RM-78-17

THE DEMOGRAPHY OF LABOR FORCE PARTICIPATION

Frans Willekens

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Preface

The Human Resources and Services Theme within the Human Settlements and Services Area is currently conducting research on health care systems and on nutrition. As part of a general exploratory evaluation to determine whether research on manpower should be accorded Research Task status within this Theme in the future, the Area held a small and informal task-force meeting in February, 1978, on which occasion this paper was presented.

The February task-force meeting led to the conclusion that the principal objective of manpower research in HSS should be the development of models and theoretical explanations of aspects of manpower supply, manpower demand, and manpower forecasting, with a focus on national and sectoral problems in both the more developed and the less developed countries of the world today. Expected results could be improved models and a better understanding of problems related to changing labor force composition, shortages of manpower in critical service sectors such as health, the rising cost of pensions, and the declining confidence of policymakers in the usefulness of manpower forecasting models.

This paper, the first of a series on manpower analysis, focuses on the demographics of manpower. It demonstrates how the analytical apparatus of the demographer can be applied in studies of manpower supply and stands as a contribution to the current state of the art in manpower analysis.

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Human Settlements
and Services Area

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Papers of the Manpower Study

1. Frans Willekens, *The Demography of Labor Force Participation*, RM-78-17
2. Anatoli Propoi, *Models for Educational and Manpower Planning: A Dynamic Linear Programming Approach*, forthcoming RM

Abstract

This paper illustrates the demographer's perspective of labor force analysis and shows how recent methodological developments in demography, in particular in multiregional demography, can fruitfully be applied to manpower studies. First, curves of age-specific labor force participation rates are investigated and it is shown how their universal features enable one to describe the schedules by a limited number of parameters. Next, an increment-decrement table of working life is developed and compared with the conventional technique of working life table construction. Finally, an improved labor force projection model is presented.

Acknowledgements

I gratefully acknowledge the suggestions and comments of Andrei Rogers and the research assistance of Oleko Tambashe.

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The Demography of Labor Force Participation

0. INTRODUCTION

Demographers have been interested in manpower analysis for many years. They have developed their interpretation and perspective on the study of the manpower segment of the population. Manpower is equivalent to the economically active population or labor force. This interpretation of manpower is based on the economists' distinction between producers and consumers. The active population can be distinguished from other segments of the population by its function of producing goods and services for the whole of the population, (United Nations, 1973, p.293). Within this framework, one can identify the active population, not only as an economic category, but also as a demographic one thus allowing the research apparatus of demography to be applied to manpower studies.

It is the purpose of this paper to illustrate the (mathematically oriented) demographer's approach to manpower analysis. The first section describes the parameters used in demographic investigations, namely the age-specific activity or labor force participation (LFP) rates. The regularities in labor force participation schedules have induced demographers to represent these schedules by a limited number of indices. The following sections of the paper review the concept of working life tables. It is not surprising that an early attempt of demographers to study the labor force included the calculation of life tables of the working, or active population. Today, many countries publish working life tables. Surprisingly, however, the technique of constructing such tables has not been improved since the design of the first tables in the late forties. It is only very recently that one can observe methodological innovations. Hoem and Fong (1976a), have developed a working life table which does not rely on the unrealistic assumptions required in conventional working life tables. Their increment-decrement life table approach may be reformulated in simpler terms and with less restrictive assumptions, by applying recent findings of multi-regional demography. In the fifth and final section, we review existing techniques for labor force projection and propose a new projection model.

Before proceeding to the study of the labor force participation schedule, however, some attention should be devoted to measurement problems of the LFP-rates. The economically active population, or labor force, has been defined by its function. According to United Nations recommendations, it consists of those individuals who furnish the supply of labor for production of economic goods and servides (United Nations, 1967, pp.61-63).* Economic goods and services are those items represented in the national accounts. Hence, the members of the labor force are the producers of the national income. Included are persons who work for wages or salaries (in civilian jobs or in the armed forces); self-employed persons, and employers who work for profit; and persons who assist without pay in a familial income-producing enterprise, such as a farm, or shop (United Nations, 1973, p.293; Durand, 1975, p.8; ILO Yearbook of Labor Statistics, 1976, p.3).** International standards include unemployed persons

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* Sometimes, a distinction is made between active population and economically active population. The first category includes then, all the persons in the active age groups (generally 15-64 years for men and 15-59 for women). In this paper, active population and economically active population are treated as being interchangeable.

** This is the "gainful worker" approach to identifying members of the active population. It considers as economically active any person who usually works at an income-producing job or assists in the production of marketable goods. The second or "labor force" approach considers as a member of the labor force any person employed or seeking employment regardless of his usual activities (Denti, 1968, p.526).

as well as those actually employed.* Not included in the labor force are those persons who do not work at income-producing jobs: housewives, students, retired and disabled workers, volunteer workers, etc. The goods and services they produce are not considered as income, because they are generally not paid for their work. Not all countries, however, follow the same practice in measuring the labor force, and a list of important divergences is given by United Nations (1958), and by Durand (1975, pp.9-14). For the definition of labor force, followed by the U.S. Bureau of Labor Statistics, see Bowen and Finegan (1969, pp.7-8).

The purpose of collecting labor force statistics is to provide information on the amount of human resources available for productive purposes. The number of persons in the labor force is only an approximate measure of the productive capacity of the economy. The real labor input per member of the active population depends on the unemployment level, the share of persons engaged in part-time or seasonal work, weeks of employment per year, and the average working hours per week; in short, the degree of involvement of members of the labor force in activities that contribute to the production of income.** The real labor input (utilization of available labor) is, of course, influenced by labor demand or employment considerations. Bowen and Finegan (1969, Chapters 4 and 6), have shown that the labor force may expand considerably in response to increasing labor demand measured by relative wages, and may contract when wages fall and unemployment grows. Gregory and Sheehan (1973), prove that labor force participation rates are sensitive to cyclical movements in the economy and in particular to the state of the labor market. High unemployment discourages job

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* In socialist countries, the definition "labor force" excludes the unemployed (or unemployment is nil).

** Bowen and Finegan (1969, pp.92-96), calculate full-time equivalent LFP rates. They reflect both the labor force participation and hours worked dimensions of labor supply. Assuming 40 hours as a "full-time" working week, a full time equivalent LFP rate is the product of LFP rates and the ratio of mean hours worked per week and 40 hours.

seekers and they may drop out of the labor force. This phenomenon is often referred to as the discouraged worker effect. It is particularly relevant for the female labor force since women, when they become unemployed, withdraw more easily from the labor force than men.

In addition to the quantitative aspect, there is the qualitative aspect of the labor supply, i.e., the determinants of the labor productivity: skills, experience, aptitudes, education, health and motivation. Together, the quantitative and the qualitative aspects determine the productive capacity of the economy. In order to measure the size of the producers in proportion to the consumers or total population accurately, both aspects should be taken into account. In the first instance, however, there is a need to include in the labor force measures the actual degree of participation in productive activities by members of the labor force (Durand, 1975, p.159).

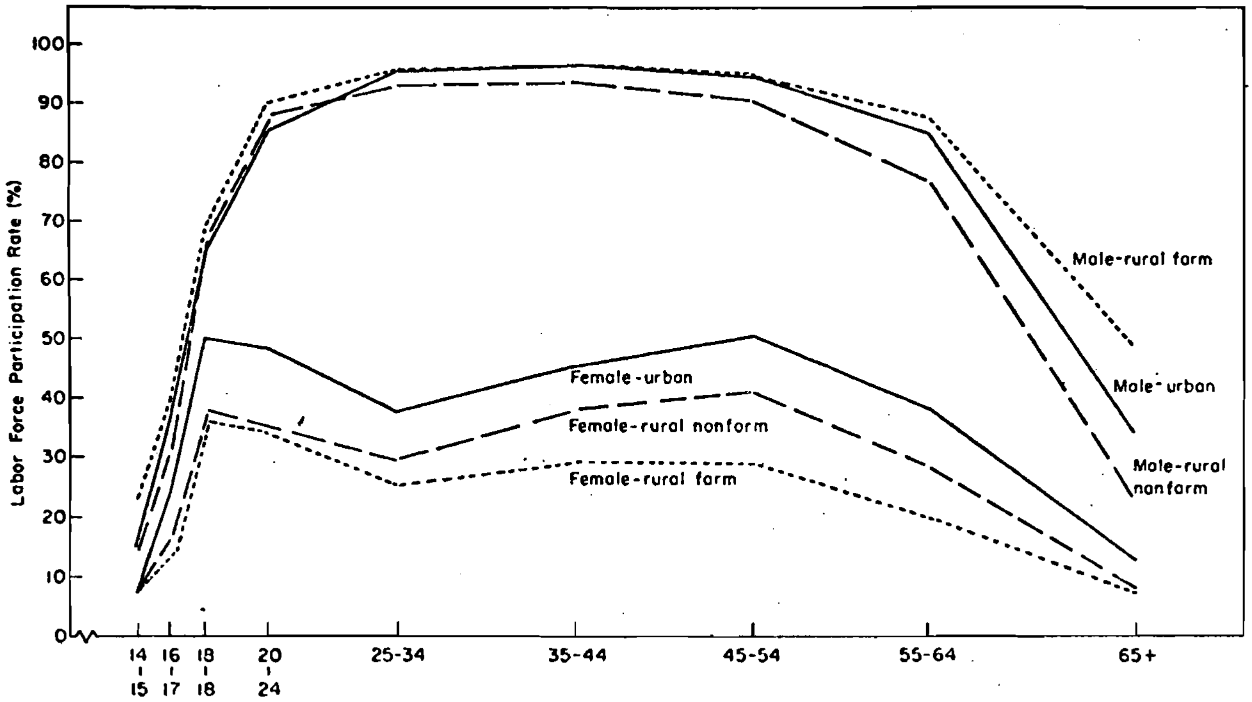
1. LABOR FORCE PARTICIPATION (LFP) SCHEDULE

The relative size of the labor force is measured by the ratio of producers to consumers. Whereas everyone is a consumer, only the persons involved in income producing activities are defined as producers. The ratio of producers to consumers is represented by the crude activity rate (CAR). It is defined by Durand (1975, p.15) as the number of labor force members, ten years of age and over, per 100 of the total population. Crude activity rates confound the effect of age- and sex-specific activity rates, and of the sex and age composition of the population. It is a weighted average of age- and sex-specific rates, the weights being the shares of each age group and sex in the total population. Changes in crude activity rates may, therefore, be caused by changes in the age composition, sex structure, or in the labor force participation by age. In a study of the dynamics of labor force participation it is necessary to isolate the effect of each individual component of change.

This section is devoted to curves of age- and sex-specific activity rates. First, we investigate empirical regularities of LFP schedules, and review ways to represent these curves by a limited number of parameters. Next, we summarize the major patterns of change of LFP curves during the course of socio-economic transformation.

1.1 Characteristics and Representation

Age-specific labor force participation rates show a universal pattern (Durand and Miller, 1968, p.133). However, unlike fertility and mortality schedules, LFP schedules show more variations. In particular, the sex-specificity of the LFP schedules is important. Figure 1 shows male and female age-specific activity rates for urban and rural (farm-nonfarm) United States. The differences in the shapes of the male and female schedules require sex-specific analyses of LFP rates.



Age Group

Figure 1. Labor force participation rates by place of residence, sex, and age, census week of 1960.

Source : Bowen and Finegan (1969) p.548

The male LFP curve has a very regular pattern. Labor force participation starts at about 15, and reaches a peak around age 30. Between the mid-thirties and the mid-fifties, the proportion of the male population in the labor force drops very gradually, but then declines rapidly due to retirement. This regularity led Durand (1975, p.22) to distinguish three phases in the typical cycle of working life for a cohort of male population:

- the ages of entry into the labor force, defined as those younger age groups in which the activity rate has not yet reached its maximum (i.e., up to about age 30);
- the prime working ages, when the activity rate remains on a high plateau (between 30 and 45 years);* and
- the ages of retirement, when the activity rate drops off, gradually at first, but rapidly for higher age groups.

Since most of the entries into the labor force take place before the age of 20, Durand divides the ages of entry into primary ages of entry (under 20 years) and the ages of late entry (20-29). Similarly the ages of retirement are divided into ages of early retirement (45 to 65), and primary ages of retirement (beyond 65).

Regularity in the female LFP schedules is much less than in the male curves. The female curve is generally not unimodal, but has two peaks. This is due to the pattern of entry into the labor force and of retirement, which is related to the life cycle of marriage and fertility. Although, as for males, the ages under 20 are primarily labor force entry ages, girls may drop out of the labor force at young ages to marry and have children. In several countries, on the other hand, female workers enter the labor force at later ages, when the children no longer need their close attention. Divorce or widowhood are also events determining the age of (re-) entry into the labor force. The importance of the differences of LFP rates by marital status is illustrated in Table 1 and Figure 2.

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* Prime-age males are defined by Bowen and Finegan (1969, p.39), as those men 25 to 54 years of age.

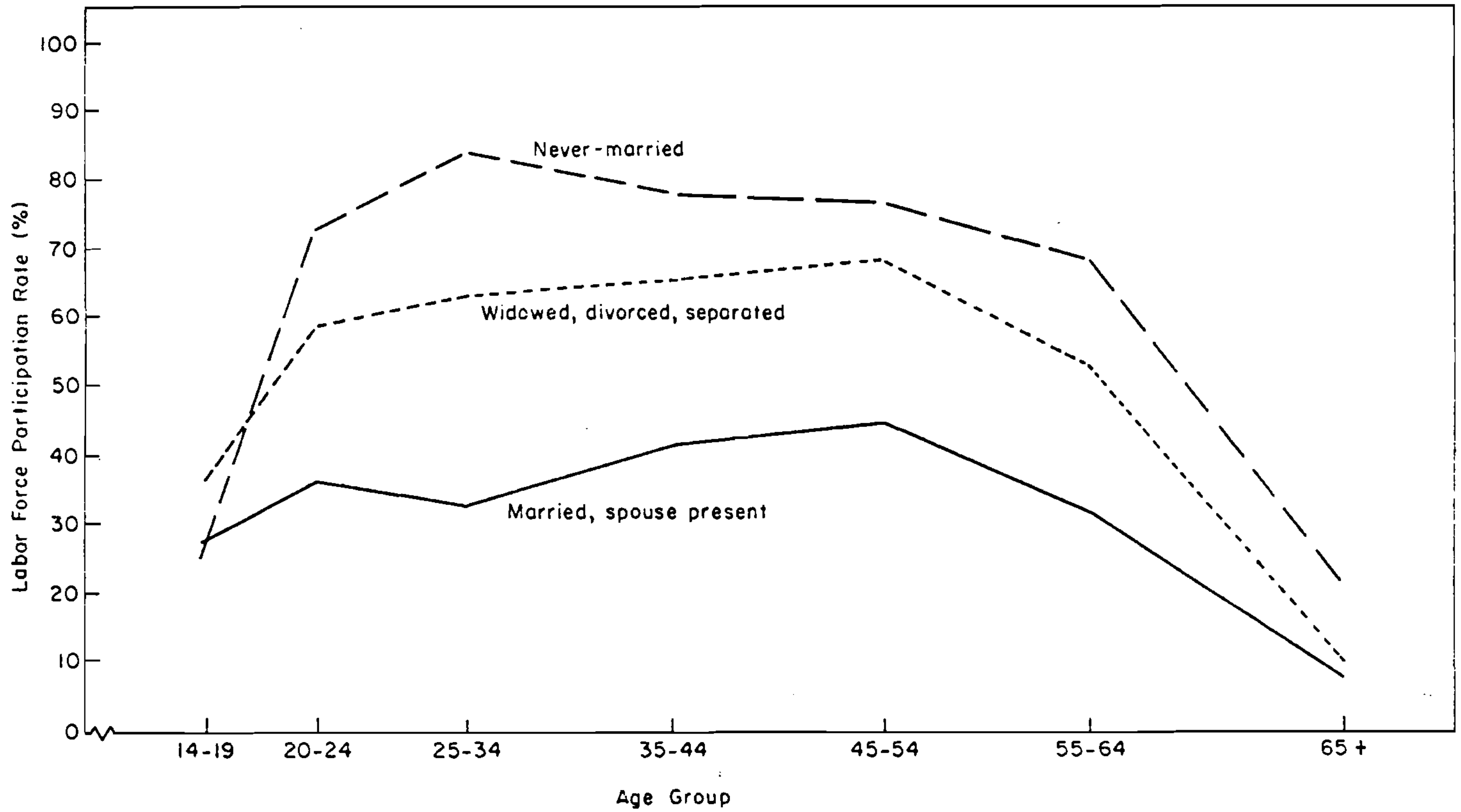

Figure 2. Female labor force participation rates by age and marital status, March 1965.

Source: Manpower Report of the President(1966), and Bowen and Finegan(1969),p.544

The LFP schedule may be analyzed in the same way demographers treat mortality, fertility and migration schedules. Instead of representing the schedule by a large number of parameters (one age-specific rate for each group), one may try to describe the curve by a function in only a few parameters. These so-called model schedules are not only useful for the comparison of different schedules, but also form a tool for the estimation of age-specific rates in cases when data do not exist or are inadequate. The usefulness of this has been mentioned by Shryock, Siegel, et al. (1973, pp.855). However, the regularities of LFP schedules do not compare with those of, for example, fertility curves, which may accurately be represented by two parameters: one measuring the level (gross rate of reproduction) and another expressing the shape (mean age of childbearing). A number of ways have been proposed to represent the level and the shape of LFP curves.

The level of the curve may be measured by the <u>gross years of active life-index</u> (Farooq, 1975, p.44; Durand, 1975, p.226):

$$GYA = \int_0^{\omega} \bar{w}(x)\,dx,$$

where ω is the highest age and $\bar{w}(x)$ is the activity rate or LFP rate of the age group x to x + dx. This expression may be numerically evaluated as follows:

$$GYA = 5 \sum_0^{z} \bar{w}(x) \quad ,$$

where the age groups are of 5 years, z is the highest age group, and w(x) is the activity rate of age group (x,x + 4). The concept of gross years of active life is analogous to the gross migraproductive rate in migration analysis. Note that, if $\bar{\alpha}$ is the lowest age group of the active population (10-14, say) and $\bar{\beta}$ is the highest age group (55-59, say) then GYA is given by

$$GYA = 5 \sum_{x=\alpha}^{\bar{\beta}} w(x) \quad .$$

The GYA represents the expected number of years in the labor force in the absence of mortality, i.e., no deaths occur before or during the active ages and therefore, no working years are lost by mortality. An index, taking mortality into account is the net years of active life index (Farooq, 1975, p.45).

The advantage of the gross years of active life index as a summary measure of the level of the age-specific activity rates is that it is independent of the age structure of the population. In this regard it is a better index of the level of labor force participation than the crude activity rate (CAR), which combines the effect of age composition and age-specific LFP rates. A comparison of the GYA index and the CAR is given in Table 2.

An alternative measure of the level of the labor force participation which is free from the effect of the age distribution of the population, is the standardized activity rate (SAR). The SAR is a weighted average of age-specific activity rates or LFP rates of a given sex. The weights are defined according to the age composition of a model population. This measure has been used by Denti (1968, p.535), and Durand (1975, p.20) among others. Durand calculated the sex-specific SAR's by weighting the age-specific LFP rates of each sex with the age structure of a population of ten years and over that has approximately the same age profile as the world population in 1960. The relative age composition of the model population was derived from the Coale-Demeny tables on stable populations (Durand, 1975, p.224).* In another study, the United Nations (1976, Table 3.4),

* For countries where age classifications of population and labor force were lacking, Durand simply used the percentage of the labor force among the population ten years of age and over. The percentage is labeled the refined activity rate (RAR). Differences between RAR and SAR are small for males but can be substantial for females.

