Models for Educational and Manpower Planning:
A Dynamic Linear Programming Approach

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Preface

The Human Resources and Services Theme within the Human Settlements and Services Area is currently conducting research on health care systems and on nutrition. As part of a general exploratory evaluation to determine whether research on manpower should be accorded Research Task status within this Theme in the future, the Area held a small and informal task-force meeting in February, 1978, on which occasion this paper was presented.

The February task-force meeting led to the conclusion that the principal objective of manpower research in HSS should be the development of models and theoretical explanations of aspects of manpower supply, manpower demand, and manpower forecasting, with a focus on national and sectoral problems in both the more developed and the less developed countries of the world today. Expected results could be improved models and a better understanding of problems related to changing labor force composition, shortages of manpower in critical service sectors such as health, the rising cost of pensions, and the declining confidence of policy makers in the usefulness of manpower forecasting models.

This paper, the second of a series on manpower analysis, illustrates how the concepts of dynamic linear programming (DLP) can be applied to the normative modeling of manpower problems. In it, Dr. Propoi demonstrates the power and broad generality of the DLP approach while, at the same time, exposing certain weaknesses of the linear fixed-coefficient framework that call for further study.

Andrei Rogers
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Papers of the Manpower Study

1. Frans Willekens, *The Demography of Labor Force Participation*, RM-78-00.

2. Anatoli Propoi, *Models for Educational and Manpower Planning: A Dynamic Linear Programming Approach*, RM-78-00.

ABSTRACT

The purpose of this paper is to show that many optimization
problems for educational and manpower planning models can be written
in a standard dynamic linear programming form. A basic model
of educational planning is described and extensions of the model
(investment and vocational training submodels and a three level
educational model) are given.

When describing models, two basic models are singled out
using two different controls: recruitment in the first and pro-
motion in the second. Finally, an integrated model of economy-
manpower interaction is considered.

The possibilities and limitations of DLP as applied to man-
power and educational planning problems are discussed.

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Models for Educational and Manpower Planning:
A Dynamic Linear Programming Approach

1. INTRODUCTION

The most important features in manpower and educational
planning are, first, the large number of variables and conditions
which should be taken into account, and second, the dynamics of
the planning process -- training of specialists needs time. Ab-
solute or relative increases in the total number of manpower does
not necessarily imply an increase in the operational quality of
a system, and therefore, elaborate planning of different categories
of specialists is required. Manpower and educational planning
"has the basic purpose of producing the correct numbers of the
correct types of people in the correct jobs at the appropriate
times" (Grinold and Marshall, 1977).

The most efficient technique which can handle a large number
of variables and constraints is linear programming (LP) (Dantzig,
1963). In order to allow proper phasing of the decision process
over time, this technique can be extended to dynamic linear pro-
gramming (DLP) (Propoi, 1976). Another feature common to manpower
and educational models is their stochastic nature (Bartholomew,
1973). However, in many practical cases, dealing with expecta-
tions, we can stay within the deterministic approach and leave
the stochastic technique for the operational control stage. Thus,
it can be argued, that in order to solve real, large-scale opti-
mization problems in manpower and education, LP and its extension,
DLP, should be used.

There is some justification of the DLP approach to manpower
and educational optimal planning problems. The purpose of this
paper, however, is not to go further in this direction, but to
consider different optimization models in manpower and educational
planning using a unified approach.

This paper consists of three parts. In the first two parts, different types of models are described; first educational models and second, manpower planning models. These parts consider first a basic model and then some modifications and applications to this model. Finally, the third part discusses briefly an integrated manpower-economy model.

2. EDUCATIONAL PLANNING MODELS

First we consider educational planning models. Though sometimes it is difficult to make explicit differences between education and manpower planning models, we shall refer to the educational process as a process of training specialists as opposed to the promotion and recruitment processes in manpower planning models.

2.1 The Basic Model

In formulating DLP models, it is useful to consider separately (Propoi, 1976):

1. *State equations* of the system with the distinct separation of *state* and *control* variables;

2. *Constraints* imposed on these variables;

3. *Planning period* T -- the number of steps during which the system is considered and the *length* of each step t;

4. *Performance index* (objective function) which quantifies the quality of a plan (control).

State equations: The general scheme of the basic model considered is presented in Figure 1.

Let

$x_i(t)$ be the number of specialists of type i (i = 1,...,n) (speciality, grade, etc.) at step t (each step equals one year, three years, six years, etc.); and

$u_k(t)$ be the number of entrants to the educational
system of type k $(k = 1,\ldots,r)$ (schools, insti-
tutes, faculties, etc.).

It is assumed that τ_k steps are needed for graduating from
the educational system of type k.

The vector $x(t) = \{x_i(t)\}$ represents the distribution of spe-
cialists at step t (manpower stock) and vector $u(t) = \{u_k(t)\}$
represents the distribution of new enrollments at step t over
different types of educational systems. Vector $x(t)$ is the state
of the system and vector $u(t)$ is the control variable.

The state equations describing the development of the man-
power system can be written as follows:

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{r} b_{ik}(t - \tau_k)u_k(t - \tau_k)$$

$$(i = 1,\ldots,n; \quad t = 0,1,\ldots,T-1) \qquad (1)$$

or in matrix form

$$x(t+1) = A(t)x(t) + B(t-\tau)u(t-\tau) \quad, \qquad (2)$$

where the notation $\tau = \{\tau_k\}$ is used.

In the state equations (1):

$a_{ij}(t)$ is the coefficient which shows how many specialists
of type j progress to group i between steps t and
$t+1$ $(i,j = 1,\ldots,n; \quad t = 0,1,\ldots,T-1)$. In many cases

$$a_{ij}(t) = \begin{cases} 1 - \tilde{a}_{ii}(t) & , \quad i = j \\ \\ 0 & , \quad i \neq j \quad ; \end{cases}$$

where $1 - \tilde{a}_{ii}(t)$ is called the manpower stock attrition
rate, $A(t)$ becomes a diagonal matrix.

$b_{ik}(t-\tau_k)$ is the coefficient which shows how many entrants of type k at step $t - \tau_k$ will obtain the speciality i at step t ($k = 1,\ldots,r$; $i = 1,\ldots,n$; $t = 0,1,\ldots,T-1$, $\tau_k < T$). These coefficients denote the ratio of graduates of type i to the total number of students enrolled in the type k educational system.

It is assumed that for the state equations (1) the initial conditions are

$$\begin{aligned} x_i(0) &= x_i^0 \\ u_k(t - \tau_k) &= u_k^0(t - \tau_k) \end{aligned} \tag{3}$$

where x_i^0 ($i = 1,\ldots,n$), $u_k^0(t - \tau_k)$ ($k = 1,\ldots,r$; $0 \le t \le \tau_k - 1$) are given numbers.

If the length of each step t is equal to the maximum duration of training in the educational system, then the state equations (1) with time delay τ will be transformed into state equations without time delays:

$$x(t + 1) = A(t)x(t) + B(t)u(t) \quad , \tag{4}$$

where the length of each step is equal to the duration of train-ing (for example, five years). Matrices A(t) and B(t) should be recalculated in this case.

For many practical cases, the matrices A(t) and B(t) are constant:

$$x(t + 1) = Ax(t) + Bu(t - \tau)$$

and frequently in (4) B(t) is an identity matrix:

$$x(t + 1) = Ax(t) + u(t) \quad . \tag{5}$$

In this case, the vector u(t) may be interpreted as the in-crease in the manpower stock during time period t.

In some cases it is necessary to take into account the flows of specialists into and out of the system. Then the state equations (2) are transformed into the following:

$$x(t + 1) = A(t)x(t) + B(t - \tau)u(t - \tau) + s^+(t) - s^-(t) \quad ,$$

where the vectors $s^+(t)$ and $s^-(t)$ can be considered either as given exogenous variables or as additional control variables.

Definitions: The sequence of vectors $u = \{u(0), \ldots, u(T-1-\tau)\}$ denotes the *control* of the system (2) (or the *enrollment plan* for a given planning horizon T). The sequence of vectors $x = \{x(0), \ldots, x(T)\}$ is the *trajectory* of the system (or the *manpower plan*).

Choosing different enrollment controls u, we can define with the state equations (1)-(3) the corresponding manpower trajectory x. The problem is to find such enrollments over time as will satisfy all the constraints of the system and be optimal in some sense. Thus the second stage of the DLP model building is to delineate the constraints on the variables.

Constraints: Basically constraints on the variables may be broken down into three types: physical, resource and goal.

Physical Constraints   It is evident that the number of people cannot be negative:

$$x_i(t) \geq 0 \qquad (i = 1, \ldots, n) \quad , \tag{6}$$

$$u_k(t) \geq 0 \qquad (k = 1, \ldots, r) \quad . \tag{7}$$

In the physical sense, the variables $x_i(t)$ and $x_k(T)$ are integers. As the number of people in the system is usually large, for practical reasons this restriction may not be taken into account. (The running of the model in an integer programming form is rather costly).

Resource Constraints   These constraints can be written as

$$\sum_{k=1}^{r} d_{sk}(t) u_k(t) \leq f_s(t) \qquad (s = 1,\ldots,m) \qquad (8)$$

where $f(t) = \{f_1(t),\ldots,f_m(t)\}$ is the vector of given resources
(educational facilities) for training (teachers, buildings, equip-
ment, etc.); the coefficients $d_{sk}(t)$ $(s = 1,\ldots,m;\ k = 1,\ldots,r)$
show the amount of resources of type s needed per unit for
education of type k at step t.   In matrix form

$$D(t) u(t) \leq f(t) \quad . \qquad (9)$$

At times, it is more convenient to evaluate the required re-
sources by the total number of students of each type $k(k = 1,\ldots,r)$
at current time period t.   In this case, the constraints (8)
are replaced by

$$\sum_{k=1}^{r} \sum_{\tau=0}^{\tau_k-1} d_{sk}(t,\tau) u_k(t - \tau) \leq f_s(t) \quad . \qquad (10)$$

In many cases it is necessary to single out the constraint on
the availability of teachers or instructors of different types.

Let the $y_j(t)$ $(j = 1,\ldots,J)$ be the number of available teachers
of type j at step t and $g_{jk}(t)$ be the ratio of required teachers
of type j to students enrolled in the educational system of type k.
Then the constraints on the teachers' availability can be written
in a similar (8) form:

$$\sum_{k=1}^{r} g_{jk}(t) u_k(t) \leq y_j(t) \qquad (j = 1,\ldots,J) \quad . \qquad (11)$$

Usually, the teachers (or some group of the teachers) constitutes
the part of the manpower stock.   In this case

$$y_j(t) = \sum_i h_{ji}(t) x_i(t)$$

where $h_{ji}(t) = 1$, if the i-th type specialists are full-time teachers and $0 \leq h_{ji}(t) \leq 1$ are part-time teachers. Then the constraints (11) should be rewritten as

$$\sum_{k=1}^{r} g_{jk}(t)u_k(t) \leq \sum_{j=1}^{J} h_{ij}(t)x_i(t) \tag{12}$$

or in matrix form

$$G(t)u(t) \leq H(t)x(t) \quad . \tag{13}$$

In the simple models, the numbers $f_s(t)$ $(s = 1,...,m)$ and $y_j(t)$ $(j = 1,...,J)$ are the given exogenous variables. In more detailed models these variables are considered as state variables which are governed by some controllable activities in training teachers and building other educational facilities (see Sections 2.2 and 2.4).

