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A DEMOECONOMIC MODEL OF INTERREGIONAL GROWTH RATE DIFFERENCES

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PREFACE

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. Recently this interest has given rise to a concentrated research effort focusing on migration dynamics and settlement patterns. Four subtasks form the core of this research effort:

I. The study of spatial population dynamics;

II. The definition and elaboration of a new research area called demometrics and its application to migration analysis and spatial population forecasting;

III. The analysis and design of migration and settlement policy;

IV. A comparative study of national migration and settlement patterns and policies.

Consistent demoeconomic modeling of multiregional systems is an important component of demometrics. It requires the determination of labor force participation, migration and unemployment rates simultaneously and endogenously in the model. This paper presents an important contribution to regional modeling. Jacques Ledent and Peter Gordon elaborate on a recently published model of interregional growth and show how the demometric approach alleviates several problems inherent in conventional modeling.

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Leader
Migration and Settlement Task

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ABSTRACT

This paper sets forth a *demoeconomic* approach to interregional development along non-neoclassical lines. This objective is carried out by elaborating on a recently published model of interregional growth rate differences (Dixon and Thirlwall, 1975).

First, a critical review of this model suggests the implausibility of its main result, i.e., the possibility of steady growth by a pair of regions over the long run. It is shown that

a) the omission of migration which would eventually dampen the implied income divergence, and

b) the linear structure of the model

cause such a result.

Thus, an extension of this model is proposed which includes migration as well as other demographic aspects of development (labor force participation and unemployment), endogenously and simultaneously determined. Interestingly enough, the nature of these variables provides an impetus for reconsidering linearity: the proper modeling of demoeconomic effects necessarily introduces nonlinearities.

Non-static long-term rates of change are shown to emerge from the simulation of this extended model: as a consequence of population shifts due to migration, there appear regional cycles accompanied by cycles of divergence and convergence of income.

A Demoeconomic Model of Interregional Growth Rate Differences

One of the most interesting models of interregional growth is that of Dixon and Thirlwall (1975)--hereafter referred to as DT. They attempt to formalize Kaldor's thoughts on development along non-neoclassical lines. Their formal model includes a price mark-up equation, in place of a marginal cost determined competitive price, as well as a positive feedback between the region's rate of technical innovation and regional economic growth rates (the Verdoorn effect). Competition between a pair of regions is taken care of by a relationship between relative regional prices and export demand.

The DT model is useful for studying the possibilities of income divergence or convergence between regions over the long term. Yet, the cited model is linear in the rates of change of all included variables and, not at all surprisingly, yields an outcome of stable growth rates in the long run. The authors cite this as an example of equilibrium characterized by an absence of divergence or convergence. Their conclusion is faulty for several reasons. First, the literature on regional convergence and divergence looks at long term income trends and not growth rate trends. Thus, stable growth rates for a pair of regions can easily be associated with an ever widening divergence of incomes. We can hardly expect this to be a long-term equilibrium. Given enough of an income gap, people will move from the poor to the rich region. This brings us to the second point which has to do with the secondary equilibrating <u>and</u> disequilibrating effects of migration. Simple models of factor price equalization cite the migration response as an equilibrating force which puts a brake on interregional income divergence. Yet, over shorter time spans, migration may well have an agglomerative effect (for example, only the most skilled and non risk averse may migrate) which accelerates income divergence. Thus, we claim that the stable growth equilibrium which DT cite is not only due to the linearity of their model but is also due to the omission of a demographic sector.

In order to put this assertion into focus, we will suggest the following: first, a truly interesting model of interregional development ought to be demoeconomic, i.e., to cover both economic and demographic aspects of development; second, such a demoeconomic model cannot be totally linear in the rates of change; and third, non-static long-term rates of change should automatically emerge from the simulation of such a model. This means that, as a consequence of population shifts due to migration, there should appear regional cycles accompanied by cycles of divergence and convergence of incomes.

To recapitulate,

1) DT should not be surprised that their *linear* model leads to constant growth rates in the long run;
2) they should not confuse steady growth with an absence of divergence or convergence of incomes;
3) the implausibility of the DT result (steady growth by a pair of regions over the long term) evokes the absence of migration and calls for a demoeconomic approach;
4) the migration response would eventually dampen the implied income divergence, and
5) the proper modeling of demoeconomic effects introduces non-linearities.

Our objective in this paper is to demonstrate these points with the help of an interregional demoeconomic model built on the DT model, which constitutes a useful reference point from which interregional demoeconomics can proceed along the non-neoclassical path.

Beyond the specific model that is developed in the following pages, we also hope to indicate the methodological gains that are suggested by the demoeconomic approach. Because economic and demographic variables interact, regional models that are either purely economic <u>or</u> demographic in nature are unsatisfactory. Yet, the demoeconomic synthesis is not trivial. Looking at the labor market in spatial terms, we treat the decision to migrate as endogenous. This extends the notion of job search

(Miron, 1978). The central idea is that labor force participation, migration and unemployment rates are endogeneous and simultaneously determined. Yet, it has been shown by Ledent (1978) that any model including variables of this sort is likely to generate preposterous unemployment and/or labor force participation rates without a proper modeling of the relationship between comparable variables of the economic and demographic sides: employment and labor force respectively. This is referred to as the consistency problem which is particularly acute if unemployment and labor force participation rates are defined as residuals. Also, when these variables are dependent variables, a linear model eventually develops population and labor force dimensions which imply unrealistic unemployment and labor force participation rates. This suggests that a demoeconomic model will have to be non linear.

In the next section, we present an augmented DT model, along demoeconomic lines. We then specify reasonable parameter values for the two-region case and suggest that the results of a long-term simulation of the expanded model are much more plausible than the growth equilibrium of DT. Finally, we comment on the costs and benefits of following the demoeconomic approach to regional analysis.

FORMULATION OF THE MODEL

In what follows, we present a two-region model which extends the DT model by allowing migration between the two regions.

It consists of three blocks which describe successively:

i) the impact of demographic forces on regional income growth rates,

ii) the impact of economic forces on regional population growth rates, and

iii) the relationships linking employment and labor force variables, ensuring the consistency between the economic and demographic sides of the model.

The first equation of the first block relates a two-element vector of regional income growth rates to the growth in the region's exports as well as in the region's population and labor force. The export-base approach was suggested by DT. We add the other elements to bring in the impact of demographic factors on growth, emphasizing the role of households as consumers as well as of suppliers of labor. Thus,

$$(g_t) = \underset{\sim}{\Gamma}(x_t) + \underset{\sim}{\phi}^1(n_t) + \underset{\sim}{\phi}^2(l_t + n_t) \qquad (1)$$

where, (g_t) is the vector of regional growth rates,
(x_t) is the vector of export growth rates,
(l_t) is the vector of labor force participation rate changes
(n_t) is the vector of population growth rates,
$\underset{\sim}{\Gamma}$, $\underset{\sim}{\phi}^1$ and $\underset{\sim}{\phi}^2$ are diagonal matrices of coefficients*.