Table 1. Average age-specific activity rates of females classified by marital status in twelve industrialized countries,[a] according to results of censuses between 1948 and 1956.

Source : United Nations (1973), p.305

Age (years)	Single	Married	Widowed, divorced or separated
15 and over	63.8	19.2	27.8
15-19	55.7	24.3	b
20-24	80.9	25.4	b
25-34	79.6	20.1	63.5
35-44	74.2	21.1	61.7
45-54	69.1	20.4	51.6
55-64	52.1	14.1	30.3
65 and over	18.5	6.5	8.5

[a] Unweighted means of percentages of economically active among female population of specified age and marital status: censuses of Australia, Canada, England and Wales, France, Federal Republic of Germany, Ireland, Israel, Japan, New Zealand, Norway, Sweden and the United States of America.

[b] Numbers too small for calculation of reliable activity rates.

Table 2. Urban and rural activity rates and gross years of active life for selected countries.

Source : United Nations (1973), p.300

Areas		Crude activity rate (labour force as percentage of total population)			Gross years of active life in age range 15-74 years	
		Both sexes	Male	Female	Male	Female
Guatemala, 1950:	Urban	37.7	57.1	19.6	53.9	15.7
	Rural	33.0	59.5	5.1	57.3	4.5
Indonesia, 1961:	Urban	32.8	48.8	16.7	47.8	16.0
	Rural	36.5	53.3	20.1	54.0	20.7
Japan, 1955:	*Shi* of 50,000 or more inhabitants	42.2	56.6	37.1	49	21
	Gun	41.1	54.7	39.9	56	36
United States, 1960:	Urban	40.6	55.1	27.0	47.1	22.5
	Rural-farm	36.0	54.7	15.4	50.0	14.1
Poland, 1960:	Urban	41.8	53.8	31.1	45.7	24.7
	Rural	52.4	56.4	48.7	54.6	44.2

used the age composition of the Netherlands population as of 1947 to compute standardized activity rates.

Another approach to the study of the impact of the population's age composition on the labor force participation, is stable population analysis.

The age structure of a stable population is uniquely determined by a set of age-specific mortality and fertility rates. By using model mortality and fertility schedules, the age structure of the population may be represented by a single fertility and a single mortality parameter. This is particularly interesting for the study of the indirect effects on labor force participation of changing mortality and fertility levels. This approach is illustrated in Table 3. It gives the values of a number of LFP statistics for various levels of fertility and mortality, and for two different LFP schedules.

Table 3 shows that the long term effects of changing activity rates and mortality levels, are outweighed by those of changing fertility.

Different model populations would, of course, yield different values of the SAR for a given LFP schedule. Therefore, the use of the GYA index may be supported, since it is not affected by an arbitrary choice of a standard age distribution . It has , however, the disadvantage of giving equal weight to the activity rates for all groups, and therefore does not take into account the relative importance of age groups. The effect of the age-sex structure of the population is measured by the difference between the CAR and the GYA or the SAR. The difference between the CAR and the SAR of the total (both sexes) population is called the age-sex index (ASI) (Durand, 1975, p.81; the components of ASI are analyzed in Appendix E). The ASI are a measure of the economic impact of the age structure, preferable to the dependency ratio, which

Table 3. Total male activity rates, annual replacement rates and ratios for the male labor force and annual rates of entrance, retirement, and losses by death, in stable population models with various levels of fertility and mortality and two schedules of male age-specific activity rates.

Source : United Nations (1973),p.310 and p.320

Gross reproduction rate	30 years expectation of life at birth A	30 years expectation of life at birth B	50 years expectation of life at birth A	50 years expectation of life at birth B	70 years expectation of life at birth A	70 years expectation of life at birth B
Male labour force as percentage of total male population						
4.0	52	46	49	43	46	40
3.0	58	52	55	49	52	46
2.0	66	60	63	57	61	54
1.0	78	71	76	67	74	65
Replacement rate (entrants less retirements and deaths) per 1,000 active males						
4.0	24.0	21.9	39.1	36.9	47.6	45.0
3.0	14.1	12.2	28.8	26.9	37.4	35.6
2.0	– 0.8	– 1.4	14.2	13.3	22.9	22.0
1.0	– 25.2	– 24.8	– 10.3	– 10.4	– 1.7	– 1.8
Entrants into the labour force per 1,000 active males						
4.0	44.8	44.7	49.3	48.9	52.4	51.4
3.0	37.1	37.6	41.0	41.0	43.7	43.7
2.0	27.5	28.8	30.3	31.4	32.7	33.7
1.0	14.7	16.5	16.1	17.8	17.5	19.2
Retirements from the labour force per 1,000 active males						
4.0	0.7	2.5	0.9	2.8	0.9	2.9
3.0	0.9	3.3	1.3	3.9	1.3	3.9
2.0	1.7	5.0	2.3	6.0	2.7	6.2
1.0	3.3	9.2	5.1	11.4	6.3	12.1
Deaths per 1,000 active males						
4.0	20.1	20.3	9.3	9.2	3.9	3.5
3.0	22.1	22.1	10.9	10.2	4.9	4.2
2.0	26.6	25.1	13.8	12.1	7.1	5.6
1.0	36.6	32.2	21.4	16.8	12.9	9.0
Replacement ratio (entrants into the labour force per 100 retirements and deaths)[a]						
4.0	220	200	480	410	1090	800
3.0	160	150	340	290	700	540
2.0	100	100	190	170	330	290
1.0	40	40	60	60	90	90

Note: Column A: Average age-specific activity rates for agricultural countries.

Column B. Average age-specific activity rates for industrialized countries.

a Ratios rounded to the nearest multiple of 10.

is used by Coale (1969). This measure is defined by the age structure of the population only. It is the ratio of the population outside active age groups to the population inside active age groups. The dependency ratio does not depend upon the extent to which persons participate in the labor force.

The shape of the curve of age-specific activity rates has a uniform pattern (Figure 1). However, a single parameter, such as the mean age of the LFP schedule, provides a weak representation of the shape of the curve, in particular for the female LFP schedules. Durand (1975), considers different types of shapes of the male and the female curves. The fundamental idea is to divide the varying observed age patterns of LFP rates into more homogenous categories. The procedure is analogous to the one followed by Coale and Demeny (1966) in classifying observed mortality curves into four groups (North, East, South, West) and the one followed by Rogers and Castro (1976), and Rogers, Raquillet and Castro (1977), in grouping migration curves into "families" of schedules. The grouping of LFP schedules in homogenous categories is a first step to the development of model LFP schedules.

Male LFP curves are characterized by the mean age entry and the mean age of retirement. In other words, it is the levels of labor force participation in the first phase (primary ages of entry) and the last phase (primary ages of retirement) that are important to describe the shape of the male LFP curve (Figure 3). Durand (1975, pp.24-25) distinguishes several shapes of male LFP schedules, dependent on the activity rate in the first and last phase. An activity rate is classified as high (H), medium (M), or low(L), according to whether it is in the upper, middle, or lower third of the average distribution of all countries. The consideration of three classes in each of the two phases leads to nine different types of age patterns of LFP rates. Each type is denoted by two letters. The first relates to the primary ages of entry, and the second relates to the primary ages of retirement. For example, a combination of

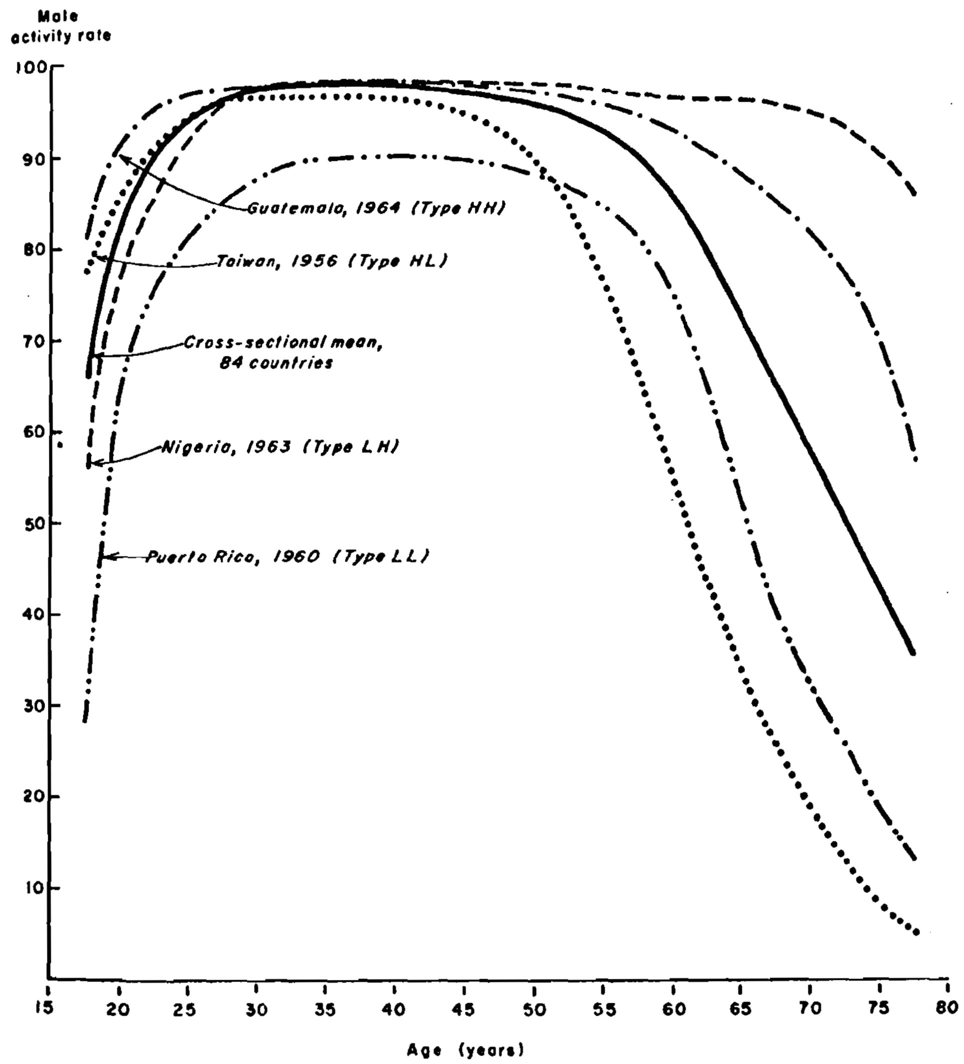

Figure 3. Age patterns of male activity rates.

Source: Durand (1975), p.23

a high activity rate for ages 15 to 19, with a low rate for 65 and over, is denoted by HL. A classification of eighty-four countries in these nine types gives the following results (Durand, 1975, p.25):

HH. 14; MH. 10; LH. 4; HM. 8; MM. 11;

LM. 9; HL. 6; ML. 7; LL. 15.

The shapes of the female LFP curves are classified in eight types, four principal types and four subtypes (Durand, 1975, pp.38-39). The classification is based on the occurrence of a single or a double peak in the LFP curve and on the location of the peak(s):

A.	Central peak or plateau	(14)
B.	Late peak	(8)
C.	Early peak	
	C-1 Without shoulder	(21)
	C-2 Peak and shoulder	(14)
D.	Double peak	
	D-1 Early peak higher	(19)
	D-2 Late peak higher	(8)

The distribution between the types of the eighty-four countries studied is given in parentheses. Type A resembles the typical pattern of male activity rates (high peak between 30 and 44 years). In type B, the peak is located at an age above 45 years, whereas in type C, the peak is at an age below 30 years. Finally, type D is characterized by two peaks, separated by a trough. The distinction between C-2 (peak and shoulder) and type D (double peak) is somewhat arbitrary. A second hump is considered to be a peak if the activity rate declines by more than one-tenth in consecutive age groups from an early peak and rises by more than one-tenth to a later peak. The various types are illustrated in Figure 4. The different patterns may be related to differences in the way marriage and motherhood affect labor force participation.

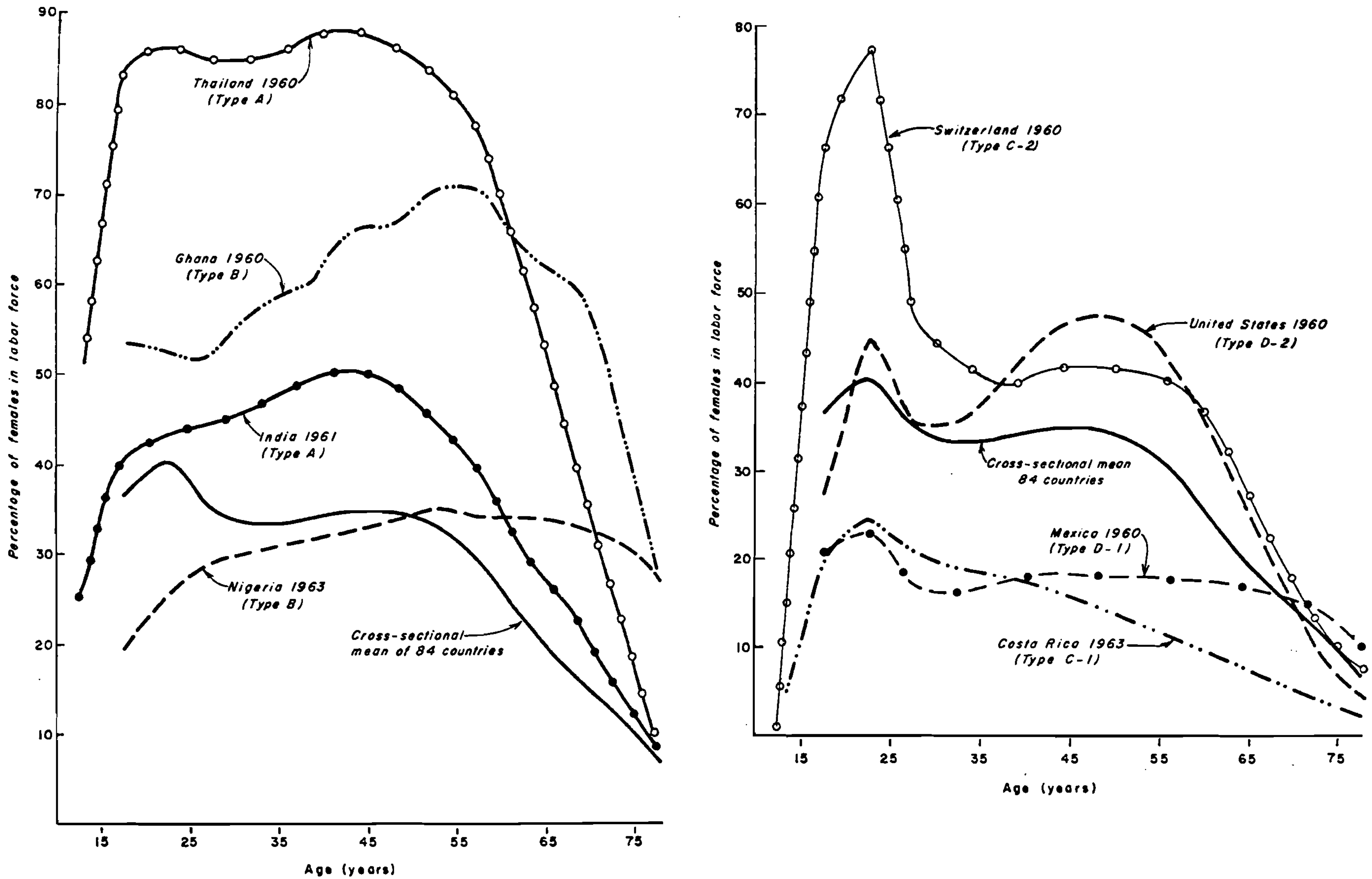

Figure 4. Types of age-patterns of female activity rates.

Source: Durand (1975), pp.40-41

1.2 Changes in LFP Schedules

Variations in male LFP schedules occur chiefly in the age distribution of the entries and retirements, in particular, in the primary ages of entry and retirement. Since entry into the labor force is largely determined by graduation from school, the curve of age-specific LFP rates can be linked to a curve of age-specific school enrollment rates. Similarly, the LFP schedule may be linked to a curve of age-specific retirement rates. As school enrollment increases and extends over higher ages, and as retirement occurs at younger ages, the width of the LFP schedule will decrease and fewer people will be available for the productive sector. This process has been observed in the postwar period (Durand, 1975; United Nations, 1973, pp.301-303; Bowen and Finegan, 1969, Chapters 11 and 14).

Recent changes in the female LFP rates in countries around the world have a common feature. As with males, the female activity rates are declining for ages under 20 and over 60. Changes in activity rates for ages between 20 and 60 are less regular. In a number of countries including the United States, one could observe a shift from a peak and shoulder to a double peak pattern (Figure 5). An equal number of countries experienced a shift from a peak and shoulder to a single peak pattern, (United Nations, 1973, pp.307-309).

Primary ages of entry and retirement are important components-of-change of labor force participation. Not only recent changes in LFP rates, but also variations during the course of economic development are largely limited to the primary ages. This is especially true for males. The curtailment of men's working life at both ends during the development process contributes to a decline in overall labor force participation.*

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* It should be stressed that the predominant component-of-change of the overall labor force participation is the age distribution of the population. Fertility decline with economic development has therefore an indirect, but large impact on labor force participation. A drop in the proportion of child to adult population tends to raise the CAR. However, the effect of changing fertility is demonstrated only in the long run.

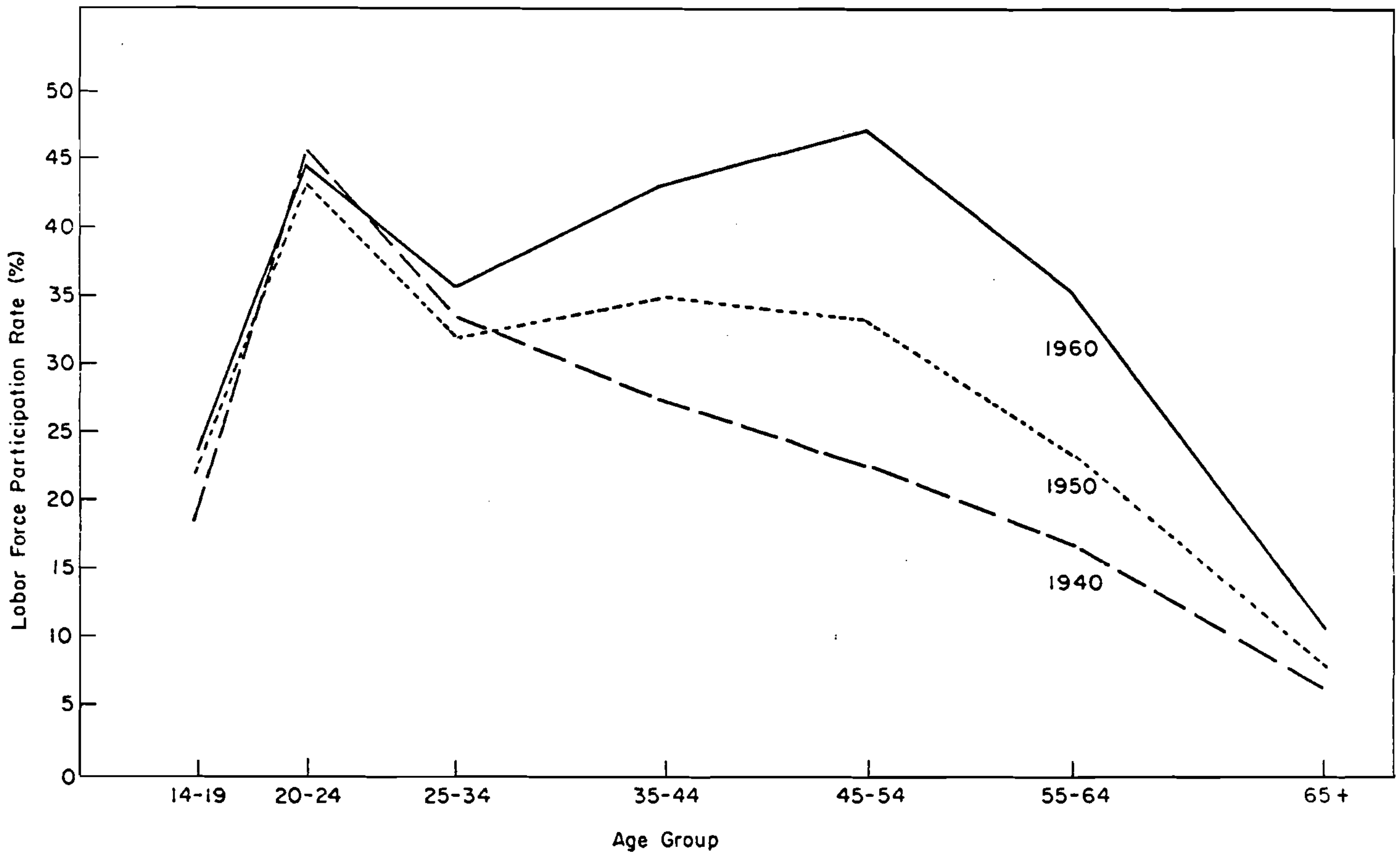

Figure 5. Female labor force participation rates by age: United States, 1940, 1950, 1960.