Goal Constraints Usually goals for the control of a system are associated with the value of an objective function. However, at times only some of them are introduced into the objective function. The others are considered as additional constraints on the system. For example, the numbers of specialists of some types $i \in I_1 \subseteq U = \{1,...,n\}$ must be kept at given levels:

$$x_i(t) = \bar{x}_i(t) \qquad i \in I_1 \subset I \quad , \tag{14}$$
or
$$x_i(t) \geq \bar{x}_i(t) \qquad i \in I_2 \subset I \quad . \tag{15}$$

In some cases, one of the goals may be to bring the system to the desired distribution of specialists at the end of planning period

$$x(T) = x_T \quad , \tag{16}$$

where x_T is a given vector (terminal conditions).

To satisfy constraints (14) in explicit form is rather costly, if not infeasible. Therefore, instead of (14), it is more reasonable to consider this type of constraint in the form

$$x_i(t) + \xi_i(t) - \eta_i(t) = \bar{x}_i(t) \quad i \in I_1 \subseteq I \tag{17}$$

where the variables $\xi_i(t) \geq 0$, $\eta_i(t) \geq 0$ denote respectively the shortage and surplus of specialists of type i in time period t and $\xi_i(t)$ and $\eta_i(t)$ are additional control variables which are introduced to the objective function. The approach is usual in goal programming technique (Charnes and Cooper, 1961).

The general form of constraints on the system's variables can be given in the form (Propoi, 1976):

$$G(t)x(t) + D(t)u(t) \leq f(t)$$
$$x(t) \geq 0, \quad u(t) \geq 0 \tag{18}$$

where $f(t)$ is the given m-vector, and $G(t)$ and $D(t)$ are the given matrices with dimension $(m \times n)$ and $(m \times r)$ respectively.

Objective Function: The ultimate goal of a manpower supply model is to meet the projected demand in manpower, thus increasing the quality of the system. In the models considered, the projected figures of required specialists of all types i are supposed to be known for each step t of the planning period T, that is, the numbers $\bar{x}_i(t)$ are given for each $i = 1, \ldots, n$ and $t = 1, \ldots, T$. The goal of control of the system is to bring the manpower stock trajectory $\{x_i(t)\}$ as close as possible, under given dynamic (1), (3) and static (6)-(18) constraints, to the desired distribution of specialists $\{\bar{x}_i(t)\}$. This closeness can be evaluated by the objective functions

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i(t) \left| x_i(t) - \bar{x}_i(t) \right| . \tag{19}$$

The $\alpha_i(t)$ are the given weight coefficients, $|x|$ is the absolute value of x. In more general form,

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \phi_i^t(x_i(t) - \bar{x}_i(t)) \quad , \tag{20}$$

where

$$\phi_i^t(x_i) = \begin{cases} \alpha_i^1(t) x_i & , \quad \text{if } x_i \geq 0 \\ \alpha_i^2(t) x_i & , \quad \text{if } x_i \leq 0 \end{cases} .$$

If $\alpha_i^1(t) = -\alpha_i^2(t)$ for all i and t, then the objective function (20) reduces to (19).

Problems with the objective functions (19) and (20) can easily be reduced to a linear case (Charnes and Cooper, 1961). For this, the objective function is introduced

$$J = \sum_{t=0}^{T-1} \sum_{i=1}^{n} [\alpha_i^1(t) \xi_i(t) + \alpha_i^2(t) \eta_i(t)] \tag{21}$$

with additional constraints (17).

It is evident that both $\xi_i(t)$ and $\eta_i(t)$ cannot be positive at the same time. If the shortage of specialists is not desirable:

$$x_i(t) \geq \bar{x}_i(t) \qquad (i = 1, \ldots, n; \ t = 0, 1, \ldots, T) \quad ,$$

then the objective function (19) can be directly written in linear form:

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i(t) [x_i(t) - \bar{x}_i(t)] \quad , \tag{22}$$

with additional constraints $x_i(t) - \bar{x}_i(t) \geq 0$.

The other group of objective functions is associated with the minimization of expenditures for education. If $\beta_k(t)$ is the cost of training per student of speciality k at year t, then the total expenditure for education will be the following:

$$J(u) = \sum_{t=0}^{T-1-\tau} \sum_{k=1}^{r} \beta_k(t) u_k(t) \quad . \tag{23}$$

Finally, if it is necessary to develop a special program for training the greatest feasible number of specialists of the given group $I_1 \subset I$ by the end of this program, then the problem can be a maximization of the objective function

$$J(u) = \sum_{i \in I_1} \alpha_i(T) x_i(T) \quad , \tag{24}$$

where $\alpha_i(T)$, $i \in I_1$ are weight coefficients for the eligible specialities.

Summarizing, we can state the problem for the considered model as follows.

Problem 2.1: Given the initial conditions

$$x_i(0) = x_i^0 \qquad (i = 1,\ldots,n)$$
$$u_k(t - \tau_k) = u_k^0(t - \tau_k) \qquad (k = 1,\ldots,r; \; 0 \le t \le \tau_k - 1)$$

and the state equations

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t) x_j(t) + \sum_{k=1}^{r} b_{ik}(t - \tau_k) u_k(t - \tau_k)$$

$$(i = 1,\ldots,n; \; t = 0,1,\ldots,T-1)$$

with constraints

$$\sum_{k=1}^{r} d_{sk}(t) u_k(t) \le f_s(t)$$

$$(s = 1,\ldots,m; \; t = 0,1,\ldots,T-1-\tau_k)$$

$$x_i(t) \ge 0 \qquad (i = 1,\ldots,n; \; t = 1,\ldots,T)$$

$$u_k(t) \ge 0 \quad , \quad (k = 1,\ldots,r; t = 0,1,\ldots,T-\tau_k-1)$$

find a control

$$u = \{u_k(0), u_k(1), \ldots, u_k(T-1-\tau_k) \quad (k = 1, \ldots, r)\}$$

and a corresponding trajectory

$$x = \{x_i(0), x_i(1), \ldots, x_i(T)\} \quad (i = 1, \ldots, n)$$

which minimize the performance index

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i(t) |x_i(t) - \bar{x}_i(t)| \quad .$$

This basic model is flexible enough and allows various modifications and extensions. But even in this simple form, the model is useful in practice, as it takes into account in some optimal way the main features of manpower planning models: the dynamic of training specialists and the limits of available resources (see Section 2.5).

It should be noted that the model considered above can be interpreted either on a national/regional (macro) level or on an institutional planning (micro) level. Below, some examples and extensions of this basic model will be considered.

2.2 Investment Submodel

In the model considered above, the values of training facilities (buildings, equipments, etc.) were supposed to be given beforehand, that is, the variables $f_s(t)$ in (8) were considered to be exogenous. In many practical cases, it is preferable to incorporate into the manpower model the subsystem for planning the development of training facilities (e.g. construction of buildings).

Let $z(t) = \{z_n(t)\}$ be the vector of available training facilities at step t ($n = 1, \ldots, N$). The vectors $x(t)$ may conincide with the vectors $f(t)$ or be connected with them by some linear transformation.

There are M options (activities) in increasing training facilities. Let

$v_m(t)$ $(m = 1, \ldots, M)$ be the funds allocated to the m-th activity at step t; and

$q_{nm}(t)$ be the increase of the n-th facility per unit of m-th activity at step t $(n = 1, \ldots, N; \ m = 1, \ldots, M)$.

Then the state equations which describe the development of the training facilities will be the following:

$$z_n(t + 1) = (1 - \delta_n(t))z_n(t) + \sum_{m=1}^{M} q_{nm}(t)v_m(t - \tau_m) \qquad (25)$$

$$(n = 1, \ldots, N; \ t = 0, 1, \ldots, T - \tau_m - 1)$$

where $1 - \delta_n(t)$ is the depreciation rate for the n-th facility, and τ_m is the time lag for investments in the m-th activity.

In matrix form equations (25) can be rewritten as

$$z(t + 1) = (I - \Delta(t))z(t) + Q(t)v(t - \tau) \quad . \qquad (26)$$

The initial conditions

$$z_n(0) = z_n^0 \qquad (n = 1, \ldots, N) \qquad (27)$$

and

$$v_m(t - \tau_m) = v_m^0(t - \tau_m) \quad (m = 1, \ldots, M; \ 0 \leq t \leq \tau_m - 1) \qquad (28)$$

are supposed to be known.

Clearly,

$$z_n(t) \geq 0 \quad , \qquad v_m(t) \geq 0 \quad . \qquad (29)$$

The budget constraints can be given either for each step t:

$$\sum_{m=1}^{M} v_m(t) \leq c(t) \qquad (30)$$

or for the total planning period:

$$\sum_{m=1}^{M} \sum_{t=0}^{T-\tau_m-1} v_m(t) \leq C \quad , \tag{31}$$

where C is the given budget for the whole planning period.

In the considered case, $v_m(t)$ are additional control variables, and constraints (8) should be replaced by the constraints

$$\sum_{k=1}^{r} d_{nk}(t) u_k(t) \leq z_n(t) \qquad (n = 1, \ldots, N) \tag{32}$$

where $z(t) = \{z_n(t)\}$ is the additional state vector of the system.

The development of capital stock for educational systems can be described in a slightly different way:

$$z(t+1) = (I - \Delta(t)) z(t) + v(t) \tag{33}$$

where vector $v(t) = \{v_n(t)\}$ denotes the total increase of capital stock vector $z(t) = \{z_n(t)\}$ in time period t. The constraints on vectors $v(t)$ are written in the form

$$R(t) v(t) \leq r(t); \quad v(t) \geq 0 \tag{34}$$

where matrix $R(t)$ specifies the resource requirements for educational capital stock development, and $r(t)$ is the vector of exogenously given resources.

2.3 Vocational Training Submodel

Systems for vocational training or for improving professional skill play an important role in educational systems. As in many manpower systems, it is desirable to take into account the continuing process in education of the specialists.

The general scheme of professional skill improvement is presented in Figure 2.

Let all manpower be broken down into n different grades
(groups). The transition of a specialist from one group to another
depends on whether this specialist enters courses for improving
his qualification or not.

Let

$x_i(t)$ be the total number of specialists of grade (group)
i (i = 1,...,n) at the beginning of step t;

$u_i^k(t)$ be the number of specialists of grade i who at
step t enter courses for improving qualifications
of type k (k = 1,...,r);

$A(t) = a_{ij}(t)$ be the transition matrix for specialists
who do not enter any courses for improving
qualifications at step t,

$B^k(t) = \{b_{ij}^k(t)\}$ be the transition matrix for specialists
who enter courses at step t, τ_k being the train-
ing time for courses of type k, and

$v_i(t)$ be the new entrants to the type i manpower stock
at step t.