The second relationship expresses the growth of exports in terms of changes in relative prices and world demand. We have,

$$(x_t) = \underset{\sim}{\eta}(p_t) + z\underset{\sim}{\varepsilon}(i) \qquad (2)$$

*Because all the variables are expressed in their growth rates, the coefficients are elasticities.

where, (p_t) is the vector of regional export price changes,
(i) is the two-element vector of ones, and
z is the change in world demand.

Note that $\underset{\sim}{\varepsilon}$ is a diagonal matrix of coefficients, unlike $\underset{\sim}{\eta}$ whose off-diagonal elements represent the impact of a region's price change on the growth of the other region's exports.

Prices are explained by a cost mark-up equation, just as in the DT paper, so that we have:

$$(p_t) = (w_t) - (r_t) + (\tau) \tag{3}$$

where, (w_t) is the vector of regional wage rate changes,
(r_t) is the vector of regional rates of technological change, and
(τ) is the exogeneous vector of regional rates of change of cost mark-up.

The next equation explains regional technical innovation in terms of an endogenous and an exogenous element,

$$(r_t) = (\bar{r}) + \underset{\sim}{\lambda}(g_t) \tag{4}$$

where, $(\bar{r})$ is the vector of the exogenous elements and
$\underset{\sim}{\lambda}$ is a diagonal matrix of coefficients.

Just as in the DT paper, the second term represents the Verdoorn effect.

At this point, it may be noted that substituting (4) into (3) and the result into (2) reveals a particular impact of one region's growth on the other region's export growth. This reflects a competitive effect in that growth in region i diminishes the export demand growth of region j through an impact on relative export prices. Another growth effect on export demand growth could be included with a positive impact via the traditional income-consumption linkage. Clearly, the two effects work in opposite directions and are of different magnitudes. In the former case we emphasize competition between regions and in the latter case we would emphasize trade. The two cases are probably differentiable in terms of the sizes of the regions vis-a-vis rest-of-the-world demand.

We retain the (implicit) small-but-competitive region example of the DT model. We do this for the sake of continuity and simplicity. Also, we wish to highlight the demoeconomic effects and it makes no difference which case is studied to make that point.

The next equation concerns the wage rate which, unlike DT, we chose to make partially endogenous. Thus,

$$(w_t) = (\bar{w}) + \underset{\sim}{\psi}_t (l_t) \quad . \tag{5}$$

A time subscript is attached to the diagonal matrix $\underset{\sim}{\psi}_t$ because its elements, representing each region's wage elasticity with respect to labor force participation rate (LFPR) are not taken as constants. It is hypothesized that the absolute value of each element ψ_{it}, which by the way has a negative sign, increases with the value of the beginning-of-the-period LFPR. Thus, supposing in addition that each region's labor force participation rate can take on values within a range of (ρ^l, ρ^r) where ρ^l is a low enough LFPR so as to have no impact on wage rate change and ρ^r is a high enough LFPR so as to have an infinite impact on wage rate change, we have:

$$\psi_{it} = d_i \frac{\rho_{it} - \rho^l}{\rho_{it} - \rho^r} \; ; \qquad \forall i = 1,2 \tag{6}$$

or, in compact form,

$$\underset{\sim}{\psi}_t = \underset{\sim}{D} \left(\underset{\sim}{\rho}_t - \rho^l \underset{\sim}{I} \right) \left(\underset{\sim}{\rho}_t - \rho^r \underset{\sim}{I} \right)^{-1} \tag{6'}$$

where $\underset{\sim}{\rho}_t$ is a diagonal matrix of the beginning-of-the period LFPR

$\underset{\sim}{I}$ is the two by two identity matrix

$\underset{\sim}{D}$ is a diagonal matrix of coefficients.

The last equation of the first block relates a region's rate of income growth to its rate of change in employment level.

$$(e_t) = \underset{\sim}{\mu}(g_t) \tag{7}$$

where (e_t) is the vector of regional employment growth rates,
$\underset{\sim}{\mu}$ is a diagonal matrix of coefficients.

Note, that the rationale for this equation is the availability of an economic variable directly comparable with a variable from the demographic side (labor force) to ensure the aforementioned consistency.

The next block of the model describes the impact of economic forces on population growth through migration. The demographic model underlying this block is the so-called components-of-change model of population growth and distribution (Rogers, 1968). Thus, we have:

$$N_{i,t+1} = N_{it} + b_i N_{it} - m_{it} N_{it} + m_{jt} N_{it} \; ; \; \forall i = 1,2 \tag{8}$$

where N_{it} is population in region i at time t,
b_i is region i's exogenous rate of natural increase
m_{it} is the migraton rate from region i to the other region in period (t,t + 1).

Rewritten, this relationship yields,

$$n_{it} = \frac{N_{i,t+1} - N_{it}}{N_{it}} = b_i - m_{ij} + m_{ji} \frac{N_{jt}}{N_{it}} \; ; \forall i = 1,2 \tag{8'}$$

as, in a more compact form:

$$(n_t) = (b) - \underset{\sim}{N}_t^{-1} \underset{\sim}{P} \underset{\sim}{N}_t (m_t) \tag{8''}$$

where (n_t) is the vector of regional population growth rates
$\underset{\sim}{P}$ is the matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
(b), and $\underset{\sim}{N}_t$ are vector or matrix equivalents of previously defined variables.

To assure a demoeconomic model, it is necessary to specify the way in which economic forces cause migration rates to change.

We suggest that,

$$m_{it} = \alpha_i \frac{N_{it}}{N_{it}+N_{jt}} \left[1 + \beta_i \left(\frac{e_{jt}}{u_{jt}} - \frac{e_{it}}{u_{it}}\right)\right]; \forall i,j = 1,2 \; (j \neq i) \quad (9)$$

That is, the migration rate out of each region is proportional to the attractiveness of the other region--measured by the part of the total population living in this region--and is related to the difference in the economic opportunities offered by the two regions. Note, that the index of regional economic opportunities used here is a slight variation of Todaro's probability that a migrant finds a job (Todaro, 1976): it is the ratio of employment growth rate e_{it} to the beginning-of-the-period unemployment rate u_{it}. (The latter is defined below).

Equation (9) can be rewritten in a more compact form as:

$$(m_t) = \frac{1}{N_{\cdot t}} \underset{\sim}{\alpha} \underset{\sim}{N}_t \left[(i) - \underset{\sim}{\beta} \underset{\sim}{p} \underset{\sim}{N}_t^{-1} (e_t)\right] \quad (9')$$

where $N_{\cdot t}$ is the total population of the system at time t,
$\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ are diagonal matrices of coefficients,
$\underset{\sim}{u}_t$ is the matrix of regional unemployment rates at time t.