Source: Bowen and Finegan (1969), p.564

A cross section of census statistics shows a limited increase of age-of-entry and an important decline of the retirement age with economic development, (Table 4; Figure 6). A large proportion of the decline in GYA with development is due to declining age-specific LFP rates in age groups above 65. In the prime working ages, the LFP rates remain on a high plateau during the development process. Therefore, it is not that fewer men take part in the labor force, but that the average working life decreases.

Durand (1975, p.110) found that the decrease of male activity rates in ages of labor force entry and retirement during the course of economic development typically follows a logistic curve. The drop is slow, if at all, during the early stages of development, more rapid during middle stages, and more slow again during advanced stages. This implies a transition from one relatively stable state to another. The transition of the LFP level shows an analogy with the demographic transition in which mortality and fertility levels go from high to low along a logistic path. Because of this analogy, the change of the LFP level during the course of economic development may be labeled the LFP transition.

As in the case of males, female LFP rates in the primary age of entry and retirement decrease with economic development, although the pattern is less regular than for males. In contrast to males, female LFP rates in the prime working ages may either increase or decrease during the course of development, depending on cultural settings and other circumstances. The result is a mixed picture of rising trends in LFP rates and GYA in some countries and falling trends in others. No distinct pattern in the individual countries can be observed, although the average result shows some regularity. However, neither cross-sectional nor time-series data fully support the so called U-hypothesis, which states that female activity rates decrease during early stages of economic development and increase again during later stages. Durand (1975, p.150) concludes, therefore, that the

U-shaped pattern observed in some data, in part reflects influences of extraneous factors (unrelated to economic development) as well as errors and biases in the measurements.

More important than economic development for variations in female LFP rates, are social and cultural norms and their susceptibility to changes (see also United Nations, 1973, p.305). For example, the female SAR is 11% in Arab countries; 45% in tropical Africa, where women are dominant in the agricultural labor force; 30% in North West Europe; and 50% in East Europe, where the participation of women in the production process is a matter of social policy. A constancy in the observation is, however, that in countries where great increases in female activity rates have been recorded there is a greater participation of married women in the labor force. Besides changing attitudes, a growing freedom from maternal cares and an expanding demand for labor in occupations regarded as suitable for women have contributed to this trend.

Table 4. Levels of male and female age-specific activity rates in countries at different levels of economic development, cross-sectional census data.

Source : Durand (1975), p.95 and 133

	a. Male rates (%) Level of Development						b. Female rates (%) Level of Development					
Age Group	Total	I	II	III	IV	V	Total	I	II	III	IV	V
15-19	65.6	75.5	69.6	62.8	60.5	61.6	36.4	50.6	28.1	22.1	34.0	51.5
20-24	90.5	91.5	90.8	90.0	89.7	90.5	40.8	52.6	29.4	24.4	45.6	55.6
25-29	96.3	96.5	96.4	96.0	96.1	96.5	34.4	52.6	27.2	20.1	39.9	37.2
30-34	97.4	97.3	97.5	97.0	97.3	97.9	33.2	53.5	27.3	19.1	37.9	33.5
35-39	97.5	97.7	97.7	97.0	97.3	97.8	33.9	54.1	28.1	19.6	38.1	34.8
40-44	97.0	97.5	97.5	96.5	96.8	96.9	34.8	54.6	29.4	20.0	39.0	36.1
45-49	96.4	97.3	97.2	95.5	95.7	96.7	34.5	53.5	29.5	19.6	38.3	36.3
50-54	94.4	96.0	95.8	92.6	92.9	95.0	32.5	50.5	28.6	18.0	35.2	34.6
55-59	90.4	93.8	93.6	87.7	86.8	91.0	28.2	43.6	26.6	15.9	28.6	30.1
60-64	80.5	86.7	88.0	78.4	72.3	78.3	21.8	33.4	22.8	13.2	21.6	20.6
65-69	65.0	78.6	80.0	66.4	56.1	45.8	16.4	26.6	19.5	10.7	15.3	11.9
70-74	50.7	64.2	69.9	53.4	40.3	27.4	11.7	19.5	16.2	7.8	10.4	6.2
75+	35.2	47.5	53.2	37.1	24.5	15.2	7.2	12.4	11.6	4.8	5.2	2.9
GYA	52.86	55.99	56.36	52.51	50.31	49.54	18.31	27.89	16.21	10.76	19.48	19.57
YA_{65}	7.55	9.51	10.16	7.85	6.05	4.43	1.77	2.93	2.37	1.16	1.55	1.05
SAR	76.8	81.0	81.1	76.0	73.7	73.1	32.0	47.8	26.3	19.5	32.0	34.2

GYA: Gross years of active life.

YA_{65}: Gross years of active life beyond the age of 65. (5 times the sum of LFP rates of age-groups above 65 years).

SAR: Standardized activity rate (assuming an age structure close to that of the world population in 1960).

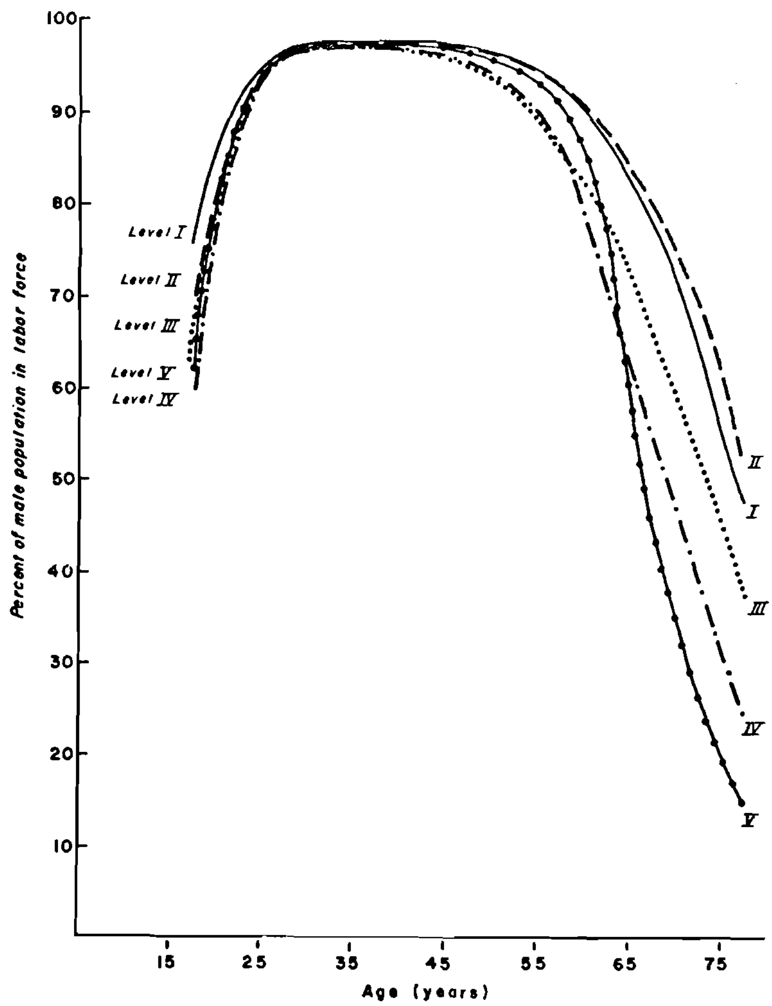

Figure 6. Age-specific activity rates of males in countries at different levels of development (mean values of rates according to cross-sectional censuses).

Source: Durand (1975),p.96

Based on a cross section of a hundred countries. The level of development is measured by an index composed of two indicators: energy consumption per head and the percent share of the nonagricultural sector in total employment or labor force. The countries are ranked in ascending order of each indicator, and the rank numbers are added. The countries are then divided in quintile groups (Durand, 1975, p.78).

2. THE TABLE OF WORKING LIFE

The working life table shows several similarities with a conventional life table. It represents the life history of a hypothetical population or cohort. In addition to the mortality experience, the working life table describes the labor force participation pattern. It focuses on a subgroup of the population, namely the labor force, and gives a number of useful statistics for this subgroup. In this section, we will review the conventional technique of constructing working life tables which is based on a number of unrealistic assumptions. The next section will be devoted to a new approach to working life table construction. It starts from the idea of increment-decrement life tables and applies the methodology developed in multiregional demography.

The table of working life was developed by Wolfbein in 1949. It is a modification of a conventional life table which summarizes the mortality experiences of a population, and therefore all its losses, at a point in time. The table of working life, in addition to representing the mortality experience of a population, shows the effects of entering and leaving the labor force. Their usefulness has been described by Durand and Miller as follows:

> (working life tables)are useful in studying the processes of growth and structural change in the labour force, estimating such quantities as lifetime expectations of earnings, evaluating returns from investment in human capital, assessing economic implications of change in activity rates and age structures of the populations, etc. (Durand and Miller, 1968, p.19)

Working life tables have been produced for several countries. An illustrative list of national tables is given by the United Nations (1973, p.318) and by Hoem and Fong (1976a, pp.6-7).

Conventional tables of working life are based on three general assumptions (Kpedekpo, 1974, p.292):

i. Persons who enter the labor force, do so prior to the age at which the activity rate reaches its maximum. This implies that the LFP curve has a maximum, i.e., is unimodal.

ii. Prior to the age of maximum labor force participation, no survivors retire from the labor force and become members of the inactive population. Retirement only occurs at ages beyond the age of maximum activity rate. Once a person has left the labor force, he can never return.

iii. The rates of mortality at each age are the same for economically active and inactive persons.

The first two assumptions are usually satisfied for males, but not for females. The female LFP curve is frequently bimodal, in particular in North American and European countries. To get around this problem, Garfinkle (1967) constructs working life tables for women by family status, and hence eliminates the problem of bimodality. The third assumption is not true since the age-specific mortality of an active population generally exceeds that of an inactive population. These three assumptions may be dropped when constructing increment-decrement tables of working life.

2.1. Construction of a Table of Working Life

Table 5 is a typical working life table. All the columns are derived by applying the mortality rates and LFP rates to a hypothetical population or cohort. The LFP rates may be replaced by rates of labor force entry and of separation, using assumption (i) and (ii) (Fullerton, 1971, pp.51-52). The computational procedure of constructing a working life table has been given by Wolfbein (1949), Durand and Miller (1968, Annex A), and by Fullerton (1971, pp.52-54) among others.

Number of Persons Living [L(x)]

The elements L(x) denote the number of persons aged x to x + 1 in the stationary population. They are computed from the age-specific mortality rates only, and appear in all standard life tables. In column 2, L(x) is expressed per 100,000 births

Table 5. Selected portions of a complete table of working life, for the male population of the United Arab Republic, 1960.

Source: Shryock, Siegel et al. (1973), p.457; adapted from Durand and Miller (1968, Annex.)

Age (years)	Percent of population in labor force	Of 100,000 born alive, number living in year of age: In the population	Of 100,000 born alive, number living in year of age: In the labor force		Of 100,000 born alive, number living and in labor force at beginning of year of age	Number of man-years in the labor force remaining in the year of age and later years	Average remaining years of active life for survivors in labor force at beginning of year of age	Complete expectation of life at beginning of year of age	Mortality rate per 1,000 living in year of age	Accessions to the labor force per 1,000 living in year of age	Separations from the labor force per 1,000 in the labor force in year of age: Due to all causes	Due to death	Due to retirement
x	w_x	L_x	Lw_x	Lw_x^{*1}	lw_x^*	Tw_x^*	$\mathring{e}w_x^*$	$\mathring{e}_x$	$1{,}000\ Q_x$	$1{,}000\ A_x$	$1{,}000\ Q_x^s$	$1{,}000\ Q_x^d$	$1{,}000\ Q_x^r$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
14......	47.2	76,186	35,960	74,738	74,815	3,704,009	49.5	53.1	2.1	80.8	2.1	2.1	-
15......	55.3	76,026	42,042	74,582	74,660	3,629,271	48.6	52.2	2.1	69.9	2.1	2.1	-
16......	62.3	75,866	47,265	74,425	74,504	3,554,689	47.7	51.3	2.1	60.9	2.1	2.1	-
17......	68.4	75,703	51,781	74,265	74,345	3,480,264	46.8	50.4	2.3	48.9	2.3	2.3	-
18......	73.3	75,532	55,365	74,097	74,181	3,405,999	45.9	49.5	2.3	40.9	2.3	2.3	-
19......	77.4	75,355	58,325	73,923	74,010	3,331,902	45.0	48.6	2.4	32.9	2.4	2.4	-
20......	80.7	75,170	60,662	73,442	73,833	3,257,979	44.1	47.7	2.5	30.9	2.5	2.5	-
21......	83.8	74,979	62,832	73,554	73,648	3,184,237	43.2	46.9	2.7	27.9	2.7	2.7	-
22......	86.6	74,780	64,759	73,359	73,457	3,110,683	42.3	46.0	2.7	24.9	2.7	2.7	-
23......	89.1	74,575	66,446	73,158	73,259	3,037,324	41.5	45.1	2.9	21.9	2.9	2.9	-
24......	91.3	74,362	67,893	72,949	73,054	2,964,166	40.6	44.2	2.9	17.9	2.9	2.9	-
32......	97.6	72,377	70,640	71,002	71,140	2,387,015	33.6	37.3	4.0	1.0	4.0	4.0	-
33......	97.7	72,087	70,429	70,717	70,860	2,316,013	32.7	36.4	4.2	1.0	4.2	4.2	-
34......	97.8	71,785	70,206	70,421	70,569	2,245,296	31.8	35.6	4.4	1.0	4.4	4.4	-
35......	97.9	71,469	69,968	70,111	70,266	2,174,875	31.0	34.7	4.6	1.0	4.6	4.6	-
36......	98.0	71,141	69,718	69,789	69,950	2,104,764	30.1	33.9	4.8	1.0	4.8	4.8	-
37......	98.1	70,799	69,454				29.2	33.1	5.0	-	6.0	5.0	1.0
38......	98.0	70,445	69,036				28.4	32.2	5.2	-	6.2	5.2	1.0
39......	97.9	70,079	68,607				27.6	31.4	5.4	-	6.4	5.4	1.0
40......	97.8	69,698	68,165				26.7	30.5	5.7	-	6.8	5.7	1.0
41......	97.7	69,297	67,703				25.9	29.7	6.1	-	7.1	6.1	1.0
60......	89.9	55,475	49,872				11.6	15.1	22.2	-	48.3	21.9	26.4
61......	87.5	54,241	47,461				11.1	14.5	24.3	-	48.8	23.9	24.9
62......	85.3	52,923	45,143				10.6	13.8	26.3	-	51.4	26.0	25.4
63......	83.1	51,532	42,823				10.2	13.1	29.1	-	54.8	28.7	26.1
64......	80.9	50,032	40,476				9.7	12.5	32.9	-	59.2	32.6	26.6
65......	78.7	48,387	38,081				9.3	11.9	36.5	-	63.4	36.0	27.4
66......	76.5	46,622	35,666				8.8	11.3	40.2	-	67.8	39.6	28.2
67......	74.3	44,750	33,249				8.4	10.7	44.2	-	72.5	43.6	28.9
68......	72.1	42,774	30,840				8.0	10.2	48.4	-	77.4	47.7	29.7
69......	69.9	40,706	28,453				7.7	9.6	52.8	-	82.6	52.0	30.6

— Represents zero.
[1] Lw_x^* is based on activity rate (w_x) at age 37 and stationary population (L_x) at each age.

(i.e., radix is 100,000). Besides the number-of-persons interpretation of L(x), L(x) may be thought of as the number of years expected to be lived between ages x and x + 1 by the cohort. The two interpretations of L(x), (number of people and person years), are widely used in demography.

Number of Persons in Labor Force [Lw(x)]

The age composition of the labor force in the stationary population is derived as the product:

$$Lw(x) = w(x).L(x) \quad , \qquad (1)$$

where w(x) is the age-specific activity rate.* Note that Lw(x) also represents the expected time spent in active life between ages x and x + 1 by the cohort.

Summing the expected time spent in active life between two ages over all ages beyond age x gives

$$Tw(x) = \sum_{y=x}^{z} Lw(y) \quad . \qquad (2)$$

It is the total expected time spent in active life beyond age x by the cohort of 100,000. The variable Tw(x) is analogous to the total-person-years lived beyond age x in the conventional life table. For ages below $\bar{\alpha}$, Tw(x) is identical and equal to Tw(0). The index Tw($\bar{\alpha}$), when expressed per unit cohort, has been labeled the net years of active life. The difference between GYA and Tw($\bar{\alpha}$) represents the loss in working life due to mortality. It has been estimated for males as 4.8 years in industrialized countries, 8.5 years in semi-industrialized, and 11.4 years in agricultural countries (United Nations, 1973, p.319).

Expectation of Working Life [ew(x)]

The average remaining years of working life or expectation of worklife is

$$ew(x) = \left[\sum_{y=x}^{z} Lw(y) \right] / \ell w(x) \quad . \qquad (3)$$

It defines the average number of years of working life remaining to a person in the labor force at exact age x. The value of ℓw(x) is computed as follows (Wolfbein, 1949, p.291; Fullerton, 1971, p.54). If w(k) is the maximum labor force participation

*Although most authors derive the working life table from the L(x) column of the life table, some use the l(x) column, i.e., the number of people at exact ages x. (Fullerton, 1971,1972; Kpedekpo, 1969).

rate, attained at age k, then the value of $\ell w(x)$ is given by:

for $x > k$

$$\ell w(x) = \frac{1}{2}\left[Lw(x-1) + Lw(x)\right] \tag{4}$$

for $x \leq k$

$$\ell w(x) = \frac{1}{2}\left[L(x-1) + L(x)\right] w(k) \quad . \tag{5}$$

The consideration of the maximum activity rate for ages below k or equal to k, is to eliminate the effect of entries into the labor force in the years following age x. The working life expectancy at age x refers to the cohort of active population $\ell w(x)$. Therefore, additional entries into the labor force after x may not be considered. The implicit assumption is that all work is done by a distinct cohort of workers. The application of w(k) for $w \leq k$ implies the assumption that all entries into the labor force occur at the youngest labor force age $\bar{\alpha}$ (14, say). Between ages $\bar{\alpha}$ and k, no person is supposed to leave the labor force (assumption ii). Hence, the active population of exact age k is smaller than at age $\bar{\alpha}$ due to mortality in the intervening years.