Then the equations which describe the dynamic of the dis-
triubtion of specialists over different groups will be the
following:

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)\left[x_j(t) - \sum_{k=1}^{r} u_j^k(t)\right] + \sum_{j=1}^{n}\sum_{k=1}^{r} b_{ij}^k(t-\tau_k)u_j^k(t-\tau_k) + v_i(t) \tag{35}$$

or in matrix form

$$x(t+1) = A(t)\left[x(t) - \sum_{k=1}^{r} u^k(t)\right] + \sum_{k=1}^{r} B^k(t-\tau_k)u^k(t-\tau_k) + v(t) . \tag{36}$$

Equation (36) can also be written in the form

$$x(t+1) = A(t)x(t) + \sum_{k=1}^{r}\left[B^k(t-\tau_k)u^k(t-\tau_k) - A(t)u^k(t)\right] + v(t) .$$

Here $x(t)$ is the vector of state variables and $u^k(t)$ are the vectors of control variables.

A simpler version of the model is ($r = 1$, $\tau_k = 0$):

$$x_i(t+1) = \sum_{j=1}^{n} \left[a_{ij}(t) \, x_j(t) - u_j(t) \right] + \sum_{j=1}^{n} b_{ij}(t) u_j(t) \quad . \quad (37)$$

The initial conditions for (35):

$$x_i(0) = x_i^0 \quad ,$$
$$u_j^k(t - \tau_k) = u_j^k(t - \tau_k) \quad .$$

The constraints can be written in the usual form:

$$\sum_{k=1}^{r} \sum_{j=1}^{n} d_{sj}^k(t) u_j^k(t) \leq f_s(t)$$

where $\{f_s(t)\}$ is the vector of given resources [see (8)] and

$$x_j(t) - \sum_{k=1}^{r} u_j^k(t) \geq 0$$

$$x_j(t) \geq 0; \quad u_j^k(t) \geq 0 \quad .$$

With this model, different objective functions can be considered. One is to meet given demand as closely as possible (22). Other objective functions of practical interest are, for example, to produce as many specialists as possible (24) (under given resources and other limitations), or to minimize total expenditure for education (23) with given requirements for manpower supply (14).

2.4 Three-Level Educational Planning Model

In this section, we consider the three-level educational planning model, which incorporates three subsystems of specialist training. This model is an optimization version of the Tinbergen model (Correa and Tinbergen, 1962).

The three levels in education are primary, secondary and higher education (Fig. 3); or, in a health care system (Propoi, (1978) nurses who graduate from medical schools, practical physicians who graduate from medical institutes, and medical specialists of high level who are trained in special professional courses (for example, postgraduates); or, in an industry, technicians, engineers and researchers (teachers).

Some of the second-level specialists can be teachers for the first-level educational subsystem, and all the third-level specialists are supposed to be instructors either for the second level or for the third level educational subsystem.

We now consider the subsystems separately.

First Level: Let

$x_i^1(t)$ be the number of specialists of the first level type i ($i \in I^1$) at time period t (the first level manpower stock);

$u_k^1(t)$ be the number of entrants to the first level educational system (schools) of type k ($k \in K^1$) at time period t;

$a_i^1(t)$ be the proportion of specialists of the first level type i ($i \in I^1$) who leave the stock of the first level labor force during time t because of death or retirement (the first level manpower stock attrition rate),

$b_{ik}^1(t)$ be the ratio of graduated specialists of the first level type i to the total number of students enrolled in the first level schools of type k at time t (the first level graduating ratio),

$u_{ki}^{21}(t)$ be the number or entrants to the second level educational system (institutes) of type k ($k \in K^2$) from the first educational level of type i ($i \in I^1$) at time period t,

τ_k^1 be the training period for the first educational level of type k $(k \in K^1)$.

Then the state equations for the first educational level will be as follows $(i \in I^1)$:

$$x_i^1(t+1) = (1 - a_i^1(t))x_i^1(t) + \sum_{k \in K^1} b_{ik}^1(t-\tau_k^1)u_k^1(t-\tau_k^1) - \sum_{k \in K^2} u_{ki}^{21}(t). \quad (38)$$

We have the usual conditions and constraints on variables $x_i^1(t)$, $u_k^1(t)$ (see Section 2.1):

$$x_i^1(0) = \bar{x}_i^1 \qquad (i \in I^1)$$
$$u_k^1(t - \tau_k^1) = \bar{u}_k^1(t - \tau_k^1) \qquad (0 \leq t \leq \tau_k^1 - 1; \ k \in K^1) \qquad (39)$$

and

$$x_i^1(t) \geq 0 \qquad (t = 1,\ldots,T; \ i \in I^1)$$

$$\qquad (40)$$

$$u_k^1(t) \geq 0 \qquad (t = 0,\ldots,T - \tau_k^1 - 1; \ k \in K^2)$$

$$\sum_{k \in K^1} d_{sk}^1(t)u_k^1(t) \leq f_s^1(t) \qquad (s = 1,\ldots,S^1) \qquad (41)$$

and in addition,

$$\sum_{k \in K^2} u_{ki}^{21}(t) \leq \alpha_i^1(t)x_i^1(t) \quad , \qquad u_{ki}^{21}(t) \geq 0 \quad (i \in I^1) \qquad (42)$$

where coefficients $\alpha_i^1(t)$ specify the availability of the first level specialists for further education at the second level.

In (41), $\{f_s^1(t)\}$ is the vector of given resources (facilities) for the first level educational subsystem (excluding teachers), and $d_{sk}^1(t)$ is the amount of resource of type s which is needed for the education of each specialist of type k $(k \in K^1)$ at time t.

The given resources vector $\{f_s^1(t)\}$ does not now include the number of teachers available for the first level educational system. In the three level model under consideration, these constraints can be treated separately.

It is supposed that some of the second level specialists are teachers for the first level educational system. The set of all such specialities is denoted by I_1^2. The set I^2 of all second level specialities contains the teachers set I_1^2: $I^2 \supset I_1^2$. Let also $x_{i_1}^2(t)$ be the number of teachers available in speciality $i_1 \in I_1^2 \subset I^2$ at time t.

Generally speaking, a teacher spares only part of his working time for teaching; the other part may be left for practicing.

Let

$\delta_{i_1}^2(t)$ be the ratio of teaching time to total working time for the second level specialists of type $i_1 \in I_1^2$ $(0 \leq \delta_{i_1}^2(t) \leq 1)$ and

$g_{i_1 k}^{21}(t)$ be the ratio of required teachers of type i_1 to students enrolled in the first educational level of type k $(i_1 \in I_1^2; k \in K^1)$.

Then the requirements for teachers of specialities of type $i_1 \in I_1^2$, which are necessary for training the first level students, can be written in the form

$$\sum_{k \in K^1} g_{i_1 k}^{21}(t) u_k^1(t) \leq \delta_{i_1}^2(t) x_{i_1}^2(t) \quad (i_1 \in I_1^2 \subset I^2) \quad . \tag{43}$$

Second level: Let

$x_i^2(t)$ be the number of second level specialists of type i $(i \in I^2)$ at time t (the second level manpower stock);

$u_k^2(t)$ be the number of entrants to the institutes of type k $(k \in K^2)$;

$a_i^2(t)$ be the proportion of second level specialists of type i $(i \in I^2)$ who leave the stock of the second level specialists during time period t (the second level manpower stock attrition rate);

$b_{ik}^2(t)$ be the ratio of second level graduates of type i ($i \in I^2$) to the total number of students enrolled in the institutes of type k ($k \in K^2$) at time t;

$u_{ki}^{32}(t)$ be the number of second level specialists of type i who enter the third level educational subsystem at time t ($i \in I^2$, $k \in K^3$);

τ_k^2 be the training time for the second educational level of type k ($k \in K^2$).

Then the state equations for the second educational level subsystem will be the following ($i \in I^2$):

$$x_i^2(t+1) = (1 - a_i^2(t))x_i^2(t) \tag{44}$$

$$+ \sum_{k \in K^2} b_{ik}^2(t - \tau_k^2)\left[u_k^2(t - \tau_k^2) + \sum_{i \in I^1} u_{ki}^{21}(t - \tau_k^2)\right] - \sum_{k \in K^3} u_{ki}^{32}(t) \quad .$$

Equations (44) are valid if the teachers (for the first educational level, $i \in I_1^2$) divide their working time between practicing and teaching. In this case, the real manpower stock should be introduced by:

$$\tilde{x}_i(t) = \begin{cases} x_i(t) \quad , & \text{if } i \notin I_1^2 \quad , \\ \\ [1 - \delta_i^2(t)]x_i(t) \quad , & \text{if } i \in I_1^2 \quad . \end{cases} \tag{45}$$

If the teachers do not practice and thus really leave the second level manpower stock, then the right side of the equation (44) should be reduced by the terms $\delta_i^2(t)x_i(t)$ for $i \in I_1^2$.

The initial conditions

$$x_i^2(0) = \bar{x}_i^2 \qquad (i \in I^2)$$

$$u_k^2(t - \tau_k^2) = \bar{u}_k^2(t - \tau_k^2) \quad (0 \leq t \leq \tau_k^2 - 1; \; k \in K^2) \quad , \tag{46}$$

and constraints

$$x_i^2(t) \geq 0 \quad (i \in I^2; \ t = 1, \ldots, T)$$

$$u_k^2(t) \geq 0 \quad (k \in K^2; \ t = 0, \ldots, T - \tau_k^2 - 1)$$

(47)

$$\sum_{k \in K^2} d_{sk}^2(t) u_k^2(t) \leq f_s^2(t) \quad (s = 1, \ldots, S^2)$$

(48)

$$\sum_{k \in K^3} u_{ki}^{32}(t) \leq \alpha_i^2(t) x_i^2(t); \ u_{ki}^{32}(t) \geq 0 \quad (i \in I^2)$$

(49)

have similar form to those for the first level subsystem. (Constraints in teaching facilities of the second level subsystem will be considered later).

Third level: The third level subsystem is the highest educational level in the considered model. Let

$x_i^3(t)$ be the number of third level specialists of type i ($i \in I^3$) at time t;

$a_i^3(t)$ be the proportion of third level specialists of type i ($i \in I^3$) who leave the stock of the third level labor force during time period t because of death or retirement (the third level manpower stock attrition rate);

$b_{ik}^3(t)$ be the ratio of third level graduates of type i ($i \in I^3$) to the total number of second level specialists enrolled in the third level educational subsystem of type k ($k \in K^3$) at time t;

τ_k^3 be the training time for third level specialists of type $k \in K^3$.

Then the state equations for the third level educational subsystem will be the following ($i \in I^3$):

$$x_i^3(t+1) = (1 - a_i^3(t)) x_i^3(t) + \sum_{k \in K^3} b_{ik}^3(t - \tau_k^3) \sum_{j \in I^2} u_{kj}^{32}(t - \tau_k^3)$$

(50)

It is supposed that there is not any enrollment to the third level educational subsystem outside of the system.

The initial conditions for the third level subsystem are:

$$x_i^3(0) = \bar{x}_i^3 \qquad (i \in I^3)$$

$$u_{kj}^{32}(t - \tau_k^3) = \bar{u}_{kj}^{32}(t - \tau_k^3) \qquad (0 \le t \le \tau_k^3 - 1; \; k \in K^3, \; j \in I^2) \quad . \tag{51}$$

Each specialist at the third level is supposed to be a teacher either for the second level or for the third level educational subsystem. Let

$\delta_i^3(t)$ be the ratio of teaching time to total working time for the third level specialists of type i $(i \in I^3)$;

$g_{ik}^{32}(t)$ be the ratio of required third level teachers of type i $(i \in I^3)$ to students enrolled in the second educational level of type k $(k \in K^2)$ at time t;

$g_{ik}^3(t)$ be the ratio of third level teachers of type i $(i \in I^3)$ required for training the second level specialists enrolled in the third educational level of type k $(k \in K^3)$ at time t.