The last block of the model defines the labor force and unemployment variables. The first equation of this block posits a behavioral basis for the change in the LFPR

$$(l_t) = \underset{\sim}{\gamma}_t (\underset{\sim}{I} - \underset{\sim}{u}_t)^{-1} \left[(u_{t+1}) - (u_t)\right] \quad (10)$$

in which $\underset{\sim}{\gamma}_t$ is a diagonal matrix introducing further non-linearity into the model. It is hypothesized that the value of each element γ_{it}, which, by the way, has a negative sign, is smaller when the unemployment rate takes on extreme values, either low or high, and much larger for unemployment rate values intermediate between those extremes. We have,

$$\gamma_{it} = a_i (u_{it} - u^l)(u_{it} - u^r); \quad \forall i = 1,2 \quad (11)$$

where u^l and u^r are the extreme values of the range in which u_{it} falls, and, in more compact form,

$$\underset{\sim}{\gamma}_t = \underset{\sim}{A}\ (\underset{\sim}{U}_t - u^l \underset{\sim}{I})(\underset{\sim}{U}_t - u^r \underset{\sim}{I}) \qquad (11')$$

where $\underset{\sim}{A}$ is a diagonal matrix of coefficients.

The last equation of this block is the following relationship:

$$(e_t) = (l_t) - (\underset{\sim}{I} - \underset{\sim}{U}_t)^{-1} \left[(u_{t+1}) - (u_t)\right] + (n_t) \qquad (12)$$

obtained by differentiating (logarithmically) the identity relating employment levels (E_t) and population levels (N_t),

$$(E_t) = \underset{\sim}{\rho}_t(\underset{\sim}{I} - \underset{\sim}{U}_t)(N_t) \qquad (13)$$

As shown in Appendix 2, various substitutions permit one to reduce each of the three blocks of the system to a single equation in three variables (e_t) [or (g_t)], (l_t) and (n_t). This leads to a simple model of three equations in three unknowns that can be analytically solved in spite of the non-linearities introduced into the model. As also shown in Appendix 2, the derivation of the reduced form equations of the model is tractable because the coefficients of the endogenous variables are known variables (either constant or depending on lagged variables).

It is clear, from these reduced form equations, that the introduction of the equations of population change have added difference equations which make the model much more dynamic than the DT model. Also, a radical departure from linearity has been introduced in the process. We note again that non-linearity is almost implicit in the demoeconomic approach.

SIMULATION OF THE MODEL

From the three reduced form equations concerning (e_t), (l_t) and (n_t), it is easy to develop a simulation of the time paths of these variables and then of all the other variables. So as to be of maximal policy interest, the simulation was conducted for an hypothetical pair of regions where the one is economically advanced and the other is developing. As already mentioned, these are competing regions, whose primary trade is with the rest of the world.

It will be seen that the time paths of growth rate changes that result fluctuate over patterns of convergence and divergence. As suggested at the outset, since non-linearities and a migration response have been added to the DT model we would not expect anything like steady state growth rates and the associated diverging regional income levels. Though our results simply indicate a simulation result, we have based the simulation on reasonable assumptions and parameter choices. In defending this sort of approach to model building, Nelson and Winter (1977) assert that,

> Simulation... can be a useful adjunct to an analytical approach. It can establish, with the same finality as a theorem, the logical consistency of the model's assumptions with a set of proportions about its behavior. And while it offers a way around the tractability constraints of analytic methods, it imposes its own constructive discipline of modeling dynamic systems: the program must contain a complete specification of how the system at t + 1 depends on that at t and exogenous factors, or it will not run.

The earlier discussion on labor force participation rates reflects precisely this point. The problems cited were not evident in the original DT model and only become apparent once the long-term demoeconomic interactions were modeled and simulated.

Our results, as indicated, follow from defensible values of the parameters. Table 1 provides a summary of these values. Many of them are similar in order of magnitude to those employed by DT. The export elasticity with respect to regional income growth is lower in the developing region (region 2) because a younger

Table 1. Summary of parameter values and initial conditions.

Parameter	Advanced Region (Region 1)	Developing Region (Region 2)
ELASTICITIES		
Elasticity of export growth wrt income growth (1)	$\gamma_1 = 0.60$	$\gamma_2 = 0.55$
Elasticity of population growth wrt income growth (1)	$\phi_1^1 = 0.65$	$\phi_2^1 = 0.70$
Elasticity of labor force growth wrt income growth (1)	$\phi_1^2 = 0.10$	$\phi_2^2 = 0.10$
Price Change elasticity wrt export growth (2)	$n_{11} = -1.50$ $n_{21} = 1.50$	$n_{12} = 1.50$ $n_{22} = -1.50$
Elasticity of world demand change wrt export growth (2)	$\varepsilon_1 = 1.00$	$\varepsilon_2 = 1.10$
Elasticity of income growth wrt technological change (4)	$\lambda_1 = 0.50$	$\lambda_2 = 0.70$
Elasticity of income growth wrt employment growth (7)	$\mu_1 = 0.30$	$\mu_2 = 0.40$
OTHER COEFFICIENTS		
Coefficient in determination of elasticity of labor force participation rate change wrt wage rate change (6)	$d_1 = 3.00$	$d_2 = 2.00$
Coefficients in determination of the migration rates (9)	$\alpha_1 = 0.0700$ $\beta_1 = 0.25$	$\alpha_2 = 0.0725$ $\beta_1 = 0.30$
Coefficient in determination of elasticity of unemployment rate change wrt labor force participation rate change (11)	$a_1 = 6000$	$a_2 = 3000$
OTHER PARAMETERS		
Price mark-up factor (3)	$\tau_1 = 0.015$	$\tau_2 = 0.015$
Exogenous rate of technological change (4)	$\bar{r}_1 = 0.025$	$\bar{r}_2 = 0.025$
Exogenous element of the wage growth rate (5)	$\bar{w}_1 = 0.015$	$\bar{w}_2 = 0.015$
Rate of natural increase (8)	$b_1 = 0.01$	$b_2 = 0.013$
INITIAL CONDITIONS		
Initial population (in thousands)	$N_{10} = 7{,}500$	$N_{20} = 2{,}500$
Initial unemployment rate	$u_{10} = 0.05$	$u_{20} = 0.035$
Initial labor force part. rate	$\rho_{10} = 0.35$	$\rho_{20} = 0.37$
NON-REGIONALIZED PARAMETERS		
Bounds on labor force part. rate (6)	$\rho^l = 0.30$ $\rho^r = 0.42$	
Bounds on unemployment rate (11)	$u^l = 0$ $u^r = 0.10$	
Rate of change of world demand (2)	$z = 0.04$	

region is usually more trade dependent, causing smaller internal foreign trade multiplier effects. The elasticity of regional population growth with respect to income growth is slightly larger in the developing region, suggesting that the developing region has greater (dynamic) opportunities for import substitution.

All price elasticities of export demand are greater, in absolute value, than unity. In fact, DT invoke values of 1.5 for these, as we do. The justification for a price elasticity in the elastic part of the demand curve rests on the small region (vis-a-vis the rest of the world) assumption: as the region's export price rises by one percent, the demand for its exports falls by about 1.5 percent. Yet, since the cross-elasticities are also elastic, this assumption must be tempered. Since any price increase is met by a fall in "own" demand and an almost equivalent rise in the competing region's demand, we have the case of close substitutability of the export, most of which is supplied by these two regions.

The next difference in parameter values involves the elasticity of world demand change with respect to export growth. This parameter is larger for the growing region, showing a greater orientation to external demand. Also, regional growth has a stronger effect on induced innovation in the younger region which has far less durable capital to depreciate before innovation can proceed.