Several authors use two measures of working life in their analysis (see e.g., Wolfbein, 1949, p.293; Fullerton and Byrne, 1970, p.35; Farooq, 1975, p.46). The first, so-called <u>labor force-based measure</u>, is identical to ew(x) and is sometimes called the <u>average remaining years of active life</u>. The second, <u>population based measure</u> is the ratio $[\sum_{y=x}^{z} Lw(y)][\ell(x)]^{-1}$, where $\ell(x)$ is the total number of people of exact age x in the life table and is known as the <u>expectation of active life</u>. The second measure assumes that all perons in the population, currently active or inactive, have an equal probability of participating in the labor force. Both measures serve different purposes. If one is interested in the working life expectancy of a person not yet in the labor force, or of a person regardless of his labor force status, the population-based measure is appropriate. This approach has been used by Durand (1948, pp. 259-265), in estimating what he calls "the average number of years in the labor force". However, if one is interested in the remaining years of work of a currently active person, the labor force based measure will be more accurate (see e.g., Durand and Miller, 1968, pp.24-27). For example, the expected working life at birth is

$$ew(0) = \sum_{y=0}^{z} Lw(y)\,\ell^{-1}(0) \quad . \tag{6}$$

Accessions to the Labor Force [A(x)]

This measure shows the net accessions to the life table labor force between ages x to x + 1 as a ratio to the life table population L(x). It gives the proportion of the population aged x to x + 1 in a life table cohort, not in the labor force, who will engage in labor activity in the next year. The ratio of the net accessions to the life table labor force is (Wolfbein, 1949, p.288)

$$A(x) = \frac{Lw(x + 1) - Lw(x)[1 - M(x)]}{L(x)} \quad , \tag{7}$$

where M(x) is the age-specific mortality rate, and the product Lw(x) . M(x) is the mortality in the labor force between x and x + 1. The quantity A(x) is not computed for ages above k, since it has been assumed that people enter the labor force only up to an age k, at which the LFP rate is maximum.

Rate of separation [MW(x)]

The rate of separation due to all causes (mortality and retirement), is defined as the ratio of the net separation from the labor force between ages x and x + 1, to the stationary labor force LW(x):

$$MW(x) = \frac{\ell w(x) - \ell w(x + 1)}{Lw(x)} \quad . \tag{8}$$

This is very similar to the death rate in a conventional life table. Before age k, it is assumed that withdrawal from the labor force is due to mortality only. After age k, two types of separation occur: mortality and retirement. The rate of separation due to mortality is

$$MW_{\delta}(x) = \frac{\ell w(x)\ q(x)}{Lw(x)} \quad , \qquad (9)$$

where q(x) is the probability of dying between ages x and x + 1 (identical for an active and an inactive population). The rate of separation due to retirement is a residual:

$$MW_{r}(x) = MW(x) - MW_{\delta}(x) \quad . \qquad (10)$$

Separation rates are important for manpower planning because they permit the calculation of expected losses from active life due to death and retirement (see for example, Garfinkle, 1967). The difference between the total accession rate and the total separation rate is the labor force replacement rate (United Nations, 1973, p.319; Farooq, 1975, p.52). The ratio of accessions to separations is the replacement ratio, a measure of pressure on the labor market. A ratio of less than one shows that not all vacancies by death and retirement are being filled.

2.2 Applications of a Table of Working Life

Tables of working life, applying economic data such as annual earnings and consumption by age and sex, have been used in studies of the economic consequences of changes in the working life expectancy (United Nations, 1973, p.321). This approach answers such questions as the effects of changing activity rates and population structures upon the consumers/producers ratio or dependency burden, the money value of man, and the costs of mortality as a loss in human capital. The costs of mortality are both in terms of loss of investments in the education of children and in terms of loss of earnings of workers who die before retirement age. The latter aspect is of particular relevance to life insurance companies (Smith, 1977).

3. THE INCREMENT-DECREMENT TABLE OF WORKING LIFE: METHODOLOGY

The conventional method for constructing tables of working life is based on a number of unrealistic assumptions, such as, the unimodality of the LFP curve. Data availability and existent methodology may have required these assumptions. The method presented here introduces more realism into the working life table, but at the same time increases the data requirements. Today, most countries do not publish all the necessary data; therefore, in the last part of this section, some attention is devoted to procedures for estimating missing data.

The working life table proposed in this study is an increment-decrement life table. Recently there has been interest in increment-decrement life tables (e.g., Schoen, 1975) and their formulation by the application of the theory of multiregional mathematical demography (Rogers and Ledent, 1976). Also in this paper multiregional demographic techniques are used for deriving increment-decrement tables of working life. The method presented has a number of conceptual similarities with a technique developed recently by Hoem and Fong (1976a); however, the mathematics are much simpler and regional differences in mortality can be easily handled.

3.1 The Activity System

The population by age and sex is partitioned into two groups: active population (labor force), and inactive population. These are the two states of the system. The analysis will be performed for a single sex (male, female or total); however, the extension into a two-sex model is straightforward. In life, everyone starts out as a member of the inactive population, state 1. The labor force status is denoted by state 2. State 3, denoted by δ, is the state of death. In each age group, people may move between state 1 and state 2 (transient states), and into state δ (absorbing state). The age-specific gross flows in and out of the labor force are explicitly taken into account. The focus on mobility represents a fundamental difference from the conventional technique of working life table construction, with its emphasis on the size (stock) of the labor force and on changes in the stock through net flows. The assumption, made in conventional working life tables, that everyone enters the labor force before age k and retires at ages over k, is therefore no longer necessary. Consequently, the need for a unimodal LFP curve no longer exists.

3.2 The Increment-Decrement Table of Working Life

The increment-decrement table of working life represents the mortality and mobility experience of a cohort population. The mobility signifies the movement into and out of the labor force. All life table statistics are derived from a set of age-specific mobility and mortality probabilities.

We follow the usual procedure of life table construction and express first the probabilities in terms of instantaneous rates of mobility and mortality, (also known as intensities or forces of mobility and mortality). Let $\eta_{12}(x)$ denote the instantaneous rate of accession to the labor force at age x: the rate at which people of age x enter the labor force. The instantaneous rate is the limiting value of the accession rate as the age interval becomes infinitesimal. The instantaneous rate of separation from the labor force at age x is $\eta_{21}(x)$, and the force of mortality is $\eta_{i\delta}(x)$, where i = 1,2. The force of mortality of the active population may be different from that of the inactive population. The assumption of equal mortality rates, which is implicit in conventional tables of working life, is therefore no longer needed.

The Transition Probabilities

The transition from one state to another is governed by the Kolmogorov equation. Consider the cohort of people who are now of exact age y. Denote the probability that an individual of age y in state 1 will be in state 2, n years later by ${}_{1y}\ell_2(y + n)$. Denoting y + n by x ($x \geq y$), this probability may be written as ${}_{1y}\ell_2(x)$. The probability that a person in 1 at age y, will be in 2 at age (x + dx) is:

$$ {}_{1y}\ell_2(x + dx) = {}_{1y}\ell_1(x)\eta_{12}(x)dx + {}_{1y}\ell_2(x)\eta_{22}(x)dx \quad , \tag{11} $$

where $\eta_{12}(x)dx$ is the probability that an x year old member of the inactive population will become active within time dx, and $\eta_{22}(x)dx$ is the probability that an x year old member of the labor force will still be active, dx year (or time-units) later.

The transition probabilities are independent of the status of the individual at age y, but do depend on its status at exact age x, i.e., at the beginning of the interval (x,x + dx) (Markovian assumption). The quantity $\eta_{22}(x)dx$ is equal to:

$$ \eta_{22}(x)dx = 1 - [\eta_{2\delta}(x) + \eta_{21}(x)]dx \quad . \tag{12} $$

Hence (11) may be written as follows:

$$ {}_{1y}\ell_2(x + dx) = {}_{1y}\ell_1(x)\eta_{12}(x)dx + {}_{1y}\ell_2(x) - {}_{1y}\ell_2(x)[\eta_{2\delta}(x) + \eta_{21}(x)]dx $$

or

$$ \frac{{}_{1y}\ell_2(x + dx) - {}_{1y}\ell_2(x)}{dx} = {}_{1y}\ell_1(x)\eta_{12}(x) - {}_{1y}\ell_2(x)[\eta_{2\delta}(x) + \eta_{21}(x)] $$

which is the Kolmogorov differential equation:

$$ \frac{d}{dx}\, {}_{1y}\ell_2(x) = {}_{1y}\ell_1(x)\eta_{12}(x) - {}_{1y}\ell_2(x)[\eta_{2\delta}(x) + \eta_{21}(x)] \quad . \tag{13} $$

Equation (13), which may also be found in Hoem and Long (1976 a, p.69), describes the changes in ${}_{1y}\ell_2(x)$ in function of the instantaneous rates of mobility and mortality and of the initial condition, i.e., the state of a person at exact age x.

Analogously to the derivation of (13), we may obtain expressions for changes in ${}_{1y}\ell_1(x)$, ${}_{2y}\ell_1(x)$, and ${}_{2y}\ell_2(x)$. The result may be written in matrix form:

$$\frac{d}{dx}\, {}_y\underset{\sim}{\ell}(x) = \underset{\sim}{\eta}(x)\; {}_y\underset{\sim}{\ell}(x) \quad , \tag{14}$$

where $\underset{\sim}{\eta}(x)$ is the matrix of instantaneous rates, arranged in the following way:

$$\underset{\sim}{\eta}(x) = \begin{bmatrix} \eta_{1\delta}(x) + \eta_{12}(x) & -\eta_{21}(x) \\ & \\ -\eta_{12}(x) & \eta_{2\delta}(x) + \eta_{21}(x) \end{bmatrix}$$

and

$$ {}_y\underset{\sim}{\ell}(x) = \begin{bmatrix} {}_{1y}\ell_1(x) & {}_{2y}\ell_1(x) \\ & \\ {}_{1y}\ell_2(x) & {}_{2y}\ell_2(x) \end{bmatrix}$$

with ${}_{iy}\ell(x)$ the probability that a person aged y in state i will be in state j n years later, i.e., at exact age x ($n = x - y$). The age y denotes the cohort. If $y = 0$, the cohort considered is the birth cohort or radix. Another interesting cohort is $y = \bar{\alpha}$, the lowest age of labor force participation ($\bar{\alpha}$ is around 16 years of age).

To solve (14), we may try the expression:

$$\underset{\sim}{y}^{\ell}(x) = e^{\int_y^x -\underset{\sim}{\eta}(t)dt} \underset{\sim}{y}^{\ell}(y) \quad . \tag{15}$$

However, (15) is a solution to (14) only if $\underset{\sim}{\eta}(t)$ commutes for all t. A solution in this form may be obtained if one introduces the assumption that $\underset{\sim}{\eta}(t)$ is constant in the interval $x - h$ to x, (i.e., $\underset{\sim}{\eta}(t) = \underset{\sim}{\eta}(x - h)$ for $x - h \leq t < x$). In this case we may write:

$$\underset{\sim}{y}^{\ell}(x) = e^{-h\underset{\sim}{\eta}(x-h)} \underset{\sim}{y}^{\ell}(x - h) \quad .$$

This assumption means that the continuous function $\underset{\sim}{\eta}(x)$ is approximated by a step function. The height of the step between exact ages $x - h$ and x is given by $\underset{\sim}{\eta}(x - h)$.

Another approach to solving (14), is followed in this paper. We use the following theorem, stated without proof (Brauer et al., 1970, pp.312-313).*

<u>Theorem</u>: Solving the system of differential equations

$$\frac{d\underset{\sim}{Y}(t)}{dt} = \underset{\sim}{A}(t)\underset{\sim}{Y}(t) \quad , \quad \underset{\sim}{Y}(t_o) \text{ given}$$

is equivalent to finding a continuous matrix function $\underset{\sim}{Z}(t)$, such that

$$\underset{\sim}{Z}(t) = \underset{\sim}{Y}(t_o) + \int_{t_o}^{t} \underset{\sim}{A}(\tau)\underset{\sim}{Z}(\tau)d\tau \quad .$$

---

*A simplified solution to (14) and (13) may be derived if one assumes no mortality or equal mortality in both states (Appendix).

Replace (14) by the integral equation

$$\underset{\sim}{y}\ell(x) = \underset{\sim}{y}\ell(y) - \int_0^n \underset{\sim}{\eta}(y+t)\,\underset{\sim}{y}\ell(x+t)\,dt \quad ,$$

where $n = x - y$.

The problem of solving the system of Kolmogorov differential equations (14) has been replaced by the problem of evaluating the integral $\int_0^n \underset{\sim}{\eta}(y+t)\,\underset{\sim}{y}\ell(y+t)\,dt$. This may be done in steps of length h, say:

$$\underset{\sim}{y}\ell(y+h) = \underset{\sim}{y}\ell(y) - \int_0^h \underset{\sim}{\eta}(y+t)\,\underset{\sim}{y}\ell(y+t)\,dt$$

$$\underset{\sim}{y}\ell(y+2h) = \underset{\sim}{y}\ell(y+h) - \int_0^h \underset{\sim}{\eta}(y+h+t)\,\underset{\sim}{y}\ell(y+h+t)\,dt$$

$$\vdots$$

$$\underset{\sim}{y}\ell(x+h) = \underset{\sim}{y}\ell(x) - \int_0^h \underset{\sim}{\eta}(x+t)\,\underset{\sim}{y}\ell(x+t)\,dt \quad . \qquad (16)$$

Define the matrix of conditional probabilities:

$$\underset{\sim}{P}(x) = \begin{bmatrix} p_{11}(x) & p_{21}(x) \\ & \\ p_{12}(x) & p_{22}(x) \end{bmatrix}$$

where an element $p_{ij}(x)$ denotes the probability that a person alive in state i at age x will be in state j, h years later, i.e., at age x + h. The probabilities $p_{ij}(x)$ are assumed to be independent of the state at age y, and only depend on the state in the beginning of the interval h. The probability of dying is obtained by subtraction:

$$q_i(x) = 1 - p_{i1}(x) - p_{i2}(x) \quad .$$

For ${}_{y}\underset{\sim}{\ell}(x)$ nonsingular, $\underset{\sim}{P}(x)$ is given by the following expression:

$$\underset{\sim}{P}(x) = {}_{y}\underset{\sim}{\ell}(x+h)\,[{}_{y}\underset{\sim}{\ell}(x)]^{-1} \quad . \tag{17}$$

The nonsingularity condition is generally satisfied for ages $x \geq \bar{\alpha}$. Since no persons are in the labor force in the younger ages, $p_{21}(x) = p_{12}(x) = p_{22}(x) = 0$, and $p_{11}(x)$ is simply the probability of surviving from age x to x + h . This probability is equal to

$$p_{11}(x) = \frac{{}_{y1}\ell_1(x+h)}{{}_{y1}\ell_1(x)} \qquad \text{for } x < \bar{\alpha} \quad .$$

In other words, the matrix expression (17) reduces to a scalar expression.

Equation (17) shows that ${}_{y}\underset{\sim}{\ell}(x+h) = \underset{\sim}{P}(x)\,{}_{y}\underset{\sim}{\ell}(x)$. A comparison of this expression with (16) yields the following for $\underset{\sim}{P}(x)$:

$$\underset{\sim}{P}(x) = \left[\underset{\sim}{I} - \int_0^h \underset{\sim}{\eta}(x+t)\,{}_{x}\underset{\sim}{\ell}(x+t)\,[{}_{y}\underset{\sim}{\ell}(x)]^{-1}dt\right] \quad . \tag{18}$$

From (16), we may also derive an expression for the annual age-specific rates in the increment-decrement life table

$$ {}_{y}\underset{\sim}{\ell}(x+h) - {}_{y}\underset{\sim}{\ell}(x) = -\int_0^h \underset{\sim}{\eta}(x+t)\,{}_{y}\underset{\sim}{\ell}(x+t)dt$$

$$[\int_0^h {}_{y}\underset{\sim}{\ell}(x+t)dt]^{-1}[\int_0^h {}_{y}\underset{\sim}{\ell}(x+t)dt] \quad . \tag{19}$$

The expression $\int_0^h {}_y\underset{\sim}{\ell}(x+t)dt$ denotes the number of years lived in each state between ages x and x + h, per person in each state at age y. Denote this by ${}_y\underset{\sim}{L}(x)$:

$$ {}_y\underset{\sim}{L}(x) = \int_0^h {}_y\underset{\sim}{\ell}(x+t)dt \quad . \tag{20} $$

The matrix ${}_yL(x)$ may also be looked upon as representing the number of people by state in age group x to x + h, per person in each state at age y. Hence it gives the age distribution of the stationary life table population. The expression

$$ \left[\int_0^h \underset{\sim}{\eta}(x+t)\,{}_y\underset{\sim}{\ell}(x+t)dt\right]\left[\int_0^h {}_y\underset{\sim}{\ell}(x+t)dt\right]^{-1} = \underset{\sim}{m}(x) \; , \tag{21} $$

is then the matrix of age-specific life table rates. Equation (19) becomes, therefore

$$ {}_y\underset{\sim}{\ell}(x+h) - {}_y\underset{\sim}{\ell}(x) = -\,\underset{\sim}{m}(x)\,{}_y\underset{\sim}{L}(x) \quad . $$

To derive an expression of $\underset{\sim}{P}(x)$ in terms of life table rates, recall that ${}_y\underset{\sim}{\ell}(x+h) = \underset{\sim}{P}(x)\;{}_y\underset{\sim}{\ell}(x)$. Hence

$$ [\underset{\sim}{P}(x) - \underset{\sim}{I}]\,{}_y\underset{\sim}{\ell}(x) = -\,\underset{\sim}{m}(x)\,{}_y\underset{\sim}{L}(x) $$

$$ \underset{\sim}{P}(x) = \underset{\sim}{I} - \underset{\sim}{m}(x)\,{}_y\underset{\sim}{L}(x)\,[{}_y\underset{\sim}{\ell}(x)]^{-1} \quad . \tag{22} $$

This expression may also be derived directly from (18) by applying (20) and (21).

Numerical Approximation of $\underset{\sim}{P}(x)$

All life table statistics may be derived from the knowledge of transition probability matrices $\underset{\sim}{P}(x)$. The expression of $\underset{\sim}{P}(x)$ in terms of instantaneous rates is given in (17) and (22). On the other hand, Rogers and Ledent (1976), have shown that $\underset{\sim}{P}(x)$ may be written as follows:

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{h}{2}\underset{\sim}{M}(x)]^{-1}[\underset{\sim}{I} - \frac{h}{2}\underset{\sim}{M}(x)] \quad , \tag{23}$$

where $\underset{\sim}{M}(x)$ is the matrix of observed (empirical) rates, associated with age group x to x + h, and with the same format as the $\underset{\sim}{\eta}(x)$ matrix.

The transition from (22) to (23) involves a number of assumptions. The Markovian assumption, already introduced, states that the transition probability during a certain interval depends only on the state in which the person is at the beginning of the interval and is independent of previous states.

Another assumption is that transitions and deaths are distributed uniformly over the interval x to x + h. The uniform distribution implies that the integral $\int_0^h {}_y\underset{\sim}{\ell}(x + t)dt$ or ${}_y\underset{\sim}{L}(x)$ may be approximated by the linear interpolation:

$${}_y\underset{\sim}{L}(x) = \frac{h}{2}[{}_y\underset{\sim}{\ell}(x) + {}_y\underset{\sim}{\ell}(x + h)] \quad . \tag{24}$$

Introducing (24) in the expression for $\underset{\sim}{P}(x)$ yields

$$\underset{\sim}{P}(x) = \underset{\sim}{I} - \frac{h}{2}\underset{\sim}{m}(x)[{}_y\underset{\sim}{\ell}(x) + {}_y\underset{\sim}{\ell}(x + h)][{}_y\underset{\sim}{\ell}(x)]^{-1}$$

$$\underset{\sim}{P}(x) = \underset{\sim}{I} - \frac{h}{2}\underset{\sim}{m}(x)[\underset{\sim}{I} + \underset{\sim}{P}(x)]$$

$$\underset{\sim}{P}(x) + \frac{h}{2}\underset{\sim}{m}(x)\underset{\sim}{P}(x) = \underset{\sim}{I} - \frac{h}{2}\underset{\sim}{m}(x)$$

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{h}{2}\underset{\sim}{m}(x)]^{-1}[\underset{\sim}{I} - \frac{h}{2}\underset{\sim}{m}(x)] \quad . \tag{25}$$

To compute $\underset{\sim}{P}(x)$ from observed data, we introduce a third assumption: annual age-specific life table rates are equal to annual age-specific rates of the observed population, i.e., $\underset{\sim}{m}(x) = \underset{\sim}{M}(x)$. This assumption makes (25) equal to (23).