Then the requirements for teachers for both the second and the third level educational subsystems can be written in the form $(i \in I^3)$

$$\sum_{k \in K^2} g_{ik}^{32}(t) u_k^2(t) + \sum_{k \in K^3} g_{ik}^3(t) \sum_{j \in I^2} u_{kj}^{32}(t) \le \delta_i^3(t) x_i^3(t) \quad . \tag{52}$$

The real manpower stock of third level specialists will be defined by the expression

$$\tilde{x}_i^3(t) = [1 - \delta_i^3(t)] x_i^3(t) \quad , \qquad (i \in I^3) \quad . \tag{53}$$

Other constraints for the third level subsystem are written in the ordinary form:

$$x_i^3(t) \geq 0 \qquad (i \in I^3)$$
$$u_{kj}^{32}(t) \geq 0 \qquad (j \in I^2; \ k \in K^3; \ t = 0,1,\ldots,T - \tau_k^3 - 1) \qquad (54)$$

$$\sum_{k \in K^3} d_{sk}^3(t) \sum_{j \in I^2} u_{kj}^{32}(t) \leq f_s^3(t) \quad (s = 1,\ldots,S^3) \qquad (55)$$

where $f^3(t) = \{f_s^3(t)\}$ is the vector of given resources (facilities) for the third level educational subsystem (excluding teachers) and $d_{sk}^3(t)$ is the amount of the resource of type s, which is needed for training one specialist of type k at time t.

Objective function: Considering the state equations and constraints for these three levels, one can see that if the number of steps T for state variables $\{x^1(t), x^2(t), x^3(t)\}$ is fixed, then the duration of control sequences will be different for controls of each subsystem (see Figure 4 where shading denotes the given a priori values). There are several ways to treat this situation. For example, one can choose large T and consider all the variables only for the period which is equal to $\min \{T - 1 - \tau_k^1, \ T - 1 - \tau_k^2, \ T - 1 - \tau_k^3\}$, or assume that the number of steps for all state variables is the same and is equal to T (Propoi, 1978).

We shall now formulate the performance index for this model, which will quantify the quality of a chosen plan of enrollment for all three educational subsystem levels.

It is supposed that the projected figures of demand for each level of specialists are available for all time periods t of planning period T, that is, the numbers

$$X_{i_1}^1(t), \ X_{i_2}^2(t), \ X_{i_3}^3(t) \quad (i_1 \in I^1; \ i_2 \in I^2; \ i_3 \in I^3) \qquad (56)$$

are given for each $t = 1,\ldots,T$.

The objective in planning the three-level model under consideration is to determine a plan for enrollment to all three educational subsystems

$$u^1_{k_1}(t), \; u^{21}_{k_2 i_1}(t), \; u^2_{k_2}(t), \; u^{32}_{k_3 i_2}(t) \quad (k_1 \in K^1; \; k_2 \in K^2; \; i_1 \in I^1; k_3 \in K^3; \; i_2 \in I^2)$$

for each $t = 0, 1, \ldots, T^\nu - 1$ where T^ν is different for each ν-th ($\nu = 1,2,3$) subsystem (Figure 4), which satisfies all the dynamic and static constraints of the system and yields the manpower stock [see (45) and (53)]:

$$x^1_{i_1}(t), \; \tilde{x}^2_{i_2}(t), \; \tilde{x}^3_{i_3}(t) \quad (i_1 \in I^1; \; i_2 \in I^2; \; i_3 \in I^3)$$

as close to demand (56) as possible for the whole planning horizon $t = 1, \ldots, T$. Thus the objective function can be written as

$$
\begin{aligned}
J = \; & \sum_{t=1}^{T} \; \sum_{i \in I^1} \; \alpha^1_i(t) \, |x^1_i(t) - X^1_i(t)| \\[2mm]
& + \sum_{t=1}^{T} \; \sum_{i \in I^2} \; \alpha^2_i(t) \, |\tilde{x}^2_i(t) - X^2_i(t)| \\[2mm]
& + \sum_{t=1}^{T} \; \sum_{i \in I^3} \; \alpha^3_i(t) \, |\tilde{x}^3_i(t) - X^3_i(t)| \; ,
\end{aligned}
\tag{57}
$$

where $\alpha^1_i(t)$ $(i \in I^1)$, $\alpha^2_i(t)$ $(i \in I^2)$, $\alpha^3_i(t)$ $(i \in I^3)$ are some weighting coefficients, and $\tilde{x}^2_i(t)$ and $\tilde{x}^3_i(t)$ are defined by (45) and (53).

<u>Remarks</u> The three level model, like Problem 1, allows different modifications and variants.

a. The simple case of this three level system is when types of education specialities directly correspond to the specialities in manpower stock. Then the state equations (38) (44) and (50) are replaced by the following [cf. (5)]:

$$x_{i_1}^1(t+1) = (1-a_{i_1})x_{i_1}^1(t) + u_{i_1}^1(t) - \sum_{i_2} u_{i_2 i_1}^{21}(t)$$

$$x_{i_2}^2(t+1) = (1-a_{i_2})x_{i_2}^2(t) + u_{i_2}^2(t) + \sum_{i_1} u_{i_2 i_1}(t) - \sum_{i_3} u_{i_3 i_2}^{32} \qquad (58)$$

$$x_{i_3}^3(t+1) = (1-a_{i_3})x_{i_3}^3(t) + \sum_{i_2} u_{i_3 i_2}^{32}(t) \qquad (i_1 \in I^1, i_2 \in I^2, i_3 \in I^3) \ .$$

The state equations (58) yield DLP problems of a transportation type (Propoi, 1976).

b. If in the state equations (38), (44) and (50) the transitions from one group i to another j are permitted, then the terms $(1 - a_i(t))x_i(t)$ in these equations should be replaced by the $\sum_j a_{ij}(t)x_j(t)$ [cf. (1)].

c. It was supposed above, that each specialist who enters the next educational level, graduates from it. If we assume that $(1 - b_{ik}^2(t))$ and $(1 - b_{ik}^3(t))$ are the attrition rates for the second and third educational levels, then the dropouts

$$\sum_{k \in K^2} [1-b_{i_1 k}^2(t-\tau_k^2)] [u_k^2(t-\tau_k^2) + \sum_{i \in I^1} u_{ki}^{21}(t-\tau_k^2)] = z_{i_1}^1(t), \ i_1 \in I^1$$

and

$$\sum_{k \in K^3} (1-b_{i_2 k}^3(t-\tau_k^3)] \sum_{j \in I^2} u_{kj}^{32}(t-\tau_k^3) = z_{i_2}^2(t), \ i_2 \in I^2$$

should be added to the second and third level manpower stock in time period t. Here $z_{i_1}^1(t)$, $i_1 \in I^1$ ($z_{i_2}^2(t)$, $i_2 \in I^2$) is the number of specialists of type i_1 (i_2) who do not graduate from the second (third) level educational subsystem and therefore return to the first (second) level manpower stock.

d. As was mentioned above, the considered model is an optimization version (see also Propoi, 1978) of the Tinbergen econometric model (Correa and Tinbergen, 1962 and Bowles 1969). In the latter case, there is no freedom in the choice of enrollment to all three educational levels, and the state equations are

used to derive a set of required enrollments for an exogenously
given rate of economic growth. This economic growth determines
the demand for labor force (56), and the problem then is to cal-
culate from the equations (38), (44) and (50) such enrollments
for which

$$x^{\nu}_{i\nu}(t) = x^{\nu}_{i\nu}(t); \quad i_{\nu} \in I^{\nu}, \ \nu = 1,2,3,$$

hold.

2.5 Some Applications and Comments

Above, the basic model and its modifications and variants
were considered. These modifications can either be reduced to the
basic Problem 1 or DLP methods can be directly used for their
solutions. Now we consider some applications of these models
(see also the survey of McNamara, 1973).

a. On the national level, a DLP model for a developing coun-
try's educational system was considered by Bowles (1969) (and has
been applied in Canada). The control variables include enrollments
and resources used at the various educational levels (primary,
secondary, technical and higher). The model allows the inclusion
of the import of a number of types of educated labor and the
sending abroad of students for their education. Thus for some
types of labor, there are three options: the production of labor
to a given level of schooling within the system; the production
of labor outside the system; or the importation of labor possess-
ing the required educational attainments from outside the system.

Constraints reflect the availability of different types of
teachers, student enrollments and transfers among the various
types of schools, school construction and current facility usage,
teacher recruitment or importation, and legal restrictions. Pro-
duction processes in the rest of the economy are not included in
the model explicitly; that is, the demands for the outputs of the
educational system and the supply for educational inputs are given
exogenously.

The time period of only eight years was chosen for a planning horizon, but the model is operated on a year-by-year sequential basis. (That is, for each current state of the system, which is considered as initial, a new 8 year plan is computed. The process is then repeated for the next year).

From a mathematical point of view, there are no state equations and state variables in explicit form (in Bowles, 1969). However, the model can be easily reformulated in the DLP format of Problem 1.

b. Optimal investment policies for education were considered by Ritsen (1975 and 1976). The investment problem considers the distribution of a given budget over courses (Ritsen, 1976) or the balanced increase of the number of teachers and the volume of the buildings (Ritsen, 1975). The objective in the first case is to minimize the deviation of the actual manpower available from the manpower targets. In the second case, it is the minimization of total funds allocated to the system. These models were formulated in quadratic forms but can be reformulated in DLP terms.

c. A simple multiperiod model for a training program was considered by Balinsky (1974). The objective function is composed of an educational cost and manpower shortage or surplus cost components. The model may be used for planning a training program for high school dropouts, or high school graduates or a Master's degree program of a particular speciality. Due to the small dimension of the problem considered, a dynamic programming method was used for its solution.

d. On the institutional level, a DLP model was considered by Walters, Mangold and Haran (1976) which was designed to aid the administrator of a school in planning and decision-making for a five-year horizon. There was no constraint, implicit or explicit, put on the structure of the school to be modeled. The school may be interdisciplinary, or organized by departments, and thus the model can represent the university itself with a little modification. The optimization model was formulated in goal

programming terms and for the prototype case, has a dimension of 436 constraints and 966 variables. The problem was solved on an IBM 370/175 using MPSX.

A simple DLP model was used by Averill (1975) to analyze admission policies for Yale University under the conventional two-term operation and with the proposed summer term. The objective of the model was to determine the admission policy, which maximizes the number of students in residence (therefore minimizing the number of vacancies) subject to constraints on enrollment capacity and admission mix. The model's advantage is that the basic data required to run it is normally available. Thus, the model can be used by other universities to aid in their enrollment planning.

e. Other educational planning models are considered, for example, in Bartholomew and Morris (1971), Bermant (1972 and 1975), Khan (1971), Menges and Elstermann (1971), Law (1977), McNamara (1973), Riordon and Mason (1971), Sinha and de Cenzo (1975), and Smith (1971). It should be noted that frequently the models are used on a simulation basis. For example, enrollment of new entrants is determined by a multivariable regression model and then the state equations of the (1) type are used for the forecasting of manpower or total students stock [see, for example, Law (1977)].