Employment growth is more sensitive to economic development in region two ($\mu_2 > \mu_1$) since it is entirely plausible that growth in that region would include labor intensive processes.

The coefficient d_i in equation (6) has a greater value for the advanced region. This means that the elasticity of wage rate change with respect to labor force participation rate change is more sensitive to fluctuations in the levels of the LFPR in the advanced region. At the same time, market institutions in the advanced economy may be more developed, permitting greater scope in these wage adjustments or less wage rigidity than in the traditional but emerging region. Perhaps the most important of these institutional differences is in information channels that underlie the labor market and aid the job search process.

The outmigration rates from the developing region are thought to be slightly more sensitive to economic conditions since the younger population of that region is probably made up of more economic opportunity seekers. Thus, $\alpha_2 > \alpha_1$ and $\beta_2 > \beta_1$.

Turning to equation (11), the coefficient a_i is significantly larger for the first region. This is because the labor force participation rate varies more in a region where pensions and other non-labor incomes are possible. In other words, the more advanced region is thought to have a social service apparatus which makes leaving the labor force more plausible.

The rate of natural increase is, of course, slightly larger in the developing region with its younger population. The remaining regional parameters are common to the two regions.

Turning to the initial conditions, the older region has three times the population of the developing region. Its initial unemployment rate is larger and its labor force participation rate is lower for the reason that its population contains more older people. The bounds on the labor force participation and unemployment rates used in the formulation of the nonlinear equations (6) and (11) are the same in the two regions.

Finally, the rate of change of world demand which drives the model is taken equal to 4 percent, as in the DT model. Results of the simulations are shown in Tables 2, 3 and 4.* In discussing these results of the simulation, it is difficult to identify simple cause and effect relationships because of the large number of second-order effects. Most important among these are the interregional feedback effects. Also, since migration and population levels appear as independent as well as dependent variables throughout the model, it is almost impossible to

*Additional simulation results are shown in Appendix 1.

Table 2. Annual growth rates of main variables.

T	G1	E1	N1	RHO1	U1	G2	E2	N2	RHO2	U2
1	0.0351	0.0108	0.0091	0.0016	-0.0021	0.0329	0.0131	0.0158	-0.0023	0.0095
2	0.0360	0.0111	0.0093	0.0017	-0.0021	0.0316	0.0126	0.0151	-0.0022	0.0086
3	0.0367	0.0113	0.0095	0.0017	-0.0022	0.0307	0.0123	0.0146	-0.0020	0.0079
4	0.0373	0.0115	0.0096	0.0017	-0.0022	0.0299	0.0120	0.0141	-0.0019	0.0073
5	0.0378	0.0116	0.0097	0.0018	-0.0022	0.0292	0.0117	0.0137	-0.0018	0.0068
6	0.0382	0.0117	0.0099	0.0018	-0.0023	0.0287	0.0115	0.0134	-0.0017	0.0064
7	0.0385	0.0118	0.0100	0.0018	-0.0023	0.0283	0.0113	0.0131	-0.0016	0.0060
8	0.0387	0.0119	0.0100	0.0018	-0.0023	0.0279	0.0112	0.0129	-0.0015	0.0057
9	0.0390	0.0120	0.0101	0.0018	-0.0023	0.0276	0.0110	0.0127	-0.0015	0.0054
10	0.0392	0.0120	0.0102	0.0018	-0.0023	0.0273	0.0109	0.0125	-0.0014	0.0051
11	0.0393	0.0121	0.0102	0.0018	-0.0023	0.0271	0.0108	0.0124	-0.0013	0.0049
12	0.0394	0.0121	0.0103	0.0017	-0.0023	0.0269	0.0108	0.0122	-0.0013	0.0046
13	0.0395	0.0122	0.0103	0.0017	-0.0023	0.0268	0.0107	0.0121	-0.0012	0.0044
14	0.0396	0.0122	0.0104	0.0017	-0.0023	0.0266	0.0106	0.0120	-0.0012	0.0042
15	0.0397	0.0122	0.0104	0.0017	-0.0022	0.0265	0.0106	0.0119	-0.0011	0.0040
20	0.0399	0.0123	0.0105	0.0016	-0.0022	0.0262	0.0105	0.0115	-0.0009	0.0032
25	0.0399	0.0123	0.0106	0.0016	-0.0021	0.0261	0.0104	0.0113	-0.0008	0.0026
30	0.0397	0.0122	0.0106	0.0015	-0.0020	0.0261	0.0104	0.0112	-0.0006	0.0022
35	0.0395	0.0122	0.0107	0.0014	-0.0019	0.0262	0.0105	0.0111	-0.0005	0.0018
40	0.0393	0.0121	0.0107	0.0013	-0.0018	0.0264	0.0106	0.0111	-0.0004	0.0014
45	0.0391	0.0120	0.0107	0.0012	-0.0018	0.0267	0.0107	0.0111	-0.0004	0.0011
50	0.0388	0.0119	0.0107	0.0012	-0.0017	0.0269	0.0108	0.0111	-0.0003	0.0009
55	0.0386	0.0119	0.0107	0.0011	-0.0016	0.0272	0.0109	0.0111	-0.0002	0.0007
60	0.0383	0.0118	0.0107	0.0011	-0.0015	0.0274	0.0110	0.0112	-0.0002	0.0005
65	0.0380	0.0117	0.0106	0.0010	-0.0015	0.0277	0.0111	0.0112	-0.0001	0.0004
70	0.0378	0.0116	0.0106	0.0009	-0.0014	0.0280	0.0112	0.0113	-0.0001	0.0002
75	0.0375	0.0115	0.0106	0.0009	-0.0013	0.0282	0.0113	0.0113	-0.0000	0.0001
80	0.0373	0.0115	0.0106	0.0008	-0.0013	0.0285	0.0114	0.0114	-0.0000	0.0000
85	0.0370	0.0114	0.0106	0.0008	-0.0012	0.0288	0.0115	0.0115	0.0000	-0.0000
90	0.0368	0.0113	0.0105	0.0007	-0.0011	0.0290	0.0116	0.0116	0.0000	-0.0001
95	0.0366	0.0113	0.0105	0.0007	-0.0011	0.0293	0.0117	0.0117	0.0001	-0.0002
100	0.0363	0.0112	0.0105	0.0007	-0.0010	0.0295	0.0118	0.0117	0.0001	-0.0002
110	0.0359	0.0110	0.0104	0.0006	-0.0009	0.0300	0.0120	0.0119	0.0001	-0.0003
120	0.0355	0.0109	0.0103	0.0005	-0.0009	0.0305	0.0122	0.0121	0.0001	-0.0003
130	0.0351	0.0108	0.0103	0.0005	-0.0008	0.0310	0.0124	0.0123	0.0001	-0.0003
140	0.0347	0.0107	0.0102	0.0004	-0.0007	0.0314	0.0126	0.0125	0.0001	-0.0003
150	0.0343	0.0106	0.0102	0.0004	-0.0006	0.0318	0.0127	0.0126	0.0001	-0.0003
160	0.0340	0.0105	0.0101	0.0004	-0.0006	0.0323	0.0129	0.0128	0.0001	-0.0003
170	0.0336	0.0104	0.0100	0.0003	-0.0005	0.0327	0.0131	0.0130	0.0001	-0.0003
180	0.0333	0.0102	0.0099	0.0003	-0.0005	0.0331	0.0132	0.0131	0.0001	-0.0002
190	0.0329	0.0101	0.0099	0.0003	-0.0004	0.0335	0.0134	0.0133	0.0001	-0.0002
200	0.0326	0.0100	0.0098	0.0002	-0.0004	0.0339	0.0136	0.0135	0.0001	-0.0002
210	0.0321	0.0099	0.0097	0.0002	-0.0004	0.0343	0.0137	0.0137	0.0001	-0.0002
220	0.0317	0.0098	0.0095	0.0002	-0.0003	0.0348	0.0139	0.0139	0.0001	-0.0002
230	0.0311	0.0096	0.0094	0.0002	-0.0003	0.0354	0.0142	0.0141	0.0000	-0.0002
240	0.0304	0.0094	0.0092	0.0002	-0.0003	0.0362	0.0145	0.0144	0.0000	-0.0001
250	0.0294	0.0090	0.0089	0.0001	-0.0002	0.0373	0.0149	0.0149	0.0000	-0.0001