Other Life Table Statistics

Several useful statistics of the working life table may be derived from the matrix of probabilities $_{y}\underset{\sim}{\ell}(x)$ and $\underset{\sim}{P}(x)$. The procedures are analogous to the construction of a multiregional life table, in which states refer to regions (Rogers, 1975, chapter 2). The matrix $_{y}\underset{\sim}{L}(x)$ has already been derived. For example, the average number of years spent in the labor force between ages x and x + h by an active person of exact age y, is

$$_{2y}L_2(x) = \int_0^h {}_{2y}\ell_2(x + t)dt,$$

which may be approximated by the linear interpolation $\frac{h}{2}[_{2y}\ell_2(x) + {}_{2y}\ell_2(x + h)]$.

The total number of years expected to be lived in the labor force beyond age y, by a person of age y in the labor force is

$$_{2y}e_2(y) = \int_0^{\bar{\beta}-y} {}_{2y}\ell_2(y + t)dt \quad ,$$

where $\bar{\beta}$ is the highest age of active life (about 70 years). The working life expectancy of a person not active at age y is

$$_{1y}e_2(y) = \int_0^{\bar{\beta}-y} {}_{1y}\ell_2(y + t)dt \quad .$$

In an analogous way, we may formulate the expected lifetime spent outside the labor force before age $\bar{\beta}$, by a person who is active (inactive) at age y. The result may be combined in the matrix formulation:

$$ {}_{y}\underset{\sim}{e}(y) = \int_{0}^{\bar{\beta}-y} {}_{y}\underset{\sim}{\ell}(y+t)dt \quad , \tag{26} $$

with

$$ {}_{y}\underset{\sim}{e}(y) = \begin{bmatrix} {}_{1y}e_{1}(y) & {}_{2y}e_{1}(y) \\ & \\ {}_{1y}e_{2}(y) & {}_{2y}e_{2}(y) \end{bmatrix} \quad . $$
$$ \overline{{}_{1y}e_{.}(y)} \qquad \overline{{}_{2y}e_{.}(y)} $$

The column sum ${}_{1y}e_{.}(y)$, denotes the total life expectancy of a person outside the labor force at age y. A part of this remaining lifetime the person will spend being inactive [${}_{1y}e_{1}(y)$]. The column sum ${}_{2y}e_{.}(y)$ refers to the life expectancy of a person in the labor force at age y. He will spend respectively, ${}_{2y}e_{1}(y)$ and ${}_{2y}e_{2}(y)$ years in inactive and active life. The measure of ${}_{2y}e_{2}(x)$ is analogous to the labor force based measure of active life , developed in the conventional life tables.

The population based measure of working life is defined as follows:

$$ {}_{.y}e_{2}(y) = \int_{y}^{\bar{\beta}-y} [{}_{1y}\ell_{2}(y+t) + {}_{2y}\ell_{2}(y+t)]dt \quad . \tag{27} $$

It is clear that for $y < \bar{\alpha}$, ${}_{.y}e_{2}(y) = {}_{1y}e_{2}(y)$.

The formulas (26) and (27) refer to the life expectancy beyond age y of people aged y years. What is the life expectancy beyond age x of people in a given state at age y? Since the state at age x is of no importance, we first compute the distribution of the x-year old people by their state n years ago, i.e., at age y. This is a vector $\{_{y\tilde{}}\ell_{.}(x)\}$ computed as follows:

$$\{_{y\tilde{}}\ell_{.}(x)\} = [_{y\tilde{}}\ell(x)]'\{\tilde{1}\} \quad ,$$

where ´ denotes transpose, and $\{\tilde{1}\}$ is a vector of ones. The life expectancy beyond age x by future state and by state at age y is

$$_{y\tilde{}}\hat{e}(x) = [\int_0^{\bar{\beta}-x} {}_{y\tilde{}}\ell(x+t)dt][_{y\tilde{}}\hat{\ell}(x)]^{-1}$$

where $_{y}\hat{\ell}(x)$ is a diagonal matrix with the elements of $\{_{y\tilde{}}\ell_{.}(x)\}$ in the diagonal. It is analogous to the matrix formula of the life expectancy by place of birth in the multiregional life table (Willekens, 1977a, p.656). Instead of regions and birth cohorts, we are considering here states and cohorts of people aged y at a given point in time. This shows how the state-specific life expectancy of a given cohort changes as age increases.

Knowing the values $_{y\tilde{}}\ell(y+t)$, $t \geq 0$ for a given cohort, one may derive the life expectancy of a person in a given state at age x (and not y). This measure is analogous to the expectation of life by place of residence in multiregional demography. In working life tables, the place of residence at age y + n is replaced by the state at age x. The life expectancy by state at age x is

$$_{x\tilde{}}e(x) = [\int_0^{\bar{\beta}-x} {}_{y\tilde{}}\ell(x+t)dt][_{y\tilde{}}\ell(x)]^{-1} \quad .$$

This expression enables one to derive an interesting recursive expression for the expectation of life (Hoem and Fong, 1976a). One may rewrite (26) as follows:

$$
{}_x\underset{\sim}{e}(x) = \left[\int_0^h {}_y\underset{\sim}{\ell}(x+t)\,dt + \int_0^{\bar{\beta}-x-h} {}_y\underset{\sim}{\ell}(x+h+t)\,dt\right]\left[{}_y\underset{\sim}{\ell}(x)\right]^{-1}
$$

$$
= {}_x\underset{\sim}{L}(x) + \left[\int_0^{\bar{\beta}-x-h} {}_y\underset{\sim}{\ell}(x+h+t)\,dt\right]\left[{}_y\underset{\sim}{\ell}(x+h)\right]^{-1}
$$

$$
\left[{}_y\underset{\sim}{\ell}(x+h)\right]\left[{}_y\underset{\sim}{\ell}(x)\right]^{-1}
$$

or

$$
{}_x\underset{\sim}{e}(x) = {}_x\underset{\sim}{L}(x) + {}_{x+h}\underset{\sim}{e}(x+h)\underset{\sim}{P}(x) \; .
$$

4. THE INCREMENT-DECREMENT TABLE OF WORKING LIFE: NUMERICAL ILLUSTRATIONS

The increment-decrement table of working life may be derived from a set of age-specific accession rates, separation rates, and mortality rates. The required data for such a life table have been provided by Hoem and Fong (1976b, pp.7 and 12) and are based on Danish labor force panel surveys in the period 1972-1974. Although sex-specific data are provided, we perform the analysis for the male population only. The data are repeated in the first three columns of Table 6, and are identical to the first three columns of Table B1 in Appendix B. We have assumed no mortality differentials between the active and the inactive population. Although this assumption is not required, it has been made to allow comparison of our results with those published by Hoem and Fong (1976b).

Transition probabilities are shown in columns four to seven of Table 6.* They are computed by formula (23). The transition probabilities associated with age group x only depend on the mobility and mortality rates of the same age group. For example, the probability matrix $\underset{\sim}{P}(x)$ for age group 20 to 21 is

$$P(20) = [I + \frac{1}{2}\underset{\sim}{M}(20)]^{-1}[I - \frac{1}{2}M(20)]$$

where

$$\underset{\sim}{M}(20) = \begin{bmatrix} 0.001221 + 0.457690 & -0.092260 \\ \\ -0.457690 & 0.001221 + 0.092260 \end{bmatrix} .$$

*The computer program used for the calculation of the increment-decrement table of working life is a slightly modified version of the multiregional life table program, listed in Willekens and Rogers (1976).

Hence,

$$\underset{\sim}{P}(20) = \begin{bmatrix} 0.640191 & 0.072283 \\ & \\ 0.358589 & 0.926497 \end{bmatrix}$$

The probability that a person of age 20 will enter the labor force in the next year is 35.86%. The probability that he will survive but remain inactive is 64.02%. For our calculations it is important to note two points. First, the probability of separating for age 16 is not zero. This is caused by the fact that people enter the labor force and drop out in the same year. The drop-out of young people is relatively high (Figure 7). Second, for some age groups the transition probabilities $p_{12}(x)$ and $p_{21}(x)$ differ considerably from those obtained by Hoem and Fong. However, the probabilities of dying, obtained as a residual, are very similar.

The life history of the initial cohort is given in Table 7. From a cohort of births of 100,000, 97,562 children survive to age 16. At this age they are all inactive. The transitions during single years of age x, x + 1, are computed by applying transition probabilities to the people at exact age x. For example, the number of people entering the labor force at age 20 is

$$_{20,1}\ell_2(21) = p_{12}(20)\,\ell_1(20)$$

or

$$8355 = 0.358589 \times 23301\,.$$

The total number of people in the labor force at age 21 is

$$.\ell_2(21) = {}_{20,1}\ell_2(21) + {}_{20,2}\ell_2(21)$$

or

$$76780 = 8355 + 68424\,.$$

Table 8 gives the number of years spent in active and inactive life within the age interval x to x + 1 per unit cohort of age 16. For example,

$$_{16,1}L_2(20) = \frac{1}{2}[_{16,1}\ell_2(20) + {}_{16,1}\ell_2(21)]/_{16,1}\ell_1(16)$$

or

$$0.77198 = \frac{1}{2}[73853 + 76780]/97562 \quad .$$

The fraction of a year spent in the labor force between ages 20 and 21 per unit born is,

$$0.77198 \frac{97562}{100000} = 0.75316 \quad .$$

The difference between 0.77198 and 0.75316 is due to mortality before the active ages.

An important statistic of the working life table is the number of years in active life. Two approaches have been followed: the population based measure of active life and the labor-force based measure. The population based measure is given in Table 8. Since at age 16, everyone is inactive, the population based measure is identical to the expression of the life expectancy by state at age 16. In other words, computing an increment-decrement life table by state at age 16 yields a population based measure of the expected number of years in active and inactive life. The same measure is obtained by computing the life expectancy at birth. A 16 year old is expected to spend 42 years in active life and about 11 years as inactive. At birth, a baby is expected to spend 42 x 0.97562 = 40.98 years in the labor force. As before, the difference is due to mortality at ages below 16 years. The population based measure of the expected remaining numbers of active years of a 20 year old is

$$_{\cdot}e_1(20) = \sum_{y=20}^{74} {}_{\cdot}L_1(y)]/_{\cdot}\hat{\ell}_{\cdot}(20)$$

where ${}_{\cdot}\hat{\ell}_{\cdot}(20) = {}_{\cdot}\ell_{\cdot}(20)/{}_{\cdot}\ell_{\cdot}(16)$. It is equal to 97153/97562 = 0.99581. Hence ${}_{\cdot}e_1(20)$ is

$$40.321 = 40.1518/0.9958 \quad .$$

To obtain numerical values for the labor force based measure of active life, we calculated the "multiregional" life table "by place of residence" or current state. The life history of the population by current state is identical to Table 7. This is as expected since Table 7 also does not distinguish between different cohorts. (Taking the cohort of the inactive population of age 16 implies the total population because at age 16 everyone is inactive.) The fraction of a year spent inside and outside of the labor force between ages x and x + 1 by a person who is in (or outside) the labor force <u>at exact age x</u> is given in Table 9. For example, a male inactive person aged 20 is expected to spend ${}_{20,1}L_2(20) = 0.1793$ years in the labor force and to spend an average of ${}_{20,1}L_1(20) = 0.8201$ years in inactive life before reaching the age of 21. On the other hand, a person already in the labor force at age 20, spends an average of ${}_{20,2}L_2(20) = 0.9632$ years in the labor force and of ${}_{20,2}L_1(20) = 0.0361$ years in inactive life before reaching 21 years. Table 9 gives the average time spent in each state during the following year per active and inactive person of exact age x. The result may be compared with columns one through three of Table B2 in Appendix B.

Instead of considering the average time spent in each state during one year intervals, we may calculate the time spent in each state beyond age x. The result is represented in Table 10. It gives the remaining lifetime inside and outside of the labor force per active and inactive person of exact age x. The right side of the table shows the labor force based measures of active and inactive life. For example, an active person of 20 years may expect to spend 40.8 years in the labor force and 8.0 years in inactive life. The left side of the table contains the inactive life based measure of active and inactive life. A 20 year old person who is inactive, is expected to spend 38.7 and 10.1 years in active and inactive life respectively. Table 10 begins with age 17, because no person of age 16 is in the labor force and, therefore, a labor force based measure of active life below this age is not meaningful. Table 10 may be compared with the results obtained by Hoem and Fong, (see columns 4 to 6 of Table B2 in Appendix B).

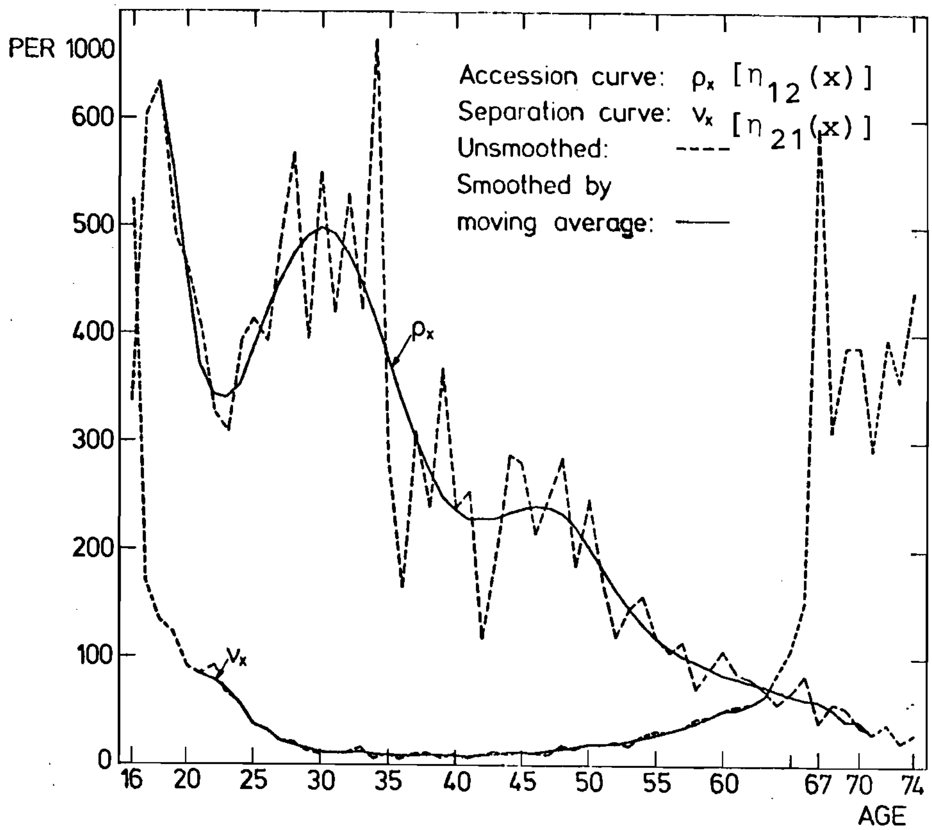

Figure 7. Labor force accession and separation curves. Denmark 1972-74, Males.

Source: Hoem and Fong (1976a, p.24).

Table 6. Age-specific rates of mortality, accession and separation, and transition probabilities.

	AGE-SPECIFIC RATES *			TRANSITION PROBABILITIES			
AGE	MORTALITY	1 TO 2	2 TO 1	1 TO 1	1 TO 2	2 TO 1	2 TO 2
16	0.000733	0.339080	0.525690	0.762690	0.236577	0.366775	0.632492
17	0.000934	0.605350	0.171700	0.563450	0.435616	0.125557	0.875509
18	0.001202	0.635730	0.135850	0.540524	0.458275	0.097929	0.900869
19	0.001327	0.557140	0.121990	0.583244	0.415430	0.090961	0.907712
20	0.001221	0.457690	0.092260	0.640191	0.356589	0.072263	0.926497
21	0.001016	0.373170	0.085200	0.695673	0.303312	0.069252	0.929734
22	0.000944	0.345440	0.082600	0.714524	0.284532	0.066389	0.932667
23	0.000997	0.341970	0.071500	0.715877	0.283126	0.059197	0.939807
24	0.001014	0.355140	0.057370	0.704844	0.294143	0.047516	0.951471
25	0.000932	0.393230	0.041280	0.676296	0.322773	0.033884	0.965185
26	0.000901	0.423520	0.032250	0.654463	0.344636	0.026243	0.972856
27	0.000980	0.449790	0.024180	0.635724	0.363296	0.019534	0.979490
28	0.001033	0.474040	0.019460	0.619131	0.379867	0.015594	0.983374
29	0.001039	0.492260	0.015090	0.606674	0.392287	0.012025	0.986936
30	0.001066	0.499390	0.012050	0.601623	0.397312	0.009567	0.989347
31	0.001198	0.495120	0.012120	0.604276	0.394527	0.009658	0.989146
32	0.001319	0.478300	0.012530	0.615089	0.383592	0.010049	0.988633
33	0.001284	0.451480	0.012780	0.632720	0.365997	0.010360	0.988356
34	0.001260	0.416880	0.010740	0.655688	0.343053	0.008838	0.989903
35	0.001377	0.379320	0.009370	0.681443	0.317181	0.007835	0.990789
36	0.001604	0.340640	0.008730	0.708843	0.289554	0.007421	0.990976
37	0.001854	0.304750	0.009660	0.735252	0.262896	0.008333	0.989815
38	0.002089	0.274010	0.010370	0.758176	0.239737	0.009060	0.988854
39	0.002220	0.251820	0.010570	0.775633	0.222149	0.009325	0.988458
40	0.002331	0.237500	0.008190	0.786621	0.211051	0.007278	0.990394
41	0.002559	0.230650	0.008670	0.791942	0.205502	0.007725	0.989720
42	0.002869	0.229460	0.009600	0.792730	0.204406	0.008552	0.988584
43	0.003341	0.231850	0.010500	0.790524	0.206140	0.009336	0.987329
44	0.003679	0.236230	0.012440	0.786950	0.209378	0.011026	0.985302
45	0.003875	0.240450	0.011710	0.783383	0.212749	0.010361	0.985771
46	0.004171	0.242860	0.012200	0.781291	0.214546	0.010778	0.985060
47	0.004482	0.241200	0.013880	0.782513	0.213015	0.012258	0.983270
48	0.004900	0.234270	0.015370	0.787799	0.207313	0.013601	0.981511
49	0.005483	0.221660	0.017530	0.797573	0.196959	0.015577	0.978956
50	0.006406	0.204470	0.019970	0.810888	0.182726	0.017846	0.975768
51	0.007386	0.185040	0.019070	0.825913	0.166728	0.017183	0.975458
52	0.008307	0.165660	0.020620	0.841635	0.150390	0.018719	0.973306
53	0.008411	0.147960	0.023110	0.856418	0.135207	0.021113	0.970507
54	0.008972	0.132980	0.026080	0.868943	0.122126	0.023951	0.967117
55	0.009876	0.120890	0.029230	0.878736	0.111437	0.026023	0.964150
56	0.010748	0.111010	0.032840	0.886815	0.102494	0.030321	0.958989
57	0.011871	0.103250	0.036300	0.892782	0.095417	0.033546	0.954652
58	0.013511	0.097030	0.040470	0.896967	0.089613	0.037376	0.949203
59	0.015083	0.091630	0.046700	0.900565	0.084465	0.043028	0.941982
60	0.016280	0.086834	0.051160	0.903890	0.079961	0.047113	0.936738
61	0.017890	0.082600	0.052510	0.906218	0.076050	0.048346	0.933922
62	0.020671	0.078570	0.057910	0.907440	0.072101	0.053142	0.926399
63	0.023411	0.074720	0.066760	0.908630	0.068237	0.060962	0.915898
64	0.025597	0.070650	0.086570	0.910811	0.063915	0.078317	0.896409
65	0.028280	0.065750	0.109690	0.913275	0.058840	0.098162	0.873553
66	0.031012	0.063300	0.152780	0.913982	0.055479	0.133904	0.835537
67	0.033397	0.062050	0.502000	0.921732	0.045419	0.453331	0.533624
68	0.036293	0.053820	0.310740	0.920319	0.044135	0.254244	0.710109
69	0.039273	0.043810	0.387380	0.926699	0.034764	0.307573	0.653914
70	0.042486	0.044910	0.386590	0.922875	0.035522	0.307355	0.651049
71	0.046823	0.031620	0.293400	0.928214	0.026053	0.241747	0.712520
72	0.051322	0.039470	0.390030	0.919038	0.030924	0.311852	0.635110
73	0.055770	0.023390	0.356970	0.927961	0.018682	0.285113	0.660650
74	0.061222	0.034290	0.440420	0.917381	0.023216	0.337560	0.603037

1 = inactive life (outside labor force)

2 = active life (inside labor force)

* Rates are from Hoem and Fong (1976b, p.7; see Table B1 in Appendix B).

