.

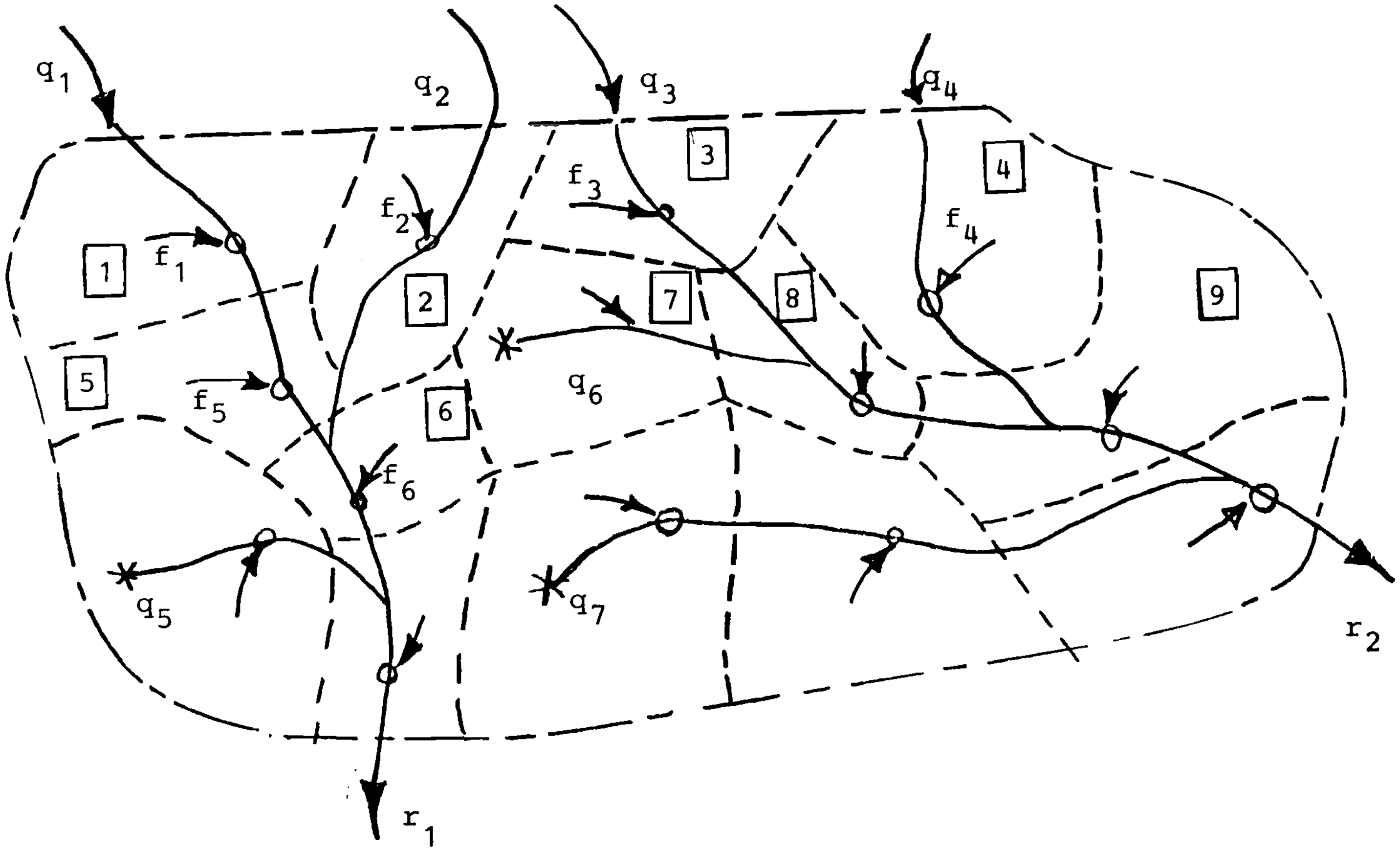


Figure 2

Geographical Division of a Region into Districts

2. The regional inflows, α in number, being watercourses or tributaries, are given. This means that for each inflow j, the sequence of historical or synthetic streamflows $\{q_j^s(\tau)\}$ $(s = 1,\dots,N)$ is specified. Everywhere argument τ corresponds to a change of time during the year, and s is the number of a year in the sequence of streamflows. In other words, $q_j^s(\tau)$ is the hydrograph recorded in the regional input site of tributary j and corresponding to the s-th year in streamflow sequence.

3. The regional outflows, β in number, are given. More precisely, the lower limits $r_1(\tau),\dots r_\beta(\tau)$ of the regional outflows are specified.

4. The on-site runoff is given so that each district i is characterized by the only inflow f_i (see Figure 2). Specifically, for each district i, the sequence of historical inflows $\{f_i^s(\tau)\}$ $(s = 1,\dots,n)$ is specified.

5. The possible locations (sites) of reservoirs and their maximum useful capacities $\bar{V}_i$ are given. The costs associated with the creation of reservoirs are supposed to be known functions $E_i(V_i)$ of their useful capacities.

6. The choice of a water supply system is determined mainly by the water requirements and does not depend on the water pollution level.

7. The planning horizon T is equal to 15-20 years. During the planning time period [O,T] the production and reservoir capacities arising once, are not changed.

Below, we confine ourselves to the deterministic statement of the problem and consider all sequences of flows as coordinated, i.e. corresponding to the same set of years.

2. LINKAGE OF THE WR AND RD MODELS

As a submodel in the system of RD models, the WR model should work in interaction with them. Such an interaction is organized as follows. The input data for the WR model from the RD models are the water demands and the water treatment cost functions which are spatially distributed and correspond to different kinds of production units. Each district, taken separately, is considered as an indivisible production complex having the aggregate water economy characteristics. Moreover, they are distributed in time during the planning period [O,T].

The WR model processing this input data according to its inner algorithms should answer the following questions:

a) Is it possible to provide the water required?
b) What are the total costs associated with the creation of a regional water supply system?
c) What are the unit district water costs?
d) What are the total costs associated with regional water treatment?
e) What are the unit district costs of wastewater treatment?

Actually, the answers to these questions are the WR model outputs. The unit district costs, above, are necessary in order to pass on to the next RD alternative.

Obviously, the assigned links with the RD model essentially influence the WR model structure. In a general form, the structural scheme of the WR model is shown in Figure 3.