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G = INCOME GROWTH RATE
E = EMPLOYMENT GROWTH RATE
N = POPULATION GROWTH RATE
RHO = LABOR FORCE PARTICIPATION GROWTH RATE
U = UNEMPLOYMENT GROWTH RATE

Table 3. Levels of main economic variables.

T	POP1	EMP1	LF1	LFPR1	UR1	POP2	EMP2	LF2	LFPR2	UR2
0	7500.0	2493.8	2625.0	0.3500	0.0503	2500.0	892.6	925.0	0.3700	0.0350
1	7567.9	2520.7	2655.1	0.3506	0.0499	2539.6	904.3	937.4	0.3691	0.0353
2	7638.2	2540.7	2682.2	0.3512	0.0498	2578.0	915.7	949.6	0.3683	0.0356
3	7710.6	2577.5	2712.3	0.3518	0.0497	2615.5	926.9	961.5	0.3676	0.0359
4	7784.8	2607.2	2743.1	0.3524	0.0496	2652.4	938.0	973.2	0.3669	0.0362
5	7863.7	2637.5	2774.7	0.3530	0.0495	2688.9	948.9	984.8	0.3662	0.0364
6	7939.2	2668.5	2807.1	0.3536	0.0493	2724.9	959.8	996.3	0.3656	0.0367
7	8017.1	2700.2	2840.0	0.3542	0.0492	2760.8	970.6	1007.8	0.3650	0.0369
8	8097.6	2732.4	2873.6	0.3549	0.0491	2796.4	981.4	1019.3	0.3645	0.0371
9	8179.4	2765.2	2907.8	0.3555	0.0490	2831.9	992.3	1030.7	0.3640	0.0373
10	8262.5	2798.6	2942.5	0.3561	0.0489	2867.4	1003.1	1042.2	0.3635	0.0375
11	8346.9	2832.5	2977.8	0.3568	0.0488	2902.9	1014.0	1053.6	0.3630	0.0377
12	8432.6	2866.9	3013.6	0.3574	0.0487	2938.4	1024.9	1065.2	0.3625	0.0378
13	8519.6	2901.9	3050.0	0.3580	0.0486	2973.9	1035.8	1076.7	0.3621	0.0380
14	8607.8	2937.3	3086.9	0.3586	0.0485	3009.6	1046.8	1088.4	0.3616	0.0382
15	8697.2	2973.3	3124.3	0.3592	0.0483	3045.4	1057.9	1100.1	0.3612	0.0383
20	9162.2	3160.3	3319.0	0.3622	0.0478	3227.1	1114.6	1159.8	0.3594	0.0390
25	9656.7	3359.3	3526.1	0.3651	0.0473	3415.1	1173.9	1222.2	0.3579	0.0396
30	10184.9	3570.4	3745.8	0.3679	0.0468	3611.1	1236.3	1287.9	0.3567	0.0400
35	10735.5	3793.7	3978.2	0.3706	0.0464	3816.5	1302.4	1357.3	0.3556	0.0404
40	11321.3	4029.7	4223.7	0.3731	0.0459	4032.8	1372.5	1430.8	0.3548	0.0407
45	11939.4	4278.8	4482.9	0.3755	0.0455	4260.9	1447.0	1508.8	0.3541	0.0410
50	12591.2	4541.6	4756.2	0.3777	0.0451	4502.1	1526.2	1591.8	0.3536	0.0412
55	13279.1	4818.5	5044.3	0.3799	0.0448	4757.6	1610.6	1680.1	0.3531	0.0414
60	14001.5	5110.2	5347.8	0.3819	0.0444	5028.6	1700.5	1774.1	0.3528	0.0415
65	14763.2	5417.5	5667.4	0.3839	0.0441	5316.3	1796.5	1874.4	0.3526	0.0416
70	15564.8	5741.0	6003.8	0.3857	0.0438	5622.1	1898.8	1981.3	0.3524	0.0416
75	16408.2	6081.3	6357.9	0.3875	0.0435	5947.5	2008.1	2095.4	0.3523	0.0417
80	17295.3	6439.5	6730.3	0.3891	0.0432	6293.8	2124.8	2217.2	0.3523	0.0417
85	18228.1	6816.1	7122.1	0.3907	0.0430	6662.8	2249.5	2347.2	0.3523	0.0417
90	19208.7	7212.2	7533.9	0.3922	0.0427	7055.5	2382.7	2486.2	0.3523	0.0416
95	20239.3	7628.6	7966.9	0.3936	0.0425	7476.0	2525.0	2634.7	0.3524	0.0416
100	21322.1	8066.1	8421.9	0.3950	0.0422	7920.0	2677.3	2793.4	0.3525	0.0416
110	23654.4	9048.9	9442.3	0.3975	0.0418	8913.6	3014.3	3144.7	0.3528	0.0415
120	26225.9	10048.9	10483.5	0.3997	0.0415	10044.4	3400.3	3546.9	0.3531	0.0414
130	29259.0	11195.0	11675.1	0.4018	0.0411	11336.6	3842.7	4007.9	0.3535	0.0412
140	32177.7	12457.0	12987.2	0.4036	0.0408	12622.6	4350.5	4536.9	0.3538	0.0411
150	35697.9	13805.5	14430.7	0.4053	0.0406	14525.6	4933.8	5144.5	0.3542	0.0410
160	39377.7	15371.7	16017.4	0.4068	0.0403	16483.5	5604.6	5843.3	0.3545	0.0408
170	43516.6	17247.7	17759.7	0.4081	0.0401	18736.6	6376.9	6647.7	0.3548	0.0407
180	48056.0	18886.2	19670.8	0.4093	0.0399	21333.4	7267.3	7575.0	0.3551	0.0406
190	53028.3	20902.2	21766.4	0.4104	0.0397	24332.9	8295.1	8645.5	0.3553	0.0405
200	58466.2	23133.2	24054.4	0.4114	0.0395	27796.9	9483.8	9883.5	0.3556	0.0404
210	64401.3	25547.9	26554.2	0.4123	0.0394	31813.4	10861.4	11318.2	0.3558	0.0404
220	70861.2	28125.8	29275.4	0.4131	0.0393	36481.1	12462.4	12985.7	0.3560	0.0403
230	77864.9	30964.6	32226.3	0.4139	0.0391	41926.1	14330.3	14931.0	0.3561	0.0402
240	85411.4	34024.6	35406.9	0.4145	0.0390	48314.1	16522.0	17213.6	0.3563	0.0402
250	93455.5	37288.0	38798.8	0.4152	0.0389	55876.7	19117.9	19916.9	0.3564	0.0401

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POP = POPUPATION LEVEL
EMP = EMPLOYMENT LEVEL
LF = LABOR FORCE LEVEL
LFPR = LABOR FORCE PARTICIPATION RATE
UR = UNEMPLOYMENT RATE

Table 4. Migration-related variables.