3. MANPOWER PLANNING MODELS

The structure of this section, in which manpower models are considered, is similar to the previous section on educational models: first, basic models are introduced (there are two models here: one is with recruitment as a control and the second is with promotion as a control), then different modifications and applications are discussed.

3.1 Basic Model I

State Equations: Consider a population which is partitioned into n grades (ranks, classes, age groups, regions, specialities). Let $x_i(t)$ be the number of people in grade i (i = 1,...,n) at time period t; $x_i(t)$ is the stock of manpower grade i at time t; coefficient $a_{ij}(t)$ be the proportion of people in grade i at time t + 1 who were in grade j at time t, and $u_i(t)$ be the number of recruitments in grade i at time t. Then the state equations which describe the dynamics of the system can be written as follows (i = 1,...,n; t = 0,1,...,):

$$x_i(t + 1) = \sum_{j=1}^{n} a_{ij}(t)x_j(t) + u_i(t) \qquad (59)$$

or in vector form

$$x(t + 1) = A(t)x(t) + u(t) \quad . \qquad (60)$$

Here $x(t) = \{x_1(t),...,x_n(t)\}$ is the state vector, and $u(t) = \{u_1(t),...,u_n(t)\}$ is the control vector.

In many cases, $A^T(t) = P$, P being the transfer (promotion) matrix with elements p_{ij} (probabilities with which members of grade i move to grade j). In this case, state equation (60) is replaced by

$$x(t + 1) = x(t)P + u(t) \quad , \qquad (61)$$

x(t) being a row vector.

Hence, the equation (59) allows stochastic interpretation with $x_i(t)$ as the expected number of people in grade i at time t (Bartholomew, 1973; Grinold and Marshall, 1977).

A more general form of the equation (59) is when recruitment vector u(t) has the other partition in comparison with the manpower stock vector x(t): $\{u(t) = u_k(t)\}$ (k = 1,...,r). In this case, the matrix $B(t) = \{b_{ik}(t)\}$ is introduced which shows the distribution of recruitments of type k over the manpower stock grades i (i = 1,...,n) and the state equations (59) is replaced by [see (28)]:

$$x(t+1) = A(t)x(t) + B(t)u(t) \quad . \tag{62}$$

Initial state in the state equation (59) is supposed to be given

$$x(0) = x^0 \quad . \tag{63}$$

Using (59) and (63), the state variable $x(t)$ can be expressed as a function of controls $u(t)$ in an explicit form (Propoi, 1973):

$$x(t) = \Phi(t,0)x^0 + \sum_{\tau=0}^{t-1} \Phi(t-1,\tau+1)u(\tau) \quad , \tag{64}$$

where

$$\Phi(t,\tau) = A(t) \ldots A(\tau);$$

$$\Phi(t,t+1) = I, \quad I \text{ is an identity matrix.}$$

We can rewrite (64) as

$$x(t) = \bar{x}(t) + \sum_{\tau=0}^{t-1} W(t,\tau)u(\tau) \tag{65}$$

where

$$\bar{x}(t) = \Phi(t,0)x^0 \quad ,$$

$$W(t,\tau) = \Phi(t-1,\tau+1) \quad ,$$

and in matrix $W(t,\tau)$ coefficient $w_{ij}(t,\tau)$ is the proportion of people entering grade j at time period τ who are counted in grade i at time period t ($t > \tau$; $i,j = 1,\ldots,n$); $\bar{x}_i(t)$ is the legacy in grade i at time t, which denotes the contribution of past inputs ($t < 0$) to the stock at future time t.

Frequently, the matrix A(t) is constant over time: A(t) = A. In this case

$$\Phi(t,\tau) = A^{t-\tau+1}$$

$$W(t,\tau) = A^{t-\tau-1} = W(t-\tau); \quad \bar{x}(t) = A^t x^0$$

and instead of (65) we have

$$x(t) = \bar{x}(t) + \sum_{\tau=0}^{t-1} W(t-\tau)u(\tau) \quad . \tag{66}$$

If we assume that M is the maximum number of time-steps with which an individual is encountered, then only (M + 1) matrices W(0),...,W(M) are needed for describing the flows through the system. In this case, (66) is replaced by

$$x(t) = \sum_{\theta=t-M}^{t} W(t-\theta)u(\theta) = \sum_{\tau=0}^{M} W(\tau)u(t-\tau) \quad . \tag{67}$$

Models which are described by the state equations (59) are called cross-sectional models in (Grinold, Marshall, 1977), as compared to longitudinal models, which are described by equations (65) or (67), in which state variables are excluded. The con- nection between the two types of models are given by (64) (see also Zadeh and Desoer, 1963). The longitudinal models attempt to describe the flow of a group (grade) through the manpower system over time and are based on the entire history of the group (67).

In the cross-sectional models, which are described by the state equations, the history of the group, if necessary, is in- cluded in the definition of the state of the system. For example, if the partition of manpower on the ranks, specialities, etc. is not sufficient for predicting the future behavior of a manpower system, then a new component -- the length of service (LOS) of an individual -- should be introduced to the state of the system. In this case, the state equation (59) is replaced by (Merchant, 1977):

$$x_i^{\alpha+1}(t+1) = \sum_{j=1}^{n} a_{ij}^{\alpha}(t)x_j^{\alpha}(t) + u_i(t) \qquad (68)$$

where $x_j^{\alpha}(t)$ is the number of people of LOS α in grade j at time period t ($\alpha = 1, \ldots, N$), N being the maximum length of service in the system. Clearly, (68) can be rewritten in the matrix form (59), where the matrix A(t) is now:

$$A(t) = \begin{bmatrix} 0 & 0 & \cdots\cdots & 0 \\ A^1(t) & 0 & \cdots\cdots & 0 \\ 0 & A^2(t) & \cdots\cdots & 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 & 0 & & A^N(t) \end{bmatrix}$$

and $x(t) = \{x_i^{\alpha}(t)\}$ ($i = 1, \ldots, n$; $\alpha = 1, \ldots, N$).

Thus, it can be argued that the description of the model in the form (59) is more preferable because it is much simpler to solve the problem with the state equation (59) as well as to analyze and implement the solution than to use equation (65) (see Propoi, 1973 and 1976).

Another advantage of the state-space description of the system is that it allows us to introduce a notion of feasible (attainable) sets, which is useful for the analysis of the manpower system behavior. But before doing that, we have to describe the constraints on the system.

Constraints: Clearly,

$$u_i(t) \geq 0 \quad \text{and} \quad x_i(t) \geq 0 \quad , \quad (i = 1, \ldots, u) \qquad (69)$$

(the remark in Section 2.1 on the integrity of the variables is also valid for manpower models). If both hiring and firing are used in the system, then $u_i(t)$ is not sign-restricted and

$$u_i(t) = u_i^+(t) - u_i^-(t) \; ; \quad u_i^+(t) \geq 0 \quad , \quad u_i^-(t) \geq 0 \quad .$$

The resource constraints are usually associated with the limitations on the total size of the organization:

$$\sum_{i=1}^{n} x_i(t) = X(t) \tag{70}$$

or for each grade:

$$x_i(t) \leq \bar{x}_i(t) \quad (i = 1,\ldots,n) \tag{71}$$

where $X(t)$ $(\bar{x}_i(t))$ are given, and/or on the size of recruitment:

$$\sum_{i=1}^{n} u_i(t) \leq U(t) \quad, \tag{72}$$

or

$$u_i(t) \leq \bar{u}_i(t) \quad, \quad (i = 1,\ldots,n) \quad . \tag{73}$$

The goal constraints for manpower models have the same form as in (14)-(17). The constraints considered can be written in the general form:

$$G(t)x(t) + D(t)u(t) \leq f(t) \quad, \tag{74}$$

$$x(t) \geq 0 \quad, \quad u(t) \geq 0 \quad .$$

After delineating all the constraints on the system, we can define the feasible sets of the system. Let X be the state space of the state variables $x(t)$. We shall call the sequences of vectors $u = \{u(0),\ldots,u(T-1)\}$ and $x = \{x(0),\ldots,x(T)\}$ by the feasible control and feasible trajectory, if they satisfy all the constraints of the system.

The feasible sets of the system, described by the state equation (59) and constraints (74), are the sets of all states in X, which can be attained from the given initial state at the given number of steps. We denote by $R_t(x^0)$ the feasible set of all states $x(t)$, which are attainable from x^0 at t steps. Formally, the feasible sets $R_t(x^0)$ can be defined by recurrent formulas (Propoi, 1973):

$$R_1(x^0) = \{x(1) \mid x(t) = A(t)x^0 + B(t)u(0); G(0)x^0 + D(0)u(0) \leq f(0); \; u(0) \geq 0\}$$

$$R_{t+1}(x^0) = \{x(t+1) \mid x(t+1) = A(t)x(t) + B(t)u(t); G(t)x(t) + Du(t) \leq f(t);$$

$$x(t) \in R_t(x^0); \; x(t) \geq 0; \; u(t) \geq 0\} \quad (t = 2,3,\ldots) \quad . \tag{75}$$

Obviously, feasible sets in this case are convex polyhedrons.

Objective Function: We can single out two basic types of objectives in manpower systems: minimization of the total deviation between feasible and required manpower for a whole planning horizon, which can be expressed in (19)-(22) form; or minimization of the total expenditures for operation and development of the manpower system, which can be expressed in the form

$$J = \sum_{t=0}^{T-1} \sum_{i=1}^{n} [c_i^1(t)x_i(t) + c_i^2(t)u_i(t)] \quad . \tag{76}$$

Thus, we can formulate the following basic model.

Problem 3.1: Given the initial state

$$x(0) = x^0$$

and the state equations

$$x(t+1) = A(t)x(t) + B(t)u(t) \quad (t = 0,\ldots,T-1)$$

with constraints

$$G(t)x(t) + D(t)u(t) \leq f(t) \quad ,$$

$$x(t) \geq 0, \quad u(t) \leq 0 \quad ,$$

find a control $u = \{u(0),\ldots,u(t-1)\}$ and a corresponding trajectory $x = \{x(0),\ldots,x(T)\}$, which minimize the objective function

$$J = \sum_{t=0}^{T-1} \sum_{i=1}^{n} \alpha_i(t) \mid x_i(t) - \bar{x}_i(t) \mid \quad .$$

This basic model is applied to the manpower control problems by recruitment (Bartholomew, 1973). Modifications of the basic model as well as some applications are considered in Section 3.4.

3.2 Basic Model II

In the basic model considered above (Problem 3.1), the control variables are recruitments. Another type of governing of a manpower system development is the control of transients from grade to grade over time. To describe models of this type, we have to express the internal flows in the system (see Fig. 5).

State Equations: We use the same partition of the manpower into n grades as in the previous section. Thus, $x_i(t)$ is the manpower stock in grade i (i = 1,...,n) at time t. Let $u_{ij}(t)$ be the number of people in grade i at time t + 1 who were in grade j at time t (flow from j to i in time period t). Then the state equations of the manpower system will be the following

$$x_i(t + 1) = x_i(t) + \sum_{j=1}^{n} u_{ij}(t) - \sum_{j=1}^{n} u_{ji}(t) + z_i(t) \qquad (77)$$

with initial condition

$$x_i(0) = x_i^0 \qquad (i = 1,...,n) \quad . \qquad (78)$$

Here $z_i(t)$ expresses the external change in manpower stock. It may be, for example, the exogenously given change in the population of the system due to deaths and births:

$$z_i(t) = z_i^+(t) - z_i^-(t) \quad ; \qquad (79)$$

where $z_i^+(t) \geq 0$ and $z_i^-(t) \geq 0$, or may also include the changes in manpower due to recruitment and dismissal:

$$z_i(t) = z_i^+(t) - z_i^-(t) + u_{i0}(t) - u_{0i}(t) \qquad (80)$$

where $u_{i0}(t) \geq 0$ and $u_{0i}(t) \geq 0$ are the controllable flows from outside the system to grade i and outside the system from grade i

in time period t. In this case, the model includes both types
of controls: "promotion" and "recruitment".