3. PRELIMINARY DESCRIPTION AND PROBLEMS OF THE WR MODEL

Let some RD alternative be given. If the intradistrict location scheme is chosen, all water users of a district can be characterized according to the total water demand, consumption and pollution as an indivisible production complex. The problem of the intradistrict location of water users is supposed to be solved separately for each district beforehand and independently of the general regional problems. In doing so, it is necessary to take into account the within-year non-uniformity of water economy characteristics of the different water users. For example, irrigated agriculture, fish production, water transportation and population are rather non-uniform within the year for the water requirements. As a result, for

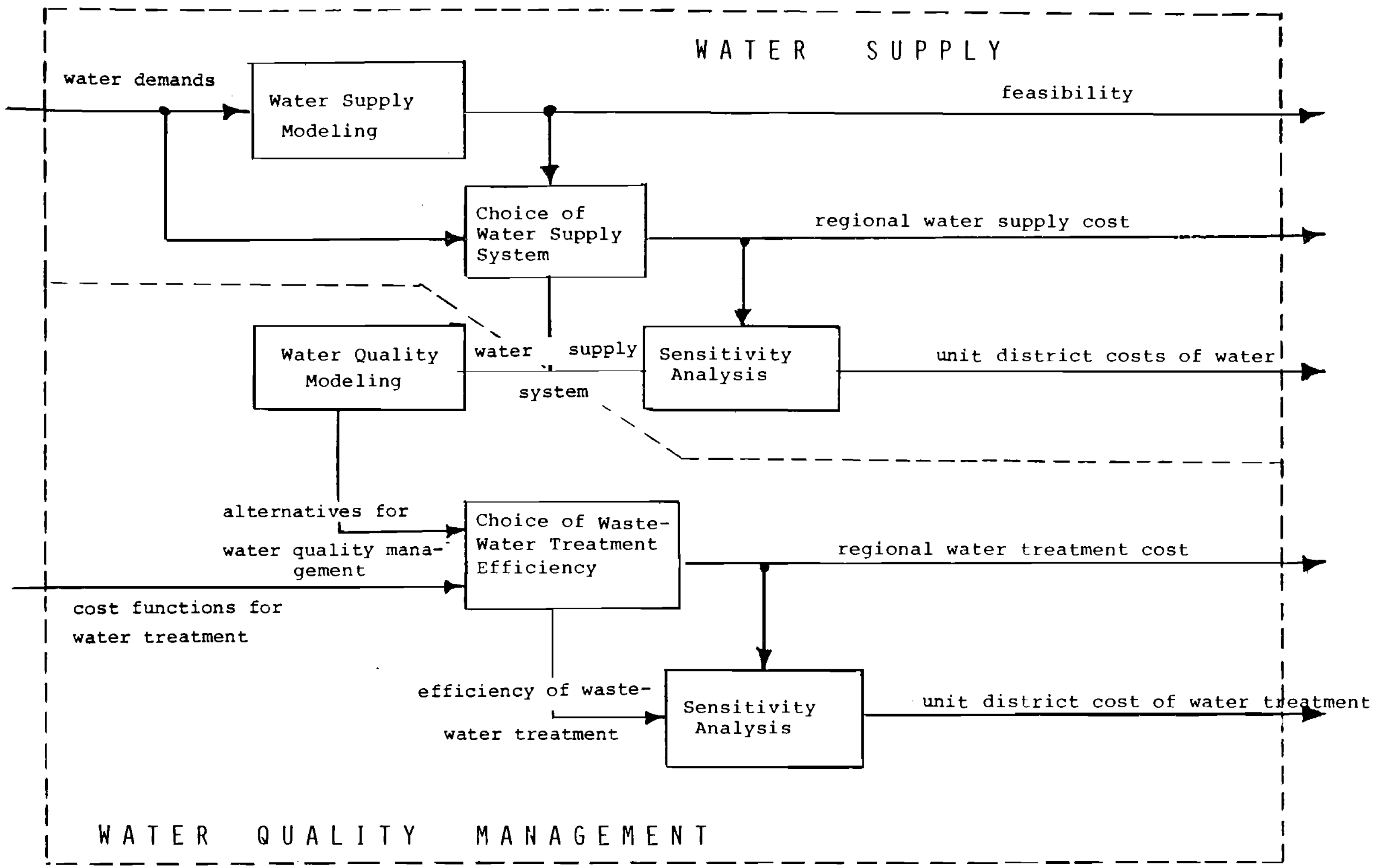


Figure 3

Structural Scheme of the WR Model

each district we can obtain the aggregate water economy characteristics. Such an aggregation allows the simulation (from the water economy point of view) of a whole district as a point in a river system.

As is obvious from the structural scheme, the WR model consists of two interconnected systems: water supply and management for water quality. However, from assumption 6 it follows that this connection is one-sided, so that the creation of a regional water supply system does not depend on the water pollution level. It enables the description of a water supply system independently of problems of water quality management.

A. Water Supply System

Thus, giving some RD alternative implies specifying the water demands $\{w_i(\tau,t)\}$ and consumption-withdrawal ratios $\{\lambda_i(\tau,t)\}$ both distributed in space and time. Spatial distribution is characterized by index i (the district number). Argument t [O,T], the number of a year, reflects the dynamics of water economy characteristics in the planning period. For example, $w(\tau,t)$ is the value of variable w at time τ in year t.

The main task of WR modeling consists in assessing the dynamic water demands $\{w_i(\tau,t), \lambda_i(\tau,t)\}$ from the point of view of their feasibility and the costs associated with the creation of an adequate water supply system. It is expedient to divide this problem into a number of stages. The main stage is the assessment of the final water demands $\{w_i(\tau), \lambda_i(\tau)\}$, corresponding to t = T. Everywhere $w_i(\tau) = w_i(\tau,t)$ and $\lambda_i(\tau) = \lambda(\tau,T)$ by definition.

The first question from the above concerns the possibility of meeting the given water demands $\{w_i(\tau), \lambda_i(\tau)\}$. The rough answer to this question can be obtained on the basis of analyzing the water balances in a river network; this will be the necessary conditions rather than the exact solution of the feasibility problem for water demands $\{w_i(\tau), \lambda_i(\tau)\}$.

Let us illustrate this statement by a specific example. For simplification, assume the region has the only river system, α inflows and one outflow (see Figure 4). We numerate all n districts in the order in which they are located on the tributaries and river reaches. Let n_i and m_j be the serial numbers of the last districts located on the i-th tributary and j-th river reach, respectively, so that:

$$0 \leq n_1 \leq m_1 \leq n_2 \leq m_2 \leq \ldots, \leq n_j \leq m_j = n .$$

Introduce the following notations:

$$Q_j = \frac{1}{N} \sum_{s=1}^{N} \int_0^1 q_j^s(\tau)d\tau$$ - normal annual runoff in the input site of tributary j;

$$F_i = \frac{1}{N} \sum_{s=1}^{N} \int_0^1 f_i^s(\tau)d\tau$$ - normal annual on-site runoff in district i;

$$W_i = \int_0^1 w_i(\tau)d\tau$$ - mean annual water demand of district i;