T	Y1	O1	OUT1	Y2	O2	OUT2	DY	NET
1	0.2158	0.0182	136.5	0.3755	0.0518	129.4	0.1596	7.1
2	0.2220	0.0182	137.6	0.3580	0.0521	132.2	0.1359	5.4
3	0.2270	0.0182	138.9	0.3440	0.0523	134.8	0.1171	4.0
4	0.2311	0.0182	140.2	0.3327	0.0525	137.3	0.1016	2.9
5	0.2345	0.0182	141.6	0.3233	0.0526	139.6	0.0888	1.9
6	0.2374	0.0182	143.0	0.3154	0.0528	141.9	0.0779	1.1
7	0.2399	0.0182	144.4	0.3086	0.0529	144.2	0.0686	0.4
8	0.2421	0.0182	145.9	0.3027	0.0529	146.2	0.0606	-0.2
9	0.2441	0.0182	147.4	0.2977	0.0530	148.3	0.2536	-0.8
10	0.2458	0.0182	149.0	0.2932	0.0531	150.3	0.0474	-1.3
11	0.2473	0.0182	150.6	0.2894	0.0531	152.4	0.0420	-1.8
12	0.2487	0.0182	152.2	0.2859	0.0532	154.4	0.2372	-2.2
13	0.2500	0.0182	153.8	0.2829	0.0532	156.4	0.0329	-2.6
14	0.2511	0.0182	155.4	0.2802	0.0533	158.4	0.0291	-3.0
15	0.2521	0.0183	157.1	0.2778	0.0533	160.4	0.0256	-3.3
20	0.2561	0.0183	165.7	0.2691	0.0534	170.4	0.0130	-4.7
25	0.2587	0.0183	174.9	0.2641	0.0535	182.6	0.0053	-5.7
30	0.2606	0.0183	184.6	0.2614	0.0535	191.1	0.0008	-6.5
35	0.2619	0.0183	194.9	0.2602	0.0535	222.0	-0.0018	-7.2
40	0.2629	0.0184	205.7	0.2600	0.0535	213.4	-0.0029	-7.7
45	0.2636	0.0184	217.3	0.2605	0.0535	225.4	-0.0031	-8.1
50	0.2642	0.0184	229.5	0.2616	0.0535	238.0	-0.0026	-8.5
55	0.2645	0.0185	242.4	0.2630	0.0534	251.3	-0.0015	-8.9
60	0.2648	0.0165	256.2	0.2647	0.0534	265.3	-0.0001	-9.2
65	0.2650	0.0165	270.7	0.2667	0.0533	280.1	0.0018	-9.4
70	0.2651	0.0166	286.2	0.2689	0.0532	295.8	0.0038	-9.6
75	0.2651	0.0186	302.7	0.2712	0.0531	312.4	0.0061	-9.8
80	0.2651	0.0187	320.1	0.2737	0.0530	330.0	0.0086	-9.9
85	0.2650	0.0188	338.7	0.2762	0.0529	348.6	0.0112	-9.9
90	0.2648	0.0189	358.5	0.2788	0.0528	368.4	0.0140	-10.0
95	0.2647	0.0189	379.5	0.2815	0.0527	389.4	0.0168	-9.9
100	0.2644	0.0190	401.8	0.2842	0.0526	411.7	0.0197	-9.9
110	0.2639	0.0193	450.9	0.2896	0.0523	460.4	0.0257	-9.5
120	0.2632	0.0195	506.6	0.2951	0.0519	515.5	0.0319	-9.0
130	0.2624	0.0198	569.7	0.3004	0.0516	577.8	0.0380	-8.1
140	0.2615	0.0201	641.3	0.3057	0.0512	648.3	0.0442	-6.9
150	0.2604	0.0205	722.7	0.3108	0.0508	728.0	0.0504	-5.3
160	0.2593	0.0209	815.1	0.3158	0.0503	818.3	0.0565	-3.2
170	0.2581	0.0214	920.1	0.3207	0.0498	920.6	0.0626	-0.5
180	0.2566	0.0218	1039.4	0.3255	0.0492	1036.4	0.0688	3.0
190	0.2550	0.0224	1175.1	0.3303	0.0486	1167.6	0.0752	7.5
200	0.2532	0.0230	1329.5	0.3351	0.0480	1316.1	0.0820	13.3
210	0.2510	0.0236	1505.3	0.3402	0.0473	1484.2	0.0893	21.1
220	0.2482	0.0243	1705.8	0.3458	0.0465	1674.1	0.0976	31.6
230	0.2447	0.0251	1934.8	0.3522	0.0457	1888.5	0.1076	46.4
240	0.2398	0.0260	2197.6	0.3603	0.0447	2129.6	0.1205	68.0
250	0.2322	0.0270	2501.6	0.3716	0.0436	2399.0	0.1394	102.6

****

Y = REGIONAL INDEX
O = OUTMIGRATION RATE
OUT = OUTMIGRATION FLOW
DY = DIFFERENTIAL IN REGIONAL INDICES
RNET = NET MIGRATION FLOW FROM REGION ONE TO REGION 2

isolate the causal influences on net migratory flows; while migration is responding to economic conditions, it is also fostering many of them.

Yet, it is important to note that the model does generate oscillations in many of the important growth rates (such as output, employment and population). The same applies to the growth rates of the labor force participation rate which peaks in the first region between the fifth and the eleventh time periods while hitting lows in region two between 75 and 90, and again at the end of the simulation.

Table 3 shows that the actual labor force participation and unemployment rate levels for region two oscillate. Moreover, both regions' rates stay within ranges of values which are entirely reasonable and also consistent. Thus, although we see, from Table 2, that actual levels of population, employment and labor force increase regularly, labor force participation and unemployment rates do not take on implausible values.

Net migration oscillates too. Initially, there exists a net flow of migrants from the advanced to the developing region in which employment opportunities were better (higher employment growth, lower unemployment rate). But as employment opportunities worsen in the developing region, this flow tends to diminish leading to a reversal in the direction of the net flow of migrants between the two regions. But, toward the end of the simulation, the developing region regains a better position and the direction of the net migration flow is once more reversed.