Table 7. Life history of initial cohort.

AGE	DEATHS INACTIVE	DEATHS ACTIVE	TRANSITIONS 1 TO 1	TRANSITIONS 2 TO 1	TRANSITIONS 1 TO 2	TRANSITIONS 2 TO 2	NUMBER OF PEOPLE AT EXACT AGE X INACTIVE	NUMBER OF PEOPLE AT EXACT AGE X ACTIVE	NUMBER OF PEOPLE AT EXACT AGE X TOTAL
16	71.	0.	74410.	0.	23081.	0.	97562.	0.	97562.
17	69.	22.	41926.	2852.	32414.	20208.	74410.	23081.	97492.
18	54.	63.	24204.	5153.	20521.	47405.	44778.	52622.	97399.
19	39.	90.	17122.	6179.	12196.	61657.	29357.	67926.	97282.
20	28.	90.	14917.	5338.	8355.	68424.	23301.	73853.	97153.
21	21.	78.	14091.	5317.	6144.	71385.	20255.	76780.	97035.
22	18.	73.	13868.	5147.	5522.	72308.	19408.	77528.	96935.
23	19.	78.	13612.	4607.	5384.	73146.	19015.	77830.	96845.
24	18.	80.	12842.	3731.	5359.	74718.	18219.	78529.	96748.
25	15.	75.	11208.	2713.	5349.	77289.	16573.	80077.	96650.
26	13.	74.	9111.	2169.	4798.	80396.	13922.	82639.	96562.
27	11.	83.	7171.	1664.	4098.	83446.	11280.	85194.	96473.
28	9.	90.	5470.	1365.	3356.	86089.	8835.	87544.	96379.
29	7.	93.	4146.	1076.	2681.	88276.	6835.	89445.	96279.
30	6.	97.	3142.	872.	2075.	89988.	5222.	90957.	96179.
31	5.	110.	2425.	889.	1584.	91064.	4014.	92063.	96077.
32	4.	122.	2039.	931.	1271.	91594.	3315.	92647.	95962.
33	4.	119.	1879.	962.	1087.	91784.	2970.	92866.	95836.
34	4.	117.	1863.	821.	975.	91934.	2841.	92871.	95713.
35	4.	128.	1829.	728.	851.	92053.	2684.	92908.	95592.
36	4.	149.	1812.	689.	740.	92065.	2557.	92904.	95461.
37	5.	172.	1839.	773.	658.	91860.	2502.	92806.	95307.
38	5.	193.	1981.	838.	626.	91487.	2613.	92518.	95131.
39	6.	204.	2187.	859.	626.	91050.	2819.	92113.	94932.
40	7.	213.	2396.	667.	643.	90796.	3046.	91676.	94722.
41	8.	234.	2426.	706.	629.	90499.	3063.	91439.	94501.
42	9.	261.	2483.	779.	640.	90088.	3132.	91126.	94262.
43	11.	303.	2579.	847.	672.	89578.	3262.	90728.	93990.
44	13.	331.	2696.	995.	717.	88924.	3426.	90251.	93676.
45	14.	347.	2891.	929.	785.	88366.	3691.	89641.	93332.
46	16.	371.	2985.	961.	820.	87819.	3820.	89151.	92971.
47	18.	396.	3087.	1087.	840.	87156.	3946.	88639.	92584.
48	20.	430.	3288.	1197.	865.	86370.	4174.	87996.	92172.
49	25.	477.	3577.	1359.	883.	85399.	4485.	87235.	91720.
50	32.	551.	4003.	1540.	902.	84192.	4936.	86282.	91218.
51	41.	626.	4578.	1462.	924.	83005.	5542.	85094.	90636.
52	48.	669.	5083.	1571.	908.	81689.	6040.	83929.	89969.
53	56.	692.	5699.	1744.	900.	80161.	6654.	82597.	89251.
54	66.	724.	6468.	1942.	909.	78395.	7443.	81061.	88504.
55	83.	779.	7389.	2064.	937.	76461.	8409.	79304.	87713.
56	101.	827.	8383.	2347.	969.	74224.	9453.	77398.	86851.
57	127.	887.	9580.	2522.	1024.	71783.	10730.	75193.	85923.
58	162.	977.	10855.	2721.	1084.	69109.	12102.	72807.	84909.
59	203.	1051.	12226.	3022.	1147.	66121.	13576.	70193.	83769.
60	246.	1086.	13783.	3169.	1219.	63012.	15248.	67267.	82515.
61	301.	1139.	15362.	3105.	1289.	59987.	16952.	64231.	81183.
62	378.	1254.	16758.	3256.	1332.	56766.	18467.	61276.	79743.
63	463.	1344.	18186.	3542.	1366.	53212.	20014.	58098.	78112.
64	549.	1379.	19789.	4274.	1389.	48923.	21727.	54577.	76304.
65	671.	1403.	21977.	4939.	1416.	43970.	24064.	50312.	74376.
66	822.	1386.	24600.	6077.	1493.	37923.	26916.	45386.	72302.
67	1008.	1295.	28277.	17080.	1393.	21041.	30678.	39416.	70094.
68	1617.	800.	41743.	5704.	1997.	15931.	45357.	22434.	67791.
69	1827.	691.	43969.	5514.	1650.	11724.	47447.	17928.	65375.
70	2059.	556.	45667.	4111.	1758.	8707.	49483.	13374.	62857.
71	2276.	479.	46204.	2530.	1297.	7456.	49777.	10465.	60242.
72	2439.	438.	44788.	2730.	1507.	5585.	48734.	8753.	57487.
73	2578.	355.	44052.	2022.	888.	4686.	47518.	7093.	54611.
74	2737.	331.	42267.	1881.	1073.	3361.	46074.	5573.	51647.

1 = inactive life

2 = active life

Table 8. Expected number of years lived within age interval x to x + 1, and beyond age x.

EXPECTED NUMBER OF YEARS LIVED

AGE	WITHIN AGE INTERVAL X TO X+1			LIFE EXPECTANCY AT AGE X		
	INACTIVE	ACTIVE	TOTAL	INACTIVE	ACTIVE	TOTAL
16	0.881345	0.118288	0.999634	10.639007	42.002472	52.641479
17	0.610830	0.387971	0.998801	9.764822	41.914911	51.679733
18	0.379936	0.617799	0.997735	9.162096	41.565453	50.727551
19	0.269867	0.726607	0.996474	8.792289	40.995669	49.787956
20	0.223222	0.771983	0.995205	8.532760	40.320637	48.853397
21	0.203273	0.790821	0.994093	8.318747	39.593710	47.912457
22	0.196914	0.796206	0.993119	8.122615	38.838022	46.960636
23	0.190822	0.801334	0.992156	7.931918	38.072613	46.004532
24	0.178310	0.812849	0.991159	7.747403	37.302513	45.049915
25	0.156285	0.833911	0.990196	7.575267	36.519821	44.095088
26	0.129157	0.860131	0.989289	7.424421	35.711296	43.135719
27	0.103087	0.885271	0.988358	7.300501	34.873661	42.174162
28	0.080306	0.907058	0.987364	7.203347	34.011715	41.215323
29	0.061791	0.924551	0.986342	7.129375	33.127720	40.257095
30	0.047333	0.937971	0.985304	7.074138	32.224319	39.298457
31	0.037557	0.946633	0.984189	7.033590	31.306232	38.339825
32	0.032206	0.950745	0.982952	7.003835	30.381329	37.385162
33	0.029780	0.951893	0.981673	6.980293	29.453558	36.433853
34	0.028314	0.952111	0.980425	6.958928	28.521122	35.480050
35	0.026857	0.952277	0.979134	6.938782	27.585342	34.524124
36	0.025925	0.951751	0.977676	6.920893	26.650101	33.570995
37	0.026212	0.949775	0.975987	6.905466	25.718622	32.624088
38	0.027838	0.946227	0.974065	6.891397	24.792294	31.683691
39	0.030056	0.941913	0.971969	6.877197	23.871695	30.748896
40	0.031305	0.938455	0.969760	6.861522	22.954592	29.816114
41	0.031748	0.935644	0.967392	6.845213	22.039307	28.884520
42	0.032769	0.932001	0.964771	6.829891	21.127353	27.957244
43	0.034275	0.927505	0.961780	6.815496	20.220621	27.036117
44	0.036473	0.921937	0.958411	6.802609	19.322319	26.124928
45	0.038495	0.916303	0.954798	6.789555	18.429813	25.219368
46	0.039799	0.911165	0.950964	6.775522	17.539831	24.315353
47	0.041612	0.905247	0.946859	6.761904	16.652996	23.414900
48	0.044378	0.898051	0.942429	6.748230	15.769596	22.517826
49	0.048283	0.889267	0.937550	6.734170	14.891798	21.625968
50	0.053702	0.878293	0.931994	6.719553	14.022563	20.742117
51	0.059358	0.866233	0.925591	6.704934	13.167277	19.872211
52	0.065056	0.853439	0.918495	6.690274	12.325556	19.015831
53	0.072249	0.838738	0.910987	6.672945	11.491736	18.164682
54	0.081243	0.821862	0.903105	6.649662	10.664216	17.313878
55	0.091544	0.803092	0.894636	6.619227	9.846185	16.465412
56	0.103437	0.782022	0.885459	6.582089	9.041780	15.623869
57	0.117012	0.758492	0.875504	6.535768	8.251552	14.787293
58	0.131600	0.732868	0.864468	6.479370	7.478553	13.957923
59	0.147723	0.704478	0.852201	6.414241	6.726749	13.140990
60	0.165022	0.673923	0.838945	6.337058	5.996038	12.333097
61	0.181521	0.643218	0.824739	6.242756	5.284564	11.527321
62	0.197216	0.611784	0.809000	6.133368	4.593016	10.726385
63	0.213923	0.577452	0.791375	6.015152	3.920629	9.935781
64	0.234677	0.537551	0.772228	5.884119	3.279477	9.163596
65	0.261266	0.490449	0.751716	5.728854	2.659362	8.388216
66	0.295163	0.434608	0.729771	5.540644	2.073873	7.614517
67	0.389673	0.316981	0.706654	5.304348	1.534278	6.838626
68	0.475613	0.206857	0.682470	4.923709	1.132206	6.055915
69	0.496759	0.160422	0.657180	4.395929	0.863281	5.259210
70	0.508703	0.122171	0.630874	3.800993	0.648868	4.449862
71	0.504863	0.098490	0.603353	3.142143	0.479177	3.621320
72	0.493283	0.081208	0.574491	2.435915	0.334992	2.770907
73	0.479653	0.064911	0.544564	1.682966	0.207557	1.890524
74	0.462387	0.051269	0.513655	0.873451	0.096847	0.970295

Table 9. Number of years lived in each state between ages x and x + 1.

State at Age x

	........Inactive........			........Active........		
	TOTAL	INACTIVE	ACTIVE	TOTAL	INACTIVE	ACTIVE
17	0.999533	0.781725	0.217808	0.999533	0.061779	0.937755
18	0.999399	0.770262	0.229137	0.999399	0.048965	0.950435
19	0.999337	0.791622	0.207715	0.999337	0.045481	0.953856
20	0.999390	0.820096	0.179295	0.999390	0.036142	0.963248
21	0.999493	0.847836	0.151656	0.999493	0.034625	0.964867
22	0.999528	0.857262	0.142266	0.999528	0.033194	0.966334
23	0.999502	0.857939	0.141563	0.999502	0.029598	0.969903
24	0.999493	0.852422	0.147071	0.999493	0.023758	0.975735
25	0.999534	0.838148	0.161386	0.999534	0.016942	0.982593
26	0.999550	0.827231	0.172318	0.999550	0.013122	0.986428
27	0.999510	0.817862	0.181648	0.999510	0.009765	0.989745
28	0.999484	0.809550	0.189934	0.999484	0.007797	0.991687
29	0.999481	0.803337	0.196144	0.999481	0.006013	0.993468
30	0.999467	0.800811	0.198656	0.999467	0.004793	0.994674
31	0.999402	0.802138	0.197264	0.999402	0.004829	0.994573
32	0.999341	0.807545	0.191796	0.999341	0.005024	0.994316
33	0.999358	0.816360	0.182998	0.999358	0.005180	0.994178
34	0.999371	0.827844	0.171527	0.999371	0.004419	0.994951
35	0.999312	0.840721	0.158591	0.999312	0.003918	0.995394
36	0.999199	0.854422	0.144777	0.999199	0.003710	0.995488
37	0.999074	0.867626	0.131448	0.999074	0.004167	0.994907
38	0.998957	0.879088	0.119869	0.998957	0.004530	0.994427
39	0.998891	0.887817	0.111075	0.998891	0.004662	0.994229
40	0.998836	0.893311	0.105526	0.998836	0.003639	0.995197
41	0.998722	0.895971	0.102751	0.998722	0.003862	0.994860
42	0.998568	0.896365	0.102203	0.998568	0.004276	0.994292
43	0.998332	0.895262	0.103070	0.998332	0.004668	0.993664
44	0.998164	0.893475	0.104689	0.998164	0.005513	0.992651
45	0.998066	0.891691	0.106375	0.998066	0.005180	0.992886
46	0.997919	0.890646	0.107273	0.997919	0.005389	0.992530
47	0.997764	0.891257	0.106508	0.997764	0.006129	0.991635
48	0.997556	0.893900	0.103656	0.997556	0.006801	0.990756
49	0.997266	0.898787	0.098479	0.997266	0.007788	0.989478
50	0.996807	0.905444	0.091363	0.996807	0.008923	0.987884
51	0.996321	0.912957	0.083364	0.996320	0.008591	0.987729
52	0.996013	0.920817	0.075195	0.996012	0.009360	0.986653
53	0.995812	0.928209	0.067603	0.995812	0.010559	0.985253
54	0.995534	0.934471	0.061063	0.995534	0.011976	0.983559
55	0.995086	0.939368	0.055718	0.995086	0.013011	0.982075
56	0.994655	0.943408	0.051247	0.994655	0.015160	0.979494
57	0.994099	0.946391	0.047708	0.994099	0.016773	0.977326
58	0.993290	0.948483	0.044806	0.993290	0.018688	0.974602
59	0.992515	0.950283	0.042233	0.992515	0.021524	0.970991
60	0.991926	0.951945	0.039981	0.991926	0.023556	0.968369
61	0.991134	0.953109	0.038025	0.991134	0.024173	0.966961
62	0.989770	0.953720	0.036050	0.989770	0.026571	0.963199
63	0.988430	0.954315	0.034115	0.988430	0.030481	0.957949
64	0.987363	0.955406	0.031957	0.987363	0.039159	0.948205
65	0.986057	0.956637	0.029420	0.986057	0.049081	0.936976
66	0.984731	0.956991	0.027740	0.984731	0.066952	0.917779
67	0.983576	0.960866	0.022710	0.983576	0.216665	0.766910
68	0.982177	0.960159	0.022017	0.982177	0.127122	0.855055
69	0.980742	0.963350	0.017392	0.980742	0.153785	0.826957
70	0.979199	0.961438	0.017761	0.979199	0.153679	0.825520
71	0.977134	0.964107	0.013027	0.977134	0.120874	0.856260
72	0.974981	0.959519	0.015462	0.974981	0.155926	0.819055
73	0.972871	0.963530	0.009341	0.972871	0.142556	0.830315
74	0.970298	0.958690	0.011608	0.970298	0.168780	0.801519

Table 10. Expectations of inactive and active life by state at age x.

	State at age x					
	......Inactive.......			........Active........		
	TOTAL	INACTIVE	ACTIVE	TOTAL	INACTIVE	ACTIVE
17	51.679741	10.091800	41.587940	51.679718	8.710689	42.969028
18	50.727539	9.974118	40.753422	50.727524	8.471107	42.256416
19	49.787930	10.025299	39.762630	49.787937	8.259106	41.528831
20	48.853386	10.107892	38.745495	48.853371	8.235796	40.617574
21	47.912441	10.113490	37.798950	47.912445	7.845270	40.067177
22	46.960621	9.980295	36.980324	46.960583	7.657566	39.303017
23	46.004517	9.790188	36.214329	46.004520	7.477929	38.526592
24	45.049908	9.581586	35.468323	45.049896	7.321855	37.728043
25	44.095081	9.379037	34.716045	44.095085	7.201945	36.893139
26	43.135704	9.230736	33.904968	43.135704	7.120119	36.015585
27	42.174133	9.122972	33.051163	42.174149	7.059196	35.114952
28	41.215015	9.054308	32.160706	41.215008	7.016503	34.198505
29	40.257084	9.032094	31.224991	40.257088	6.983978	33.273109
30	39.298431	9.063311	30.235119	39.298416	6.959900	32.338516
31	38.339817	9.149629	29.190187	38.339809	6.941334	31.398476
32	37.385151	9.294801	28.090351	37.385155	6.921873	30.463284
33	36.433849	9.495400	26.938448	36.433842	6.899861	29.533981
34	35.480026	9.740889	25.739136	35.480022	6.873801	28.606222
35	34.524117	10.009489	24.514627	34.524120	6.850283	27.673838
36	33.570995	10.276551	23.294443	33.570995	6.828543	26.742451
37	32.624088	10.511168	22.112921	32.624084	6.808267	25.815819
38	31.683689	10.690245	20.993444	31.683685	6.784113	24.899572
39	30.748894	10.803880	19.945013	30.748878	6.757017	23.991861
40	29.816113	10.857284	18.958828	29.816113	6.728783	23.087330
41	28.884512	10.866362	18.018150	28.884514	6.710516	22.173998
42	27.957247	10.853383	17.103863	27.957243	6.691607	21.265635
43	27.036112	10.840350	16.195761	27.036114	6.670781	20.365332
44	26.124920	10.846522	15.278397	26.124928	6.649107	19.475821
45	25.219372	10.886061	14.333311	25.219368	6.620879	18.598490
46	24.315353	10.966657	13.348697	24.315353	6.595925	17.719427
47	23.414904	11.092699	12.322205	23.414898	6.569129	16.845770
48	22.517828	11.258001	11.259827	22.517829	6.534315	15.983513
49	21.625977	11.447408	10.178568	21.625969	6.491840	15.134129
50	20.742121	11.635970	9.106151	20.742117	6.438294	14.303823
51	19.872217	11.796898	8.075318	19.872208	6.373277	13.498931
52	19.015835	11.903229	7.112606	19.015829	6.315141	12.700688
53	18.164686	11.932237	6.232449	18.164684	6.249238	11.915446
54	17.313879	11.874830	5.439049	17.313881	6.169877	11.144005
55	16.465412	11.736391	4.729022	16.465412	6.076617	10.388796
56	15.623866	11.528908	4.094958	15.623869	5.977901	9.645967
57	14.787298	11.259058	3.528240	14.787301	5.861757	8.925544
58	13.957922	10.937880	3.020043	13.957927	5.738279	8.219647
59	13.140989	10.576465	2.564524	13.140989	5.609210	7.531780
60	12.333097	10.176309	2.156787	12.333099	5.466784	6.866314
61	11.527322	9.734447	1.792874	11.527323	5.321239	6.206083
62	10.726383	9.254311	1.472073	10.726386	5.192785	5.533600
63	9.939981	8.744025	1.195956	9.939983	5.575073	4.364911
64	9.163596	8.200426	0.963170	9.163598	4.961992	4.201606
65	8.388235	7.615787	0.772449	8.388237	4.826353	3.561884
66	7.614514	6.989913	0.624601	7.614513	4.681179	2.933335
67	6.838626	6.327039	0.511587	6.838626	4.508382	2.330244
68	6.053916	5.651748	0.402168	6.053916	3.451800	2.602115
69	5.259211	4.958716	0.300494	5.259211	2.906528	2.352682
70	4.449862	4.228028	0.221833	4.449862	2.220981	2.228882
71	3.621320	3.480605	0.140715	3.621320	1.532181	2.089139
72	2.770907	2.681055	0.089852	2.770907	1.071285	1.699622
73	1.890524	1.855448	0.035076	1.890524	0.527392	1.363132
74	0.970298	0.958690	0.011608	0.970298	0.168780	0.801519

5. LABOR FORCE PROJECTIONS

Traditionally, labor force projections are made by applying trends of LFP rates to population projections by sex and age. The conventional procedure focuses on the stock of the labor force. In order to include people who enter and leave the labor force at each age, Cohn, Nelson and Neumann (1974), developed a model denoted here as the CNN model. However, new entrants into the labor force are not generated by the model but must be projected separately. A model that projects the active and inactive population simultaneously and that accounts for the differences and the interactions between these two states is proposed in the final part of this section. It is analogous to the multiregional demographic growth model (Rogers, 1975, Chapter 3). This section is only a first attempt to develop such a model. Elaboration and a numerical illustration will be given in a later paper.