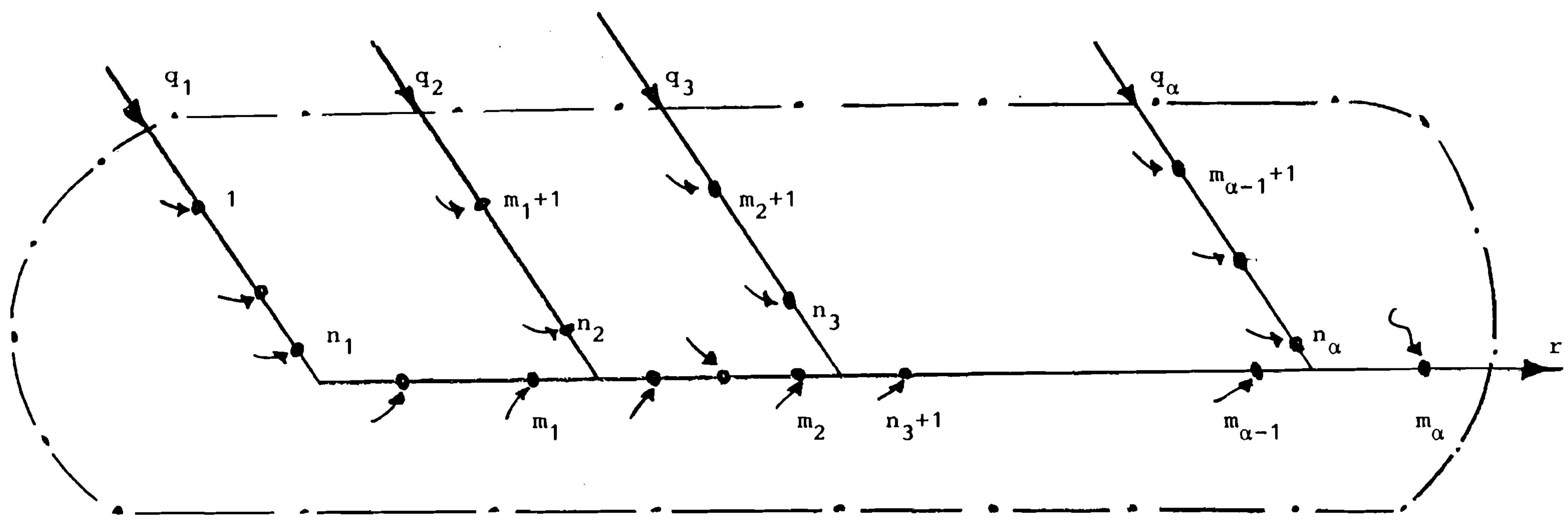
$$\Lambda_i = \frac{1}{X_i} \int_0^1 \lambda_i(\tau)w_i(\tau)d\tau$$ - mean annual consumption-withdrawal ratio for district i.

Using the balance relations in the river network and omitting the intermediate calculations we can write the following constraints:

For districts located on tributaries:

$$W_i - \sum_{k=m_{j-1}+1}^{i-1} (1-\Lambda_k)W_k \leq Q_j + \sum_{k=m_{j-1}+1}^{i} F_k , \qquad (1)$$

$$j = 1,\ldots,\alpha \quad , \qquad i = m_{j-1}+1,\ldots,n_j \quad ;$$



<u>Figure 4</u>

Numeration Scheme of Districts in the One-River-Basin Regions

For districts located on river reaches:

$$W_i - \sum_{k=1}^{i-1} (1-\Lambda_k)W_k \leq \sum_{p=1}^{i} Q_p + \sum_{k=1}^{i} F_k \quad , \tag{2}$$

$$j = 1,\ldots,\alpha \quad , \qquad i = n_m+1,\ldots,m_j \quad .$$

For the regional outflow:

$$\sum_{j=1} Q_j + \sum_{i=1} (F_i-\Lambda_i W_i) \leq \int_0^1 r(\tau)d\tau \quad . \tag{3}$$

A necessary condition for the given water demands to be provided is that their mean annual values $\{W_i, \Lambda_i\}$ satisfy the set of inequalities (1)-(3). Theoretically, in this case, it is possible to create an adequate regional water supply system. However, this is, in practice, a more complicated matter. Indeed, relations (1)-(3) do not take into account the upper limits for the values of regulated flow for at least some sites in a river basin. Such limits are associated with the on-site topographical or environmental conditions and can even originate from the economic constraints. So, strictly speaking, satisfying the relations (1)-(3) is not sufficient for the existence of a water supply system meeting the regional water demands. Nevertheless, we confine the analysis of the feasibility of the water demands $\{w_i(\tau), \lambda_i(\tau)\}$ by checking the inequalities (1)-(3), especially since in the next stage we will choose a certain regional water supply system.

If the water demands $\{w_i(\tau), \lambda_i(\tau)\}$ are feasible, there exist many adequate water supply systems distinguished from each other by the different variants for creating a storage reservoirs system.

It is now time to state the second question concerning the economic assessment of feasible water supply systems. It is highly reasonable to choose the best among them, proceeding from the minimization of total costs associated with the creation of a regional water supply system.

Problem 1. Let water demands $\{w_i(\tau), \lambda_i(\tau)\}$, the possible locations for reservoirs and their maximal useful capacities $\{\bar{V}_i\}$ be given. Find the feasible reservoirs system $\{V_i\}$ for which the total costs $E = \sum_{i=1}^{n} E_i(V_i)$ associated with its creation, have a minimum.

For the present we confine ourselves solely to the statement of such an optimization problem. Note however, that considering the time-discrete analogy for Problem 1, we can reduce it to a problem of nonlinear programming. From the practical point of view, this is a good approximation for the initial time-continuous Problem 1 and its numerical solvability depends only on the convexity properties of cost functions $E_i(V_i)$ and the dimension of the problem.

So far, we have dealt with the assessment of the regional water supply system corresponding to the end of the planning period $t = T$. A similar analysis of the considered RD alternative can also be carried out at every time $t \in (0,T)$. For all $t < T$ the problem of the feasibility of water demands $\{w_i(\tau,t), \lambda_i(\tau,t)\}$ has a trivial solution in the following sense. On the strength of assumption 7, the feasibility of regional water demands at time $t = T$ entails the feasibility of these at every time $t < T$. Thus, it is not necessary to analyze the feasibility of water demands $\{w_i(\tau,t), \lambda_i(\tau,t)\}$ for all $t < T$.

At the same time, there remains the uncertainty of the development of the regional water supply system on time interval $t \in (T,0)$, or in other words, what the functions $V_i(t)$, $(i = 1,\dots,n)$ are. So far, we only know the boundary values for them: $\{V_i(0)\}$ as the initial state of the reservoirs system and $\{V_i(T)\}$ as the solution to Problem 1. Assume each reservoir (as a regulator of flow) is put into operation only after the completion of its construction and its complete filling. As before, we give preference to the economic criterion during the whole planning period. For generality, formulate the problem of determining the water supply system on time interval $(0,T)$ as follows.