Sometimes it is more convenient to express the uncontrollable
component of $z_i(t)$ as a proportion of the current manpower stock:

$$z_i^+(t) - z_i^-(t) = (1 - a_i(t))x_i(t) \tag{81}$$

where $a_i(t)$ is the attrition rate in the manpower stock of grade
i at time t.

Constraints: Clearly,

$$x_i(t) \geq 0 \quad , \qquad u_{ij}(t) \geq 0 \quad .$$

The upper bounds on the control variables can be given either
in the form

$$0 \leq u_{ij}(t) \leq \bar{u}_{ij}(t) \tag{82}$$

where $\bar{u}_{ij}(t)$ are given numbers, or in the form

$$0 \leq u_{ij}(t) \leq \alpha_{ij}(t)x_i(t) \tag{83}$$

where $0 \leq \alpha_{ij}(t) \leq 1$. The upper bounds on the state variables are
given in the form (70) or (71).

The resource constraints usually have the form [cf. (8)]

$$\sum_{i,j} d_{kij}(t)u_{ij}(t) \leq f_k(t) \tag{84}$$

where $d_{kij}(t)$ is the required amount of resource k per transition
of unit (e.g. 100 persons) from grade i to grade j at time t; and
$f_k(t)$ is the given amount of resource k at time t.

Objective Function: The objectives for this type of model
are the same as for the model considered in Section 3.1. Thus,
we can formulate the following basic model for control by promo-
tion.

Problem 3.2: Given the initial state

$$x_i(0) = x_i^0 \quad .$$

and the state equations

$$x_i(t+1) = x_i(t) + \sum_{j=1}^{n} u_{ij}(t) - \sum_{j=1}^{n} u_{ji}(t) + z_i(t)$$

with constraints

$$\sum_{i,j} d_{kij}(t) u_{ij}(t) \leq f_k(t)$$

$$0 \leq u_{ij}(t) \leq \alpha_{ij}(t) x_i(t)$$

$$\sum_{i=1}^{n} x_i(t) \leq X(t); \qquad x_i(t) \geq 0$$

which minimize the objective function

$$J = \sum_{t=0}^{T-1} \sum_{i=1}^{n} \alpha_i(t) \, |x_i(t) - \bar{x}_i(t)| \quad .$$

The problem 3.2 is a DLP problem of the transportation type
(Propoi, 1976) in comparison with the problem 3.1, which is a
DLP problem of the general type.

We will now consider some extensions and modifications of
these basic problems.

3.3 General Manpower Planning Model

The basic models (Problems 1 and 2) allow us to build the
general manpower planning model which comprises all major fea-
tures of a manpower system. This model was developed by Ivanilov
and Petrov, 1971.

We consider a system with its population partitioned into
some groups (grades). Let I be the set of these groups, $x_i(t)$
be the number of people in group $i \in I$ at time period t. Consider-
ing the state equations (77), we note that in fact not all

transitions from one group to another are permitted. In addition, it is necessary to distinguish ways of transition from i to j.

Thus, we denote by $u_{ij}^s(t)$ the number of individuals of group j at time t who begin the transition from j to i, using the s-th way of transition, τ_{ij}^s being the time for this transition.

Let $J^+(i)$ be the set of groups j, the representatives of which can transit to group i; $J^-(i)$ be the set of groups to which transitions from grade i are allowed and $S(i,j)$ be the set of all possible transitions from j to i. We assume that transition from group i to group j by way s takes τ_{ij}^s time steps.

Then the state equations (77) with (80) can be generalized as follows:

$$x_i(t+1) = x_i(t) + \sum_{j \in J^+(i)} \sum_{s \in S(i,j)} u_{ij}^s(t - \tau_{ij}^s)$$

$$- \sum_{j \in J^-(i)} \sum_{s \in S(j,i)} u_{ji}^s(t) + u_{i0}(t)$$

$$\text{(85)}$$

$$- u_{0i}(t) + z_i^+(t) - z_i^-(t) \quad ,$$

$$(i \in I, \ t = 0,1,\ldots,T-1) \quad .$$

The initial state

$$x_i(0) = x_i^0 \qquad (i \in I) \tag{86}$$

and preplanning controls $u_{ij}^s(t)$, t < 0, are known.

The equations (85) represent the general type of state equations and include the case (59), when

$$x_i(t) + z_i^+(t) - z_i^-(t) = \sum_{j=1}^n a_{ij}(t) x_j(t) \quad ,$$

$$u_i(t) = u_{i0}(t); \quad u_{0i}(t) = 0;$$

and (77), when

$$u_{ij}(t) = \sum_{s \in S(i,j)} u_{ij}^s(t) \quad ,$$

$$\tau_{ij}^s = 0 \quad ,$$

and all transitions from i to j are possible. As usual, all state and control variables are nonnegative:

$$x_i(t) \geq 0, \quad u_{ij}^s(t) \geq 0, \quad u_{0i}(t) \geq 0, \quad u_{0i}(t) \geq 0 \quad . \quad (87)$$

The upper bounds on transitions can be given in the form

$$0 \leq u_{ij}^s(t) \leq \bar{u}_{ij}^s(t) \tag{88}$$

where $\bar{u}_{ij}^s(t)$ are given, or in the form

$$0 \leq u_{ij}^s(t) \leq \alpha_{ij}^s(t) x_i(t) \quad , \tag{89}$$

where coefficients $\alpha_{ij}^s(t)$, $0 \leq \alpha_{ij}^s(t) \leq 1$ are given [cf. (82) and (83)]. The upper bound constraints on variables $x_i(t)$ and $u_{0i}(t)$, $u_{0i}(t)$ are written in the form (70)-(72).

The resource constraints are a generalization of constraints (10) and (84):

$$\sum_{i \in I} \sum_{j \in J^+(i)} \sum_{s \in S(i,j)} \sum_{\tau=0}^{\tau_{ij}^s - 1} d_{kij}^s(t,\tau) u_{ij}^s(t-\tau) \leq f_k(t) \ (k \in K) \tag{90}$$

where $d_{kij}^s(t,\tau)$ is the required amount of resource k per unit transition from group i to j, when the s-th transition is used; K is the total set of required resources.

The resource constraints (90) are usually associated with facilities for the people's training (e.g. building and equipment). In addition to (90), we can single out the constraints on teachers' (instructors') availability. Among all groups I of the system population, we single out the set I_0 of the groups of teachers (instructors): $I_0 \subseteq I$. Let $g_{kij}^s(t, \tau)$ be the number of required teachers of group $k \in I_0$ for insuring the unit transition of people from group i to group j when the way s is used. Then the constraints on teachers' availability can be written in the form similar to (90)

$$\sum_{i \in I} \sum_{j \in J^+(i)} \sum_{s \in S(i,j)} \sum_{\tau=0}^{\tau_{ij}^s - 1} g_{kij}^s(t, \tau) u_{ij}^s(t-\tau) \leq x_k(t) \quad (k \in I_0) \tag{91}$$

$x_k(t)$ is calculated from the state equations (85).

The goal constraints are analogous to the conditions (14)-(17). We can consider different objectives with this model. These objectives can be broken down into the following groups:

1) To bring the system (manpower stock $\{x_i(T)\}$) as close as possible to the desired state $\bar{x}_i(T)$ ($i \in I$) at the end of the planning period, or to keep during the whole planning period $t = 1, \ldots, T$ the manpower stock $\{x_i(t)\}$ as close as possible to the given demand in manpower $\bar{x}_i(t)$ ($i \in I, t = 1, \ldots, T$) under given resources and other constraints. The objective functions for this case can be expressed as in (19)-(22).

2) To minimize the total expenditure for the system's transition to the given targets, which are expressed by (14)-(16). The objective function for this case can be written in the form [cf. (23)]:

$$J = \sum_{t=0}^{T-1} \sum_{i \in I} \sum_{j \in J^+(i)} \sum_{s \in S(i,j)} \sum_{\tau=0}^{\tau_{ij}^s - 1} c_{ij}^s(t,\tau) u_{ij}^s(t-\tau) \qquad (92)$$

3) To maximize outputs in eligible groups of specialists to the end of the planning period. The objective function for this case has the form of (24).

4) To combine planning (or policy analysis) in manpower and economy development. In this case, the objectives should be of economic character rather than bound by an "optimal" development of only manpower systems (see Section 4).

3.4 Some Examples and Applications

Now we consider some examples and applications of the models described above.

a. Control by Recruitment These types of models relate to the basic Model I. In the simple versions, the constraints have the form (70):

$$\sum_{i=1}^{n} x_i(t) = X(t) \quad ,$$

$$x_i(t) \geq 0 \quad , \quad u_i(t) \geq 0 \quad ,$$

where $X(t)$ is the given size of the organization, and the objective, for example, is to minimize the total salary bill

$$J = \sum_{t=1}^{T} \sum_{i=1}^{n} c_i(t) x_i(t)$$

where $c_i(t)$ is the average salary cost in grade i at time period t, or to minimize the total under- and over-manning in various time periods and grades. The objective function is in this case expressed by (21) which is equivalent to (19).

These types of models were considered by Bartholomew (1973); Charnes, et al. (1969 and 1971); Davies (1973 and 1976); Grinold and Marshall (1977), and many others.

It should be noted that many models considered in the references mentioned above are not formulated in the explicit form of Problem 1, but can be easily reduced to Problem 1.

b. Attainable Structures In a number of models, objective functions are given in inexplicit form. For this case, we introduce feasible sets $R_t(x^0)$ $(t = 0,1,2,...)$, that is, the sets of all states $x(t)$, which are attainable by feasible controls $\{u(0),...,u(t-1)\}$ at t steps [see (75)].

Using this notion of the feasible sets, one can formulate the following problems:

1. Find a structure x_T which can be obtained from the initial state x^0 in T steps. Thus, the problem is reduced to finding such a state x_T which belongs to the feasible set $R_T(x^0)$ (T and x^0 are given):

$$x_T \in R_T(x^0) \quad .$$

This problem, in turn, is equivalent to finding an arbitrary feasible control $\{u(0),...,u(t-1)\}$ for basic Problem 1 for given x^0 and T.

2. Find an initial structure x^0, from which the given terminal structure x_T can be obtained [that is, find x^0 for which $X_T \in R_T(x^0)$]. This problem can be reduced to Problem 1 if the additional control variable x^0 is introduced and is equivalent to finding a feasible control $\{x^0,u(0),...,u(T-1)\}$ in Problem 1 with additional condition $x(T) = x_T$.

3. Find reattainable in T steps structure x (that is, find x, for which $x \in R_T(x)$, and T is given).

This problem is equivalent to finding a feasible control in Problem 1 with the additional condition

$$x(0) = x(T) = x \quad ,$$

where x is a new control variable.