5.1 Traditional Techniques

The general approach to labor force projections is to project the population by age and sex and to apply the trend of LFP rates to the projected population. Let $\{\underset{\sim}{K}(t)\}$ denote the age-composition of the population (by sex), $\underset{\sim}{G}(t)$ the Leslie growth matrix and $\underset{\sim}{W}(t)$ a diagonal matrix of age-specific LFP rates. The labor force by age group at time t + 1 then becomes

$$\{\underset{\sim}{E}(t+1)\} = \underset{\sim}{W}(t+1)\{\underset{\sim}{K}(t+1)\}$$

$$= \underset{\sim}{W}(t+1)\ \underset{\sim}{G}(t)\{\underset{\sim}{K}(t)\} \quad . \tag{28}$$

If $\underset{\sim}{W}(t)$ and $\underset{\sim}{G}(t)$ remain constant, we have $\{\underset{\sim}{E}(t+1)\} = \underset{\sim}{W}\underset{\sim}{G}\{\underset{\sim}{K}(t)\}$ $= \underset{\sim}{H}\{\underset{\sim}{K}(t)\}$. The matrix $\underset{\sim}{H}$ is of the same form as $\underset{\sim}{G}$. The subdiagonal elements are products of $w(x+5)\ s(x)$.

If the rate of change of labor force participation rates is fixed, then $\underset{\sim}{W}(t) = \underset{\sim}{D}^t\underset{\sim}{W}(0)$, where $\underset{\sim}{D}$ is a diagonal matrix with the non zero elements equal to one plus the rate of change of the LFP rate, and $\underset{\sim}{W}(0)$ is the matrix of LFP rates in the base years (Fullerton and Prescott, 1975, p.69). A logistic curve has been used by Im and Ramachandran (1961).

This approach is also being used by the U.S. Bureau of Labor Statistics (see for example Johnston, 1973a, pp.11; 1973b, pp.15) and by the OEEC (1961, 1966). Current labor force projections reflect changes in the age composition of the population and changes in the age-specific LFP rates. Johnston clearly states which of the two components is more important:

> The predominant factor in these projections is the anticipated changes in the size and age-sex distribution of the population; projected changes in participation rates play a relatively minor role, (Johnston, 1973b, p.17).

As far as the LFP rates are concerned, the most frequent assumptions made are (Johnston, 1973a, p.11; Rosenblum, 1972).

- full employment (generally favorable demand situation, or the demand for labor is completely elastic);
- no significant change in the size of the armed forces;
- social and political stability;
- continuation of education trends; and
- no change in the definition of "labor force", "employment" or "unemployment".

5.2 The CNN Model

The method of applying age-specific labor force participation rates to the projected population gives an estimate of the future population in active life. It does not distinguish between new entrants and people leaving the labor force. A projection procedure which explicitly considers new entrants and leavers is proposed by Cohn, Nelson and Neumann (1974) for the forecasting of the aggregate supply of coalminers. The model considers five year age groups and projection intervals of 5 years. The procedure consists of two parts. The first part is the estimation of the expected labor force for each of the projection years, disregarding new entrants and the second part is the estimation of the expected number of entrants into the labor force by age.

To estimate the expected labor force disregarding new entrants, the authors use the concept of labor force retention rate, which is one minus the total separation rate. The expected number of people, active at time t and aged x to x + 4, who will still be active 5 years later is

$$E_s^{(t+1)}(x+5) = [1 - MW(x)]\ E^{(t)}(x) \tag{29}$$

where $E_s^{(t)}(x)$ is the surviving active population in age group x to x + 4 at time t, $E^{(t)}(x)$ is the total labor force aged x to x + 4 at t, and [1 - MW(x)] is the labor force retention rate. Writing (29) for several age groups, we have the matrix expression,

$$\begin{bmatrix} E^{(t+1)}(\bar{\alpha}) \\ E^{(t+1)}(\bar{\alpha}+5) \\ \vdots \\ E^{(t+1)}(x-5) \\ E^{(t+1)}(x) \\ \vdots \\ E^{(t+1)}(\bar{\beta}-5) \\ E^{(t+1)}(\bar{\beta}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ RW(\bar{\alpha}) & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & & \ddots & \vdots & \vdots \\ 0 & & RW(x-5) & \vdots & \vdots \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & & \vdots & \ddots & \vdots \\ 0 & & 0 & RW(\bar{\beta}-5) & 0 \end{bmatrix} \begin{bmatrix} E^{(t)}(\bar{\alpha}) \\ E^{(t)}(\bar{\alpha}+5) \\ \vdots \\ E^{(t)}(x-5) \\ E^{(t)}(x) \\ \vdots \\ E^{(t)}(\bar{\beta}-5) \\ E^{(t)}(\bar{\beta}) \end{bmatrix}$$

where RW (x) = [1 - MW(x)], or

$$\{\underset{\sim}{E}_s^{(t+1)}\} = \underset{\sim}{R}\ \{\underset{\sim}{E}^{(t)}\} \quad . \tag{30}$$

The proposed method to estimate the expected number of entrants into the labor force by age is based on two premises. First, the new entries in the five year period around the base year are assumed to be equal to the difference between the actual labor force at the end of the interval and the expected labor force at the end of the interval (using the retention rate method). Second, it is assumed that these entry levels by age will remain constant, i.e., at each projection interval the same number of people enter the labor force and their age composition is fixed. The number of new entrants by age are computed as follows:

$$E_N^{(t+1)}(x) = E^{(t+1)}(x) \quad \text{for } x = \bar{\alpha}$$
$$E_N^{(t+1)}(x) = E^{(t+1)}(x) - E^{(t)}(x-5)[1 - MW(x)] \quad \text{for } x > \bar{\alpha} \quad . \tag{31}$$

In other words, the labor force in age group $\bar{\alpha}$ at time t + 1 is completely made up of new entrants (between t and t + 1). The labor force in age group (x, x + 4), for $x > \bar{\alpha}$ at time t, is composed of people who were in the labor force at t and survived, and new entrants between t and t + 1:

$$E^{(t+1)}(x) = E_S^{(t+1)}(x) + E_N^{(t+1)}(x) \quad \text{for } x > \bar{\alpha}$$
$$= [1 - MW(x)]\, E^{(t)}(x-5) + E_N^{(t+1)}(x) \quad . \tag{32}$$

If we know the age structure of the labor force at two consecutive points in time we may compute $E_N^{(t)}(x)$:

$$E_N^{(t+1)}(x) = E^{(t+1)}(x) - [1 - MW(x)]E^{(t)}(x-5) \quad . \tag{33}$$

If $E^{(t+1)}(x)$ is less than $E^{(t)}(x)$, then $E_N^{(t+1)}(x) = 0$. The total labor force at time t + 1 is therefore,

$$\{\underset{\sim}{E}^{(t+1)}\} = \underset{\sim}{R}\{\underset{\sim}{E}^{(t)}\} + \{\underset{\sim}{E}_N^{(t+1)}\} \quad , \tag{34}$$

where $\{\underset{\sim}{E}_N^{(t+1)}\}$ is the vector of new entrants during the time interval t and t + 1.

An illustration of the necessary data for labor projection by this method is given in Table 11.

Table 11. Data for labor force projection.
Source: Cohn, Nelson and Neumann (1974, pp. 294, 295 and 298)

Age Group	Labor force 1970	Separation Rate	Retention Rate	Entrants 1968-1972
15-19	1,542	0.0017	0.9983	1,964
20-24	11,632	0.0022	0.9978	18,703
25-29	14,855	0.0020	0.9980	14,217
30-34	13,734	0.0025	0.9975	7,012
35-39	12,893	0.0044	0.9956	3,790
40-44	17,237	0.0065	0.9935	2,202
45-49	20,741	0.0109	0.9891	-
50-54	21,301	0.0184	0.9816	-
55-59	17,237	0.0337	0.9663	-
60-64	7,848	0.0941	0.9059	-
65-69	1,121	0.1763	0.8237	-

5.3 The Two-state Projection Model

The labor force projection model proposed in this section proceeds from our analysis of increment-decrement tables of working life, and uses findings of multiregional demography (Rogers, 1975; Chapter 3). The active and inactive populations by age (and sex) are projected simultaneously. This method enables one to consider mortality and fertility differences between the active and inactive population, and to examin the interactions between these subsets of the population.

The necessary data for labor force projection by this method consist of:

- active and inactive population by age in the base year (all ages),
- age-specific rates of mortality of both active and inactive population and rates of accession and separation, and
- age-specific birth rates for the active and inactive population.

The model is presented for the case of a single sex population, an extension to a two-sex population being straightforward. Active and inactive population by age at time t is contained in the vector {K(t)}:

$$\{\underset{\sim}{K}(t)\} = \begin{bmatrix} \{\underset{\sim}{K}^{(t)}(0)\} \\ \{\underset{\sim}{K}^{(t)}(1)\} \\ \vdots \\ \{\underset{\sim}{K}^{(t)}(x)\} \\ \vdots \\ \{\underset{\sim}{K}^{(t)}(\omega)\} \end{bmatrix} \text{, with } \{\underset{\sim}{K}^{(t)}(x)\} = \begin{bmatrix} K_1^{(t)}(x) \\ K_2^{(t)}(x) \end{bmatrix} .$$

An element $K_i^{(t)}(x)$ denotes the number of people of age x to x + 1 in state i at time t (i = 1 for the inactive, and i = 2 for the active population). At ages below $\bar{\alpha}$ and above $\bar{\beta}$, $K_2(x)$ will be zero. Of the people at age x to x + 1 in the labor force at time t, some may remain in the labor force during the time period (t, t + 1), some may become inactive, and some may die. On the other hand, the people of age x + 1 to x + 2 in the labor force at time t + 1 are composed of active population survivors of time t and of new entrants into the labor force:

$$K_2^{(t+1)}(x + 1) = s_{22}(x)K_2^{(t)}(x) + s_{12}(x)K_1^{(t)}(x) \quad . \qquad (35)$$

where $s_{22}(x)$ and $s_{12}(x)$ are survivorship proportions, which may be derived as part of the working life table or computed directly from age-specific rates of mortality, accession and separation:

$$\underset{\sim}{S}(x) = [\underset{\sim}{I} + \frac{1}{2}\underset{\sim}{M}(x+1)]^{-1}[\underset{\sim}{I} - \frac{1}{2}\underset{\sim}{M}(x)] \quad .$$

The inactive population aged x + 1 to x + 2 at time t + 1 is

$$K_1^{(t+1)}(x+1) = s_{11}(x)\ K_1^{(t)}(x) + s_{21}(x)\ K_2^{(t)}(x) \quad . \qquad (36)$$

Equations (35) and (36) may be combined into the following matrix expression

$$\{\underset{\sim}{K}^{(t+1)}(x+1)\} = \underset{\sim}{S}(x)\{\underset{\sim}{K}^{(t)}(x)\} \quad . \qquad (37)$$

The number of children in the first age (from 0 to 1) at time t + 1 is given by the surviving births during the interval (t,t + 1). Let $F_1(x)$ and $F_2(x)$ be the age-specific annual birth rate of a member of the labor force and of the inactive group, respectively. It is realistic to allow for different fertility schedules for the active and the inactive population. Assuming a uniform distribution of births over the time interval the number of births between t and t + 1 to mother aged x to x + 4 at t is equal to

$$\sum_{i=1,2} \frac{1}{2}[F_i(x)\ K_i^{(t)}(x) + F_i(x+1)\ K_i^{(t+1)}(x+1)] \quad .$$

Not all of these babies born will survive to become members of the first age group at time t + 1. However, all the survivors will be in state 2, i.e., inactive. Assuming that infant mortality is independent of the activity status of the parent, the number of children aged 0 to 1 years at time t + 1 is

$$K_2^{(t+1)}(0) = \sum_x \frac{1}{2} \hat{P} \sum_{i=1,2} [F_i(x) K_i^{(t)}(x) + F_i(x+1) K_i^{(t+1)}(x+1)]$$

$$K_1^{(t+1)}(0) = 0$$

where $\hat{P}$ denotes the proportion of babies born during the interval (t, t + 1), that will survive to time t + 1.

In matrix notation, we may write

$$\{\underset{\sim}{K}^{(t+1)}(0)\} = \sum_x \frac{1}{2} \underset{\sim}{\hat{P}} [\underset{\sim}{F}(x) \{\underset{\sim}{K}^{(t)}(x)\} + \underset{\sim}{F}(x+1) \underset{\sim}{K}^{(t+1)}(x+1)] \tag{38}$$

where

$$\underset{\sim}{F}(x) = \begin{bmatrix} F_1(x) & 0 \\ & \\ 0 & F_2(x) \end{bmatrix} \quad \underset{\sim}{\hat{P}} = \begin{bmatrix} \hat{p} & \hat{p} \\ & \\ 0 & 0 \end{bmatrix} .$$

Equation (38) may be reformulated as follows:

$$\{\underset{\sim}{K}^{(t+1)}(0)\} = \sum_x \frac{1}{2} \underset{\sim}{\hat{P}} [\underset{\sim}{F}(x) + \underset{\sim}{F}(x+1) \underset{\sim}{S}(x)] \{\underset{\sim}{K}^{(t)}(x)\}$$

$$\{\underset{\sim}{K}^{(t+1)}(0)\} = \sum_x \underset{\sim}{B}(x) \{\underset{\sim}{K}^{(t)}(x)\} \quad . \tag{39}$$

Equations (37) and (39) constitute the complete growth model of the active and inactive population.

$$
\begin{bmatrix}
\{\underset{\sim}{K}^{(t+1)}(0)\} \\
\{\underset{\sim}{K}^{(t+1)}(1)\} \\
\{\underset{\sim}{K}^{(t+1)}(2)\} \\
\vdots \\
\{\underset{\sim}{K}^{(t+1)}(z)\}
\end{bmatrix}
=
\begin{bmatrix}
\underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{B}(\hat{\alpha}) & \cdots & \underset{\sim}{B}(\hat{\beta}) \cdots \underset{\sim}{0} & \underset{\sim}{0} \\
\underset{\sim}{S}(0) & \underset{\sim}{0} & \underset{\sim}{0} & \cdots & \underset{\sim}{0} & \underset{\sim}{0} \\
\underset{\sim}{0} & \underset{\sim}{S}(1) & \underset{\sim}{0} & \cdots & \underset{\sim}{0} & \underset{\sim}{0} \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
\underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} & \cdots & \underset{\sim}{S}(z-1) & \underset{\sim}{0}
\end{bmatrix}
\begin{bmatrix}
\{\underset{\sim}{K}^{(t)}(0)\} \\
\{\underset{\sim}{K}^{(t)}(1)\} \\
\{\underset{\sim}{K}^{(t)}(2)\} \\
\vdots \\
\{\underset{\sim}{K}^{(t)}(z)\}
\end{bmatrix}
$$

or

$$
\{\underset{\sim}{K}(t+1)\} = \underset{\sim}{G}\{\underset{\sim}{K}(t)\} \quad . \qquad (40)
$$

The ages $\hat{\alpha}$ and $\hat{\beta}$ are respectively the lowest and highest ages of the reproductive period.

6. CONCLUSION

Labor force analyses have been carried out by economists, operations researchers and demographers, each with its own point of view of the important issues and of the relevant approach. This paper illustrates the demographer's perspective of labor force analysis and shows how the research apparatus of demography can be applied to manpower studies. Two aspects of labor force analysis are particularly important to the demographic approach. The first is the study of the age-specificity of labor force participation. Where aggregate measures of labor force participation (such as the CAR), may fluctuate considerably in time and space, curves of sex and age-specific LFP rates have a universal shape and their changes in time and space show a high degree of regularity. As a consequence the study of LFP schedules may benefit from experiences and insights obtained in studying fertility, mortality and migration schedules. Particularly relevant is the description of the LFP schedule by a few parameters. This may be applied to the development of model LFP schedules, and to the study of variations in labor force participation during the course of economic development, as illustrated in Section 1.

The second aspect of labor force analysis that has attracted the interest of demographers is the separation of the effect of the population's age and sex composition and of the basic schedules of mortality, fertility and labor force participation. To isolate the effect of mortality and LFP schedules, working life tables are constructed. The life table statistics are independent of the observed population structure but do depend on the mortality and LFP experience of the population as expressed by the age-specific rates. Conventional techniques for constructing working life tables are based on three important but unrealistic assumptions: unimodality of the LFP curve, entrance into the labor force occurring only at ages left of the peak and retirement only at ages right of the peak and independence of mortality of the labor force status. These assumptions are implicit in the working life tables that are published up to date in a large number of countries.

A new technique which does not require any of these assumptions is presented in this paper. It applies the theory of multiregional demography to the construction of increment-decrement tables of working life. The fundamental difference between this technique and the conventional method is its focus on flows instead of stocks. In the conventional method, only changes in the stock (net flow) of the active population by age are considered The new technique focuses on gross flows in and out of the labor force at each age. A drawback of this is the large data requirement. Today, most countries do not publish the data required to build increment-decrement tables of working life. However, here too, multiregional demography and migration research may contribute. Procedures developed to estimate age-specific migration flows by origin and destination from incomplete data may be applied to infer age-specific gross flows in and out of the labor force, i.e., between the inactive and active states, (United Nations, 1970; Rogers, 1975; Rees, 1977; Willekens, 1977b).