Problem 2. Let the states $\{V_i(t_1)\}$ and $\{V_i(t_2)\}$ of the reservoirs system be known. For some time $t\in(t_1,t_2)$ find the state $\{V_i(t)\}$ satisfying the water demands $\{w_i(\tau,t),\ \lambda_i(\tau,t)\}$ so that total costs $E(t) = \sum_{i=1}^{n} E_i[V_i(t)]$ associated with the creation of the reservoirs system have a minimum.

The solution of this problem exists because under assumption 7, the reservoirs system $\{V_i(t)\} = \{V_i(t_2)\}$ is able to satisfy the water demands $\{w_i(\tau,t),\ \lambda_i(\tau,t)\}$. Show that Problem 2 is simpler than Problem 1 and can be reduced to the reset of a number of reservoirs systems. Indeed, according to the foregoing, we can establish the following property:

$$V_i(t) = 0 \quad \text{or} \quad V_i(t_2) \qquad \text{for all } i = 1,\dots,n \text{ and } t < t_2 \ . \tag{4}$$

Denote by I the set of indexes i for which $V_i(t_2) \neq 0$ and $V_i(t_1) = 0$ simultaneously. Applying property (4) to pair t_1 and t, we come to the following conclusion:

$$V_i(t) = V_i(t_2) \qquad \text{for all } i\bar{\in} I \ ,$$

$$V_i(t) = 0 \quad \text{or} \quad V_i(t_2) \qquad \text{for all } i\in I \ .$$

Thus, the system of functions $\{V_i(t)\}$ is not determined only on the set of indexes $i\in I$, where each $V_i(t)$ can take on only two values 0 or $V_i(t_2)$. Giving the functions $V_i(t)$ the values 0 or $V_i(t_2)$ for all $i\in I$, we obtain a finite number of variants. By analyzing each of them on satisfying the water demands $\{w_i(\tau,t),\ \lambda_i(\tau,t)\}$ and calculating the total costs $E(t)$, we find the solution of Problem 2.

So, if $\{V_i(t_1)\}$ and $\{V_i(t_2)\}$ are known, to find $\{V_i(t)\}$ for $t_1 < t < t_2$ offers no difficulty. In doing so, we should observe the only condition on points t_1 and t_2: at any point $t\in(t_1,t_2)$ the state of the reservoirs system $\{V_i(t)\}$ is not known. Then, step by step, we can find functions $V_i(t)$ and $E(t)$. That is what in outline the approach to determining the dynamics of the regional water supply system and the total costs associated with its creation is.

At this point the exact water economy analysis of a given RD alternative is over. However, the RD modeling is not exhausted by the only RD alternative. On the contrary, on the basis of a comprehensive, but not solely water economy analysis, the other RD alternative--better than the previous one--should be chosen. In order to jump to the next RD alternative, the WR model should give additional information, in particular the unit district costs of water which would point out the common tendency of the spatial redistribution of regional production units.

It is reasonable to define the unit district water costs as marginal costs of water resources in accordance with the following procedure. Varying the water demands only in the k-th district and determining the change of total costs associated with the creation of the water supply system, we can calculate the unit district cost C_k sought. Define this notion more exactly.

For this, introduce the concept of the partial variation of water demands $\{w_i(\tau), \lambda_i(\tau)\}$ at the end of the planning period t = T. Vary the water demands only in the k-th district by means of introducing the additional water consumption δ_k distributed uniformly during the year so that the increments

$$\Delta w_k(\tau) = \delta_k, \quad \text{and} \quad \Delta\lambda_k(\tau) = \frac{\delta_k[1-\lambda_k(t)]}{w_k(\tau) + \delta_k} .$$

If the constant $\delta_k < 0$, the varied water demands $\{w_i(\tau) + \Delta w_i(\tau),\ \lambda_i(\tau) + \Delta\lambda_i(\tau)\}$, where $w_i(\tau) = \Delta\lambda_i(\tau) = 0$ for all $i \neq k$, are feasible. Suppose the same also holds subject to $\delta_k > 0$.

It is correct to state Problem 1 for the varied water demands. The solution of such a problem gives us the minimum total costs E' which are some function of δ_k. If the function $E'(\delta_k)$ is differentiable, the strict definition of the unit water cost for the k-th district can be given as follows:

$$C_k = \frac{\partial E'}{\partial \delta_k} . \qquad (5)$$

The meaning of C_k consists in the fact that the additional irreversible water withdrawal entails the additional cost C_k per water unit, and vice versa; the net saving of water causes the effect C_k per water unit.

If for $\delta_k > 0$ the varied water demands are not feasible, one can only speak about the left-side derivative $\frac{\partial E'(-0)}{\partial \delta_k}$.
As in the previous case, this is the unit district cost of water. However, introducing the additional water consumption in the k-th district in inadmissible.

In practice, in order to calculate the district water cost C_k, it is sufficient to solve Problem 1 once more with the varied water demands for a small enough fixed δ_k and to use the approximate formula:

$$C_k \simeq \frac{E' - E}{\delta_k}$$

On the strength of the smallness of δ_k it is possible to resort to another approach based on the linear approximation of Problem 1 with respect to parameter δ_k. This can be much simpler.

In actual fact, the unit district water costs defined above correspond to the end of the planning period $t = T$. Analogously to the foregoing (see Problem 2), we can develop the procedure of determining the dynamic district water costs $C_i(t)$ at every time $t \in [0,T]$.

The unit district costs of water $\{C_i(t)\}$ carry rather useful information while choosing the next RD alternative, namely, that it is preferable to locate the production with a higher level of water demand in a district with a lower water cost. Note that the district water costs have a local character and therefore may essentially depend on the district development level. This means that their use is correct when two sequential RD alternatives are close enough.

B. Water Quality Management

The essential development of a WR model consists in taking into account water pollution in a region. The management of water quality in a river system is of great importance in planning the economic development of river regions. Generally speaking, under assumption 6, this problem is solved independently of the water supply problem. Here, we will point out two approaches to the regional management of water quality.

One of these is purely normative, requiring the compulsory observance of given streamflow standards for water quality in different sites of a river system. The other approach is based only on the economic assessment of water pollution consequences. It does not mean that in this case water quality standards are completely ignored. Rather, this approach can be used in establishing the economically justified standards for water quality in those river sites where such standards are absent. Each district is supposed to have its own water quality standards, if any.

The wastewater treatment is considered below as a main means of water quality management. Therefore the problem is to determine the efficiencies of wastewater treatment in the different districts. The objective function is structured in terms of costs associated with wastewater treatment and use of polluted water. The latter means both the cost of the withdrawal water treatment and the cost associated with the additional facilities for the direct use of polluted water. In addition, the unit district costs associated with wastewater treatment are introduced.