Problems of this type were considered, for example, by Vajda (1975) and Bartholomew (1973). In some of these papers, properties of the feasible sets were investigated or simple numerical examples were considered. As one can see from above, all such problems are reduced to finding a feasible control in a corresponding DLP problem and, therefore, standard LP or DLP methods can be directly used for their solution.

c. <u>Control by Promotion</u> This type of problem is associated with the basic Problem 2. However, frequently both types of control -- recruitment and promotion -- are used simultaneously (see for example, Forbes, 1971). In this case, the models are formulated as described in Section 3.3. The models of these types may be used both on a national/regional (macro) level or an institutional (micro) level.

On a regional (national) level, the flows of people from one group to another are usually caused by some training or educational course (for example, vocational training). In this sense, these models represent a flow type educational planning model and in fact, were described in Section 2.4 (see also Ivanilov and Petrov, 1971).

On an institutional level, the problem is to analyze the best promotion and recruitment policies in an organization under given requirements in manpower, and budget and other resource constraints.

An interesting model of this type is described by Billionnet (1977). This model is operational at the Direction Centrale de l'Après-vente of the Régie Nationale des Usines Renault and allows one to study organizations including approximately 50 jobs and a few thousand persons. With this model one can determine answers to many questions regarding a large or small organization in the short and middle terms. In the short term, it allows finding the best movements to face an unexpected departure, a promotion, or other individual hazards. In the middle term, it allows allocation movements over time in order to face the expansion, the stagnation, and the regression of the different departments of the organization.

Different manpower models using promotion as a control were considered by Bartholomew (1973); Holl, Legrat and Benayon (1971); Morgan (1971); Stanford (1976) and others.

4. LINKAGE WITH ECONOMY MODEL

In the considered educational or manpower models, the demand for manpower and resource constraints for education are given exogenously. Of great interest is the analysis of interrelations between manpower and economy development models.

When the interaction between manpower and economy development is analyzed, two major options should be taken into account: development of some sectors in an economy in order to absorb the projected surplus in manpower of certain types and development of educational facilities in order to fill up possible shortages in manpower for other sectors of an economy. Besides, we have to add possibilities of labor force migration into and out of the system.

The problem should be disaggregated on major economic activities (various industrial sectors, agriculture, construction, transportation, public administration and other services) and on the levels of education (primary, secondary, higher).

Below, we consider a simple integrated optimization model of economy-manpower interaction.

Let $m(t)$ be the vector of skilled manpower at time t, $n(t)$ be the vector (of the same dimension) of the manpower increase during time period t, and $P(t)$ be the transition matrix. Then the state equations for the manpower/educational subsystem will be the following [see (5)]:

$$m(t + 1) = P(t)m(t) + n(t) \quad . \tag{93}$$

The training of people requires resources; first of all, teachers [see (13)]:

$$n(t) \leq \Phi m(t) \tag{94}$$

and second, buildings and equipment (8)

$$n(t) \leq y_e(t) \tag{95}$$

where $y_e(t)$ is the vector of capital stock for the educational sub-system. The development of this subsystem can be expressed in the same terms as development of the production system:

$$y_e(t + 1) = D_e(t)y_e(t) + v_e(t) \tag{96}$$

$$y(t + 1) = D(t)y(t) + v(t) \tag{97}$$

where the subscript e refers to the educational subsystem, $D_e(t)$; $D(t)$ are depreciation matrices. The balance of goods production and their consumption for the total system will be the following

$$(I - A(t))z(t) = B(t)v(t) + B_e(t)v_e(t) + w(t) \tag{98}$$

with constraints

$$z(t) \leq y(t) \tag{99}$$

and

$$L(t)z(t) \leq m(t) \tag{100}$$

where the matrix $L(t)$ specifies requirement in skilled labor for each sector of economy, $z(t)$ is the vector of gross outputs.

The connection between consumption vector $w(t)$ and manpower vector $m(t)$ is assumed to be given as

$$w(t) = g(t) + F(t)m(t) \tag{101}$$

where $g(t)$ is the exogenously given vector of governmental consumption, and the matrix $F(t)$ expresses the consumption profile for different categories of manpower. Conditions (100) and (101) describe the linkage between the educational (93)-(96) and economy (97)-(99) submodels.

The development of the educational system is upper bounded by the growth of population

$$n(t) \leq Hx(t) = \bar{n}(t) \tag{102}$$

where $x(t)$ is the population (sex/age) distribution vector and can be obtained from demography models (see, for example, Rogers 1975), and H is a matrix.

With the model (93)-(102), optimal policies with different objective functions can be analyzed. Here, for certainty, the objective is to maximize the discounted consumption vector

$$j = \sum_{t=0}^{T-1} \beta(t)(c(t),w(t)) \quad . \tag{103}$$

Thus, the problem is to choose such control vectors $\{v(t),z(t)\}$ and $\{v_e(t),n(t)\}$ and corresponding state vectors $y(t)$, $y_e(t)$ and $m(t)$, which maximize (103) subject to (93)-(102).

Two basic approaches can be singled out when separate sub-models are incorporated into a whole system. The first approach is the integration of separate models into an optimization problem with a corresponding objective function. The second approach is the investigation of linkage between submodels considering these submodels on an independent basis, each with its own objective function. Both approaches naturally have their own advantages and drawbacks. The major advantage of the first "machine" approach is that it allows one to take into account all the constraints and interactions between many factors influencing the decision and combining them into some optimal mix. However, the building of an integrated model evidently leads to a very large optimization problem, which though it is sometimes possible to solve, is always very difficult to interpret. The "manual" approach -- when information obtained from one submodel is interpreted by an analyst and is supplied as an input to another submodel -- becomes more attractive, but is more time consuming and sometimes may lead to an uncertainty whether the "true optimal" solution for a whole system has been obtained or not.

Considering combined manpower-economy models, it is natural to include demographic aspects into a whole system [see (102)]. Such models were described in Rogers and Ledent (1971) and Grandinetti, Pezzella and LaBella (1977).

5. CONCLUSION

One can see from above, that many educational and manpower planning models can be written in a standard DLP form. Hence, for the solution of these problems, either standard LP algorithms (Dantzig, 1963) or special DLP methods (Glassey, 1970; Ho and Manne, 1974; Propoi and Krivonozhko, 1977), which take into account the dynamic features of the problems, can be used.

Clearly, not all manpower and educational planning problems can be written in the DLP format. In those cases, one may use computer simulation methods (an interesting comparison of these two approaches was made by Merchant, 1977) or may try to apply a more sophisticated technique, such as stochastic optimization.
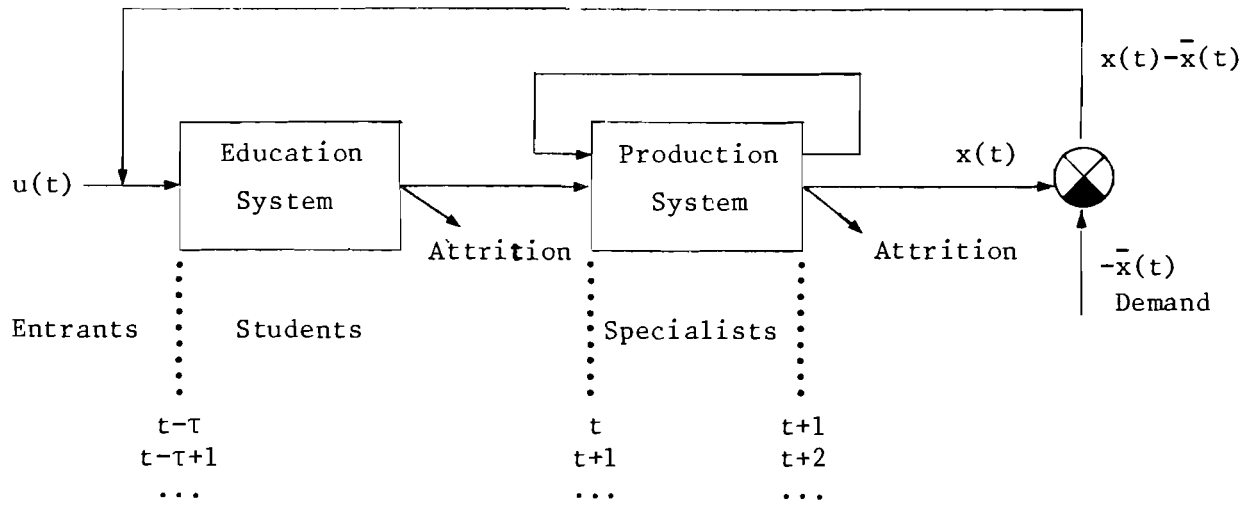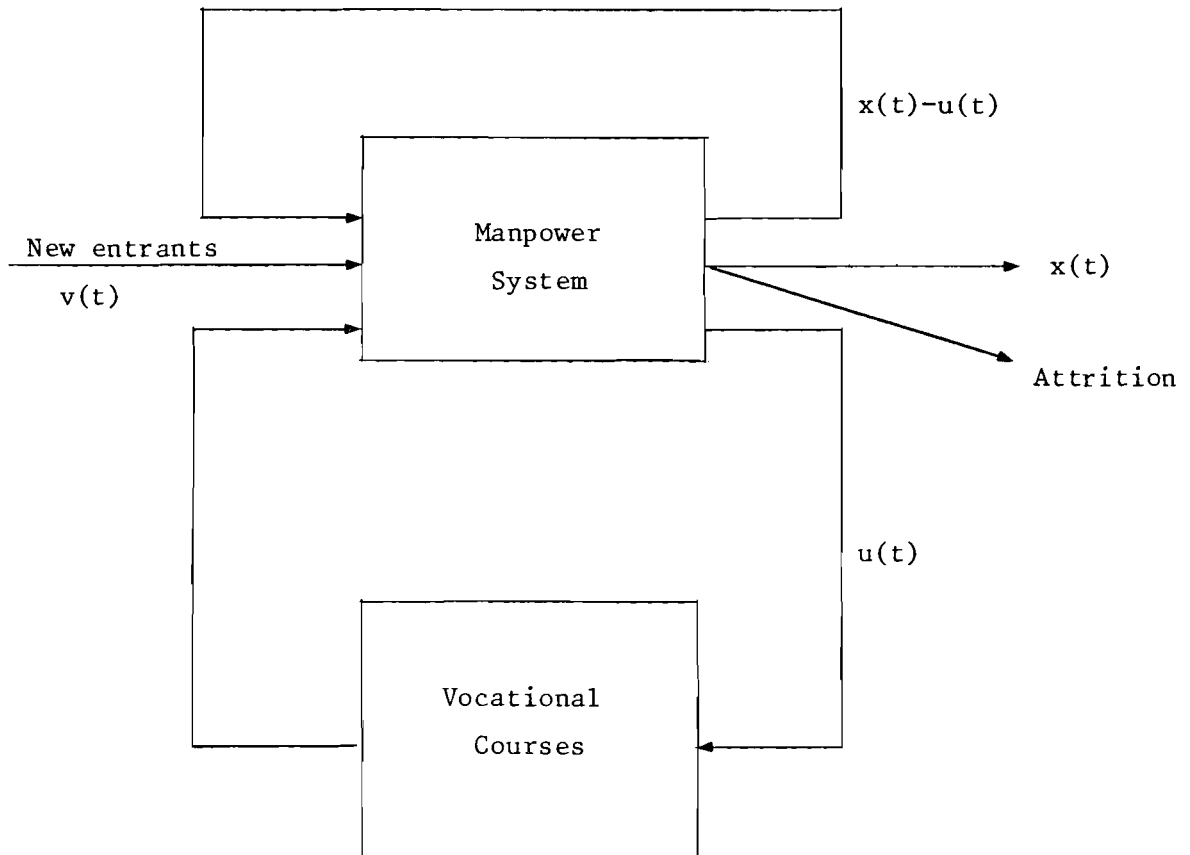
Figure 1. The Basic Educational Model