To see how the direction of the net flow of migrants depends on the relative economic conditions of both regions, we can, from equation (9), formulate an expression for the net migratory flow from region 1 to region 2. Substituting (9) into the identity $RNET_t = m_{it}N_{it} - m_{jt}N_{jt}$ leads to

$$RNET_t = \frac{N_{it}N_{jt}}{N_{\cdot t}} \left[\alpha_i - \alpha_j + (\alpha_i\beta_i + \alpha_j\beta_j)\left(\frac{e_{jt}}{u_{jt}} - \frac{e_{it}}{u_{it}}\right)\right] \quad . \quad (9a)$$

Thus, there is a net flow of migrants from the advanced region to the developing region as long as the difference between the two regional indices appearing in (9a) remains higher than $\frac{\alpha_j - \alpha_i}{\alpha_i \beta_i - \alpha_j \beta_j}$, i.e., 0.064 (see the last two columns of Table 4 for a confirmation of the result). Yet, it must be recalled, that through its effect on regional population growth and through that effect on regional output growth (equation 1), we have a more complex situation than (9a) might imply. In fact, as we have seen, the oscillation of net migration is a response to, as well as a cause of, other fluctuations.

The main point suggested by this simulation is, then, that the two regions' growth rates are induced to also fluctuate, ruling out the possibility of evermore income divergence over the long run. Thus, the demoeconomic extension of the DT model has been the impetus for a non-linear approach which, in turn, has released us from the implausible inexorable income divergence of the DT model.

ADVANTAGES AND DISADVANTAGES OF THE DEMOECONOMIC APPROACH

In compiling a ledger on the demoeconomic approach, we note immediately that linearity and tractable reduced form results, as obtained by DT, are unlikely. On the benefit side, a more believable result is obtained. That is, we should not expect any two regions to settle on steady state growth rates over the long term and our demoeconomic model shows that this will not occur. We have seen that demoeconomics obviates much of the linearity of the DT model. This is so because steady state growth of employment and population could distort the labor force participation rate which is often defined in the model as a residual quantity . By forcing us to reconsider linearity or to respecify labor force participation, the demoeconomic approach aids in model building. As usual, we pay for an increment in realism by surrendering an amount of simplicity.

In addition, the inclusion of a transition matrix from interregional demography necessarily introduces difference equations; any demoeconomic model would have to be dynamic. This is surely a benefit as is the notion that, rather than taking migration rates as fixed, we make them endogenous. In fact, the model allows us to observe how migration rates and labor force participation rates interact with each other and with unemployment rates. This allows for a superior analysis of labor markets (it makes them spatial) and job search.

The model did not deal in terms of an age-sex specific breakdown of cohorts, and we did not model the effect that changes in the age compositon would have on the economic variables. That would be the obvious next step. The population does age inexorably and this momentum has well known economic consequences. In fact, the demoeconomic approach also has the potential for introducing age-sex detail into regional economics. Just as regional economists prize the sectoral detail of input-output model results, so ought they to value demographic detail. For example, such detail can give policy makers some idea of how formidable a task regional development or revival are likely to be in specific regions.

Finally, by the proper choice of regions, even the parameters of natural population growth can be made endogenous. What this means is that since the demographic transition seems to be a function of urbanization and since urbanization is endogenous in a demoeconomic model which happens to deal with an urban and rural region (or regions), the natural rate of increase could be made endogenous.

All of this appears to be an important break with the sort of regional modeling that has been done heretofore. We hope that the next few years will witness increasing interest in regional and interregional demoeconomics.

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Appendix 1. Annual growth rates of other economic variables.

T	X1	P1	R1	W1	X2	P2	R2	W2
1	0.0469	-0.0160	0.0425	0.0115	0.0371	-0.0114	0.0480	0.0216
2	0.0481	-0.0167	0.0430	0.0113	0.0359	-0.0113	0.0471	0.0209
3	0.0491	-0.0172	0.0434	0.0112	0.0349	-0.0111	0.0465	0.0203
4	0.0499	-0.0176	0.0437	0.0110	0.0341	-0.0110	0.0459	0.0199
5	0.0505	-0.0180	0.0439	0.0109	0.0335	-0.0110	0.0455	0.0195
6	0.0510	-0.0183	0.0441	0.0108	0.0330	-0.0109	0.0451	0.0192
7	0.0514	-0.0185	0.0442	0.0107	0.0326	-0.0109	0.0448	0.0189
8	0.0517	-0.0188	0.0444	0.0106	0.0323	-0.0109	0.0445	0.0186
9	0.0520	-0.0190	0.0445	0.0105	0.0320	-0.0109	0.0443	0.0184
10	0.0523	-0.0191	0.0446	0.0104	0.0317	-0.0110	0.0441	0.0182
11	0.0524	-0.0193	0.0447	0.0104	0.0316	-0.0110	0.0440	0.0180
12	0.0526	-0.0194	0.0447	0.0103	0.0314	-0.0110	0.0438	0.0178
13	0.0527	-0.0196	0.0448	0.0102	0.0313	-0.0111	0.0437	0.0177
14	0.0528	-0.0197	0.0448	0.0102	0.0312	-0.0111	0.0436	0.0175
15	0.0529	-0.0198	0.0449	0.0101	0.0311	-0.0112	0.0436	0.0174
20	0.0530	-0.0202	0.0449	0.0098	0.0310	-0.0115	0.0433	0.0169
25	0.0529	-0.0204	0.0449	0.0095	0.0311	-0.0118	0.0432	0.0165
30	0.0527	-0.0206	0.0449	0.0093	0.0313	-0.0121	0.0433	0.0162
35	0.0523	-0.0207	0.0448	0.0091	0.0317	-0.0124	0.0434	0.0159
40	0.0520	-0.0207	0.0447	0.0089	0.0320	-0.0128	0.0435	0.0157
45	0.0516	-0.0208	0.0445	0.0088	0.0324	-0.0131	0.0437	0.0156
50	0.0511	-0.0208	0.0444	0.0086	0.0329	-0.0134	0.0438	0.0155
55	0.0507	-0.0208	0.0443	0.0085	0.0333	-0.0137	0.0440	0.0153
60	0.0503	-0.0208	0.0441	0.0083	0.0337	-0.0139	0.0442	0.0153
65	0.0499	-0.0208	0.0440	0.0082	0.0341	-0.0142	0.0444	0.0152
70	0.0495	-0.0208	0.0439	0.0081	0.0345	-0.0145	0.0446	0.0151
75	0.0491	-0.0208	0.0438	0.0080	0.0349	-0.0147	0.0448	0.0151
80	0.0488	-0.0208	0.0436	0.0079	0.0352	-0.0149	0.0450	0.0150
85	0.0484	-0.0208	0.0435	0.0078	0.0356	-0.0152	0.0451	0.0150
90	0.0481	-0.0207	0.0434	0.0077	0.0359	-0.0154	0.0453	0.0149
95	0.0477	-0.0207	0.0433	0.0076	0.0363	-0.0156	0.0455	0.0149
100	0.0474	-0.0207	0.0432	0.0075	0.0366	-0.0158	0.0457	0.0149
110	0.0467	-0.0207	0.0430	0.0073	0.0373	-0.0162	0.0460	0.0149
120	0.0461	-0.0206	0.0427	0.0071	0.0379	-0.0165	0.0464	0.0148
130	0.0456	-0.0205	0.0425	0.0070	0.0384	-0.0168	0.0467	0.0148
140	0.0450	-0.0205	0.0424	0.0069	0.0390	-0.0172	0.0470	0.0148
150	0.0445	-0.0204	0.0422	0.0067	0.0395	-0.0174	0.0473	0.0148
160	0.0440	-0.0204	0.0420	0.0066	0.0400	-0.0177	0.0476	0.0149
170	0.0435	-0.0203	0.0418	0.0065	0.0405	-0.0180	0.0479	0.0149
180	0.0430	-0.0203	0.0416	0.0064	0.0410	-0.0183	0.0481	0.0149
190	0.0425	-0.0202	0.0415	0.0062	0.0415	-0.0185	0.0484	0.0149
200	0.0420	-0.0202	0.0413	0.0061	0.0420	-0.0188	0.0487	0.0149
210	0.0415	-0.0201	0.0411	0.0060	0.0425	-0.0191	0.0490	0.0149
220	0.0408	-0.0200	0.0408	0.0058	0.0432	-0.0195	0.0494	0.0149
230	0.0401	-0.0200	0.0406	0.0056	0.0439	-0.0199	0.0498	0.0149
240	0.0392	-0.0199	0.0402	0.0053	0.0448	-0.0204	0.0503	0.0149
250	0.0379	-0.0197	0.0397	0.0050	0.0461	-0.0212	0.0511	0.0149