Let us state the problem more precisely. Characterize the quality of water withdrawn in the i-th district by vector y_i of pollutant concentrations. Vector y_i should include all of the most important pollutants or even some groups of them. By u_i denote the vector of concentrations of the same components in the wastewater discharged into receiving waters by the i-th district. Introduce the concept of cost p_i associated with the use of polluted water and the wastewater treatment in the i-th district. The withdrawal of polluted water entails the neccessity in either the treatment of water withdrawn or the creation of some additional facilities for the direct use of polluted water or, most probably, the reasonable combination of both. In general, cost p_i depends on the quantity and quality of the withdrawal and discharge waters of the i-th district:

$$p_i = g_i(y_i, u_i, w_i, (1-\lambda_i) w_i) \quad . \qquad (6)$$

In a word, function p_i characterizes the i-th district as a whole from the point of view of the costs associated with water pollution. Here, we do not concern ourselves with the identification of function p_i and only note that its form depends essentially on the production specialization of a district. The functions $\{p_i\}$ so introduced are inputs of the WR model (see Figure 3).

The problem of water quality management, distinct from the water supply system, is solved for conditions of a "dry year" which is determined in a sequence of historical streamflow as a year with a given runoff frequency. This means that in "wet years" the pollution level in a river system will be lower.

Generally speaking, each district is characterized by two distinctive sites: for water withdrawal and wastewater discharge. It should be noted that we can directly change the pollutant concentrations $\{u_i\}$ in districts' wastewaters by properly changing the efficiencies of their treatment. With regard to water quality $\{y_i\}$ in the different withdrawal sites, we can influence these concentrations only indirectly by changing $\{u_i\}$. Therefore, u_i (below) are interpreted as control parameters.

Some words should be said about the relation between water quality in different sites and pollutant discharges. In river reaches located between the distinctive sites in different districts, complex physical and chemical-biological processes occur. Here, the well-known techniques for water quality modeling can be used. However, for our purposes, the modeling results should be presented in a somewhat non-traditional form, namely that of the explicit functions of pollutant discharges. According to assumption 6, all flows in river reaches, water withdrawals, and wastewater discharges are determined beforehand, while developing the water supply system, and are now known. Therefore, at same time τ, the pollutant concentrations in the i-th district withdrawal site depend on the pollutant concentrations only in those discharges that are located upstream, that is:

$$y_i = \Phi_i(u_1,\ldots,u_{i-1},\tau) \quad . \tag{7}$$

Note that these relations actually should also include the time delays in running the flows from one site to the other. However, at small rates of change in the concentrations u_i, such time delays can be omitted. It should be stressed that the relation (7) corresponds to the conditions of a dry year.

Assume that functions Φ_i are known. Since the water demands $\{w_i(\tau), \lambda_i(\tau)\}$ at the end of the planning period $t = T$ are specified, the relation (6), taking into account equation (7) for y_i, can be rewritten as follows:

$$p_i = \Psi_i(u_1,\ldots,u_i,\tau) \quad . \tag{8}$$

In other words, the i-th district cost associated with the receiving water pollution also depends only on the wastewater discharges located up-stream.

We can now turn to the statement of the optimization problem concerning the determination of wastewater treatment efficiencies for the different districts. As a start, let us suppose that there are no constraints on water quality in the district withdrawal sites. More specifically, we take into account some of these constraints indirectly by choosing the proper form of the cost functions p_i. The control parameters $u_i(t)$ unknown, are determined as follows.

Problem 3. Find the vector functions $u_1(\tau),\ldots,u_n(\tau)$ of pollutant concentrations in the district wastewaters so that the total annual costs $P = \sum_{i=1}^{n} \int_0^1 \Psi_i(u_1(\tau),\ldots,u_n(\tau),\tau)d\tau$ associated with the receiving water pollution have a minimum.

It is not, in fact, a problem for calculus of variations. Indeed, it is easy to see that Problem 3 is reduced to the following problem of unconstrained optimization with the only parameter τ: at every $\tau\in[0,1]$ find the values of vectors $u_1,\ldots,u_n$ for which the function $\sum_{i=1}^{n} \Psi_i(u_1,\ldots,u_n,\tau)$ has a minimum. Here, the classical search and gradient methods for unconstrained optimization can be used.

Now assume the predetermined streamflow standards for water quality in the district withdrawal sites should be strictly observed. If the functions (7) are found, this means the following constraints on the control paramters $u_1,\ldots,u_n$ should hold:

$$\Phi_i(u_1,\ldots,u_{i-1},\tau) \leq B_i \quad , \tag{9}$$

where B_i is a vector of the maximal admissible concentration of pollutants in the i-th district withdrawal site. In this case we can formulate the analogous optimization problem.

Problem 3'. Among all systems $\{u_i(\tau)\}$ of vector functions satisfying the constraints (9) find that system for which the total annual costs $P = \sum_{i=1}^{n} \int_0^1 \Psi_i(u_1(\tau),\ldots,u_n(\tau),\tau)d\tau$ have a minimum.

The pollutant concentrations $u_1(\tau),\ldots,u_n(\tau)$ in wastewaters and the total costs P obtained as a result of the solution of Problem 3 or 3' are the solution of the problem of regional management for water quality. This is an additional assessment of the considered RD alternative from the water pollution point of view. The total annual costs P associated with the receiving water pollution are of special interest to regional planners and are therefore one of the outputs for the WR model. In addition, for decisionmaking on the choice of the next RD alternative, it is necessary to have the unit district costs of wastewater treatment. In defining these, we use the same methodology that was employed for the district water costs.

The concept of the unit district costs of wastewater treatment is introduced separately for each pollutant. Let h_k^j be a within-year constant variation of the j-th pollutant flow in the k-th district wastewater. This entails the increment

$\frac{h_k^j}{(1-\lambda_k)w_k}$ in the concentration of the j-th pollutant in the k-th district wastewater. Let R_k be the additional water treatment cost associated with maintaining the j-th pollutant concentration in wastewater discharged by district k on the previous level u_k. Since $w_k(\tau)$ and $\lambda_k(\tau)$ are known at some time τ, the additional cost R_k depends only on h_k and u_k:

$$R_k = \omega_k \ (u_k, \ h_k^j, \ \tau) \quad ,$$

where ω_k is assumed to be a known function such that $\omega_k(u_k, 0, \tau) = 0$. In the presence of the additional source of pollutant j, it is convenient to characterize the k-th district by the varied cost function p_k' associated with water pollution:

$$p_k' = \Psi_k \ (u_1, \ldots, u_k, \tau) + \omega_k \ (u_k, h_k^j, \tau) \quad . \tag{10}$$