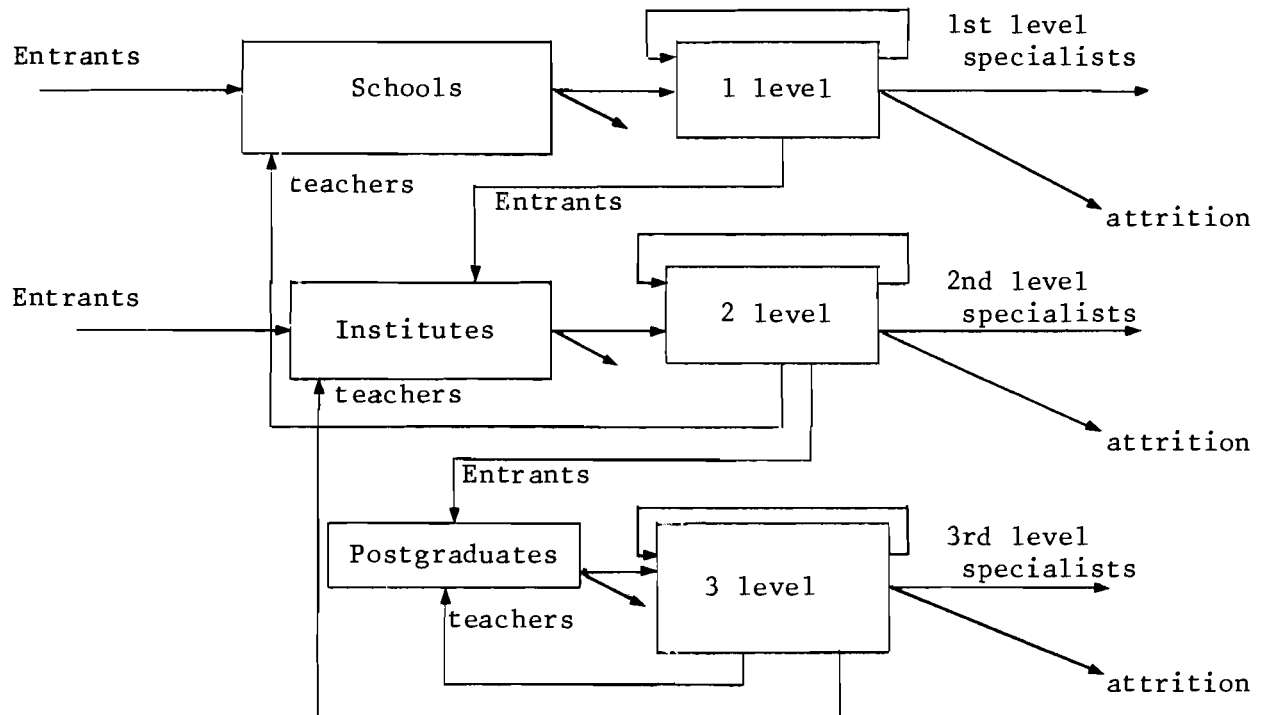Figure 2. Model of the Professional Skill Improvement

Figure 3. Three-Level Educational Planning Model

Control variables

Controls for the
1st level subsystem

Controls for the 2nd
level subsystem

Control for the 3rd
level subsystem

States of the
subsystems

$u_k^1(-\tau_k^1)$

$u_k^1(T-1-\tau_k^1)$

$u_{ki}^{21}(T-1)$

$-\tau_k^1$

$T-1$

$u_k^2(-\tau_k^2)$

$u_k^2(T-1-\tau_k^2)$

$u_{ki}^{32}(T-1)$

$-\tau_k^2$

$T-1$

$u_{kj}^{32}(-\tau_k^3)$

$u_{kj}^{32}(T-1-\tau_k^3)$

$-\tau_k^3$

$T-1$

State Variables
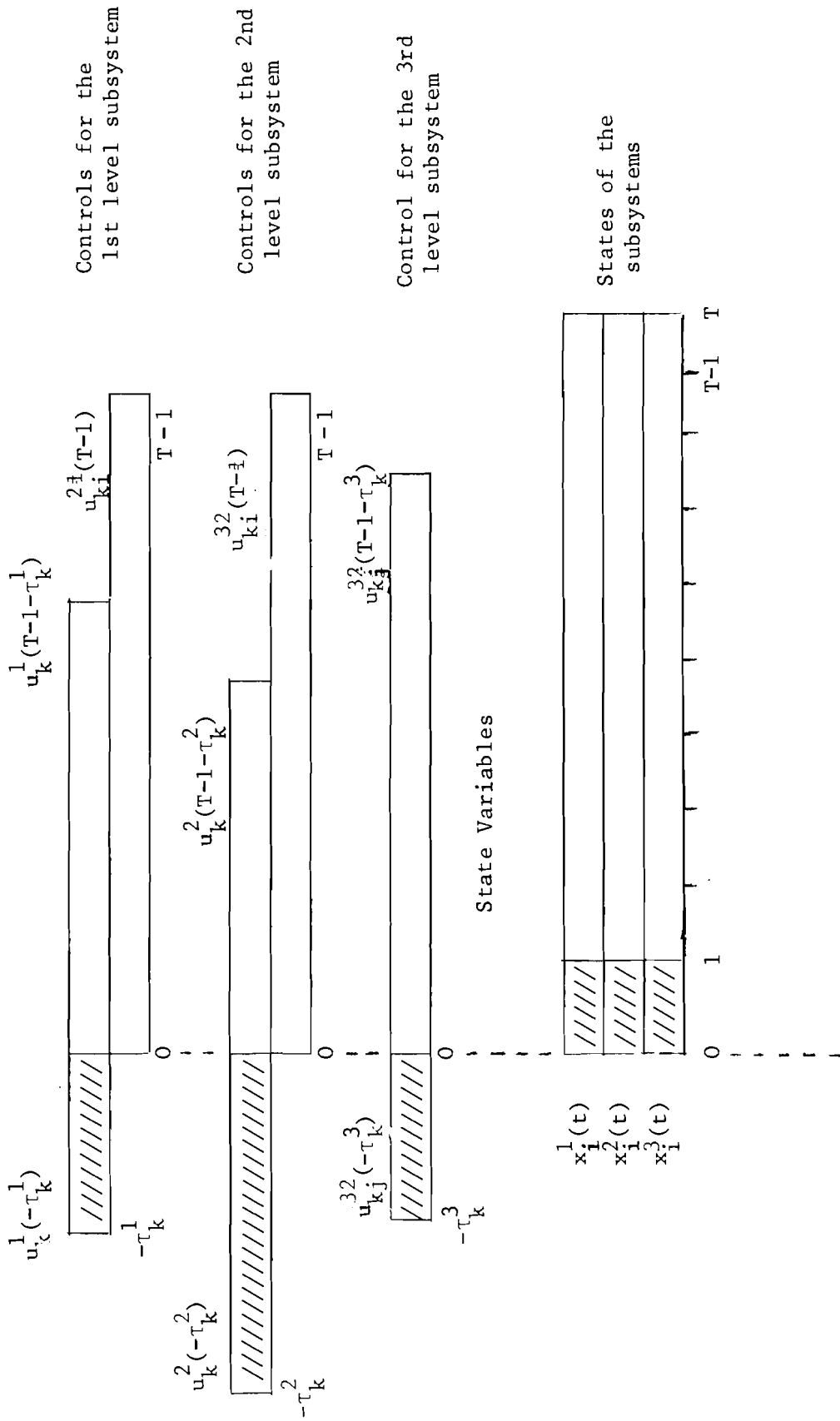
$x_i^1(t)$
$x_i^2(t)$
$x_i^3(t)$

0 1

T-1 T

Figure 4. Durations of control and states variables for different level subsystems
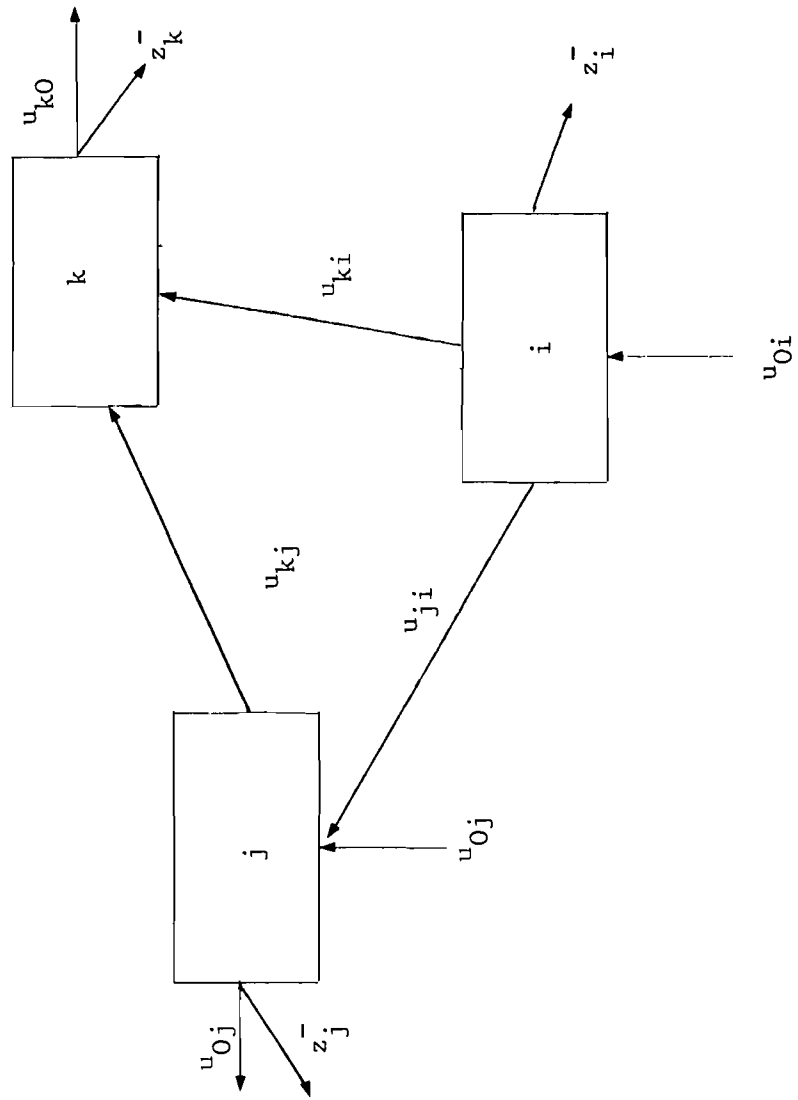
Figure 5. Flows in Manpower Models

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