****

X = EXPORT GROWTH RATE
P = PRICE GROWTH RATE
R = INNOVATION GROWTH RATE
W = WAGE GROWTH RATE

Appendix 2. Derivation of the solution of the model.

Combining equations (1) through (7) of the first block leads to:

$$\underset{\sim}{E}(e_t) = (h) + \underset{\sim}{F}(n_t) + \underset{\sim}{G}_t(l_t) \qquad \text{(A1)}$$

in which

$$\underset{\sim}{E} = [\underset{\sim}{I} + \underset{\sim}{\Gamma}\underset{\sim}{n}\underset{\sim}{\lambda}]\ \underset{\sim}{\mu}^{-1}$$

$$\underset{\sim}{F} = \underset{\sim}{\phi}^{1} + \underset{\sim}{\phi}^{2}$$

$$\underset{\sim}{G}_t = \underset{\sim}{\Gamma}\underset{\sim}{n}\underset{\sim}{D}(\underset{\sim}{\rho}_t - \rho^{l}\underset{\sim}{I})(\underset{\sim}{\rho}_t - \rho^{r}\underset{\sim}{I})^{-1} + \underset{\sim}{\phi}^{2}$$

$$(h) = \underset{\sim}{\Gamma}\underset{\sim}{n}[(\tau) + (\bar{w}) - (\bar{r}) + z\underset{\sim}{E}(i)] \quad .$$

In the second block [equations (8'') and (9'), by substituting (9') in (8''), we have

$$(n_t) = (k_t) + \underset{\sim}{J}_t(e_t) \qquad \text{(A2)}$$

in which

$$\underset{\sim}{J}_t = \frac{1}{N_{\cdot t}}\ \underset{\sim}{N}_t^{-1}\ \underset{\sim}{P}\underset{\sim}{N}_t\underset{\sim}{\alpha}\underset{\sim}{N}_t\underset{\sim}{\beta}\underset{\sim}{P}\underset{\sim}{u}_t^{-1}$$

$$(k_t) = (b) - \frac{1}{N_{\cdot t}}\ \underset{\sim}{N}_t^{-1}\ \underset{\sim}{P}\underset{\sim}{N}_t\underset{\sim}{\alpha}\underset{\sim}{N}_t(i) \quad .$$

Finally, the third block [equations (10), (11') and (12)] yields

$$(e_t) = (n_t) + \underset{\sim}{M}_t(l_t) \qquad \text{(A3)}$$

in which

$$\underset{\sim}{M}_t = \underset{\sim}{I} - [\underset{\sim}{U}_t - u^{r}\underset{\sim}{I})(\underset{\sim}{U}_t - u^{l}\underset{\sim}{I})^{-1}\underset{\sim}{A}^{-1} \quad .$$

Thus, our demoeconomic model reduces to a three-equation system in three unknowns such that the coefficients of the endogenous variables are known in each period: they are either constant (independent of time) or depend on lagged variables. Then, by combining (A1) through (A3), it is simple to obtain the three reduced form equations of the model concerning (e_t), (n_t) and

(l_t):

$$(e_t) = \left[\underset{\sim}{E} - \underset{\sim}{F}\underset{\sim}{J}_t - \underset{\sim}{G}_t\underset{\sim}{M}_t^{-1}(\underset{\sim}{I} - \underset{\sim}{J}_t)\right]^{-1} \left[(h) + \left(\underset{\sim}{F} - \underset{\sim}{G}_t\underset{\sim}{M}_t^{-1}(\underset{\sim}{I} - \underset{\sim}{J}_t)\right) (k_t)\right] \tag{A4}$$

$$(n_t) = \left[\underset{\sim}{E}\underset{\sim}{J}_t^{-1} - \underset{\sim}{F} + \underset{\sim}{G}_t\underset{\sim}{M}^{-1}(\underset{\sim}{I} - \underset{\sim}{J}_t^{-1})\right]^{-1}\left[(h) + \left(\underset{\sim}{E} - \underset{\sim}{G}_t\underset{\sim}{M}_t^{-1}\right) \underset{\sim}{J}_t^{-1} (k_t)\right] \tag{A5}$$

$$(l_t) = \underset{\sim}{M}_t^{-1} \left[(\underset{\sim}{E} - \underset{\sim}{F}\ \underset{\sim}{J}_t)(\underset{\sim}{I} - \underset{\sim}{J}_t)^{-1} - \underset{\sim}{G}_t\underset{\sim}{M}_t^{-1}\right]^{-1} \left[(h) + \left(\underset{\sim}{F} + \underset{\sim}{G}_t\underset{\sim}{M}_t^{-1}\underset{\sim}{J}_t - (\underset{\sim}{E} - \underset{\sim}{F}\underset{\sim}{J}_t)(\underset{\sim}{I} - \underset{\sim}{J}_t)^{-1}\right)(k_t)\right] \ . \tag{A6}$$

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1. Nathan Keyfitz, *Understanding World Models*, RM-77-18.

2. Andrei Rogers, *Migration, Urbanization, Resources, and Development*, RR-77-14.

3. Roman Kulikowski, *Optimization of Rural-Urban Development and Migration*, RM-77-41.

4. Frans Willekens, *Spatial Population Growth in Developing Countries: With a Special Emphasis on the Impact of Agriculture*, internal working paper, 1977, forthcoming as a Research Report.

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6. Allen Kelley and C. Swartz, *The Impact of Family Structure on Household Decision Making in Developing Countries: A Case Study in Urban Kenya*, internal IIASA working paper, WP-78-18, published in the Proceedings of the IUSSP Conference on *Economic and Demographic Change: Issues for the 1980's*, 1978.

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