Thus, if the concentration of the j-th pollutant in the k-th district wastewater changes and we want to maintain the water quality in a river system, only at the expense of district k, the additional cost $\omega_k(u_k, h_k^j, \tau)$ is inevitable. But it is perhaps advisable to distribute the additional water treatment between the different districts and/or to change the water quality in a river system. Rightly, these possibilities are taken into account by the introduction of unit wastewater treatment cost ρ_k^j which is defined as an increment of the total annual cost P corresponding to the unit increment of the j-th pollutant flow in the k-th district wastewater. Let $u_1, \ldots, u_n$, and the total cost P' be the solution of Problem 3 or 3' (depending on the approach) with the varied cost function p_k' for the k-th district. Obviously, the minimal total cost $P' = \sum_{i=1}^{n} p_i + \omega_k$ is a function of h_k^j. Under the assumption that P' is differentiable with respect to h_k^j, the unit district cost ρ_k^j is defined as follows:

$$\rho_k^j = \frac{\partial P'}{\partial h_k^j} \quad .$$

In actual fact, it is a minimal cost (or net saving) in the whole region while introducing the additional pollution in only one of the districts. For a simplified calculation of the unit district costs of wastewater treatement it is sufficient to solve again Problem 3 (or 3') with the varied cost function p_k' for a small enough h_k^j and to use the approximate formula:

$$\rho_k^j = \frac{P-P'}{h_k^j} \quad .$$

Note that the introduced unit costs $\{\rho_k^j\}$ correspond to the regional water demands at the end of the planning period and to the runoff conditions of a dry year. The matrix $\{\rho_k^j\}$ gives us the information on the geographical distribution of the unit costs associated with wastewater treatment for each pollutant separately. Such information can be used when planning the location for both production units or their wastewater discharges in a river system.

In brief, that is the main contents of the WR modeling in a regional context.

IV. WATER SUPPLY MODEL FOR THE SILISTRA CASE STUDY

Here the first version of the water supply model for the Silistra region in Bulgaria will be presented. With respect to methodology, this model is developed in close cooperation with the Water Demand Model for Silistra successfully developed by the Regional Water Management Task in 1977. One of the goals in modeling the water supply was to emphasize the essential dependence of marginal costs of water both with respect to the season and to the location of the water user. The water supply model set out below is constructed on the basis of preliminary data.

Silistra is not a large region, 2700 km^2 in area, located in the North-Eastern part of Bulgaria. The soil quality and number of days of sun per year point to the fact that the region is favourable for intensive agricultural development. At the same time, it has a pronounced deficit of water resources. However, even in the absence of irrigation, Silistra is mainly an agricultural region. Practically speaking, the Danube river is the only source of water for agriculture, population and industry consumption.

The Silistra water supply system, in essence, is divided into two separate systems: irrigation water supply and water supply for population and industry consumption. The reason for such a division is the essential distinction of the water quality demand for the different water users. With regard to industry, the Silistra region has mainly food enterprises except for some others in the city of Silistra. However, being situated on the Danube riverside, they have their own water supply systems small enough on a region scale. As is known, both the food industry and the population require that the water quality be of drinking-water standards which, of course, is of higher quality than needed for agriculture. Only 8-10% of the total water demands in the region fall to the share of population and industry. The only source of water for these is the stream terrace waters which are limited in quantity. Furthermore, the drinking water supply requires the creation of watermains. Irrigation water is withdrawn from the Danube streamflow. All the above leads to the conclusion that the irrigation water supply is in fact a separate and the most important part of the Silistra water supply system. Therefore, we will be solely concerned with the irrigation water supply system, in some cases identifying it, in a certain sense, with the whole regional water supply system.

From the geographical point of view, the water supply system for the Silistra region is also divided into three disconnected irrigation systems corresponding to the Tutrakan, M. Preslavetch and the Silistra districts. Among others, the M. Preslavetch irrigation system is the most representative with respect to both the irrigated area (about 50%) and such typical elements as reservoirs, pumping stations, canals, etc. For this reason, the water supply model constructed for the M. Preslavetch district can afterwards be transferred to other irrigation systems in the Silistra region. Below we deal only with the water supply system for the M. Preslavetch district.

The main goal of the modeling of water supply consists in determining the basic parameters of the system: capacities of reservoirs, capacities of pumping stations and discharge capacities of canals. While constructing the water supply model for the Silistra region, we are proceeding from the following assumptions.

1. The water supply system designed is determined by the end of the planning period (~1990).
2. The water resources available are unlimited because the quantity of water withdrawn in the Silistra site from the Danube does not exceed 5-7% even during the peak period.
3. The water resources are used only for irrigation characterized by consumptive water use.
4. Proceeding from the topographical conditions, the structure of the water supply system is fixed (see Figure 5). It includes the pumping station on the Danube streamflow, three storage reservoirs, the canal network and five point water withdrawals for irrigation.
5. Water demands are specified. The model should permit the variations of water demands.
6. All agricultural areas are divided into two classes: areas irrigated directly from the Danube river through canals only, and areas allowing a mixed water supply (both from the river directly and from storage reservoirs).