It has been an important recent observation that applicability of multiregional demography is not limited to the study of a system of regions, but extends to the analysis of any system of states or categories of age-specific populations (marital status; health status, etc.) for which increment-decrement life tables may be developed. In addition to life table construction, multiregional demography may be used for the development of better labor force projection models as seen in the two-state projection model proposed in Section 5 of this paper.

Appendix A

This Appendix gives the solution to (13) and (14) derived by Hoem and Fong (1976a, pp.68-70), assuming no mortality or equal mortality in both states.

No Mortality

Without mortality $[\eta_{i\delta}(x) = 0,\ i = 1,2]$, all people in state i at age y will be either in state i or j at age x, $(x = y + n)$; $(i,l = 1,2)$. Therefore, ${}_{iy}\bar{\ell}_i(x) = 1 - {}_{iy}\bar{\ell}_j(x)$ $(i \neq j)$ for a unit cohort, where the bar denotes the absence of mortality. Introducing this in equation (13) gives:

$$\frac{d}{dx}\, {}_{1y}\bar{\ell}_2(x) = [1 - {}_{1y}\bar{\ell}_2(x)]\eta_{12}(x) - {}_{1y}\bar{\ell}_2(x)\eta_{21}(x) \quad ,$$

or

$$\frac{d}{dx}\, {}_{1y}\bar{\ell}_2(x) = \eta_{12}(x) - \gamma(x)\ {}_{1x}\bar{\ell}_2(x) \quad , \tag{A.1}$$

where $\gamma(x) = \eta_{21}(x) + \eta_{12}(x)$. The solution to (A.1) is:

$${}_{1y}\bar{\ell}_2(x) = \int_0^n \eta_{21}(y + t)\ \exp\left[- \int_t^n \gamma(y + u)du\right]dt \quad . \tag{A.2}$$

This formula is numerically evaluated by replacing the continuous function $\eta_{21}(y + t)$ and $\gamma(y + t)$ by step functions. If $\eta_{21}(y + t) = \bar{\eta}_{21}(y)$ and $\gamma(y + t) = \bar{\gamma}(y)$ in the interval from y to $x = y + n$, then*

$$* \int_0^n \exp\left[-a \int_s^n du\right]ds = \int_0^n \exp[-a(n - s)]ds$$

$$= \int_0^n \exp[-an]\exp[as]ds$$

$$= \exp[-an]\int_0^n \exp[as]ds$$

$$= \exp[-an]\left[\frac{1}{a}\exp[an] - \frac{1}{a}\exp[0]\right]$$

$$= \frac{1}{a}[1 - \exp[-an]] \quad .$$

$$ {}_{1y}\bar{\ell}_2(x) = \frac{\bar{\eta}_{21}(y)}{\bar{\gamma}(y)} \left[1 - \exp\left[-\bar{n}\gamma(y)\right]\right] \quad . \tag{A.3} $$

Equal Mortality

In the case of equal mortality, the differential equation describing the dynamics of mortality is, for example,

$$ \frac{d}{dx}\, {}_{1y}\ell_\delta(x) = [1 - {}_{1y}\ell_\delta(x)]\,\eta_\delta(x) \quad . \tag{A.4} $$

Note that for differential mortality in both states, equation (A.4) does not hold. If mortality depends on the state (active/inactive), one must consider the state of the person at exact age x. The variable ${}_{1y}\ell_\delta(x)$ measures the probability that a person in the labor force at age y will die before reaching the age x. Since mortality is equal in both states, it is not important where the person will be at the time of death.

The solution to (A.4) is

$$ {}_{1y}\ell_\delta(x) = 1 - \exp\left[- \int_0^n \eta_\delta(y + t)dt\right] = {}_yq(x) \quad . $$

Note that the solution is independent of the state at age y. The quantity ${}_yq(x)$ measures the probability of dying between ages y and x = x + n.

To derive an expression for ${}_{1y}\ell_2(x)$ in the case of mortality, we introduce,

$$ {}_{1y}\ell_1(x) = 1 - {}_yq(x) - {}_{1y}\ell_2(x) $$

into (13) and solve for ${}_{1y}\ell_2(x)$:

$$ {}_{1y}\ell_2(x) = {}_{1y}\bar{\ell}_2(x)[1 - {}_yq(x)] \quad . $$

Hence the probability of changing states in the case of mortality is the product of the probability in the case of no mortality and the survivorship probability $[1 - {}_yq(x)]$. In general we may write

$$ {}_{iy}\ell_j(x) = {}_{iy}\bar{\ell}_j(x)[1 - {}_yq(x)] \quad . $$

This formula separates the mortality and the mobility effects. Note that this approach to solving (13) and (14) differs from ours. We did not assume constant instantaneous rates in the interval (y, y + n). These differences partly explain the deviations between Hoem and Fong's results, shown in Appendix B, and those given in Tables 6 to 10.

Appendix B. Increment-decrement table of working life for males, Denmark 1972 - 1974.

Table Bl. Observed Rates, Probabilities and Life Histories.

Source: Hoem and Fong (1976b, p.7)

AGE X	MU(X) ×1000	NU(X) ×1000	RHO(X) ×1000	L(X)	LW(X)	W(X) ×100	P12(X) ×100	P21(X) ×100	Q(X) ×1000	D12(X)	D21(X)	D13(X)	D23(X)
16	.7332	525.69	339.08	97562	0	.00	35.16	22.68	.73	.00	22127.56	.00	71.50
17	.9339	171.70	605.35	97490	22128	22.70	11.93	42.05	.93	2638.93	31687.96	20.65	70.35
18	1.2020	135.85	635.73	97399	51156	52.52	9.46	44.25	1.20	4837.24	20463.04	61.45	55.55
19	1.3269	121.99	557.14	97282	66720	68.58	8.84	40.39	1.33	5899.88	12342.69	88.47	40.53
20	1.2205	92.26	457.69	97153	73075	75.22	7.09	35.16	1.22	5179.54	8466.58	89.13	29.37
21	1.0156	85.20	373.17	97035	76273	78.60	6.83	29.90	1.02	5207.52	6208.69	77.43	21.08
22	.9444	80.60	345.44	96936	77196	79.64	6.56	28.10	.94	5061.83	5547.20	72.87	18.63
23	.9970	71.50	341.97	96845	77609	80.14	5.85	27.98	1.00	4540.23	5382.37	77.33	19.17
24	1.0135	57.37	355.14	96748	78374	81.01	4.70	29.07	1.01	3600.81	5341.58	79.39	18.61
25	.9316	41.28	393.23	96650	79955	82.73	3.35	31.86	.93	2674.76	5319.74	74.45	15.55
26	.9014	32.25	423.52	96560	82526	85.47	2.59	33.98	.90	2135.27	4769.37	74.36	12.65
27	.9800	24.18	449.79	96473	85085	88.20	1.92	35.79	.98	1636.71	4075.31	83.35	11.15
28	1.0329	19.46	474.04	96379	87441	90.73	1.53	37.38	1.03	1341.41	3340.72	90.27	9.23
29	1.0392	15.09	492.26	96279	89350	92.80	1.18	38.57	1.04	1056.39	2672.52	92.80	7.20
30	1.0663	12.05	499.39	96179	90873	94.48	.94	39.05	1.07	856.19	2072.15	96.85	5.65
31	1.1977	12.12	495.12	96077	91992	95.75	.95	38.79	1.20	873.47	1584.27	110.11	4.89
32	1.3191	12.53	478.30	95962	92593	96.49	.99	37.75	1.32	915.95	1271.68	122.06	4.44
33	1.2843	12.78	451.48	95835	92826	96.86	1.02	36.07	1.28	948.15	1085.26	119.14	3.86
34	1.2598	10.74	416.88	95712	92844	97.00	.87	33.88	1.26	810.15	971.51	116.89	3.61
35	1.3766	9.37	379.30	95592	92889	97.17	.78	31.38	1.38	720.11	848.24	127.78	3.72
36	1.6041	8.73	340.64	95460	92889	97.31	.74	28.70	1.60	683.44	737.95	148.88	4.12
37	1.8536	9.66	304.75	95307	92795	97.36	.83	26.20	1.85	767.57	655.72	171.85	4.65
38	2.0888	10.37	274.41	95131	92511	97.25	.90	23.83	2.09	833.43	624.21	193.03	5.47
39	2.2198	10.57	251.82	94932	92109	97.03	.93	22.10	2.22	854.25	623.93	204.24	6.26
40	2.3306	8.19	237.50	94722	91674	96.78	.72	21.01	2.33	664.08	640.18	213.41	7.09
41	2.5588	8.67	230.65	94501	91437	96.76	.77	20.46	2.56	703.11	626.93	233.67	7.83
42	2.8685	9.60	229.46	94260	91127	96.68	.85	20.35	2.86	775.96	637.47	261.03	8.97
43	3.3411	10.50	231.85	93990	90728	96.53	.93	20.52	3.34	843.07	669.38	302.62	10.89
44	3.6790	12.44	236.23	93676	90251	96.34	1.10	20.84	3.67	990.43	713.66	331.42	12.58
45	3.8754	11.71	240.45	93332	89643	96.05	1.03	21.17	3.87	923.96	780.98	346.73	14.27
46	4.1713	12.20	242.86	92971	89153	95.89	1.07	21.35	4.16	956.22	814.96	371.11	15.89
47	4.4816	13.88	241.20	92584	88641	95.74	1.22	21.19	4.47	1080.95	835.70	396.37	17.63
48	4.8997	15.37	234.27	92170	87999	95.48	1.35	20.63	4.89	1191.20	860.43	430.12	20.38
49	5.4828	17.53	221.66	91720	87238	95.11	1.55	19.61	5.47	1352.74	878.59	477.00	24.50
50	6.4063	19.97	204.47	91218	86287	94.59	1.78	18.20	6.39	1533.41	897.27	551.01	31.49
51	7.3863	19.07	185.04	90636	85100	93.89	1.71	16.61	7.36	1457.46	919.67	626.27	40.74
52	8.0070	20.62	165.66	89969	83936	93.30	1.87	14.99	7.98	1566.61	904.50	669.39	46.11
53	8.4105	23.11	147.96	89251	82605	92.55	2.11	13.49	8.38	1739.95	896.34	691.83	55.67
54	8.9719	26.08	132.98	88504	81069	91.60	2.39	12.18	8.93	1937.11	905.87	724.10	66.40
55	9.8760	28.23	120.89	87713	79314	90.42	2.60	11.12	9.83	2059.77	934.04	779.46	82.54
56	10.7482	32.84	111.01	86851	77409	89.13	3.03	10.23	10.69	2342.51	965.82	827.56	100.95
57	11.8714	36.30	103.25	85923	75204	87.53	3.35	9.52	11.80	2518.03	1020.75	887.51	126.49
58	13.5110	40.47	97.03	84909	72820	85.76	3.73	8.94	13.42	2716.52	1081.20	977.26	162.24
59	15.0826	46.70	91.63	83769	70207	83.81	4.30	8.43	14.97	3015.92	1143.21	1050.98	203.07
60	16.2800	51.16	86.83	82515	67283	81.54	4.70	7.98	16.15	3163.60	1215.40	1086.53	245.97
61	17.8903	52.51	82.60	81183	64249	79.14	4.82	7.59	17.73	3099.50	1285.22	1139.23	300.77
62	20.6709	57.91	78.57	79743	61295	76.87	5.30	7.19	20.46	3250.22	1327.16	1254.07	377.43
63	23.4109	66.76	74.72	78112	58118	74.40	6.08	6.81	23.14	3534.22	1360.82	1344.85	462.65
64	25.5974	86.57	70.65	76304	54600	71.56	7.81	6.37	25.27	4263.39	1383.06	1379.95	548.55
65	28.2798	109.69	65.75	74376	50339	67.68	9.78	5.86	27.89	4923.19	1409.14	1403.74	670.26
66	31.0123	152.78	63.30	72302	45422	62.82	13.32	5.52	30.54	6050.57	1483.44	1387.12	820.88
67	33.3975	592.00	62.05	70094	39467	56.31	42.02	4.41	32.85	16586.00	1349.08	1296.46	1006.04
68	36.2932	310.74	53.82	67791	22934	33.83	25.11	4.35	35.65	5759.14	1951.02	817.51	1598.59
69	39.2729	387.38	43.81	65375	18308	28.01	30.26	3.42	38.52	5539.31	1610.37	705.18	1812.82
70	42.4865	388.59	44.91	62857	13674	21.75	30.22	3.49	41.60	4132.42	1717.72	568.89	2046.11
71	46.8028	293.40	31.62	60242	10691	17.75	23.90	2.58	45.73	2555.48	1276.33	488.91	2266.09
72	51.3219	398.03	39.47	57487	8923	15.52	30.62	3.04	50.04	2732.54	1474.76	446.47	2430.03
73	55.7704	356.97	23.39	54610	7218	13.22	28.08	1.84	54.20	2027.04	871.97	391.55	2571.35
74	61.2217	440.42	30.29	51647	5672	10.98	33.04	2.27	59.40	1874.01	1044.62	336.91	2731.69
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)

Table B2. Mean Remaining Lifetime and Mean Time in Labor Force by Age and Labor Force status.

Source: Hoem and Fong (1976b, p.8).

AGE X	IN SINGLE YEAR AGE INTERVAL: MEAN TIME IN L.F. IN L.F. AT AGE X	IN SINGLE YEAR AGE INTERVAL: MEAN TIME IN L.F. OUTSIDE L.F. AT AGE X	IN SINGLE YEAR AGE INTERVAL: MEAN LIFETIME	UP TO FINAL AGE 75: MEAN TIME IN L.F. IN L.F. AT AGE X	UP TO FINAL AGE 75: MEAN TIME IN L.F. OUTSIDE L.F. AT AGE X	UP TO FINAL AGE 75: MEAN LIFETIME	MEAN REMAINING LIFETIME UP TO AGE 90
16	.7987	.1296	.9996	43.2492	41.9943	52.6415	56.6658
17	.9322	.2373	.9995	42.9713	41.5796	51.6797	55.7070
18	.9461	.2495	.9994	42.2594	40.7425	50.7275	54.7586
19	.9501	.2247	.9993	41.5312	39.7546	49.7879	53.8239
20	.9607	.1919	.9994	40.8188	38.7429	48.8534	52.8947
21	.9627	.1610	.9995	40.0677	37.7995	47.9124	51.9587
22	.9644	.1505	.9995	39.3034	36.9818	46.9606	51.0110
23	.9682	.1496	.9995	38.5269	36.2161	46.0045	50.0587
24	.9744	.1554	.9995	37.7283	35.4700	45.0499	49.1081
25	.9816	.1709	.9995	36.8935	34.7171	44.0951	48.1574
26	.9856	.1828	.9996	36.0161	33.9051	43.1357	47.2018
27	.9891	.1931	.9995	35.1155	33.0502	42.1741	46.2439
28	.9912	.2023	.9995	34.1992	32.1582	41.2150	45.2888
29	.9931	.2091	.9995	33.2738	31.2208	40.2571	44.3351
30	.9944	.2119	.9995	32.3393	30.2293	39.2984	43.3806
31	.9943	.2103	.9994	31.3993	29.1830	38.3398	42.4264
32	.9940	.2042	.9993	30.4641	28.0824	37.3852	41.4766
33	.9939	.1943	.9994	29.5348	26.9307	36.4338	40.5307
34	.9947	.1815	.9994	28.6071	25.7325	35.4800	39.5822
35	.9952	.1671	.9993	27.6748	24.5096	34.5241	38.6314
36	.9953	.1519	.9992	26.7432	23.2910	33.5710	37.6840
37	.9947	.1374	.9991	25.8165	22.1108	32.6241	36.7437
38	.9942	.1249	.9990	24.9003	20.9923	31.6837	35.8109
39	.9940	.1154	.9989	23.9925	19.9444	30.7489	34.8847
40	.9951	.1094	.9988	23.0880	18.9583	29.8161	33.9612
41	.9947	.1065	.9987	22.1747	18.0175	28.8845	33.0392
42	.9941	.1059	.9986	21.2663	17.1029	27.9572	32.1276
43	.9935	.1069	.9983	20.3660	16.1944	27.0361	31.2134
44	.9924	.1086	.9982	19.4765	15.2767	26.1249	30.3162
45	.9927	.1104	.9981	18.5992	14.3313	25.2194	29.4261
46	.9923	.1114	.9979	17.7202	13.3466	24.3154	28.5384
47	.9914	.1106	.9978	16.8465	12.3202	23.4149	27.6556
48	.9905	.1076	.9976	15.9842	11.2582	22.5178	26.7770
49	.9892	.1021	.9973	15.1348	10.1777	21.6260	25.9067
50	.9876	.0946	.9968	14.3044	9.1061	20.7421	25.0464
51	.9874	.0861	.9963	13.4994	8.0762	19.8722	24.2041
52	.9864	.0775	.9960	12.7012	7.1141	19.0158	23.3799
53	.9849	.0696	.9958	11.9159	6.2345	18.1647	22.5638
54	.9832	.0627	.9955	11.1444	5.4414	17.3139	21.7501
55	.9817	.0572	.9951	10.3891	4.7316	16.4654	20.9417
56	.9791	.0526	.9947	9.6463	4.0976	15.6239	20.1445
57	.9769	.0489	.9941	8.9259	3.5310	14.7873	19.3568
58	.9741	.0460	.9933	8.2200	3.0227	13.9579	18.5820
59	.9704	.0434	.9925	7.5322	2.5672	13.1410	17.8280
60	.9677	.0411	.9919	6.8668	2.1593	12.3331	17.0913
61	.9663	.0391	.9911	6.2067	1.7953	11.5273	16.3636
62	.9624	.0371	.9898	5.5344	1.4742	10.7264	15.6500
63	.9570	.0351	.9884	4.8660	1.1977	9.9400	14.9664
64	.9469	.0330	.9874	4.2032	.9645	9.1636	14.3091
65	.9352	.0305	.9861	3.5643	.7732	8.3882	13.6672
66	.9149	.0289	.9847	2.9371	.6245	7.6145	13.0449
67	.7479	.0247	.9836	2.3369	.5104	6.8386	12.4401
68	.8472	.0234	.9822	2.5963	.4023	6.0539	11.8456
69	.8163	.0186	.9807	2.3465	.3006	5.2592	11.2650
70	.8147	.0190	.9792	2.2202	.2222	4.4499	10.6962
71	.8490	.0138	.9771	2.0758	.1415	3.6213	10.1388
72	.8076	.0166	.9750	1.6864	.0907	2.7709	9.6008
73	.8204	.0100	.9729	1.3480	.0361	1.8905	9.0801
74	.7881	.0125	.9703	.7881	.0125	.9703	8.5724
	(1)	(2)	(3)	(4)	(5)	(6)	(7)

Table B1, (1) mortality rate

(2) separation rate

(3) accession rate

(4) total number of survivors at exact age x from birth cohort of 100,000

(5) number of persons in the labor force at exact age x

(6) labor force participation rate: (6) = (5)/(4)

(7) probability of leaving the labor force (state 1 in Hoem and Fong) to become inactive (state 2) between ages x and x + 1

(8) probability of entering the labor force between x and x + 1

(9) probability of dying between x and x + 1

(10) number of transitions from active to inactive life by people of age x

(11) number of transitions from inactive to active life

(12) number of persons in the labor force and x years old who die before reaching age x

(13) number of persons in inactive life and x years old who die before reaching age x

Table B2, (1) number of years spent in the labor force between ages x and x + 1 per person who is in the labor force at age x

(2) as (1) but per person in inactive life at age x

(3) total number of years lived between ages x and x + 1 per person at age x

(4) expectation of active life of a person in the labor force at age x

(5) expectation of active life of a person outside the labor force at age x

(6) total life expectancy of a person at age x

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