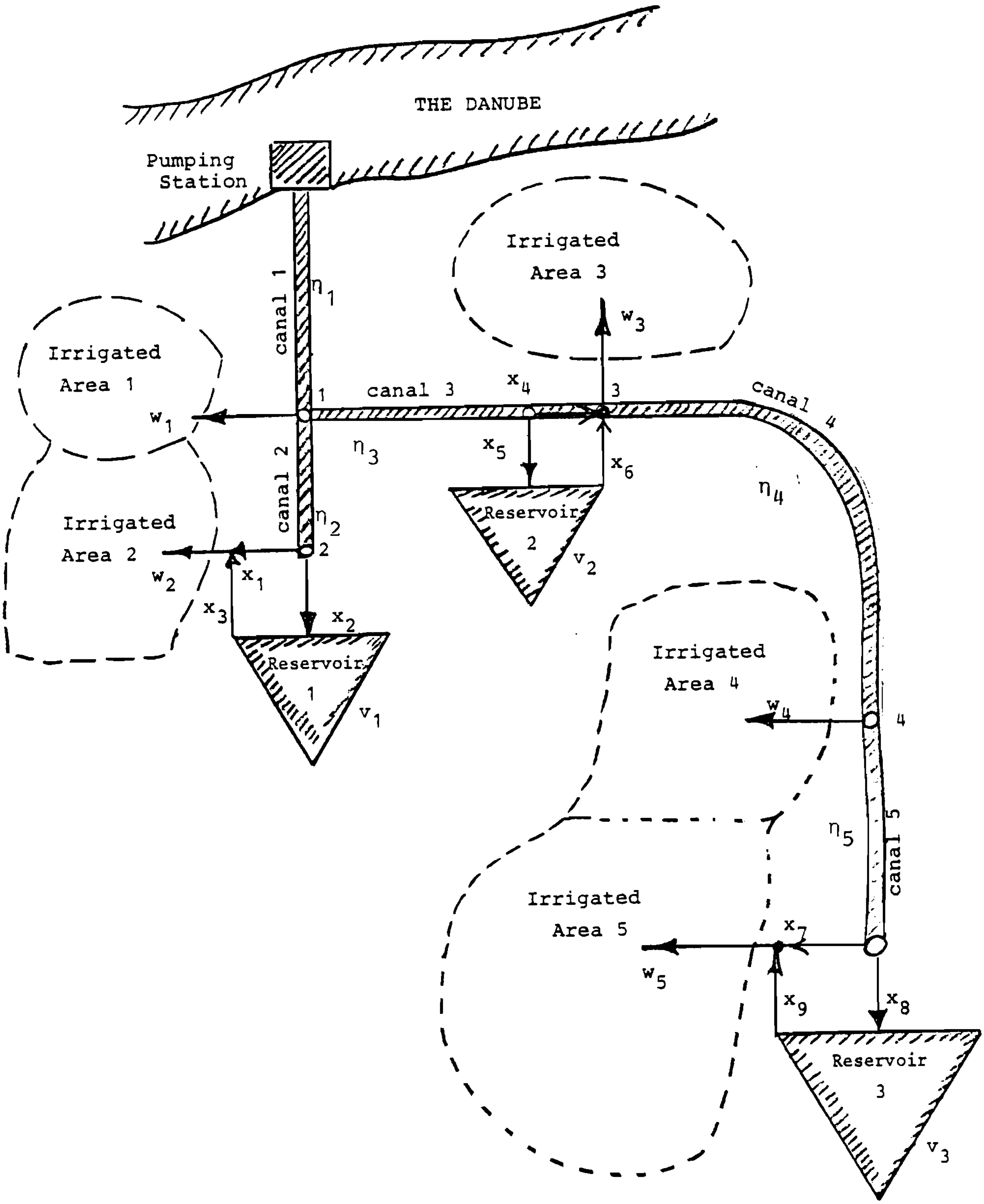


Figure 5

Structural Scheme of the WS System in the Silistra Region

7. Only within-year regulation of water resources is considered.
8. For each irrigated area the generalized irrigation timetables are specified. The profiles of these timetables can vary according to the crops.
9. On the strength of the small size of the Silistra region, the transit time delays for canals are not taken into account.
10. The basic water supply system parameters are determined by the minimization of its generalized annual costs taking into account both capital investments and operating costs.

Let us go on to the mathematical description of the model. For this, number the reservoirs, the main canals and the point water withdrawals as shown in Figure 5. The most compact description of the water supply model corresponds to its continuous analogy from which we start. Later, while computing the model, we shall go on to the discrete analogy of this. The following notations will be used in the model:

$w_i(t)$ - water flow intaken in the i-th water withdrawal ($i = 1,\ldots,5$) at time t;

$\eta_j(t)$ - water flow in the j-th canal ($j = 1,\ldots,5$) at time t;

$S_k(t)$ - active water storage in the k-th reservoir ($k = 1,2,3$) at time t;

$X_p(t)$ - water flow in the p-th distributing canal ($p = 1,\ldots,9$) at time t;

Z_j - discharge capacity of the j-th canal;

v_k - capacity of the k-th reservoir.

In essence, all the constraints in the model are the equations for conservation of mass in different nodes of the water network at any time t (see Figure 5) and can be presented as follows:

$$\begin{aligned}
\eta_1(t) &= \eta_2(t) + \eta_3(t) + w_1(t) \\
\eta_2(t) &= x_1(t) + x_2(t) \\
\eta_3(t) &= x_4(t) + x_5(t) \\
\eta_4(t) &= \eta_5(t) + w_4(t) \\
\eta_5(t) &= x_7(t) + x_8(t) \\
x_1(t) + x_3(t) &= w_2(t) \\
x_4(t) + x_6(t) - \eta_4(t) &= w_3(t) \\
x_7(t) + x_9(t) &= w_5(t) \quad .
\end{aligned} \tag{11}$$

The conditions of filling these three reservoirs can be written in the form of the following ordinary differential equations:

$$\frac{dS_k(t)}{dt} = x_{3k-1}(t) - x_{3k}(t) \quad , \quad k = 1,2,3 \quad . \tag{12}$$

By definition, the capacities for reservoirs and canals are:

$$\begin{aligned}
Z_j &= \max_t \eta_j(t) \quad , \quad j = 1,\ldots,5 \quad , \\
V_k &= \max_t S_k(t) \quad , \quad k = 1,2,3 \quad .
\end{aligned} \tag{13}$$

Lastly, the physical constraints on the signs of variables are:

$$x_p(t) \geq 0 \quad , \quad \eta_i(t) \geq 0 \quad , \quad S_k(t) \geq 0 \quad . \tag{14}$$

In relations (11)-(14) the values $w_i(t)$ are the water demands given and $x(t)$, $\eta_i(t)$, $S_k(t)$, Z_j and V_k are decision variables.

As is obvious from assumption 10, we proceed from an economic criterion. We assume that it is a linear function of the decision variables or the capacities for pumping stations, reservoirs, and canals. While constructing the optimization criterion we need the following notations:

$e_kV_k + f_k$ - generalized annual cost associated with the creation and operation of the k-th reservoir;

$a_1Z_1 + b_1$ - generalized annual cost associated with the creation of a pumping station;

a_2 - unit cost of electrical energy pumping station;

γ_j - unit cost associated with the water conveyance through the j-th canal while it operates in complete capacity;

μ_j - an increment of unit cost above, due to the operation of the j-th canal in incomplete capacity.

Here, all cost coefficients e_k, f_k, a_1, b_1, a_2, λ_j and μ_j are specified. In these terms, the generalized annual costs for the whole water supply system are written as follows:

$$E = \sum_{k=1}^{3} (e_kV_k + f_k) + a_1Z_1 + b_1 + \int_0^1 a_2\eta_1(t)dt$$

$$+ \sum_{i=1}^{5} \int_0^1 [\gamma_j\eta_j(t) + \mu_j(Z_j - \eta_j(t)] \, dt \quad . \tag{15}$$

Thus, the set of relations (11)-(14) and functional (15) are the continuous mathematical model for water supply. The appropriate optimization problem can be stated as follows.

Problem 4. Find the system of the time-varying functions $\eta_i(t)$, $S_k(t)$, $x_p(t)$, Z_j and V_k satisfying constraints (11)-(14) for which functional (15) has a minimum.

Hence, the continuous statement of the problem for the water supply system is a purely variational problem and should formally be solved by methods for calculus of variation. In practice, however, the matter is much simpler. Namely, in the discrete statement, Problem 4 is reduced to one of linear programming. For this it is enough to write the constraints (11)-(14) and the functional (15) in discrete form. We do not do this here, although some results below correspond to the discrete model.

Before showing some modeling results, introduce the concept of the seasonal unit costs of water for the different irrigated areas. In doing so, we follow, in many respects, the concept of the variational derivative of the functional.

For this purpose, vary the water demand $w_k(t)$ for the only irrigated area at some time t_o. Specifically, give $w_i(t)$ an increment $\Delta_k(t)$ which is different from zero only in the neighborhood of point t_o. The increment of water quantity withdrawn for the k-th irrigated area is $\sigma = \int_0^1 \Delta_k(t)dt$. For water demands varied in such a way it is correct to state Problem 4. Let E' be a value of functional (10) corresponding to the solution of such a problem. Calculate the ratio $\rho = \frac{E - E'}{\sigma}$. Next, let σ go to zero in such a way that both $\max |\Delta_k(t)|$ and the length of the interval in which $\Delta_k(t)$ is nonvanishing go to zero. Then, under the assumption that the ratio ρ converges to a limit such as $\sigma \to 0$, which we call the unit cost of water in the k-th irrigated area at time t_o and denoted by $\rho_k(t_o)$. In essence it is an additional cost or net saving, associated with supplying the k-th area with the additional unit of water in time t_o.

The discrete version of the model was run on the PDP-11/70 computer at IIASA.* The preliminary data corresponding to dividing the year into 7 periods τ^i are shown in Figure 6.

Besides the basic variant of the initial data two others were considered. The first corresponds to water demands w_i reduced twice, and the other to the absence of storage reservoirs ($v_i = v_2 = v_3 = 0$). For all variants some modeling results are presented in Figure 7. Figure 8 shows the timetables of fillings and releases from reservoirs for the basic variant. It follows from Figure 8 that there exist two typical time points: the beginning of the second period where all three reservoirs are filled completely, and the end of the fifth period where those are completely empty, disregarding their dead storage.

Since the initial data are preliminary, qualitative results of modeling can be more important than quantitative ones. In this respect, the seasonal unit costs of water are of interest. In Figure 9, for all variants of initial data, the seasonal unit ρ_k^i of water are shown as functions of the season and the geographical location of water withdrawals. As is obvious from Figure 9, the unit costs of water depend essentially on the season and what the irrigated area is.

Three tendencies are clearly observed. Firstly, at the period of the most intensive irrigation (i = 5) the unit costs of water are much more than at all other periods. Secondly, the unit costs of water increase when the irrigated area is moved away from the Danube river. Thirdly, the unit water cost corresponding to the period of the most intensive irrigation increases strongly when the reservoirs are absent.

*The program for running the discrete model was prepared by W. Sikorski.

Period Number i	1	2	3	4	5	6	7	
τ^i	4	1	1	0.5	0.5	1	2	month
w_1^i	0	3	1	6	9	1	0	m^3/sec
w_2^i	0	2	0.7	5	8	0.5	0	m^3/sec
w_3^i	0	1	0.4	4	6	0.3	0	m^3/sec
w_4^i	0	2	0.8	4.5	7	0.6	0	m^3/sec
w_5^i	0	5	3	12	26	2	0	m^3/sec

Reservoir Number k	1	2	3	
e_k	0.032	0.024	0.015	lv/m^3
f_k	$2 \cdot 10^6$	$2 \cdot 10^6$	$5 \cdot 10^6$	lv

Canal Number j	1	2	3	4	5	
γ_j	0.009	0.005	0.0105	0.013	0.013	lv/m^3
μ_j	0.004	0.002	0.005	0.007	0.007	lv/m^3

$a_1 = 0.104 \cdot 10^6$ $lv/m^3/sec$

$b_1 = 2 \cdot 10^6$ lv

$a_2 = 0.009$ lv/m^3

Figure 6

Initial Data for the Silistra WS System

Variant	Basic	Reduced Demands	Without Reservoirs	
Capacity of Pumping Station	9	5	56	m^3/sec
Capacities of Reservoirs				$10^6 \cdot m^3$
v_1	14.7	6.9	0	
v_2	30.7	14.7	0	
v_3	50.8	25.4	0	
Capacities of Canals				m^3/sec
z_1	9	5	56	
z_2	1.65	0.67	8	
z_3	4.92	2.41	39.2	
z_4	7	3.49	32.9	
z_5	3.08	1.54	26	
Annual cost E	21.6	16.6	43	$10^6 \cdot lv$

Figure 7

Basic Parameters of the Silistra WS System

Period Number i	1	2	3	4	5	6	7	
τ^i	4	1	1	0.5	0.5	1	2	month
S_1^i *)	11.2	14.7	12.4	14.7	10.4	0	2.87	$10^6 \cdot m^3$
S_2^i	11.9	30.7	27.7	29.3	16.8	0	2.39	$10^6 \cdot m^3$
S_3^i	18.8	50.8	45.8	46	33.7	0	2.81	$10^6 \cdot m^3$

*) S_k^i - an active water storage in the k-th reservoir at the beginning of period i

Figure 8

Timetables of Fillings and Releases for Reservoirs

Seasonal Unit Costs of Water: \ Period Number i		1	2	3	4	5	6	7	
ρ_1^i :	b*)	1.4	3.1	1.4	4.01	11.4	1.4	1.4	
	r*)	1.4	1.4	1.4	3.82	4.6	1.4	1.4	10^{-2}lv/m^3
	o*)	1.4	1.4	1.4	1.4	17.4	1.4	1.4	
ρ_2^i :	b	1.7	3.41	3.41	4.9	4.9	1.7	1.7	
	r	1.7	2.51	2.51	4.9	4.9	1.7	1.7	10^{-2}lv/m^3
	o	1.7	1.7	1.7	1.7	21.7	1.7	1.7	
ρ_3^i :	b	2.16	4.56	4.56	4.56	4.56	2.16	2.16	
	r	1.97	4.37	4.37	4.37	4.37	1.97	1.97	10^{-2}lv/m^3
	o	1.95	1.95	1.95	1.95	27.95	1.95	1.95	
ρ_4^i :	b	2.76	5.16	5.16	5.34	19.1	2.76	2.76	
	r	2.57	4.97	4.97	5.05	18.89	2.57	2.57	10^{-2}lv/m^3
	o	2.55	2.55	2.55	2.55	42.55	2.55	2.55	
ρ_5^i :	b	4.34	5.84	5.84	5.84	5.84	4.34	4.34	
	r	4.15	5.65	5.65	5.65	5.65	4.15	4.15	10^{-2}lv/m^3
	o	3.15	3.15	3.15	3.15	57.15	3.15	3.15	

*) b corresponds to the basic variant of the initial data
r corresponds to reduced water demands
o corresponds to the absence of reservoirs

Figure 9

Seasonal Unit Costs of Water for the Different Irrigated Areas

For example, for the first district, the cost of water varies from period III (May) to Season V (the end of June) nearly eight times (1,4 and 11,4 respectively). It is clear that this variation is much greater than for many other factors influencing regional and intraregional location and far beyond the rate of "normal error" in usual economic calculations (10-15%).

The same can be said for the space influence: the variation of cost for different irrigated areas for period III varies from 1,4 to 5,84 lv/m^3, which also means that the use of average data leads to grave error.

Everything mentioned above can be explained without difficulty in terms of capital and operation costs associated with the creation of a water supply system.

Thus the change of the unit water costs in time and space should essentially influence the allocation of regional production units and the choice of production specialization of the region. At the same time, the distributed unit costs of water call for care in handling such concepts as the annual unit cost of water or average regional unit cost of water while planning the agricultural development of the Silistra region.

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